AEROSPACE INFORMATION REPORT

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Cargo Compartments
Maximum Package Size Calculation Methodology

RATIONALE

The SAE AGE-2A committee determined that the technology/practice defined in this standard has reached a level of maturity such that it is unlikely to change in the foreseeable future; therefore, it does not require regular 5-year reviews by the committee and it is appropriate for it to be stabilized.

STABILIZED NOTICE

This document has been declared "Stabilized" by the SAE AGE-2A Cargo Handling Committee and will no longer be subjected to periodic reviews for currency. Users are responsible for verifying references and continued suitability of technical requirements. Newer technology may exist.

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INTRODUCTION

For a given aircraft and each cargo compartment, the maximum package size tables determine the maximum size of rectangular shaped package that can be accommodated, due to cargo door and compartment geometry and sizes.

Published by each airframe manufacturer for information, these data are verified and adapted by the individual airline in line with loading methods practiced and training provided. They serve to accept such loads for air cargo transport.

When such a package is received and checked, the reliability of the maximum package size table data and pertaining assumptions are the basis for further considerations, i.e. weight, manpower, multiple packages, loading and restraint, to realize a smooth loading operation.

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1. SCOPE:

The methodology for maximum package size loading is based on a mathematical method allowing the calculation of maximum package size tables.

This method does not in principal differentiate between bulk loading and cargo system loading.

However, some restrictions have to be considered:

- Some cargo systems generate pre-determined pallet trajectories. Envelope curves depending on the pallet size and the possible trajectories have to be determined first.
- Door geometric limitations (with or without cargo loading system)
- Turning limitations due to weight, load geometry and conveyance capability
- Securing requirements

This document is not intended for airline operational use. It should be used by engineers performing calculations or developing computer programs to produce Maximum Package Size tables specified in AS1825.

1.1 Purpose:

The purpose of this document is to provide mathematical support to establish and verify max. package loadability and max. package size tables for given cargo hold dimensions.

This standardized methodology shall be:

- applicable for any given aircraft and each compartment
- based on a defined and practicable loading procedure
- exact for the range of maximum size packages within a rectangular contour envelope
- referenced as basic methodology, to identify geometrical variants

2. REFERENCES:

2.1 Applicable Documents:

The following publications form a part of this document to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this document and references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

2.1.1 SAE Publications: Available from SAE, 400 Commonwealth Drive, Warrendale, PA 15096-0001.

AS1825	Methodology of Calculating Aircraft Volumes
ARP5595	Cargo Restraint Straps - Utilization Guidelines

2.1.2 IATA Publications:

Orders from North, Central, South America, Asia, Australasia and the Pacific: International Air Transport Association, 800 Place Victoria, P.O. Box 113, MONTREAL, Quebec, Canada H4Z1M1.

Orders from Europe, Africa and the Middle East: International Air Transport Association, IATA Centre, 33 route de l'aéroport, P.O. Box 672, CH-1215 GENEVA 15 Airport, Switzerland.

IATA - AHM 671	securing of loads
IATA - AHM 673	bulk compartment load limitations
IATA - AHM 676	handling / bulk loading of HEAVY items
IATA - AHM 677	handling and loading of BIG OVERHANG items

of the IATA AIRPORT HANDLING MANUAL, latest edition

2.2 Definitions:

RECTANGULAR COMPARTMENT: In this document and for practical reasons, a cargo compartment with a constant cross section and a rectangular horizontal section will be called rectangular compartment.

TAPERED SHAPE COMPARTMENT: In this document and for practical reasons, a cargo compartment with a variable cross section and/or a tapered horizontal section will be called tapered compartment.

2.3 Abbreviations:

A/D: Aircraft Datum Line
CC: Cargo Compartment
MPS: Maximum Package Size

3. METHODOLOGY:

3.1 Method of Loading:

Loading a maximum size rectangle shaped package follows on both ends simultaneously the guideline (A) and the intersection line (B) of the cargo compartment floor/lining, passing the corner (C) when manipulated in contact with the floor at a critical angle (the maximum height is the one of the lower deck, all clearances respected).

When lifted by the distance L parallel to the floor, B_L and C_L apply. The package length might be then significantly larger but the maximum height reduced by L.

NOTE: The load is positioned at the cargo compartment door sill, perpendicular to the aircraft datum line (A/D).

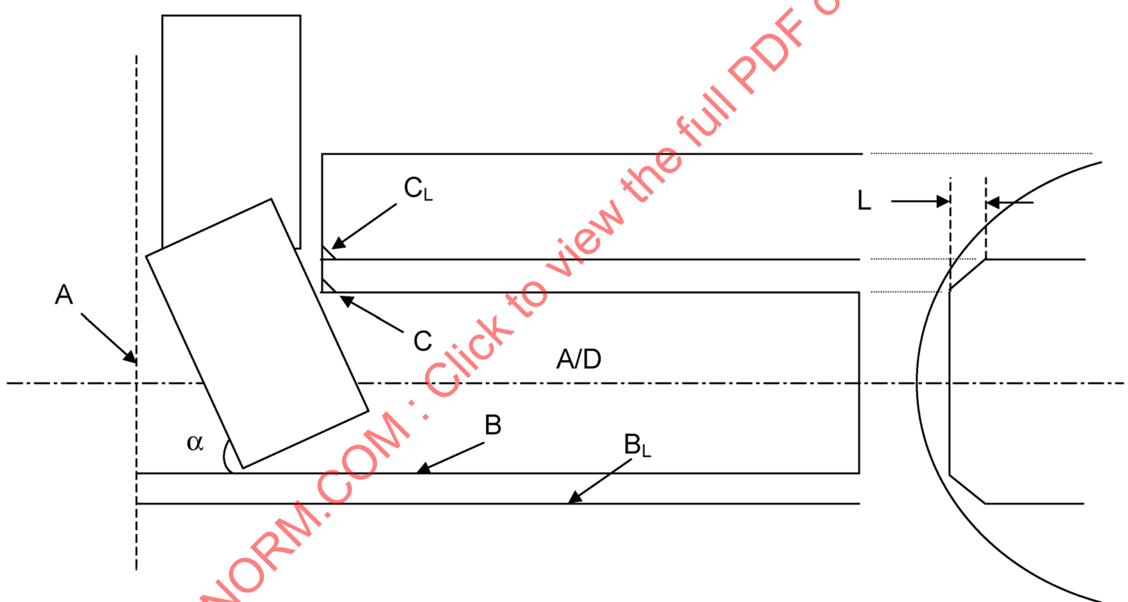


FIGURE 1

3.2 Loading Parameters:

3.2.1 For bulk loading and manipulation with floor contact, the following has to be used:

- Door width minus 50.8 mm (2 in) clearance on both sides (= maximum width)
- Floor width minus 50.8 mm (2 in) clearance on both sides
- Compartment height minus 50.8 mm (2 in) clearance on top (= maximum height)
- Package length (at defined package width):
  - defined by the door-/floor width dimensions
  - at corner C - see 3.1
  - above standard clearances considered

3.2.2 For bulk loading and manipulation with the packages lifted parallel to the floor, the following has to be used:

- Compartment width replaces floor width
- Package length (at defined package width):
  - defined by the door-/compartment width dimensions
  - at corner C_L - see 3.1
  - above standard clearances considered

3.2.3 For containerised/palletised compartments, manipulation with floor contact.

- 3.2.1 applies
- However cargo loading system components i.e.:
  - guides
  - latches (Z-restraint),
  - Power Drive Units,
  - etc.

  and possible trajectories plus adequate clearances have to be taken into account.
- This could be waived, when i.e. the package is loaded with auxiliary equipment, to pass above those cargo loading system components, which reduce the usable height.

3.2.4 For containerised/palletised compartments, manipulation with packages lifted parallel to floor.

- 3.2.2 applies

3.2.5 The package length can be limited by the compartment length, clearances to bulkheads of 50.8 mm (2 in) accounted for.

NOTE: The Maximum Package Size tables are assuming loading/positioning parallel to A/D.

3.2.6 Constraints which have to be considered for practical loading are:

- Weight,
- Possible trajectories and turning limitations,
- Manpower,
- Weight distributions,
- Securing requirements,
- Conveyance capability,
- Access and restraints especially when loading multiple packages

3.3 Mathematical Solution:

3.3.1 Introduction: For the mathematical purpose, it is assumed that a rectangular package is to be loaded into a room through an opening located at the longitudinal side. Heights of package and loading room (i.e., the third dimension) are not considered. Loading is done by pushing the package without any tilting or lifting (rectangular movement).

The mathematical theory is provided in the annexes.

3.3.2 "Rectangular compartment" (constant cross section + rectangular horizontal section)

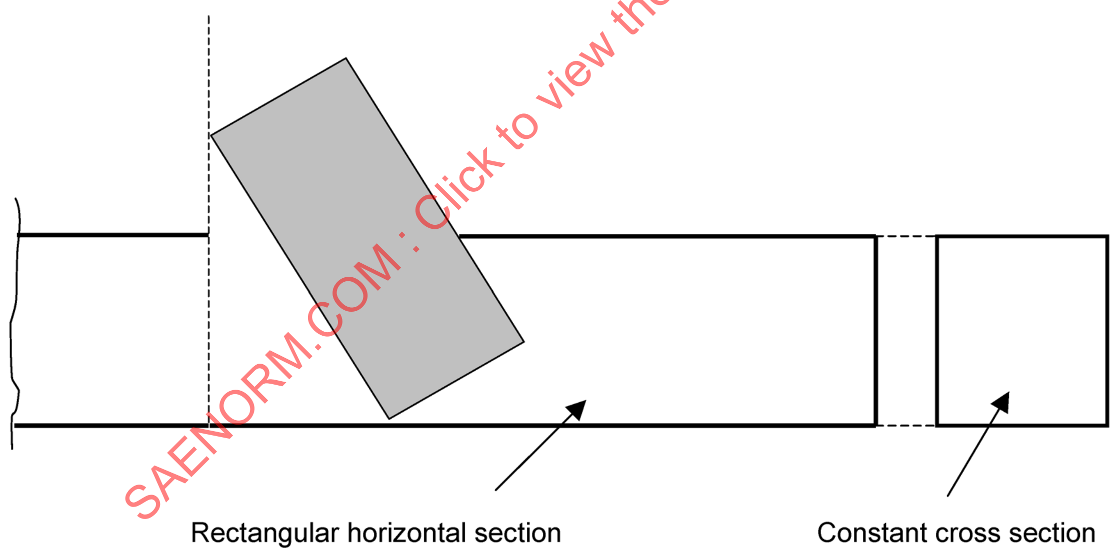


FIGURE 2

The method using the "rectangular compartments" does not necessarily apply to a perfect rectangular horizontal section compartment. In fact, the rectangular section has to be inscribed in the dimensions of the compartment.

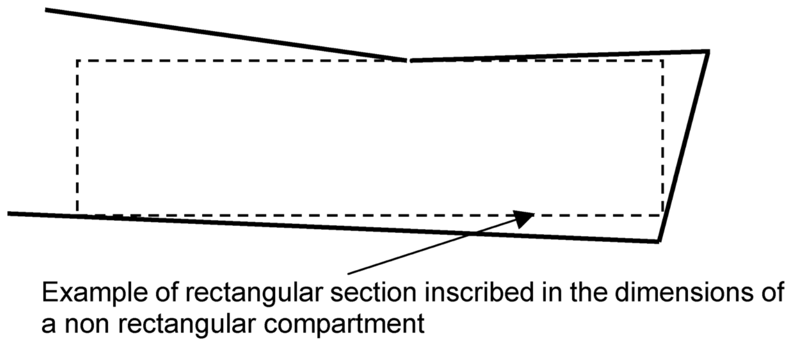


FIGURE 3

3.3.3 "Tapered shape Compartment" (variable cross section + tapered horizontal section)

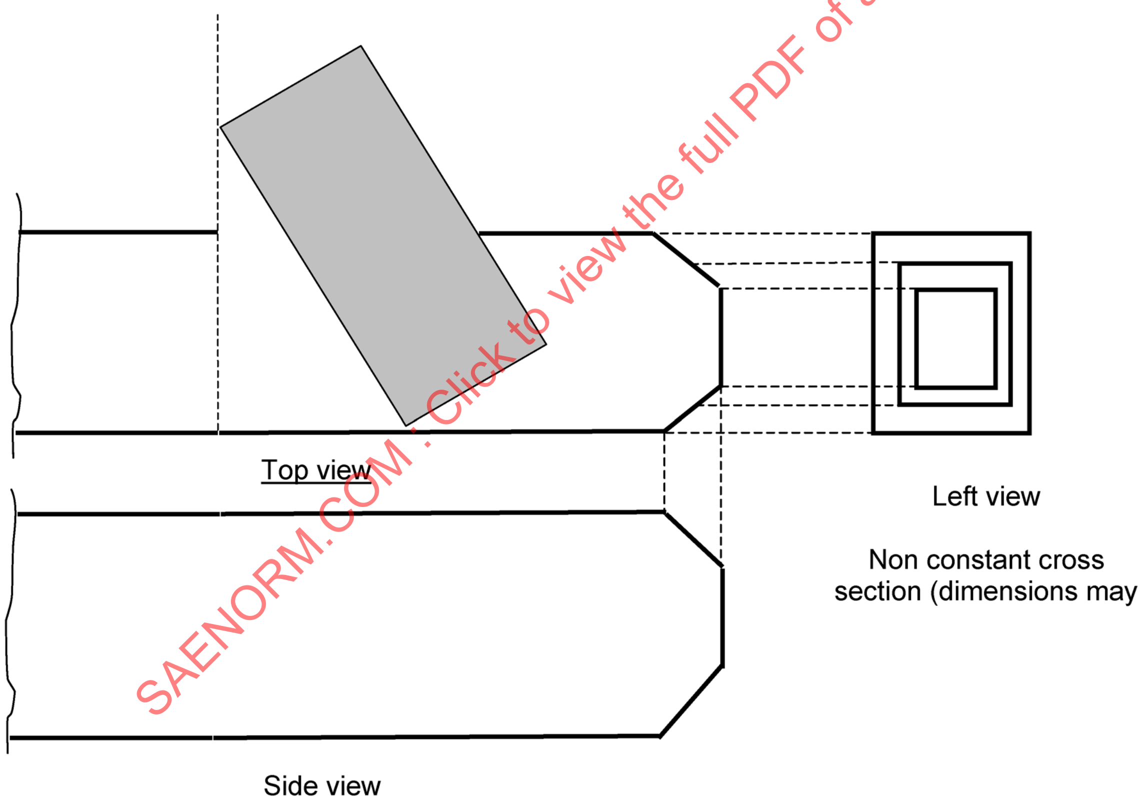


FIGURE 4

The method using the tapered cargo compartment refers to a cross section that may change from frame to frame. Therefore, the cross section dimensions have to be given if necessary for each frame.

3.4 Influences of Compartment Width:

The main factor for all cargo compartment max. package size calculations is the floor width:

The floor width cannot be exceeded by the package bearing base width.

3.5 Influences of Modified Clearances:

Modifying clearances results in new Door-/Floor width proportions that can be recalculated.

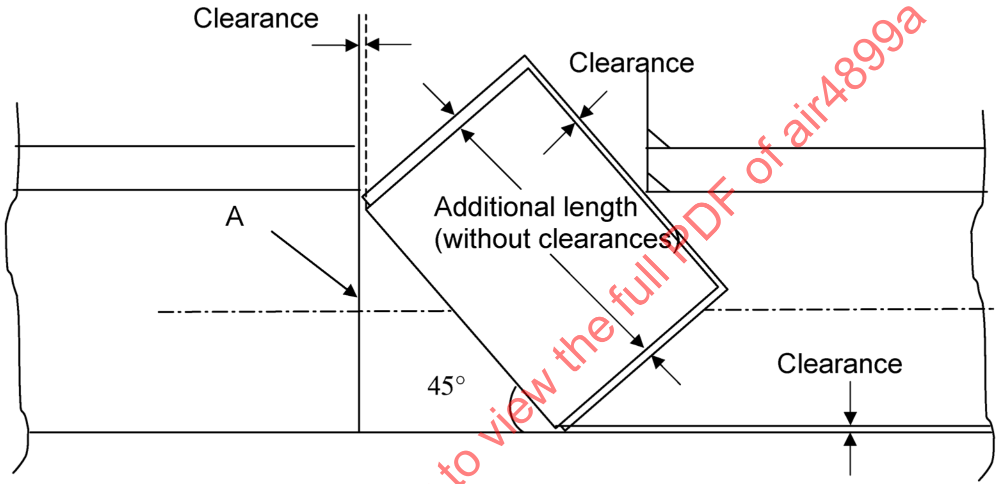


FIGURE 5

4. RECTANGULAR CARGO COMPARTMENT EXAMPLE:

4.1 Rectangular Cargo Compartment Dimensions:

The maximum dimensions of the packages that are loadable into the rectangular cargo compartments are shown in the tables below. The following dimensions are approximate and referring to rectangular package sizes.

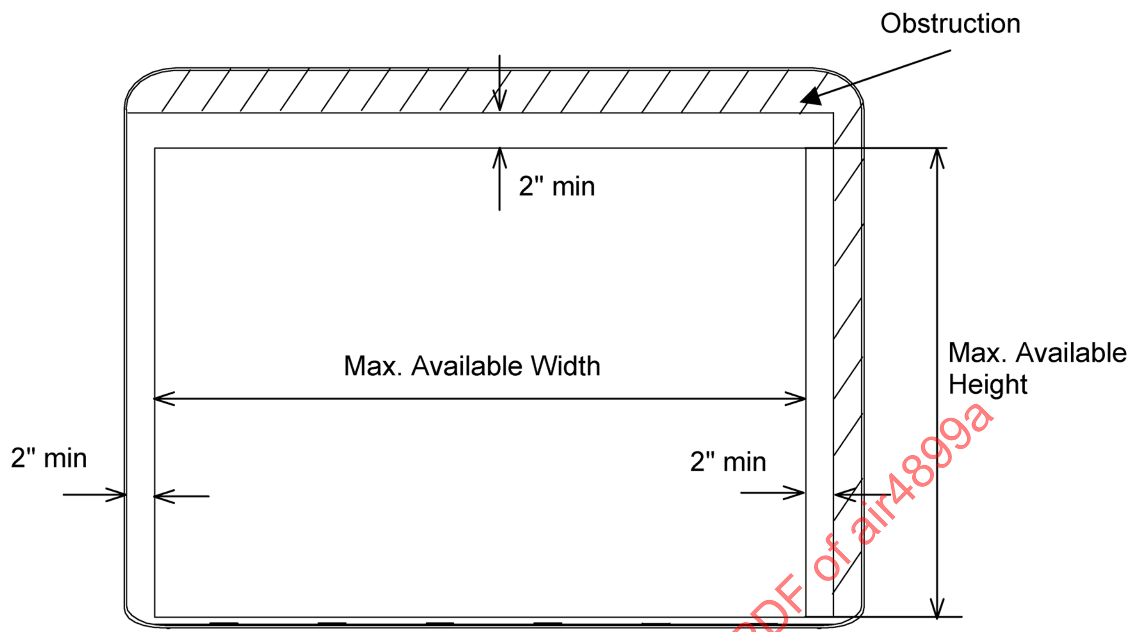


FIGURE 6

Cargo Compartment, usable length 7984 mm

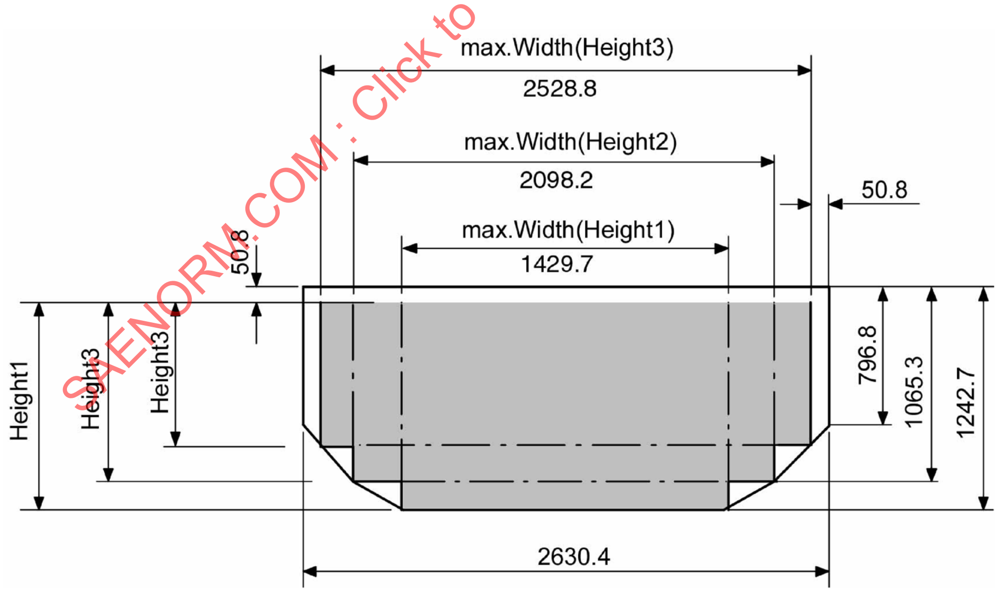


FIGURE 7

4.2 Rectangular Cargo Compartment MPS Tables:

	RANGE 1		RANGE 2		RANGE 3	
PACK HEIGHT MAX. (mm)	1184.8		1014.5		746	
FLOOR WIDTH MAX. (mm)	1430		2098		2529	
DOOR WIDTH MAX. (mm)	1651		1651		1651	
FLOOR LENGTH (mm)	7984		7984		7984	
PACK WIDTH	PACK LENGTH	ANGLE	PACK LENGTH	ANGLE	PACK LENGTH	ANGLE
0	4353	43.6	5290	47.3	5867	49.1
100	4153	43.5	5089	47.4	5665	49.3
200	3953	43.4	4888	47.5	5463	49.5
300	3752	43.3	4687	47.7	5260	49.7
400	3552	43.2	4486	47.9	5057	50.0
500	3351	43.0	4285	48.1	4854	50.3
600	3151	42.8	4084	48.3	4650	50.6
700	2950	42.6	3883	48.5	4446	51.0
800	2749	42.3	3681	48.9	4241	51.5
900	2548	41.9	3479	49.2	4036	52.0
1000	2347	41.4	3277	49.7	3829	52.6
1100	2145	40.7	3074	50.2	3621	53.4
1200	1942	39.7	2870	50.9	3411	54.3
1300	1738	38.0	2665	51.8	3198	55.5
1400	1651	0.0	2458	53.1	2982	57.2
1500			2247	55.1	2758	59.6
1600			2098	90.0	2529	90.0

FIGURE 8

4.3 Rectangular Cargo Compartment MPS Curves:

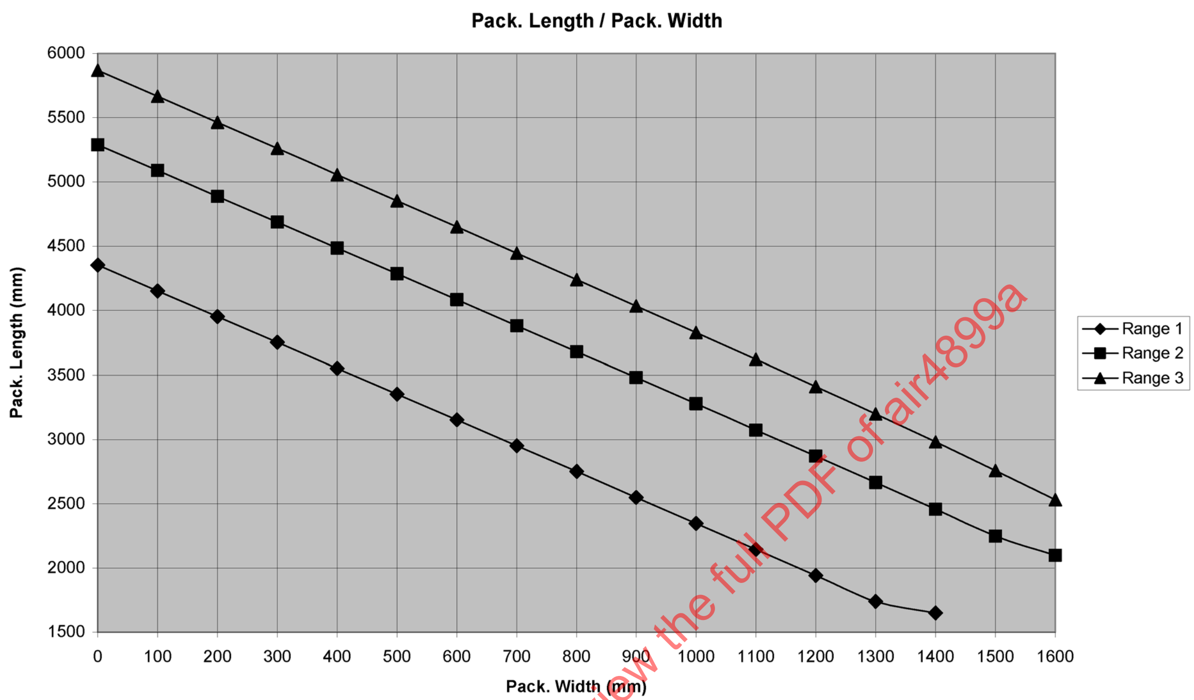
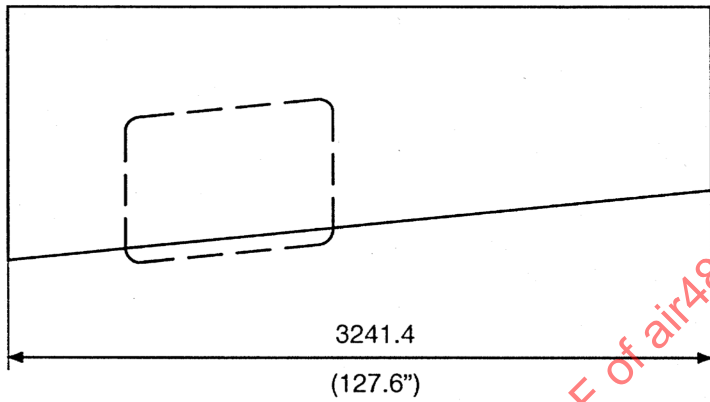


FIGURE 9

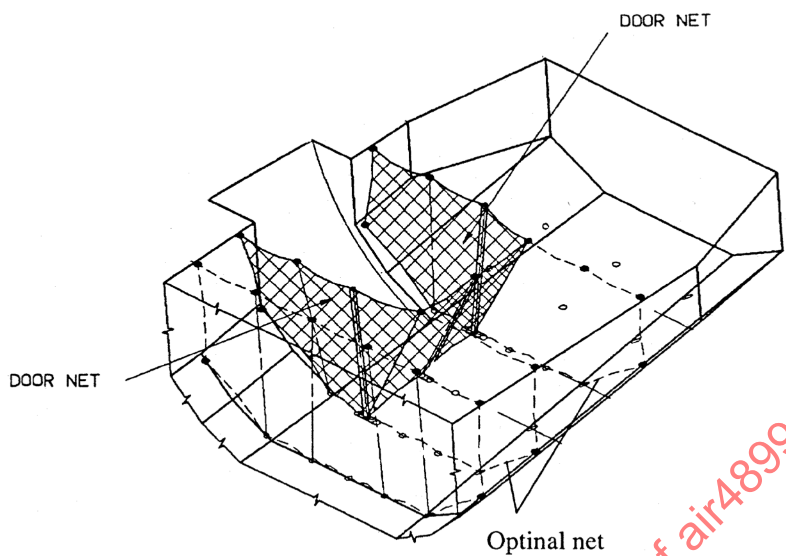
5. TAPERED CARGO COMPARTMENT EXAMPLE:

5.1 Tapered Compartment Dimensions:



Cargo Door Example

FIGURE 10



Tapered Cargo Compartment Design Example

FIGURE 11

Cargo Door Dimensions

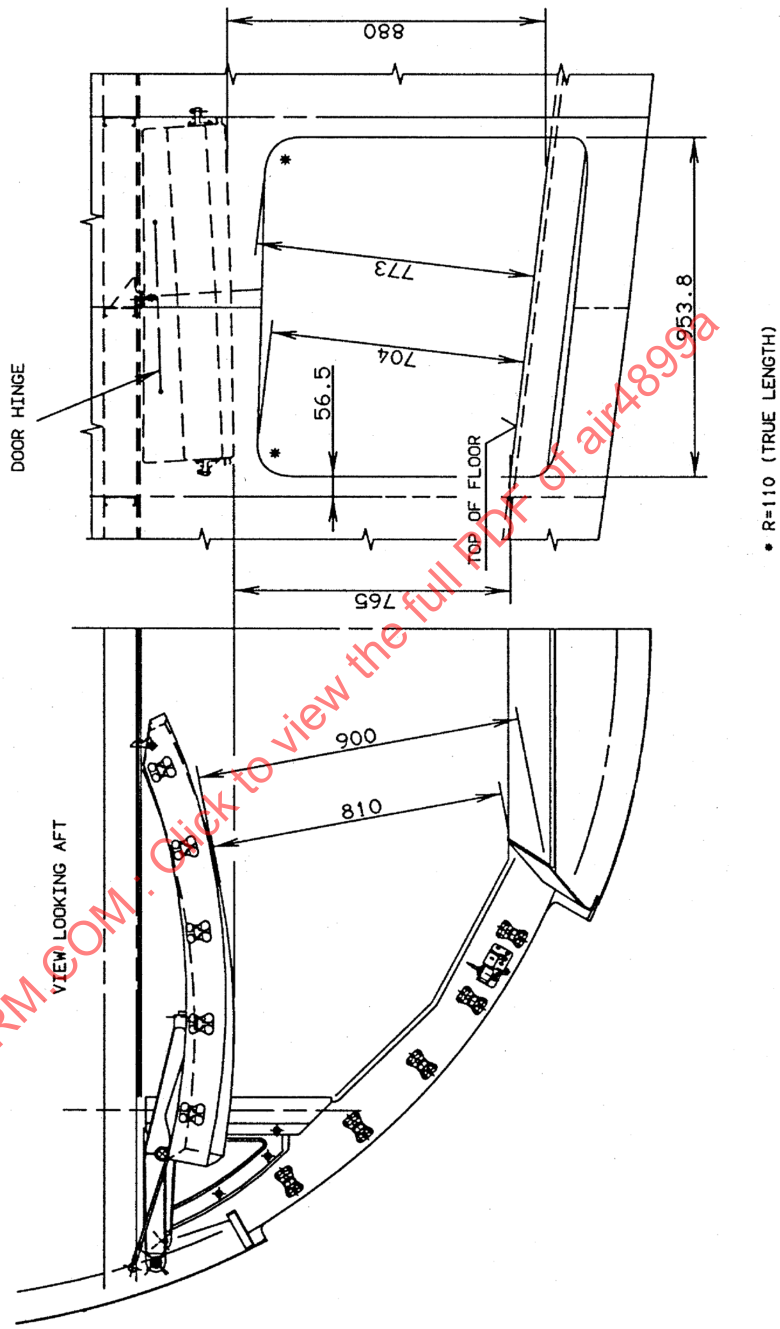
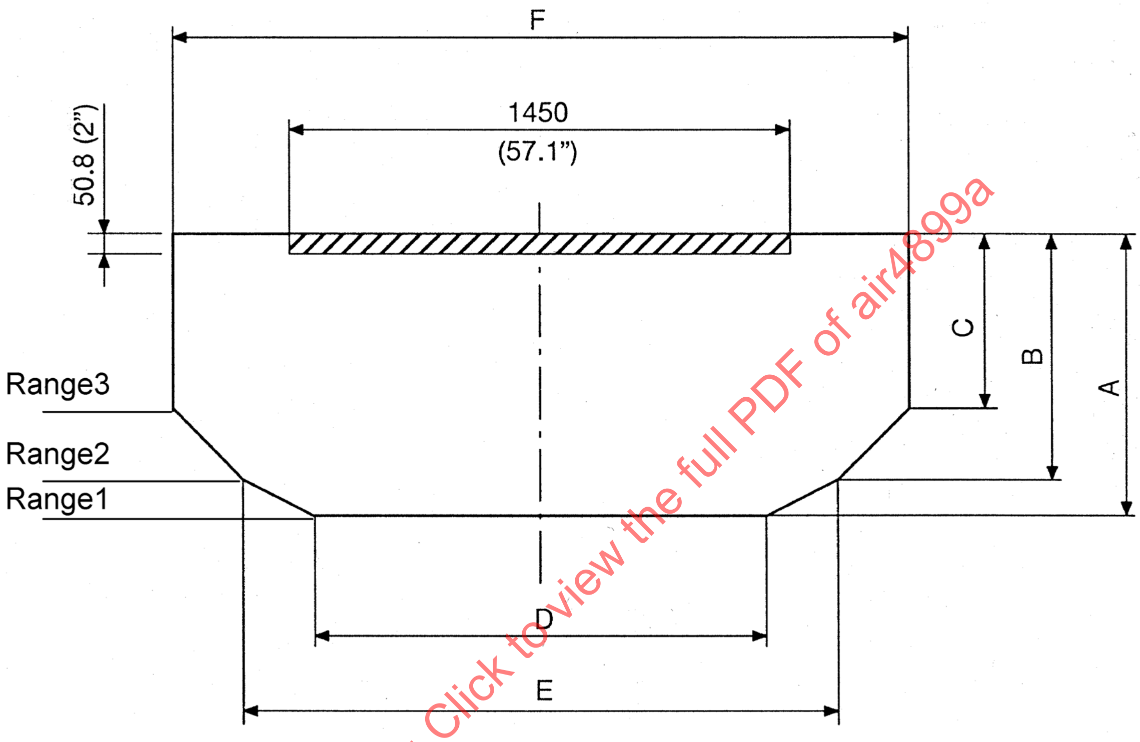


FIGURE 12

5.1 (Continued):

In the Tapered Cargo Compartment the cross section dimensions change from frame to frame. Therefore the cross section dimensions are given in a table referring to each frame.



Frame	A	B	C	D	E	F
X1	1211	1027.4	756	1408.2	2100.4	2630.4
X2	1147.5	953.2	681.9	1368.2	2100.4	2630.4
X3	1084	879	607.7	1328.2	2100.4	2630.4
X4	1020.4	804.8	533.5	1288	2100.4	2630.4
X5	956.9	730.7	459.4	1248	2100.4	2630.4
X6	891.2	698.6	557.9	1113.4	1839.3	2112.6
X7	820.5	NA	664	968.4	NA	1558.2

NA: Not Applicable

FIGURE 13

	RANGE 1		RANGE 2		RANGE 3	
PACK. HEIGHT MAX. (mm)	A		B		C	
FLOOR WIDTH MAX. (mm)	D		E		F**	
DOOR WIDTH MAX. (mm)	762		762		762	
FLOOR LENGTH * (mm)	2708		2708		2708	
PACK. WIDTH	PACK. LENGTH	ANGLE	PACK. LENGTH	ANGLE	PACK. LENGTH	ANGLE
100	2640	48.3	3067	29.9	3321	38.6
200	2455	49.0	3021	30.4	3282	39.6
300	2269	49.9	2977	30.9	3246	40.7
400	2081	51.1	2933	31.6	3214	42.0
500	1891	52.7	2891	32.5	3189	43.5
600	1695	55.1	2552	63.6	3171	45.3
700	1484	63.1	2284	68.7	3150	47.4

* at the aft door frame
** 1 in. clearances (both side) included in the calculation

FIGURE 14

5.1 (Continued):

NOTE: Considering a rectangular loading parallel to the floor, the full width of the door is not taken into account for the calculations. (see figure)

In case of short packages, the maximum usable door width can be increased by loading them horizontally.

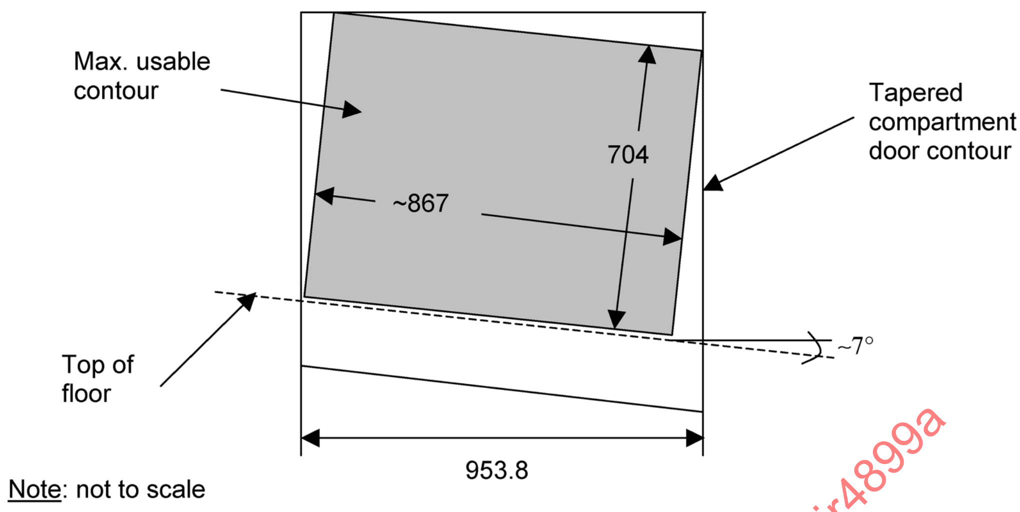


FIGURE 15

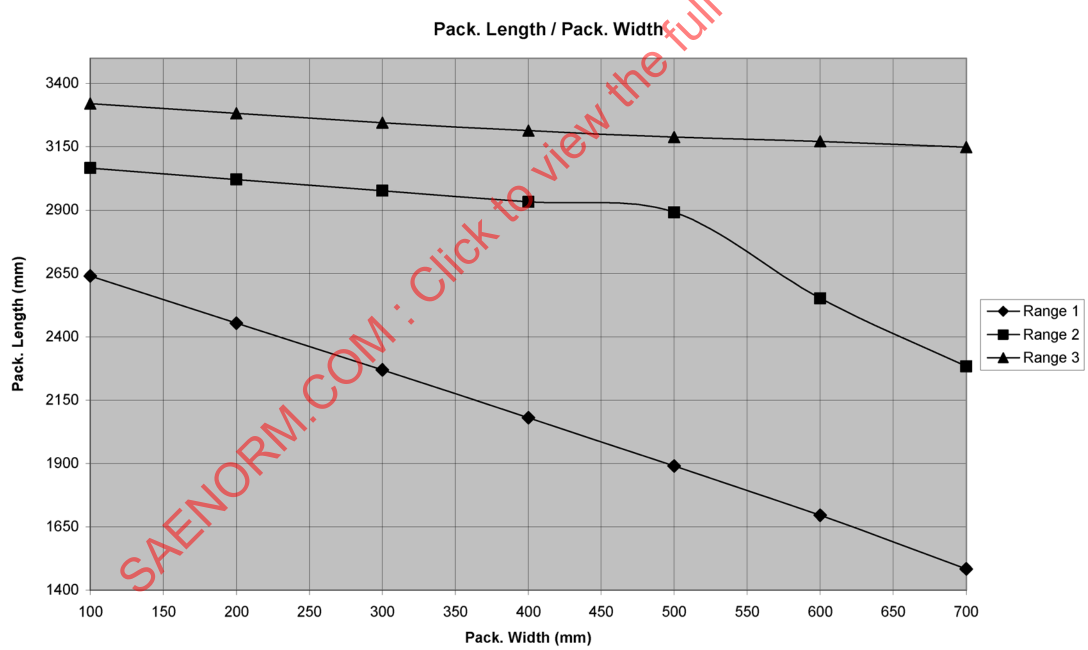


FIGURE 16

APPENDIX A
CALCULATION OF MAXIMUM PACKAGE LENGTHS FOR RECTANGULAR AND TAPERED COMPARTMENTS (THEORY)

A.1 INTRODUCTION:

A rectangular package is to be loaded into a (rectangular) room through a door located at the end of the longitudinal side. Heights of package and loading room (i.e., the third dimension) are not considered. Loading is done by pushing the package without any tilting or lifting (rectangular movement).

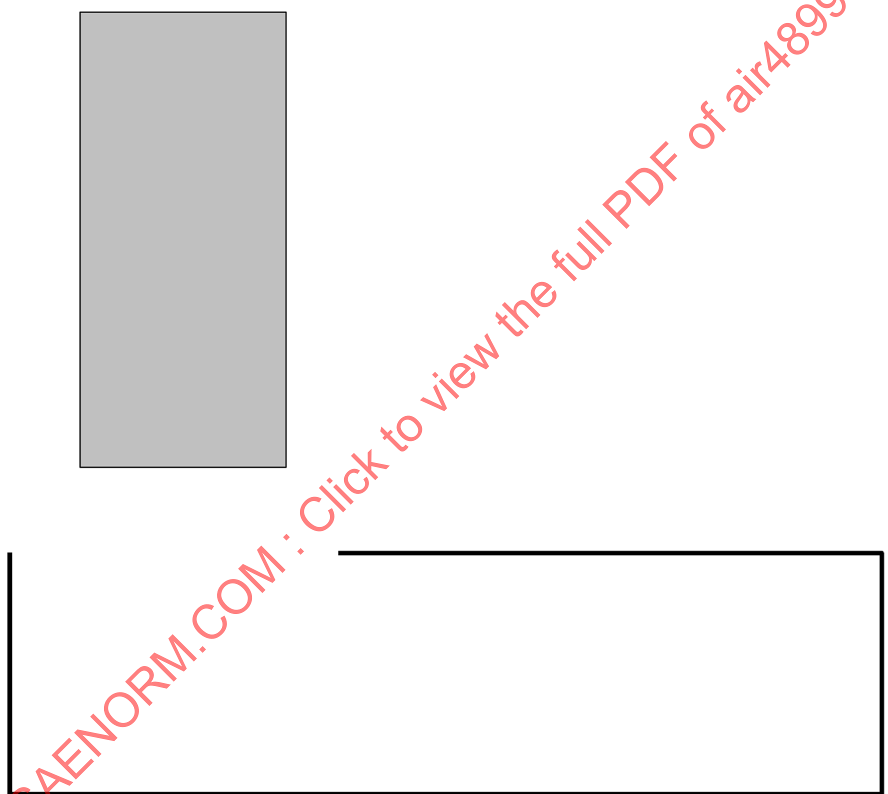


FIGURE A1

Given any values for loading room dimensions and package width, what is the maximum package length so that this package is still loadable?

In the following sections this problem is investigated for airplane rectangular and tapered compartments as loading rooms. Mathematical formulas are derived for calculation of the optimal package length, and for each compartment type an algorithm is presented which supports writing a corresponding computer program.

A.1 (Continued):

Some general fundamentals about the loading technique shall be mentioned in this introduction. The package is pushed straight to the door first and then moved around the doorframe until it reaches a longitudinal position in the room. As soon as the rear edge (i.e., the corner) of the package reaches the doorframe, the package length is definitely computable; but to get finite values before, the package length is previously limited by prolongation of the compartment wall. In practice, such a limit is achieved by using belt loaders.

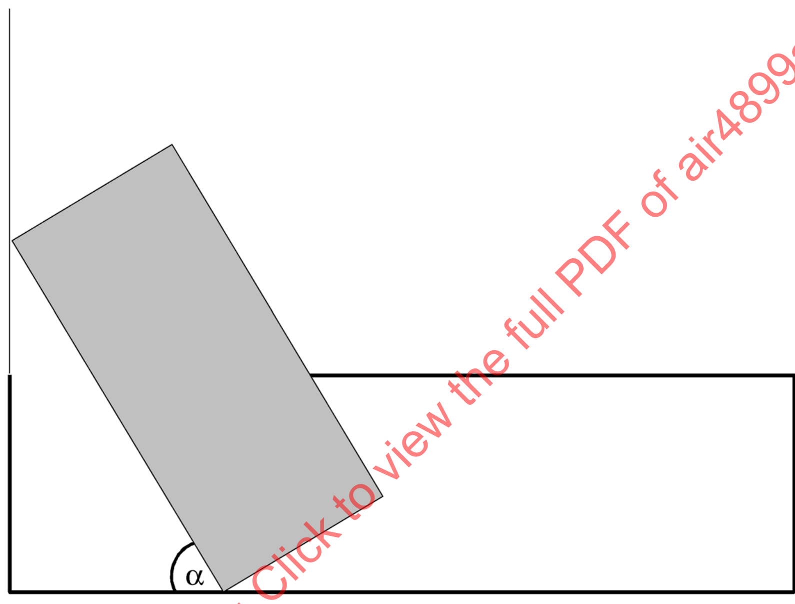


FIGURE A2

Let α denote the angle between the longitudinal edge of the package and the room wall. The package length can be computed by a mathematical function dependent only on α, and to solve the maximum package size problem one has to determine the angle in which the package has its minimum length.

A.2 LOADING INTO RECTANGULAR COMPARTMENTS:

This section deals with a rectangular loading room, as in case of an airplane rectangular compartment. Given are length (f) and width (a) of the loading room and width (b) of the door opening. Let (p) denote the width of the package which is to be loaded. Necessary conditions for passing the door and getting completely into the room are $p<b$ and $p<a$.

First investigation: 'special' package with width p=0.

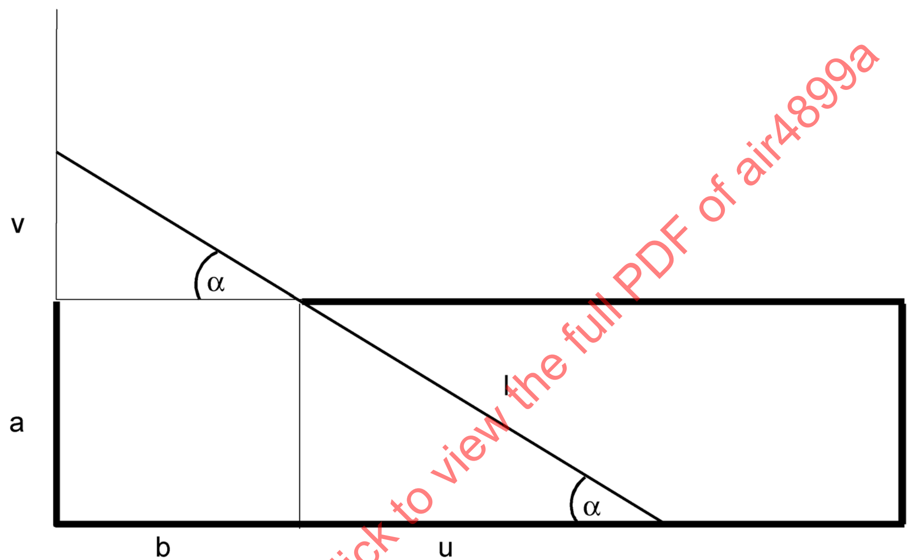


FIGURE A3

As long as the package moves along the longitudinal compartment wall, length l can be computed using Pythagoras' Theorem:

$$l = \sqrt{a^2 + u^2} + \sqrt{b^2 + v^2}$$

With

$$u = a \cot \alpha, \; v = b \tan \alpha$$

$$\Rightarrow l = a\sqrt{1 + \cot^2\alpha} + b\sqrt{1 + \tan^2\alpha} = \frac{a}{\sin \alpha} + \frac{b}{\cos \alpha}$$

After reaching the end of the compartment, anaother formula for the calculation of l becomes applicable.

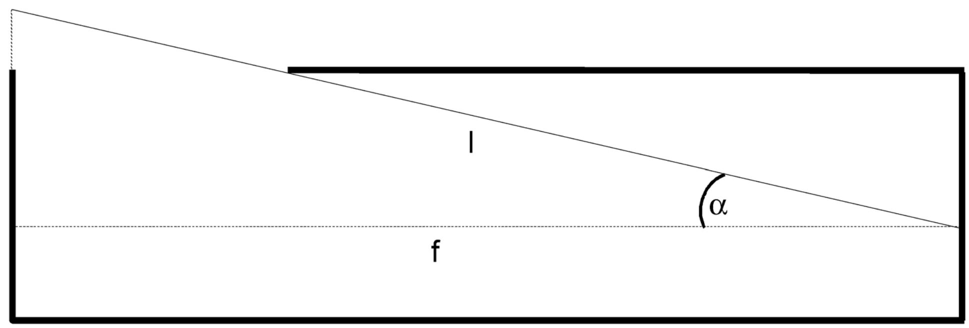


FIGURE A4

A.2 (Continued):

$$\cos \alpha = \frac{f}{l} \Rightarrow l = \frac{f}{\cos \alpha}$$

Calculation of the package length does not depend anymore on the fact that the package has contact to the door frame.

changing from the first formula to the second one is necessary when the package reaches the corner of the compartment.

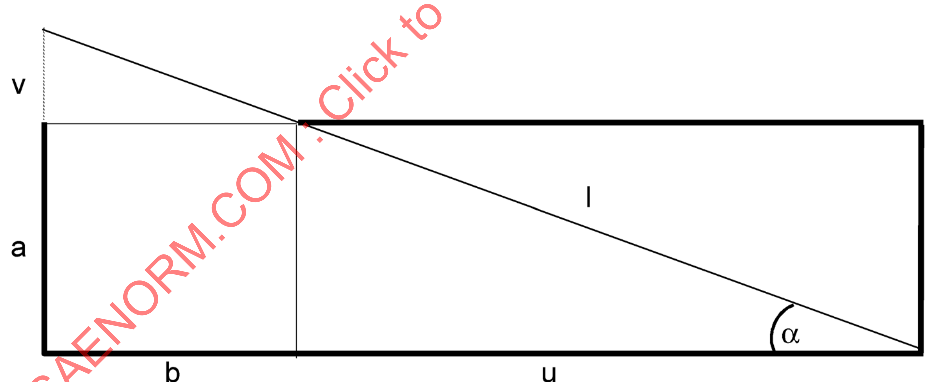


FIGURE A5

The corresponding angle $\tilde{\alpha}$ can be determined:

$$b + u = f \Rightarrow b + a \cot \tilde{\alpha} = f \rightarrow \tilde{\alpha} = \text{arc cot}\left(\frac{f - b}{a}\right)$$

A.2 (Continued):

The computation of the package length for the case p=0 can be concluded as:

$$l = \frac{a}{\sin\alpha} + \frac{b}{\cos\alpha} \qquad \text{if } \alpha > \tilde{\alpha}$$

$$l = \frac{f}{\cos\alpha} \qquad \text{if } \alpha \leq \tilde{\alpha}$$

In the general case p=0, two situations cause two different formulas for the package length:

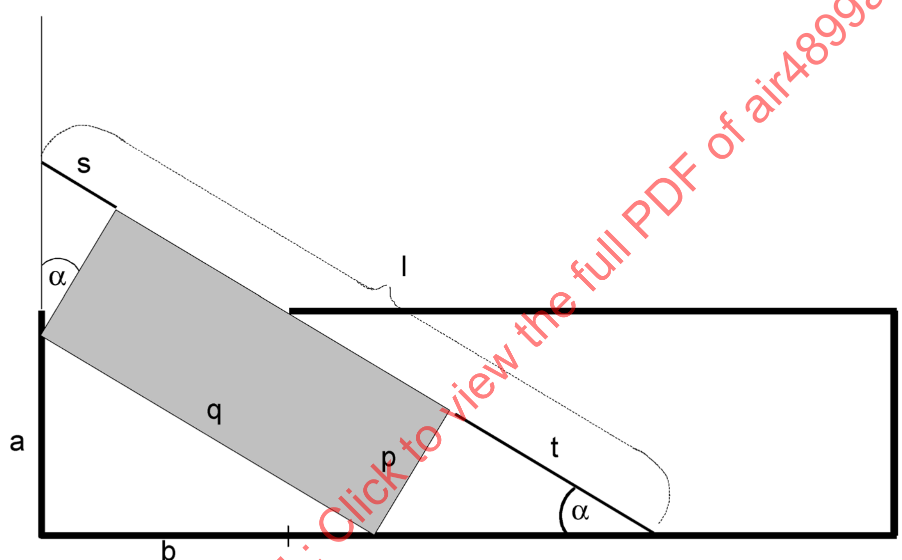


FIGURE A6

In the situation above the package length q can be expressed as

$$q = l - s - t$$

For variable l refer to the case p=0, variables s and t can be determined using trigonometrical relations:

$$s = p \tan\alpha, \; t = p \cot\alpha = p\,\frac{1}{\tan\alpha}$$

A.2 (Continued):

The first formula for the package length q dependent on angle α is:

$$q = l - s - t$$

$$= \frac{a}{\sin\alpha} + \frac{b}{\cos\alpha} - p\tan\alpha - \frac{p}{\tan\alpha}$$

$$= \frac{a}{\sin\alpha} + \frac{b}{\cos\alpha} - \frac{p}{\sin\alpha\cos\alpha}$$

The situation when the package has reached the end of the compartment leads to the second formula:

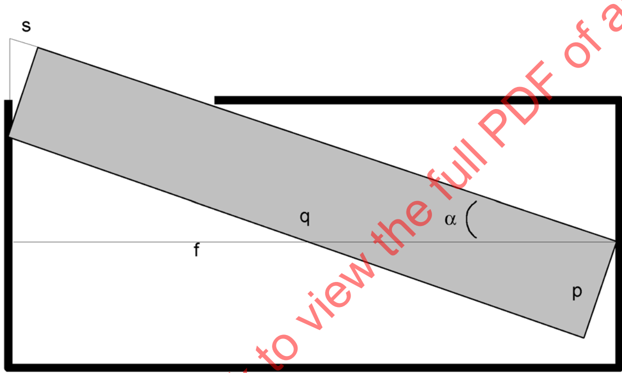


FIGURE A7

Here (compare the case p=0):

$$q = l - s = \frac{f}{\cos\alpha} - \frac{p\sin\alpha}{\cos\alpha} = \frac{f - p\sin\alpha}{\cos\alpha}$$

The situation, in which calculation of q changes, is shown below. One corner of the package still has contact to the longitudinal compartment wall, the other corner already touches the end of the compartment.

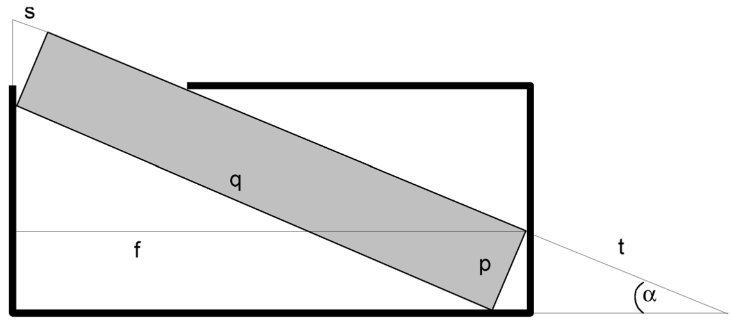


FIGURE A8

A.2 (Continued):

The corresponding angle $\tilde{\alpha}$ can be derived from the fact that in this situation both formulas for the package length q are valid.

$$q = l - s - t \wedge q = \frac{f}{\cos\alpha} - s$$

$$\Rightarrow \frac{a}{\sin\alpha} + \frac{b}{\cos\alpha} - \frac{p}{\tan\alpha} = \frac{f}{\cos\alpha}$$

$$\Leftrightarrow \frac{a\cos\alpha + (b-f)\sin\alpha - p\cos^2\alpha}{\sin\alpha\cos\alpha} = 0$$

The root of the numerator is the required angle $\tilde{\alpha}$.

F_{SC} denotes the function, which computes to given α the corresponding package length q:

$$F_{SC}(\alpha) = \frac{a}{\sin\alpha} + \frac{b}{\cos\alpha} - \frac{p}{\sin\alpha\cos\alpha} \quad \text{if } a > \tilde{\alpha}$$

$$F_{SC}(\alpha) = \frac{f - p\sin\alpha}{\cos\alpha} \quad \text{if } a \leq \tilde{\alpha}$$

To search for the global minimum of F_{SC}, the minima of both partial functions of F_{SC} has to be computed to take the lower one.

For minimization we need the roots of the first derivation of F_{SC}:

A.2 (Continued):

For $a > \tilde{\alpha}$:

$$F'_{S'C}(\alpha) = \frac{-a\cos^3\alpha + b\sin^3\alpha + p(\cos^2\alpha - \sin^2\alpha)}{\sin^2\alpha \cdot \cos^2\alpha}$$

$$= \frac{-a\cos^3\alpha + b\sin^3\alpha + p(1 - 2\sin^2\alpha)}{\sin^2\alpha\cos^2\alpha}$$

In this case determining the root of $F'_{S'C}$ leads to the equation:

$$-a\cos^3\alpha + b\sin^3\alpha + p(1 - 2\sin^2\alpha) = 0$$

For $\alpha \leq \tilde{\alpha}$

$$F'_{S'C}(\alpha) = -p + \frac{(f - p\sin\alpha)\sin\alpha}{\cos^2\alpha}$$

To get the second minimum the equation has to be solved:

$$-p\cos^2\alpha + (f - p\sin\alpha)\sin\alpha = 0$$

For any given values a,b,f,p and considering the condition $0<\alpha<90$ both equations can be solved by using an adequate numerical method (for example Brent's Method).

An alternative to calculating minima of both partial functions is the Golden Section Method: this method computes a minimum of a function within a given interval by only using function values; so, the partial derivations are not needed.

α_{min} is denoted as the angle in which the function F_{SC} has its (global) minimum. The corresponding package length $q_{min} := F_{SC}(\alpha_{min})$ is the upper limit for packages that can be loaded; any package longer than q_{min} will not fit into the compartment.

At least two further conditions should be checked: if $q_{min} < a$, the package does not have to be moved around the door frame; it would be more efficient to push it straight into the compartment and then move it aside. In this case a is the upper limit for package lengths.

If $q_{min} < b$, the package is first turned around by 90° before being pushed straight into the compartment. In this case b is the upper limit.

Handling packages with a small width (compared to the compartment dimensions), it may be possible to achieve a higher limit for the package length by making a "parallel move" which finally leads to a diagonal position:

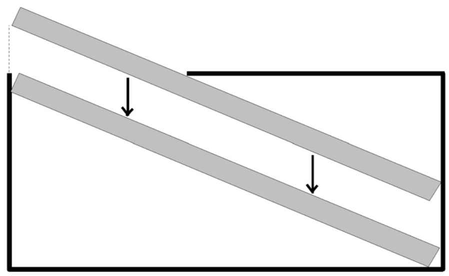


FIGURE A9

A.2 (Continued):

The angle α_P, in which this parallel movement is possible, has to be determined first.

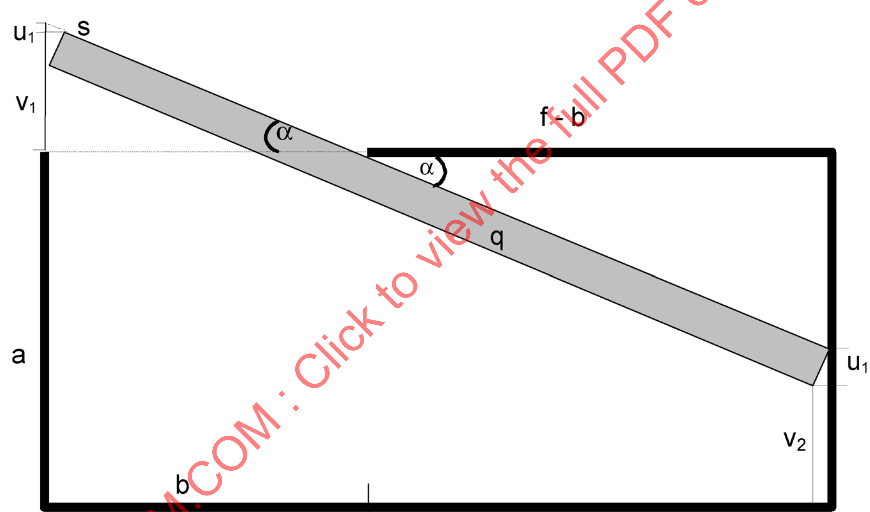


FIGURE A10

Using the notations of the illustration above:

$u_1 = s \cdot \sin\alpha$, $v_1 = b \cdot \tan\alpha - u_1 = b \cdot \tan\alpha - s \cdot \sin\alpha = b \cdot \tan\alpha - p \cdot \tan\alpha \cdot \sin\alpha$
$u_2 = p \cos\alpha$, $v_2 = a - (f - b) \tan\alpha - p \cos\alpha$

A.1 (Continued):

The "earliest" parallel move so that the package fits completely into the compartment is possible if we have $v_1 = v_2$:

$b \tan \alpha - p \tan \alpha \sin \alpha = a - (f - b) \tan \alpha - p \cos \alpha$
$\Leftrightarrow f \tan \alpha + p \cos \alpha - p \tan \alpha \sin \alpha - a = 0$

α_P is the solution of this equation. The parallel move is only efficient (i.e., leads to a higher limit for package lengths) if $\alpha_P > \alpha_{min}$. If $\alpha_P < \alpha_{min}$, the calculated maximum package size q_{min} is achieved before parallel moving is possible.

The following algorithm concludes all the results of this section. Naming of variables corresponds to above-mentioned notations.

ALGORITHM 1: Max. package Sizes in Rectangular Compartments

1. READ a,b,f,p (with p<a, p<b).
2. Calculate $\tilde{\alpha} \in [0, 90]$ as root of $a \cos \alpha + (b - f) \sin \alpha - p \cos^2 \alpha$
3. Define:

$$F_{SC}(\alpha) = \frac{a}{\sin \alpha} + \frac{b}{\cos \alpha} - \frac{p}{\sin \alpha \cos \alpha} \quad \text{if } a > \tilde{\alpha}$$

$$F_{SC}(\alpha) = \frac{f - p \sin \alpha}{\cos \alpha} \quad \text{if } a \le \tilde{\alpha}$$

4. Calculate $\alpha_1 \in [\tilde{\alpha}, 90]$ as minimum of $\frac{a}{\sin \alpha} + \frac{b}{\cos \alpha} - \frac{p}{\sin \alpha \cos \alpha}$
5. Calculate $\alpha_2 \in [0, \tilde{\alpha}]$ as minimum of $\frac{f - p \sin \alpha}{\cos \alpha}$
6. IF $F_{SC}(\alpha_1) < F_{SC}(\alpha_2)$ THEN $\alpha_{min} := \alpha_1$ ELSE $\alpha_{min} := \alpha_2$
7. Calculate $\alpha_P \in [\tilde{\alpha}, 90]$ as root of $f \tan \alpha + p \cos \alpha - p \tan \alpha \sin \alpha - a$
8. IF $\alpha_P > \alpha_{min}$ THEN $q_{min} := F_{SC}(\alpha_P)$ ELSE $q_{min} := F_{SC}(\alpha_{min})$
9. IF $q_{min} < a$ THEN $q_{min} := a$
10. IF $q_{min} < b$ THEN $q_{min} := b$
11. RETURN q_{min}

A.3 LOADING INTO TAPERED COMPARTMENTS:

The end section of tapered compartments narrow, those compartments type need a separate investigation.

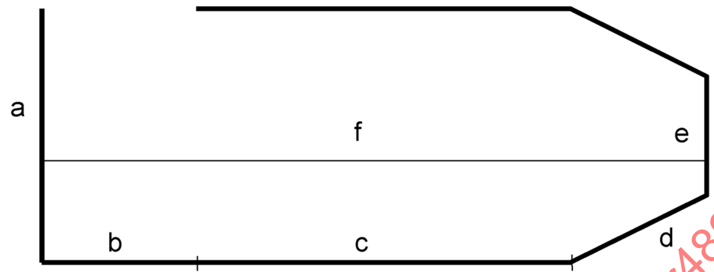


FIGURE A11

Given the values a,b,c,d,e the dimensions of a tapered compartment are completely described. The total floor length calculates as

$$f = b + c + \sqrt{d^2 - \frac{(a-e)^2}{4}}$$

Case p=0:

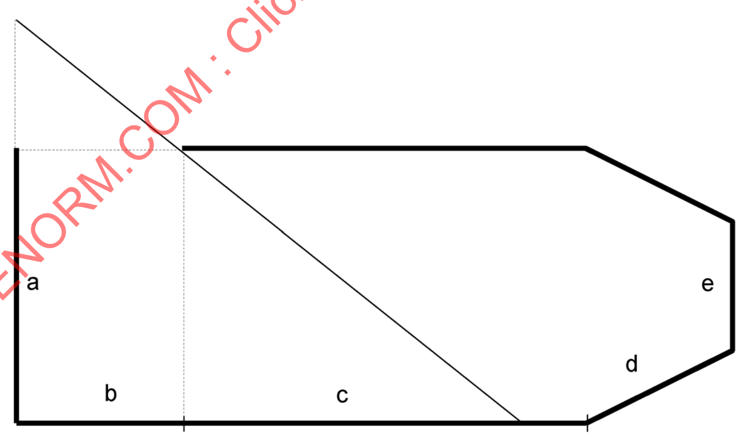


FIGURE A12

A.3 (Continued):

As long as the package moves on edge c, the package length can be computed as in a rectangular compartment.

Moving along edge d introduces a new variable:

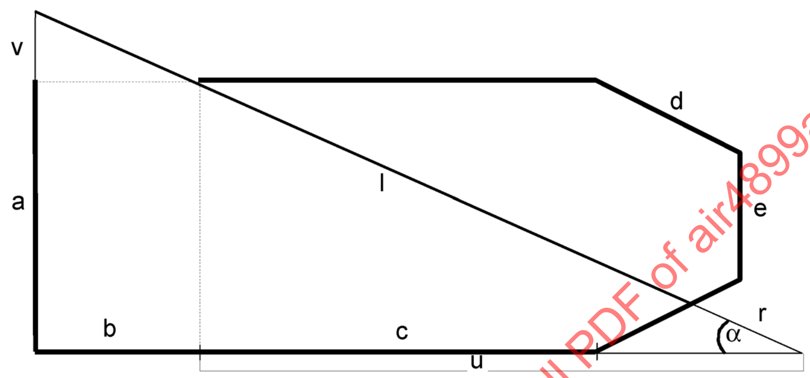


FIGURE A13

The prolongation of the package is from edge d to the base line (= prolongation of line c), u and v correspond to the same variables as in the case rectangular compartment.

Length l is computed as:

$$l = \sqrt{a^2 + u^2} + \sqrt{b^2 + v^2} - r = \frac{a}{\sin \alpha} + \frac{b}{\cos \alpha} - r$$

The following figure illustrates the calculation of r:

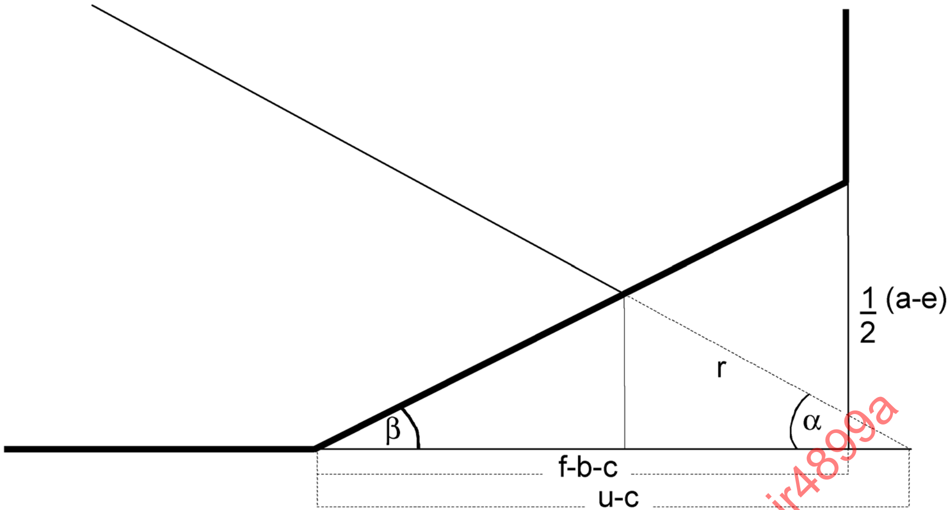


FIGURE A14

A.3 (Continued):

Using sinus relations for angle β and the angle opposite to line u-c:

$$\frac{r}{\sin\beta} = \frac{u - c}{\sin(180 - \alpha - \beta)} \Rightarrow r = \frac{(u - c)\sin\beta}{\sin(180 - \alpha - \beta)}$$

simplification

$$\sin(180 - \alpha - \beta) = \sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

and

$$\sin\beta = \frac{\frac{1}{2}(a - e)}{d} = \frac{(a - e)}{2\,d} \qquad \cos\beta = \frac{(f - b - c)}{d}$$

The formula for r is only dependent on angle α:

$$r = \frac{(u - c)(a - e)}{2\,d\left(\frac{f - b - c}{d}\sin\alpha + \frac{(a - e)}{2\,d}\cos\alpha\right)} = \frac{(a\cot\alpha - c)(a - e)}{2\,(f - b - c)\sin\alpha + (a - e)\cos\alpha}$$

This value r has to be considered as the package moves along d. The change from liine c to d (i.e., the point where the tapered compartment narrows) at angle α_{cd} can be determined:

$$u = c \Leftrightarrow a\cot\alpha_{cd} = c \Leftrightarrow \alpha_{cd} = \text{arc}\cot\frac{c}{a}$$

A.3 (Continued):

If the package has arrived on line e, $l = \frac{f}{\cos \alpha}$.

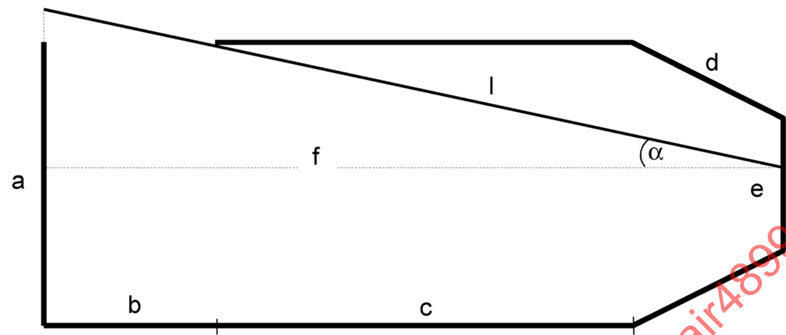


FIGURE A15

The package can only be loaded completely if it is not fixed anymore at the doorframe. For computation of the package length fixing is not required because the last formula for l does not depend on Pythagoras but only on angle α.

Also the general case p>0 is partially analogous to the case rectangular compartment. Without any restrictions $p \leq e$ is assumed.

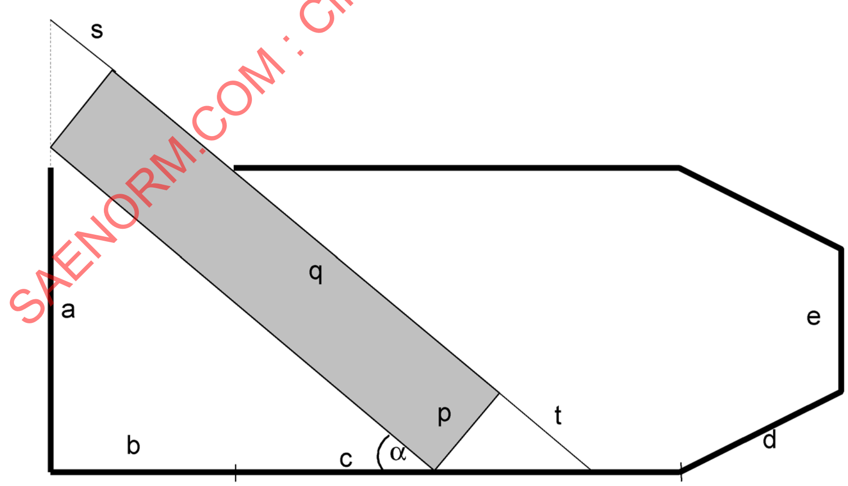


FIGURE A16

A.3 (Continued):

As long as the prolongation t hits line c, the package length q is computed with the formula

$$q = l - s - t = \frac{a}{\sin \alpha} + \frac{b}{\cos \alpha} - \frac{p}{\sin \alpha \cos \alpha}$$

This formula is also valid in the following situation:

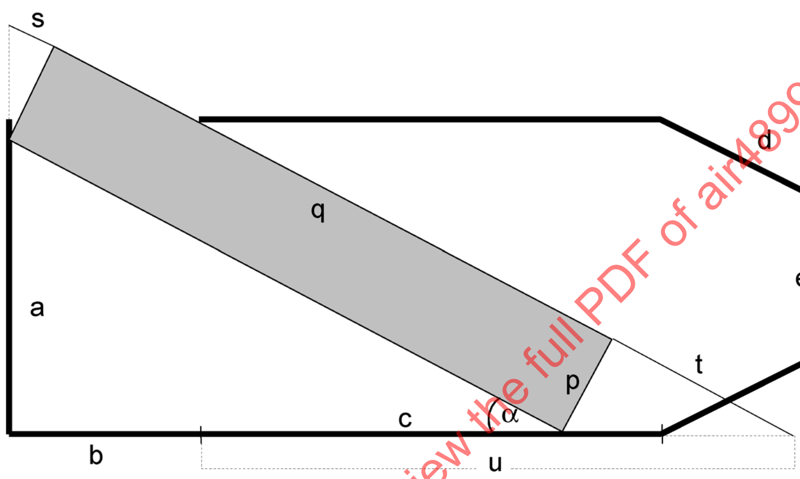


FIGURE A17

Prolongation of line c leads to the same situation as in a rectangular compartment, so the same formula can be used.

Just as the package moves along line d, the considered variable r has to be subtracted:

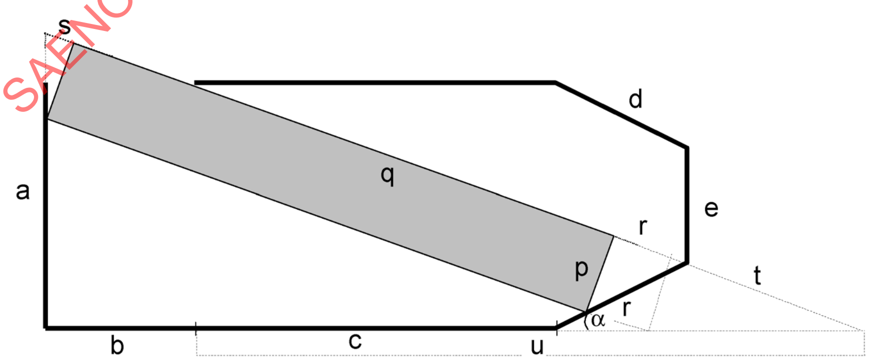


FIGURE A18

A.3 (Continued):

$q = l - s - t - r$

For p>0 the formula for variable r is more complicated. See the following figure:

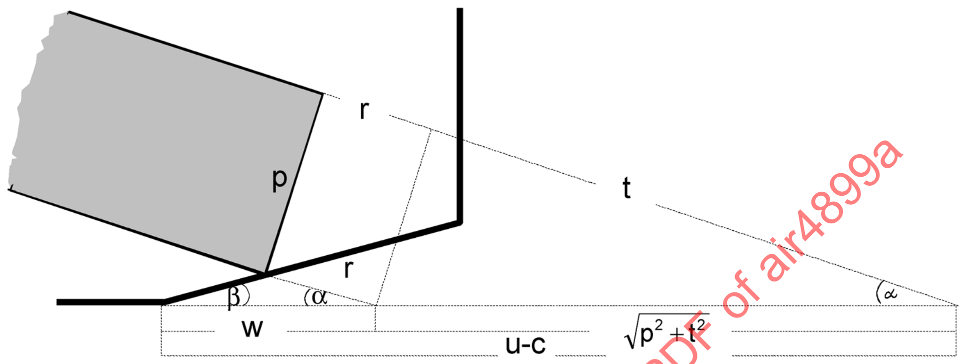


FIGURE A19

Using the same trigonometrical relations as before, the equation is:

$$\frac{r}{\sin\beta} = \frac{w}{\sin(180 - \alpha - \beta)} \Rightarrow r = \frac{w \sin\beta}{\sin(180 - \alpha - \beta)}$$

again with

$$\sin\beta = \frac{(a - e)}{2\,d} \qquad \cos\beta = \frac{(f - b - c)}{d}$$

The new variable w can be determined as

$$w = u - c - \sqrt{p^2 + t^2} = a \cot\alpha - c - \sqrt{p^2 + (p \cot\alpha)^2}$$

$$= a \cot\alpha - c - p\sqrt{1 + \cot^2\alpha} = a \cot\alpha - \frac{p}{\sin\alpha} - c$$

Finally,

$$r = \frac{w \sin\beta}{\sin(180 - \alpha - \beta)} = \frac{\left(a \cot\alpha - \frac{p}{\sin\alpha} - c\right)(a - e)}{2\,d\,(\sin\alpha \cos\beta + \cos\alpha \sin\beta)} = \frac{\left(a \cot\alpha - \frac{p}{\sin\alpha} - c\right)(a - e)}{2\,(f - b - c)\sin\alpha + (a - e)\cos\alpha}$$

and the formula for package length q is complete.

A.3 (Continued):

To determine angle α_{cd} (where the package leaves from line c to line d) the following relation is used:

$$w = 0 \iff a \cot \alpha - \frac{p}{\sin \alpha} - c = 0$$

$$\Rightarrow \alpha = 2 \arctan \left(- \frac{c \pm \sqrt{a^2 + c^2 - p^2}}{a + p} \right)$$

As $p \leq a$, $0 < \alpha_w < 90$ the solution is definitely

$$\alpha_{cd} = 2 \arctan \left(\frac{\sqrt{a^2 + c^2 - p^2} - c}{a + p} \right)$$

Moving the package along line e finally results in

$$q = \frac{f - p \sin \alpha}{\cos \alpha}$$

Remember that in this situation the package is not fixed anymore at the doorframe.

Changing from line d to line e means that the package touches both lines simultanaeously:

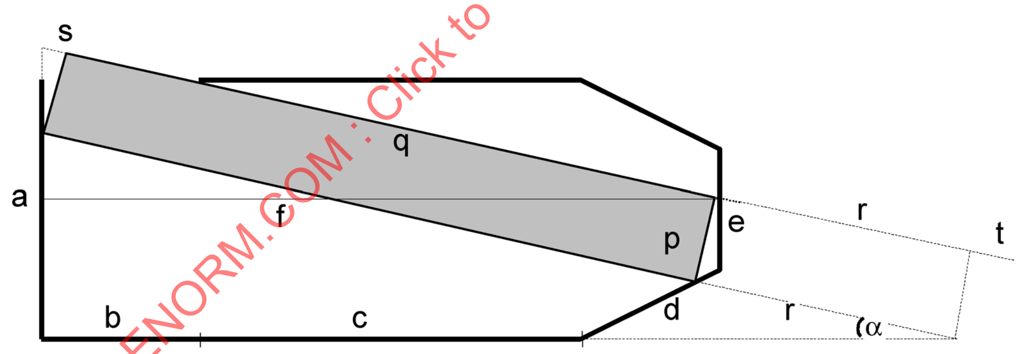


FIGURE A20