SAE International

SURFACE VEHICLE STANDARD

SAE J2609

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Multi-Dimensional Thermal Properties of Insulated Heat Shield Material Systems

1. Scope

This test method measures the system material properties of an insulated formed heat shield under in-vehicle conditions. While the material properties of the individual components can often be determined via existing test methods, the system properties of the entire composite is typically much harder to ascertain (especially for multi-layer shields). System material properties include thermal conductivity in the lateral or in-plane (x) direction, thermal conductivity through the thickness or perpendicular (y), surface emissivity on the top and bottom sides of the shield and specific heat of the shield material.

1.1 All properties are determined for the entire shielding material specimen as a composite of the entire structure. Properties are determined using a testing apparatus that allows for two-dimensional heat flow through the specimen. Due to this, the material property results from this test method may not agree with one-dimensional heat flow type testing methods, but is representative of most heat shield materials performance tested with a centralized heat source. Therefore, material property results from this test method may be more suited for multi-dimensional analytical studies.

1.2 This standard sets forth the general guidelines to construct and operate the testing apparatus to acquire a satisfactory set of test data. Designs conforming to this standard are included and must ***not*** be deviated from for sensitivity reasons that will be discussed in more detail later. Test parameters that cannot be deviated from include, but are not limited to; specimen size, distance between the source and the specimen, source diameter and environmental conditions around the apparatus.

1.3 This method ultimately determines the shield material properties by using the test data along with an analytical scheme, see Section 8.1.

1.4 This test method will evaluate both isotropic and anisotropic insulated shielding materials. This may also include multi-layer shielding structures which include embossed/corrugated solids, porous, fibrous, granulated and coated materials.

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1.5 Limitations

This test method does have limitations in the type of insulated shielding materials that can be evaluated. However, many of the limitations apply to materials that would not typically be suitable in a heat shielding function or the properties can be derived by simpler one-dimensional hot plate methods (SAE J1361, ASTM C 177).

Limitations include:

a) Materials where the radiant transmissivity through the material cannot be assumed as zero. Materials of this type are classified as translucent or transparent.

b) Materials that do *not* have an insulating characteristic in at least one axis; (*i.e.*, single wall stamped metal shielding). This includes shielding materials where lateral thermal conductivity (x) and thermal conductivity through the thickness (y) are the same and considered high (in the order of 25 W/m-C) when compared to metallic materials. These types of single sheet metallic shields are not included in the standard because the properties of these materials are typically well known and do not require a procedure to determine them.

c) Materials where the lateral thermal conductivity (x) is less than the thermal conductivity through the thickness (y).

d) Testing exposes the shielding material to temperatures up to 250°C. Materials with limits below this level should *not* use this method.

1.6 Safety

This method involves a test apparatus that exposes the operator to very high temperatures. This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.

1.7 The attached Appendix A provides a detailed discussion of the analytical technique used in calculating the insulated shielding material properties from the test data. The Appendix A also presents the theoretical sensitivity study of the analytical method.

1.8 This test method requires two specific pieces of test instrumentation. A portable emissometer as outlined in ASTM C1371 and a radiosity meter or infrared camera with the ability to set the emissivity to 1.0.

2. References

2.1 Related Publications

The following publications are for informational purposes only. **The ShieldProp** and **ShieldTherm** programs, available free from ThermoAnalytics at http://www.thermoanalytics.com/products/shieldtherm/index.html were written specifically to solve the equations for this standard and give examples.

2.1.1 SAE PUBLICATIONS

Available from SAE, 400 Commonwealth Drive, Warrendale, PA 15096-0001.

SAE J1361 Hot Plate Method for Evaluating Heat Resistance and Thermal Insulation Properties of Materials

2.1.2 ASTM PUBLICATIONS

Available from ASTM, 100 Barr Harbor Drive, West Conshohocken, PA 19428-2959.

ASTM C177 Standard Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot Plate Apparatus

ASTM C1371 Standard Test Method for Determination of Emittance of Materials Near Room Temperature Using Portable Emissometers

3. Summary

This method describes a testing procedure and equipment, coupled with a computer analyses to directly measure or calculate the base material properties of an insulated shielding specimen. These material properties include thermal conductivity in the lateral or in-plane (x) direction, thermal conductivity through the thickness (y), surface emissivity on the top and bottom sides of the shield, density, and specific heat of the shield material.

3.1 Figure 1 is a slice through the thickness of a shield to illustrate a typical multi-dimensional heat flow through an insulated shielding specimen with a centralized heat source. This arrangement is very typical of actual in-vehicle usage and is the basis for this test method.

3.2 This test method is designed to induce a multi-dimensional heat flow pattern into the insulated shield test specimen. After collecting temperature data on both the shielding material and ambient with a predetermined arrangement, the following composite shield material properties can be calculated: thermal conductivity in the lateral or in-plane (x) direction, thermal conductivity through the thickness (y), and specific heat of the shield. Surface emissivity on the top and bottom sides of the shield are obtained through the use of ASTM Standard C 1371 and not analytically calculated as part of this test method, although, it is a required piece of data for the analytical software program. Also, heat source radiosity is a required input to the analytical software program. Both surface emissivity of the shield, heat source radiosity and floor radiosity are obtained through the use of additional equipment as outlined in Section 4. Since specific heat is one of the properties the test method calculates, the test procedure and the data collected will be done in a transient mode.

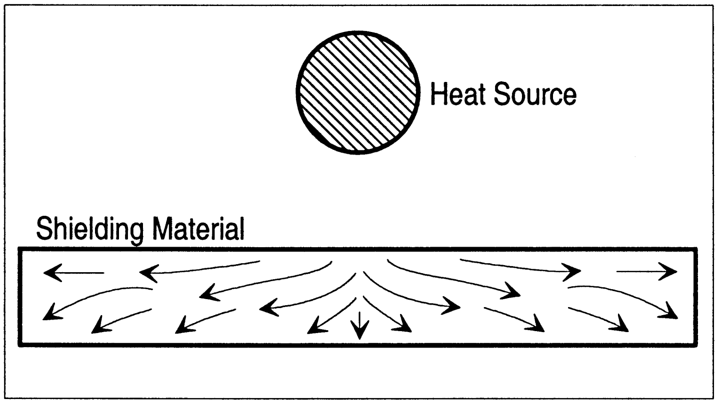

FIGURE 1—MULTI-DIMENSIONAL HEAT FLOW THROUGH SHIELDING MATERIAL WITH CENTRAL HEAT SOURCE

3.3 Figure 2 illustrates the main components of the test set-up. Two shield specimen sizes are required to be tested independently during this method, 22.5 cm x 45 cm and 45 cm square. Both are required to satisfy the sensitivity requirements of the analytical calculation. The insulated shield test specimen is thermocoupled on the top and the bottom, identically, in the arrangement displayed in Figures 3 and 4. It is important that the thermocouples be completely shielded from direct radiant energy from the heat source. Also, an ambient thermocouple, similarly shielded, is set to measure the ambient air temperature in the test area that would represent T_∞. The ambient thermocouple is intended to measure the average temperature of the air at a distance sufficiently outside the convective boundary layer of the shield specimen. A cylindrical heat source of 5.08 cm in diameter is positioned 25.4 mm above and centered on the specimen. The heat source is held at a constant, average temperature of 400° C (as measured across the length of the test specimen) during the entire transient test. As soon as the heat source is at steady state, the insulated shield specimen is moved quickly into position and the data collection is started.

3.4 Data collection is started and continued until steady state is reached on the insulated shield test specimen. Data sampling rates should be a least one per minute. Data should continue to be taken, after the specimen has reached steady state, for at least ten minutes. Data is saved to a predetermined comma delimited (.csv) format, Table 1.

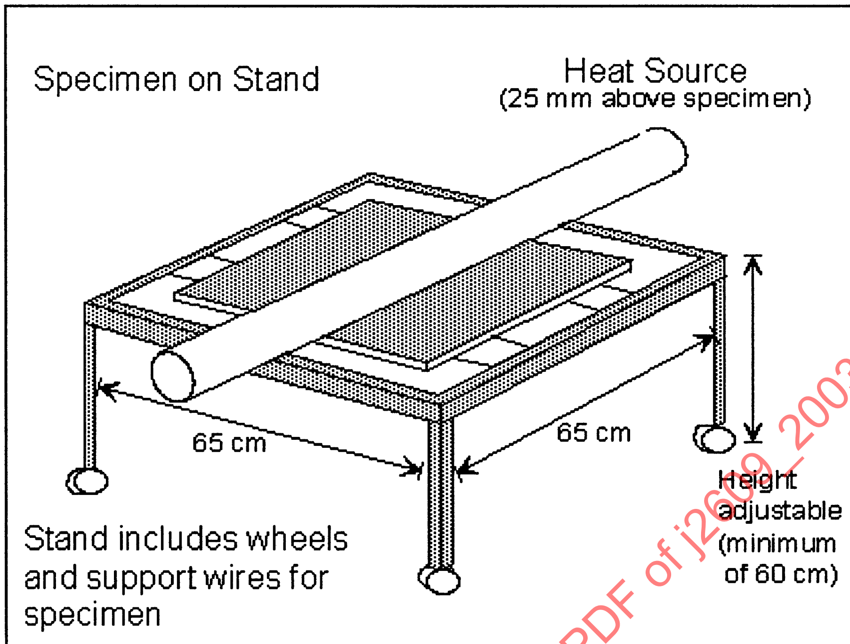

FIGURE 2—MAIN COMPONENTS OF EXPERIMENTAL SETUP

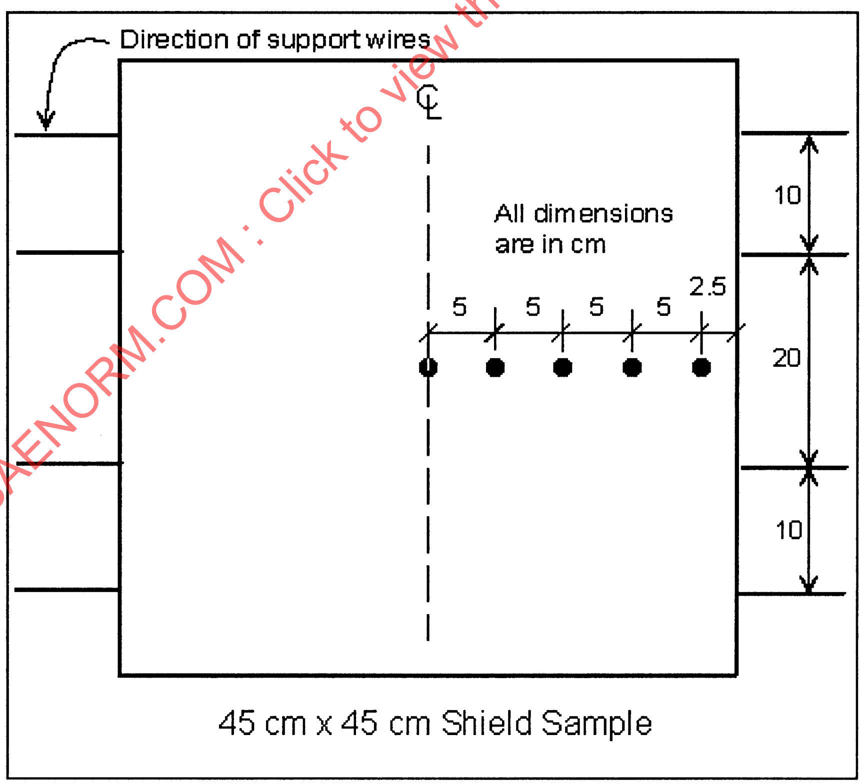

FIGURE 3—THERMOCOUPLE ARRANGEMENT ON 45 cm SAMPLE

TABLE 1—EXPERIMENTAL RESULTS, FORMAT OF THE .CSV DATA FILE

Column #	Test Data
1	Time (seconds)
2	Temp Heat Source (C)
3	Temp Room Ambient (C)
4	Temp Top of Specimen (center, #1 position)
5	Temp Top of Specimen (#2 position)
6	Temp Top of Specimen (#3 position)
7	Temp Top of Specimen (#4 position)
8	Temp Top of Specimen (#5 position)
9	Temp Bottom of Specimen (center, #1 position)
10	Temp Bottom of Specimen (#2 position)
11	Temp Bottom of Specimen (#3 position)
12	Temp Bottom of Specimen (#4 position)
13	Temp Bottom of Specimen (#5 position)

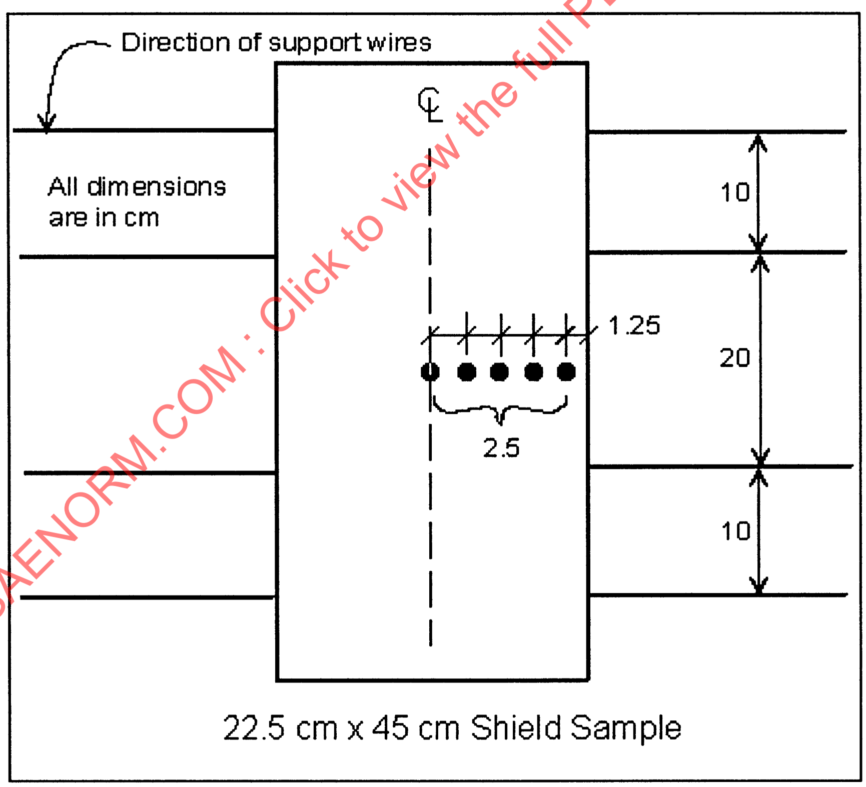

FIGURE 4—THERMOCOUPLE ARRANGEMENT ON 22.5 cm SAMPLE

3.5 Both the 22.5 cm and 45 cm width specimen sizes are tested using the same procedure and the data saved to separate .csv files.

3.6 The data is read into an available analytical software program that calculates the base material properties of the insulated shield material.

4. Equipment

4.1 Heated pipe

The radiant heat source for this test is a heated cylindrical pipe that has a 5.08 ±0.013 cm (2.0 ± 0.005 inch) outer diameter. The pipe should be made from 310 stainless steel for durability and because the analytical program calculates pipe growth due to heating, which will change the distance between the heat source and the specimen. The pipe may be also made from 316 stainless steel which has equivalent thermal expansion as 310 but the durability may not be as good. It must be capable of continuous operation at 400°C without drooping or serious degradation. Discoloration of the heat source is acceptable as long as it is uniform across the surface of the pipe. The pipe must be at least 61 cm (24 inches) long and extend at least 7.6 cm (3 inches) beyond each end of the heat shield specimen.

4.2 Heat source

The method of heating the pipe is up to the discretion of the user. It must be capable of heating the pipe uniformly to 400°C. Since the data and analytical calculation are taken at a perpendicular slice to the heat source, at the centerline of the specimen, the temperature of the heat source can drop up to 25°C from one end of the pipe to the other (over the specimen length) without affecting the accuracy of the results.

4.3 RTD temperature sensors

The shield temperatures will be measured using RTD temperature sensors with an accuracy of ±0.12%. Omega thin film RTD series F elements that are 1 mm thick, 2.3 mm long and 2 mm wide meet the requirements. For materials where the temperature sensors cannot be tack welded directly to the surface, special epoxies can be used. Omegabond OB-500 air-dry cement (Omega Engineering Inc., Stanford, Connecticut) or equivalent should be used.

4.4 Temperature recorder

A minimum 13-channel digital data recorder with a 0.1°C readout capability. The recorder must have the ability to output to a file that can be formatted to the column designations required for the analytical software program used in the standard.

4.5 Test stand

The shield specimen is supported on a test stand as shown in Figure 2. The test stand is a framed design with an open center area of 65 cm on each side. The shield is centered and supported in the middle of this section by 4 wires running perpendicular to the heated pipe. The wires are 0.81 mm diameter stainless steel Safety Lock Wire and are stretched tight between the sides of the test stand frame, although any small diameter wire will serve the same purpose. The wires are spaced at 10 cm increments starting 10 cm on each side of the center of the test stand opening. The purpose of the wires is to allow the specimen to float in space during the testing with little to no energy being transferred from the stand to the specimen or vice-a-verse.

4.6 Shield location

It is very important that the center of the shield and the corresponding RTD sensor be located directly under the pipe during the test. Since the shield must be positioned quickly and accurately under the hot pipe during the test, locator devices providing positive and repetitive positional accuracy should be used to be sure the exact position is obtained.

4.7 Emissometer

An emissometer as outlined in ASTM C1371 is used to measure the emissivity of the shield surfaces, top and bottom, prior to the test. Emissometer Model AE from Devices & Services or equivalent.

4.8 Radiation pyrometer or radiosity meter

A meter capable of measuring the pipe radiosity temperature using an emissivity setting of 1.0 is necessary. The Oakton Model U-35629-30 Infrapro sold by Cole-Palmer or equivalent will suffice.

4.9 Analytical software

Any analytical software program capable of performing the methods described in Section 8.2, see Section 2 Related References for software.

4.10 Computer

The minimum requirements should be based on the software selected in Section 4.9. The minimum requirements to use the software in the first reference of section 2 are as follows: IBM PC compatible computer, 125 MB RAM, 200 MHz, 3 MB Hard Disk Space, Windows 95 or higher.

5. Test requirements

5.1 Specimen

There are two test specimens used for this standard: 45x45cm square and 22.5x45 cm rectangle with a tolerance of ±0.5 cm. The test specimens must be flat (except for any surface embossment pattern, etc.) and its materials and construction must be representative of the final production part, except for any edge treatments. The specimen must be free of any visible defects that would affect the test results. Samples should not contain crimped or rolled edges. Edges of this type help short circuit the energy to the back side of the specimen and do not give a representative performance of the material. The analytical software in Related References section 2.0 does not assume a short circuit; therefore under these conditions, less effective thermal conductivity values will result from this phenomenon.

5.2 Specimen surface condition

The surface emissivity of each shield will be measured four times. The first and second measurements will be taken in the as received (new) condition on both the top and the bottom of the shield specimen. These are the values that will be used to satisfy the surface emissivity property requirement. Then for testing and calculation purposes, the specimen will be painted with a high temperature flat black on the top and high temperature flat gray on the bottom and measured a third (top) and fourth (bottom) time. The emissivity on the top should be greater than 0.8 and on the bottom in the range of 0.5 to 0.7. The shield will only be tested under the heat source in the painted condition. For an accurate value of emissivity to be obtained in both the as received and painted conditions, multiple measurements over the surface should be taken and averaged. The specimen is painted and tested to satisfy the calculation assumption of a uniform emissivity over the surface of the specimen. The paint does not change the results of the thermal conductivity values calculated. Using the assumption that the paint is 5 microns thick and the base specimen has a thermal conductivity as high as 10 W/m C, the added paint would have less than a 1% affect on the overall specimen thermal conductivity. It is the painted emissivity values that are used in the calculation for the specimen thermal properties. To assure accurate calculations and even paint coverage, the variation of measured emissivity on a given side of the painted shield should not be more than 0.005.

5.3 Pipe surface and geometry

The outside of the pipe must have uniform surface coloration to provide a uniform radiosity. Heat-treating the pipe to a high temperature is one method of obtaining this uniformity but other approaches may be used. The pipe will grow in diameter during the heating process. The analytical program takes this into account and calculates this pipe growth. The pipe must be at least 61 cm (24 inches) long and extend at least 7.6 cm (3 inches) beyond each end of the heat shield specimen. Bowing or deflection of the pipe must not exceed 1% of the pipe length. Pipe surface emissivity must be greater than 0.8.

5.4 Pipe temperature

The pipe temperature is set to 400°C by the control sensor directly over the center of the shield and must be at steady state as noted by at least 10 consecutive readings taken at least 1 minute apart which are within 5°C of each other and not showing an upward or downward trend.

5.5 Pipe radiosity temperature measurement

The radiosity temperature is the temperature of the pipe as measured by radiation methods assuming that the emissivity is 1.0. This measurement will be taken at the middle of the pipe directly over the center of the specimen.

5.6 Shield temperature measurement

The shield temperatures will be measured at the locations shown in Figures 3 & 4 on both the top and bottom of the shield using RTD temperature sensors. The wires from the RTD mounting location will run parallel to the heated pipe for at least 3 cm to minimize interference with the shield gradient. The preferred method of mounting the sensors to the shield surface is by tack welding, however if the surface of the specimen does not allow this, special epoxies may be used. For materials where the temperature sensors cannot be tack welded directly to the surface, special epoxies can be used. Omegabond OB-500 air-dry cement (Omega Engineering Inc., Stanford, Connecticut) or equivalent should be used. The temperatures will be recorded to an accuracy of 0.1° C.

5.7 Temperature recording

All temperatures are to be recorded simultaneously to avoid errors since this is a transient test. Recorders that do rapid sequential recording are acceptable. Manual recording of the data is not acceptable since this causes skewing of the data, which will cause errors in the calculated material properties. The recording device must be able to record in 0.1° C increments.

5.8 Distance pipe to specimen

The distance between the top of the shield and the bottom of the pipe must be 25.4 $\pm$ 1.5 mm while the setup is in the cold position. Adjustments to the pipe or the shield stand must be made prior to the start of the test to assure the correct distance. An inside caliper measurement tool can be used for this placement. The analytical program calculates the pipe growth due to heating, which will change the distance between the heat source and the specimen.

5.9 Distance specimen to floor

The specimen must be held at least 60 cm above the floor.

5.10 Ambient airflow

Test in a "passive hood" environment. External airflow must not influence test measurements. There can be no forced airflow anywhere in the region of the test setup.

5.11 Ambient air temperature

A standard K-Type thermocouple shall be mounted at least 60 cm away from the pipe and at approximately the same height as the pipe to measure the room air temperature.

6. Calibration

6.1 The instrument manufacturer or certified agent should calibrate all recorders to National Institute for Science and Technology (NIST) for accuracy claimed and the appropriate label displayed on the instrument. Calibration by the test facility is appropriate if substantiated by data measured by calibrated instruments traceable to NIST.

7. Procedure

7.1 Prepare two specimens of the same material. One 45 x 45 cm and one 22.5 x 45 cm. Measure the length and width of each specimen to be sure that it meets the specifications in Section 5.

7.2 Weigh the specimens in grams. Measure the thickness of the specimens using a micrometer. Multiple measurements (minimum of 4 for simple construction and 10 for complex structures) should be taken to assure that a "total surface area" average thickness can be derived.

7.3 Using the emissometer, measure the emissivity of the specimens on the top and bottom surfaces in the as-received condition (prior to painting). Since surface emissivity can change over the surface of the specimen, multiple readings over the surface of the specimen should be taken and averaged. These readings should be recorded as the as-received measurements.

7.4 Install the RTD temperature sensors per Figures 3 and 4. The preferred method of mounting the sensors to the shield surface is by tack welding, however if the surface of the specimen does not allow this, Appendix B presents a study that evaluated different epoxy materials for adhering the sensors and how well they performed.

7.5 For testing and calculation purposes, the specimens will be painted with a high temperature flat black on the top (facing the heat source) and high temperature flat gray on the bottom. After the paint has dried, measure the emissivity again using the emissometer. (The averaged readings must agree within 0.005 to ensure uniform paint coverage.) The emissivity will be measured on both the top and bottom of the specimen. The shield will only be tested under the heat source in the painted condition.

7.6 With the specimen in place under the heat source and the heat source cold, measure the distance from the top of the specimen to the bottom of the pipe. Adjust this distance until the shield is 25.4 mm below the heat source. When the heat source is centered over the specimen, install locators on the floor so that when the test stand is rolled out, the correct position will be known.

7.7 Remove the specimen from under the heat source. Having the test stand on wheels will allow easy movement of the specimen during the test. Turn on the heat source and bring the pipe to steady state at 400°C.

7.8 Quickly but carefully roll the test stand with the specimen on it under the pipe so that the center RTD sensor is directly below the heat source pipe (use the floor locators) with the pipe 25.4 mm above the specimen. Using the data acquisition system, start recording the temperature data immediately.

7.9 Record the 13 columns of data as specified in Table I, at least one-minute intervals. Continue taking data until the specimen has reached steady state. Steady state is achieved when the temperatures on the specimen do not increase more than 1 °C during a two-minute period. The data will be recorded and stored in a file in a comma delimited (.csv) format as described in Table I.

7.10 Measure the floor radiosity temperature directly beneath the specimen. Use either the IR pyrometer (emissivity set to 1.0) or the radiosity meter.

7.11 Using the IR pyrometer (emissivity set to 1.0) or the radiosity meter, manually measure the radiosity temperature of the heat source at the center of the specimen. Record this measurement as the Pipe Radiosity Temperature.

7.12 Shut down the test and repeat step 7.2 thru 7.11 with the other specimen size. Data sets from both specimen sizes, 45 x 45 cm and 22.5 x 45 cm, are required for the calculation. Due to the natural repeatability issues of any experiment as outlined in Appendix C, it is prudent to repeat this test on the same specimen multiple times to gain confidence that the results are representative of the average material.

8. Calculation

8.1 From the data, the calculation of the properties of interest; normal (through thickness) conductivity, lateral (in-plane) conductivity, specific heat, and density is somewhat complex. The equations are given in Appendix A and can be solved using any mathematical computer program using the method described below or the software in the first reference of section 2 written specifically for this standard.

8.2 Solving the System of Equations

First, the steady state equations are solved to determine conductivities. Afterwards, the transient equations are solved for specific heat. The ten steady state equations and four unknowns (k_x, k_y, h_T, and h_B) constitute a 10 x 4 overdetermined system of equations. Overdetermined systems cannot, in general, be satisfied exactly, and thus the standard procedure is to solve the system in a least-squares sense. The Normal Equations method is used to find the least-squares solution to the system [1].

9. Report

9.1 The report file includes all of the inputted data from the program and the calculated material properties: normal (through thickness) conductivity, lateral (in-plane) conductivity, specific heat and density.

9.2 A complete description of the specimen including all input data (including but not limited to the top and bottom emissivity) to the computer program plus special construction methods and any other pertinent information related to the material.

PREPARED BY THE SAE FORMED HEAT SHIELD COMMITTEE

APPENDIX A
EQUATION SOLUTION AND SENSITIVITY ANALYSES

A.1 Introduction

This appendix is provided to describe both the method used to solve the equations and the sensitivity analysis that ultimately helped define many of the test setup parameters, see Section 2. References.

A.2 Symbols

A	Element surface area
A_c	Element cross-sectional area
C_p	Specific heat
$F_{i\text{-}j}$	View factor (surface i to surf. j)
h	Convection coefficient
J	Radiosity emitted
k_x	Lateral conductivity
k_y	Normal conductivity
m	Mass
T	Temperature
t_s	Specimen thickness
t	time
w_e	Element width
w_s	Specimen width
ε	Emissivity
σ	Stefan-Bolzman constant

Subscripts

A	Ambient air
T	Top of specimen
B	Bottom of specimen
P	Radiant heat source
W	Room walls
s	Specimen
e	Element
1	Centermost element
5	Outermost element

A.3 System of equations

The shield specimen is divided into nine elements on each side. Due to symmetry, four of the nine elements are redundant; hence, only five elements on each side need to be considered. An RTD temperature sensor is located at the center of each of the ten unique elements. The elements are designated as 1T, 2T, 3T, 4T 5T, 1B, 2B, 3B, 4B, and 5B. A "T" indicates that an element is on the top, a "B" indicates that an element is on the bottom, a "1" indicates that an element is on the shield centerline, and a "5" indicates that an element is on the edge. An energy balance can be formed for each element.

The energy balance for element 1T is

$$0 = \varepsilon_T A\left(\sigma T_{1T}^4 - J_P F_{1T-P} - J_W F_{1T-W}\right) + 2k_x \frac{A_c}{w_e}(T_{1T} - T_{2T}) + h_T A(T_{1T} - T_A) + k_y \frac{A}{t_s}(T_{1T} - T_{1B})$$

The energy balance for element iT (where i = 2, 3, or 4) is

$$0 = \varepsilon_T A\left(\sigma T_{iT}^4 - J_P F_{iT-P} - J_W F_{iT-W}\right) + k_x \frac{A_c}{w_e}(2T_{iT} - T_{(i-1)T} - T_{(i+1)T}) + h_T A(T_{iT} - T_A) + k_y \frac{A}{t_s}(T_{iT} - T_{iB})$$

The energy balance for element 5T is

$$0 = \varepsilon_T A\left(\sigma T_{5T}^4 - J_P F_{5T-P} - J_W F_{5T-W}\right) + k_x \frac{A_c}{w_e}(T_{5T} - T_{4T}) + h_T A(T_{5T} - T_A) + k_y \frac{A}{t_s}(T_{5T} - T_{5B})$$

The energy balance for element 1B is

$$0 = \varepsilon_B A\left(\sigma T_{1B}^4 - J_W\right) + 2k_x \frac{A_c}{w_e}(T_{1B} - T_{2B}) + h_B A(T_{1B} - T_A) + k_y \frac{A}{t_s}(T_{1B} - T_{1T})$$

The energy balance for element iB (where i = 2, 3, or 4) is

$$0 = \varepsilon_B A\left(\sigma T_{iB}^4 - J_W\right) + k_x \frac{A_c}{w_e}(2T_{iB} - T_{(i-1)B} - T_{(i+1)B}) + h_B A(T_{iB} - T_A) + k_y \frac{A}{t_s}(T_{iB} - T_{iT})$$

The energy balance for element 5B is

$$0 = \varepsilon_B A\left(\sigma T_{5B}^4 - J_W\right) + k_x \frac{A_c}{w_e}(T_{5B} - T_{4B}) + h_B A(T_{5B} - T_A) + k_y \frac{A}{t_s}(T_{5B} - T_{5T})$$

Because the system is nominally two-dimensional, view factors are evaluated using two-dimensional analytical formulas.

The aforementioned equations form a 10X4 matrix, which we multiplied each side of the matrix equation by the transpose of this matrix, $A^T AX = A^T B$, allowing for reduction to a 4X4 system of equations with an equivalent solution through Gaussian elimination.

The transient energy balance for element 1T is

$$m_{1T} C_p \frac{T_{1T}^{i+1} - T_{1T}^{i-1}}{t_{i+1} - t_{i-1}} = \varepsilon_T A\left(\sigma T_{1T}^4 - J_P F_{1T-P} - J_W F_{1T-W}\right) + 2k_x \frac{A_c}{w_e}(T_{1T} - T_{2T}) + h_T A(T_{1T} - T_A) + k_y \frac{A}{t_s}(T_{1T} - T_{1B})$$

where the temperatures on the right hand side are at time *i.*

The transient energy balance for element iT (where i = 2, 3, or 4) is

$$m_{iT} C_p \frac{T_{iT}^{i+1} - T_{iT}^{i-1}}{t_{i+1} - t_{i-1}} = \varepsilon_T A\left(\sigma T_{iT}^4 - J_P F_{iT-P} - J_W F_{iT-W}\right) + k_x \frac{A_c}{w_e}(2T_{iT} - T_{(i-1)T} - T_{(i+1)T}) + h_T A(T_{iT} - T_A) + k_y \frac{A}{t_s}(T_{iT} - T_{iB})$$

The transient energy balance for element 5T is

$$m_{5T} C_p \frac{T_{5T}^{i+1} - T_{5T}^{i-1}}{t_{i+1} - t_{i-1}} = \varepsilon_T A\left(\sigma T_{5T}^4 - J_P F_{5T-P} - J_W F_{5T-W}\right) + k_x \frac{A_c}{w_e}(T_{5T} - T_{4T}) + h_T A(T_{5T} - T_A) + k_y \frac{A}{t_s}(T_{5T} - T_{5B})$$

The transient energy balance for element 1B is

$$m_{1B} C_p \frac{T_{1B}^{i+1} - T_{1B}^{i-1}}{t_{i+1} - t_{i-1}} = \varepsilon_B A\left(\sigma T_{1B}^4 - J_W\right) + 2k_x \frac{A_c}{w_e}(T_{1B} - T_{2B}) + h_B A(T_{1B} - T_A) + k_y \frac{A}{t_s}(T_{1B} - T_{1T})$$

The transient energy balance for element iB (where i = 2, 3, or 4) is

$$m_{iB}C_p \frac{T_{iB}^{i+1} - T_{iB}^{i-1}}{t_{i+1} - t_{i-1}} = \varepsilon_B A\left(\sigma T_{iB}^4 - J_W\right) + k_x \frac{A_c}{w_e}(2T_{iB} - T_{(i-1)B} - T_{(i+1)B}) + h_B A(T_{iB} - T_A) + k_y \frac{A}{t_s}(T_{iB} - T_{iT})$$

The transient energy balance for element 5B is

$$m_{5B}C_p \frac{T_{5B}^{i+1} - T_{5B}^{i-1}}{t_{i+1} - t_{i-1}} = \varepsilon_B A\left(\sigma T_{5B}^4 - J_W\right) + k_x \frac{A_c}{w_e}(T_{5B} - T_{4B}) + h_B A(T_{5B} - T_A) + k_y \frac{A}{t_s}(T_{5B} - T_{5T})$$

The transient set of equations creates a 10X1 matrix and the same solution method is used to determine the value for specific heat.

A.4 Solving the system of equations

Typically, the energy balance equations are solved to find the temperature distribution and/or history, so that there is one unknown temperature for each equation. Although the system is nonlinear, numerous techniques are available which can easily provide the steady or transient solution. What is needed here is sometimes described as solving the equations “backwards.” Knowing the temperatures, we need to determine the unknown values: k_x, k_y, C_p, and any others that are not directly measured.

First, the steady state equations are solved to determine conductivities. Afterwards, the transient equations are solved for specific heat. It was decided that the direct measurement of temperature, radiosity, and emissivity, and the subsequent calculation of conductivity, specific heat, and convection coefficients provides the most reasonable balance between cost and sensitivity. For example, while numerical calculation of radiosity from the other known data is possible, it would significantly increase sensitivity. The ten steady state equations and four unknowns (k_x, k_y, h_T, and h_B) constitute a 10 x 4 overdetermined system of equations. Overdetermined systems cannot, in general, be satisfied exactly, and thus the standard procedure is to solve the system in a least-squares sense. The Normal Equations method is used to find the least-squares solution to the system [1].

The equations are solved for both sizes of heat shields and the eigenvalues for the solutions are determined. Check the eigenvalues for both size shields and select the one that is the most stable to use for the property calculations.

A.5 Sensitivity studies

The sensitivity of the procedure to perturbations in the input data was studied. Such perturbations may be introduced through measurement errors or normal variability in the experimental setup from site to site. The sensitivity studies were performed analytically for a given specimen of known conductivities in one of two ways:

1. the steady equations are solved for the temperatures and radiosities,
2. a temperature or radiosity value is perturbed to simulate measurement error,
3. the equations are solved in a least-squares sense to estimate conductivities,

or

1. a geometric parameter is perturbed from its nominal value to simulate variability in the experimental setup,
2. the steady equations are solved for the temperatures and the radiosities,
3. the geometric parameters are returned to their nominal values,
4. the equations are solved in a least-squares sense to estimate conductivities.

The percent error in the estimated conductivities is the propagated error due to the perturbation. It is a measure of sensitivity to the given perturbation. For example, an inaccurate measurement of emissivity by 1% could cause a 5% error in the estimated values of conductivity.

The results of the steady-state sensitivity studies are presented in Tables A1-A20. The percentage shown at the top and bottom of each box is the error in computing k_y and k_x, respectively. For example, for a shield with $k_x = 0.5$ and $k_y = 0.1$, Table A1 shows that a temperature mis-measurement at location 1T will cause a 2.6% and 5.3% error in the calculation of k_y and k_x, respectively. For one who carefully follows the standard setup and tolerance requirements, a bound on the error is given in Tables A1-A10. Only the most sensitive (Table A1) and least sensitive (Table A2) elements to temperature measurement accuracy are shown. The average error and maximum error are shown in Tables A9 and A10, respectively. The purpose of Tables A12-A20 is to give the user some understanding of why parameters were specified the way they were, why tolerances are as small as they are, and why two shield specimens are required.

The results shown in each box of Tables A1-A10 & A14-A20 correspond to the width (22.5 cm or 45 cm) which is expected to have a lower sensitivity. The software from the Related References section 2.0 performs this step automatically by comparing the minimum eigenvalue of the normal equations [2] as soon as the data from both the 22.5 cm and the 45 cm specimens are available. Table A11 shows, for each case, which specimen width (22.5 cm or 45 cm) was chosen by the software. The propagated error using only a 22.5 cm specimen is provided in Table A12, whereas the propagated error using only a 45 cm specimen is given in Table A13. For large values of k_x, the error associated with the 45 cm specimen is much smaller than that associated with the 22.5 cm specimen. On the other hand, the error associated with the 22.5 specimen is less than that associated with the 45 cm specimen for large values of k_y and small values of k_x. Since the conductivities are unknown before testing, data from both a 22.5 cm and a 45 cm specimen are necessary to satisfy sensitivity requirements.

Tables A14 and A15 are provided to illustrate the importance of achieving the error tolerances required by this standard. First, very accurate RTD temperature sensors, which are accurate within ±0.12%, are required in Section 4.3. If the RTD sensors were replaced with less accurate type-K thermocouples, which are only accurate to the larger of ±1.1°C and ±0.375%, the material properties would be computed to a lower accuracy. This is shown in Table A14 which can be compared to Table A1. Second, the vertical distance from the top of the shield to the bottom of the heat source is required to be 25.4 mm ± 1.5 mm in Section 5.8

The user is required to know whether the specimen in question is isotropic or anisotropic. The large errors in Table A16 (as compared to Table A10) show why the anisotropic model is not usable for isotropic specimens.

The sensitivity also depends on the nominal values of various parameters: H, h_T, h_B, J_p, ε_T, and ε_B.

Thus a significant effort was made to use parameter design to reduce the sensitivity of the system. The results showed that, in general, sensitivity can be reduced by increasing ε_T or J_p. See Tables A17 and A18, respectively. Emissivity is a parameter because the specimen can be painted to achieve a specific value. Heat source radiosity J_p increases to the fourth power of T_p. The results further showed that as either H or convection coefficients are reduced the sensitivity is likewise reduced. See Tables A19 and A20, respectively. However, H was not reduced beyond 25.4 mm to keep the heat source thermal boundary layer sufficiently far from the specimen. The convection coefficients can be reduced, to that corresponding to natural convection from a heated plate, by minimizing forced airflow over the shield.

The analysis considers neither the error or sensitivity due to numerical truncation nor that due to neglecting spatial variation of convection coefficients.

A.6 References

[1] B. N. Datta, *Numerical Linear Algebra and Applications*, Pacific Grove, California: Brooks/Cole Publishing Company, 1995.

[2] J. V. Beck, *Parameter Estimation in Engineering and Science*, New York: John Wiley & Sons, 1977.

TABLE A1—PROPAGATED ERROR DUE TO A TEMPERATURE MISMEASUREMENT AT LOCATION 1T OF ±0.12%.

k_y k_x	0.1	0.5	2	10	40	200
0.1	3.6% 3.6%	2.6% 5.3%	1.7% 1.7%	1.0% 1.0%	0.7% 0.8%	0.1% 1.4%
0.5		4.8% 4.8%	3.2% 1.4%	1.8% 0.8%	1.5% 0.7%	1.3% 1.4%
2			7.8% 7.8%	4.8% 0.8%	3.8% 1.2%	3.1% 1.3%
10				13.7% 13.7%	13.9% 1.3%	10.5% 1.2%

TABLE A2—PROPAGATED ERROR DUE TO A TEMPERATURE MISMEASUREMENT AT LOCATION 2B OF ±0.12%.

k_y \ k_x	0.1	0.5	2	10	40	200
0.1	0.4% 0.4%	0.3% 0.1%	0.2% 0.0%	0.1% 0.0%	0.3% 0.0%	1.7% 0.1%
0.5		0.4% 0.4%	0.5% 0.1%	0.4% 0.0%	0.2% 0.1%	0.5% 0.2%
2			0.5% 0.5%	1.3% 0.0%	0.9% 0.1%	0.1% 0.2%
10				0.3% 0.3%	2.9% 0.2%	0.1% 0.1%

TABLE A3—PROPAGATED ERROR DUE TO A MISMEASUREMENT OF THE SHIELD THICKNESS BY ±0.127 MM.

k_y \ k_x	0.1	0.5	2	10	40	200
0.1	2.4% 2.4%	2.5% 2.5%	2.5% 2.5%	2.5% 2.5%	2.5% 2.5%	2.5% 2.5%
0.5		2.1% 2.1%	2.5% 2.5%	2.5% 2.5%	2.5% 2.5%	2.5% 2.5%
2			1.3% 1.3%	2.5% 2.5%	2.5% 2.5%	2.5% 2.5%
10				0.3% 0.3%	2.5% 2.5%	2.5% 2.5%

TABLE A4—PROPAGATED ERROR DUE TO SHIELD WIDTH (NOMINALLY 22.5 OR 45 CM) TOO LARGE OR TOO SMALL BY 1% (REDISTRIBUTING THE TEMPERATURE SENSORS)

k_y \ k_x	0.1	0.5	2	10	40	200
0.1	10.5% 10.5%	6.7% 25.9%	3.8% 8.5%	1.4% 3.7%	0.4% 2.8%	0.5% 2.2%
0.5		6.1% 6.1%	3.0% 7.5%	1.2% 3.4%	0.5% 2.6%	0.1% 2.2%
2			3.8% 3.8%	1.2% 3.3%	0.6% 2.6%	0.0% 2.1%
10				2.1% 2.1%	0.6% 2.6%	0.1% 2.1%

TABLE A5—PROPAGATED ERROR DUE TO HEAT SOURCE DIAMETER (NOMINALLY 50.8 MM) TOO LARGE OR TOO SMALL BY 0.127 MM.

k_y \ k_x	0.1	0.5	2	10	40	200
0.1	1.0% 1.0%	0.6% 2.2%	0.3% 0.5%	0.0% 0.1%	0.1% 0.1%	0.2% 0.1%
0.5		0.5% 0.5%	0.2% 0.4%	0.0% 0.0%	0.1% 0.1%	0.1% 0.1%
2			0.2% 0.2%	0.0% 0.0%	0.1% 0.1%	0.1% 0.1%
10				0.0% 0.0%	0.1% 0.1%	0.1% 0.1%

TABLE A6—PROPAGATED ERROR DUE TO SPECIMEN-TO-SOURCE DISTANCE (NOMINALLY 25.4 MM) TOO LARGE OR TOO SMALL BY 0.127 MM.

k_y \ k_x	0.1	0.5	2	10	40	200
0.1	3.3% 3.3%	2.1% 7.7%	1.3% 2.3%	0.6% 0.8%	0.4% 0.5%	0.1% 0.3%
0.5		1.9% 1.9%	1.1% 2.0%	0.6% 0.7%	0.4% 0.5%	0.2% 0.3%
2			1.2% 1.2%	0.6% 0.7%	0.4% 0.4%	0.3% 0.3%
10				0.6% 0.6%	0.4% 0.4%	0.3% 0.3%

TABLE A7—PROPAGATED ERROR DUE TO MISMEASUREMENT OF HEAT-SOURCE RADIOSITY TEMPERATURE BY ±0.5°C OR ±0.5% (WHICHEVER IS LARGER).

k_y k_x	0.1	0.5	2	10	40	200
0.1	3.3% 3.3%	2.3% 8.3%	1.8% 3.1%	1.4% 1.6%	1.3% 1.4%	1.3% 1.3%
0.5		2.1% 2.1%	1.5% 2.8%	1.3% 1.6%	1.3% 1.4%	1.2% 1.3%
2			1.8% 1.8%	1.3% 1.5%	1.3% 1.3%	1.2% 1.3%
10				1.4% 1.4%	1.3% 1.3%	1.2% 1.3%

TABLE A8—PROPAGATED ERROR DUE TO MISMEASUREMENT OF ROOM-WALL RADIOSITY TEMPERATURE BY ±3°C OR ±3% (WHICHEVER IS LARGER).

k_y k_x	0.1	0.5	2	10	40	200
0.1	2.0% 2.0%	0.5% 15.1%	0.4% 4.0%	0.2% 0.9%	0.2% 0.9%	0.2% 0.3%
0.5		1.9% 1.9%	0.6% 3.8%	0.3% 0.9%	0.3% 0.9%	0.2% 0.3%
2			1.4% 1.4%	0.3% 0.9%	0.2% 0.3%	0.2% 0.3%
10				0.6% 0.6%	0.2% 0.3%	0.2% 0.3%

TABLE A9—AVERAGE PROPAGATED ERROR WITH PRESENT SETUP.

k_y k_x	0.1	0.5	2	10	40	200
0.1	2.2% 2.2%	1.5% 4.7%	1.0% 1.4%	0.6% 0.6%	0.4% 0.5%	0.9% 0.5%
0.5		2.3% 2.3%	1.5% 1.3%	0.8% 0.6%	0.6% 0.5%	0.7% 0.5%
2			3.1% 3.1%	2.0% 0.6%	1.5% 0.5%	1.1% 0.5%
10				4.7% 4.7%	5.6% 0.5%	3.6% 0.5%

TABLE A10—MAXIMUM PROPAGATED ERROR USING PRESENT SETUP.

k_y k_x	0.1	0.5	2	10	40	200
0.1	10.5% 10.5%	6.7% 25.9%	3.8% 8.5%	1.4% 3.7%	1.3% 2.8%	4.2% 2.2%
0.5		6.1% 6.1%	3.2% 7.5%	1.8% 3.4%	1.5% 2.6%	2.4% 2.2%
2			7.8% 7.8%	4.8% 3.3%	3.8% 2.6%	3.4% 2.1%
10				13.7% 13.7%	13.9% 2.6%	11.7% 2.1%

TABLE A11—PREFERRED SPECIMEN WIDTH (IN CM) BASED ON EXPECTED SYSTEM SENSITIVITY.

k_y \ k_x	0.1	0.5	2	10	40	200
0.1	22.5	22.5	22.5	22.5	45	45
0.5		22.5	22.5	22.5	45	45
2			22.5	22.5	22.5	45
10				22.5	22.5	45

TABLE A12—MAXIMUM PROPAGATED ERROR USING ONLY A 22.5 CM WIDE SPECIMEN.

k_y \ k_x	0.1	0.5	2	10	40	200
0.1	10.5% 10.5%	6.7% 25.9%	3.8% 8.5%	1.4% 3.7%	3.6% 2.7%	47.2% 4.6%
0.5		6.1% 6.1%	3.2% 7.5%	1.8% 3.4%	2.3% 2.7%	15.0% 4.6%
2			7.8% 7.8%	4.8% 3.3%	3.8% 2.6%	8.6% 4.6%
10				13.7% 13.7%	13.9% 2.6%	14.5% 4.8%

TABLE A13—MAXIMUM PROPAGATED ERROR USING ONLY A 45 CM WIDE SPECIMEN.

k_y k_x	0.1	0.5	2	10	40	200
0.1	7.8% 7.8%	6.0% 40.9%	4.2% 12.3%	1.7% 4.5%	1.3% 2.8%	4.2% 2.2%
0.5		5.6% 5.6%	4.3% 10.7%	2.4% 4.1%	1.5% 2.6%	2.4% 2.2%
2			11.6% 11.6%	7.2% 3.9%	3.9% 2.5%	3.4% 2.1%
10				19.8% 19.8%	18.2% 2.5%	11.7% 2.1%

TABLE A14—PROPAGATED ERROR DUE TO A TEMPERATURE MISMEASUREMENT AT LOCATION 1T OF ±1.1°C OR ±0.375% (WHICHEVER IS GREATER).

k_y k_x	0.1	0.5	2	10	40	200
0.1	13.6% 13.6%	10.0% 20.5%	6.7% 7.1%	4.3% 4.3%	3.8% 4.2%	1.9% 9.3%
0.5		19.2% 19.2%	13.6% 6.5%	8.6% 4.2%	8.3% 4.1%	8.7% 9.9%
2			32.1% 32.1%	22.8% 4.4%	20.4% 8.2%	20.7% 9.9%
10				54.9% 54.9%	55.8% 8.9%	51.6% 9.9%

TABLE A15—PROPAGATED ERROR DUE TO SPECIMEN-TO-SOURCE DISTANCE (NOMINALLY 25.4 MM) TOO LARGE OR TOO SMALL BY 1.5875 MM.

k_y \ k_x	0.1	0.5	2	10	40	200
0.1	42% 42%	27% 98%	17% 29%	8% 10%	5% 6%	2% 4%
0.5		24% 24%	13% 25%	7% 9%	5% 6%	3% 4%
2			15% 15%	7% 8%	5% 5%	3% 4%
10				7% 7%	5% 5%	3% 4%

TABLE A16—MAXIMUM PROPAGATED ERROR OF AN ISOTROPIC SPECIMEN USING AN ANISOTROPIC ANALYTICAL MODEL.

k_y \ k_x	0.1	0.5	2	10	40	200
0.1	8% 118%					
0.5		5% 22%				
2			9% 7%			
10				20% 3%		

TABLE A17—MAXIMUM PROPAGATED ERROR USING AN UNPAINTED SPECIMEN HAVING AN EMISSIVITY OF 0.20 ON BOTH SIDES.

k_y k_x	0.1	0.5	2	10	40	200
0.1	17.8% 17.8%	6.9% 12.9%	2.8% 5.3%	1.8% 3.0%	1.8% 2.5%	10.0% 2.8%
0.5		8.9% 8.9%	5.3% 4.9%	2.9% 2.9%	2.6% 2.4%	5.3% 2.8%
2			11.4% 11.4%	8.5% 2.9%	7.7% 2.5%	7.7% 2.7%
10				23.0% 23.0%	27.4% 2.6%	27.6% 2.7%

TABLE A18—MAXIMUM PROPAGATED ERROR WITH HEAT SOURCE NOMINALLY SET AT 300°C, RATHER THAN 500°C.

k_y k_x	0.1	0.5	2	10	40	200
0.1	6.5% 6.5%	6.1% 36.9%	3.2% 10.3%	1.7% 3.4%	1.7% 2.8%	8.1% 2.3%
0.5		8.6% 8.6%	4.6% 10.0%	2.5% 3.2%	2.3% 2.8%	4.3% 3.2%
2			10.8% 10.8%	7.2% 3.1%	6.1% 2.6%	6.1% 2.2%
10				19.9% 19.9%	21.9% 2.5%	21.6% 2.2%