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Vehicle Dynamics Terminology
SAE J670d

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Handbook Supplement

Vehicle Dynamics Terminology

SAE J670d

Report of Vehicle Dynamics Committee approved July 1952
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National Standard Acoustical Terminology ANS Z24.1

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FOREWORD

This revision of "Vehicle Dynamics Terminology—SAE J670" has been expanded by the Vehicle Dynamics Committee to encompass terminology related to directional control of vehicles. Revisions have also been made to update the original terminology. An alphabetical index is appended to facilitate location of definitions.

The function of uniform terminology is to promote understandable and exact communication. A great deal of effort has been expended to make these definitions suit this purpose. It is recognized that this terminology, like other dictionaries, must be revised periodically to reflect current usage and changing needs. The Vehicle Dynamics Committee therefore solicits suggestions for improvements and additions to be considered in future revisions. Comments should be directed to SAE Headquarters.

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VEHICLE DYNAMICS TERMINOLOGY

NOTE: Italicized words and phrases appearing in a definition are themselves defined elsewhere in this Terminology.

1. MECHANICAL VIBRATION-QUALITATIVE TERMINOLOGY

1.1 VIBRATION (OSCILLATION), GENERAL - Vibration is the variation with time of the displacement of a body with respect to a specified reference dimension when the displacement is alternately greater and smaller than the reference. (Adapted from ANS Z24.1-1951, item 1.040.)

1.2 FREE VIBRATION - Free Vibration of a system is the *vibration* during which no variable force is externally applied to the system. (Adapted from ANS Z24.1-1951, item 2.135.)

1.3 FORCED VIBRATION - Forced vibration of a system is *vibration* during which variable forces outside the system determine the *period* of the vibration. (Adapted from ANS Z24.1-1951, item 2.130.)

1.3.1 *Resonance* - A *forced vibration* phenomenon which exists if any small change in *frequency* of the applied force causes a decrease in the *amplitude* of the vibrating system. (Adapted from ANS Z24.1, item 2.105.)

1.4 SELF-EXCITED VIBRATION - *Vibrations* are termed self-excited if the vibratory motion produces cyclic forces which sustain the *vibration*.

1.5 SIMPLE HARMONIC VIBRATION - *Vibration* at a point in a system is simple harmonic when the displacement with respect to time is described by a simple sine function.

1.6 STEADY-STATE VIBRATION - Steady state vibration exists in a system if the displacement at each point recurs for equal increments of time. (Adapted from ANS Z24.1-1951, items 11.005 and 1.045.)

1.7 PERIODIC VIBRATION - Periodic vibration exists in a system when recurring *cycles* take place in equal time intervals.

1.8 RANDOM VIBRATION - Random vibration exists in a system when the *oscillation* is sustained but irregular both as to *period* and *amplitude*.

1.9 TRANSIENT VIBRATION - Transient vibration exists in a system when one or more component *oscillations* are discontinuous.

2. MECHANICAL VIBRATION-QUANTITATIVE TERMINOLOGY

2.1 PERIOD - Period of an *oscillation* is the smallest increment of time in which one complete sequence of variation in displacement occurs. (Adapted from ANS Z24.1-1951, item 1.050.)

2.2 CYCLE - Cycle of *oscillation* is the complete sequence of variations in displacement which occur during a *period*. (Adapted from ANS Z24.1-1951, item 1.055.)

2.3 FREQUENCY - Frequency of *vibration* is the number of *periods* occurring in unit time. (Adapted from ANS Z24.1-1951, item 1.060.)

2.3.1 *Natural Frequency* - Natural frequency of a body or system is a frequency of free vibration. (Same as ANS Z24.1-1951, item 2.140.)

2.3.2 *Exciting Frequency* - Exciting frequency is the *frequency* of variation of the exciting force.

2.3.3 *Frequency Ratio* - The ratio of *exciting frequency* to the *natural frequency*.

2.3.4 *Resonant Frequency* - *Frequency* at which *resonance* exists. (Same as ANS Z24.1-1951, item 2.110.)

2.4 AMPLITUDE - Amplitude of displacement at a point in a *vibrating system* is the largest value of displacement that the point attains with reference to its equilibrium position. (Adapted from ANS Z24.1-1951, item 1.070.)

2.4.1 *Peak-to-Peak Amplitude (Double Amplitude)* - Peak-to-Peak amplitude of displacement at a point in a *vibrating system* is the sum of the extreme values of displacement in both directions from the equilibrium position. (Adapted from ANS Z24.1-1951, item 1.075.)

2.4.2 *Static Amplitude* - Static amplitude in *forced vibration* at a point in a system is that displacement of the point from its specified equilibrium position which would be produced by a static force equal to the maximum value of exciting force.

2.4.3 *Amplitude Ratio (Relative Magnification Factor)* - The ratio of a *forced vibration* amplitude to the *static amplitude*.

2.5 VELOCITY - Velocity of a point in a *vibrating system* is the time rate of change of its displacement. (Adapted from ANS Z24.1-1951, item 1.345.)

In *simple harmonic vibration*, the maximum velocity,

$$v_m = \omega x$$

where:

$$\omega = 2\pi f$$

$$f = \text{frequency}$$

$$x = \text{amplitude}$$

2.6 ACCELERATION - Acceleration of a point is the time rate of change of the *velocity* of the point. (Same as ANS Z24.1-1951, item 1.355.)

In *simple harmonic vibration*, the maximum acceleration,

$$a_m = \omega^2 x$$

2.7 **JERK** - "Jerk" is a concise term used to denote the time rate of change of *acceleration* of a point.

In *simple harmonic motion*, the maximum jerk,

$$j_m = \omega^3 x$$

2.8 **TRANSMISSIBILITY** - Transmissibility in *forced vibration* is the ratio of the transmitted force to the applied force.

3. VIBRATING SYSTEMS

3.1 **DEGREE OF FREEDOM** - The number of degrees of freedom of a *vibrating system* is the sum total of all ways in which the masses of the system can be independently displaced from their respective equilibrium positions.

EXAMPLES: A single rigid body constrained to move only vertically on supporting springs is a system of one degree of freedom. If the same mass is also permitted angular displacement in one vertical plane, it has two degrees of freedom: one being vertical displacement of the center of gravity; the other, angular displacement about the center of gravity.

3.2 **LINEAR** - Linear *vibrating systems* are those in which all the variable forces are directly proportional to the displacement, or to the derivatives of the displacement, with respect to time.

3.3 **NONLINEAR** - Nonlinear *vibrating systems* are those in which any of the variable forces are not directly proportional to the displacement, or to its derivatives, with respect to time.

EXAMPLE: A system having a variable *spring rate*.

3.4 **UNDAMPED** - Undamped systems are those in which there are no forces opposing the vibratory motion to dissipate energy.

3.5 **DAMPED** - Damped systems are those in which energy is dissipated by forces opposing the vibratory motion.

Any means associated with a *vibrating system* to balance or modulate exciting forces will reduce the vibratory motion, but are not considered to be in the same category as damping. The latter term is applied to an inherent characteristic of the system without reference to the nature of the excitation.

3.5.1 **Viscous Damping** - Damping in which the force opposing the motion is proportional and opposite in direction to the *velocity*.

3.5.2 **Critical Damping** - The minimum amount of *viscous damping* required in a *linear system* to prevent the displacement of the system from passing the equilibrium position upon returning from an initial displacement.

3.5.3 **Damping Ratio** - The ratio of the amount of *viscous damping* present in a system to that required for *critical damping*.

3.5.4 **Coulomb Damping** - Damping in which a constant force opposes the vibratory motion.

3.5.5 **Complex Damping** - Damping in which the force opposing the vibratory motion is variable, but not proportional, to the *velocity*.

In the field of aircraft flutter and vibration, complex damping is also used to denote a specific type of damping in which the damping force is assumed to be *harmonic* and in phase

with the *velocity* but to have an *amplitude* proportional to the *amplitude* of displacement.

4. COMPONENTS AND CHARACTERISTICS OF SUSPENSION SYSTEMS

4.1 VIBRATING MASS AND WEIGHT -

4.1.1 **Sprung Weight** - All weight which is supported by the suspension, including portions of the weight of the suspension members.

In the case of most vehicles, the sprung weight is commonly defined as the total weight less the weight of *unsprung parts*.

4.1.2 **Sprung Mass** - Considered to be a rigid body having equal mass, the same center of gravity, and the same moments of inertia about identical axes as the total *sprung weight*.

4.1.3 **Dynamic Index** - (k^2/ab ratio) is the square of the radius of gyration (k) of the *sprung mass* about a transverse axis through the center of gravity, divided by the product of the two longitudinal distances (a and b) from the center of gravity to the front and rear *wheel centers*.

4.1.4 **Unsprung Weight** - All weight which is not carried by the suspension system, but is supported directly by the tire or wheel, and considered to move with it.

4.1.5 **Unsprung Mass** - The unsprung masses are the equivalent masses which reproduce the inertia forces produced by the motions of the corresponding unsprung parts.

4.2 **SPRING RATE** - The change of load of a spring per unit deflection, taken as a mean between loading and unloading at a specified load.

4.2.1 **Static Rate** - Static rate of an elastic member is the rate measured between successive stationary positions at which the member has settled to substantially equilibrium condition.

4.2.2 **Dynamic Rate** - Dynamic rate of an elastic member is the rate measured during rapid deflection where the member is not allowed to reach static equilibrium.

4.3 RESULTANT SPRING RATE -

4.3.1 **Suspension Rate (Wheel Rate)** - The change of wheel load, at the *center of tire contact*, per unit vertical displacement of the *sprung mass* relative to the wheel at a specified load.

If the *wheel camber* varies, the displacement should be measured relative to the lowest point on the rim centerline.

4.3.2 **Tire Rate (Static)** - The *static rate* measured by the change of wheel load per unit vertical displacement of the wheel relative to the ground at a specified load and inflation pressure.

4.3.3 **Ride Rate** - The change of wheel load, at the *center of tire contact*, per unit vertical displacement of the *sprung mass* relative to the ground at a specified load.

4.4 STATIC DEFLECTION -

4.4.1 **Total Static Deflection** - Total static deflection of a loaded suspension system is the overall deflection under the static load from the position at which all elastic elements are free of load.

4.4.2 **Effective Static Deflection** - Effective static deflection of a loaded suspension system equals the static load

divided by the *spring rate* of the system at that load.

Total *static deflection* and effective static deflection are equal when the *spring rate* is constant.

4.4.3 *Spring Center* - The vertical line along which a vertical load applied to the *sprung mass* will produce only uniform vertical displacement.

4.4.3.1 Parallel Springing - Describes the suspension of a vehicle in which the *effective static deflections* of the two ends are equal; that is, the *spring center* passes through the center of gravity of the *sprung mass*.

4.5 DAMPING DEVICES - As distinct from specific types of damping, damping devices refer to the actual mechanisms used to obtain damping of suspension systems.

4.5.1 *Shock Absorber* - A generic term which is commonly applied to hydraulic mechanisms for producing damping of suspension systems.

4.5.2 *Snubber* - A generic term which is commonly applied to mechanisms which employ dry friction to produce damping of suspension systems.

5. VIBRATIONS OF VEHICLE SUSPENSION SYSTEMS

5.1 SPRUNG MASS VIBRATIONS -

5.1.1 *Rigid Body Modes* -

5.1.1.1 Bounce - The *oscillation* of the *sprung mass* which consists primarily of vertical displacement.

5.1.1.2 Pitch - The *oscillation* of the *sprung mass* which consists primarily of angular displacement about a transverse axis.

5.1.1.3 Roll - Angular displacement of the *sprung mass* about a central longitudinal axis with respect to its static position.

5.1.2 *Structural Modes (Shake)* - A mode of *vibration* of the entire *sprung mass* as a flexible body occurring generally in the *frequency* of 5-25 cps.

5.1.2.1 Torsional Shake - A mode of *vibration* of the entire *sprung mass* involving predominately twisting deformation of the structure.

5.1.2.2 Beaming - A mode of *vibration* of the entire *sprung mass* involving predominately bending deformation of the entire structure.

5.2 UNSPRUNG MASS VIBRATIONS -

5.2.1 *Wheel Vibration Modes* -

5.2.1.1 Hop - The vertical oscillatory motion of a wheel between the road surface and the *sprung mass*.

5.2.1.1.1 Parallel hop is the form of wheel hop in which a pair of wheels hop in phase.

5.2.1.1.2 Tramp is the form of wheel hop in which a pair of wheels hop in opposite phase.

5.2.1.2 Brake Hop - An oscillatory hopping motion of a single wheel or of a pair of wheels which occurs when brakes are applied in forward or reverse motion of the vehicle.

5.2.1.3 Power Hop - An oscillatory hopping motion of a single wheel or of a pair of wheels which occurs when *tractive force* is applied in forward or reverse motion of the vehicle.

5.2.2 *Axle Vibration Modes* -

5.2.2.1 Axle Side Shake - Oscillatory motion of an axle

which consists of transverse displacement.

5.2.2.2 Axle Fore-and-Aft Shake - Oscillatory motion of an axle which consists purely of longitudinal displacement.

5.2.2.3 Axle Yaw - Oscillatory motion of an axle around the vertical axis through its center of gravity.

5.2.2.4 Axle Windup - Oscillatory motion of an axle about the horizontal transverse axis through its center of gravity.

5.2.3 *Steering System Vibrations* -

5.2.3.1 Wheel Flutter - Forced *oscillation* of steerable wheels about their steering axes.

5.2.3.2 Wheel Wobble - A self-excited *oscillation* of steerable wheels about their steering axes, occurring without appreciable tramp.

5.2.3.3 Shimmy - A self-excited *oscillation* of a pair of steerable wheels about their steering axes, accompanied by appreciable *tramp*.

5.2.3.4 Wheelight - A rotary disturbance of the steering wheel produced by forces acting on the steerable wheels.

6. SUSPENSION GEOMETRY

6.1 KINGPIN GEOMETRY -

6.1.1 *Wheel Plane* - The central plane of the tire, normal to the *spin axis*.

6.1.2 *Wheel Center* - The point at which the *spin axis* of the wheel intersects the *wheel plane*.

6.1.3 *Center of Tire Contact* - The intersection of the *wheel plane* and the vertical projection of the *spin axis* of the wheel onto the road plane. (See Note 1.)

6.1.4 *Kingpin Inclination* - The angle in front elevation between the steering axis and the vertical.

6.1.5 *Kingpin Offset* - Kingpin offset at the ground is the horizontal distance in front elevation between the point where the steering axis intersects the ground and the *center of tire contact*.

The kingpin offset at the *wheel center* is the horizontal distance in front elevation from the *wheel center* to the steering axis.

6.2 WHEEL CASTER -

6.2.1 *Caster Angle* - The angle in side elevation between the steering axis and the vertical. It is considered positive when the steering axis is inclined rearward (in the upward direction) and negative when the steering axis is inclined forward.

6.2.2 *Rate of Caster Change* - The change in *caster angle* per unit vertical displacement of the *wheel center* relative to the *sprung mass*.

6.2.3 *Caster Offset* - The distance in side elevation between the point where the steering axis intersects the ground, and the *center of tire contact*. The offset is considered positive when the intersection point is forward of the tire contact center and negative when it is rearward.

6.2.4 *Centrifugal Caster* - The unbalance moment about the steering axis produced by a lateral acceleration equal to gravity acting at the combined center of gravity of all the steerable parts. It is considered positive if the combined center of gravity is forward of the steering axis and negative if rearward of the steering axis.

6.3 WHEEL CAMBER -

6.3.1 *Camber Angle* - The inclination of the *wheel plane* to the vertical. It is considered positive when the wheel leans outward at the top and negative when it leans inward.

6.3.2 *Rate of Camber Change* - The change of *camber angle* per unit vertical displacement of the *wheel center* relative to the *sprung mass*.

6.3.2.1 *Swing Center* - That instantaneous center in the transverse vertical plane through any pair of *wheel centers* about which the wheel moves relative to the *sprung mass*.

6.3.2.2 *Swing-Arm Radius* - The horizontal distance from the *swing center* to the *center of tire contact*.

6.3.3 *Wheel Track (Wheel Tread)* - The lateral distance between the *centers of tire contact* of a pair of wheels. For vehicles with dual wheels, it is the distance between the points centrally located between the *centers of tire contact* of the inner and outer wheels. (See SAE J693.)*

6.3.4 *Track Change* - The change in *wheel track* resulting from vertical suspension displacements of both wheels in the same direction.

6.3.5 *Rate of Track Change* - The change in *wheel track* per unit vertical displacement of both *wheel centers* in the same direction relative to the *sprung mass*.

6.4 WHEEL TOE -

6.4.1 *Static Toe Angle (deg)* - The static toe angle of a wheel, at a specified wheel load or relative position of the *wheel center* with respect to the *sprung mass*, is the angle between a longitudinal axis of the vehicle and the line of intersection of the *wheel plane* and the road surface. The wheel is "toed-in" if the forward portion of the wheel is turned toward a central longitudinal axis of the vehicle, and "toed-out" if turned away.

6.4.2 *Static Toe (in (mm))* - Static toe-in or toe-out of a pair of wheels, at a specified wheel load or relative position of the *wheel center* with respect to the *sprung mass*, is the difference in the transverse distances between the *wheel planes* taken at the extreme rear and front points of the tire treads. When the distance at the rear is greater, the wheels are "toed-in" by this amount; and where smaller, the wheels are "toed-out." (See Note 2.)

6.5 COMPRESSION - The relative displacement of *sprung* and *unsprung masses* in the suspension system in which the distance between the masses decreases from that at static condition.

6.5.1 *Ride Clearance* - The maximum displacement in *compression* of the *sprung mass* relative to the *wheel center* permitted by the suspension system, from the normal load position.

6.5.2 *Metal-to-Metal Position (Compression)* - The point of maximum *compression* travel limited by interference of substantially rigid members.

6.5.3 *Bump Stop* - An elastic member which increases the *wheel rate* toward the end of the *compression* travel.

The bump stop may also act to limit the *compression* travel.

6.6 REBOUND - The relative displacement of the *sprung* and *unsprung masses* in a suspension system in which the distance between the masses increases from that at static condition.

6.6.1 *Rebound Clearance* - The maximum displacement in *rebound* of the *sprung mass* relative to the *wheel center* permitted by the suspension system, from the normal load position.

6.6.2 *Metal-to-Metal Position (Rebound)* - The point of maximum *rebound* travel limited by interference of substantially rigid members.

6.6.3 *Rebound Stop* - An elastic member which increases the *wheel rate* toward the end of the *rebound* travel.

The *rebound stop* may also act to limit the rebound travel.

6.7 CENTER OF PARALLEL WHEEL MOTION - The center of curvature of the path along which each of a pair of *wheel centers* moves in a longitudinal vertical plane relative to the *sprung mass* when both wheels are equally displaced.

6.8 TORQUE ARM

6.8.1 *Torque-Arm Center in Braking* - The instantaneous center in a vertical longitudinal plane through the *wheel center* about which the wheel moves relative to the *sprung mass* when the brake is locked.

6.8.2 *Torque-Arm Center in Drive* - The instantaneous center in a vertical longitudinal plane through the *wheel center* about which the wheel moves relative to the *sprung mass* when the drive mechanism is locked at the power source.

6.8.3 *Torque-Arm Radius* - The horizontal distance from the *torque-arm center* to the *wheel center*.

7. TIRES AND WHEELS

7.1 GENERAL NOMENCLATURE - Values of tire/wheel dimensions, forces, moments of relationships between any of these are valid at only the *vertical load* and inflation pressure for which they are stated.

7.1.1 *Standard Loads and Inflations* - Those combinations of loads and inflations up to the maximum load and inflation recommended by the Tire and Rim Association and published in the yearly editions of the Tire and Rim Association Year Book.

7.1.2 *Rim Diameter* - The diameter at the intersection of the *bead seat* and the flange. (See Tire and Rim Association Year Book.) Nominal rim diameter (i.e., 14, 15, 16.5, etc.) is commonly used.

7.1.3 *Rim Width* - The distance between the inside surfaces or the rim flanges. (See Tire and Rim Association Year Book.)

7.1.4 *Tire Section Width* - The width of the unloaded new tire mounted on specified rim, inflated to the normal recommended pressure, including the normal sidewalls but not including protective rib, bars, and decorations. (See Tire and Rim Association Year Book.)

7.1.5 *Tire Overall Width* - The width of the unloaded new tire, mounted on specified rim, inflated to the normal recommended pressure, including protective rib, bars, and decora-

*Published in the SAE Handbook, available from Society of Automotive Engineers, Inc., Two Pennsylvania Plaza, New York, N. Y. 10001.

tions. (See Tire and Rim Association Year Book.)

7.1.6 *Tire Section Height* - Half the difference between the tire outside diameter and the nominal rim diameter.

7.1.7 *Outside Diameter* - The maximum diameter of the new unloaded tire inflated to the normal recommended pressure and mounted on a specified rim. (See Airplane Section, Tire and Rim Association Year Book.)

7.1.8 *Flat Tire Radius* - The distance from the *spin axis* to the road surface of a loaded tire on a specified rim at zero inflation.

7.1.9 *Deflection (Static)* - The radial difference between the undeflected tire radius and the static loaded radius, under specified loads and inflation.

7.1.9.1 *Percent Deflection* - The static deflection expressed as a percentage of the unloaded section height above the top of the rim flange.

7.1.10 *Tire Rate (Static)* - See paragraph 4.3.2.

7.1.11 *Sidewall* - The portion of either side of the tire which connects the *bead* with the *tread*.

7.1.11.1 *Sidewall Rib* - A raised circumferential rib located on the *sidewall*

7.1.12 *Bead* - The portion of the tire which fits onto the rim of the wheel.

7.1.12.1 *Bead Base* - The approximately cylindrical portion of the *bead* that forms its inside diameter.

7.1.12.1 *Bead Toe* - That portion of the *bead* which joins the *bead base* and the inside surface of the tire.

7.1.13 *Tread (Tire)* - The peripheral portion of the tire, the exterior of which is designed to contact the road surface.

7.1.13.1 *Tread Contour* - The cross sectional shape of tread surface of an inflated unloaded tire neglecting the *tread pattern* depressions.

7.1.13.2 *Tread Radius* - The radius or combination of radii describing the tread contour.

7.1.13.3 *Tread Arc Width* - The distance measured along the tread contour of an unloaded tire between one edge of the tread and the other. For tires with rounded tread edges, the point of measurement is that point in space which is at the intersection of the tread radius extended until it meets the prolongation of the upper sidewall contour.

7.1.13.4 *Tread Chord Width* - The distance measured parallel to the *spin axis* of an unloaded tire between one edge of the tread and the other. For tires with rounded tread edges, the point of measurement is that point in space which is at the intersection of the tread radius extended until it meets the prolongation of the upper sidewall contour.

7.1.13.5 *Tread Contact Width* - The distance between the extreme edges of road contact at a specified load and pressure measured parallel to the Y' axis at zero *slip angle* and zero *inclination angle*.

7.1.13.6 *Tread Contact Length* - The perpendicular distance between the tangent to edges of the leading and following points of road contact and parallel to the *wheel plane*.

7.1.13.7 *Tread Depth* - The distance between the base of a tire *tread* groove and a line tangent to the surface of the two adjacent *tread* ribs or rows.

7.1.13.8 *Gross Contact Area* - The total area enclosing the

pattern of the tire *tread* in contact with a flat surface, including the area of grooves or voids.

7.1.13.9 *Net Contact Area* - The area enclosing the pattern of the tire tread in contact with a flat surface, excluding the area of grooves or other depressions.

7.1.13.10 *Tread Pattern* - The molded configuration on the face of the *tread*. It is generally composed of ribs, rows, grooves, bars, lugs, and the like.

7.2 ROLLING CHARACTERISTICS

7.2.1 *Loaded Radius* - Same as 7.3.5 SAE J670.

7.2.2 *Static Loaded Radius* - The *loaded radius* of a stationary tire inflated to normal recommended pressure. Note: In general, static loaded radius is different from the radius of slowly rolling tire. Static radius of a tire rolled into position may be different from that of the tire loaded without being rolled.

7.2.3 *Spin Axis* - Same as 7.4.4 SAE J670.

7.2.4 *Spin Velocity* - (Ω) The angular velocity of the wheel on which the tire is mounted, about its *spin axis*. Positive *spin velocity* is shown in Fig. 1.

7.2.5 *Free-Rolling Tire* - A loaded rolling tire operated without application of *driving* or *braking torque*.

7.2.6 *Straight Free-Rolling Tire* - A *free-rolling tire* moving in a straight line at zero *inclination angle* and zero *slip angle*.

7.2.7 *Longitudinal Slip Velocity* - The difference between the *spin velocity* of the driven or braked tire and the *spin velocity* of the *straight free-rolling tire*. Both spin velocities are measured at the same linear velocity at the wheel center in the X' direction. A positive value results from *driving torque*.

7.2.8 *Longitudinal Slip (Percent Slip)* - The ratio of the *longitudinal slip velocity* to the *spin velocity* of the *free straight-rolling tire* expressed as a percentage.

Note: This quantity should not be confused with the slip number that frequently appears in kinematic analysis of tires in which the *spin velocity* appears in the denominator.

7.2.9 *Effective Rolling Radius* - Same as 7.4.3 SAE J670.

7.2.10 *Wheel Skid* - The occurrence of sliding between the tire and road interface which takes place within the entire *contact area*. Skid can result from braking, driving and/or cornering.

7.3 TIRE FORCES AND MOMENTS

7.3.1 *Tire Axis System (Fig. 1)* - Same as 7.5.1 SAE J670.

7.3.2 Tire Angles

7.3.2.1 *Slip Angle* - (α) The angle between the X' axis and direction of travel of the *center of tire contact*.

7.3.2.2 *Inclination Angle* - (γ) The angle between the Z' axis and the *wheel plane*.

7.3.3 *Tire Forces* - The external force acting on the tire by the road having the following components:

7.3.3.1 *Longitudinal Force* (F_x): The component of the *tire force* vector in the X' direction.

7.3.3.2 *Driving Force* - The *longitudinal force* resulting from *driving torque* application.

7.3.3.3 *Driving Force Coefficient* - The ratio of the *driving force* to the *vertical load*.

7.3.3.4 *Braking Force* - The negative *longitudinal force* re-

sulting from *braking torque* application.

7.3.3.5 *Braking Force Coefficient (Braking Coefficient)* - The ratio of the *braking force* to the *vertical load*.

7.3.3.6 *Rolling Resistance Force* - The negative *longitudinal force* resulting from energy losses due to deformations of a rolling tire.

Note: This force can be computed from the forces and moments acting on the tire by the road.

$$F_r = \frac{M_y \cos \gamma + M_z \sin \gamma}{R\ell}$$

7.3.3.7 *Rolling Resistance Force Coefficient (Coefficient of Rolling Resistance)* - The ratio of the *rolling resistance* to the *vertical load*.

7.3.3.8 *Lateral Force (F_y)* - The component of the *tire force* vector in the *Y'* direction.

7.3.3.9 *Lateral Force Coefficient* - The ratio of the *lateral force* to the *vertical load*.

7.3.3.10 *Slip Angle Force* - The *lateral force* when the *inclination angle* is zero and *plysteer* and *conicity* forces have been subtracted.

7.3.3.11 *Camber Force (Camber Thrust)* - The *lateral force* when the *slip angle* is zero and the *plysteer* and *conicity* forces have been subtracted.

7.3.3.12 *Normal Force (F_z)* - The component of the *tire force* vector in the *Z'* direction.

7.3.3.13 *Vertical Load* - Same as 7.5.3.4 SAE J670.

7.3.3.14 *Central Force* - The component of the *tire force* vector in the direction perpendicular to the direction of travel of the *center of tire contact*. *Central Force* is equal to *lateral force* times cosine of *slip angle* minus *longitudinal force* times sine of *slip angle*.

7.3.3.15 *Tractive Force* - The component of the *tire force* vector in the direction of travel of the *center of tire contact*. *Tractive force* is equal to *lateral force* times sine of *slip angle* plus *longitudinal force* times cosine of *slip angle*.

7.3.3.16 *Drag Force* - The negative *tractive force*.

7.3.4 *Tire Moments* - The external moments acting on the tire by the road having the following components:

7.3.4.1 *Overturning Moment (M_x)*: The component of the *tire moment* vector tending to rotate the tire about the *X'* axis, positive clockwise when looking in the positive direction of the *X'* axis.

7.3.4.2 *Rolling Resistance Moment (M_y)* - The component of the *tire moment* vector tending to rotate the tire about the *Y'* axis, positive clockwise when looking in the positive direction of the *Y'* axis.

7.3.4.3 *Aligning Torque (Aligning Moment) (M_z)* - The component of the *tire moment* vector tending to rotate the tire about the *Z'* axis, positive clockwise when looking in the positive direction of *Z'* axis.

7.3.4.4 *Wheel Torque (T)* - The external torque applied to the tire from the vehicle about the spin axis; positive *wheel torque* is shown in Fig. 1.

7.3.4.5 *Driving Torque* - The positive *wheel torque*.

7.3.4.6 *Braking Torque* - The negative *wheel torque*.

7.4 TIRE FORCE AND MOMENT STIFFNESS (may be evaluated at any set of operating conditions).

7.4.1 *Cornering Stiffness* - The negative of the rate of change of *lateral force* with respect to change in *slip angle*, usually evaluated at zero *slip angle*.

7.4.2 *Camber Stiffness* - The rate of change of *lateral force* with respect to change in *inclination angle*, usually evaluated at zero *inclination angle*.

7.4.3 *Braking (Driving) Stiffness* - The rate of change of *longitudinal force* with respect to change in *longitudinal slip*, usually evaluated at zero *longitudinal slip*.

7.4.4 *Aligning Stiffness (Aligning Torque Stiffness)* - The rate of change of *aligning torque* with respect to change in *slip angle*, usually evaluated at zero *slip angle*.

7.5 NORMALIZED TIRE FORCE AND MOMENT STIFFNESSES (COEFFICIENTS)

7.5.1 *Cornering Stiffness Coefficient (Cornering Coefficient)* - The ratio of *cornering stiffness* of a *free straight-rolling tire* to the *vertical load*.

Note: Although the term *cornering coefficient* has been used in a number of technical papers, for consistency with definitions of other terms using the word *coefficient*, the term *cornering stiffness coefficient* is preferred.

7.5.2 *Camber Stiffness Coefficient (Camber Coefficient)* - The ratio of *camber stiffness* of a *free straight-rolling tire* to the *vertical load*.

7.5.3 *Braking (Driving) Stiffness Coefficient* - The ratio of *braking (driving) stiffness* of a *free straight-rolling tire* to the *vertical load*.

7.5.4 *Aligning Stiffness Coefficient (Aligning Torque Coefficient)* - The ratio of *aligning stiffness* of a *free straight-rolling tire* to the *vertical load*.

7.6 TIRE TRACTION COEFFICIENTS

7.6.1 *Lateral Traction Coefficient* - The maximum value of *lateral force coefficient* which can be reached on a *free-rolling tire* for a given road surface, environment and operating condition.

7.6.2 *Driving Traction Coefficient* - The maximum value of *driving force coefficient* which can be reached on a given tire and road surface for a given environment and operating condition.

7.6.3 *Braking Traction Coefficient* - The maximum of the *braking force coefficient* which can be reached without locking a wheel on a given tire and road surface for a given environment and operating condition.

7.6.3.1 *Sliding Braking Traction Coefficient* - The value of the *braking force coefficient* of a tire obtained on a locked wheel on a given tire and road surface for a given environment and operating condition.

7.7 TIRE ASSOCIATED NOISE AND VIBRATIONS

7.7.1 *Tread Noise* - Airborne sound (up to 5000 Hz) except squeal and slap produced by the interaction between the tire and the road surface.

7.7.1.1 *Sizzle* - A *tread noise* (up to 4000 Hz) characterized by a soft frying sound, particularly noticeable on a very smooth road surface.

7.7.2 *Squeal* - Narrow band airborne tire noise (150-800 Hz) resulting from either *longitudinal slip* or *slip angle* or both.

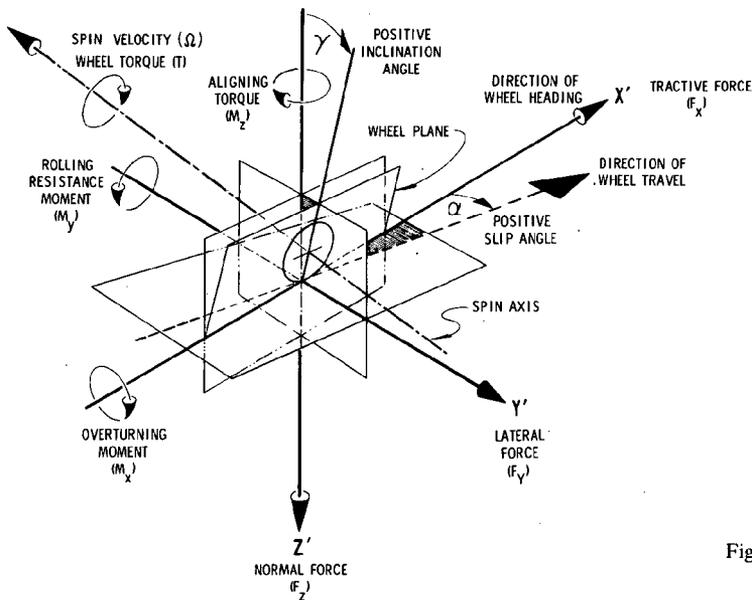


Fig. 1 - Tire axis system

7.7.2.1 *Cornering Squeal* - The *squeal* produced by a *free-rolling tire* resulting from *slip angle*.

7.7.2.2 *Braking (Driving) Squeal* - The *squeal* resulting from *longitudinal slip*.

7.7.3 *Thump* - A periodic vibration and/or audible sound generated by the tire and producing a pounding sensation which is synchronous with wheel rotation.

7.7.4 *Roughness* - Vibration (15-100 Hz) perceived tactily and/or audibly, generated by a rolling tire on a smooth road surface and producing the sensation of driving on a coarse or irregular surface.

7.7.5 *Harshness* - Vibrations (15-100 Hz) perceived tactily and/or audibly, produced by interaction of the tire with road irregularities.

7.7.6 *Slap* - Airborne smacking noise produced by a tire traversing road seams such as tar strips and expansion joints.

7.8 TIRE AND WHEEL NON-UNIFORMITY CHARACTERISTICS

7.8.1 Radial Run-Out

7.8.1.1 *Peak-to-Peak Radial Wheel Run-Out* - The difference between the maximum and minimum values of the wheel *bead seat radius*, measured in a plane perpendicular to the *spin axis* (measured separately for each *bead seat*).

7.8.1.2 *Peak-to-Peak Unloaded Radial Tire Run-Out* - The difference between maximum and minimum undeflected values of the tire radius, measured in plane perpendicular to the *spin axis* on a true running wheel.

7.8.1.3 *Peak-to-Peak Loaded Radial Tire Run-Out* - The difference between maximum and minimum values of the *loaded radius* on a true running wheel.

7.8.2 Lateral Run-Out

7.8.2.1 *Peak-to-Peak Lateral Wheel Run-Out* - The difference between maximum and minimum indicator readings, measured parallel to the *spin axis* on the inside vertical portion of a rim flange (measured separately for each flange).

7.8.2.2 *Peak-to-Peak Lateral Tire Run-Out* - The difference between maximum and minimum indicator readings, mea-

sured parallel to the *spin axis* at the point of maximum *tire section*, on a true running wheel (measured separately for each sidewall).

7.8.3 *Radial Force Variation* - The periodic variation of the *normal force* of a loaded *straight free-rolling tire* which repeats each revolution at a fixed *loaded radius*, given mean *normal force*, constant speed, given inflation pressure and test surface curvature.

7.8.3.1 *Peak-to-Peak (Total) Radial Force Variation* - The difference between maximum and minimum values of the *normal force* during one revolution of the tire.

7.8.3.2 *First Order Radial Force Variation* - The peak-to-peak amplitude of the fundamental frequency component of the Fourier series representing *radial force variation*. Its frequency is equal to the rotational frequency of the tire.

7.8.4 *Lateral Force Variation* - The periodic variation of lateral force of a *straight free-rolling tire* which repeats each revolution, at a fixed *loaded radius*, given mean *normal force*, constant speed, given inflation pressure and test surface curvature.

7.8.4.1 *Peak-to-Peak (Total) Lateral Force Variation* - The difference between the maximum and minimum values of the *lateral force* during one revolution of the tire.

7.8.4.2 *First Order Lateral Force Variation* - The peak-to-peak amplitude of the fundamental frequency component of the Fourier series representing *lateral force variation*. Its frequency is equal to the rotational frequency of the tire.

7.8.5 *Lateral Force Offset* - The average *lateral force* of a *free straight-rolling tire*.

7.8.5.1 *Ply Steer Force* - The component of *lateral force offset* which does not change sign (with respect to the *Tire Axis System*) with a change in direction of rotation (positive along positive *Y'* axis). The force remains positive when it is directed away from the serial number on the right side tire and toward the serial number on the left side tire.

7.8.5.2 *Conicity Force* - The component of *lateral force offset* which changes sign (with respect to the *Tire Axis Sys-*

tem) with a change in direction of rotation (positive away from the serial number or toward the whitewall). The force is positive when it is directed away from the serial number on the right side tire and negative when it is directed toward the serial number on the left side tire.

8. KINEMATICS: FORCE AND MOMENTS NOTATION

8.1 EARTH-FIXED AXIS SYSTEM (X, Y, Z) - This system is a right-hand orthogonal axis system fixed on the earth. The trajectory of the vehicle is described with respect to this earth-fixed axis system. The X and Y-axis are in a horizontal plane and the Z-axis is directed downward.

8.2 VEHICLE AXIS SYSTEM (x, y, z) - This system is a right-hand orthogonal axis system fixed in a vehicle such that with the vehicle moving steadily in a straight line on a level road, the x-axis is substantially horizontal, points forward, and is in the longitudinal plane of symmetry. The y-axis points to the driver's right and the z-axis points downward. (See Fig. 2.)

8.3 ANGULAR ORIENTATION - The orientation of the vehicle axis system (x, y, z) with respect to the earth-fixed axis system (X, Y, Z) is given by a sequence of three angular rotations. The following sequence of rotations (see Note 6), starting from a condition in which the two sets of axis are initially aligned, is defined to be the standard:

- (1) A yaw rotation, Ψ , about the aligned z and Z-axis.
- (2) A pitch rotation, θ , about the vehicle y-axis.
- (3) A roll rotation, ϕ , about the vehicle x-axis.

8.4 MOTION VARIABLES -

8.4.1 Vehicle Velocity - The vector quantity expressing velocity of a point in the vehicle relative to the earth-fixed axis system (X,Y,Z). The following motion variables are components of this vector resolved with respect to the moving vehicle axis system (x,y,z).

8.4.1.1 Longitudinal Velocity (u) of a point in the vehicle is the component of the vector velocity in the x-direction.

8.4.1.2 Side Velocity (v) of a point in the vehicle is the component of the vector velocity in the y-direction.

8.4.1.3 Normal Velocity (w) of a point in the vehicle is the component of the vector velocity in the z-direction.

8.4.1.4 Forward Velocity of a point in the vehicle is the component of the vector velocity perpendicular to the y-axis and parallel to the road plane.

8.4.1.5 Lateral Velocity of a point in the vehicle is the component of the vector velocity perpendicular to the x-axis and parallel to the road plane.

8.4.1.6 Roll Velocity (p) - The angular velocity about the x-axis.

8.4.1.7 Pitch Velocity (q) - The angular velocity about the y-axis.

8.4.1.8 Yaw Velocity (r) - The angular velocity about the z-axis.

8.4.2 Vehicle Acceleration - The vector quantity expressing the acceleration of a point in the vehicle relative to the earth-fixed axis system (X,Y,Z). The following motion variables are components of this vector, resolved with respect to the moving vehicle axis system.

8.4.2.1 Longitudinal Acceleration - The component of the vector acceleration of a point in the vehicle in the x-direction.

8.4.2.2 Side Acceleration - The component of the vector acceleration of a point in the vehicle in the y-direction.

8.4.2.3 Normal Acceleration - The component of the vector acceleration of a point in the vehicle in the z-direction.

8.4.2.4 Lateral Acceleration - The component of the vector acceleration of a point in the vehicle perpendicular to the vehicle x-axis and parallel to the road plane. (See Note 7.)

8.4.2.5 Centripetal Acceleration - The component of the vector acceleration of a point in the vehicle perpendicular to the tangent to the path of that point and parallel to the road plane.

8.4.3 Heading Angle (Ψ) - The angle between the trace on the X-Y plane of the vehicle x-axis and the X-axis of the earth-fixed axis system. (See Fig. 3.)

8.4.4 Sideslip Angle (Attitude Angle) (β) is the angle between the traces on the X-Y plane of the vehicle x-axis and the vehicle velocity vector at some specified point in the vehicle. Sideslip angle is shown in Fig. 3 as a negative angle.

8.4.5 Sideslip Angle Gradient - The rate of change of sideslip angle with respect to change in steady-state lateral acceleration on a level road at a given trim and test conditions.

8.4.6 Course Angle (ν) is the angle between the trace of the

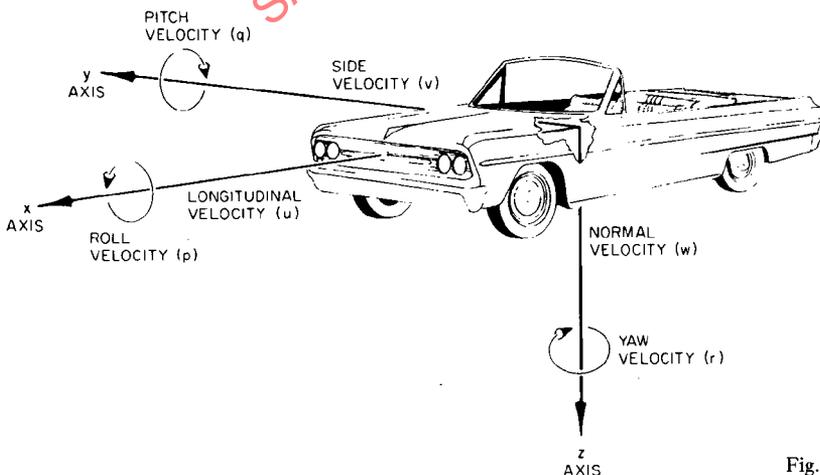


Fig. 2 - Directional control axis system

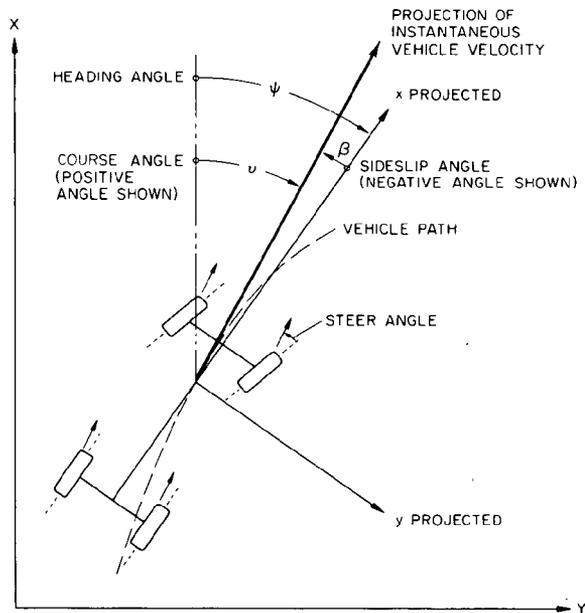


Fig. 3 - Heading, sideslip, and course angles

vehicle velocity vector on the X-Y plane and X-axis of the earth-fixed axis system. A positive course angle is shown in Fig. 3. Course angle is the sum of heading angle and sideslip angle ($\nu = \Psi + \beta$).

8.4.7 **Vehicle Roll Angle** - The angle between the vehicle y-axis and the ground plane.

8.4.8 **Vehicle Roll Gradient** - The rate of change in vehicle roll angle with respect to change in steady-state lateral acceleration on a level road at a given trim and test conditions.

8.4.9 **Vehicle Pitch Angle** - The angle between the vehicle x-axis and the ground plane.

8.5 **FORCES** - The external forces acting on the automobile can be summed into one force vector having the following components:

8.5.1 **Longitudinal Force (F_x)** is the component of the force vector in the x-direction.

8.5.2 **Side Force (F_y)** is the component of the force vector in the y-direction.

8.5.3 **Normal Force (F_z)** is the component of the force vector in the z-direction.

8.6 **MOMENTS** - The external moments acting on the automobile can be summed into one moment vector having the following components:

8.6.1 **Rolling Moment (M_x)** is the component of the moment vector tending to rotate the vehicle about the x-axis, positive clockwise when looking in the positive direction of the x-axis.

8.6.2 **Pitching Moment (M_y)** is the component of the moment vector tending to rotate the vehicle about the y-axis, positive clockwise when looking in the positive direction of the y-axis.

8.6.3 **Yawing Moment (M_z)** is the component of the moment vector tending to rotate the vehicle about the z-axis,

positive clockwise when looking in the positive direction of the z-axis.

9. DIRECTIONAL DYNAMICS

9.1 CONTROL MODES -

9.1.1 **Position Control** - That mode of vehicle control wherein inputs or restraints are placed upon the steering system in the form of displacements at some control point in the steering system (front wheels, Pitman arm, steering wheel), independent of the force required.

9.1.2 **Fixed Control** - That mode of vehicle control wherein the position of some point in the steering system (front wheels, Pitman arm, steering wheel) is held fixed. This is a special case of position control.

9.1.3 **Force Control** - That mode of vehicle control wherein inputs or restraints are placed upon the steering system in the form of forces, independent of the displacement required.

9.1.4 **Free Control** - That mode of vehicle control wherein no restraints are placed upon the steering system. This is a special case of force control.

9.2 **VEHICLE RESPONSE** - The vehicle motion resulting from some internal or external input to the vehicle. Response tests can be used to determine the stability and control characteristics of a vehicle.

9.2.1 **Steering Response** - The vehicle motion resulting from an input to the steering (control) element. (See Note 8.)

9.2.2 **Disturbance Response** - The vehicle motion resulting from unwanted force or displacement inputs applied to the vehicle. Examples of disturbances are wind forces or vertical road displacements.

9.2.3 **Steady-State** - Steady-state exists when periodic (or constant) vehicle responses to periodic (or constant) control and/or disturbance inputs do not change over an arbitrarily long time. The motion responses in steady-state are referred to as steady-state responses. This definition does not require the vehicle to be operating in a straight line or on a level road surface. It can also be in a turn of constant radius or on a cambered road surface.

9.2.4 **Transient State** - Transient state exists when the motion responses, the external forces relative to the vehicle, or the control positions are changing with time. (See Note 9.)

9.2.5 **Trim** - The steady-state (that is, equilibrium) condition of the vehicle with constant input which is used as the reference point for analysis of dynamic vehicle stability and control characteristics.

9.2.6 **Steady-State Response Gain** - The ratio of change in the steady-state response of any motion variable with respect to change in input at a given trim.

9.2.7 **Steering Sensitivity (Control Gain)** - The change in steady-state lateral acceleration on a level road with respect to change in steering wheel angle at a given trim and test conditions.

9.3 STABILITY - (See Note 10.)

9.3.1 **Asymptotic Stability** - Asymptotic stability exists at a prescribed trim if, any small temporary change in disturbance or control input, the vehicle will approach the motion defined by the trim.

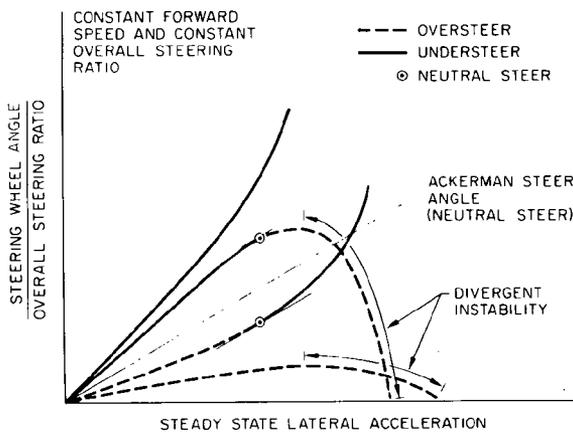


Fig. 4 - Steer properties (see note 17)

9.3.2 *Neutral Stability* - Neutral stability exists at a prescribed *trim* if, for any small temporary change in disturbance or *control input*, the resulting motion of the vehicle remains close to, but does not return to, the motion defined by the *trim*.

9.3.3 *Divergent Instability* - Divergent instability exists at a prescribed *trim* if any small temporary disturbance or *control input* causes an ever increasing *vehicle response* without *oscillation*. (See Note 11.)

9.3.4 *Oscillatory Instability* - Oscillatory instability exists if a small temporary disturbance or *control input* causes an oscillatory *vehicle response* of ever increasing amplitude about the initial *trim*. (See Note 12.)

9.4 SUSPENSION STEER AND ROLL PROPERTIES - (Fig. 4) - (See Note 13.)

9.4.1 *Steer Angle* (δ) - The angle between the projection of a longitudinal axis of the vehicle and the line of intersection of the *wheel plane* and the road surface. Positive angle is shown in Fig. 3.

9.4.2 *Ackerman Steer Angle* - (δ_a) is the angle whose tangent is the wheelbase divided by the radius of turn.

9.4.3 *Ackerman Steer Angle Gradient* - The rate of change of *Ackerman steer angle* with respect to change in steady-state *lateral acceleration* on a level road at a given *trim* and test conditions. (See Note 14.)

9.4.4 *Steering Wheel Angle* - Angular displacement of the steering wheel measured from the straight-ahead position (position corresponding to zero average *steer angle* of a pair of steered wheels).

9.4.5 *Steering Wheel Angle Gradient* - The rate of change in the *steering wheel angle* with respect to change in steady-state *lateral acceleration* on a level road at a given *trim* and test conditions.

9.4.6 *Overall Steering Ratio* - The rate of change of *steering wheel angle* at a given steering wheel *trim* position, with respect to change in average *steer angle* of a pair of steered wheels, assuming an infinitely stiff steering system with no roll of the vehicle (Note 15).

9.4.7 *Understeer/Oversteer Gradient* - The quantity obtained by subtracting the *Ackerman steer angle gradient* from the ratio of the *steering wheel angle gradient* to the *overall steering ratio*.

9.4.8 *Neutral Steer* - A vehicle is neutral steer at a given *trim* if the ratio of the *steering wheel angle gradient* to the *overall steering ratio* equals the *Ackerman steer angle gradient*.

9.4.9 *Understeer* - A vehicle is understeer at a given *trim* if the ratio of the *steering wheel angle gradient* to the *overall steering ratio* is greater than the *Ackerman steer angle gradient*.

9.4.10 *Oversteer* - A vehicle is oversteer at a given *trim* if the ratio of the *steering wheel angle gradient* to the *overall steering ratio* is less than the *Ackerman steer angle gradient*.

9.4.11 *Steering Wheel Torque* - The torque applied to the steering wheel about its axis of rotation.

9.4.12 *Steering Wheel Torque Gradient* - The rate of change in the *steering wheel torque* with respect to change in steady-state *lateral acceleration* on a level road at a given *trim* and test conditions.

9.4.13 *Characteristic Speed* - That forward speed for an understeer vehicle at which the steering sensitivity at zero *lateral acceleration* trim is one-half the steering sensitivity of a neutral steer vehicle.

9.4.14 *Critical Speed* - That forward speed for an oversteer vehicle at which the steering sensitivity at zero *lateral acceleration* trim is infinite.

9.4.15 *Neutral Steer Line* - The set of points in the x-z plane at which external lateral forces applied to the *sprung mass* produce no steady-state *yaw velocity*.

9.4.16 *Static Margin* - The horizontal distance from the center of gravity to the *neutral steer line* divided by the wheelbase. It is positive if the center of gravity is forward of the *neutral steer line*.

9.4.17 *Suspension Roll* - The rotation of the vehicle *sprung mass* about the x-axis with respect to a transverse axis joining a pair of *wheel centers*.

9.4.18 *Suspension Roll Angle* - The angular displacement produced by *suspension roll*.

9.4.19 *Suspension Roll Gradient* - The rate of change in the *suspension roll angle* with respect to change in steady-state *lateral acceleration* on a level road at a given *trim* and test conditions.

9.4.20 *Roll Steer* - The change in *steer angle* of front or rear wheels due to *suspension roll*.

9.4.20.1 *Roll Understeer* - *Roll steer* which increases vehicle *understeer* or decreases vehicle *oversteer*.

9.4.20.2 *Roll Oversteer* - *Roll steer* which decreases vehicle *understeer* or increases vehicle *oversteer*.

9.4.21 *Roll Steer Coefficient* - The rate of change in *roll steer* with respect to change in *suspension roll angle* at a given *trim*.

9.4.22 *Compliance Steer* - The change in *steer angle* of front or rear wheels resulting from compliance in suspension and steering linkages and produced by forces and/or moments applied at the tire-road contact.

9.4.22.1 *Compliance Understeer* - *Compliance steer* which increases vehicle *understeer* or decreases vehicle *oversteer*.

9.4.22.2 *Compliance Oversteer* - *Compliance steer* which decreases vehicle *understeer* or increases vehicle *oversteer*.

9.4.23 *Compliance Steer Coefficient* - The rate of change in *compliance steer* with respect to change in forces or moments applied at the tire-road contact.

9.4.24 *Roll Camber* - The camber displacements of a wheel resulting from *suspension roll*.

9.4.25 *Roll Camber Coefficient* - The rate of change in wheel *inclination angle* with respect to change in *suspension roll angle*.

9.4.26 *Compliance Camber* - The camber motion of a wheel resulting from compliance in suspension linkages and produced by forces and/or moments applied at the tire-road contact.

9.4.27 *Compliance Camber Coefficient* - The rate of change in wheel *inclination angle* with respect to change in forces or moments applied at the tire-road contact.

9.4.28 *Roll Center* - The point in the transverse vertical plane through any pair of *wheel centers* at which lateral forces may be applied to the sprung mass without producing *suspension roll*. (See Note 16.)

9.4.29 *Roll Axis* - The line joining the front and rear *roll centers*.

9.4.30 *Suspension Roll Stiffness* - The rate of change in the restoring couple exerted by the suspension of a pair of wheels on the *sprung mass* of the vehicle with respect to change in *suspension roll angle*.

9.4.31 *Vehicle Roll Stiffness* - Sum of the separate *suspension roll stiffnesses*.

9.4.32 *Roll Stiffness Distribution* - The distribution of the *vehicle roll stiffness* between front and rear suspension expressed as percentage of the *vehicle roll stiffness*.

9.5 TIRE LOAD TRANSFER -

9.5.1 *Tire Lateral Load Transfer* - The *vertical load transfer* from one of the front tires (or rear tires) to the other that is due to acceleration, rotational, or inertial effects in the lateral direction.

9.5.2 *Tire Lateral Load Transfer Distribution* - The distribution of the total *tire lateral load transfer* between front and rear tires expressed as the percentage of the total.

9.5.3 *Tire Longitudinal Load Transfer* - The *vertical load transfer* from a front tire to the corresponding rear tire that is due to acceleration, rotational, or inertial effects in the longitudinal direction.

9.5.4 *Overtuning Couple* - The overturning moment on the vehicle with respect to a central, longitudinal axis in the road plane due to lateral acceleration and roll acceleration.

9.5.5 *Overtuning Couple Distribution* - The distribution of the total *overturning couple* between the front and rear suspensions expressed as the percentage of the total.

10. AERODYNAMIC NOMENCLATURE

10.1 AERODYNAMIC MOTION VARIABLES -

10.1.1 *Ambient Wind Velocity* (v_a) is the horizontal component of the air mass velocity relative to the earth-fixed axis system in the vicinity of the vehicle.

10.1.2 *Ambient Wind Angle* (ν_a) is the angle between the X axis of the *earth-fixed axis system* and the *ambient wind velocity vector*. A positive ambient wind angle is shown in Fig. 5.

10.1.3 *Resultant Air Velocity Vector* (v_r) is the vector difference of the *ambient wind velocity vector* and the projection of the velocity vector of the vehicle on the X-Y plane.

10.1.4 *Aerodynamic Sideslip Angle* (β_a) is the angle between the traces on the vehicle X-Y plane of the vehicle X-axis and the resultant air velocity vector at some specified point in the vehicle.

10.1.5 *Aerodynamic Angle of Attack* (α_a) is the angle between the vehicle x-axis and the trace of the resultant air velocity vector on a vertical plane containing the vehicle x-axis.

10.2 AERODYNAMIC FORCE AND MOMENT COEFFICIENTS -

10.2.1 Reference Dimensions

10.2.1.1 *Vehicle Area* (A) is the projected frontal area including tires and underbody parts.

10.2.1.2 *Vehicle Wheelbase* (ℓ) is the characteristic length upon which aerodynamic moment coefficients are based.

10.2.2 Standard Air Properties

10.2.2.1 The density of standard dry air shall be taken as 2378×10^{-6} slugs/ft at 59 F and 29.92 in. Hg.

10.2.2.2 The viscosity of Standard dry air shall be taken as 373×10^{-9} slugs/ft-sec.

10.2.3 Force Coefficients

10.2.3.1 The *Longitudinal Force Coefficient* (C_x) is based on the aerodynamic force acting on the vehicle in the x direction (as established by paragraph 8.2) and is defined as:

$$C_x = \frac{F_x}{qA}$$

where q is dynamic pressure at any given relative air velocity as given by the formula

$$q = \frac{\rho v^2}{2}$$

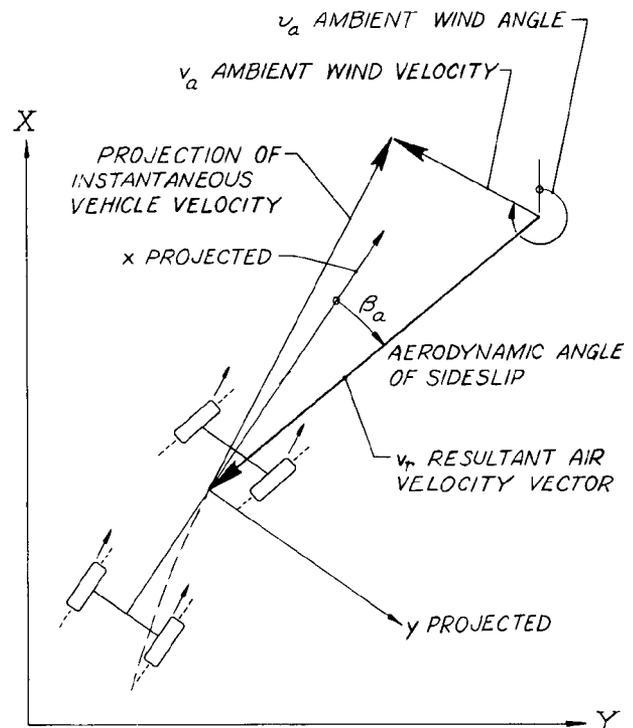


Fig. 5 - Wind vectors

10.2.3.2 Side Force Coefficient (C_y) is based on the aerodynamic force acting on the vehicle in the y-direction (as established by paragraph 8.2) and is defined as:

$$C_y = \frac{F_y}{qA}$$

where q is the dynamic pressure at any given relative air velocity as given by the formula

$$q = \frac{\rho v^2}{2}$$

10.2.3.3 The Normal Force Coefficient (C_z) is based on the aerodynamic force acting in the z direction (as established by paragraph 8.2) and is defined as:

$$C_z = \frac{F_z}{qA}$$

10.2.4 Moment Coefficients

10.2.4.1 The Rolling Moment Coefficient (C_{M_x}) is based on the rolling moment deriving from the distribution of aerodynamic forces acting on the vehicle and is defined as:

$$C_{M_x} = \frac{M_x}{qA\ell}$$

10.2.4.2 The Pitching Moment Coefficient (C_{M_y}) is based on the pitching moment deriving from the distribution of aerodynamic forces acting on the vehicle and is defined as:

$$C_{M_y} = \frac{M_y}{qA\ell}$$

10.2.4.3 The Yawing Moment Coefficient (C_{M_z}) is based on the yawing moment deriving from the distribution of aerodynamic forces acting on the vehicle and is defined as:

$$C_{M_z} = \frac{M_z}{qA\ell}$$

NOTES

1. The *center of tire contact* may not be the geometric center of the tire contact area due to distortion of the tire produced by applied forces.

2. The static toe (inches) is equal to the sum of the toe angles (degrees) of the left and right wheels multiplied by the ratio of tire diameter (inches) to 57.3.

If the toe angles on the left and right wheels are the same and the outside diameter of tire is 28.65 in (727.7 mm), the *static toe (inches)* is equal to *static toe angle (degrees)*.

3. It is important to recognize that to make axis transformations and resolve these forces with respect to the direction of vehicle motion, it is essential to measure all six force and moment components defined in paragraphs 7.5.3.1-7.5.3.3 and 7.5.4.1-7.5.4.3.

4. This *rolling resistance force* definition has been generalized so that it applies to wheels which are driven or braked. The *wheel torque* can be expressed in terms of the *longitudinal force*, *rolling resistance force*, and *loaded radius* by the equation

$$T = (F_x + F_r)R_\ell$$

For a free-rolling wheel, the rolling resistance force is therefore the negative of the *longitudinal force*.

5. For small *slip* and *inclination angles*, the *lateral force* developed by the tire can be approximated by

$$F_y = C_\alpha \alpha + C_\gamma \gamma$$

6. Angular rotations are positive clockwise when looking in the positive direction of the axis about which rotation occurs.

7. In steady-state condition, *lateral acceleration* is equal to the product of *centripetal acceleration* times the cosine of the vehicle's *sideslip angle*. Since in most test conditions the *sideslip angle* is small, for practical purposes, the *lateral acceleration* can be considered equal to *centripetal acceleration*.

8. Although the steering wheel is the primary directional control element it should be recognized that *longitudinal forces* at the wheels resulting from driver inputs to brakes or throttle can modify directional response.

9. Transient responses are described by the terminology normally employed for other dynamic systems. Some terminology is described in the "Control Engineers' Handbook,"* but a more complete terminology is contained in ANS C85.1-1963.**

10. Passenger vehicles exhibit varying characteristics depending upon test conditions and *trim*. Test conditions refer to vehicle conditions such as wheel loads, front wheel alignment, tire inflation pressure, and also atmospheric and road conditions which affect vehicle parameters. For example, temperature may change shock absorber damping characteristics and a slippery road surface may change tire cornering properties. *Trim* has been previously defined as the vehicle operating condition within a given environment, and may be specified in part by *steer angle*, *forward velocity*, and *lateral acceleration*. Since all these factors change the vehicle behavior, the vehicle *stability* must be examined separately for each environment and *trim*.

For a given set of vehicle parameters and particular test con-

*John G. Truxal (Ed.), "Control Engineers' Handbook." New York: McGraw-Hill.

**"Terminology for Automatic Control." ANS C85.1-1963. Published by American Society of Mechanical Engineers.