SURFACE VEHICLE RECOMMENDED PRACTICE	J2969™	JAN2024
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Use of the Critical Speed Formula

RATIONALE

This SAE Recommended Practice was developed to provide a guideline for the recommended use of the critical speed formula. The basic procedures and techniques for data collection and use of the method shown in this document are not to be considered all inclusive.

SAE J2969 has been reaffirmed to comply with the SAE Five-Year Review policy.

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1. SCOPE

This SAE Recommended Practice provides guidelines for procedures and practices used to obtain and record measurements and to analyze the results of the critical speed method. It is for use at accident sites using manual or electronic measurements. The method allows for many unique factors and the recommended procedure will permit a consistent use of the method in order to reduce errors and uncertainty in the results. The results from the critical speed formula should always, when possible, be compared to other accident reconstruction methodologies. When different accident reconstruction methods are used, the uncertainty of each method should be analyzed and presented.

2. REFERENCES

2.1 Applicable Publications

The following publications form a part of this specification to the extent specified herein. Unless otherwise indicated, the latest issue of SAE publications shall apply.

2.1.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA, 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or +1 724-776-4970 (outside USA), www.sae.org.

SAE J670 Vehicle Dynamics Terminology

SAE J2505 Measurement of Vehicle-Roadway Frictional Drag

2.1.2 Related Publications

More than 20 years of critical speed research has been evaluated in order to prepare this recommended practice. The majority of the historical research supports the critical speed method, although there are a few publications listed below that criticize the method. The SAE Critical Speed Task Force has evaluated these reports and has determined the method to be valid within its limits of uncertainty when used correctly, which further supports the need for a recommended practice to ensure the method is used correctly. The following publications were evaluated to prepare this recommended practice:

Amirault, G. and MacInnis, S., "Variability of Yaw Calculations from Field Testing," SAE Technical Paper 2009-01-0103, 2009, doi:10.4271/2009-01-0103.

Asay, A. and Woolley, R., "Rollover Testing of Sport Utility Vehicles (SUVs) on an Actual Highway," SAE Technical Paper 2010-01-0521, 2010, doi:10.4271/2010-01-0521.

Bartlett, W., Wright, W., Masory, O., Brach, R. et al., "Evaluating the Uncertainty in Various Measurement Tasks Common to Accident Reconstruction," SAE Technical Paper 2002-01-0546, 2002, doi:10.4271/2002-01-0546.

Bartlett, W. and Wright, W., “Summary of 56 recent critical speed yaw analysis tests, including ABS and electronic stability control on pavement, gravel, and grass: corrected,” Accident Reconstruction Journal, May/June, pp. 29-32, 2008.

Bellion, P., "Project Y.A.M. (Yaw Analysis Methodology) Vehicle Testing and Findings - Victoria Police, Accident Investigation Section," SAE Technical Paper 970955, 1997, doi:10.4271/970955.

Bohan, T. and Keierleber, J., “Speed from “Skuffs” on a Grassy Field: A Case for Dr. Daubert?” Proceedings of the American Academy of Forensic Sciences, Annual Meeting, Atlanta, GA, Feb. 11-16, 2002.

Brach, R., "An Analytical Assessment of the Critical Speed Formula," SAE Technical Paper 970957, 1997, doi:10.4271/970957.

Brach, R.M. and Brach, M., “Vehicle Accident Analysis and Reconstruction Methods,” (Warrendale, Society of Automotive Engineers, Inc., 2005), ISBN 978-0-7680-0776-3.

Cannon, J., "A Study of Errors in Yaw-Based Speed Estimates Due to Effective Braking," SAE Technical Paper 2003-01-0888, 2003, doi:10.4271/2003-01-0888.

Cliff, W., Lawrence, J., Heinrichs, B., and Fricker, T., "Yaw Testing of an Instrumented Vehicle with and without Braking," SAE Technical Paper 2004-01-1187, 2004, doi:10.4271/2004-01-1187.

Daily, J., Shigemura, N., and Daily, J., “Fundamentals of Traffic Crash Reconstruction Volume 2 of the Traffic Crash Reconstruction Series,” Institute of Police Technology and Management, University of North Florida, 2006.

Dickerson, C., Arndt, M., Arndt, S., and Mowry, G., "Evaluation of Vehicle Velocity Predictions Using the Critical Speed Formula," SAE Technical Paper 950137, 1995, doi:10.4271/950137.

Fischer, G., “Rebuttal to criticism of critical speed formula,” Proceedings of the American Academy of Forensic Sciences, Annual Meeting, Seattle, WA, Feb. 19-24, 2001.

Fischer, W., "Challenging the Critical Speed Formula In Light Of the Daubert Decision," SAE Technical Paper 2005-01-3141, 2005, doi:10.4271/2005-01-3141.

Fricke, L., “Traffic Accident Reconstruction, Volume 2 of the Traffic Accident Investigation Manual,” Northwestern University, 1990, ISBN 0-912642-07-6.

Hague, D., Lambourn, R., and Turner, D., “Critical speed studies I: The accuracy of speeds calculated from critical curve marks, and their striations,” The Institute of Traffic Accident Investigators, Proceedings of the 3rd National Conference, Telford, 14-16 November 1997.

Hague, D., Turner, D., and Williams, A., “Critical speed studies II: The generation of tyre marks by cornering vehicles,” The Institute of Traffic Accident Investigators, Proceedings of the 3rd National Conference, Telford, 14-16 November 1997.

Lambourn, R., “The calculation of motor car speeds from curved tyre marks,” Journal of Forensic Science Society, 1989, 29(6):371-386.

Lambourn, R., "Braking and Cornering Effects with and without Anti-Lock Brakes," SAE Technical Paper 940723, 1994, doi:10.4271/940723.

Manning, L. and Bentson, L., “Highway Speed vs. Sideslip (Critical Speed in a Curve),” Journal of the National Academy of Forensic Engineers, Vol. 1, No. 2, October, 1984.

Manning, L., “Critical Speed: A False Doctrine,” Proceedings of the American Academy of Forensic Sciences, Annual Meeting, Reno, NV, Feb. 21-26, 2000.

Masory, O., Delmas, S., Wright, B., and Bartlett, W., "Validation of the Circular Trajectory Assumption in Critical Speed," SAE Technical Paper 2005-01-1189, 2005, doi:10.4271/2005-01-1189.

Reveley, M., Brown, D., and Guenther, D., "A Comparison Study of Skid and Yaw Marks," SAE Technical Paper 890635, 1989, doi:10.4271/890635.

Sledge, N. and Marshek, K., "Vehicle Critical Speed Formula - Values for the Coefficient of Friction - A Review," SAE Technical Paper 971148, 1997, doi:10.4271/971148.

Yamazaki, S. and Akasaka, T., “Buckling behavior in contact area of radial tire structure and skid marks left by tires,” JSAE Review, Vol. 9, No. 3, pp. 51-55, 1988.

Yamazaki, S., “Friction coefficients of tires and skid marks left by tires,” Proceedings of the International Workshop on Traffic Accident Reconstruction, Organized by National Research Institute of Police Science, Japan, Nov. 12-13, 1998.

3. DEFINITIONS

The following terms and acronyms are defined for use herein:

ABS DECELERATION VALUE: Average deceleration value over the entire stopping test conducted with ABS activated.

ACCELERATION: Rate of change of velocity with respect to time.

CHORD: A straight line distance between two points on an arc.

COEFFICIENT OF FRICTION: A number representing the resistance to sliding of two flat surfaces in contact defined as the ratio of the resistance force to the normal force between the surfaces.

CENTRIPETAL ACCELERATION: The scalar value of the component of vehicle acceleration in the direction of the horizontal normal to the horizontal velocity vector [SAE J670].

CRITICAL SPEED: The maximum speed at which a vehicle can traverse a path with a specific radius of curvature without loss of directional control; the speed of a vehicle undergoing a turn maneuver at which the tires leave visible sideslip marks. The speed when the centripetal acceleration is limited by roadway friction. $f = \mu n \pm m$, where f is the drag factor, μ is the coefficient of friction, n is the percentage of braking, and m is the slope. This is valid for slopes less than 10°.

CRITICAL SPEED FORMULA: A formula that calculates the speed of a vehicle from the radius of curvature of its tires critical speed scuff marks, taking into account roadway grade, superelevation and frictional drag coefficient.

CRITICAL SPEED SCUFF MARK: A tire yaw mark caused by a sidesliping tire often showing a diagonal striped pattern called striations.

EQUIVALENT ROAD FRICTION: The average locked-wheel frictional drag coefficient which includes road grade.

FRICTIONAL DRAG COEFFICIENT: The ratio of the frictional drag force to the vehicle's weight at a given speed, position or instant in time [SAE J2505].

FRICTIONAL DRAG FACTOR: The frictional drag force averaged over a fully braked stop as used in the stopping distance formula in SAE J2505.

INERTIA: A physical property of a body that represents its resistance to acceleration.

MIDDLE ORDINATE: A distance measurement from the chord to the curve that is perpendicular to the chord and measured at the mid-point of the chord.

RADIUS OF CURVATURE: The measured or calculated radius of a critical speed scuff mark at the point of measurement used in the critical speed formula.

SEALED SURFACE: A roadway with a hard-sealed surface of bitumen, tar, or concrete.

SIDESLIP: Lateral/transverse translation of a vehicle perpendicular to its heading.

SIDESLIP ANGLE: The angle between the vehicle's heading and the velocity vector of its center of mass.

STRIATIONS: Periodic stripes that appear diagonal to the tire marks from a vehicle traveling at its critical speed.

SUPERELEVATION: The slope of the surface perpendicular to the critical speed scuff mark.

UNSEALED SURFACE: A roadway not sealed with bitumen, tar, or concrete (for instance a loose gravel or dirt roadway).

4. THEORY

4.1 Critical Speed Formula

The derivation of the critical speed formula is available in many references, such as Brach and Brach (2005) and Daily et al. (2006). The formula assumes a vehicle is traveling along a surface that is possibly banked (superelevation) and uphill or downhill (road grade) during the time that the critical speed scuff marks are being generated. During this time, the center of mass of the vehicle traces out a near circular arc with the center of the arc located on a horizontal plane at distance r inside of the turn. It is also assumed that there are no aerodynamic loads and that the force from friction is proportional to the normal force on the road. It is assumed that the vehicle is a point mass, traveling at an instantaneous constant speed, is in a steady state maneuver, and traveling in uniform circular motion.

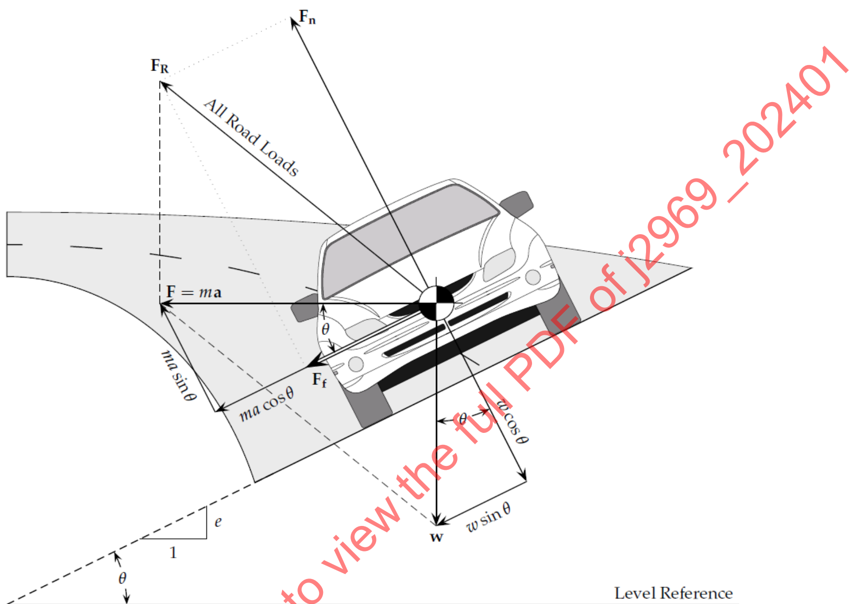

Figure 1 - Free body diagram of a car traveling on a banked, curved road with superelevation

Figure 1 shows a vehicle in a turn. When a vehicle is trying to turn, the frictional force is lateral to the vehicle's motion. If this vehicle is in a critical speed yaw, then the maximum amount of lateral frictional force is being generated by the tires. When the maximum amount of frictional force is being generated the tires are saturated and evidence of such usually manifests itself as a critical speed scuff mark on the pavement. The critical speed formula is:

$$v = \sqrt{\frac{gr(\mu_{eq} + e)}{(1 - \mu_{eq} e)}} \quad \text{(Eq. 1)}$$

where:

g = acceleration due to gravity

r = radius

e = superelevation of the road in percent

μ_{eq} = equivalent road friction

This is the general form of the critical speed formula and is valid for any unit system. However, there are restrictions on the admissible values of μ_{eq} and *e*. There are two mathematical requirements for *e* and μ_{eq}:

4.1.1 The product of μ_{eq} and e must be less than 1 (i.e., $\mu_{eq}e < 1$). Physically speaking, a critical speed cannot exist if there is enough bank and friction. If the product $\mu_{eq}e$ is greater than one, then when the maximum frictional force is developed (i.e., marks are left on the road), the vector representing all the road loads is pointing below the desired force vector, and the vehicle will turn off the road toward the inside of the curve. As such, a critical velocity cannot exist when the product $\mu_{eq}\,e$ is greater than 1.

4.1.2 The sum of μ_{eq} and e must be greater than zero (i.e., $\mu_{eq}+e > 0$). The physical significance is the fact that if the slope is negative (i.e., the curve slopes away) by an amount greater than the coefficient of friction, then the vehicle will simply slide off the road regardless of its speed. This would require either a very steep bank or a very slick surface.

For nearly level surfaces, Equation 1 simplifies to:

$$v = \sqrt{gr\,\mu_{eq}} \quad \text{(Eq. 2)}$$

5. RECOMMENDED METHOD

5.1 Vehicle Considerations

The method as outlined here does not apply to articulated trucks and has not been tested on other non-standard vehicles like motorcycles and trikes.

It is recommended to document the vehicle's tires make and model, as well as the tire's DOT number. Do not use the critical speed formula when significantly dissimilar tires are installed like a donut spare on the loaded side of the turning vehicle.

5.2 Roadway Documentation

Measure the vehicle-to-roadway frictional drag factor in accordance with SAE Recommended Practice J2505. Although the tire generates lateral friction during a critical speed maneuver, the method has been correlated through research using the average longitudinal frictional drag factor and therefore, this value should be used.

Measure the road superelevation to be used in the critical speed formula and the road grade.

When a vehicle is skid tested at the accident location, the road grade will already be included in the measured locked wheel friction. If the road friction is estimated or measured on a flat portion of the roadway, then the frictional value should be adjusted for road grade to get the equivalent road friction used in the critical speed formula. When the road grade is positive (uphill) it should be added in decimal percent to the average locked-wheel frictional drag and it should be subtracted when the road grade is negative (downhill).

5.3 Tire Mark Evaluation

Identify at least two curved tire marks representing the outer tires. Be certain that the rear tire mark is tracking outside the front tire mark and the tire marks typically show evidence of diagonal striations. Photograph the tire mark showing evidence of diagonal striations. Figure 2 shows an example of a tire mark that has clear diagonal striations while the vehicle is yawing, but turn to parallel striations when the vehicle begins to skid. The tire mark should not show any evidence of skidding in the area of the yaw maneuver measurements. In some cases, the rear tire mark may be hard to discern.

Figure 2 - Tire mark showing diagonal striations that turn to parallel striations when the tire begins to skid

Two radius measurements should be taken (or more when possible) using one of the following methods:

Method 1: Use a 15-m chord with three equally spaced ordinates. The middle ordinate should be the one used for the radius determination and the 3/4 ordinate will be larger than the 1/4 ordinate when the vehicle is decreasing in speed as it performed the critical speed maneuver.

Method 2: Use two 10-m chords with middle ordinates. The radius is calculated with both measurements and should decrease if the vehicle is slowing as the vehicle progresses through the maneuver.

Method 3: Measure the tire marks electronically using Total Station or other survey equipment. Measure the marks every meter and use the outside darkest edge of the mark to reduce position error in trying to determine the location of the center of the mark. Use the electronic measurements to determine the radius of the tire mark.

Method 4: Photogrammetry can be used to measure the tire mark as long as the photogrammetry method can quantify the accuracy of the measurements and their sensitivity to the radius calculation.

Figure 3 shows examples of Methods 1 and 2. Begin the measurement at the first visible evidence of a critical speed tire mark from the outside front tire. Use the dark outer portion of the mark as a reference to reduce subjective variability as compared to attempting to determine the center of the mark. Some research has shown that the method works best when measuring the first 15 m (49.2 feet) of the mark. In some low speed cases, a shorter critical speed scuff mark must be used. If a manual measured chord and middle ordinate approach is used to measure the radius, for higher speed (larger radius) cases it is sometimes necessary to use a longer chord length to reduce measurement uncertainty. In these cases, it is recommended to use the shortest possible chord length (15 m or greater) that achieves a middle ordinate of at least 0.15 m (6 inches). To prevent measurements where the vehicle has achieved too high of a sideslip angle, the separation of the front and rear critical speed scuff marks over the length of the chord measurement should be no more than half the track width of the vehicle.