AEROSPACE INFORMATION REPORT

AIR1678™	REV. B
Issued 1985-08 Revised 2011-05 Reaffirmed 2023-03 Superseding AIR1678B	

Uncertainty of In-Flight Thrust Determination

RATIONALE

AIR1678B has been reaffirmed to comply with the SAE Five-Year Review policy.

FOREWORD

In 1972, the Safety Standardization Advisory Committees of the SAE Aerospace Council, working with the SAE Propulsion Division suggested the need for improved knowledge of aircraft propulsion system in-flight thrust. Later, the U.S. Air Force Aeronautical Systems Division independently made a similar suggestion.

The Propulsion Division and the Aerospace Council concluded that the real need was to establish a forum where this subject could be discussed by knowledgeable experts on a technical basis. SAE undertook to do so. Dr. Robert Abernethy was commissioned by George Townsend, Propulsion Division Chairman, to organize the In-Flight Propulsion Measurement Committee E-33. Mr. Gary Adams, representing the Air Force, was also involved in organizing the E-33 Committee at this early stage. The first meeting took place in December 1978.

The E-33 Committee endeavored to gather industry-wide expertise in in-flight thrust measurement and uncertainty analysis. The committee was organized into Subcommittees A/B which concentrated on thrust-drag bookkeeping and thrust determination methodology and Subcommittee C which addressed the subject of thrust determination uncertainty.

After reviewing the industry state-of-the-art, Committee E-33 determined that it would be appropriate to assemble and publish two companion Aerospace Information Reports. Subcommittee E-33A/B was organized under Chairman John Roberts to produce AIR1703, "In-Flight Thrust Determination." Subcommittee E-33C, under Chairman Gary Adams, produced AIR1678, "Uncertainty of In-Flight Thrust Determination." Together these reports provided a comprehensive survey of in-flight thrust determination, beginning with definitions and concluding with guidelines for planning a total program and estimating the measurement errors.

The original version of AIR1678 was published in 1985 and later reaffirmed in December 1992. In 1996, Subcommittee E-33C under the chairmanship of Mike McGonigle undertook a revision to the document to bring it into compliance with other national and international standards, primarily ANSI/ASME PTC 19.1-1998 and the ISO Guide to the Expression of Uncertainty in Measurement. This work was completed by the committee under the leadership of Glenn Steele and published in 2002. Revision B continued this development and was completed in 2011.

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INTRODUCTION

Estimating the measurement error or uncertainty is an essential element in the complex process of evaluating in-flight thrust. During the pre-flight planning phase, it is necessary to assess the accuracy of the candidate methodologies for determining in-flight thrust and to select those which best satisfy program requirements and resources. Estimation of measurement error defines parameter accuracy, data acquisition, and data reduction requirements for the ground and flight test phases; and post-flight is used to examine the consistency of the test results with pretest predictions and with vehicle performance estimates.

The purpose of this document is to provide information covering methods for estimating the effects of the measurement error or uncertainty of in-flight thrust determination in aircraft employing conventional turbofan/turbojet engines. Methodologies beyond those presented are required in order to evaluate configurations such as vectored thrust or V/STOL. This document is intended to be used as a technical guide. It is not intended to be used as a standard or legal document. A companion document, Reference 2.1.1.1, presents information and guidance on the selection and use of methodologies to predict and assess propulsion system in-flight thrust. Both documents describe comprehensive procedures and tasks for implementing the methodologies. Each program would select those tasks that are appropriate to meet its particular objectives.

The term "in-flight thrust determination" is used synonymously with "in-flight thrust measurement" although in-flight thrust is not directly measured but is determined or calculated using mathematical modeling relationships between in-flight thrust and various direct measurements of physical quantities. The in-flight thrust determination process includes both Ground Testing and Flight Testing, Figure 1. The mathematical modeling relationships between the in-flight thrust and the measurements of the physical quantities are calibrated in Ground Tests. Error effect estimates for each item shown in Figure 1 are required to calculate the uncertainty of the "in-flight thrust measurement."

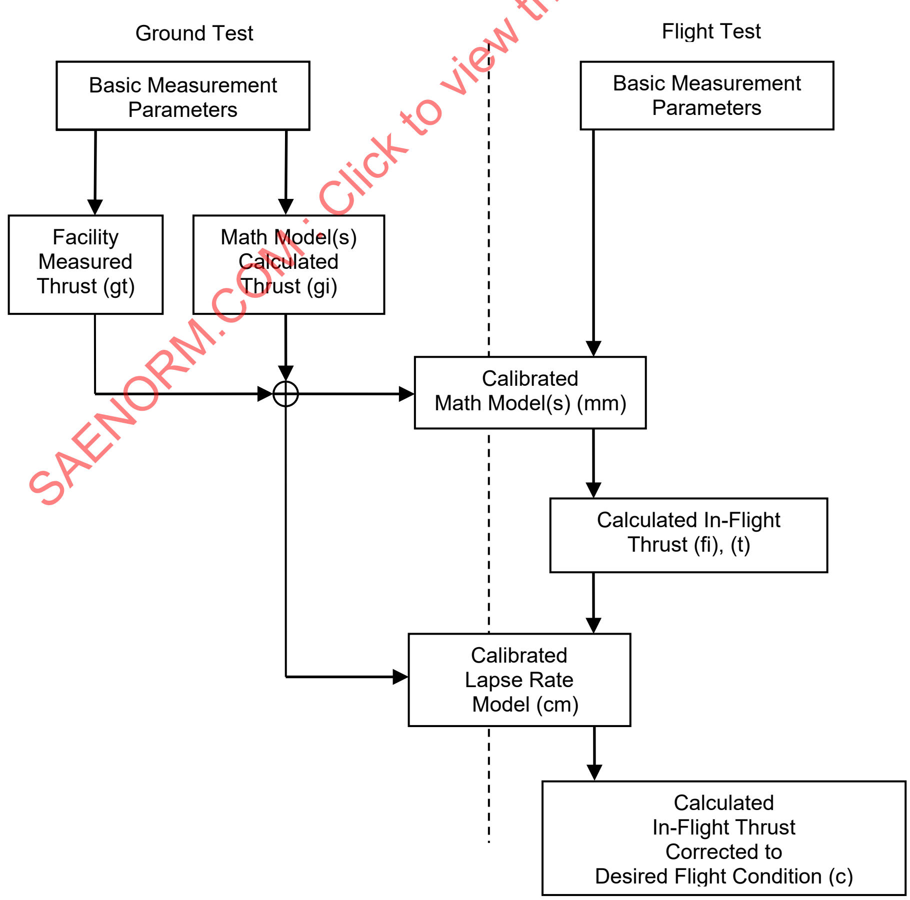

FIGURE 1 - THE IN-FLIGHT THRUST MEASUREMENT PROCESS

The text is organized into the following major topics:

Measurement Uncertainty Methodology (Section 4): This section describes the concepts involved in estimating the uncertainty of in-flight thrust. Its purpose is to provide a common basis for understanding. The methodology includes the use of statistical and engineering concepts to meet the need for an easily interpreted estimate of uncertainty (References 2.1.2.1, 2.1.4.1).

In-Flight Thrust Measurement Processes (Section 5): This section describes the ground test and flight test phases of the in-flight thrust determination process. The five sets of fundamental error components which must be evaluated to determine the uncertainty of in-flight thrust are identified.

Application of Uncertainty Methodology to the In-Flight Thrust Program (Section 6): The uncertainty methodology described in Section 4 is applied to the In-Flight Thrust Measurement Process described in Section 5. Four representative test program phases are defined, i.e., Program Definition Planning, Ground Test, Flight Test, and Results Analysis. The tasks to be accomplished in each phase are described in Reference 2.1.4.5 and shown in Table 8.

Seven Appendices are also included:

- Appendix A - Small Sample Statistical Methods

Special, small sample, statistical methods are required to determine the value for the Student’s "t_{95}" distribution when the degrees of freedom of the result is less than 10 (Reference 2.1.4.2). This appendix discusses the t value, degrees of freedom, and the propagation of degrees of freedom. A table of "t_{95}" values is presented for degrees of freedom from one to thirty.

- Appendix B - Impact of Measurement Process on Error Classification and Estimation

This appendix discusses the decision logic for the classification of error based on the actual measurement process. Reclassification is discussed and the concept of fossilized error is introduced.

- Appendix C - Example Influence Coefficient Calculations

- Appendix D - Initial Performance Survey of Thrust Measurement Options

This appendix uses the published results of the Air Launched Cruise Missile Program to illustrate the approach for conducting an initial performance survey of thrust options.

- Appendix E - Calibrated Mathematical Model and the Associated Uncertainty Components for Flight Test

The term "calibrated mathematical model" is discussed and simplified examples of two common approaches are provided.

- Appendix F – Nonsymmetric Uncertainties

- Appendix G – Background of Uncertainty Methodology

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1. SCOPE

This document defines and illustrates the process for determination of uncertainty of turbofan and turbojet engine in-flight thrust and other measured in-flight performance parameters. The reasons for requiring this information, as specified in the E-33 Charter, are:

- determination of high confidence aircraft drag;
- problem rectification if performance is low;
- interpolation of measured thrust and aircraft drag over a range of flight conditions by validation and development of high confidence analytical methods;
- establishment of a baseline for future engine modifications.

This document describes systematic and random measurement uncertainties and methods for propagating the uncertainties to the more complicated parameter, in-flight thrust. Methods for combining the uncertainties to obtain given confidence levels are also addressed. Although the primary focus of the document is in-flight thrust, the statistical methods described are applicable to any measurement process.

The E-33 Committee has endeavoured to gather industry-wide expertise in in-flight measurement and uncertainty analysis to collect and promulgate recommended practices in the subject disciplines. The Committee is organized into subcommittees to address both the analytical and test methodology for determination of in-flight thrust and also the uncertainty of the determination. This document; Uncertainty of In-flight Thrust Determination, AIR1678, addresses the process for determining the uncertainty of in-flight thrust. A companion document, In-Flight Thrust Determination, AIR1703, addresses the basic methodology for determining in-flight thrust.

The Committee, after reviewing recommended changes and clarification in definitions and application of statistical uncertainty items, made small revisions to the original document published in 1985. These changes were incorporated into AIR1678 Rev A.

This Revision B has the same Scope as preceding versions. The nomenclature and methodology used herein are now consistent with evolving world and national standards promulgated primarily by ISO and ASME.

2. REFERENCES

2.1 Applicable Documents

The following publications form a part of this document to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this document and references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

2.1.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or 724-776-4970 (outside USA), www.sae.org.

2.1.1.1 AIR1703, In-Flight Thrust Determination

2.1.1.2 831439, Application of In-Flight Thrust Determination Uncertainty, SAE Technical Paper, Adams, G. R., Thompson, J. W., Abernathy, R. B., Biesiadny, T., et al., Presented at the Aerospace Congress and Exposition, Long Beach, California, October 1983, doi:120.4271/831439.

2.1.2 ANSI Publications

Available from American National Standards Institute, 25 West 43rd Street, New York, NY 10036-8002, Tel: 212-642-4900, www.ansi.org.

2.1.2.1 ANSI/ASME PTC 19.1-2005 (Revision of ASME PTC 19.1-1998), Test Uncertainty

2.1.2.2 ISO 10012-1 1992, Quality Assurance requirements for measuring equipment - Part 1: Metrological confirmation system for measuring equipment.

2.1.2.3 ISO 1995, Guide to the Expression of Uncertainty in Measurement.

2.1.3 ASME Publications

Available from American Society of Mechanical Engineers, 22 Law Drive, P.O. Box 2900, Fairfield, NJ 07007-2900, Tel: 973-882-1170, www.asme.org.

2.1.3.1 80-GT-9, Improvement of Parameter Estimation Through Uncertainty Analysis, Afdams, G. R., ASME Paper, Presented at the Gas Turbine Conference and Products Show, New Orleans, LA, March 1980.

2.1.4 Applicable References

2.1.4.1 H. W. Coleman and W. G. Steele, "Engineering Application of Experimental Uncertainty Analysis," AIAA Journal, Vol. 33, No. 10, Oct. 1995, pp. 1888-1896.

2.1.4.2 W. G. Steele, R. A. Ferguson, R. P. Taylor, and H. W. Coleman, "Comparison of ANSI/ASME and ISO Models for Calculation of Uncertainty," ISA Transactions, Vol. 33, 1994, pp. 339-352.

2.1.4.3 P. J. Campion, J. E. Burns, and A. Williams, Section 5 Recommendations, Her Majesty's National Physical Laboratory Report "A Code of Practice for the Detailed Statement of Accuracy," 1973.

2.1.4.4 R. B. Abernethy and J. W. Thompson, Jr. "Handbook Uncertainty in Gas Turbine Measurements," AEDC-TR-73-5, February, 1973.

2.1.4.5 A. T. J. Hayward in "Repeatability and Accuracy," Mechanical Engineering Publications, Ltd.

2.1.4.6 W. G. Steele, P. K. Maciejewski, C. A. James, R. P. Taylor, and H. W. Coleman, "Asymmetric Systematic Uncertainties in the Determination of Experimental Uncertainty," AIAA Journal, Vol. 34, No. 7, July 1996, pp. 1458-1463.

2.1.4.7 K. K. Brown, H. W. Coleman, W. G. Steele, and R. P. Taylor, "Evaluation of Correlated Bias Approximations in Experimental Uncertainty Analysis," AIAA Journal, Vol. 34, No. 5, May 1996, pp. 1013-1018.

2.1.4.8 H. W. Coleman and W. G. Steele, "Implications of Correlated Bias Uncertainties in Single and Comparative Tests," Journal of Fluids Engineering, Vol. 117, Dec. 1995, pp. 552-556.

2.1.4.9 H. W. Coleman and W. G. Steele, "Experimentation, Validation, and Uncertainty Analysis for Engineers", 3nd Edition, John Wiley and Sons, New York, NY, 2009.

2.1.4.10 B. D. Couch, W. O. Boals, and B. M. Bishop "ALCM Preflight-Test Thrust Uncertainty Analysis," AEDC-TR-81-2, July, 1981.

2.1.4.11 R. B. Abernethy and Barbara Ringhiser, United Technologies Corp., "The History and Statistical Development of the New ASME-SAE-AIAA-ISO Measurement Uncertainty Methodology," AIAA paper 85-1403, July 8, 1985.

2.1.4.12 J. W. Thompson, W. F. Kimzey, and W. O. Boals, "An Overview of the Application of Standard Measurement Uncertainty Methodology to Propulsion Testing," AGARD Conference Proceedings No. 429, September 1987.

2.1.4.13 R. B. Abernethy, et al, "ICRPG Handbook for Estimating the Uncertainty in Measurements Made with Liquid Propellant Rocket Engine Systems," JANNAF (formerly ICRPG) Performance Standardization Working Group Report CPIA No. 180 (AD 851 127), April 1969.

2.1.4.14 "Uncertainty of In-Flight Thrust Determination," SAE AIR1678, 1985.

2.1.4.15 R. B. Abernethy, "SAE In-Flight Propulsion Measurement Committee E-33: Its Life and Work," SAE in Aerospace Engineering Volume 1, No. 1, July 1981.

2.1.4.16 R. B. Abernethy, et al, "Uncertainty Methodology for In-Flight Thrust Determination," SAE paper 831438, October 1983. See also SAE paper 831439 for application.

2.1.4.17 ANSI/ASME MFC-2M, "Measurement Uncertainty for Fluid Flow in Closed Conduits," 1983.

2.1.4.18 R. B. Abernethy, R. P. Benedict, and R. P. Dowdell, "ASME Measurement Uncertainty," ASME paper 83WA/FM-3.

2.1.4.19 ANSI/ASME PTC 19.1, "Measurement Uncertainty," Performance Test Codes Supplement, 1985.

2.1.4.20 AIAA Thrust-Drag Editorial Board, chaired by Dr. E. E. Covert, "Measure and Prediction of Thrust and Drag from Transports to Fighters," 1985.

2.1.4.21 D. L. Colbert, B. D. Powell, and R. B. Abernethy, "The ICRPG Measurement Uncertainty Model, "AIAA paper No. 69-734.

2.1.4.22 R. A. Abernethy and J. W. Thompson, Jr., "Uncertainty in Gas Turbine Measurements," AIAA paper No. 73-1230.

2.1.4.23 R. B. Abernethy, et al, "Uncertainty in Gas Turbine Measurements," Revised 1980 Edition, ISA I-483-3.

2.1.4.24 R. B. Abernethy and R. P. Benedict, "Measurement Uncertainty: A Standard Methodology," ISA paper 1984, 0-87664-806-5/84.

2.1.4.25 ISO TC30 SC9, "Fluid Flow Measurement Uncertainty," Draft of January 1985 (Revision to ISO 5168).

2.1.4.26 International Organization for Standardization, "International Vocabulary of Basic and General Terms in Metrology," 2nd Edition, 1993 (Geneva, Switzerland).

2.1.4.27 W. G. Steele, R. P. Taylor, R. E. Burrell, and H. W. Coleman, "Use of Previous Experience to Estimate Precision Uncertainty of Small Sample Experiments," AIAA Journal, Vol. 31, No. 10, October 1993, pp. 1891-1896.

2.1.4.28 B. N. Taylor and C. E. Kuyatt, "Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results," NIST Technical Note 1297, 1994 Edition.

2.1.4.29 Joint Committee for Guides in Metrology (JCGM), “Evaluation of Measurement Data – Supplement 1 to the “Guide to the Expression of Uncertainty in Measurement” – Propagation of Distributions using a Monte Carlo Method”, JCGM 101:2008, France, 2008.

2.1.4.30 “Measurement Uncertainty Applied to Cost-Effective Testing”, SAE AIR5925, 2002.

2.1.4.31 “Advanced Ducted Propulsor In-Flight Thrust Determination”, SAE AIR5450 2008

2.1.4.32 “Turbofan and Turbojet Gas Turbine Engine Test Cell Correlation”, SAE ARP741 Rev B, 2002.

2.2 Symbols

Greek

β	True systematic error: the fixed, systematic or constant component of the total error, δ. It is constant for every measurement and is an unknown theoretical quantity.
$\tilde{\beta}_r$	Observed systematic error between the uncalibrated calculated thrust and the facility measured thrust.
β_r	Observed systematic error between the calibrated calculated thrust and the facility measured thrust.
δ	Total error: the difference between the measurement and the true value : $\delta = \beta + \varepsilon$. It is an unknown theoretical quantity.
ε	True random error, sometimes called repeatability error or sampling error, the random component of δ. Its value is characteristic of each measurement and is an unknown theoretical quantity.
μ	Mean of the parent population (i.e. mean of the set of all population values of a parameter). An unknown theoretical quantity.
ν	Number of degrees of freedom.
σ	Standard deviation of the parent population. It is an unknown theoretical quantity.
σ^2	True variance of a population. An unknown theoretical quantity.
θ	Absolute influence coefficient or absolute sensitivity.
θ'	Relative influence coefficient or relative sensitivity.

Roman

b_X	Systematic standard uncertainty of a single measurement.
$b_{\overline{X}}$	Systematic standard uncertainty of the sample mean. Since the systematic error remains constant for every measurement, the systematic uncertainty of the sample mean equals the systematic uncertainty of a single measurement : $b_X = b_{\overline{X}}$.
B	Expanded systematic uncertainty and is the estimate of the limit of the systematic error β, for a stated confidence level, usually 95%.
B^+, B^-	Expanded systematic uncertainties or estimates of the upper and lower limits of a nonsymmetrical systematic error with a normal distribution. B^+ and B^- are the upper and lower values for the nonsymmetrical interval defined such that $[-B^-, B^+]$ contains the true systematic error with a stated confidence level, usually 95%.
F	Thrust.
f( )	Functional relationship.
K	Order of curve fit.
M_L, M_U	Range of measurements, Lower and Upper.

N_i Number of measurements or sample points or observations available for the measurement X_i (sample size).

I Number of elemental measurements used in the calculation of the performance parameter.

r Calibration model error, the absolute difference between calculated and measured performance.

r' Calibration model error, the relative difference between calculated and measured performance.

s_X Random standard uncertainty of a sample of N_i measurements.

$s_{\overline{X}}$ Random sample standard uncertainty of the mean (of a sample of N_i measurements) where $s_{\overline{X}} = \frac{s_X}{\sqrt{N_i}}$.

s_r Standard deviation of the observed calibration model error.

$\tilde{s}_r$ Standard deviation of the observed uncalibrated model error.

$\hat{s}_r$ Estimated value of calibration model error standard deviation.

t Student's t value at a specified confidence level with ν degrees of freedom, i.e, $t_{95,\nu}$.

u Combined standard uncertainty.

U Expanded uncertainty, with a defined level of confidence (usually 95 %).

U^+,U^- Upper and lower limits of a nonsymmetrical uncertainty interval.

X Value of an individual measurement.

$\overline{X}$ Mean of the sample population (also called sample mean).

Y A performance parameter.

2.3 Subscripts

c Flight test corrected to desired test conditions

cm Lapse rate correction model error

f Flight test

fi Flight instrumentation (engine)

g Ground test

gt Ground test facility instrumentation

gi Ground instrumentation (engine)

i Counter for variables (also defined as elemental measurements or basic measurements)

j Counter for the individual measurements

k Counter for sources of elemental errors and uncertainties within a category (p) p Counter for error categories

p = 1 Calibration errors
p = 2 Test Article and / or Instrumentation Installation Errors
p = 3 Data acquisition errors
p = 4 Data reduction errors
p = 5 Errors of method

m Counter for multiple results

meas Measured

mm Mathematical model error

N Net

r Calibration model error

t As-tested flight condition

2.4 Superscripts

* Fossilized random error

3. GLOSSARY

ACCURACY: This is a qualitative and not a quantitative term for the closeness or agreement between a measured value and the true value.

AVERAGE VALUE ($\overline{X}$): The arithmetic mean of N readings.

CALIBRATION: The process of comparing the response of an instrument to a standard instrument over the measurement range. Corrections may be applied in an attempt to reduce the instrument systematic error.

CALIBRATION HIERARCHY: The chain of calibrations which links or traces a measuring instrument to a primary standard.

CONFIDENCE LEVEL (COVERAGE): The probability that the true value falls within the specified limits.

DEGREES OF FREEDOM (ν): The number of independent observations used to calculate a statistic. One degree of freedom is lost for each previously calculated statistic used to calculate a new statistic.

ELEMENTAL RANDOM ERROR SOURCE: An identifiable source of random error that is a subcomponent of total random error.

ELEMENTAL RANDOM STANDARD UNCERTAINTY ($s_{\overline{X_{kp}}}$): An estimate of the standard deviation of the mean of an elemental random error source.

ELEMENTAL SYSTEMATIC ERROR SOURCE: An identifiable source of systematic error that is a subcomponent of total systematic error.

ELEMENTAL SYSTEMATIC STANDARD UNCERTAINTY ($b_{\overline{X_{kp}}}$): An estimate of the standard deviation of the mean of an elemental systematic error source.

EXPANDED UNCERTAINTY ($U_{\overline{X}}$): An estimate of the plus or minus limits of total error, with a defined level of confidence (usually 95%).

ESTIMATE: A value calculated from a sample of data as a substitute for an unknown population constant. For example, the sample standard deviation (s) is an estimate of the population standard deviation (σ).

FOSSILIZATION: Sometimes the random error from the calibration process becomes part of the fixed systematic error. This conversion from random to systematic error is indicated by an asterisk superscript.

INFLUENCE COEFFICIENT (θ): see SENSITIVITY

LABORATORY STANDARD: An instrument which is calibrated periodically against a primary standard. The laboratory standard may also be called an interlab standard.

MEASUREMENT ERROR: The difference between the measured value and the true value. It includes both systematic and random errors.

PARENT POPULATION: Total theoretical population.

PRIMARY STANDARD: A primary physical standard or one maintained at a National Standards Laboratory.

RANDOM ERROR (ε): The portion of the total error that varies randomly in repeated measurements of the true value throughout a test process.

RANDOM STANDARD UNCERTAINTY (s_X): A value that quantifies the dispersion of any measurement of the data sample

RANDOM STANDARD UNCERTAINTY OF THE SAMPLE MEAN ($s_{\overline{X}}$): A value that quantifies the dispersion of a sample mean.

REPEATABILITY ERROR: See random error.

RESIDUAL STANDARD DEVIATION : The measure of dispersion of the dependent variable about a least squares regression or curve.

RESULT: A value calculated from a number of parameters.

SAMPLE STANDARD DEVIATION : see RANDOM STANDARD UNCERTAINTY

SAMPLE SIZE (N_i): The number of independent measurements of the basic measurement X_i in a sample.

SENSITIVITY: The instantaneous rate of the change in a result due to a change in a parameter.

STANDARD DEVIATION (σ): The true measure of dispersion of a population.

STATISTIC: Any numerical quantity derived from the sample data. $\overline{X}$ and $s_{\overline{X}}$ are statistics

STUDENT'S t: A value used to estimate the uncertainty for a given confidence level.

SYSTEMATIC ERROR (β): The difference between the average of the parent population (μ) and the true value.

SYSTEMATIC STANDARD UNCERTAINTY ($b_{\overline{X}}$): A value that quantifies the dispersion of a systematic error associated with the mean.

TOTAL ERROR (δ) The true, unknown difference between the assigned value of a parameter or test result and the true value.

TRACEABILITY: The documentation of a measuring device's calibration through a chain of calibrations to a primary standard. Traceability does not guarantee accuracy but helps to assure it by the discipline of a documented calibration hierarchy.

TRANSFER STANDARD: A laboratory instrument which is used to calibrate working standards and which is periodically calibrated against the laboratory standard.

TRUE VALUE: The unknown actual value of the parameter being measured.

TYPE A UNCERTAINTY: Uncertainties are classified as Type A when data is used to calculate a standard deviation for use in estimating the uncertainty (used in Reference 2.1.2.3).

TYPE B UNCERTAINTY: Uncertainties are classified as Type B when data is not used to calculate a standard deviation, requiring the uncertainty to be estimated by other methods (used in Reference 2.1.2.3).

UNCERTAINTY INTERVAL: The interval expected to contain the true value with a defined probability: ±U for symmetrical systematic uncertainty intervals and $+U^{+}$ and $-U^{-}$ for nonsymmetrical systematic uncertainty intervals.

STANDARD UNCERTAINTY: The estimate of the expected error limit defined as $u = \sqrt{b^2 + s^2}$.

WORKING STANDARD: An instrument which is calibrated in a laboratory against an interlab or transfer standard and is used as a standard in calibrating measuring instruments.

4. MEASUREMENT UNCERTAINTY METHODOLOGY

The methodology for determining the uncertainty of in-flight thrust must be structured to combine statistical and engineering concepts in a manner which can be applied to each step in the assessment procedure. This section addresses each of these steps and formulates the appropriate methodology. The result is an easily interpreted, single value estimate of thrust uncertainty.

The essential steps in the procedure for determining thrust uncertainty are:

a. Identify all elemental error sources for each measurement in the ground test facility and in-flight related to determining thrust.

b. Preliminarily classify elemental error sources as systematic or random.

c. Estimate the magnitude of each elemental uncertainty.

d. For each flight test measurement process, make the final systematic or random classification.

e. Combine elemental uncertainties to define measurement systematic and random uncertainty components.

f. Propagate measurement uncertainty components of calculated thrust uncertainty.

g. Calculate thrust uncertainty from uncertainty components.

h. Report results.

The following section defines measurement error and its components, identifies the sources of the errors, indicates how to classify the errors and provides guidance in estimating the error components. Combining the uncertainty for the elemental error sources to define measurement uncertainty components (systematic and random) and how to propagate these uncertainty components to net thrust uncertainty is also discussed. Section 4.2 introduces the concept of measurement uncertainty obtained by combining the components of measurement uncertainties. The uncertainty interval and its interpretation are explained for a basic measurement. Guidelines are provided for validating the estimated measurement uncertainties. The final section provides guidelines for reporting the uncertainty components of the basic measurements and the uncertainty intervals of both the basic measurements and calculated thrust. Definitions are presented in Section 3.

4.1 Measurement Error

All quantities, ranging from basic measurements to calculated performance, have error. This error is the difference between the measured or calculated observable value and the true, unknown value, as illustrated in Figure 2.

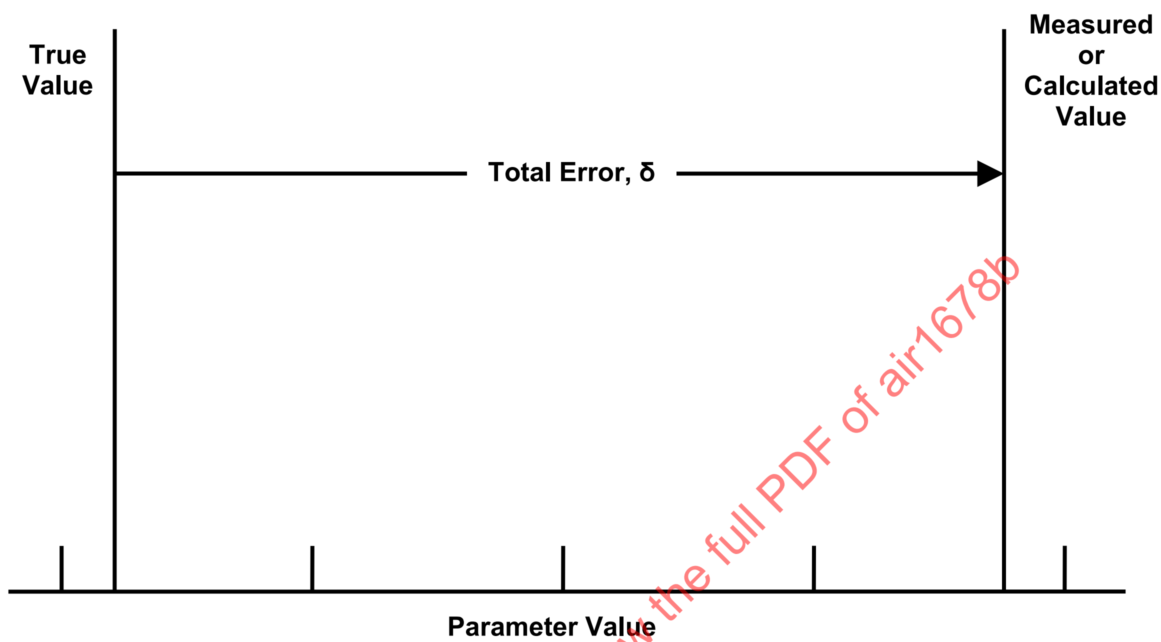

FIGURE 2 - MEASUREMENT ERROR, δ

4.1.1 Error Classification

Measurement errors can be classified as one of two components: a random error or a systematic error. Total error is the sum of a systematic error (β) and a random error (ε_j) (see Figure).

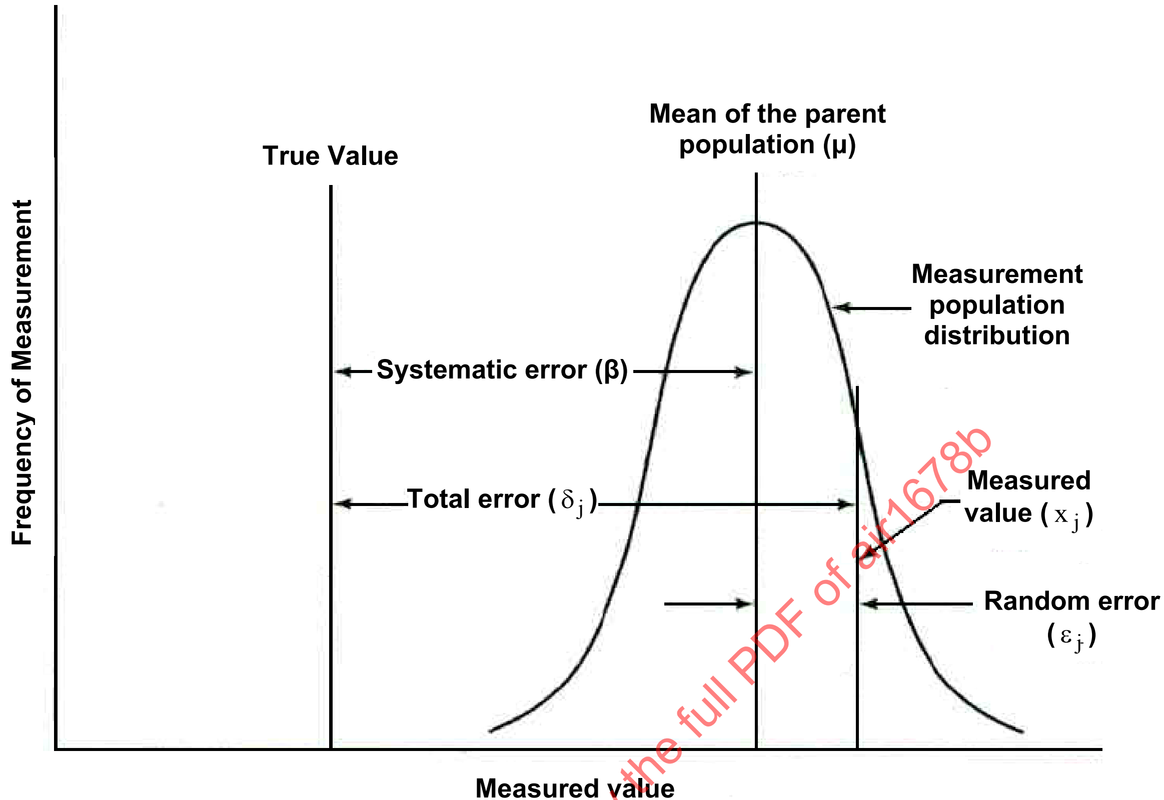

FIGURE 3 - ERROR CLASSIFICATION

A random error is one which can be estimated by statistical analysis of repeated measurements, while systematic error estimation is at least partially dependent on non-statistical methods (Reference 2.1.4.3).

An accurate measurement here is one that has small error in both random and systematic components, as is shown in Figure 2a.

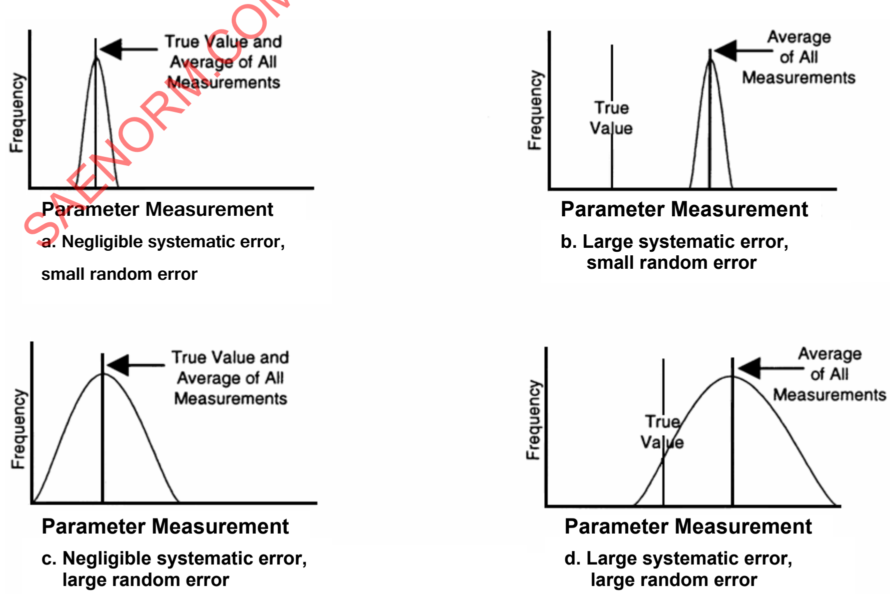

FIGURE 4 - MEASUREMENT ERROR COMPONENTS (SYSTEMATIC AND RANDOM)

RANDOM ERROR

Random errors are encountered in repeated measurements and are the differences between the observed values (each value x_j has its own random error ε_j) and the mean value of the parent population (μ). Repeated measurements at steady-state (constant) conditions are not expected to produce precisely the same data. There are numerous, small effects which may cause measurement variations. These variations tend to spread about an average value in the fashion of a normal distribution curve, Figure 5. The curve is characterized by the population standard deviation, σ.

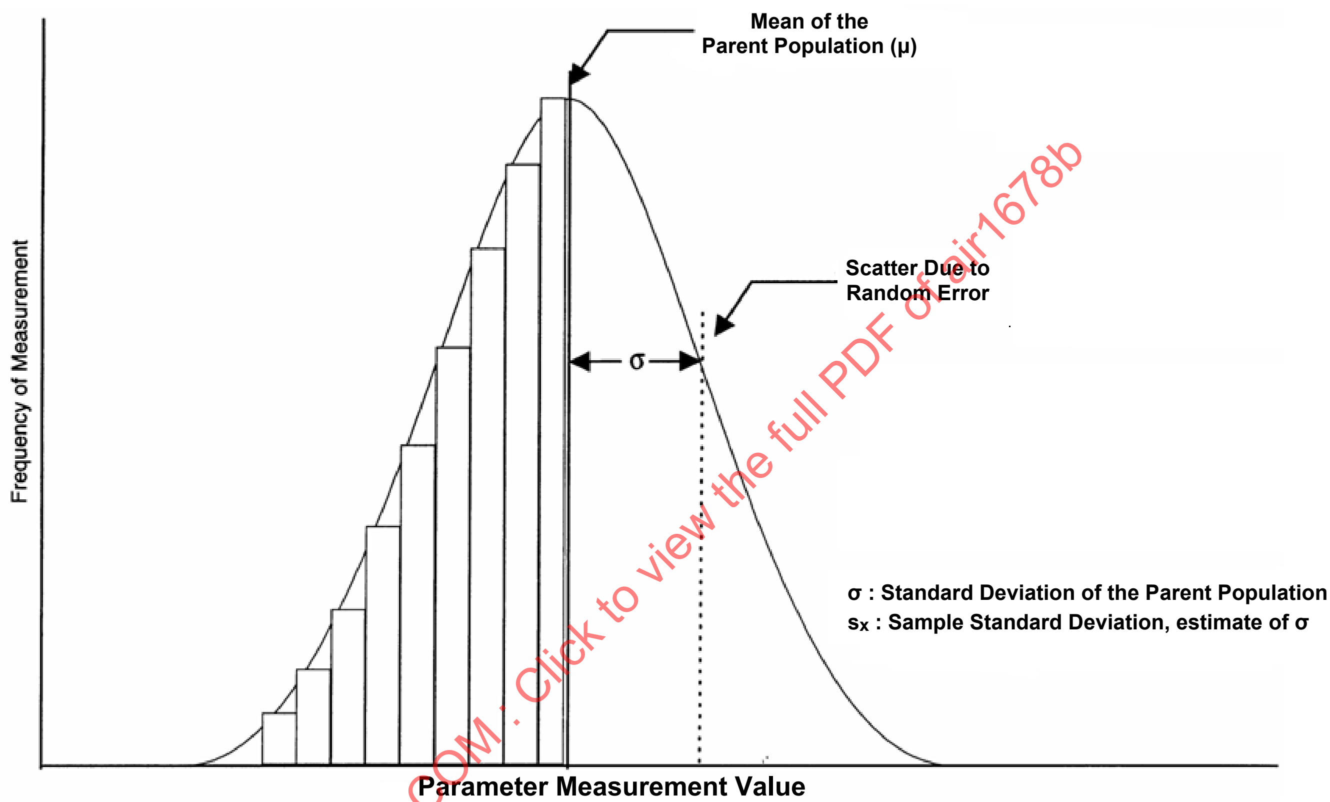

FIGURE 5 - NORMAL DISTRIBUTION CURVE RESULTING FROM TYPICAL STEADY-STATE DATA MEASUREMENT

The sample standard deviation for the variable X_i, s_{X_i}, is the computed estimate of the population standard deviation, σ, and is calculated as follows:

$$\text{Sample Standard Deviation} = s_{X_i} = \sqrt{\frac{\sum_{j=1}^{N_i}\left(X_{i_j} - \overline{X}_i\right)^2}{N_i - 1}} \quad \text{(Eq. 1)}$$

where:

N_i = Number of measurements for the variable X_i

X_{i_j} = Individual measurements of the variable X_i

$\overline{X}_i$ = Sample mean, average value of X_{i_j}, calculated as $\overline{X}_i = \frac{\sum_{j=1}^{N_i} X_{i_j}}{N_i}$ (Estimate of the mean of the parent population μ)

The goodness of the sample standard deviation as an estimate of the true population standard deviation is dependent on the number of measurements (observations) used in the initial estimation process. A large sample, thirty or more, is usually available for this estimation.

SYSTEMATIC ERROR

An uncertainty analysis assumes a carefully controlled measurement process within which every known calibration correction has been made. Because the calibration corrections are not ideal, some small systematic errors will remain.

The (unknown) systematic error β remains unchanged for the duration of a test. In repeated measurements at steady-state (constant) test conditions each measurement has the same systematic error magnitude, therefore the average measurement has the same systematic error as any single measurement. The relationship between systematic and random errors and the true value of the measured quantity is depicted in Figure 6. The systematic error β is the difference between the parent population mean μ, which can be approximated by the sample average $\overline{X}$ if the sample is large, and the true value.

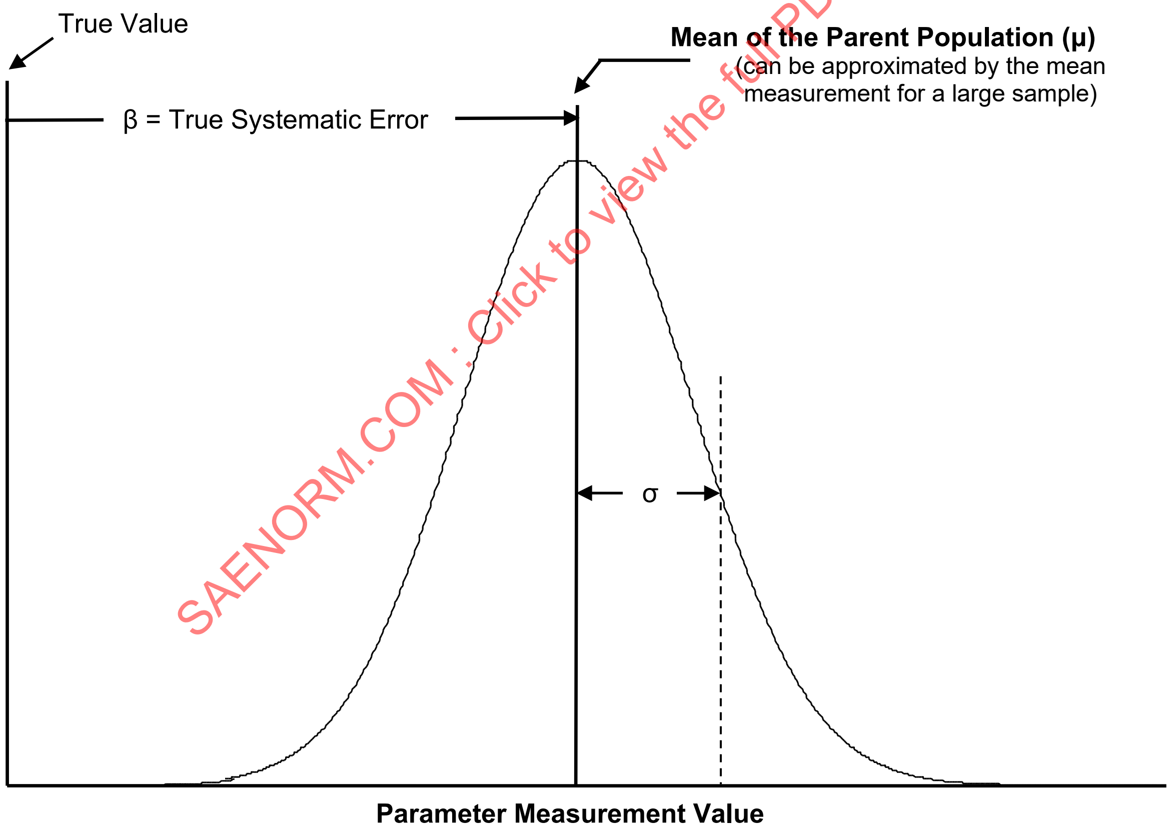

FIGURE 6 - SYSTEMATIC ERROR

4.1.2 Sources of Measurement Errors

A measurement system has many potential sources of error. The errors for each source are referred to as elemental errors. It is preferable to estimate the uncertainty for each elemental error source; however, it is permissible to estimate the uncertainty for a combination of elemental error sources. For convenience, these elemental error sources for any measurement can be divided into five categories indicated by the subscript p:

p Category

1. Calibration Errors

2. Test Article and / or Instrumentation Installation Errors

3. Data Acquisition Errors

4. Data Reduction Errors

5. Errors of Method

Elemental error sources associated with each of the above five categories are discussed in the following paragraphs.

CALIBRATION ERRORS (Category p=1)

The measurement uncertainty analysis assumes a well controlled measurement process in which there are no gross mistakes or errors. It also assumes that reasonable calibration corrections have been applied. Calibrations are performed to improve the test accuracy and to provide test measurement traceability to a national standards laboratory. By applying the calibration corrections, some systematic errors are reduced, but in the process some other errors may be introduced. The systematic error of the uncalibrated test instrument is exchanged for the systematic error of the standard or master meter. In addition, the random error of the calibration process will contribute to the calibration error. In some cases, the random error of the calibration process will contribute to the random error of the test result, and in others, it will be converted (fossilized) into systematic error. For a fuller discussion, see Appendix B.

Traceability is established and maintained through a calibration hierarchy. Each comparison in the hierarchy constitutes an elemental error source, k. Figure 7 is an example instrument calibration hierarchy.

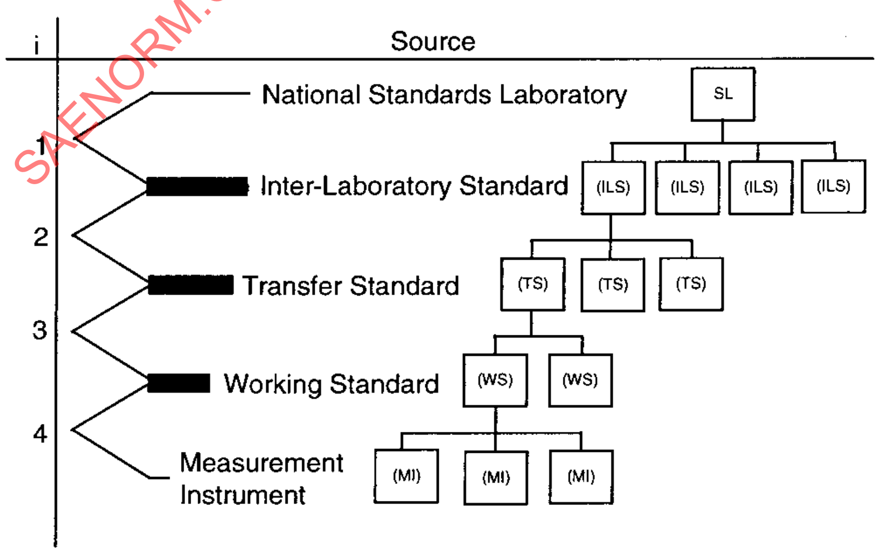

FIGURE 7 - MEASUREMENT CALIBRATION HIERARCHY

Each comparison in the calibration hierarchy has elemental errors associated with it. Estimates of these elemental errors provide standard deviations for the random and systematic uncertainties in each level of the hierarchy, as listed in Table 1.

TABLE 1 - CALIBRATION HIERARCHY ERROR SOURCES

Calibration (p=1)

k	Error source	Systematic Uncertainty	Random Uncertainty
1	NSL - ILS	b_{11}	s_{11}
2	ILS - TS	b_{21}	s_{21}
3	TS - WS	b_{31}	s_{31}
4	WS - MI	b_{41}	s_{41}

TEST ARTICLE AND / OR INSTRUMENTATION INSTALLATION (Category p=2)

Interactions between (a) test instrumentation (for instance a probe) and test article (for instance the air inlet of an engine), or between (b) the test article and the test facility can be encountered.

The case (a) can be illustrated by the measurement of the airflow in the air intake of an engine. The presence of a Pitot tube may cause intrusive disturbance effects, may affect the measurement and the calculated airflow. Also, heat transfers if any (conduction, convection and radiation) or swirl in the vicinity of the sensor may affect the measurement.

The case (b) can be illustrated with the testing of an engine in an altitude facility. If the rate of flow of secondary cooling air is changed during a test, to cool the test cell, there may be a change in thrust measurement. Such a change in measured thrust represents an error in this category.

Facility limitations for testing may require extrapolations to other conditions, which introduces a category 2 error.

Typical test article and / or instrumentation error sources are listed in Table 2.

TABLE 2 - TEST ARTICLE AND / OR INSTRUMENTATION INSTALLATION ERROR SOURCES

Interactions (p=2)

k	Error source	Systematic Uncertainty	Random Uncertainty
1	Heat transfer	b_{12}	s_{12}
2	Installation effect	b_{22}	s_{22}

DATA ACQUISITION ERRORS (Category p=3)

Table 3 illustrates some of the error sources, k, associated with the data acquisition system illustrated in Figure. Data for this example are acquired by measuring the electrical output resulting from pressure applied to a strain-gage-type pressure measurement instrument. Other error sources, such as electrical simulation, probe errors, and environmental effects are also present.

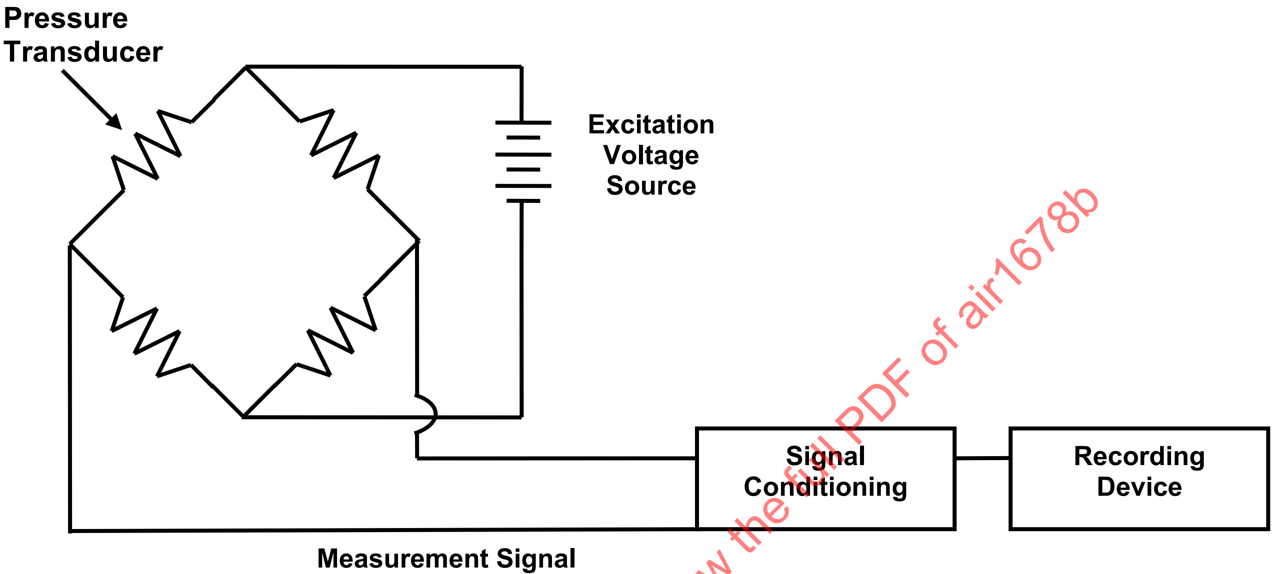

FIGURE 8 - EXAMPLE DATA ACQUISITION SYSTEM

TABLE 3 - TYPICAL DATA ACQUISITION ERROR SOURCES

Data Acquisition (p=3)

k	Error source	Systematic Uncertainty	Random Uncertainty
1	Excitation Voltage	b_{13}	s_{13}
2	Electrical Simulation	b_{23}	s_{23}
3	Signal Conditioning	b_{33}	s_{33}
4	Recording Device	b_{43}	s_{43}
5	Pressure Transducer	b_{53}	s_{53}
6	Probe Errors	b_{63}	s_{63}
7	Environmental Effects	b_{73}	s_{73}

DATA REDUCTION ERRORS (Category p=4)

Computers operate on raw data to produce output in engineering units. Typical data reduction error sources are listed in Table 4.

TABLE 4 - TYPICAL DATA REDUCTION ERROR SOURCES

Data Reduction (p=4)

k	Error source	Systematic Uncertainty	Random Uncertainty
1	Curve Fit	b_{14}	s_{14}
2	Computer Resolution	b_{24}	s_{24}
3	Assumptions or constants contained in calculation routines	b_{34}	s_{34}
4	Using approximating engineering relationships or violating their assumptions	b_{44}	s_{44}
5	Using an empirically derived correlation (fluid properties)	b_{54}	s_{54}

ERRORS OF METHOD (Category p=5)

There are other sources of errors in the measurement process not captured in categories 1 to 4. Such errors may be categorized as errors of method, or real effects. Typical errors of method are listed in Table 5.

TABLE 5 - TYPICAL ERRORS OF METHOD SOURCES

Non-Instrumentation (p=5)

k	Error source	Systematic Uncertainty	Random Uncertainty
1	Spatial Nonuniformity	b_{15}	s_{15}
2	Temporal Nonuniformity	b_{25}	s_{25}
3	Test Procedures (e.g. stabilization time)	b_{35}	s_{35}

4.1.3 Estimating Measurement Uncertainty

The following paragraphs outline approaches and considerations for estimating and classifying the random and systematic error components.

Nonetheless the best method to minimize the total uncertainty due to the effects of these different sources is to carry out overall system calibrations (end-to-end method for example) when possible.

RANDOM STANDARD UNCERTAINTY ESTIMATION

When sufficient data are available, the random standard uncertainty is taken to be the sample standard deviation s_X. For the data set to be sufficient, a large sample (over 30) should be taken over a period of time during which all random error sources are active.

In the absence of adequate data, engineering judgment can be used to estimate the random standard uncertainty.

The standard deviation, s_{X_i} is calculated from test data. When large samples are used in estimating the standard deviation, the interval defined by plus and minus the sample standard deviation around any measurement ($X_{i_j} \pm s_{X_i}$) will contain the population mean 68% of the time if the random error population is normally distributed. In other words if they are normally distributed the random errors ε_{i_j} will be contained 68% of the time in the interval [$-s_{X_i}$, $+s_{X_i}$].

For repeated or simultaneous observations, the standard deviation can often be reduced by treating the mean of the multiple independent observations as a single measurement. The distribution of the means will have a smaller standard deviation than the sample standard deviation, as indicated in Figure 9 and defined by Equation 2. Indeed if several independent samples of size N_i are taken from a large, normally distributed population, and the mean of every sample is calculated, then the standard deviation of the distribution of the means will be the standard deviation of the individual measurements divided by the square root of the size of the samples N_i.

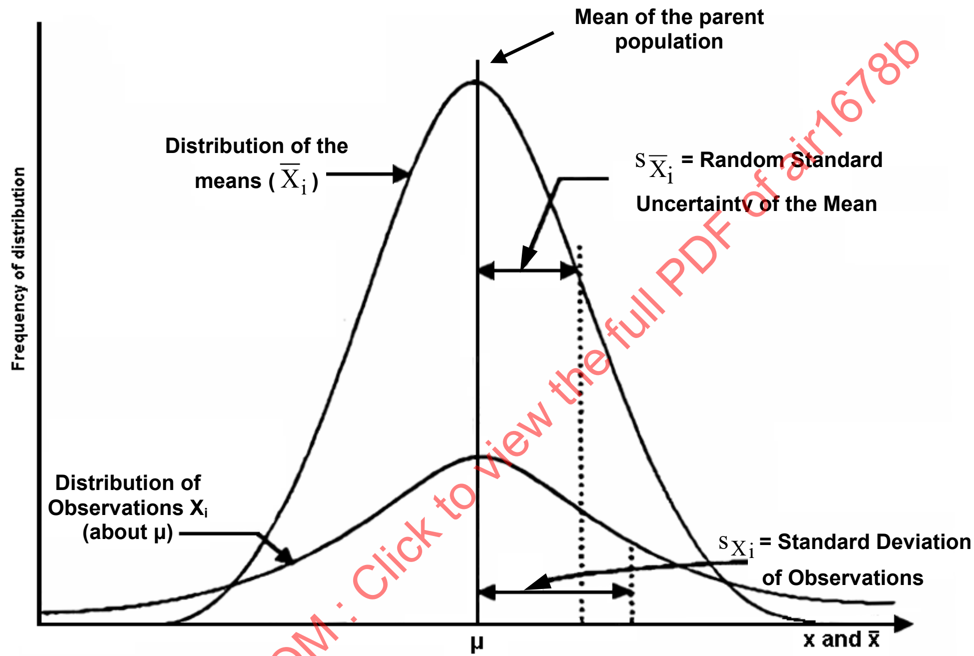

FIGURE 9 - DISTRIBUTION OF X_i AND $\overline{X}_i$

Random standard uncertainty of the sample mean = Standard deviation of the sample mean = $s_{\overline{X}_i} = \frac{s_{X_i}}{\sqrt{N_i}}$ (Eq. 2)

The random standard uncertainty of the sample mean $s_{\overline{X}_i}$ quantifies the error when using the sample mean $\overline{X}_i$ to estimate the population mean μ.

When large samples (more than thirty measurements) are used, the interval defined by plus and minus the standard deviation of the sample means around the mean of any sample ($\overline{X}_i \pm s_{\overline{X}_i}$) will contain the population mean 68% of the time if the random error population is normally distributed. If the systematic error is equal to zero then the population mean is equal to the true value, therefore this interval will contain the true value 68% of the time.

Since the standard deviation of the sample mean is lower than the standard deviation of the sample, the sample mean is a better estimate of the mean of the parent population. Therefore it is preferable to use a measurement mean in order to reduce the random uncertainty and the overall uncertainty of the measurement.

SYSTEMATIC STANDARD UNCERTAINTY ESTIMATION

Information from special tests and engineering judgment is the principal source for estimating the systematic standard uncertainty b_{X_i}. The systematic uncertainty must account for small known fixed errors, which are too difficult to correct, as well as for small unknown fixed errors. These systematic errors will include fossilized random error from the calibration process, as discussed in Appendix B. In a well controlled measurement process, the assumption is that there are no large unknown systematic errors. To ensure that a controlled measurement process exists, all measurements should be monitored with statistical quality control charts (Reference 2.1.4.4).

The magnitude of the systematic error cannot be determined unless the measurements can be compared with the true value, which is not feasible. Therefore the sytematic error is assumed to be a single value drawn from a population of possible error values. The standard deviation of this population of possible errors β, which quantifies the dispersion of the values, is the systematic standard uncertainty b_{X_i}. The systematic error remains constant for every measurement, and therefore the systematic standard uncertainty of the data sample is the same as the systematic standard uncertainty of the mean: $b_{\overline{X}_i} = b_{X_i}$.

Since the systematic error β remains constant and unknown during successive measurements, the evaluation of the standard deviation b can hardly be inferred from experiments. Engineering judgment and applicable test information can therefore be used to estimate a symmetrical interval [-B , +B] which is expected to contain the systematic error β with 95% confidence. The population of possible β values is assumed to be normally distributed (unless information indicates the contrary), and so the systematic standard uncertainty can be estimated as : $b_{\overline{X}_i} = \frac{B}{2}$, as shown in Figure 10. If the error distribution is not normal the factor used to convert the 95 % confidence level interval estimate B into the systematic standard uncertainty b may be different. For example for a rectangular distribution on the interval [-a , +a] the relation is $b_{\overline{X}_i} = \frac{a}{\sqrt{3}} = \frac{B}{0.95 \times \sqrt{3}}$.

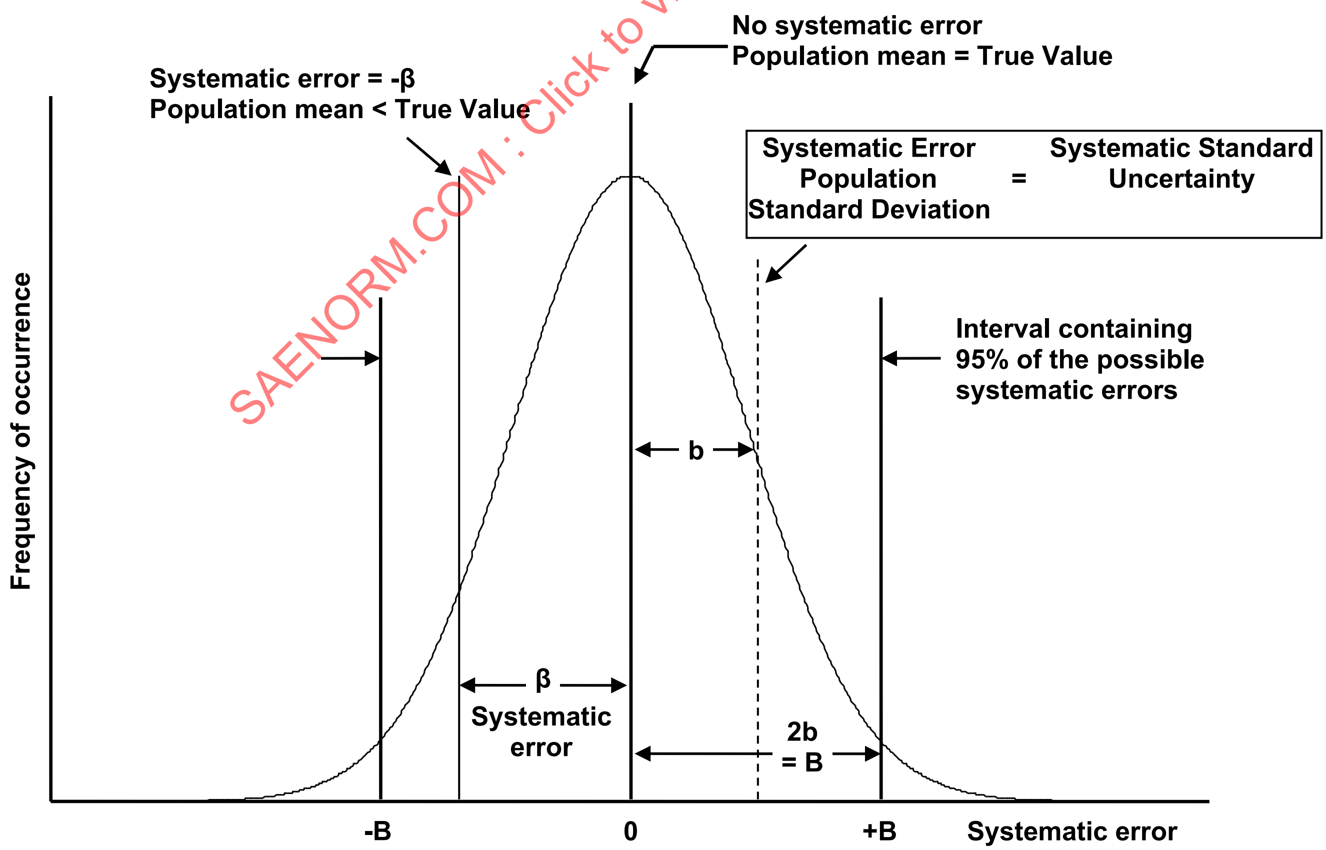

FIGURE 10 - SYSTEMATIC STANDARD UNCERTAINTY ESTIMATION (WITH A NORMAL DISTRIBUTION OF B)

The unknown systematic uncertainty intervals (±B) are assumed to be symmetrically distributed about the true value. However, sometimes the physics of the problem indicates that the systematic uncertainty intervals are nonsymmetric : in these cases there are a lower and upper value B^- and B^+ for the nonsymmetrical interval (see appendix F). The intent of the systematic uncertainty interval estimate is to identify the limits of the actual systematic error of a given measurement process.

4.1.4 Combining Measurement Uncertainties

Combination of the elemental uncertainties is essential for modeling the basic measurement uncertainty, for propagating measurement uncertainties to performance parameters, and for uncertainty reporting and validation. Uncertainties associated with all five categories of error sources are combined to define the basic measurement uncertainty.

COMBINING RANDOM UNCERTAINTIES

The random standard uncertainty (s), can be either calculated directly using Equations 1 and 2 or inferred from the root-sum-square of the elemental random uncertainties from all sources.

$$s = \sqrt{\sum_p \sum_k s_{kp}^2} \qquad \text{(Eq. 3)}$$

where p defines error categories: (1) calibration, (2) test article / instrumentation installation, (3) data acquisition, (4) data reduction, and (5) errors of method and k defines elemental error sources within a category.

For example, the random uncertainty for the calibration category (p=1) is the root-sum-square of the elemental random uncertainties (see Table).

$$s_1 = s_{cal} = \sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2} \qquad \text{(Eq. 4)}$$

In like manner, the random uncertainties for the other categories are defined by the root-sum-square of their individual elemental random uncertainties.

The basic measurement random uncertainty is then the root-sum-square of the random uncertainties of the five categories.

$$s = \sqrt{s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2}$$

$$s = \sqrt{\sum_k s_{k1}^2 + \sum_k s_{k2}^2 + \sum_k s_{k3}^2 + \sum_k s_{k4}^2 + \sum_k s_{k5}^2} = \sqrt{\sum_p \sum_k s_{kp}^2} \qquad \text{(Eq. 5)}$$

COMBINING SYSTEMATIC UNCERTAINTIES

Most measurement processes will contain a large number of systematic error sources. The systematic standard uncertainty used herein is the root-sum-square of the elemental systematic uncertainties of all categories (Reference 2.1.4.5).

$$b = \sqrt{\sum_p \sum_k b_{kp}^2} \qquad \text{(Eq. 6)}$$

The individual category error limits; calibration (b_1), test article / instrumentation errors (b_2), data acquisition (b_3), data reduction (b_4), and errors of method (b_5), are defined by the root-sum-square of their individual elemental systematic uncertainties.

The systematic uncertainty for the basic measurement process is then

$$b = \sqrt{b_1^2 + b_2^2 + b_3^2 + b_4^2 + b_5^2}$$

$$b = \sqrt{\sum_k b_{k1}^2 + \sum_k b_{k2}^2 + \sum_k b_{k3}^2 + \sum_k b_{k4}^2 + \sum_k b_{k5}^2} = \sqrt{\sum_p \sum_k b_{kp}^2} \quad \text{(Eq. 7)}$$

If any of the elemental systematic uncertainties are nonsymmetrical, the method given in appendix F must be used to obtain B^+ and B^-.

4.1.5 Propagation of Basic Measurement Uncertainties to Thrust Parameters

Basic measurements such as temperature and pressure are made and the thrust is calculated as a function of the basic measurements. Combined components of standard uncertainty (s and b) in the measurements are propagated to thrust through the mathematical algorithm. The effect of the propagation can be approximated using Taylor's series method (Reference 2.1.4.1). An alternate method using Monte Carlo techniques can also be used (Reference 2.1.4.29).

For propagating uncertainties, the concept of the "influence coefficient" is convenient. The influence coefficient is the change in the performance parameter due to a change in the basic measurement. The "influence coefficient" of each basic measurement is obtained in one of two ways:

a. Analytically - When there is a known mathematical relationship between thrust F, and the measured variables, X_1, $X_2, \ldots, X_i, \ldots, X_I$, the dimensional influence coefficient, θ_i, of the quantity, X_i, is obtained by partial differentiation.

   Thus, if F = f(X_1, $X_2, \ldots, X_I$), then

$$\theta_i = \frac{\partial F}{\partial X_i} \quad \text{(Eq. 8)}$$

b. Numerically - Where no mathematical relationship is available or when differentiation is difficult, finite increments may be used to evaluate θ_i. Here θ_i is given by

$$\theta_i = \frac{\Delta F}{\Delta X_i} \quad \text{(Eq. 9)}$$

For independent measurements, the basic measurement uncertainty components, s_{X_i} and b_{X_i} are then propagated to the thrust uncertainty components, s_F and b_F, using the influence coefficients, θ_i, as follows:

$$s_F = \sqrt{\sum_{i=1}^{I} \left(\theta_i s_{X_i}\right)^2} \quad \text{(Eq. 10)}$$

$$b_F = \sqrt{\sum_{i=1}^{I} \left(\theta_i b_{X_i}\right)^2} \quad \text{(Eq. 11)}$$

NOTE: For complex thrust calculations, the same measurement may be used more than once in the formula. For example, in several thrust models, airflow appears both in gross thrust and ram drag. If the Taylor's series relates the most elementary measurements to the ultimate result, i.e., net thrust, these "linked" relationships will be properly accounted for.

When separate measurements have systematic errors from a common source, then additional terms are needed in Equation 11 to account for these correlated systematic errors An example would be two independent temperature measurements (inlet and exit temperature) made with probes that are calibrated against the same standard. The systematic error from the calibration standard would be common for the two temperatures. The methodology in References 2.1.4.7, 2.1.4.8, and 2.1.4.9 would have to be used to properly account for these correlated systematic errors in the determination of b_F. In this case the systematic uncertainty is calculated as :

$$b_F = \sqrt{\sum_{i=1}^{I}\left[\theta_i b_{X_i}\right]^2 + 2\sum_{i=1}^{I-1}\sum_{k=i+1}^{I}\theta_i\theta_k b_{Xik}} \quad \text{(Eq. 12)}$$

With I the number of basic measurements used in the calculation of F, and b_{Xik} represents the effect of the correlated systematic errors between the basic measurements X_i and X_k .

4.2 Measurement Uncertainty

The total uncertainty in a measurement is the combination of the uncertainty due to random errors and the uncertainty due to systematic errors. The combined standard uncertainty of the measurement is expressed as:

$$\text{Combined standard uncertainty of the measurement} = u_{X_i} = \sqrt{b_{X_i}^2 + s_{X_i}^2} \quad \text{(Eq. 13)}$$

Whenever possible the random uncertainties sources will be estimated as random standard uncertainties of the measurement mean. If the sample size for the X_i measurement is N_i :

$$\text{Combined standard uncertainty of the measurement mean} = u_{\overline{X}_i} = \sqrt{b_{\overline{X}_i}^2 + s_{\overline{X}_i}^2} = \sqrt{b_{X_i}^2 + \left(\frac{s_{X_i}}{\sqrt{N_i}}\right)^2} \quad \text{(Eq. 14)}$$

It has been shown (Reference 2.1.2.1) that, assuming that the overall error distribution is normal (which is expected according to the central limit theorem with the Lindeberg conditions, if every elemental standard uncertainty is small compared to the combined standard uncertainty), a reasonable estimate for a 95% confidence level uncertainty value can be determined as:

$$\text{Expanded uncertainty of the measurement mean} = U_{\overline{X}_i} = 2 \cdot u_{\overline{X}_i} = 2 \cdot \sqrt{b_{\overline{X}_i}^2 + s_{\overline{X}_i}^2} \quad \text{(Eq. 15)}$$

The limitations on the use of this expression are given in Appendix A.

4.2.1 Uncertainty Interval Coverage

A rigorous calculation of confidence level or coverage of the true value by the uncertainty interval is not possible. However, Monte Carlo simulations using various relative sizes of the systematic and random components indicate that the coverage of U_{95} is about 95% (Reference 2.1.4.2). For very small sample sizes, see Appendix A.

4.2.2 Propagation of Uncertainty Components

The uncertainty components, systematic (b_{X_i}) and random (s_{X_i}), are propagated separately. The basic measurement (i.e. Temperature, Pressure…) uncertainty components are obtained by combining the respective elemental uncertainties as discussed previously. Then, the basic measurement uncertainty components are propagated to the performance parameter uncertainty components.

4.2.3 Pretest and Posttest Analyses

The accuracy of the test is often stipulated as part of the test requirements. The ability to meet the accuracy requirements is estimated by a pretest uncertainty analysis. Several different thrust models may be analyzed prior to flight to select the most accurate ones. If the experiment accuracy requirement is of the same size or smaller than the projected measurement uncertainty, corrective action should be taken to reduce the measurement uncertainty or else not perform the experiment.

The pretest analysis is based on data and information that exist before the test: calibration histories, previous tests with similar instrumentation, prior measurement uncertainty analyses and expert opinions. In complex tests, there are often alternatives to evaluate: different thrust models, various instrumentation layouts, and alternate calculation procedures. Pretest analysis will identify the most preferred test methods.

Posttest analysis is required to confirm the pretest estimates and to identify potential problems. Comparison of test results with the pretest analysis is an excellent data validity check. The standard deviation calculated by using repeated points or redundant instruments, should not be significantly larger than the pretest estimates. When redundant instrumentation or calculation methods are available, the individual averages should be within the pretest uncertainty interval. If there are several ways to obtain a parameter, the uncertainty intervals should overlap. The final uncertainty intervals reported for the test results should be based on the posttest analysis. There may be a pretest and posttest analysis for both the ground test and flight test phases.

4.3 Reporting Measurement Uncertainty

The uncertainty components, random standard uncertainty, systematic standard uncertainty, uncertainty interval and influence coefficients are recommended to be included in the reports on measurement uncertainty along with applicable test conditions. Two separate tables are recommended: The Elemental Uncertainty for One Basic Measurement Table and the Measurement Uncertainty Summary Table, which in our case is the Thrust Uncertainty Summary Table.

ELEMENTAL UNCERTAINTY TABLE

The elemental uncertainty sources table records all the elemental uncertainties of each basic measurement. When different conditions or ranges of conditions are assessed, it is necessary to create several tables if there are changes in the elemental uncertainties. This is the case when the uncertainty on a measurement is given as a percentage of this measurement. These tables would normally be maintained by the test center and be available for review.

The elemental contributions are required to confirm measurement uncertainty estimates and to support any corrective action needed to reduce the uncertainty or to identify data validity problems. This list of elemental error sources should be reviewed to insure against potential missed sources.

Table 6 provides a suggested reporting format and illustrates the data required to be recorded.

THRUST UNCERTAINTY SUMMARY TABLE

The suggested reporting format is shown in Table 7. The uncertainty components, s and b, are necessary to: (1) indicate corrective action if the uncertainty is unacceptably large before the test, (2) to propagate the uncertainty to more complex parameters, and (3) to substantiate the uncertainty interval.

The influence coefficients for each measurement are provided to document the calculation of the thrust uncertainty components (using Equations 10 and 11). The test condition (or range of conditions) must be identified to qualify both the uncertainty components and influence coefficients. The flight condition (actual or simulated) and engine power level must be identified in a manner that completely defines the engine operating point.

As for the elemental uncertainty tables, it can be necessary to create several performance parameter uncertainty tables if the different conditions or ranges of conditions assessed require it.

TABLE 6 - RECOMMENDED TABLE FORMAT FOR REPORTING ELEMENTAL UNCERTAINTIES FOR ONE BASIC MEASUREMENT

Note: a separate report is needed for each basic measurement.

<table>
<tr><td colspan="2">Basic measurement X_i:
Type and station
e.g., PT2</td><td colspan="2">Type of sensor
Basic description
e.g., thermocouple</td><td>Measurement range
Expected range for test condition</td></tr>
<tr><td colspan="5">Test condition(s): e.g., altitude, Mach number, ram recovery, power lever angle or engine inlet total pressure and temperature, thrust level and ambient pressure</td></tr>
<tr><td colspan="2">Elemental error source</td><td rowspan="2">Random standard uncertainty
s_{kp}</td><td rowspan="2">Systematic standard uncertainty
b_{kp}</td><td rowspan="2">Substantiation source and notes</td></tr>
<tr><td>Sub-script</td><td></td></tr>
<tr><td></td><td><u>Calibration</u></td><td></td><td></td><td rowspan="5">Identity substantiating sources for error estimates

e.g., latest calibration on file, source specification, etc.</td></tr>
<tr><td>11</td><td></td><td>s_{11}</td><td>b_{11}</td></tr>
<tr><td>21</td><td></td><td>s_{21}</td><td>b_{21}</td></tr>
<tr><td>31</td><td></td><td>.</td><td>.</td></tr>
<tr><td>41</td><td></td><td>.</td><td>.</td></tr>
<tr><td></td><td><u>Test article –
Instrumentation installation</u></td><td></td><td></td><td></td></tr>
<tr><td>12</td><td></td><td>s_{12}</td><td>b_{12}</td><td></td></tr>
<tr><td>22</td><td></td><td>s_{22}</td><td>b_{22}</td><td></td></tr>
<tr><td>32</td><td></td><td>.</td><td>.</td><td></td></tr>
<tr><td>42</td><td></td><td>.</td><td>.</td><td></td></tr>
<tr><td>.</td><td></td><td>.</td><td>.</td><td></td></tr>
<tr><td></td><td><u>Data acquisition</u></td><td></td><td></td><td></td></tr>
<tr><td>13</td><td></td><td>s_{13}</td><td>b_{13}</td><td></td></tr>
<tr><td>23</td><td></td><td>s_{23}</td><td>b_{23}</td><td></td></tr>
<tr><td>.</td><td></td><td>.</td><td>.</td><td></td></tr>
<tr><td>.</td><td></td><td>.</td><td>.</td><td></td></tr>
<tr><td>14</td><td><u>Data reduction</u></td><td>s_{14}</td><td>b_{14}</td><td></td></tr>
<tr><td>24</td><td></td><td>s_{24}</td><td>b_{24}</td><td></td></tr>
<tr><td>.</td><td></td><td>.</td><td>.</td><td></td></tr>
<tr><td>.</td><td></td><td>.</td><td>.</td><td></td></tr>
<tr><td></td><td><u>Errors of Method</u></td><td></td><td></td><td></td></tr>
<tr><td>15</td><td></td><td>s_{15}</td><td>b_{15}</td><td></td></tr>
<tr><td>25</td><td></td><td>s_{25}</td><td>b_{25}</td><td></td></tr>
<tr><td>.</td><td></td><td>.</td><td>.</td><td></td></tr>
<tr><td>.</td><td></td><td>.</td><td>.</td><td></td></tr>
<tr><td></td><td>Resultant measurement uncertainty components</td><td>$s_{X_i} = \sqrt{\sum_p \sum_k s_{kp}^2}$</td><td>$b_{X_i} = \sqrt{\sum_p \sum_k b_{kp}^2}$</td><td>Elemental uncertainty Report Number XXX</td></tr>
</table>

TABLE 7 - RECOMMENDED TABLE FORMAT FOR PERFORMANCE PARAMETER UNCERTAINTY SUMMARY

Note: The performance parameter in this table is thrust, but this table format may be used for other calculated parameters.

Test condition: e.g. ., altitude, Mach number, ram recovery, power setting or engine inlet total pressure and temperature, thrust level and ambient pressure							
Basic measurement	Range	Random standard uncertainty s_{X_i}	Systematic standard uncertainty b_{X_i}	Influence Coefficient θ_i	Combined Standard Uncertainty u_{X_i}	Expanded Uncertainty U_{X_i}	Supporting elemental uncertainties report number (from Table 6)
Basic measurement 1	ML-MU	s_{X_1}	b_{X_1}	θ_1	$u_{X_1} = \sqrt{b_{X_1}^2 + s_{X_1}^2}$	$U_{X_1} = 2\sqrt{b_{X_1}^2 + s_{X_1}^2}$	XXX1
Basic measurement 2	ML-MU	s_{X_2}	b_{X_2}	θ_2	$u_{X_2} = \sqrt{b_{X_2}^2 + s_{X_2}^2}$	$U_{X_2} = 2\sqrt{b_{X_2}^2 + s_{X_2}^2}$	XXX2
• • •	• • •	• • •	• • •	• • •	• • •	• • •	• • •
Thrust	ML-MU	$s_F = \sqrt{\sum_{i=1}^{I}\left(\theta_i s_{X_i}\right)^2}$	$b_F = \sqrt{\sum_{i=1}^{I}\left(\theta_i b_{X_i}\right)^2}$		$u_F = \sqrt{b_{X_i}^2 + s_{X_i}^2}$	$U_F = 2\sqrt{b_{X_i}^2 + s_{X_i}^2}$	
Additional performance parameters of interest • • •							

5. IN-FLIGHT THRUST MEASUREMENT PROCESSES

The in-flight thrust measurement process comprises:

- Selection of the in-flight thrust determination methods and definition of appropriate mathematical models (Reference 2.1.1.1).
- Calibration of the mathematical models by facility testing.
- Calculation of thrust using in-flight measurements and the calibrated mathematical models.

During the ground test phase, errors are introduced in measuring test facility thrust, in calibrating the mathematical model and from the instrumentation used to determine test facility flight conditions. In flight test, errors result from the instrumentation required to determine the actual flight condition. An accurate assessment of thrust uncertainty is dependent on both the precise definition of all error elements as well as a thorough understanding of how the ground test uncertainties are classified, combined and propagated to the in-flight thrust uncertainty.

The initial decision on classifying elemental error sources may be changed after the in-flight measurement process has been defined and the instrumentation system has been considered. The error source is classified as random if it increases the scatter in the test data; otherwise, it is a systematic error source. The decision logic for error classification, based on the actual measurement process, is discussed in Appendix B. Reclassification of initial random error(s) to a systematic error(s) is marked with an asterisk (*) to indicate that the random error has become a fossilized or fixed error.

Two sets of measurements are identified; the facility measurements and the engine measurements. The former consists of those measurements required to produce the facility-measured thrust, while the latter consists of those measurements, including simulated flight conditions, that are expected to be made in flight to calculate thrust. Both sets of measurements contribute uncertainties from calibration, test article / instrumentation installation, data acquisition, data reduction, and errors of method. These uncertainties are propagated through their respective thrust calculation procedures, as outlined in Section 4, to define the calculated thrust uncertainty components. The facility-measured thrust uncertainty components are referred to as s_{gt} and b_{gt}. The instrumentation uncertainties for the in-flight thrust calculation have components s_{gi} and b_{gi}.

The following paragraphs discuss ground test and flight test phases of the thrust measurement process for a ground calibration in-flight thrust determination method, such as the residual error method (Reference 2.1.1.1). The associated uncertainty components are defined for each phase and the combination and propagation of uncertainties are discussed.

5.1 Ground Test Measurement Process

The purpose of the ground test measurement process is to develop a calibrated mathematical model and define the associated random and systematic uncertainty components. The process is illustrated in Figure. The mathematical model is used to calculate thrust. In this report, its uncertainty components are defined as s_{mm} and b_{mm}. The model contains lapse rate corrections which can have uncertainty components s_{cm} and b_{cm}.

For a given in-flight thrust determination method, a calibration parameter(s) is used to minimize the difference between the measured data and the model. The uncertainty on in-flight thrust due to the fit of the calibration parameter is caracterized as an additional uncertainty (s_r).

The systematic standard uncertainty of the calibrated model is defined by:

$$b_{mm} = \sqrt{b_{gt}^2 + b_{gi}^2 + \left(\frac{s_r^*}{\sqrt{N}}\right)^2} \qquad \text{(Eq. 16)}$$

with N the number of separate thrust measurements, and assuming that the in-flight instrumentation is different from that used on the engine in the ground test (see Eq. 30).

The random standard uncertainty, s_{mm}, is usually zero but can be bounded between zero (0) and the observed calibration model error standard deviation (s_r).

The lapse rate correction error components are estimated from flight test instrumentation only.

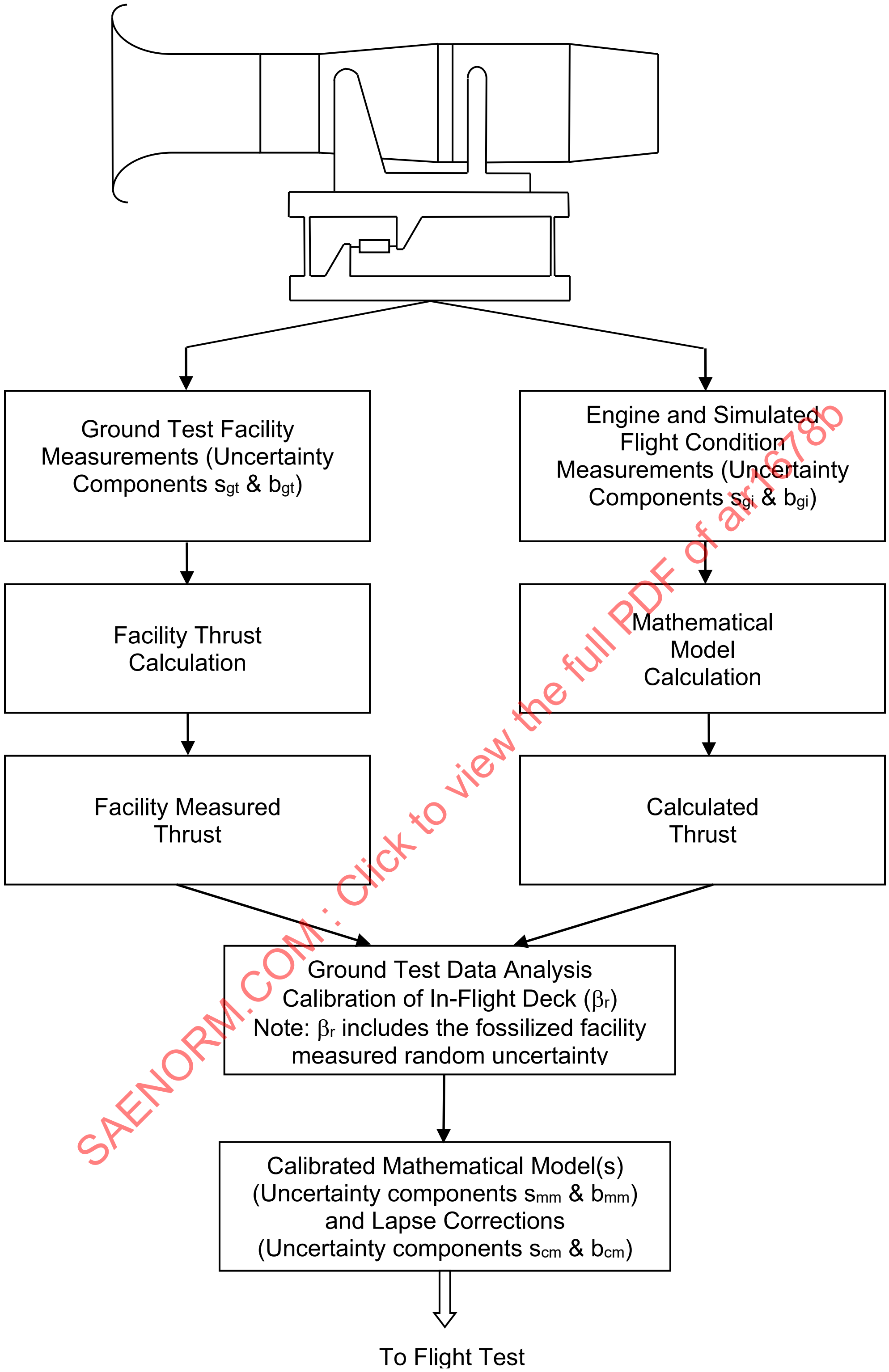

FIGURE 11 - GROUND TEST THRUST MEASUREMENT PROCESS INCLUDING DEFINITION/CALIBRATION OF MATHEMATICAL MODEL(S)

5.2 Flight Test Measurement Process

Flight tests provide calculated thrust using the mathematical model. The general thrust measurement process is illustrated in Figure 12. The flight condition and engine basic measurement uncertainties are combined to define the flight test instrumentation uncertainty components, s_{fi} and b_{fi}. These are then combined with the thrust model uncertainty components, s_{mm} and b_{mm}, to determine the as-tested in-flight thrust uncertainty components, s_t and b_t. Typically, corrections to reference conditions are included in the mathematical model (lapse corrections). This procedure may introduce uncertainties which have to be quantified from additional information. If this information is available it will be included as uncertainty components, s_{cm} and b_{cm}.

The in-flight calculated thrust, corrected to the reference flight condition has uncertainty components, s_c and b_c. These are related to instrumentation, thrust and lapse rate corrections uncertainty components, by the relationships:

$$s_c = \sqrt{s_{fi}^2 + s_{mm}^2 + s_{cm}^2} \quad \text{(Eq. 17)}$$

$$b_c = \sqrt{b_{fi}^2 + b_{mm}^2 + b_{cm}^2} \quad \text{(Eq. 18)}$$

6. APPLICATION OF UNCERTAINTY METHODOLOGY TO THE IN-FLIGHT THRUST PROGRAM

Uncertainty analyses should be carried out throughout the test program as described in AIR5925 (Reference 2.1.4.30). Table 8 shows the breakdown of activities required to support the flight test thrust measurement process.

The overall process of uncertainty estimation is iterative. The closed loop consists of estimating the error components (pretest) then re-evaluating them in light of actual test data (posttest).

Careful application of the described methodology should lead to both an improvement in the actual estimate of the in-flight thrust through better control of the elemental uncertainties and an improvement in the interpretation and understanding of the final flight test results.

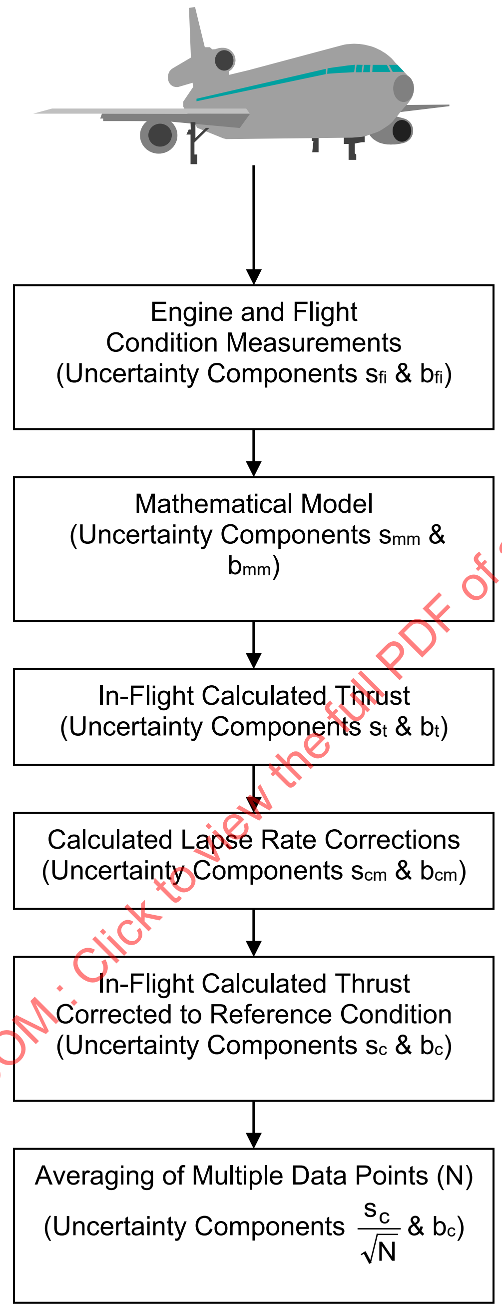

FIGURE 12 - GENERAL FLIGHT TEST THRUST MEASUREMENT PROCESS

TABLE 8 - IN-FLIGHT THRUST MEASUREMENT UNCERTAINTY ACTIVITIES

Program Phases	*Related Activities*	*Relevant Section*
Program Definition and Planning	• Establish Program In-Flight Thrust Uncertainty Requirements	6.1
	• Identify Potential Thrust Measurement Mathematical Model Options	6.1.1
	• Conduct Mathematical Model Options Survey and Select Options for use in the Program	6.1.2 6.1.3
Ground Test	*Pretest*	6.2.1
	• Estimate Engine and Facility Measurement Uncertainties (s_{gi}, b_{gi}) (s_{gt}, b_{gt})	
	• Estimate Selected Mathematical Model(s) Uncertainties (s_{mm}, b_{mm})	
	Posttest	6.2.2
	• Confirm Pretest Estimates, Adjust as Necessary	
	• Calibrate Mathematical Model(s) and Derive Uncertainties (s_{mm}, b_{mm})	
Flight Test	*Pretest*	6.3.1
	• Estimate Flight Measurement Uncertainties (s_{fi}, b_{fi})	
	• Estimate Selected Mathematical Model(s) Uncertainties (obtained from ground test estimates) (s_{mm}, b_{mm})	
	Posttest	6.3.2
	• Confirm Pretest Estimates, Adjust as Necessary	
Result Analyses	• Examine Consistency of Ground and Flight Test Data	6.4
	• Compare Thrust Options Results with Program Predictions	6.4.1
	• Resolve Test Data and Performance Discrepancy Problems	6.4.1
	• Report Test Results and Uncertainties	6.4.3

6.1 Program Definition and Planning

The first task is to establish the measurement uncertainty requirement based on the objectives of the program. The second is to consider a number of in-flight thrust measurement options and conduct pretest uncertainty estimates. The number of options is reduced with the aid of these estimates.

6.1.1 In-Flight Thrust Measurement Options

The identification of potential options to provide the desired accuracy is required. Net thrust has three separate components, gross thrust, airflow, and aircraft velocity. Each of these components may have a number of evaluation options available (Reference 2.1.1.1). Cost/accuracy trades are considered.

The measurements required are identified for each thrust option. These include the facility and flight test thrust measurements. Data from previous testing is used unless some new instrumentation is utilized.

The uncertainty components for each measurement are estimated. Table 9 shows an example format that can be used to tabulate the measurement uncertainty components. Ground and flight test measurements are identified separately.

6.1.2 Initial Survey of Thrust Measurement Options

The survey results will direct attention to the measurements which are most critical to the success of the test program. The approach to the survey is to determine the influence that each measurement has on the output net thrust, F_N.

The table of influence coefficients, θ, is established (Table 9). The influence coefficient is the ratio of the change in F_N for each measurement change, expressed in units of the measurement. The influence coefficients are obtained readily by programming the thrust measurement equations on a computer and observing the F_N output variations with successively small input perturbations, as illustrated in Appendix C. The values of θ will vary with flight condition, engine power setting and thrust option and should be established for every option under consideration. Each uncertainty component is propagated to define the random standard uncertainty and systematic standard uncertainty for the facility measured performance parameter (s_{gt} and b_{gt}), the ground instrumentation (s_{gi} and b_{gi}), and the flight instrumentation (s_{fi} and b_{fi}).

Thus:

$$s_{gt} = \sqrt{\sum_I \left(\theta_{gt,i} s_{gt,i}\right)^2} \quad \text{(Eq. 19)}$$

$$b_{gt} = \sqrt{\sum_I \left(\theta_{gt,i} b_{gt,i}\right)^2} \quad \text{(Eq. 20)}$$

$$s_{gi} = \sqrt{\sum_I \left(\theta_{gi,i} s_{gi,i}\right)^2} \quad \text{(Eq. 21)}$$

$$b_{gi} = \sqrt{\sum_I \left(\theta_{gi,i} b_{gi,i}\right)^2} \quad \text{(Eq. 22)}$$

$$s_{fi} = \sqrt{\sum_I \left(\theta_{fi,i} s_{fi,i}\right)^2} \quad \text{(Eq. 23)}$$

$$b_{fi} = \sqrt{\sum_I \left(\theta_{fi,i} b_{fi,i}\right)^2} \quad \text{(Eq. 24)}$$

where there are I measurements contributing to each systematic and random summation.

TABLE 9 - EXAMPLE PRELIMINARY PERFORMANCE SURVEY FOR NET THRUST OPTION 1

Test Condition : Altitude / Mach Number / Engine Power Setting							
Basic Measurements	Facility Measurements		Thrust Calculation Measurements				Influence Coefficient
			Ground		Flight		
	s_{gt}	b_{gt}	s_{gi}	b_{gi}	s_{fi}	b_{fi}	θ
Gross Thrust							
Facility Measurement							
• Load Cell	$s_{gt,1}$	$b_{gt,1}$	--	--	--	--	$\theta_{gt,1}$
•	$s_{gt,2}$	$b_{gt,2}$	--	--	--	--	$\theta_{gt,2}$
•	•	•	--	--	--	--	•
•	$s_{gt,N}$	$b_{gt,N}$	--	--	--	--	$\theta_{gt,N}$
Thrust Calculation							
• Exhaust Area (A)	--	--	$s_{gi,1}$	$b_{gi,1}$	$s_{fi,1}$	$b_{fi,1}$	$\theta_{gi,1}$ & $\theta_{fi,1}$
• Exhaust Press (P)	--	--	$s_{gi,2}$	$b_{gi,2}$	$s_{fi,2}$	$b_{fi,2}$	$\theta_{gi,2}$ & $\theta_{fi,2}$
•	--	--	•	•	•	•	•
•	--	--	$s_{gi,N}$	$b_{gi,N}$	$s_{fi,N}$	$b_{fi,N}$	$\theta_{gi,N}$ & $\theta_{fi,N}$
Airflow							
Facility measurement							
• Venturi Δp	$s_{gt,N+1}$	$b_{gt,N+1}$	--	--	--	--	$\theta_{gt,N+1}$
•	$s_{gt,N+2}$	$b_{gt,N+2}$	--	--	--	--	$\theta_{gt,N+2}$
•	•	•	--	--	--	--	•
Thrust Calculation							
• Fan Speed (N1)	--	--	$s_{gi,N+1}$	$b_{gi,N+1}$	$s_{fi,N+1}$	$b_{fi,N+1}$	$\theta_{gi,N+1}$ & $\theta_{fi,N+1}$
•	--	--	$s_{gi,N+2}$	$b_{gi,N+2}$	$s_{fi,N+2}$	$b_{fi,N+2}$	$\theta_{gi,N+2}$ & $\theta_{fi,N+1}$
•	--	--	•	•	•	•	•
•	--	--	$s_{gi,m}$	$b_{gi,m}$	$s_{fi,m}$	$b_{fi,m}$	$\theta_{gi,m}$ & $\theta_{fi,m}$
Velocity							
•	--	--	--	--	--	--	--
•	--	--	--	--	--	--	--

An estimate of the in-flight thrust calculation random standard uncertainty, s_c, and systematic standard uncertainty, b_c, are determined using Equations 17 and 18. The procedure is repeated for each test condition and thrust option. The final results for each can be combined to define the single value, u, representing the combined standard uncertainty. This procedure will rank the candidate options and may identify different options as being best for different flight conditions. It will also identify critical areas to be closely monitored and areas for improvement. An example is provided in Appendix D.

When the survey is completed, the options for use in the test program are selected. The selection will involve participation by aircraft, engine, flight and ground test, instrumentation, and performance analysis experts.

6.2 Ground Test

The uncertainty evaluation activities include: (1) pretest estimating of engine and facility measurement uncertainties and mathematical model uncertainties, and (2) posttest confirmation of pretest uncertainty estimates and definition of the calibrated mathematical model including model error determination. The end product is the calibrated mathematical model which will satisfy the in-flight thrust measurement accuracy requirement.

The ground test facility used to measure the thrust of an engine under test must be calibrated in order to minimise systematic error and to determine the uncertainties, s_{gt} and b_{gt}. The calibration applies to all relevant measurements used in the determination of the engine thrust, including:

- Thrust Frame Reaction Force (e.g. loadcells).
- Air Mass Flow (e.g. intake venturi).
- Local pressure and temperature instrumentation (e.g. transducers and thermocouples).
- Dimensional measurements (e.g. venturi or nozzle areas).
- Correlation of the thrust measurement to a baseline or "master" test facility, if required.

The combined uncertainty of thrust measurement must satisfy the accuracy requirement. This is covered in a number of publications; see Reference 2.1.1.1, Section 7.4, Reference 2.1.4.31, Section 7.2, and Reference 2.1.4.32.

6.2.1 Ground Pretest Uncertainty Analysis

The objective of the pretest uncertainty analysis is to ensure that the candidate measurement systems will satisfy the program accuracy requirement. A logic flow chart is shown in Figure 13.

The total measurement system, including calibration, test article and/or instrumentation installation, data acquisition, data reduction processes and method is identified using the selected thrust options. A detailed pretest uncertainty analysis, including elemental uncertainty estimates for each error category, is made following the methodology of Section 4. The elemental error sources and uncertainty estimates may be tabulated using the format shown in Table 6. An example is shown for a typical pressure measurement in Table 10.

The results of the pretest uncertainty analysis of the measurement parameters should be reported in the Performance Parameter Uncertainty Summary Report format shown in Table 7. The measurement uncertainty should be propagated to the desired performance parameters. These results would also be reported in the summary report. Performance parameter uncertainties may vary widely with test conditions. To report representative uncertainties, it may be necessary to provide a separate summary report for each expected test condition, or to provide a number of summary reports each of which cover a range of test conditions. Table 11 illustrates a typical performance parameter uncertainty summary report format. A summary of this is contained in Appendix C. The net thrust uncertainty components illustrated represent the ground instrumentation uncertainty components, s_{gi} and b_{gi}. Similar data would be required for the facility instrumentation uncertainty components, s_{gt} and b_{gt}.

Only one flight condition is shown in this example. When examining a range of test conditions, the uncertainty components for each test condition should be propagated and the results combined in an appropriate fashion.

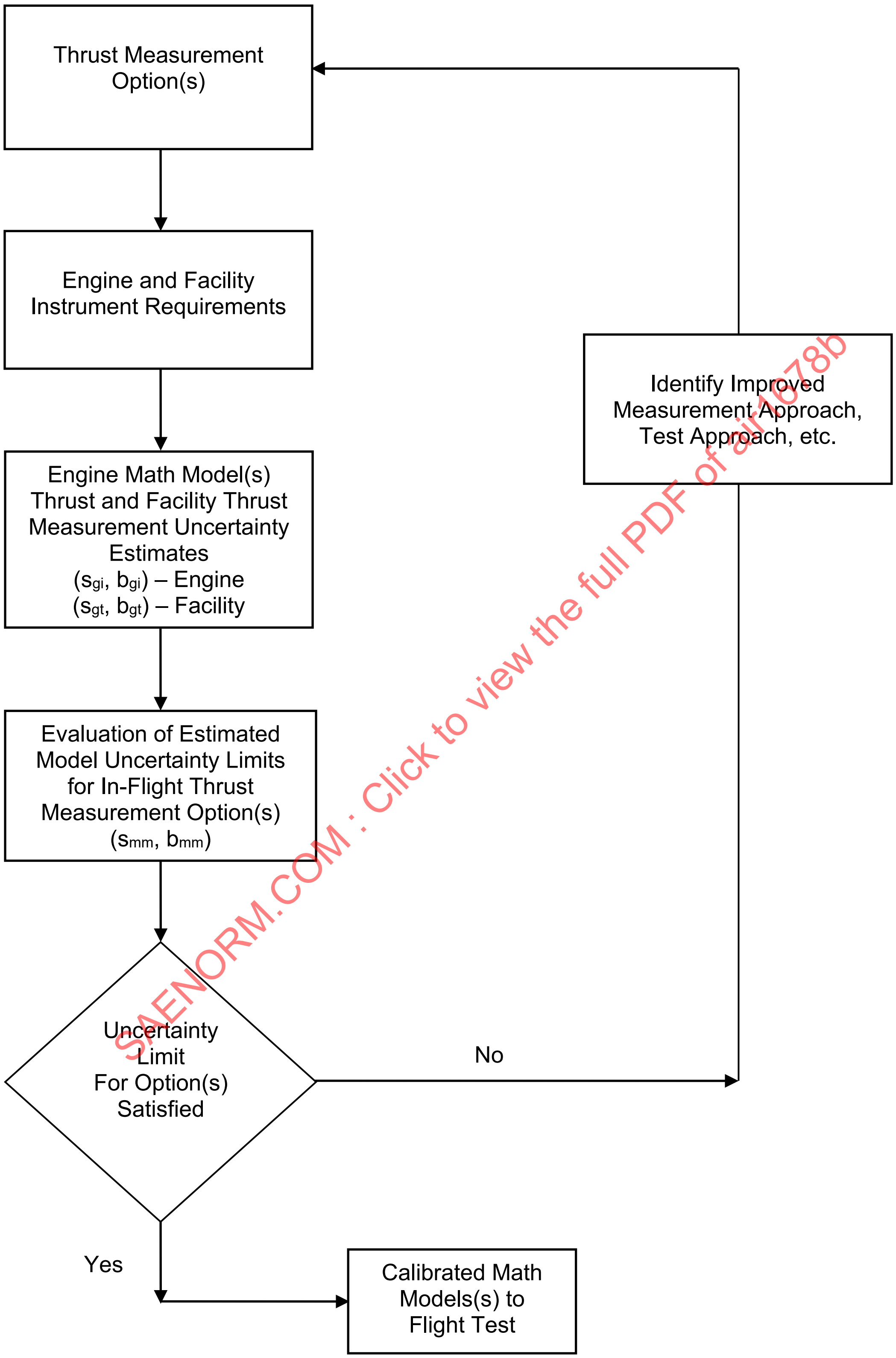

FIGURE 13 - GROUND TEST MEASUREMENT PROCESS EVALUATION CYCLE

TABLE 10 - EXAMPLE ELEMENTAL UNCERTAINTIES FOR ONE BASIC MEASUREMENT REPORT (REPORT NO. 0003)

Basic Measurement Inlet Static Pressure PS1		Type Sensor Strain Gage		Measurement Range 0 to 15 psi
Test Condition(s)				
Elemental Error Source		Random Standard Uncertainty s_{kp}	Systematic Standard Uncertainty b_{kp}	Substantiation Source and Notes
Subscript				
	Calibration			
11	SL - ILS	0.0035	0.0035	Calibration Sheet Dated 03/03/73
21	ILS – TS	0.0035	0.0035	Calibration Sheet Dated 06/06/73
31	TS – WS	0.0042	0.0052	Calibration Sheet Dated 11/26/73
41	WS - MI	0.0056	0.0069	Calibration Sheet Dated 12/01/73
	Test article – Instrumentation installation			
12	Heat Transfer	0.0035	0.0017	Special Test Sheet Dated 06/17/74
	Data acquisition			
13	Excitation Voltage	0.0028	0.0104	Special Test Sheet Dated 06/17/74
23	Electrical Simulation	0.0028	0.0104	Special Test Sheet Dated 06/17/74
33	Signal Conditioning	0.0014	0.0076	Special Test Sheet Dated 06/17/74
43	Recording Device	0.0008	0.0008	Special Test Sheet Dated 06/17/74
53	Pressure Transducer	0.0056	0.0313	Manufactured Spec
63	Probe Errors	0.0056	0.0347	Special Test Sheet Dated 06/17/74
73	Environmental Effect	0.0014	0.0174	Special Test Sheet Dated 06/17/74
	Data reduction			
14	Curve Fit	0.0010	0	Curve Fit Sheet Dated 04/17/74
24	Computer Resolution	0.0010	0	Round Off Error Analysis
	Errors of Method			
15	None			
	Resultant measurement error components	0.0130 psi	0.0535 psi	Elemental uncertainty Report Number 0003

TABLE 11 - EXAMPLE PERFORMANCE PARAMETER UNCERTAINTY SUMMARY

Test Condition : 1000' / Mach 0.65 / Cruise Power								
Basic Measurement		Range	Random Standard Uncertainty	Systematic Standard Uncertainty	Influence Coefficient	Combined Standard Uncertainty	Expanded Uncertainty	Supporting Elemental Uncertainties Report Number
Identification	Units							
Low Rotor Speed N1	(rpm)	17 - 37000	6	3	-0.0135	-	-	0001
High Rotor Speed N2	(rpm)	94 – 64000	31	3	0.0620	-	-	0002
Inlet Static Press PS1	(psi)	0 – 15	0.0130	0.0535	26.3039	-	-	0003
Inlet Delta Press DELP0	(psi)	0 – 8	0.0069	0.0292	7.9855	-	-	0004
Inlet Total Temp T2	(° F)	-110 – 220	0.35	1.35	-3.7796	-	-	0005
Net Thrust	(lbf)	409	2.36	5.30	-	5.8	11.61	
Note 1 : Elemental Uncertainties for the basic measurement PS1 (Report NO. 0003) is documented in Table 10								

6.2.2 Ground Posttest Analysis

The objectives of posttest data analysis are (1) to confirm the pretest measurement uncertainty estimates and (2) to provide a calibrated engine thrust mathematical model including lapse rate corrections and associated uncertainty components.

CONFIRMATION OF PRETEST ESTIMATES

The random standard uncertainty for each measurement can be verified from repeat data points. The systematic standard uncertainty can only be checked in a broad sense by comparison of measured thrust to predicted values. If multiple thrust options are available they may be compared to provide additional assurance that systematic uncertainties have been properly assessed.

CALIBRATION OF THE MATHEMATICAL MODEL AND ESTIMATION OF UNCERTAINTY COMPONENTS

The procedure for the derivation of the calibrated mathematical model will depend on the specific form of the mathematical model itself. Appendix E provides details. Once the calibrated mathematical model is defined, the mathematical model uncertainty components, illustrated in Figure 14, can be defined in a four-step process.

First, the ground test facility instrumentation uncertainty components are propagated to define the uncertainty components, b_{gt} and s_{gt}, of the facility performance measurement.

Second, the uncertainty components of the engine ground test instrumentation are propagated through the calibrated mathematical model to define the ground instrumentation uncertainty components, b_{gi} and s_{gi}.

Third, the uncertainty in the output from the model is determined by comparing the thrust from the calibrated mathematical model (F_{calc}) to the facility measured thrust (F_m), to define the calibration model error (r) for each ground test data point. This could be done as an absolute difference:

$$r = F_{calc} - F_m \qquad \text{(Eq. 22)}$$

or as a relative difference:

$$r' = \frac{F_{calc} - F_m}{F_m} \qquad \text{(Eq. 23)}$$

The relative difference, which is not necessarily constant through the range of F_m values, is more advantageous when examining a range of test conditions. The definition used will affect the numerical value of the error estimates.

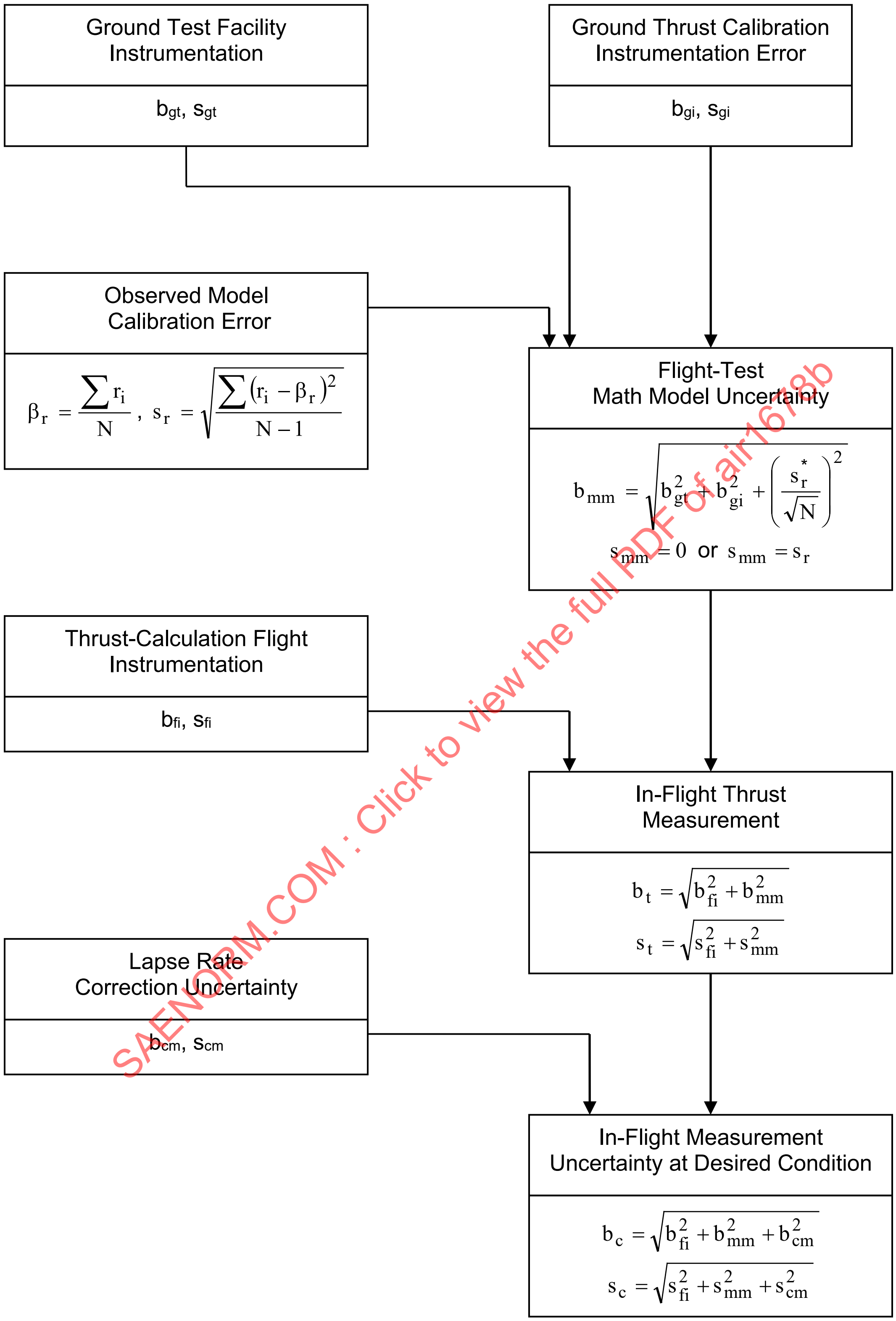

FIGURE 14 - IN-FLIGHT THRUST MEASUREMENT PROCESS UNCERTAINTY PROPAGATION

Fourth, the information from the first three steps is combined to identify the mathematical model error. The mathematical model will always have a systematic error. The limit of this error can be estimated from the information above. Initially, the observed systematic error ($\tilde{\beta}_r$) is determined and eliminated as much as possible through calibration. The calibration model error for each flight condition is the average calibration model error for the flight condition or range of conditions of interest (Figure). Care should be taken in defining the range of test conditions for which data can be combined. Appendix E provides some guidelines.

$$\tilde{\beta}_r = \frac{\sum_i \tilde{r}_i}{N} \quad \text{(Eq. 27)}$$

where $\tilde{r}_i$ is the difference between an individual measurement and the initial calibration model.

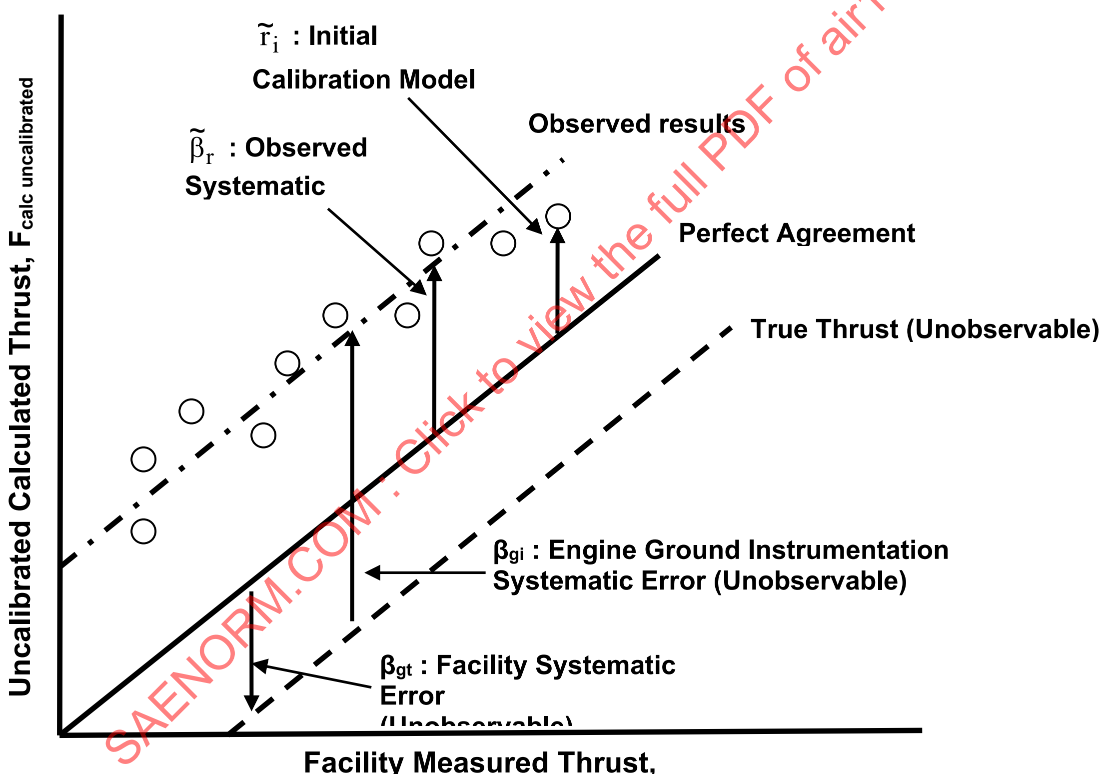

FIGURE 15 - ILLUSTRATION OF MATHEMATICAL MODEL SYSTEMATIC ERROR

The standard deviation of the observed calibration model error (s_r) must also be calculated (Figure).

$$s_r = \sqrt{\frac{\sum_i \left(\tilde{r}_i - \tilde{\beta}_r\right)^2}{N-1}} = \sqrt{\frac{\sum_i (r_i)^2}{N-1}} \quad \text{(Eq. 28)}$$

where r_i is the difference between an individual measurement and the post-calibrated model. The corrected observed systematic error is then $\beta_r = \frac{\sum_i r_i}{N} \approx 0$ (See Appendix E).

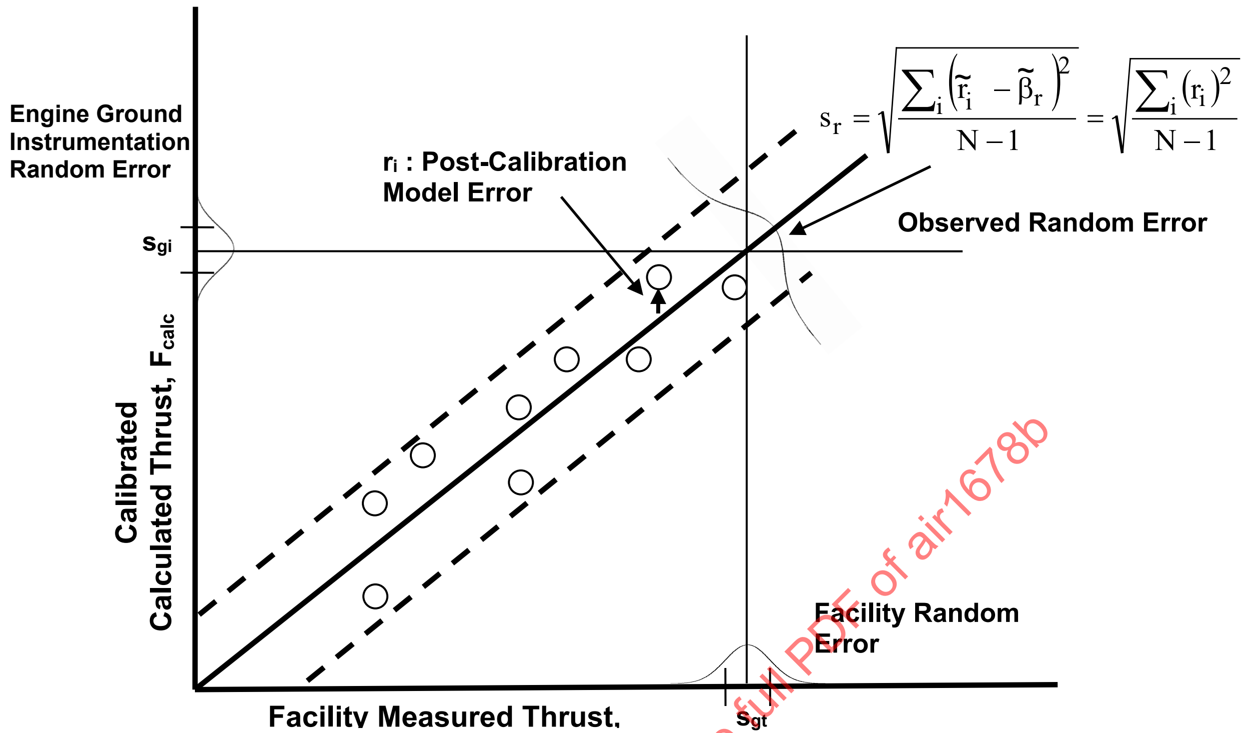

FIGURE 16 - ILLUSTRATION OF MATHEMATICAL MODEL (CORRECTED FOR $\tilde{\beta}_r$) STANDARD DEVIATION

Two cases are considered to define the mathematical model systematic uncertainty. In the simplest case, the flight test engine and its instrumentation are assumed to be those used in the ground test. If the total flight test data acquisition and reduction process is replicated, then the mathematical model systematic uncertainty is defined by the ground facility measurement systematic standard uncertainty (b_{gt}) and the fossilized random element from the calibration process: $\frac{2s_r^*}{\sqrt{N}}$, for large sample sizes (N $\geq$ 30).

The mathematical model systematic standard uncertainty is defined by root-sum-squaring these two components.

$$b_{mm} = \sqrt{b_{gt}^2 + \left(\frac{s_r^*}{\sqrt{N}}\right)^2} \qquad \text{(Eq. 29)}$$

For the case where any portion of the flight test measurement process is not identical to the ground test measurement process, the systematic uncertainty of the ground instrumentation, b_{gi}, (or at least that portion which is not carried over into flight test) must be considered. The mathematical model systematic uncertainty is then defined by:

$$b_{mm} = \sqrt{b_{gt}^2 + b_{gi}^2 + \left(\frac{s_r^*}{\sqrt{N}}\right)^2} \qquad \text{(Eq. 30)}$$

In addition to the systematic uncertainty, the random uncertainty should be investigated. The thrust calculation procedure is not normally expected to introduce any random errors. An estimate of the standard deviation of the calibration model error ($\hat{s}_r$) is calculable from the root-sum-square of the random components of the two sets of measurement standard deviations, s_{gt} and s_{gi}.

$$\hat{s}_r = \sqrt{s_{gt}^2 + s_{gi}^2} \quad \text{(Eq. 31)}$$

The actual standard deviation (s_r) is calculated from the observed data using Equation 28. The two are compared using an F test as defined in Appendix E4. If they are essentially the same, it can be assumed that the ground test process is well controlled and understood and that the mathematical modeling process does not introduce any random error. In this event, the Mathematical Model standard deviation is zero as shown in Equation 32.

$$s_{mm} = 0 \quad \text{(Eq. 32)}$$

If a significant difference exists, then the Mathematical Model standard deviation is equal to s_r.

$$s_{mm} = s_r \quad \text{(Eq. 33)}$$

Care must be taken to ensure that the derived uncertainty from the resultant calibrated mathematical model does not excessively misstate the uncertainty. The evaluation could assess only the worst case condition and provide a conservative estimate (maximum value) for the desired test envelope, or could assess data throughout the envelope to provide an average estimate. Since it may not be feasible to identify the worst case condition prior to completion of the data analysis, the latter option may be the most practical for evaluations which require broad flight envelope testing. The assumptions made in combining flight test conditions can be tested by subdividing the final combined conditions and showing that the mathematical model results remain essentially constant.

DERIVATION OF THE CALIBRATED LAPSE RATE CORRECTION MODEL AND ITS UNCERTAINTY COMPONENTS

An additional task which will be part of the posttest analysis is that of defining the procedure to correct actual flight test data to desired standard conditions. This also requires an evaluation to determine if additional uncertainties are introduced by the correction process, b_{cm} and s_{cm}.

The procedures are discussed in Appendix E3.

6.3 Flight Test

The flight test activities include pretest estimating of flight measurement uncertainties and in-flight thrust uncertainties and posttest confirmation of pretest uncertainty estimates. A logic flow chart is shown in Figure 17.

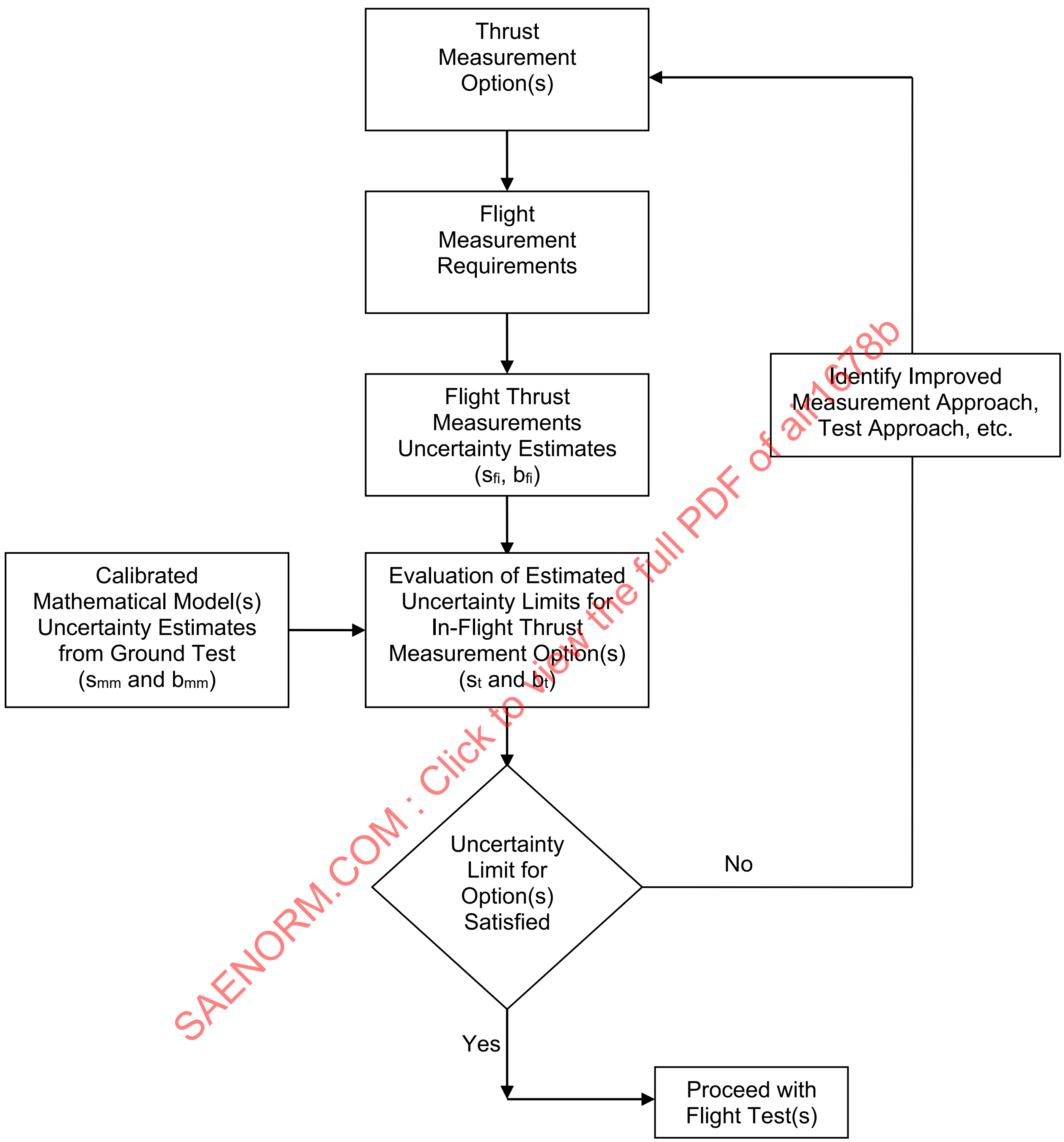

FIGURE 17 - FLIGHT TEST MEASUREMENT PROCESS EVALUATION CYCLES

6.3.1 Flight Pretest Uncertainty Analysis

The total measurement system, including calibration, test article and/or instrumentation installation, data acquisition, data reduction processes and method is identified using the selected thrust options. A detailed pretest uncertainty analysis is made as it was for ground test. Tables similar to Table 10 and Table 11 are completed. This analysis provides the flight instrumentation uncertainty components s_{fi} and b_{fi}. The results are combined with the mathematical model uncertainty components (s_{mm} and b_{mm}) and, where appropriate, the lapse rate correction uncertainty components (s_{cm} and b_{cm}) to define the flight test thrust measurement uncertainty components (s_c and b_c):

$$s_c = \sqrt{s_{fi}^2 + s_{mm}^2 + s_{cm}^2} \quad \text{(Eq. 34)}$$

$$b_c = \sqrt{b_{fi}^2 + b_{mm}^2 + b_{cm}^2} \quad \text{(Eq. 35)}$$

where b_{mm} is calculated with Equation 27.

If the same instrumentation is used in the ground test and the flight test then the common elements in b_{fi} may cancel and can be left out of Equation 35, and b_{mm} is calculated with Equation 29.

For as-tested conditions, the lapse rate correction uncertainty components would be deleted.

6.3.2 Flight Posttest Analysis

The objectives of the posttest data analysis are to confirm the pretest measurement uncertainty estimates and to calculate the in-flight thrust measurement uncertainty for the selected mathematical model options.

The random standard uncertainty for each measurement can be verified from repeat data points. The systematic standard uncertainty can only be checked in a broad sense by comparison of measured thrust to predicted values. If multiple thrust options are available they may be compared to provide additional assurance that systematic uncertainties have been properly assessed.

6.4 Results Analyses

The objective of this effort is to ensure that systematic unknown fixed errors do not go undetected in the various program processes. The program results review consists of examining the consistency of the program test results (ground and flight tests) with pretest predictions and with vehicle performance characteristics. Checks should reveal specific areas requiring detailed test data analysis. Problem areas should be resolved and, as a result, the uncertainty intervals for the in-flight thrust measurement options may require adjustment.

6.4.1 Review of Ground and Flight Test Results

The prime objective is to ensure that the ground test calibrated mathematical model(s) is viable and corresponding uncertainty components are applicable to the actual flight tests. In the review, factors that may void or alter the applicability of the mathematical model should be identified. These factors include instrumentation failures (and possibly subsequent substitutions), differences in the instrumentation type and probe location between ground and flight test, differences between the simulated flight conditions and the actual flight conditions, and differences in engines or engine configurations between ground and flight test. Adjustments to the final flight thrust results are made on the basis of this review.
Techniques for testing data validity using aircraft performance data consistency (e.g., drag polar analysis, Reference 2.1.1.1) may be employed. These techniques are useful for identifying problems.

6.4.2 Test Data Averaging

It may be advantageous to combine data points to improve the performance estimation. The standard deviation of an average is reduced by the inverse ratio of the square root of the number of independent data points in the average compared to the standard deviation of single points. This results in a standard uncertainty for the average $(u_{\overline{X}})$ defined by:

$$u_{\overline{X}} = \sqrt{b_c^2 + \left(\frac{s_c}{\sqrt{N}}\right)^2} \quad \text{(Eq. 36)}$$

Data point independence implies that all of the random error sources are active in each data point which is not usually the case for back to back readings from a data system.

If more than one independent thrust measurement option is available for a given flight test condition it may be possible to combine data to improve the estimate. However, defining the uncertainty of such combined data is very complex and beyond the scope of this document (see Reference 2.1.3.1).

6.4.3 Program Data Uncertainty Reporting

The flight test performance results should be reported following the general requirements of 3.3. The uncertainty interval and its components for each thrust option should be reported and should include supportive uncertainty information: each option's measurements uncertainty components and the corresponding elemental error sources with their uncertainty estimates. It may be useful to report intermediate performance results.

For combined data points, it is necessary to report the number of data points represented by the average value in addition to the s_c and b_c components.

APPENDIX A - SMALL SAMPLE STATISTICAL METHODS

A.1 UNCERTAINTY MODEL

When the degrees of freedom of the result being determined in a test is small, the uncertainty model given in Reference 2.1.2.3 must be used to find the uncertainty of the result (The uncertainty model used in the main body of this document, Equation 12, is the large sample version (Reference 2.1.4.1) of the ISO model)

Consider the case where a result, R, is determined from measured variables, $\overline{X}_i$, as

$$R = f(\overline{X}_1, \overline{X}_2, ..., \overline{X}_I) \quad \text{(Eq. A1)}$$

Once the systematic standard uncertainties for the variables have been obtained from Equation 7 and the standard random uncertainties have been obtained from Equation 1, 2, or 3 as appropriate, then the uncertainty of the result at a 95% confidence level is determined using the ISO model as

$$U_{R_{95}} = t_{95}\left\{\sum_{i=1}^{I}\left[\theta_i b_{X_i}\right]^2 + 2\sum_{i=1}^{I-1}\sum_{k=i+1}^{I}\theta_i\theta_k b_{X_{ik}} + \sum_{i=1}^{I}\left(\theta_i s_{X_i}\right)^2\right\}^{1/2} \quad \text{(Eq. A2)}$$

The second term on the right of Equation A2 accounts for the systematic errors that are from the same source and are, therefore, correlated. Here the b_{ik} term represents the correlated systematic errors between variables i and k as defined in 2.1.4.1. This correlated term is calculated as $b_{ik} = r*b_i*b_k$, with r being the correlation coefficient. Since there is no data to use to calculate r, we assume that the elemental systematic standard uncertainties are completely correlated and therefore r = 1. Correlated systematic uncertainties can have a significant effect on the total uncertainty of the result, and they should be carefully considered in a test program (References 2.1.4.7, 2.1.4.8, 2.1.4.9).

The t_{95} factor in Equation A2 is the 95th percentile point for the two-tailed student's "t" distribution (Table A1). The value of t_{95} depends on the degrees of freedom for the result, R.

A.2 DEGREES OF FREEDOM (ν)

In a sample, the number of degrees of freedom is the size of the sample. When a statistic is calculated from the sample, the degrees of freedom associated with the statistic is reduced by one for every estimated parameter used in calculating the statistic. For example, from a sample of size N_i, $\overline{X}_i$ is calculated:

$$\overline{X}_i = \frac{\sum_{j=1}^{N_i} X_j}{N_i} \quad \text{(Eq. A3)}$$

which has $\nu = N_i$ degrees of freedom, and

$$s_{X_i} = \sqrt{\frac{\sum_{j=1}^{N_i}\left(X_{i_j} - \overline{X}_i\right)^2}{N_i - 1}} \quad \text{(Eq. A4)}$$

which has $\nu = N_i - 1$ degrees of freedom because $\overline{X}_i$ (based on the same sample of data) is used to calculate s. In calculating other statistics, more than one degree of freedom may be lost. For example, in calculating the standard error of a curve fit, the number of degrees of freedom which are lost is equal to the number of estimated coefficients used to define the curve. Each random standard uncertainty (s_i) in Equation A2 with its associated degrees of freedom (ν_{s_i}) is used in the determination of the degrees of freedom of the result, ν_R. These random standard uncertainties can be either calculated directly from data or estimated from standard deviations from elemental random error sources as described in 3.1.4.

When random standard uncertainties of elemental random error sources are combined (root-sum-square), the degrees of freedom of the result must be determined. The Welch-Satterthwaite formula is used for this purpose. It is a function of the degrees of freedom and the magnitude of the elemental random standard uncertainties.

If,

$$s_1 = \sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2} \text{ with degrees of freedom } \nu_{11}, \nu_{21}, \nu_{31} \quad \text{(Eq. A5)}$$

Then according to the Welch-Satterthwaite formula,

$$\nu_{s_1} = \frac{\left[s_{11}^2 + s_{21}^2 + s_{31}^2\right]^2}{\frac{s_{11}^4}{\nu_{11}} + \frac{s_{21}^4}{\nu_{21}} + \frac{s_{31}^4}{\nu_{31}}} \quad \text{(Eq. A6)}$$

The general form being:

$$\nu_{s_{X_i}} = \frac{\left[\sum_p \sum_k s_{kp}^2\right]^2}{\sum_p \sum_k \frac{s_{kp}^4}{\nu_{s_{kp}}}} = \frac{\left[s_{X_i}\right]^4}{\sum_p \sum_k \frac{s_{kp}^4}{\nu_{s_{kp}}}} \quad \text{(Eq. A7)}$$

Each of the systematic standard uncertainties will also have an associated degrees of freedom based on the experimenter's judgment of the goodness of the systematic uncertainty estimates. The systematic standard uncertainty of the measurement, b_{X_i}, is the root-sum-square of the elemental systematic standard uncertainties, $(b_{X_i})_{kp}$, as

$$b_{X_i} = \sqrt{\sum_{p=1}^{5} \sum_{k=1}^{K_p} b_{kp}} \quad \text{(Eq. A8)}$$

where K_p is the number of elemental systematic error sources for the category p and the measurement $\overline{X}_i$. Reference 2.1.2.3 recommends the approximation for the degrees of freedom of b_{kp} as

$$\nu_{b_{kp}} = \frac{1}{2}\left(\frac{\Delta(b_{kp})}{b_{kp}}\right)^{-2} \quad \text{(Eq. A9)}$$

where the quantity in parenthesis is an estimate of the relative variability of the systematic standard uncertainty estimate. For instance, if one thought that the estimate of b_{kp} was reliable to within ±25%, then

$$\nu_{b_{kp}} = \frac{1}{2}(0.25)^{-2} = 8 \quad \text{(Eq. A10)}$$

Using the Welch-Satterthwaite formula, the degrees of freedom for b_i is then determined as

$$\nu_{b_{X_i}} = \frac{\left[b_{X_i}\right]^4}{\sum_p \sum_k \frac{b_{kp}^4}{\nu_{b_{kp}}}} \quad \text{(Eq. A11)}$$

The degrees of freedom of the result is then determined as

$$\nu_R = \frac{\left[\sum_{i=1}^{I}\left\{\left(\theta_{X_i} b_{X_i}\right)^2 + \left(\theta_{X_i} s_{X_i}\right)^2\right\}\right]^2}{\sum_{i=1}^{I}\left\{\frac{\left(\theta_{X_i} b_{X_i}\right)^4}{\nu_{b_{X_i}}} + \frac{\left(\theta_{X_i} s_{X_i}\right)^4}{\nu_{s_{X_i}}}\right\}} \quad \text{(Eq. A12)}$$

Note that ν_R is influenced most by the number of degrees of freedom of the largest of the $\theta_{X_i} s_{X_i}$ or $\theta_{X_i} b_{X_i}$ terms.

A.3 STUDENT'S "t" TABLE

The table of the student's "t" distribution (Table A1) presents the two-tailed 95% "t" values for the degrees of freedom from 1 to 30. Above 30, the value is rounded to 2.0.

The t_{95} value in Equation A2 for $U_{R_{95}}$ is obtained from Table A1 for the degrees of freedom ν_R from Equation A12.

The uncertainty expression in Equation A2 can then be written as

$$U_{R_{95}} = t_{95} \cdot \sqrt{b_R^2 + s_R^2} \quad \text{(Eq. A13)}$$

where

$$b_R = \left\{\sum_{i=1}^{I}\left[\theta_i b_{Xi}\right]^2 + 2\sum_{i=1}^{I-1}\sum_{k=i+1}^{I}\theta_i\theta_k b_{Xik}\right\}^{1/2} \quad \text{(Eq. A14)}$$

and

$$s_R = \left\{\sum_{i=1}^{I}\left(\theta_i s_{X_i}\right)^2\right\}^{1/2} \quad \text{(Eq. A15)}$$

For many engineering applications the degrees of freedom for the result from Equation 12 is large enough for t_{95} to be taken as 2 (References 2.1.4.1 and 2.1.4.2). This case is called the "large sample" approximation for $U_{R_{95}}$ with Equation A13 expressed as

$$U_{R_{95}} = 2 \cdot \sqrt{b_R^2 + s_R^2} \quad \text{(Eq. A16)}$$

Where b_R and s_R are given by Equations A14 and A15.

The $U_{R_{95}}$ expression in Equation A16 is the uncertainty expression used in this document (Equation 12) with the exception that correlated systematic uncertainties are not included in any of the examples in the document.

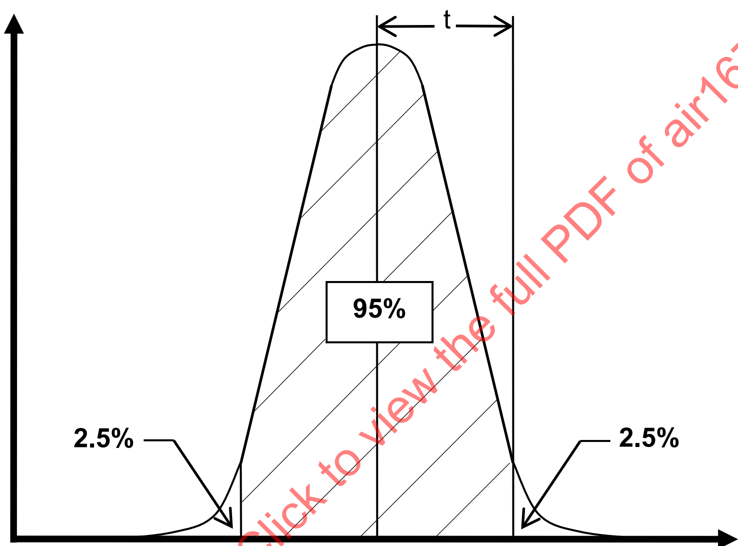

FIGURE A1 – TWO-TAILED 95% STUDENT'S "t" DISTRIBUTION

TABLE A1 - TWO-TAILED STUDENT'S "t" TABLE

Degrees of Freedom	"t" 95	*Degrees of Freedom*	"t" 95
1	12.706	16	2.120
2	4.303	17	2.110
3	3.182	18	2.101
4	2.776	19	2.093
5	2.571	20	2.086
6	2.447	21	2.080
7	2.365	22	2.074
8	2.306	23	2.069
9	2.262	24	2.064
10	2.228	25	2.060
11	2.201	26	2.056
12	2.179	27	2.052
13	2.160	28	2.048
14	2.145	29	2.045
15	2.131	30 or More use 2.0	

A.4 CONFIDENCE LEVELS OTHER THAN 95%

For a confidence level other than 95%, the appropriate student's "$t_{\%}$" value for that confidence level and for the degrees of freedom in the result determined in Equation A12 is used in Equation A2 as:

$$U_{R_{\%}} = t_{\%}\left\{\sum_{i=1}^{I}\left[\theta_i b_{X_i}\right]^2 + 2\sum_{i=1}^{I-1}\sum_{k=i+1}^{I}\theta_i\theta_k b_{X_{ik}} + \sum_{i=1}^{I}\left(\theta_i s_{X_i}\right)^2\right\}^{1/2} \quad \text{(Eq. A17)}$$

Once the degrees of freedom of the result, ν_R, is ten or more, the large sample value for $t_{\%}$ can be used (see Reference 2.1.4.9). For instance, for a large sample, the 99% confidence t value would be 2.6.

APPENDIX B - IMPACT OF MEASUREMENT PROCESS ON ERROR CLASSIFICATION AND ESTIMATION

B.1 GENERAL

Measurement errors are initially put into one of two classes, random or systematic, depending on how the error affects the measurement. The random standard uncertainty, s, is generally derived by a statistical analysis of repeated measurements (ISO Type A) while the systematic standard uncertainty, b, is at least partially estimated by non-statistical methods (ISO Type B). This approach, similar to that proposed in Reference 2.1.2.1, makes a very complex situation manageable and keeps the statistical estimate and the judgment estimate uncertainty components separate until the appropriate time to combine them.

The initial decision on classifying elemental errors may be changed after considering the defined in-flight measurement process and the instrumentation system.

Calibration errors are the most common initial error(s) which may be reclassified as a result of the defined measurement process. For example, considering the measurement of air flow for a gas turbine engine at a test facility over a long period, the uncertainty of this measurement will contain errors due to variations between calibrations, test stands, and measurement instruments. The initial classification of errors would likely remain unchanged. However, if the same measurements were from a single test stand involving one calibration, then the initial random uncertainty in the calibration process would manifest itself as a systematic uncertainty for this process. Reclassification of initial random uncertainty to a systematic uncertainty is marked with an asterisk (*) to indicate that it is a fossilized or fixed uncertainty for this process.

The uncertainty analysis for the above examples will be different from the uncertainty analysis for a comparative, back-to-back test to measure air flow on a single test stand for a single engine, which is a different measuring process. Calibration errors (systematic and random) which propagate as a systematic uncertainty into the measurement process are significantly reduced in comparative testing when the same instrumentation and equipment are used for all testing (Reference 2.1.4.8).

The planned instrumentation, type and number, is also part of the definition of the measurement process. Averaging can be used to reduce the standard deviation with repeated single measurements if the measured variable is constant, or if redundant instruments can be recorded simultaneously. The decision logic for error classification based on the actual measurement process is shown in Table B1.

In summary, errors from the calibration process merit special consideration. There are three cases to consider:

a. If the test period is long enough that instrumentation may be calibrated more than once and/or several test stands are involved, the random errors in the calibration hierarchy should be treated as contributing to the overall random uncertainty. The calibration cycle times are determined using Reference 2.1.2.2.

b. For a single set of instrumentation, calibrated only once during the test, all the calibration error is frozen or fossilized into a systematic error. The uncertainty of the calibration process is all systematic uncertainty. An asterisk is used to designate calibration random uncertainty that is fossilized, i.e., s*.

c. For comparative back-to-back tests where the test objective is the difference between two successive tests and the calibration does not vary within the timeframe of the comparison, the calibration error (systematic plus fossilized random) is constant and therefore will not contribute to the systematic error of the difference.

TABLE B1 - DECISION LOGIC DIAGRAM TO SUMMARIZE RULES FOR ERROR CLASSIFICATION

Measurement Process Defined As:	Errors Classified As:
Many Tests Over Long Periods With Many Calibrations	$b_1 = b_{CAL}$, $s_1 = s_{CAL}$ s_2, s_3, s_4, s_5, b_2, b_3, b_4, b_5 Unaffected
Single Test, One Calibration	All uncertainty from calibration process is fixed or fossilized into the systematic uncertainty $b_1 = \sqrt{b_{CAL}^2 + \left(\frac{\overset{*}{s}_{CAL}}{\sqrt{N}}\right)^2}$ where N = number of samples $s_1 = 0$ s_2, s_3, s_4, s_5, b_2, b_3, b_4, b_5 Unaffected
Comparative or Back-to-Back Test, One Calibration, Ground Test or Flight Test	Same as Single Test, One Calibration, except all systematic uncertainty is significantly reduced when first test result is subtracted from second result. $s_1 = 0$ b_1, b_2, b_3, b_4 and b_5 may be reduced s_2, s_3, s_4, and s_5 Unaffected
Flight Test Uses Same Engine Instrumentation as Ground Test Without Recalibration	Ground test calibration of thrust model will reduce engine instrumentation calibration systematic uncertainty
Pressure Differential (P_1-P_2) uses same Transducer for Both P_1 and P_2	Systematic Uncertainty for transducers cancels out if the systematic uncertainties for P_1 and P_2 are both percent of full scale.
Subscript Notation:	1 = Calibration 2 = Test Article and/or Instrumentation Installation 3 = Data Acquisition 4 = Data Reduction 5 = Errors of Method

APPENDIX C - EXAMPLE INFLUENCE COEFFICIENT CALCULATIONS

The basis of this example is found in Reference 2.1.4.10. For consistency within this document, the format and units have been changed and additional information has been provided. In this example, the in-flight thrust model uses five measurements:

TABLE C1 – MEASUREMENTS USED IN EXAMPLE

Low Rotor Speed	**N1**	**rpm**
High Rotor Speed	**N2**	**rpm**
Inlet Static Press.	**PS1**	**psi**
Inlet Delta Press. (Total minus Static)	**DELP0**	**psi**
Inlet Total Temp.	**T2**	**°R**

The exhaust nozzle area, A8, is fixed and measured. The functional relationships are as follows:

$$FN = FG - (WA)(V0)/G \quad \text{(Eq. C1)}$$

$$FG = \left[(FGP)\left(\frac{P2}{PS1}\right) - 1\right](PS1)(A8) \quad \text{(Eq. C2)}$$

$$FGP = f(N2C) \quad \text{(Eq. C3)}$$

$$N2C = (N2)\sqrt{\frac{518.67}{T2}} \quad \text{(Eq. C4)}$$

therefore

$$FGP = f(N2, T2) \quad \text{(Eq. C5)}$$

$$P2 = (P0)(ETAR) \quad \text{(Eq. C6)}$$

$$P0 = PS1\left[1. + 0.2(XM)^2\right]^{3.5} \quad \text{(Eq. C7)}$$

$$XM = \sqrt{5 \cdot \left[\left(\frac{DELP0}{PS1} + 1.\right)^{0.286} - 1\right]} \quad \text{(Eq. C8)}$$

$$ETAR = f(WAC) \quad \text{(Eq. C9)}$$

$$WAC = f(N1C) \quad \text{(Eq. C10)}$$

$$N1C = (N1)\sqrt{\frac{518.67}{T2}} \quad \text{(Eq. C11)}$$

$$P2 = f(PS1, DELP0, N1, T2) \quad \text{(Eq. C12)}$$

$$A8 = 32.08 \text{ in}^2 \quad \text{(Eq. C13)}$$

$$WA = WAC\left(\frac{P2}{14.696}\right) / \sqrt{\frac{518.67}{T2}} \quad \text{(Eq. C14)}$$

therefore

$$WA = f(PS1, DELP0, N1, T2) \quad \text{(Eq. C15)}$$

$$V0 = (49.02)(XM)\sqrt{TS0} \quad \text{(Eq. C16)}$$

$$TS0 = \frac{T2}{1.+(0.2)(XM)^2} \quad \text{(Eq. C17)}$$

therefore

$$V0 = f(DELP0, PS1, T2) \quad \text{(Eq. C18)}$$

$$G = 32.174 \quad \text{(Eq. C19)}$$

Net thrust is then related through the functional relationships represented by Equations C5, C12, C15, and C18 to the five basic measurements. As part of the calibration of the mathematical model, the first three of these functional relationships were derived from test data as follows:

$$FGP = 6.442 - 2.463 \times 10^{-4}\, N2C + 2.883 \times 10^{-9} (N2C)^2 \quad \text{(Eq. C20)}$$

$$WAC = -1.185 + 4.666 \times 10^{-4}\, N1C - 2.593 \times 10^{-10} (N1C)^2 \quad \text{(Eq. C21)}$$

$$ETAR = 0.9876 + 2.551 \times 10^{-3}\, WAC - 1.525 \times 10^{-4} (WAC)^2 \quad \text{(Eq. C22)}$$

The final functional relationship was derived from the physics of fluid flow as defined by Equations C16 and C17.

The task of deriving the influence coefficients by partial derivatives would be quite lengthy. Rather, a computer program can be written to calculate both the nominal value of the performance parameter as well as the influence coefficients for any set of inputs. Table C2 shows the results for a sample test case that could be obtained from such a program. The test case chosen was for a flight condition of 1000 ft and Mach 0.65. The rotor speeds chosen are completely arbitrary and intended only for illustrative purposes. The output lists the absolute values for the influence coefficient. That is,

$$\theta_i = \frac{\Delta FN}{\Delta X_i} \quad \text{(Eq. C23)}$$

where X_j represents the measurements. It also shows a relative influence coefficient (θ') defined by:

$$\theta_i' = \frac{\Delta FN / FN}{\Delta X_i / X_i} \quad \text{(Eq. C24)}$$

The program also provides the influence coefficients for the intermediate performance parameters (FG, WA, V0).

The exhaust area (A8) is assumed to be a known constant. Actually, the exhaust area will have two error components. The first is systematic error due to the measurement which established the value used. This error would essentially be calibrated out in the mathematical calibration model. The second error is in the assumption that it is a fixed (constant) value. The actual area will vary with temperature and pressure. This too will be calibrated out to a certain extent. If all the temperature and pressure profiles are not duplicated both internally and externally, an error could be introduced by this assumption and should be evaluated.

The resultant influence coefficients will be used to propagate the measurement uncertainty components to the performance parameter (as illustrated in Table 12 of the main text).

TABLE C2 - INFLUENCE COEFFICIENTS CALCULATED FOR THRUST OPTION 1

MATHEMATICAL MODEL INPUTS		
N1 = 35000. rpm	N2 = 63300. rpm	T2 = 98.7 °F
PS1 = 14.17 psi	DELP0 = 4.65 psi	
MATHEMATICAL MODEL CALCULATED PERFORMANCE PARAMETERS		
FG = 832. lbf	WA = 18.82 lb/s	
FN = 409. lbf	V0 = 723.2 ft/s	
INFLUENCE COEFFICIENTS:		
ABSOLUTE VALUES - DEL PP/DEL BM		

PERFORMANCE PARAMETERS BASIC MEASUREMENTS	FG	WA	V0	FN
N1	-0.0010	0.0006	0.0000	-0.0135
N2	0.060	0.0000	0.0000	0.0620
PS1	36.2831	0.9996	-21.1935	26.3039
DELP0	68.4540	1.0010	64.7534	7.9855
T2	-3.4142	-0.0006	0.6473	-3.7796

RELATIVE VALUES - (DEL PP/NOM PP) / (DEL BM/NOM BM)

PERFORMANCE PARAMETERS BASIC MEASUREMENTS	FG	WA	V0	FN
N1	-0.0430	1.0341	0.0000	-1.1557
N2	4.7127	0.0000	0.0000	9.5817
PS1	0.6176	0.7527	-0.4153	0.9103
DELP0	0.7824	0.2473	0.4164	0.0907
T2	-2.2901	-0.0175	0.4998	-5.1544

APPENDIX D - INITIAL PERFORMANCE SURVEY OF THRUST MEASUREMENT OPTIONS

The basis of this example is found in Reference 2.1.4.10. The same example is discussed in 6.2.1 of the main text and Appendix C. This is a test condition of 1000 feet, 0.65 Mach number, at an arbitrary power setting. For all the thrust options, the facility measurements would be identical. For simplicity, the facility measurements are not tabulated. The performance parameter uncertainty components are assumed to be defined as:

$$s_{gt} = 5.0 \quad \text{(Eq. D1)}$$

$$b_{gt} = 5.0 \quad \text{(Eq. D2)}$$

It is assumed that for simplicity the ground and flight test instrumentation have equivalent capabilities. Table D1 uses the general format of Table 9 in the main text modified for this specific application. It should be noted that in this case the measurements are not sub-divided into intermediate performance parameters. This is because the measurements for gross thrust, airflow and velocity are not independent, so the influence coefficients must be evaluated end-to-end with respect to the final performance parameter. Note that for Option 1, the tabulation is identical to Table 12 of the main text.

The procedure described in Appendix C is carried out for each thrust option to calculate the influence coefficients. The influence coefficients in Table D1, except for Option 1, are based on Appendix B of Reference 2.1.4.10. Average values of the measured parameter were assumed in converting from the functional form in the report to the absolute values shown in this table. The estimated uncertainty components are then propagated to define the measurement uncertainty components for each thrust option (s_{gi}, b_{gi}, s_{fi} and b_{fi}).

These uncertainty components are combined with the facility measurement uncertainty components (s_{gt} and b_{gt}) to define the complete inflight thrust measurement uncertainty components (s and b) using the relationship defined in 5.1.2 of the main text.

$$b = \sqrt{b_{gt}^2 + b_{gi}^2 + b_{fi}^2 + \frac{\left[s_{gt}^*\right]^2 + \left[s_{gi}^*\right]^2}{N_g}} \quad \text{(Eq. D4)}$$

$$s = s_{fi} \quad \text{(Eq. D5)}$$

The number of ground test data points (N_g) expected to be included in the mathematical calibration model is assumed to be 30. This example only considers a single test condition, so a single apparent "best" option can be identified. If more than one test condition is investigated, the "best" option may vary from one condition to the next. The option would be compared directly through the single value for the uncertainty limit, where N_f is the number of in-flight tests.

$$U_{95} = 2\sqrt{b^2 + \left(\frac{s}{\sqrt{N_f}}\right)^2} \quad \text{(Eq. D6)}$$

TABLE D1 - EXAMPLE PERFORMANCE SURVEY OF THRUST OPTIONS

Test Condition: 1000 ft / 0.65 Mach No. / Cruise Power										
Basic Measurement	Facility Measurement		Thrust Calculation Measure					In-Flight Thrust **		
			Ground		Flight			Random and Systematic Uncertainties		Expanded Uncertainty
	s_{gt}	b_{gt}	s_{gi}	b_{gi}	s_{fi}	b_{fi}	θ	s	b	U
Facility Net Thrust	5.0	5.0								
Thrust Option 1										
Low Rotor Speed N1			6	3.0	6	3.0	-0.0135			
High Rotor Speed N2			31	3	31	3	0.0620			
Inlet Static Pressure PS1			0.0130	0.0535	0.0130	0.0535	26.3039			
Inlet Delta Pressure DELP0			0.0069	0.0292	0.0069	0.0292	7.9855			
Inlet Total Temperature T2			0.35	1.35	0.35	1.35	-3.7796			
Net Thrust*			2.36	5.30	2.36	5.30	-	2.36	9.07	18.74
Thrust Option 2										
Low Rotor Speed N1			6	3.0	6	3.0	0.0089			
Inlet Static Pressure PS1			0.0125	0.0535	0.0125	0.0535	-155.7216			
Inlet Delta Pressure DELP0			0.0069	0.0292	0.0069	0.0292	-26.7120			
Inlet Total Temperature T2			0.35	1.35	0.35	1.35	-0.8074			
Turbine Exit Pressure P6			0.047	0.005	0.047	0.005	12.5573			
Duct Discharge Pressure P16			0.39	0.045	0.39	0.045	9.7920			
Turbine Exit Temperature T6			3.1	1.5	3.1	1.5	0.0955			
Duct Discharge Temperature T16			1.07	1.6	1.07	1.6	0.3014			
Burner Pressure P3			0.39	0.045	0.39	0.045	0.6695			
Fuel Flow WF			0.005	0.0015	0.005	0.0015	0.0001			
Net Thrust*			4.37	8.46	4.37	8.46	-	4.37	13.02	27.48
Thrust Option 3										
Low Rotor Speed N1			6	3.0	6	3.0	0.0089			
Inlet Static Pressure PS1			0.0125	0.0535	0.0125	0.0535	-152.7984			
Inlet Delta Pressure DELP0			0.0069	0.0292	0.0069	0.0292	-26.5104			
Inlet Total Temperature T2			0.35	1.35	0.35	1.35	-0.8098			
Turbine Exit Pressure P6			0.047	0.005	0.047	0.005	9.6678			
Duct Discharge Pressure P16			0.39	0.045	0.39	0.045	12.6596			
Turbine Exit Temperature T6			3.1	1.5	3.1	1.5	0.1294			
Duct Discharge Temperature T16			1.07	1.6	1.07	1.6	0.2260			
Burner Pressure P3			0.39	0.045	0.39	0.045	0.6506			
Fuel Flow WF			0.005	0.0015	0.005	0.0015	0.0002			
Net Thrust*			5.35	8.31	5.35	8.31	-	5.35	12.84	27.82
Thrust Option 4										
Low Rotor Speed N1			6	3.0	6	3.0	0.0128			
Inlet Static Pressure PS1			0.0125	0.0535	0.0125	0.0535	-116.0208			
Inlet Delta Pressure DELP0			0.0069	0.0292	0.0069	0.0292	-26.0784			
Inlet Total Temperature T2			0.35	1.35	0.35	1.35	-1.0119			
Turbine Exit Pressure P6			0.047	0.005	0.047	0.005	14.6934			
Duct Discharge Pressure P16			0.39	0.045	0.39	0.045	9.7072			
Turbine Exit Temperature T6			3.1	1.5	3.1	1.5	0.1926			
Duct Discharge Temperature T16			1.07	1.6	1.07	1.6	0.1440			
Fuel Flow WF			0.005	0.0015	0.005	0.0015	0.0001			
Net Thrust*			4.18	6.42	4.18	6.42	-	4.18	10.44	22.49

*These numbers such as 2.36 are calculated as follows: $s_{gi}(F) = \sqrt{[(6)(-0.0135)]^2 + [(31)(0.0620)]^2 + \ldots} = 2.36$ (Eq. D3)

**These values are calculated using Equations D4 and D5.

In evaluating the in-flight systematic uncertainty, the significance of the number of ground test points used in the mathematical calibration model can be quantified. Table D2 shows the effect of the final two terms in Equation D4 when fewer data points are taken. on the uncertainty.

TABLE D2 - EFFECT OF THE NUMBER OF DATA POINTS ON IN-FLIGHT SYSTEMATIC UNCERTAINTY (OPTION 1)

Number of Test Points N	Systematic Standard Uncertainty b (lbf)	Percent Increase in Systematic Uncertainty
∞	9.01	Base
30	9.07	0.6
10	9.18	1.9
5	9.34	3.7
1	10.57	17.3

For the special case where the flight test engine and instrumentation are used in the ground test, the in-flight measurement uncertainty would be essentially equal to the facility systematic standard uncertainty (b_{gt}) for a large number of test points. Until the test is completed, it should only be considered a lower limit since problems in flight test could force changes to the engine and/or instrumentation. This would require the inclusion of the b_{gi} and b_{fi} terms in Equation D4.