SAE Aerospace
An SAE International Group

AEROSPACE RECOMMENDED PRACTICE

SAE ARP4990

Issued 1997-09
Reaffirmed 2007-11

TURBINE FLOWMETER FUEL FLOW CALCULATIONS

TABLE OF CONTENTS

TO PLACE A DOCUMENT ORDER: Tel: 877-606-7323 (inside USA and Canada)
Tel: 724-776-4970 (outside USA)
Fax: 724-776-0790
Email: CustomerService@sae.org
SAE WEB ADDRESS: http://www.sae.org

TABLE OF CONTENTS (Continued)

TABLE OF CONTENTS (Continued)

1. SCOPE:

This SAE Aerospace Recommended Practice (ARP) provides to the aerospace industry a procedure for the consistent and accurate calculation of fuel flow using turbine flowmeters during development, production, and post overhaul/repair gas turbine engine testing.

2. REFERENCES:

2.1 Applicable Documents:

The following publications form a part of this specification to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this specification and references cited herein, the text of this specification takes precedent. Nothing in this specification, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

2.1.1 SAE Publications: Available from SAE, 400 Commonwealth Drive, Warrendale, PA 15096-0001.

ARP741 Turbofan and Turbojet Gas Turbine Engine Test Cell Correlation

2.1.2 API Publications: Available from API, 2101 L Street, Northwest, Washington, DC 20037.

Manual of Petroleum Measurement Standards, Ch. 4: Proving Systems

Manual of Petroleum Measurement Standards, Ch. 11.2.1: Compressibility Factors for Hydrocarbons: 0-90 API Gravity Range

Manual of Petroleum Measurement Standards, Ch. 12.2.5.1: Correction for the Effect of Temperature on Steel, Cts

Manual of Petroleum Measurement Standards, Ch. 12.2.5.2: Correction for the Effect of Pressure on Steel, Cps

Technical Data Book - Petroleum Refining, API Procedure 11A5.7, Liquid Viscosity of High-Molecular-Weight Pure and Mixed Hydrocarbons at High Pressure

2.1.3 ASTM Publications: Available from ASTM, 100 Barr Harbor Drive, West Conshohocken, PA 19428-2959.

ASTM D 240	Standard Test Method for Heat of Combustion of Liquid Hydrocarbon Fuels by Bomb Calorimeter
ASTM D 287	Standard Test Method for API Gravity of Crude Petroleum and Petroleum Products (Hydrometer Method)
ASTM D 341	Standard Viscosity-Temperature Charts for Liquid Petroleum Products
ASTM D 445	Standard Test Method for Kinematic Viscosity of Transparent and Opaque Liquids (and the Calculation of Dynamic Viscosity)
ASTM D 1018	Standard Test Method for Hydrogen in Petroleum Fractions
ASTM D 1217	Standard Test Method for Density and Relative Density (Specific Gravity) of Liquids by Bingham Pycnometer
ASTM D 1250	Standard Guide for Petroleum Measurement Tables, Volume X: Background, Development, and Implementation Procedures
ASTM D 1298	Standard Practice for Density, Relative Density (Specific Gravity), or API Gravity of Crude Petroleum and Liquid Petroleum Products by Hydrometer Method
ASTM D 2382	Standard Test Method for Heat of Combustion of Hydrocarbon Fuels by Bomb Calorimeter (High Precision Method)
ASTM D 3701	Standard Test Method for Hydrogen Content of Aviation Turbine Fuels by Low Resolution Nuclear Magnetic Resonance Spectrometry
ASTM D 4052	Standard Test Method for Density and Relative Density of Liquids by Digital Density Meter
ASTM D 4809	Standard Test Method for Heat of Combustion of Liquid Hydrocarbon Fuels by Bomb Calorimeter (Intermediate Precision Method)

2.1.4 Coordinating Research Council, Inc., Aviation Fuel Properties, CRC Report No. 530, May, 1988.

2.1.5 Craft, D. William, High Accuracy Fuel Flowmeter - Final Report - Phase IIC and Phase III, The Mass Flowrate Calibration of High Accuracy Fuel Flowmeters, NASA CR 187108, February, 1992.

2.1.6 Grabe, W., Fuel Flow Measurement in Gas Turbine Testing, National Research Council of Canada, TR-ENG-001, NRC No. 29808, 1988/08.

2.1.7 Hochreiter, H. M., Dimensionless Correlation of Coefficients of Turbine-Type Flowmeters, ASME Paper No. 57-A-63, Transactions of the ASME, October 1958.

2.1.8 IEEE Std. 268 - 1982, IEEE Standard Metric Practice, Institute of Electrical and Electronics Engineers, 1982.

2.1.9 Mattingly, G. E., The Characterization of a Piston Displacement-Type Flowmeter Calibration Facility and the Calibration and Use of Pulsed Output Type Flowmeters, Journal of Research of the National Institute of Standards and Technology, Volume 97, Number 5, September-October 1992, pp. 509-531.

2.1.10 Olivier, Paul D., Determination of Turbine Flowmeter Usable Turndown, Proceedings of National Conference of Standards Laboratories Workshop and Symposium, July 1995.

2.1.11 Ruffner, Donald F, and Olivier, Paul D., Improved Turbine Meter Accuracy by Utilization of Dimensionless Data, Proceedings of National Conference of Standards Laboratories Workshop and Symposium, August 1994.

2.1.12 Schoonover, Randall M. and Jones, Frank E., Air Buoyancy Correction in High-Accuracy Weighing on Analytical Balances, Journal of Analytical Chemistry, Volume 53, Number 6, May 1981, pp. 900-902.

2.1.13 Shafer, M. R. and Ruegg, F. W., Liquid-Flowmeter Calibration Techniques, ASME Paper No. 57-A-70, Transactions of the ASME.

3. NOMENCLATURE:

API	American Petroleum Institute
ASTM	American Society for Testing and Materials
B_o	buoyancy adjustment to convert from "in-vacuo" to "in-air" (dimensionless)
C_{pfd}	pressure correction on fuel density (dimensionless)
C_{pfv}	pressure correction on fuel viscosity (dimensionless)
C_{pk}	pressure correction factor due to flowmeter expansion to obtain K_{op} (dimensionless)
C_{pr}	pressure correction factor due to flowmeter expansion to obtain Ro_{op} (dimensionless)
C_{tk}	temperature correction factor due to flowmeter expansion to obtain K_{op} (dimensionless)
C_{tr}	temperature correction factor due to flowmeter expansion to obtain Ro_{op} (dimensionless)
D	bore diameter of flowmeter (in or m), measured at T_{cal}
E	modulus of elasticity of the flowmeter meter material (psi or bar)
f	flowmeter frequency (Hz or cycles/s)
F	compressibility factor of the fuel (1/psia or 1/bar-a)
HYC	stem correction for glass hydrometer thermal expansion (dimensionless)
K, K-factor	flowmeter coefficient (cycles/US-gal or cycles/L)
K_{op}	K-factor at operating conditions (cycles/US-gal or cycles/L)
LHV	measured lower heating value (Btu/lb or kJ/kg)
LHV_{cf}	lower heating value correction factor (dimensionless)
LHV_{ref}	reference lower heating value (Btu/lb or kJ/kg)
p_{atm}	atmospheric pressure at operating conditions (psia or bar-a)
$p_{atm, cal}$	atmospheric pressure during flowmeter calibration (psia or bar-a)
p_{cal}	absolute pressure of fuel at the flowmeter during calibration (psia or bar-a)
p_{eq}	equilibrium pressure of the fuel (psia or bar-a)
p_{gage}	gage pressure of fuel at the flowmeter during operation (psig or bar-g)
p_{op}	absolute pressure of fuel at the flowmeter during operation (psia or bar-a)
Q	fuel volume flow rate (US-gal/s or L/s)
RD	relative density formerly referred to as specific gravity (dimensionless)
RF	radio frequency
Ro	Roshko number (dimensionless)
Ro_{op}	Roshko number at operating conditions (dimensionless)

3. (Continued):

St	Strouhal number (dimensionless)
St_{op}	Strouhal number at operating conditions (dimensionless)
t	wall thickness of the flowmeter (in or m)
T_{cal}	calibration temperature of the flowmeter (°F or °C)
T_{ref}	reference temperature (60 °F or 15 °C)
T_{op}	operating temperature of the fuel at the flowmeter (°F or °C)
$T_{op,abs}$	absolute operating temperature of the fuel at the flowmeter (°R or °K)
T_s	temperature of the fuel sample (°F or °C)
$T_{s,abs}$	absolute temperature of the fuel sample (°R or °K)
UVC	universal viscosity curve
VCF	volume correction factor (dimensionless)
$W_{f,t}$	true fuel mass flow rate (lbm/h or kg/h)
$W_{f,LHV}$	LHV corrected fuel mass flow rate (lb/h or kg/h), i.e., true fuel mass flow rate referenced to a standard heat of combustion

3.1 Greek Symbols:

$\alpha_{F,T_{ref}}$	coefficient of thermal expansion of the fuel at reference temperature (T_{ref}) (1/°F or 1/°C)
α_l	coefficient of linear thermal expansion of the flowmeter (1/°F or 1/°C)
δ	(observed inlet total absolute pressure) / (absolute pressure of ISO sea level standard day reference atmosphere) (dimensionless)
μ_s	dynamic viscosity of the fuel sample (centipoise)
ν_{op}	kinematic viscosity of the fuel sample corrected for operating temperature (T_{op}) and pressure (p_{op}) (cSt)
ν_s	kinematic viscosity of the fuel sample (cSt)
ν_{Top}	kinematic viscosity of the fuel sample corrected for operating temperature (T_{op}) (cSt)
θ	(observed inlet total absolute temperature) / (absolute temperature of ISO sea level standard day reference atmosphere) (dimensionless)
ρ_a	air density (kg/m^3)
ρ_{op}	density of the fuel corrected for operating temperature (T_{op}) and pressure (p_{op}) (kg/m^3)
ρ_s	fuel sample density (kg/m^3)
$\rho_{s,hyc}$	fuel sample density corrected for glass hydrometer thermal expansion (kg/m^3)
ρ_{Top}	density of the fuel corrected for operating temperature (T_{op}) (kg/m^3)
$\rho_{Top,gcc}$	density of the fuel corrected for operating temperature (T_{op}) (g/cm^3)
ρ_{ref}	density of the fuel at reference temperature (T_{ref}) (kg/m^3)
$\rho_{ref,gcc}$	density of the fuel at reference temperature (T_{ref}) (g/cm^3)
ρ_w	balance weight density (kg/m^3)
$\rho_{water,ref}$	density of water at reference temperature in vacuo (T_{ref}) (kg/m^3)

4. BACKGROUND:

A variety of fuel flow determination methods have evolved throughout the aerospace industry. Most fuel property and flow measurement methods as well as temperature/pressure corrections have been based on established techniques published in the literature. Often, however, notable differences in measured fuel properties and calculated fuel flow have become apparent upon comparison of results between parties. A portion of these differences could be justifiably attributed to experimental uncertainty and the reproducibility of the measurement technique in different laboratories. Other disparities, due to differences in the measurement and/or correction techniques, have resulted in considerable time and effort being devoted to the justification and/or validation of each facility's methodology.

A procedure adopted by the entire industry would provide a common basis for the consistent and accurate calculation of fuel flow and help alleviate points of confusion or inconsistency. As such, the procedure defined herein is based on ASTM and API standard test methods as well as temperature and pressure correction methodologies and can be used to establish the physical quantities required in the determination of fuel flow such as relative density, viscosity, lower heating value and for calibrating turbine flowmeters.

5. INTRODUCTION:

5.1 True Fuel Mass Flow Rate:

The basic building blocks for determining fuel flow with volumetric flowmeters are shown in Equation 1. A mass per unit volume quantity (density) is multiplied by a volume per unit time quantity (volume flow rate) to obtain true mass per unit time (true mass flow rate).

$$\frac{\text{mass}}{\text{volume}} * \frac{\text{volume}}{\text{time}} = \frac{\text{mass}}{\text{time}} \qquad \text{(Eq.1)}$$

or similarly,

$$\text{density} * \text{volume flow rate} = \text{true mass flow rate} \qquad \text{(Eq.2)}$$

Fuel sample density is measured and/or reported at a reference temperature and pressure. If the fuel is subjected to conditions at the flowmeter during engine operation that are different from those which existed at reference conditions, then temperature and/or pressure corrections should be applied to the density term. The corrections to fuel density are not needed when an in-line digital density meter is used and is located sufficiently close to the flowmeter so that the fuel can be assumed to be at the same temperature and pressure at both the digital density meter and the flowmeter.

5.1 (Continued):

The volume flow rate is obtained using the flowmeter's calibration curve. The calibration curve is valid at the temperature and pressure of calibration for the flowmeter. However, during engine operation, if the flowmeter is subjected to conditions different from those at calibration, then appropriate temperature and pressure corrections to account for changes in flowmeter geometry should be applied to the calibration data.

Also, as discussed in 5.3, the calibration curve is a function of the fluid viscosity, which must be known to properly use the curve. Temperature and pressure corrections should be applied to determine fluid viscosity at engine operating conditions. The viscosity corrections are not needed when an in-line viscometer is used and is located sufficiently close to the flowmeter.

The product of density and volume flow rate then yields the true mass flow rate of the fuel.

5.2 LHV Corrected Fuel Mass Flow Rate:

In order to normalize engine performance data to a reference fuel heat content, a lower heating value correction (LHV_{cf}) may be applied to the true mass flow rate as in Equation 3.

$$\text{LHV corrected mass flow rate} = \text{true mass flow rate} * LHV_{cf} * B_o \qquad \text{(Eq.3)}$$

LHV_{cf} is defined as the ratio of measured LHV to a reference LHV. This correction is necessary because the LHV of fuel from different sources may vary significantly.

An additional correction for buoyancy (B_o) is also required when the true mass flow rate is corrected for the measured LHV of the fuel. When lower heating value measurements are made following any of the recommended test methods, the amount of fuel used is weighed "in air." However, fuel density, which is mass per unit volume, may or may not be referenced to "in vacuo" conditions. Therefore, a buoyancy adjustment (B_o) should be applied to ensure consistent reference to "in air" conditions for both LHV and fuel density, when density is referenced to "in vacuo" conditions.

NOTE: Fuel flow is also typically corrected to standard day conditions for pressure (δ) and temperature (θ^x), where the θ-exponent (x) is empirically determined by the engine manufacturer during development testing and corrects for the effect of different gaspath temperatures on specific heat. The reader should refer to SAE ARP741 (2.1.1) for guidelines on making δ and θ corrections to fuel flow rate. Some thermodynamic performance models use LHV, θ and δ in the determination of the thermodynamic mass flow rate. Any discussion of such performance models is beyond the scope of this document. This document is only applicable to direct fuel flow measurements using turbine flowmeters.

5.3 Turbine Flowmeter Calibration and Corrections:

The turbine flowmeter calibration process is critical in determining the volume flow rate during actual operation. The traditional representation of turbine flowmeter characteristics has been the K-factor curve illustrated in Figure 1. K-factor (cycles/US-gal or cycles/L) is shown as a function of flowmeter output frequency divided by kinematic viscosity (f/ν). The Universal Viscosity Curve (UVC) is a K-factor versus f/ν curve generated from calibration data of a particular flowmeter for fluids at several viscosities.

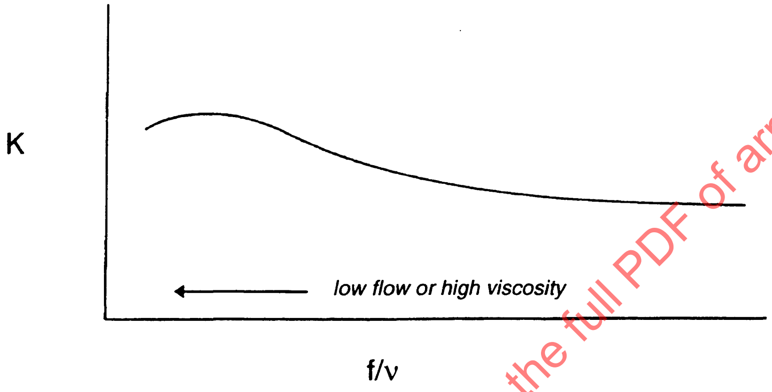

FIGURE 1 - K-factor Calibration Curve

Section 7.2 describes the use of two nondimensional parameters, Strouhal number (St) versus Roshko number (Ro), to quantify the flow characteristics of a turbine flowmeter. As will be explained in 7.2, the K-factor versus f/ν plot (or, the UVC) is a special case of the correlation of the Strouhal number versus the Roshko number. The calibration curve is valid at the calibration conditions, and so appropriate temperature and pressure corrections must be made to account for the effect of actual operating conditions on turbine flowmeter geometry and kinematic viscosity.

In general, because of the nonlinearity of turbine flowmeter characteristics, the flowmeters should not be operated beyond the range of calibration, i.e., the characteristic plot should not be extrapolated. Even with all the corrections applied, special care should be exercised at low flow or high viscosity conditions, both of which correspond to low Ro number. The concept of the UVC, and of the St-Ro plot, gives an impression that a turbine flowmeter, calibrated at one viscosity, can be used at any other viscosity as long as the operating Ro number falls within the calibration range. However, the UVC and St-Ro plot have a break-away point as shown in Figure 2, which defines the lower limit of the “usable turn-down region,” for each viscosity (2.1.10). The break-away point can be determined from plots of calibration data at different viscosities.

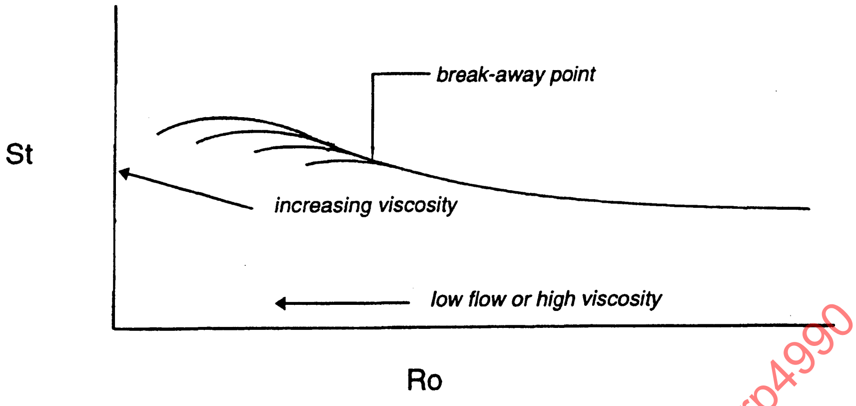

FIGURE 2 - Break-away Point for a Strouhal-Roshko Calibration Curve

5.3 (Continued):

Use of a flowmeter below the break-away point, at viscosities and/or temperatures substantially different from those at which the meter was calibrated, can lead to serious errors in flow measurements.

5.4 Fuel Flow Calculation Summary:

In general terms, the basis of calculating fuel flow is defined by Equation 2, where the true mass flow rate is equal to the product of fuel density and volume flow rate through the turbine flowmeter. After all the appropriate fuel properties are measured and corrected for pressure and temperature, Equation 4 can be used to calculate true mass flow rate ($W_{f,t}$).

$$W_{f,t} = 3600 * (f / K_{op}) * 8.345404 \text{ E-3} * (\rho_{ref} * VCF * C_{pfd}) \text{ lbm/h} \qquad \text{(US)} \qquad \text{(Eq.4)}$$

$$W_{f,t} = 3600 * (f / K_{op}) * 1.0 \text{ E-3} * (\rho_{ref} * VCF * C_{pfd}) \text{ kg/h} \qquad \text{(SI)}$$

Throughout this document, equations are provided in both US customary units (US) and SI units where appropriate. Appendix C contains the units conversion factors used. In both of the previous equations, 3600 is a time conversion of frequency, f, from Hz to cycles/h. In Equation 4 (US), 8.345404 E-3 is a density conversion of ρ_{ref} from kg/m^3 to lbm/US-gal yielding fuel mass flow rate in US customary units of lbm/h. In Equation 4 (SI), 1.0 E-3 is a density conversion of ρ_{ref} from kg/m^3 to kg/L yielding fuel flow rate in SI units of kg/h. Different units conversion factors may have to be used if the units of the measured parameters are different from those given in this document.

5.4 (Continued):

In order to normalize engine performance data to a reference fuel heat content, fuel flow rate is corrected both to a standard lower heating value (LHV_{cf}, 6.3) and for buoyancy effects (B_o, 6.4). Equation 5 can be used to calculate LHV corrected fuel mass flow rate ($W_{f,LHV}$).

$$W_{f,LHV} = W_{f,t} * LHV_{cf} * B_o \qquad \text{(Eq.5)}$$

5.5 Document Summary:

Section 6 describes the basis for each parameter in Equations 4 and 5 including procedures for the measurement of relative density, viscosity and lower heating value and the appropriate temperature and pressure corrections for relative density and viscosity.

Section 7 discusses turbine flowmeter calibration characteristics.

Section 8 recommends appropriate use of the fuel flow calculation process.

Table A1 of Appendix A lists the recommended ASTM and API measurement methods along with appropriate temperature and pressure correction methods for fuel properties in Table A2. The repeatability and reproducibility of the various test methods may be found in Table A3 of Appendix A for easy reference and comparison.

Finally, the sample calculation in Appendix B and the equations identified throughout this document are provided in both US customary and SI units to facilitate implementation of the recommended methodology.

6. FUEL PROPERTY MEASUREMENTS AND CORRECTIONS:

6.1 Density:

The density of a fuel sample is measured at a given temperature and pressure. During actual engine operation in flight or in a test cell environment, fuel passing through the turbine flowmeter may be at a significantly different temperature and/or pressure. Hence, it is not only necessary to measure the density of the fuel sample, but also to correct the measured value for the fuel temperature and pressure that exists during actual engine operation.

6.1.1 Density: Measurement:

Fuel density can be measured using the following devices:

a.	Digital Density Meter:	ASTM D 4052 (2.1.3)
b.	Bingham Pycnometer:	ASTM D 1217 (2.1.3)
c.	Hydrometer:	ASTM D 1298 (2.1.3)

6.1.1 (Continued):

Both the digital density meter and pycnometer measure fuel sample density (ρ_{ref}) at the reference temperature (T_{ref} = 60 °F or 15 °C). The Bingham Pycnometer, although most accurate of the three, is however a much more labor intensive device. If an in-line digital density meter is placed sufficiently close to the flowmeter to measure fuel density at actual engine operating conditions, then the corrections for operating temperature and pressure discussed in the next two sections are not needed.

Both the hydrometer and the digital density meter can be used for measuring fluid relative density ($RD_{Ts/Tref}$), formerly referred to as specific gravity in the literature. Relative density of the fuel sample is defined by Equation 6 (ASTM D 1250, 2.1.3) as the ratio of the fuel sample density (ρ_s) at the sample temperature (T_s), to the density of water at the reference temperature (T_{ref}). When the relative density of the fuel sample is measured, it must be converted to density (ρ_s) using Equation 6.

$$RD_{Ts/Tref} = \rho_s / \rho_{water,ref} \qquad \text{(Eq.6)}$$

where:

$\rho_{water,60F}$ = 999.012 kg/m^3 in vacuo (ASTM D 1250, 2.1.3)
$\rho_{water,15C}$ = 999.103 kg/m^3 in vacuo (ASTM D 1250, 2.1.3)

When a borosilicate glass hydrometer is used to measure fuel sample relative density, a stem correction factor for hydrometer thermal expansion (HYC) shown in Equation 8 should be applied to the sample density (ρ_s) as in Equation 9. The stem correction in Equation 8 is applicable to borosilicate glass hydrometers. The corrected sample density ($\rho_{s,HYC}$) is then referenced to T_{ref} using the iterative procedure illustrated in Appendix B.3 to obtain ρ_{ref}.

$$\Delta T_s = T_s - T_{ref} \qquad \text{(Eq.7)}$$

$$\begin{aligned} HYC &= 1 - (1.278\text{ E-5} * \Delta T_s) - (6.2\text{ E-9} * \Delta T_s^{\,2}) && \text{(US)} \qquad \text{(Eq.8)} \\ &= 1 - (2.3\text{ E-5} * \Delta T_s) - (2\text{ E-8} * \Delta T_s^{\,2}) && \text{(SI)} \end{aligned}$$

$$\rho_{s,HYC} = \rho_s * HYC \qquad \text{(Eq.9)}$$

The repeatability and reproducibility of each of the measurement methods discussed above is listed in Table A3 of Appendix A and allows users to quantify differences in measurement method results.

6.1.2 Density: Temperature Correction:

If the fuel temperature at the flowmeter during engine operation is different from the reference temperature, then a correction is made to the sample density measurement (ρ_{ref}) to account for the effect of actual fuel operating temperature (T_{op}) on density. This correction is not necessary if an in-line digital density meter is used.

6.1.2 (Continued):

A number of empirical correlations [e.g., 2.1.6] assume a constant coefficient of thermal expansion, i.e., a constant slope when density is plotted versus temperature, independent of the fuel density (ρ_{ref}). In actuality, the coefficient of thermal expansion varies not only with temperature, but also with ρ_{ref}. Therefore, assuming a constant coefficient of thermal expansion will produce less accurate results. ASTM D 1250 (2.1.3) defines the temperature correction for fuel density which accounts for the effect of actual fuel operating temperature (T_{op}) as well as the fuel sample density (ρ_{ref}) at the reference temperature (T_{ref}).

As seen in Equation 11, the fuel's thermal expansion coefficient ($\alpha_{F,T_{ref}}$) is a function of the fuel sample density (ρ_{ref}) at the reference temperature. K_0 and K_1 are fuel dependent constants that can be obtained from ASTM D 1250 (2.1.3).

$$\Delta T = T_{op} - T_{ref} \qquad \text{(Eq.10)}$$

$$\alpha_{F,T_{ref}} = K_0/(\rho_{ref}^2) + K_1/\rho_{ref} \qquad \text{(Eq.11)}$$

The thermal expansion coefficient ($\alpha_{F,T_{ref}}$) is used in conjunction with the fuel's operating temperature in the determination of the volume correction factor (VCF) in Equation 12.

$$VCF = \exp(- \alpha_{F,T_{ref}} * \Delta T * \{1 + 0.8 * \alpha_{F,T_{ref}} * \Delta T\}) \qquad \text{(Eq.12)}$$

Equation 12 is an empirical fit to density data as a function of temperature. Table A4 of Appendix A contains the predicted precision of thermal expansion properties at the 95 percent confidence level from ASTM D 1250 (2.1.3). Lighter fuels expand more with temperature than heavier fuels. If computational difficulties are encountered while using Equation 12, the exponential term on the right hand side of the equation may be expanded in a power series, and six or seven terms of the series may be retained to get the desired accuracy.

The volume correction factor (VCF) can then be multiplied with ρ_{ref} to obtain fuel density corrected for temperature (ρ_{Top}) as in Equation 13.

$$\rho_{Top} = \rho_{ref} * VCF \qquad \text{(Eq.13)}$$

Vol. VI of ASTM D 1250 (2.1.3) also provides an alternative method for greater accuracy in determining the fuel's thermal expansion coefficient ($\alpha_{F,T_{ref}}$) in lieu of using the constants K_0 and K_1 in Equation 11. The relationship between VCF and ΔT for a given fuel composition can be established by measuring fuel sample densities at a recommended minimum of 10 different temperatures and dividing each by ρ_{ref}. With VCF and ΔT known, Equation 12 can be used to empirically determine the relationship between $\alpha_{F,T_{ref}}$ and ΔT. Subsequently, for any ΔT, Equation 12 can be used in conjunction with the empirically determined relationship between $\alpha_{F,T_{ref}}$ and ΔT, to obtain the VCF. This alternative method allows users to establish their own density versus temperature relationship which is valid for a given fuel composition.

6.1.3 Density: Pressure Correction:

If elevated fuel pressures exist at the flowmeter during engine operation, then the fuel is susceptible to compressibility effects and a pressure correction should be applied to the density term. However, when line pressures are relatively low, the pressure correction is insignificant. Ch. 11.2.1 of the API Manual of Petroleum Measurement Standards (2.1.2) defines the pressure correction for fuel density. This correction is not necessary if an in-line digital density meter is used.

From Equation 16, a compressibility correction factor (F) is calculated based on the fuel sample density at the reference temperature ($\rho_{ref,gcc}$) from Equation 14, equilibrium pressure (p_{eq}) from Equation 15, and measured operating temperature (T_{op}).

$$\rho_{ref,gcc} = \rho_{ref} / 1000 \qquad \text{(Eq.14)}$$

$$p_{eq} = 0.145 * (10^{6.284 - 1991.3 / ((Top + 459.67)/1.8)}) \qquad \text{(US)} \qquad \text{(Eq.15)}$$

$$= 0.010 * (10^{6.284 - 1991.3 / (Top + 273.15)}) \qquad \text{(SI)}$$

$$F = (\exp(- 1.9947 + 0.00013427 * T_{op} + 0.79392/\{\rho_{ref,gcc}{}^2\} + 0.002326 * T_{op} /\{\rho_{ref,gcc}{}^2\})) / 100000 \qquad \text{(US)} \qquad \text{(Eq.16)}$$

$$= (\exp(- 1.62080 + 0.00021592 * T_{op} + 0.87096/\{\rho_{ref,gcc}{}^2\} + 0.0042092 * T_{op} /\{\rho_{ref,gcc}{}^2\})) / 10000 \qquad \text{(SI)}$$

The equilibrium pressure (p_{eq}) correlation of Equation 15 was determined from digitizing the Jet-A fuel curve from 2.1.4. Similar relationships may be established for other types of fuel, or from actual measurements.

The compressibility factor (F) is used to obtain the pressure correction factor (C_{pfd}) in Equation 17 by correcting the fuel first to ambient pressure (p_{atm}) and then to the elevated operating pressure (p_{op}) that exists during engine test operating conditions.

$$C_{pfd} = (1 - F * (p_{atm} - p_{eq})) / (1 - F * (p_{op} - p_{eq})) \qquad \text{(Eq.17)}$$

The pressure correction factor (C_{pfd}) can then be multiplied with the fuel density corrected to operating temperature (ρ_{Top}) to obtain fuel density corrected to both temperature and pressure (ρ_{op}) as in Equation 18.

$$\rho_{op} = \rho_{Top} * C_{pfd} \qquad \text{(Eq.18)}$$

6.2 Kinematic Viscosity:

Fuel viscosity has a significant effect on the operational characteristics of a turbine flowmeter. The flowmeter calibration curve discussed in Section 7.2 accounts for viscosity dependent flowmeter characteristics. Since kinematic viscosity also varies with temperature and pressure, appropriate corrections must be made during the flowmeter calibration process and for actual engine operating conditions. These corrections are not necessary if an in-line viscometer is used.

6.2.1 Kinematic Viscosity: Measurement:

Kinematic viscosity can be measured using the following device:

a. Viscometer: ASTM D 445 (2.1.3)

The kinematic viscosity of the fuel sample (ν_{s1} and ν_{s2}) must be measured at a minimum of two different temperatures (T_{s1} and T_{s2}) to establish the relationship between viscosity and temperature (see below). This relationship is valid for a given fuel composition. It must be re-established for different fuel types/compositions and if the source of a given fuel type is changed.

6.2.2 Kinematic Viscosity: Temperature Correction:

As in the case of fuel density, a correction must be applied to the kinematic viscosity to account for the effect of elevated fuel temperatures that may exist during actual engine operation. ASTM D 341 (2.1.3) defines the temperature correction for kinematic viscosity. This method, intended for use in computer programs, provides the most accurate correction. Where a small reduction in accuracy can be tolerated, a kinematic viscosity-temperature chart can be used to determine the corrected kinematic viscosity once two measured values are plotted. The mathematical relationship in ASTM D 341 is the preferred method for data reduction. This temperature correction is not necessary if an in-line viscometer is used.

The general logarithmic relationship between viscosity and temperature is shown in Equation 21, where the y-intercept (A) and slope (B) must be established for a given fuel composition.

$$T_{s,abs} = T_s + 459.67 \quad \text{(US)} \qquad \text{(Eq.19)}$$
$$= T_s + 273.15 \quad \text{(SI)}$$

$$T_{op,abs} = T_{op} + 459.67 \quad \text{(US)} \qquad \text{(Eq.20)}$$
$$= T_{op} + 273.15 \quad \text{(SI)}$$

$$\log \log Z_s = A - B \log T_{s,abs} \qquad \text{(Eq.21)}$$

$$Z_s = \nu_s + 0.7 + \exp\{-1.47 - 1.84^*(\nu_s) - 0.51^*(\nu_s^2)\} \qquad \text{(Eq.22)}$$

In Equation 22, for $\nu_s \geq 2.0$ cSt, the exponential term on the right hand side takes on values ≤ 0.00075. Thus, a simpler version of the equation may be used for $\nu_s \geq 2.0$ cSt.

$$Z_s \approx \nu_s + 0.7 \qquad \text{for } \nu_s \geq 2.0 \qquad \text{(Eq.23)}$$

The straight line governed by Equation 21, has a slope given by B, and an off-set given by A. At least two viscosities, ν_{s1} and ν_{s2}, are required to be measured at two different temperatures ($T_{s1,abs}$ and $T_{s2,abs}$) from the same sample in order to evaluate B using Equation 24 and A using Equation 25.

$$B = \{\log (\log Z_{s1} / \log Z_{s2}) / \log (T_{s2,abs} / T_{s1,abs})\} \qquad \text{(Eq.24)}$$

$$A = \log \log Z_{s1} + B \log T_{s1,abs} \qquad \text{(Eq.25)}$$

Once established, either B, or A, or both B and A, can be used as constants if subsequent measurements show them to be constant within a tolerance band, and/or it provides acceptable accuracy for that particular application. Periodic checks are recommended to verify fuel consistency.

In the case where both B and A are assumed constant, only the fuel operating temperature (T_{op}) is required to determine kinematic viscosity (ν_{Top}) at the actual operating temperature (T_{op}) using Equations 26 through 29.

$$Z_{Top} = (10)^{(10)^{(A - B \log Top,abs)}} \qquad \text{(Eq.26)}$$

$$AZ_{Top} = Z_{Top} - 0.7 \qquad \text{(Eq.27)}$$

$$\nu_{Top} = AZ_{Top} - \exp(- 0.7487 - 3.295{*}AZ_{Top} + 0.6119{*}AZ_{Top}{}^{2} - 0.3193{*}AZ_{Top}{}^{3})) \qquad \text{(Eq.28)}$$

In Equation 28, for $AZ_{Top} \geq 2.0$, the exponent term on the right hand side takes on values ≤ 0.00058. Thus, a simpler version of the equation may be used for $AZ_{Top} \geq 2.0$.

$$\nu_{Top} \approx AZ_{Top} \qquad \text{for } AZ_{Top} \geq 2.0 \qquad \text{(Eq.29)}$$

In the case where only the slope (B) of the kinematic viscosity - temperature relationship is assumed constant within a tolerance band and/or it provides acceptable accuracy for that particular application, then the y-intercept (A) must be determined. Kinematic viscosity of the fuel sample must first be measured and then used in Equation 22 or 23 to calculate Z_s. To minimize the effect of any slope variation, it is recommended that the sample be measured at a temperature near the actual operating temperature (T_{op}). With Z_s and B known, the y-intercept (A) is calculated from Equation 25. Equations 26 through 29 are then used as above to determine kinematic viscosity (ν_{Top}) at the actual operating temperature (T_{op}).

NOTE: Equation 21 is a straight line when plotted on a log-log scale, and thus it allows for extrapolation of data beyond the range over which the relationship is determined, i.e., beyond the range defined by $T_{s1,abs}$ and $T_{s2,abs}$. However, in order to minimize errors that may arise from extrapolation, it is crucial that the two known kinematic viscosity-temperature points be sufficiently far apart (ASTM D 341, 2.1.3). This is an important consideration especially if the difference between an extrapolated temperature and the nearest temperature of determination is greater than the difference between the two temperatures of determination. In extreme cases, an additional determination at a third temperature is advisable.

6.2.3 Kinematic Viscosity: Pressure Correction:

API Procedure 11A5.7 (2.1.2) defines the recommended pressure correction for kinematic viscosity. This correction is not necessary if an in-line viscometer is used. The gage pressure (p_{gage}) in Equation 30, the dynamic viscosity (μ_s) in Equation 32, and C_{pfd} from Equation 17 are used to calculate the pressure correction factor for fuel sample viscosity (C_{pfv}) in Equation 33.

$$p_{gage} = p_{op} - p_{atm} \qquad \text{(Eq.30)}$$

$$\rho_{Top,gcc} = \rho_{Top} / 1000 \qquad \text{(Eq.31)}$$

$$\mu_s = \rho_{Top,gcc} * \nu_{Top} \qquad \text{(Eq.32)}$$

$$\begin{aligned} C_{pfv} &= [10^{**} \{p_{gage} * (-0.0102 + 0.04042*\{\mu_s^{0.181}\}) / 1000\}] / C_{pfd} && \text{(US)} \qquad \text{(Eq.33)} \\ &= [10^{**} \{p_{gage} * (-0.1479 + 0.58624*\{\mu_s^{0.181}\}) / 1000\}] / C_{pfd} && \text{(SI)} \end{aligned}$$

The pressure correction factor (C_{pfv}) is multiplied with viscosity corrected to operating temperature (ν_{Top}) in Equation 34 to obtain viscosity corrected for temperature and pressure (ν_{op}).

$$\nu_{op} = \nu_{Top} * C_{pfv} \qquad \text{(Eq.34)}$$

6.3 Heat of Combustion / Lower Heating Value:

In order to normalize engine performance data to a reference fuel heat content, a lower heating value correction is typically applied to the true mass flow rate to account for the fact that not all fuels liberate the same amount of energy.

The lower heating value (LHV) can be measured in the following ways:

a. High Precision Method: ASTM D 2382 (2.1.3)
b. Intermediate Precision Method: ASTM D 4809 (2.1.3)
c. Low Precision Method: ASTM D 240 (2.1.3)

The lower heating value can vary significantly from one batch of fuel to another and as such, must be measured. All three methods listed above utilize bomb calorimeters in which the fuel sample is burned to measure its heat content. The formation of nitrates and sulfides is accounted for by titrating the reaction products for these acids. The distinguishing feature of the high precision method is the use of a platinum resistance thermometer capable of precise temperature measurements. Table A3 of Appendix A indicates ASTM D 2382 has the highest repeatability and reproducibility of all the bomb calorimeter measurement methods.

6.3 (Continued):

Bomb calorimeters are not insulated well enough to reduce the heat loss to under 0.05% of the heat content of the sample. Therefore, they must be calibrated with known traceable standards that have defined chemical compositions and thus calculable heats of combustion. The ASTM procedures prescribe the use of benzoic acid as a standard fuel. Iso-octane is recommended as a secondary standard to check the measurement.

The bomb calorimeter measures gross heating value, because it cools enough to condense the water formed in the reaction. During actual engine operation however, water vapor is part of the products of combustion and so lower heating value must be calculated to correct for the heat of vaporization. This calculation is made using the hydrogen content of the fuel to calculate the water formed when the hydrocarbon fuel reacts with oxygen to produce carbon dioxide and water. While hydrogen content can be calculated empirically from the gross heating value following ASTM D 1018 (2.1.3), more accurate results can be obtained by directly measuring hydrogen content following ASTM D 3701 (2.1.3).

The lower heating value correction factor (LHV_{cf}) of Equation 35 is the ratio between measured LHV and a reference LHV.

$$LHV_{cf} = LHV / LHV_{ref} \qquad \text{(Eq.35)}$$

6.4 Buoyancy Adjustment:

When lower heating value measurements are made by using any of the bomb calorimeter methods described in 6.3, the amount of fuel used is weighed “in air.” However fuel density, which is mass per unit volume, may or may not be referenced to “in vacuo” conditions. Since both these fuel properties are used in the calculation of fuel flow (Section 5.4 and Appendix B.12), an inconsistency in units may exist where LHV is referenced “in air” and density “in vacuo.” In such a case, the weight of the sample in air is not the same as its weight in vacuo, because air is displaced by both the sample being weighed and by the weights being used to balance the scales. These two buoyancy terms are unequal and their difference causes the resulting weight indication to be in error. Therefore, the buoyancy adjustment (B_o) in Equation 36 may have to be applied to ensure consistent reference to “in air” conditions for both LHV and fuel density.

$$B_o = (1 - \rho_a/\rho_{ref}) / (1 - \rho_a/\rho_w) \qquad \text{(Eq.36)}$$

where:

ρ_a = 1.217 kg/m^3 (air density at 60 °F, 760 mm Hg, 50% relative humidity) (ASTM D 1250, 2.1.3)
= 1.219 kg/m^3 (air density at 15 °C, 760 mm Hg, 50% relative humidity) (ASTM D 1250, 2.1.3)

ρ_w = 8390.9 kg/m^3 (brass weight density) (2.1.12)
= 8000 kg/m^3 (stainless steel density) (2.1.12)

7. TURBINE FLOWMETER CALIBRATION AND CORRECTIONS:

7.1 Turbine Flowmeter: Calibrators:

Turbine flowmeters are routinely calibrated with the following devices:

a. Gravimetric Flowbench ("Catch and Weigh"): ASME Paper No. 57-A-70 (2.1.13)

b. Small Volume Prover ("Ballistic Calibrator"): API Manual of Petroleum Measurement Standards, Ch. 4, Section 3 (2.1.2)

With the gravimetric flowbench, the flow used to calibrate the flowmeter is caught and weighed in a vessel on a balance. In addition to temperature and pressure corrections, the fuel density must be precisely known and buoyancy effects accounted for to get accurate results.

The small volume prover calibration method defined in (2.1.2) requires the displacement of a known volume of fluid in a precisely measured time interval through the flowmeter being calibrated. Volumetric changes in the flowmeter and calibrator due to pressure and temperature must be accounted for. With this calibration process, the Double-Chronometry Pulse Interpolation (2.1.2, Section 6) method of data acquisition provides the highest accuracy in calibration curve generation.

7.2 Turbine Flowmeter: Calibration Curve:

The traditional method of representing turbine flowmeter calibration characteristics has been the K-factor curve, which is K-factor as a function of flowmeter output frequency divided by kinematic viscosity (f/v) as shown earlier in Figure 1. The K-factor (cycles/US-gal or cycles/L) is defined in Equation 37 as flowmeter output frequency (f) divided by the volume flow rate (Q).

$$K = f / Q \qquad \text{(Eq.37)}$$

Research has shown that a more fundamental and thus accurate representation of turbine flowmeter characteristics can be obtained through the use of the nondimensional Strouhal number (St) versus Roshko number (Ro) defined by Equation 38 and Equation 39 respectively and depicted in Figure 3. (2.1.5, 2.1.7, 2.1.9).

$$St = K * D^3 \qquad \text{(Eq.38)}$$

$$Ro = (f/v) * D^2 \qquad \text{(Eq.39)}$$

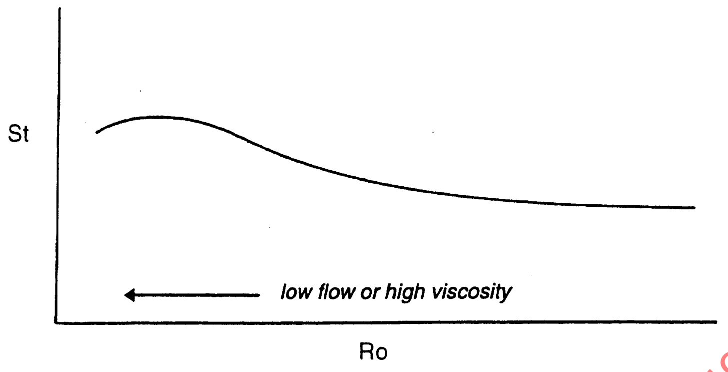

FIGURE 3 - Strouhal-Roshko Relationship

7.2 (Continued):

It may be noted that when the bore diameter term (D) remains unchanged, the factors D^3 and D^2 merely act as scaling factors for the two axes of Figure 3. Thus, with constant D, there is no meaningful difference between Figure 1 and Figure 3. Using similar reasoning, it may be seen that if the calibration conditions are used as reference, the D terms in the Strouhal and Roshko numbers defined by Equations 38 and 39 respectively, may be ignored when processing the calibration data of a flowmeter. If the D terms are dropped or set to unity while processing the calibration data, they must not be introduced when using the calibration data during actual operation of the flowmeter. In all such cases, the D terms may be set to unity without affecting the accuracy in data processing. Therefore, although both the Strouhal and Roshko numbers are defined as dimensionless parameters, in practice they take on the units of their counterparts K (cycles/US-gal or cycles/L) and f/ν (Hz/cSt), respectively.

The calibration data are unique for a specific flowmeter and should be treated as such, even within the same family of flowmeters. The D terms must not be used for scaling the calibration data obtained from one flowmeter to another flowmeter of different diameter, even though they are of the same type. In addition, since turbine flowmeters are sensitive to installation effects, they should be calibrated in as-installed plumbing. If the flowmeter is used at conditions different from calibration conditions, greater accuracy is obtained by accounting for changes in flowmeter diameter and fuel viscosity as will be discussed in the next section.

7.3 Turbine Flowmeter: Temperature and Pressure Corrections:

When using the calibration curve to determine fuel mass flow rate for actual engine operating conditions, appropriate temperature and pressure correction factors are applied to account for the differences between the calibration and operating conditions. The kinematic viscosity (ν) is first corrected to operating temperature and pressure to obtain ν_{op} (6.2.2 and 6.2.3, respectively). Temperature correction (C_{tr}) and pressure correction (C_{pr}) factors are then applied to obtain the operating Ro number (Ro_{op}) from Equation 40 (2.1.10).

$$Ro_{op} = (f / \nu_{op}) * C_{tr} * C_{pr} \qquad \text{(Eq.40)}$$

7.3 (Continued):

where:

$$C_{tr} = (1 + \alpha_l * \Delta T)^2 \approx 1 + 2 * \alpha_l * \Delta T \qquad \text{(Eq.41)}$$

$$C_{pr} = [1 + (\Delta p * D) / (E * t)]^2 \qquad \text{(Eq.42)}$$

$$\Delta p = p_{gage} - (p_{cal} - p_{atm,cal}) \qquad \text{(Eq.43)}$$

ΔT is obtained from Equation 10 where T_{cal} is substituted in place of T_{ref}.

p_{gage} is defined in Equation 30. In view of the relative magnitudes of the parameters in Equation 42, it is evident that the assumption of $p_{atm} = p_{atm,cal}$ will have no significant impact on the calculated value of C_{pr}. Thus, Equation 43 may be re-written as,

$$\Delta p = p_{op} - p_{cal} \qquad \text{(Eq.44)}$$

Once the operating Strouhal number (St_{op}) has been obtained for a given operating Roshko number from Figure 3 of a typical calibration curve, it is corrected for temperature (C_{tk}) using Equation 46, and for pressure (C_{pk}) using Equation 47. Strouhal number corrected for temperature and pressure yields the operating K-factor (K_{op}) defined in Equation 45. K_{op} in Equation 45 takes into account flowmeter dimensional changes due to the difference between the calibration and the operating conditions of temperature and pressure. Chs. 12.2.5.1 and 12.2.5.2 of the API Manual of Petroleum Measurement Standards (2.1.2) define the corrections, respectively, for temperature and pressure induced turbine flowmeter expansion.

$$K_{op} = St_{op} / (C_{tk} * C_{pk}) \qquad \text{(Eq.45)}$$

where:

$$C_{tk} = (1 + \alpha_l * \Delta T)^3 \approx 1 + 3 * \alpha_l * \Delta T \qquad \text{(Eq.46)}$$

$$C_{pk} = [1 + (\Delta p * D) / (E * t)]^3 \qquad \text{(Eq.47)}$$

The temperature correction factors (C_{tr} and C_{tk}) in Equations 41 and 46 respectively, are a function of α_l, which is the linear coefficient of thermal expansion. These equations assume that the flowmeter body and rotor materials are the same, although this is rarely the case. For the highest accuracy, the thermal expansion coefficient for the user's particular flowmeter should be used. Thermal expansion coefficients may also be obtained from the calibrating agency or manufacturer of each particular type of turbine flowmeter.

The pressure correction factors (C_{pr} and C_{pk}) in Equations 42 and 47 respectively, are a function of operating pressure (p_{gage}), flowmeter bore diameter (D), the modulus of elasticity of the material (E), and the flowmeter wall thickness (t).

7.4 Turbine Flowmeter: General Recommendations:

The following are general recommendations for calibrating flowmeters and the use of calibration results.

a. Instrument grade turbine flowmeters that utilize modulated carrier (RF) type pickoff coils and electronic signal conditioning are recommended. The procedure will not work with turbine flowmeters that attempt to provide corrections with customized signal conditioning.

b. Turbine flowmeters must be calibrated in as-installed plumbing to ensure that installation effects are accounted for during actual engine operation.

c. Because of the nonlinearity of turbine flowmeter characteristics, they should not be used beyond the range of calibration, i.e., the characteristic plot should not be extrapolated. Even with all the corrections applied, special care should be exercised at low flow or high viscosity conditions, both of which correspond to low Ro number. The concept of the UVC, and of the St-Ro plot, gives an impression that a turbine flowmeter, calibrated at one viscosity, can be used at any other viscosity as long as the operating Ro number falls within the calibration range. However, the UVC has a break-away point, which defines the lower limit of the "usable turn-down region", for each viscosity (2.1.10). The break-away point for a specific viscosity can be determined from plots of calibration data at different viscosities. Use of a flowmeter below the break-away point, at viscosities and/or temperatures substantially different from those at which the meter was calibrated, can lead to serious errors in flow measurements.

8. RECOMMENDATIONS:

This document summarizes turbine flowmeter fuel flow calculations. The pressure and temperature corrections applied to fuel property measurements and for turbine flowmeter dimensional changes are based on currently available ASTM and API methods. Although all of the pressure and temperature corrections are discussed in their entirety in Sections 6 and 7, and their use illustrated in the sample calculation found in Appendix B, all the corrections may not be significant in all cases. The impact of each correction on fuel mass flow rate depends on the specific application and the operating environment.

For example, in the simplest of applications, a flowmeter may be used with the same fuel at the same pressure and temperature conditions as it was calibrated in. In such an application, it is sufficient to correlate K-factor against frequency (f) and use this as the flowmeter characteristic calibration. Corrections for the effects of pressure and temperature on fuel density, viscosity and flowmeter geometry are thus redundant. Similarly, for low pressure applications, the effects of pressure on viscosity and density may be neglected.

It is therefore recommended that during the design of a measurement system for a specific application, each correction factor be evaluated and its impact compared against the overall measurement uncertainty target, before a decision is made about neglecting the factor. The correction methods, in their entirety, are recommended to provide an accurate and consistent means of determining fuel mass flow rate.

9. NOTES:

9.1 Revision:

None

9.2 Key Words:

Automotive, propulsion, engines, flow calculation, flow measurement, turbine flowmeter, relative density, viscosity, lower heating value, Roshko number, Strouhal number, universal viscosity curve

PREPARED BY SAE SUBCOMMITTEE EG-1E,
GAS TURBINE ENGINE TEST FACILITIES & EQUIPMENT
OF COMMITTEE EG-1, AEROSPACE PROPULSION SYSTEMS SUPPORT EQUIPMENT

APPENDIX A
ASTM AND API STANDARDS SUMMARY

TABLE A1 - Summary of Test Method Procedures

Test Method	Reference
Fuel Density	ASTM D 4052 (2.1.3)
Fuel Viscosity	ASTM D 445 (2.1.3)
Meter Calibration	API MPMS, Ch. 4 (2.1.2)
Lower Heating Value	ASTM D 2382 (2.1.3)

TABLE A2 - Summary of Correction Procedures

Correction	Reference
Fuel Density (T)	ASTM D 1250 (2.1.3)
Fuel Density (p)	API MPMS Ch. 11.2.1 (2.1.2)
Fuel Viscosity (T)	ASTM D 341 (2.1.3)
Fuel Viscosity (p)	API Technical Data Book, Proc. 11A5.7 (2.1.2)
Meter Expansion (T)	API MPMS Ch. 12.2.5.1 (2.1.2)
Meter Expansion (p)	API MPMS Ch. 12.2.5.2 (2.1.2)
Buoyancy	*See 6.4

TABLE A3 - Repeatability[1] and Reproducibility[2] of Standard Test Methods

Reference	Equipment	Repeatability	Reproducibility
ASTM D 287	hydrometer	0.0008	0.0020
ASTM D 1298	hydrometer	0.0005	0.0012
ASTM D 4052	digital density meter	0.0001	0.0005
ASTM D 1217	pycnometer	0.00002 of mean	0.00003 of mean
ASTM D 445	viscometer	0.35% of mean	0.70% of mean
ASTM D 240	bomb calorimeter (low precision)	56 Btu/lb 130 kJ/kg	172 Btu/lb 400 kJ/kg
ASTM D 4809	bomb calorimeter (intermediate)	41 Btu/lb 96 kJ/kg	139 Btu/lb 324 kJ/kg
ASTM D 2382	bomb calorimeter (high precision)	22 Btu/lb 51 kJ/kg	56 Btu/lb 130 kJ/kg

[1] Repeatability is defined as the difference between successive test results obtained by the same operator with the same apparatus under constant operating conditions on identical test material, [that] would in the long run, in the normal and correct operation of the test method, exceed the following values only in one case in twenty (ASTM D 1250, 2.1.3).

[2] Reproducibility is defined as the difference between two single and independent results, obtained by different operators working in different laboratories on identical test material, [that] would in the long run, in the normal and correct operation of the test method, exceed the following values only in one case in twenty (ASTM D 1250, 2.1.3).

TABLE A4 - Predicted Precision of Thermal Expansion Properties of Crudes and Products at 95% Confidence Level (ASTM D 1250, 2.1.3)

Fuel Temperature, °F	VCF Precision, %
100	±0.05
150	±0.15
200	±0.25
250	±0.35

APPENDIX B
SAMPLE FUEL FLOW CALCULATION

B.1 GIVEN PARAMETERS:

The following sample calculation illustrates the steps required to determine fuel flow based on the recommended procedures defined in this document. The sample calculations are carried out in both US customary and SI units. The units conversion factors found in Appendix C are used throughout.

The following quantities are considered as givens in the following calculations:

Fuel Operating Pressure:	p_{op} = 450 psia p_{op} = 31.026 bar
Fuel Operating Temperature:	T_{op} = 135 °F T_{op} = 57.22 °C
Fuel Sample Density:	ρ_{ref} = 814.794 kg/m^3 at 60 °F ρ_{ref} = 815.199 kg/m^3 at 15 °C
Fuel Sample Temperatures:	T_{s1} = 77 °F T_{s1} = 25 °C T_{s2} = 140 °F T_{s2} = 60 °C
Fuel Sample Relative Density:	$RD_{77F/60F}$ = 0.80886 $RD_{25C/15C}$ = 0.80880
Fuel Sample Viscosity:	ν_{s1} = 1.86 cSt ν_{s2} = 1.12 cSt
Measured Lower Heating Value:	LHV = 18490 Btu/lb LHV = 43008 kJ/kg
Reference Lower Heating Value:	LHV_{ref} = 18400 Btu/lb LHV_{ref} = 42798 kJ/kg
Atmospheric Pressure:	p_{atm} = 14.7 psia p_{atm} = 1.0135 bar
Fuel Meter Frequency:	f = 1360 Hz
Turbine Flowmeter Calibration Temperature:	T_{cal} = 80 °F T_{cal} = 26.67 °C

B.1 (Continued):

Turbine Flowmeter Calibration Pressure: $p_{cal} = 45$ psia
$p_{cal} = 3.103$ bar

NOTE 1: The calculations found in Appendix B.2 and B.3 for correcting fuel density and relative density, respectively, to operating temperature are based on procedures outlined in ASTM D 1250 (2.1.3) and are used here for illustrative purposes only. ASTM D 1250 contains computer programs with special rounding subroutines so that consistent results may be obtained.

NOTE 2: Measured density or relative density for a given fuel sample is different depending on the reference temperature ($T_{ref} = 60$ °F or 15 °C). The corresponding temperature and pressure corrections may also be different. However, these differences are consistent and the resulting fuel mass flow rate is the same regardless of the reference temperature.

B.2 CORRECT FUEL DENSITY TO OPERATING TEMPERATURE (DIGITAL DENSITY METER METHOD):

B.2.1 Basis:

6.1.1 (ρ_{ref}) and 6.1.2 (VCF)

B.2.2 Constants (Jet-A fuel (ASTM D 1250, 2.1.3)):

English units: $K_0 = 330.3010$
$K_1 = 0.0$

SI units: $K_0 = 594.5418$
$K_1 = 0.0$

B.2.3 Measurements:

Fuel Density: $\rho_{ref} = 814.794$ kg/m^3 at 60 °F
$\rho_{ref} = 815.199$ kg/m^3 at 15 °C
Fuel Operating Temperature: $T_{op} = 135$ °F or 57.22 °C

B.2.4 Calculations:

$\Delta T = T_{op} - T_{ref}$ (Identical to Eq.10) (Eq.B1)

$= 135 - 60$
$= 75$ °F (US)

$= 57.22 - 15$
$= 42.22$ °C (SI)

$\alpha_{F,T_{ref}} = K_0/(\rho_{ref}^2) + K_1/\rho_{ref}$ (Identical to Eq.11) (Eq.B2)

$= 330.3010/(814.794^2) + 0/814.794$
$= 4.9752$ E-4 (1/°F) (US)

$= 594.5418/(815.199^2) + 0/815.199$
$= 8.9465$ E-4 (1/°C) (SI)

$VCF = \exp(- \alpha_{F,T_{ref}} * \Delta T * \{1 + .8 * \alpha_{F,T_{ref}} * \Delta T\})$ (Identical to Eq.12) (Eq.B3)

$= \exp(-4.9752 \text{ E-4} * 75 * \{1 + .8 * 4.9752 \text{ E-4} * 75\})$
$= 0.9623008$ (US)

$= \exp(-8.9465 \text{ E-4} * 42.22 * \{1 + .8 * 8.9465 \text{ E-4} * 42.22\})$
$= 0.9618337$ (SI)

$\rho_{Top} = \rho_{ref} * VCF$ (Identical to Eq.13) (Eq.B4)

$= 814.794 * 0.9623008$ for $T_{ref} = 60$ °F
$= 784.077 \text{ kg/m}^3$

$= 815.199 * 0.9618337$ for $T_{ref} = 15$ °C
$= 784.086 \text{ kg/m}^3$

B.3 CORRECT FUEL RELATIVE DENSITY TO OPERATING TEMPERATURE (HYDROMETER METHOD):

B.3.1 Basis:

6.1.1 (ρ_{ref}) and 6.1.2 (VCF)

B.3.2 Constants (Jet-A fuel (ASTM D 1250, 2.1.3)):

English units: $K_0 = 330.3010$
$K_1 = 0.0$

SI units: $K_0 = 594.5418$
$K_1 = 0.0$

B.3.3 Measurements:

Fuel Sample Relative Density: $RD_{77F/60F} = 0.80886$
$RD_{25C/15C} = 0.80880$
Fuel Sample Temperature: $T_{s1} = 77$ °F or 25 °C
Fuel Operating Temperature: $T_{op} = 135$ °F or 57.22 °C

B.3.4 Calculations:

First calculate the sample density (ρ_{s1}) from the measured relative density ($RD_{s1/Tref}$).

$$RD_{Ts/Tref} = \rho_s / \rho_{water,ref} \qquad \text{(Identical to Eq.6)} \quad \text{(Eq.B5)}$$

$$\rho_{s1} = RD_{77F/60F} * 999.012 \qquad \text{(Eq.B6)}$$
$$= 0.80886 * 999.012$$
$$= 808.061 \text{ kg/m}^3 \qquad \text{(US)}$$

$$= RD_{25C/15C} * 999.103$$
$$= 0.80880 * 999.103$$
$$= 808.075 \text{ kg/m}^3 \qquad \text{(SI)}$$

$$\Delta T_s = T_{s1} - T_{ref} \qquad \text{(Identical to Eq.7)} \quad \text{(Eq.B7)}$$

$$= 77 - 60$$
$$= 17 \text{ °F} \qquad \text{(US)}$$

$$= 25 - 15$$
$$= 10 \text{ °C} \qquad \text{(SI)}$$

B.3.4 (Continued):

Calculate and apply the following stem correction (HYC) for the thermal expansion of the glass hydrometer.

$HYC = 1 - (1.278\text{ E-5} * \Delta T_s) - (6.2\text{ E-9} * \Delta T_s^2)$ (Identical to Eq.8) (Eq.B8)
$= 1 - (1.278\text{ E-5} * 17) - (6.2\text{ E-9} * 17^2)$
$= 0.999781$ (US)

$= 1 - (2.3\text{ E-5} * \Delta T_s) - (2.0\text{ E-8} * \Delta T_s^2)$
$= 1 - (2.3\text{ E-5} * 10) - (2.0\text{ E-8} * 10^2)$
$= 0.999768$ (SI)

Apply the hydrometer correction to the observed density.

$\rho_{s1,\,HYC} = \rho_{s1} * HYC$ (Identical to Eq.9) (Eq.B9)

$= 808.061 * 0.999781$
$= 807.884\text{ kg/m}^3$ (US)

$= 808.075 * 0.999768$
$= 807.888\text{ kg/m}^3$ (SI)

Since the equation for ρ_{ref} cannot be solved explicitly, a successive approximation iterative scheme is used to obtain the solution. The initial estimate of ρ_{ref} is the value of the density at the sample temperature ($\rho_{s1,HYC}$). This approximation is substituted into Equations B10 through B12 until ρ_{ref} converges to the desired degree of accuracy.

$\alpha_{F,1} = K_0/(\rho_{s1,HYC}{}^2) + K_1/\rho_{s1,HYC}$ (Identical to Eq.11) (Eq.B10)

$= 330.3010/(807.884^2) + 0.0/807.884$
$= 5.0607\text{ E-4 } (1/°F)$ (US)

$= 594.5418/(807.888^2) + 0.0/807.888$
$= 9.1092\text{ E-4 } (1/°C)$ (SI)

$VCF_1 = \exp(-\alpha_{F,1} * \Delta T_s * \{1 + 0.8 * \alpha_{F,1} * \Delta T_s\})$ (Identical to Eq.12) (Eq.B11)

$= \exp(-5.0607\text{ E-4} * 17 * \{1 + 0.8 * 5.0607\text{ E-4} * 17\})$
$= 0.9913750$ (US)

$= \exp(-9.1092\text{ E-4} * 10 * \{1 + 0.8 * 9.1092\text{ E-4} * 10\})$
$= 0.9908664$ (SI)

B.3.4 (Continued):

$\rho_{ref,1} = \rho_{s1,HYC} / VCF_1$ (Identical to Eq.13) (Eq.B12)

$= 807.884 / 0.9913750$
$= 814.913\ kg/m^3$ (US)

$= 807.888 / 0.9908664$
$= 815.335\ kg/m^3$ (SI)

Check for convergence, i.e. $|\rho_{ref,1} - \rho_{s1,HYC}| < 0.05$. The convergence criterion of 0.05 is obtained from ASTM D 1250 (2.1.3).

$\rho_{ref,1} - \rho_{s1,HYC} = 814.913 - 807.884$ (Eq.B13)
$= |7.029| > 0.05$ (continue) (US)

$= 815.335 - 807.888$
$= |7.447| > 0.05$ (continue) (SI)

Since the convergence criterion is not satisfied, substitute $\rho_{ref,1}$ back into Equation B10 to obtain a better approximation of α_F.

$\alpha_{F,2} = K_0/(\rho_{ref,1}^2) + K_1/\rho_{ref,1}$ (Identical to Eq.11) (Eq.B14)

$= 330.3010/(814.913^2) + 0.0/814.913$
$= 4.9738\ E\text{-}4\ (1/°F)$ (US)

$= 594.5418/(815.335^2) + 0.0/815.335$
$= 8.9436\ E\text{-}4\ (1/°C)$ (SI)

Now Equation B11 may be used to evaluate new VCF_2 and the iteration process may continue for convergence on ρ_{ref} to yield,

$\rho_{ref} = 814.794\ kg/m^3$ (US) (Eq.B15)

$= 815.199\ kg/m^3$ (SI)

B.3.4 (Continued):

Now calculate fuel density at the operating temperature.

$\Delta T = T_{op} - T_{ref}$ (Identical to Eq.10) (Eq.B16)

$= 135 - 60$
$= 75\ °F$ (US)

$= 57.22 - 15$
$= 42.22\ °C$ (SI)

$\alpha_{F,T_{ref}} = K_0/(\rho_{ref}^2) + K_1/\rho_{ref}$ (Identical to Eq.11) (Eq.B17)

$= 330.3010/(814.794^2) + 0.0/814.794$
$= 4.9752\ E\text{-}4\ (1/°F)$ (US)

$= 594.5418/(815.199^2) + 0.0/815.199$
$= 8.9465\ E\text{-}4\ (1/°C)$ (SI)

$VCF = \exp(-\ \alpha_{F,T_{ref}} * \Delta T * \{1 + 0.8 * \alpha_{F,T_{ref}} * \Delta T\}\ )$ (Identical to Eq.12) (Eq.B18)

$= \exp(-4.9752\ E\text{-}4 * 75 * \{1 + 0.8 * 4.9752\ E\text{-}4 * 75\})$
$= 0.9623008$ (US)

$= \exp(-8.9465\ E\text{-}4 * 42.22 * \{1 + 0.8 * 8.9465\ E\text{-}4 * 42.22\})$
$= 0.9618337$ (SI)

Using ρ_{ref} and VCF, the fuel density at the operating temperature may be evaluated as,

$\rho_{Top} = \rho_{ref} * VCF$ (Identical to Eq.13) (Eq.B19)

$= 814.794 * 0.9623008$
$= 784.077\ kg/m^3$ (US)

$= 815.199 * 0.9618337$
$= 784.086\ kg/m^3$ (SI)

B.4 CORRECT FUEL DENSITY TO OPERATING PRESSURE:

B.4.1 Basis:

6.1.1 (ρ_{ref}) and 6.1.3 (C_{pfd})

B.4.2 Measurements:

Atmospheric Pressure:	p_{atm} = 14.7 psia or 1.0135 bar
Fuel Operating Pressure:	p_{op} = 450 psia or 31.026 bar
Fuel Density:	ρ_{ref} = 814.794 kg/m^3 at 60 °F
	ρ_{ref} = 815.199 kg/m^3 at 15 °C
Fuel Operating Temperature:	T_{op} = 135 °F or 57.22 °C

B.4.3 Calculations:

$\rho_{ref,gcc} = \rho_{ref} / 1000$ (Identical to Eq.14) (Eq.B20)

$= 814.794 / 1000$
$= 0.814794\ \text{g/cm}^3$ (US)

$= 815.199 / 1000$
$= 0.815199\ \text{g/cm}^3$ (SI)

$p_{eq} = 0.145 * (10^{6.284 - 1991.3/((Top + 459.67)/1.8)})$ (Identical to Eq.15) (Eq.B21)
$= 0.145 * (10^{6.284-1991.3/((135+459.67)/1.8)})$
$= 0.2618$ psia (US)

$= 0.010 * (10^{6.284 - 1991.3\ /\ (Top + 273.15)})$
$= 0.010 * (10^{6.284 - 1991.3\ /\ (57.22 + 273.15)})$
$= 0.0181$ bar (SI)

$F = (\exp(- 1.9947 + 0.00013427{*}T_{op} + 0.79392/\rho_{ref,gcc}{}^2$ (Identical to Eq.16) (Eq.B22)
$+ 0.002326{*}T_{op}/\rho_{ref,gcc}{}^2)) / 100000$
$= (\exp(- 1.9947 + 0.00013427{*}135 + 0.79392/0.814794^2$
$+ 0.002326{*}135/0.814794^2)) / 100000$
$= 7.3512\ \text{E-6}$ (1/psia) (US)

$= (\exp(- 1.62080 + 0.00021592{*}\ T_{op} + 0.87096/\{\rho_{ref,gcc}{}^2\}$
$+ 0.0042092{*}\ T_{op} /\{\rho_{ref,gcc}{}^2\})) / 10000$
$= (\exp(- 1.62080 + 0.00021592{*}\ 57.22 + 0.87096/0.815199^2$
$+ 0.0042092{*}\ 57.22 /0.815199^2)) / 10000$
$= 1.0667\ \text{E-4}$ (1/bar) (SI)

B.4.3 (Continued):

C_{pfd} $= (1 - F * (p_{atm} - p_{eq})) / (1 - F * (p_{op} - p_{eq}))$ (Identical to Eq.17) (Eq.B23)

$= (1 - 7.3512\text{ E-6} * (14.7 - 0.2618))$
$/ (1 - 7.3512\text{ E-6} * (450 - 0.2618))$
$= 1.0032$ (US)

$= (1 - 1.0667\text{ E-4} * (1.0135 - 0.0181))$
$/ (1 - 1.0667\text{ E-4} * (31.026 - 0.0181))$
$= 1.0032$ (SI)

B.5 CORRECT FUEL VISCOSITY TO OPERATING TEMPERATURE:

B.5.1 Basis:

6.2.1 (ν_s) and 6.2.2 (ν_{Top})

B.5.2 Measurements:

Fuel Operating Temperature: $T_{op} = 135$ °F or 57.22 °C
Fuel Sample Temperature: $T_{s1} = 77$ °F or 25 °C
$T_{s2} = 140$ °F or 60 °C
Fuel Sample Viscosity: $\nu_{s1} = 1.86$ cSt
$\nu_{s2} = 1.12$ cSt

B.5.3 Calculations:

$T_{s1,abs}$ $= T_{s1} + 459.67$ (Identical to Eq.19) (Eq.B24)
$= 77 + 459.67$
$= 536.67$ °R (US)

$= T_{s1} + 273.15$
$= 25 + 273.15$
$= 298.15$ °K (SI)

$T_{s2,abs}$ $= T_{s2} + 459.67$ (Identical to Eq.19) (Eq.B25)
$= 140 + 459.67$
$= 599.67$ °R (US)

$= T_{s1} + 273.15$
$= 60 + 273.15$
$= 333.15$ °K (SI)