

**Aerospace-Particle Count Data  
Conversion and Extrapolation**

**1. SCOPE:**

This SAE Aerospace Information Report (AIR) describes a mathematical model that can be used to analyze particle count data. Particle counts that fit the model can be graphically displayed, converted from one counting size-frequency range to another, and extrapolated to estimate counts beyond the measured range. Derivation, applications, and calculations are described.

**1.1 Outline of Method:**

Cumulative particle counts larger than stated size-range are fitted against a modified log-normal distribution function by plotting on special log-log<sup>2</sup> graph paper. Many real particle distributions approximate a straight line, showing good fit for this log-normal model. The resultant plot provides a basis for further analysis, including size-range conversion and extrapolation.

**2. REFERENCES:**

**2.1 Applicable Documents:**

The following publications form a part of this document to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this document and references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

**2.1.1 SAE Publications: Available from SAE, 400 Commonwealth Drive, Warrendale, PA 15096-0001.**

|        |  |
|--------|--|
| ARP598 | The Determination of Particulate Contamination in Liquids by the Particle Count Method |
| AS4059 | Cleanliness Classification for Hydraulic Fluids  |

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3. MODIFIED LOG-NORMAL MODEL:

Many real particle counts have been shown to approximate a Gaussian distribution with a logarithmic variate. This cumulative distribution function is represented by Equation 1 having the form:

$$n_c = \frac{\Sigma n}{\sqrt{2\pi} \ln \sigma} \int_{\ln x}^{\infty} \exp - \left[ \frac{\ln x - \ln M}{\sqrt{2} \ln \sigma} \right]^2 d \ln x \quad (\text{Eq. 1})$$

where:

- $n_c$  = Cumulative particle count
- $\Sigma n$  = Total number of particles
- $\ln \sigma$  = Natural logarithm of the geometric standard deviation
- $M$  = Geometric mean particle size
- $x$  = Particle size variate

Assuming that  $\ln M$  is small compared to  $\ln x$ , which means that about half of the particles are smaller than  $1 \mu\text{m}$  in size, the rather formidable expression in Equation 1 may be reduced to a simpler equivalent expression using lumped constants:

$$\log n_c = \log A - 2.303 B \log^2 x \quad (\text{Eq. 2})$$

where:

- $n_c$  = Cumulative particle count expressed as a base -10 logarithm
- $x$  = Particle size variable
- $A$  = Characteristic intercept-slope constant that describes quantity
- $B$  = Characteristic intercept-slope constant that describes distribution

If a particle count fits the mathematical model in Equation 2, data plots on special log-log<sup>2</sup> graph paper are linear and provide a basis for further analysis. The necessary graph paper is made by laying out a logarithmic n-axis and an x-axis proportional to the square of the same logarithmic modulus. (A sample of log-log<sup>2</sup> graph paper is shown in Appendix A.)

4. DATA PLOTTING:

Contamination particle counts are plotted on log-log<sup>2</sup> graph paper as cumulative counts larger than each lower size-range limit. This method provides a discrete size to be plotted, rather than some "average" value of the size range, and tends to correct classification errors for the smaller size ranges.

EXAMPLE: Contamination analysis according to ARP598 was performed on 100 mL of hydraulic oil. The following microscopic particle counts were obtained.

TABLE 1

|                 |      |        |       |       |      |
|-----------------|------|--------|-------|-------|------|
| Size Range (μm) | >100 | 50-100 | 25-50 | 15-25 | 5-15 |
| No. Particles   | 4    | 19     | 150   | 450   | 3780 |

These frequency counts are converted to corresponding cumulative counts by successive addition from left to right.<sup>1</sup>

TABLE 2

|                 |      |     |     |     |      |
|-----------------|------|-----|-----|-----|------|
| Size Limit (μm) | >100 | >50 | >25 | >15 | >5   |
| No. Particles   | 4    | 23  | 173 | 623 | 4403 |

The greater number of significant digits in the cumulative counts (4403 particles > 5 μm) are not meaningful and may be rounded off (4400) for convenience. These cumulative counts are plotted as shown in Figure 1.

Many such contamination plots are essentially linear on log-log<sup>2</sup> graph paper showing good fit to the modified log-normal function. The example count has been faired to a straight line because all data points except the largest (4 particles > 100 μm) fit quite closely. The largest particle count size-range usually exhibit the greatest counting error and should be less heavily weighted in drawing the mean curve.

1. Total particles greater than 100 μm is 4.  
 Total particles greater than 50 μm is 4 + 19 = 23.  
 Total particles greater than 25 μm is 23 + 150 = 173, etc.

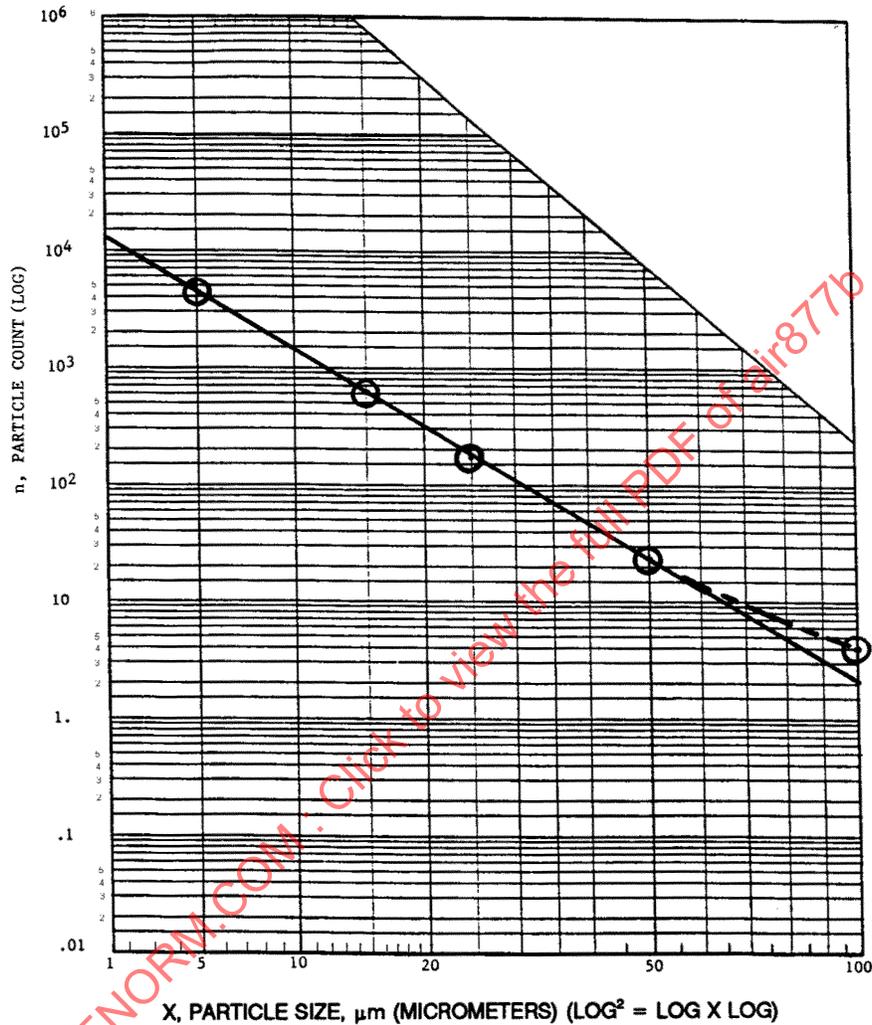


FIGURE 1 - Modified Log-Normal Contamination Plot

4. (Continued):

Deviations from linearity are also significant when based on accurate count data. Upward-concave curves may imply a closed distribution within the counting range or a mechanical breakdown which generates large particles. Downward-concave curves may imply the action of a wear process which preferentially generates smaller particles. Inflected or multiple curves may imply the presence of more than one type of particle population showing contamination from different sources.

## 5. CALCULATIONS:

The cumulative particle count plot demonstrated in Figure 1 may be used for further analysis or modifications of the original data. Data conversion to other particle count size-ranges or extrapolation of data to uncounted size-ranges may be accomplished analytically or graphically.

EXAMPLE: The particle counts given in Section 4 and plotted on Figure 1 must be converted to equivalent frequency counts in the different micrometer size-ranges of >80, 40 to 80, 20 to 40, 10 to 20, and 5 to 10. Cumulative particle counts corresponding to these lower micrometer size-range limits of >80, >40, >20, >10, and >5 are read directly from the plot of Figure 1. (If determination of a cleanliness class is desired, size ranges per AS4059 should be used.)

TABLE 3

|                              |     |     |     |      |      |
|------------------------------|-----|-----|-----|------|------|
| Size Limit ( $\mu\text{m}$ ) | >80 | >40 | >20 | >10  | >5   |
| No. Particles                | 5   | 45  | 320 | 1450 | 4400 |

Counts are read from the straight line plot, not the dashed curve (4.5 particles >80, not 6.5) and are rounded off. Frequency counts are obtained by the reverse process given in Section 4 by successive subtraction from left to right.

TABLE 4

|                              |     |       |       |       |      |
|------------------------------|-----|-------|-------|-------|------|
| Size Range ( $\mu\text{m}$ ) | >80 | 40-80 | 20-40 | 10-20 | 5-10 |
| No. Particles                | 5   | 40    | 275   | 1130  | 2950 |

These frequency counts are equivalent to the original ARP598 frequency counts.

Data extrapolation may also be estimated from the plot of Figure 1. For example, suppose that a pump-wear study or an electrohydraulic servovalve investigation required knowledge of the number of particles in the 1 to 5  $\mu\text{m}$  size-range. ARP598 data is limited to the size-range above 5  $\mu\text{m}$ .

Extrapolation of this data according to Figure 1 shows a cumulative count of 14 000 particles greater than 1  $\mu\text{m}$ . Subtraction of 4400 particles greater than 5  $\mu\text{m}$  ( $14\ 000 - 4400 = 9600$ ) gives 9600 particles in the 1 to 5  $\mu\text{m}$  size-range.

Extrapolated data should be treated with caution since calculated values may differ significantly from actual particle counts.