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AEROSPACE
INFORMATION
REPORT

AIR 1678

Issued August 1985
Revised

UNCERTAINTY OF IN-FLIGHT THRUST DETERMINATION

PREFACE

The growing need for determining in-flight propulsion system thrust employing turbofan and turbojet engines has been apparent for several years. The reasons for requiring this information, as specified in the E-33 charter, are:

- Determination of aircraft drag
- Problem rectification if aircraft performance is low
- Interpolation of measured thrust and aircraft drag over a range of flight conditions by validation and development of analytical models
- Establishment of a baseline for future aircraft and engine modifications.

In 1972, the Safety Standardization Advisory Committees of the SAE Aerospace Council, working with the SAE Propulsion Division suggested the need for improved knowledge of aircraft propulsion system in-flight thrust. Later, the U.S. Air Force Aeronautical Systems Division independently made a similar suggestion.

The Propulsion Division and the Aerospace Council concluded that the real need was to establish a forum where this subject could be discussed by knowledgeable experts on a technical basis. SAE undertook to do so. Dr. Robert Abernethy was commissioned by George Townsend, Propulsion Division Chairman, to organize the In-Flight Propulsion Measurement Committee E-33. Mr. Gary Adams, representing the Air Force, was also involved in organizing the E-33 Committee at this early stage. The first meeting took place in December, 1978.

PREPARED BY
SAE COMMITTEE E-33
IN-FLIGHT PROPULSION MEASUREMENT

Printed in U.S.A.

The E-33 Committee endeavored to gather industry-wide expertise in in-flight thrust measurement and uncertainty analysis. The Committee was organized into Subcommittees A/B which concentrated on thrust/drag bookkeeping and thrust determination methodology, and Subcommittee C which addressed the subject of thrust determination uncertainty.

After reviewing the industry state-of-the-art, Committee E-33 determined that it would be appropriate to assemble and publish two companion Aerospace Information Reports. Subcommittee E-33A/B was organized under Chairman John Roberts to produce AIR 1703, "In-Flight Thrust Determination." Subcommittee E-33C, under Chairman Gary Adams, produced AIR 1678, "Uncertainty of In-Flight Thrust Determination." Together these reports provide a comprehensive survey of in-flight thrust determination, beginning with definitions and concluding with guidelines for planning a total program and estimating the measurement errors.

Each member of this committee worked diligently for many years to produce these two reports. We are indebted to them for their extraordinary efforts. Special mention is due the Sponsors, Bill Wallace and Jim Thompson and the Arnold Engineering Development Center that produced the figures. The member sponsoring organizations are to be commended for their support. Finally the British MIDAP Group, the AIAA Thrust-Drag Editorial Board, and our Consultants provided valuable liaison.

R. B. Abernethy John Roberts Gary Adams John Steurer

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SYMBOLS

DESCRIPTION

Greek

β	True bias error: the fixed, systematic or constant component of the total error, δ.
δ	Total error: the difference between the measurement and the true value.
ε	True precision error, sometimes called repeatability error or sampling error, the random component of δ.
σ	The true standard deviation of the population.
θ	Absolute Influence Coefficient
θ'	Fractional Influence Coefficient

Roman

B	The estimate of the upper limit of the bias error, β.
B^+, B^-	Upper and lower limits of a nonsymmetrical bias error.
F	Thrust
f	( ) Functional Relationship
n	The number of measurements available (the sample size).
K	Order of Curve Fit
r	Residual error, the absolute difference between calculated and measured performance.
r'	Residual error, the fractional difference between calculated and measured performance.
S	An estimate of the standard deviation, σ, called the precision index.
$\hat{S}_r$	Estimated Value of Residual Error Precision Index
U	Uncertainty, the largest expected error limit. Defined arbitrarily as: Additive Model: $U_{ADD}=B+2S$ Root-Sum-Square Model: $U_{RSS}=\sqrt{B^2+(2S)^2}$

SYMBOLS (Cont'd.)

U^+, U^-	Upper and lower limits of a nonsymmetrical uncertainty interval.
X	An individual measurement, sometimes called an observation or reading.
$\bar{X}$	Average of a set of individual measurements.
$\bar{X}_\infty$	Average of the total population
Y	A performance parameter

SUBSCRIPTS

c	flight test corrected to desired test conditions
cm	lapse rate correction model error
fi	Flight instrumentation (engine)
gt	Ground test facility instrumentation
gi	Ground instrumentation (engine)
i	The number of the elemental error source within the error category
j	The number of the error category which arbitrarily is assigned as: $j = 1$ Calibration errors, $j = 2$ Data acquisition errors, $j = 3$ Data reduction errors,
k	The number of the individual measurement
m	Measured
N	Net
mm	math model error
r	Residual error
t	as-tested flight condition

SUPERSCRIPTS

*	Fossilized precision error

1. INTRODUCTION:

Estimating the measurement error or uncertainty is an essential element in the complex process of evaluating in-flight thrust. During the pre-flight planning phase, it is necessary to assess the accuracy of the candidate methodologies for determining in-flight thrust and to select those which best satisfy program requirements and resources. It defines parameter accuracy, data acquisition, and data reduction requirements for the ground and flight test phases. Post-flight, it is used to examine the consistency of the test results with pretest predictions and with vehicle performance estimates.

At present, there are no industry or government standards for determining in-flight thrust or the associated uncertainty. The purpose of this document is to provide information covering methods for estimating the measurement error or uncertainty of in-flight thrust determination in aircraft employing conventional turbofan/turbojet engines. Methodologies beyond those presented are required in order to evaluate configurations such as vectored thrust or V/STOL. The document is intended to be used as a technical guide. It is not intended to be used as a standard or legal document. A companion document, SAE AIR 1703, presents information and guidance on the selection and use of methodologies to predict and assess propulsion system in-flight thrust. Both documents describe comprehensive procedures and tasks for implementing the methodologies. Each program would select those tasks that are appropriate to meet its particular objectives.

The term "in-flight thrust determination" is used synonymously with "in-flight thrust measurement" although in-flight thrust is not directly measured but is determined or calculated using math modeling relationships between in-flight thrust and various direct measurements of physical quantities. The in-flight thrust determination process includes both Ground Testing and Flight Testing, Figure 1.1. The math modeling relationships between the in-flight thrust and the measurements of the physical quantities are calibrated in Ground Tests. Error estimates for each item shown in Figure 1.1 are required to calculate the uncertainty of the "in-flight thrust measurement".

The text is organized into the following major topics:

Measurement Uncertainty Methodology (Section 2): This section describes the concepts involved in estimating the uncertainty of in-flight thrust. Its purpose is to provide a common basis for understanding. The methodology includes the use of statistical and engineering concepts to meet the need for an easily interpreted estimate of uncertainty (1.1)*.

In-Flight Thrust Measurement Processes (Section 3): This section describes the ground test and flight test phases of the in-flight thrust determination process. The five sets of fundamental error components which must be evaluated to determine the uncertainty of in-flight thrust are identified.

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*Numbers in parenthesis designate references listed in Section 5.

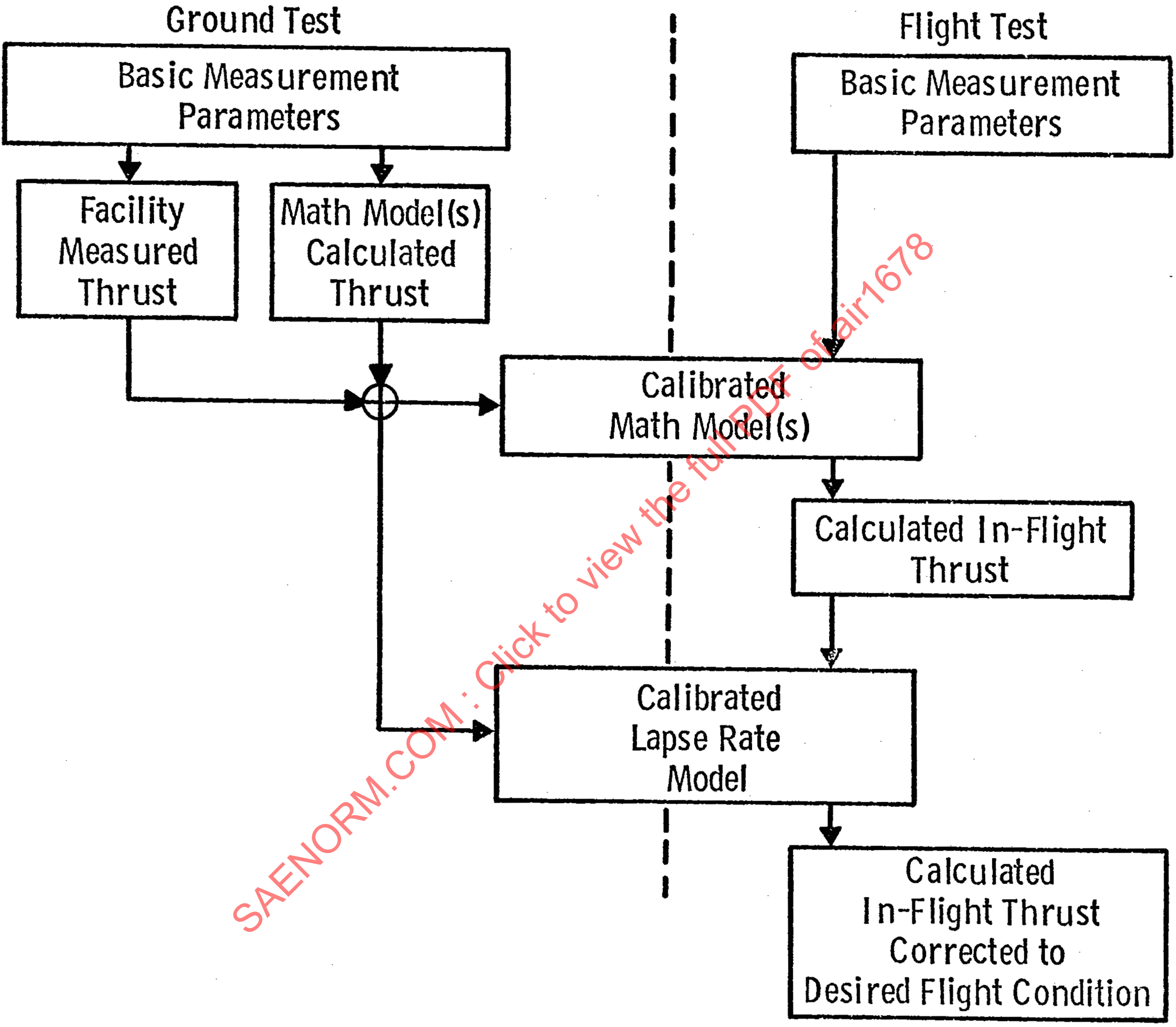


Figure 1.1 - The In-Flight Thrust Measurement Process

Application of Uncertainty Methodology to the In-Flight Thrust Program (Section 4): The uncertainty methodology described in Section 2 is applied to the In-Flight Thrust Measurement Process described in Section 3. Four representative test program phases are defined, i.e., Program Definition Planning, Ground Test, Flight Test, and Results Analysis. The tasks to be accomplished in each phase are described (1.2).

Seven Appendices are also included:

o Appendix A - Definitions

o Appendix B - Small Sample Statistical Methods

Special, small, statistical methods are required to determine the value for the Student's t_{95} distribution when the sample size is less than thirty. This appendix discusses the t value, degrees of freedom, and the propagation of degrees of freedom. A table of "t_{95}" values is presented for degrees of freedom from one to thirty.

o Appendix C - Impact of Measurement Process on Error Classification and Estimation

This appendix discusses the decision logic for the classification of error based on the actual measurement process. Reclassification is discussed and the concept of fossilized error is introduced.

o Appendix D - A Monte Carlo Comparison of Alternative Uncertainty Methods

Using a Monte Carlo simulation, this appendix compares the coverage provided by the additive model and the root-sum-squares model for the calculation of measurement uncertainty.

o Appendix E - Example Influence Coefficient Calculations

o Appendix F - Initial Performance Survey of Thrust Measurement Options

This appendix uses the published results of the Air Launched Cruise Missile Program to illustrate the approach for conducting an initial performance survey of thrust options.

o Appendix G - Calibrated Math Model and the Associated Uncertainty Components for Flight Test

The term "calibrated math model" is discussed and simplified examples of two common approaches are provided.

2. MEASUREMENT UNCERTAINTY METHODOLOGY:

The methodology for determining the uncertainty of in-flight thrust must be structured to combine statistical and engineering concepts in a manner which can be systematically applied to each step in the assessment procedure. The section addresses each of these steps and formulates the appropriate methodology. The result is an easily interpreted, single value estimate of thrust uncertainty.

The essential steps in the procedure for determining thrust uncertainty are:

1. Identify all elemental errors for each measurement in the ground test facility and in-flight related to determining thrust.
2. Preliminarily classify elemental error sources as bias or precision.
3. Estimate the magnitude of each elemental error.
4. For each flight test measurement process, make the final bias or precision classification.
5. Combine elemental errors to define measurement bias and precision error components and tabulate elemental errors.
6. Propagate measurement error components to calculated thrust.
7. Calculate uncertainty limit from error components.
8. Report results.

The following section defines measurement error and its components, identifies the sources of the errors, indicates how to classify the errors and provides guidance in estimating the error components. Combining the elemental error sources to define measurement error components (bias and precision) and how to propagate (combine) these error components to net thrust is also discussed. Section 2.2 introduces the concept of measurement uncertainty obtained by combining the components of measurement errors. The uncertainty interval and its interpretation is explained for a basic measurement. Guidelines are provided for validating the estimated measurement uncertainties. The final section provides guidelines for reporting the error components of the basic measurements and the uncertainty intervals of both the basic measurements and calculated thrust. Definitions are presented in Appendix A.

2.1 Measurement Error: All quantities, ranging from basic measurements to calculated performance, have error. This error is the difference between the measured or calculated observable value and the true non-observable value, as illustrated in Figure 2.1. The true value for any measurement is defined arbitrarily as the value that would be measured by a specific metrology laboratory such as the United States National Bureau of Standards. For calculated values, the true value is defined arbitrarily as that value that would be calculated using "true" measurements.

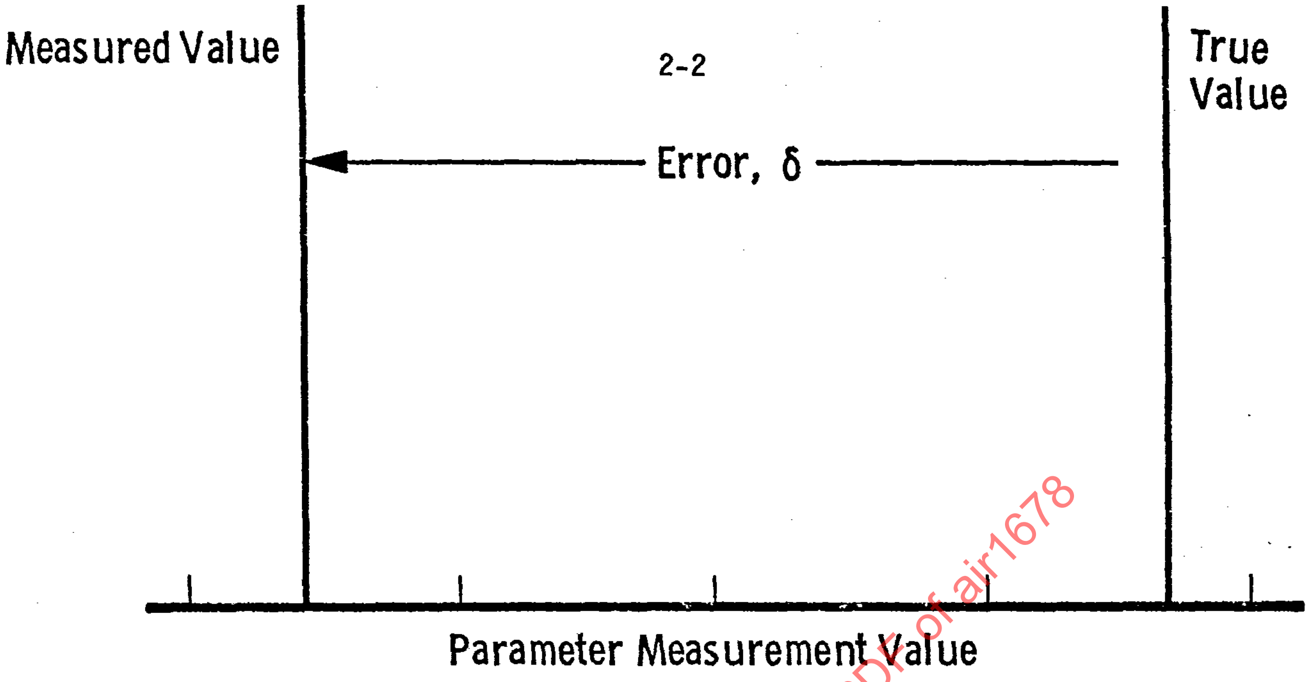
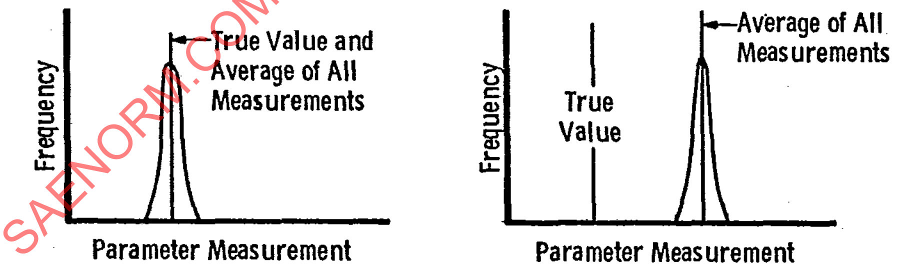


Figure 2.1 - Measurement Error, δ

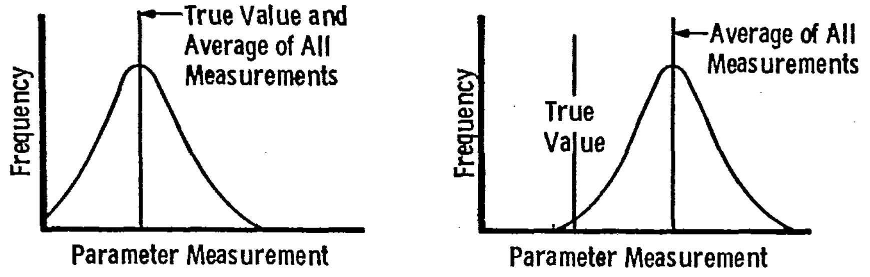
2.1.1 Error Classification: Measurement errors can be classified as one of two components: a precision (random) error or a bias (fixed) error. A precision error is one which can be estimated by statistical analysis of repeated measurements, while estimating the bias error is at least partially dependent on non-statistical methods (2.1).

An accurate measurement herein is one that has small error in both precision and bias, as is shown in Figure 2.2.



a. Unbiased, Precise, Accurate

b. Biased, Precise, Inaccurate



c. Unbiased, Imprecise, Inaccurate

d. Biased, Imprecise, Inaccurate

Figure 2.2 - Measurement Error (Bias, Precision, and Accuracy)

PRECISION ERROR

Random errors are encountered in repeated measurements and are the differences between the observed values and the average value of a very large sample. Repeated measurements at steady state (constant) conditions are not expected to produce precisely the same data. There are numerous, small effects which may cause measurement variations. These variations tend to spread about an average value in the fashion of a normal distribution curve, Figure 2.3. The curve is characterised by the standard deviation, σ.

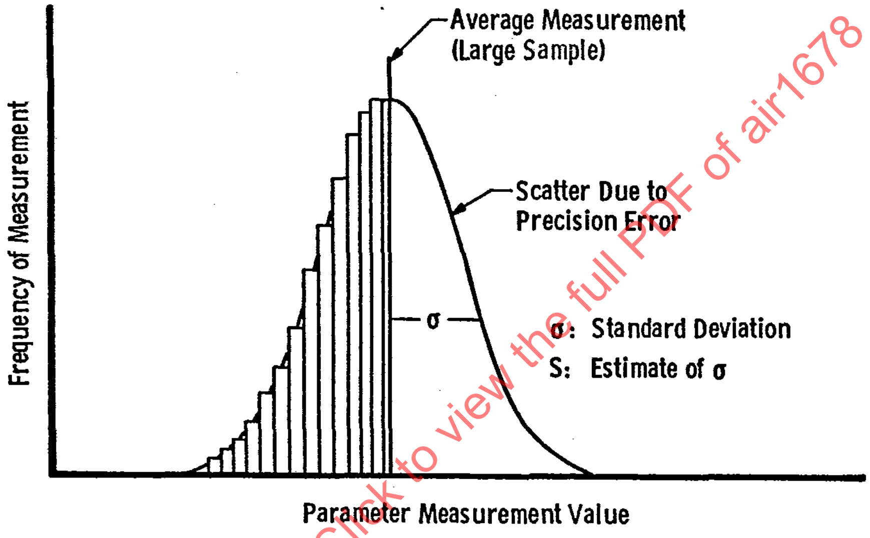


Figure 2.3 - Normal Distribution Curve Resulting from Typical Steady State Data Measurement

The Precision Index, S, is the computed estimate of the Standard Deviation, σ, of the data and is calculated as follows:

$$\text{Precision Index} \doteq S = \sqrt{\frac{\sum_{k=1}^{n} (X_k - \bar{X})^2}{n-1}} \qquad (2.1)$$

where:

n = number of measurements

X_k = individual measurements

$\bar{X}$ = average value of $X_k = \frac{\sum_{k=1}^{n} X_k}{n}$

The goodness of the precision index as an estimate of the true standard deviation is dependent on the number of measurements (samples) used in the initial estimation process. Large samples, thirty or more, are usually available for this estimation. When large samples are used in estimating the precision index, the interval defined by plus and minus two times the precision index (±2S) will contain the true value 95% of the time if the bias is negligible. Therefore, two times the precision index (2S) is used as the recommended upper limit of the precision error uncertainty.

When large samples are not available, the analysis will require special, small sample, statistical methods. These are described in Appendix B.

BIAS ERROR

The bias component is a fixed or systematic error (β) which remains unchanged for the duration of a test. In repeated measurements at steady state (constant) test conditions each measurement has the same bias magnitude. The relationship between bias, precision and the true value of the measured quantity is depicted in Figure 2.4. The magnitude of the bias cannot be determined unless the measurements can be compared with the true value, which is not feasible. Therefore, bias limits (±B) are estimated using applicable test information and engineering judgment. Note that an uncertainty analysis assumes a carefully controlled measurement process within which every known calibration correction has been made. Because the calibration corrections are not ideal, some small fixed systematic errors will remain. The bias limits are estimates of the largest possible remaining systematic error after all corrections have been made.

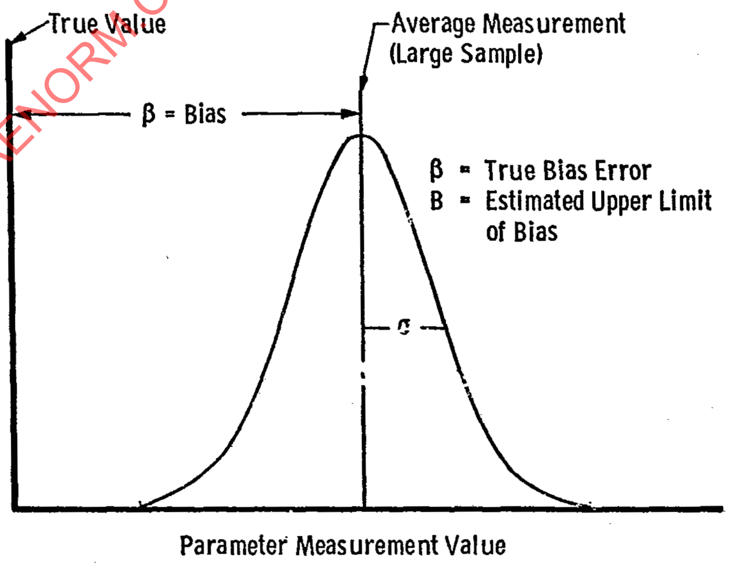


Figure 2.4 - Bias Error

2.1.2 Sources of Measurement Errors: A measurement system has many potential sources of error. The errors for each source are referred to as elemental errors. It is preferable to estimate the error for each elemental error source; however, it is permissible to estimate the error for a combination of elemental errors. For convenience, these elemental error sources for any measurement can be divided into three categories indicated by the subscript j:

j	Category
1.	Calibration Errors
2.	Data Acquisition Errors
3.	Data Reduction Errors

Elemental error sources associated with each of the above three categories are discussed in the following paragraphs.

CALIBRATION ERRORS (Category j=1)

The measurement uncertainty analysis assumes a well controlled measurement process in which there are no gross mistakes or errors. It also assumes that reasonable calibration corrections have been applied. Calibrations are performed to improve the test accuracy and to provide test measurement traceability to the national standards laboratory. By applying the calibration corrections, some biases are reduced but in the process some other errors may be introduced. The bias error of the uncalibrated test instrument is exchanged for the bias of the standard or master meter. In addition, the precision error of the calibration process will contribute to the calibration error. In some cases, the precision of the calibration process will contribute to the precision of the test result and in others, it will be converted into fixed bias error. For a fuller discussion, see Appendix C.

Traceability is established and maintained through a calibration hierarchy. Each comparison in the hierarchy constitutes an elemental error source, i. Figure 2.5 is an example instrument calibration hierarchy.

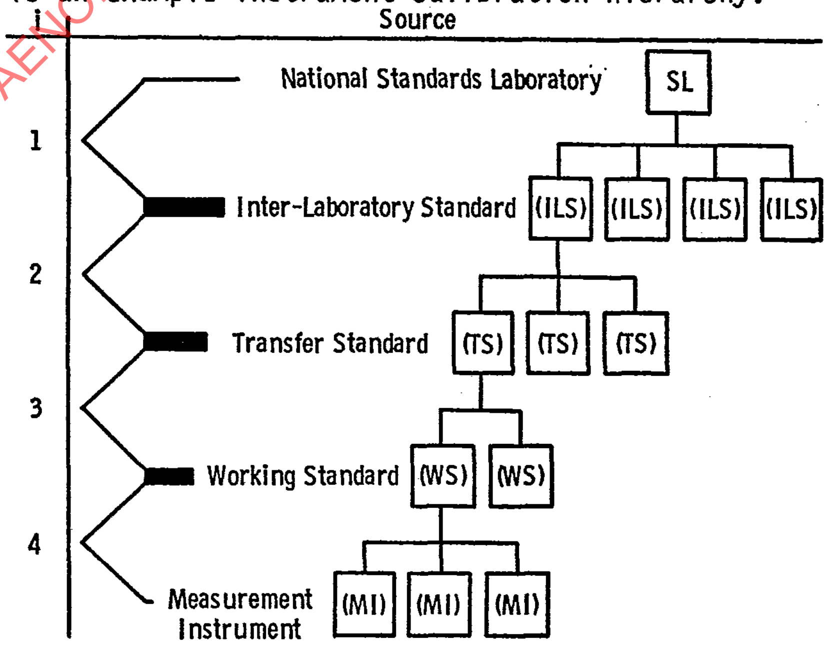


Figure 2.5 - Measurement Calibration Hierarchy

Each comparison in the calibration hierarchy has elemental errors associated with it. Estimates of these elemental errors provide precision indices and bias limits in each level of the hierarchy, as listed in Table 2.1.

Calibration (j=1)

i	Error Source	Bias Limit	Precision Index
1	SL-ILS	B_{11}	S_{11}
2	ILS-TS	B_{21}	S_{21}
3	TS-WS	B_{31}	S_{31}
4	WS-MI	B_{41}	S_{41}

TABLE 2.1 Calibration Hierarchy Error Sources

DATA ACQUISITION ERRORS (Category j=2)

Table 2.2 illustrates some of the error sources, i, associated with the data acquisition system illustrated in Figure 2.6. Data for this example are acquired by measuring the electrical output resulting from pressure applied to a strain-gage-type pressure measurement instrument. Other error sources, such as electrical simulation, probe errors, and environmental effects, are also present.

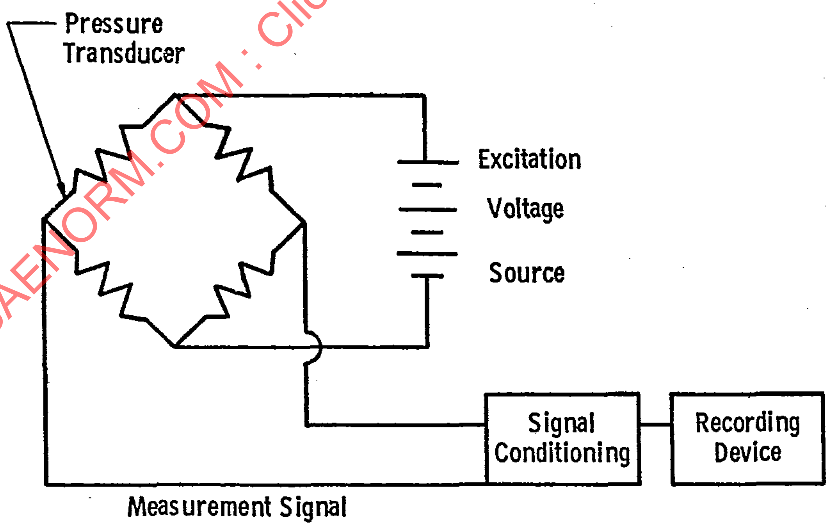


Figure 2.6 - Example Data Acquisition System

Data Acquisition (j=2)

i	Error Source	Bias Limit	Precision Index
1	Excitation Voltage	B_{12}	S_{12}
2	Electrical Simulation	B_{22}	S_{22}
3	Signal Conditioning	B_{32}	S_{32}
4	Recording Device	B_{42}	S_{42}
5	Pressure Transducer	B_{52}	S_{52}
6	Probe Errors	B_{62}	S_{62}
7	Environmental Effects	B_{72}	S_{72}

TABLE 2.2 Typical Data Acquisition Error Sources

DATA REDUCTION ERRORS (Category j=3)

Computers operate on raw data to produce output in engineering units. Typical errors in this process stem from curve fits and computer resolution. Typical data reduction error sources are listed in Table 2.3.

Data Reduction (j=3)

i	Error Source	Bias Limit	Precision Index
1	Curve Fit	B_{13}	S_{13}
2	Computer Resolution	B_{23}	S_{23}

TABLE 2.3 Typical Data Reduction Error Sources

2.1.3 Estimating Measurement Errors: The following paragraphs outline approaches and considerations for estimating and classifying the precision and bias error components.

PRECISION ERROR ESTIMATION

The precision index, S, is calculated from test data.

For repeated or simultaneous observations the precision index can often be reduced by treating the averages of the multiple observations as a single measurement. The distribution of the averages will have a smaller precision index, as indicated in Figure 2.7 and defined by Equation 2.2.

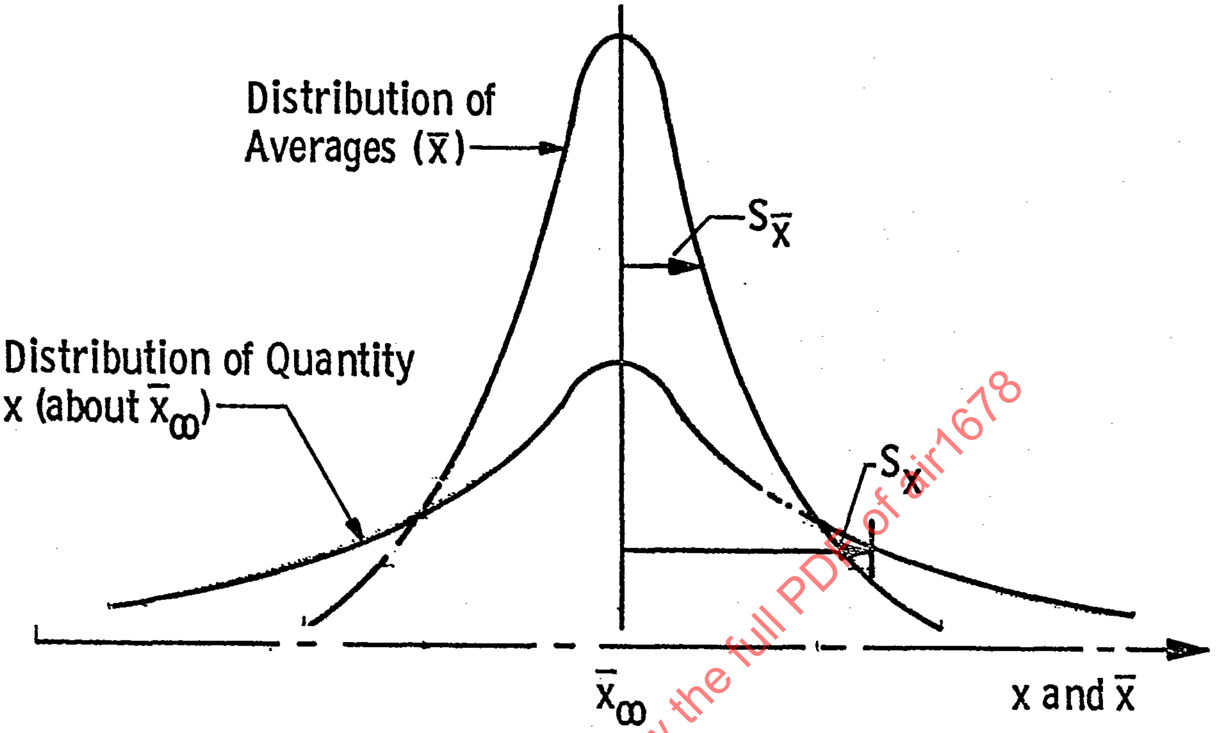


Figure 2.7 - Distribution of x and $\overline{x}$

$$S_{\overline{x}} \cong \frac{S_x}{\sqrt{n}} \qquad (2.2)$$

BIAS ERROR UPPER LIMIT ESTIMATION

Information from special test and engineering judgment is the principal source for estimating the bias limit, B. The bias limit must account for small known biases, which are too difficult to correct out, as well as for small unknown biases. These bias errors will include fossilized precision error from the calibration process, as discussed in Appendix C. In a well controlled measurement process, the assumption is that there are no large unknown biases. To ensure that a controlled measurement process exists, all measurements should be monitored with statistical quality control charts (2.2).

The unknown bias error is commonly assumed to be equally distributed (B^- and B^+) about the measurement. Sometimes the physics of the problem provides knowledge of the sign but not the magnitude of the bias, resulting in nonsymmetric bias limits. The intent of the bias limit estimate is to identify the largest value of the actual bias error of a given measurement process.

2.1.4 Combining Measurement Errors: After the elemental error sources are identified they must be combined into the basic measurement error components, precision and bias. Combining the elemental errors into separate components is essential for modelling the basic measurement uncertainty, for propagating measurement uncertainties to performance parameters, and for uncertainty reporting and validation.

Typically, only calibration, data acquisition, and data reduction error sources are involved. These are combined to define the basic measurement error.

COMBINING PRECISION INDICES

The precision index (S) is the root-sum-square of the elemental precision indices from all sources.

$$S = \sqrt{\sum_j \sum_i S_{ij}^2} \qquad (2.3)$$

where j defines error categories: (1) calibration, (2) data acquisition, and (3) data reduction and i defines elemental error sources within a category.

For example, the precision index for the calibration category is the root-sum-square of the elemental precision indices (reference Table 2.1).

$$S_1 = S_{cal} = \sqrt{S_{11}^2 + S_{21}^2 + S_{31}^2 + S_{41}^2} \qquad (2.4)$$

In like manner, the precision index for the data acquisition category, S_2, and the precision index for the data reduction category, S_3, are defined by the root-sum-square of their individual elemental precision indices.

The basic measurement precision index is then the root-sum-square of the precision indices of the three categories.

$$\begin{aligned} S &= \sqrt{S_1{}^2 + S_2{}^2 + S_3{}^2} \qquad (2.5) \\ &= \sqrt{\sum_i S_{i1}^2 + \sum_i S_{i2}^2 + \sum_i S_{i3}^2} \\ &= \sqrt{\sum_j \sum_i S_{ij}^2} \end{aligned}$$

COMBINING BIAS LIMITS

Most measurement processes will contain a large number of bias error sources. The bias limit used herein is the root-sum-square of the elemental bias error limits of all categories (2.3).

$$B = \sqrt{\sum_j \sum_i B_{ij}^2} \qquad (2.6)$$

The individual category error limits; calibration (B_1), data acquisition (B_2), and data reduction (B_3), are defined by the root-sum-square of their individual elemental bias limits.

The bias limit for the basic measurement process is then

$$B = \sqrt{B_1^2 + B_2^2 + B_3^2} \qquad (2.7)$$

$$= \sqrt{\sum_i B_{i1}^2 + \sum_i B_{i2}^2 + \sum_i B_{i3}^2}$$

$$= \sqrt{\sum_j \sum_i B_{ij}^2}$$

If any of the elemental bias limits are nonsymmetrical separate root-sum-squares are used to obtain B^+ and B^-, (2.2).

2.1.5 Propagation of Basic Measurement Errors to Thrust Parameters: Basic measurements such as temperature and pressure are made and the thrust is calculated as a function of the basic measurements. Basic components of uncertainty (S and B) in the measurements are propagated to thrust through the math model. The effect of the propagation can be approximated using the Taylor's series method (2.2).

For propagating errors, the concept of the "influence coefficient" is convenient. This is the error propagated to the performance parameter due to a unit error in the basic measurement. The "influence coefficient" of each basic measurement is obtained in one of two ways:

a. Analytically - When there is a known mathematical relationship between thrust, F, and the measured variables, X_1, X_2 ...., X_k, the dimensional influence coefficient, θ_k, of the quantity X_k, is obtained by partial differentiation.

Thus, if $F = f(X_1, X_2, ..., X_k)$, then

$$\theta_k = \frac{\partial F}{\partial X_k} \qquad (2.8)$$

b. Numerically - Where no mathematical relationship is available or when differentiation is difficult, finite increments may be used to evaluate θ_k. Here θ_k is given by

$$\theta_k = \frac{\Delta F}{\Delta X_k} \tag{2.9}$$

For independent measurements, the basic measurement error components, S_{Xk} and B_{Xk} are then propagated to the thrust error components, S_F and B_F, using the influence coefficients, θ_k, as follows:

$$S_F = \sqrt{\sum_k \quad (\theta_k S_{Xk})^2} \tag{2.10}$$

$$B_F = \sqrt{\sum_k \quad (\theta_k B_{Xk})^2} \tag{2.11}$$

Note: Care should be taken to check that the errors are independent. For complex thrust calculations, the same measurement may be used more than once in the formula. For example, in several thrust models, airflow appears both in gross thrust and ram drag. This may increase or decrease the error, depending on whether the sign of the measurement is the same or opposite. If the Taylor's series relates the most elementary measurements to the ultimate result, i.e. net thrust, these "linked" relationships will be properly accounted for.

2.2 Measurement Uncertainty: A single number (some combination of bias and precision) is needed to express a reasonable limit for error. This single number, called uncertainty U, must have a simple interpretation and be useful. It is impossible to define a single rigorous statistic because the bias is an upper limit involving judgment which has unknown characteristics. Any function of these two numbers must be a hybrid combination of an unknown quantity (bias) and a statistic (precision). However, the need for a single number to measure error is so great that the use of an arbitrary standard is warranted.

Two options are in use: U_{ADD} and U_{RSS}. The former is the additive model in which two times the precision index is added to the bias limit, thus:

$$U_{ADD} = (B + 2S) \tag{2.12}$$

The latter is the root-sum-squares combination:

$$U_{RSS} = \sqrt{B^2 + (2S)^2} \tag{2.13}$$

Either the U_{ADD} or the U_{RSS} option is viable; however, for ease of presentation, U_{ADD} will be used for illustration in the remainder of this document. Appendix D provides a comparison of these two definitions using Monte Carlo results.

Figure 2.8 illustrates the U_{ADD} limits which provide an estimate of the largest error that may reasonably be expected for that measurement process. The probability that the "true" values lie within the uncertainty limits is known as the Coverage.

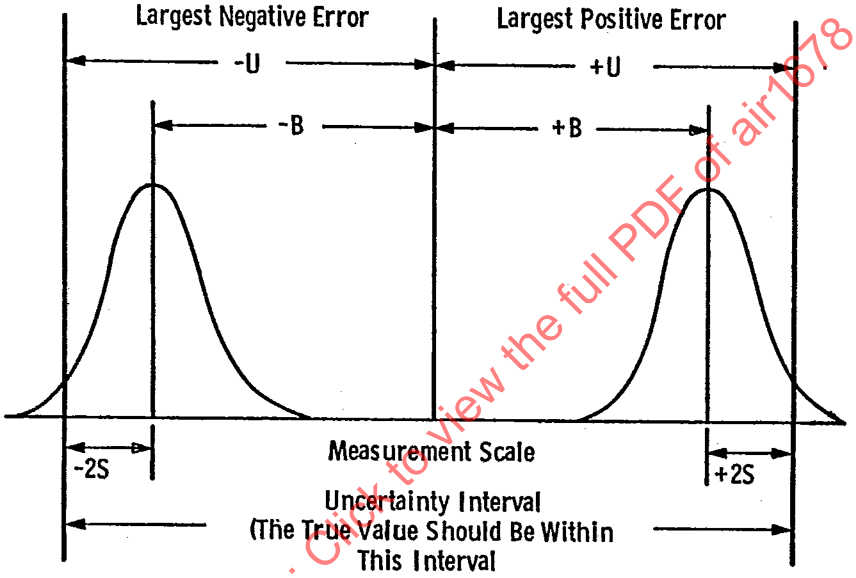


Figure 2.8 - Measurement Uncertainty Interval (Additive Model)
Symmetrical Bias, Large Sample

If the bias is nonsymmetric, ie. $B^+ \neq B^-$, then "each side" of the interval should be treated separately. Using equations similar to (2.12):

$U^-_{ADD} = B^- - 2S$ and $U^+_{ADD} = B^+ + 2S$.

Nonsymmetric uncertainty intervals are discussed in reference (2.2).

2.2.1 Uncertainty Interval Coverage: A rigorous calculation of confidence level or coverage of the true value by the uncertainty interval is not possible. However, Monte Carlo simulations using various relative sizes of the bias and precision components indicates that the coverage of U_{ADD} is about 99% while that of U_{RSS} is about 95%. Additional details are covered in Appendix D.

2.2.2 Propagation of Uncertainty Components: The error components, bias (B) and precision (S), are propagated separately. The basic measurement error components are obtained by combining the respective elemental errors as discussed previously. Then the basic measurement error components are propagated to the performance parameter error components.

2.2.3 Pretest And Posttest Analyses: The accuracy of the test is often stipulated as part of the test requirements. The ability to meet the accuracy requirements is estimated by a pretest uncertainty analysis. Several different thrust models may be analyzed prior to flight to select the most accurate ones. If the experiment accuracy requirement is of the same size or smaller than the projected measurement uncertainty, corrective action should be taken to reduce the measurement uncertainty or else not perform the experiment.

The pretest analysis is based on data and information that exist before the test: calibration histories, previous tests with similar instrumentation, prior measurement uncertainty analyses and expert opinions. In complex tests there are often alternatives to evaluate: different thrust models, various instrumentation layouts, and alternate calculation procedures. Pretest analysis will identify the most preferred test methods.

Posttest analysis is required to confirm the pretest estimates and to identify potential problems. Comparison of test results with the pretest analysis is an excellent data validity check. The precision index calculated by using repeated points or redundant instruments should not be significantly larger than the pretest estimates. When redundant instrumentation or calculation methods are available, the individual averages should be within the pretest uncertainty interval. If there are several ways of obtaining a parameter, the uncertainty intervals should overlap. The final uncertainty limits reported for the test results should be based on the posttest analysis. There may be a pretest and posttest analysis for both the ground test and flight test phases.

2.3 Reporting Measurement Error: The uncertainty components, precision index, bias limit, uncertainty limit and influence coefficients are included in the reports on measurement error along with applicable test conditions. Two separate reports are recommended: The Measurement Uncertainty Summary Report and the Elemental Error Sources Report.

MEASUREMENT UNCERTAINTY SUMMARY REPORT

The suggested reporting format is shown in Table 2.4. The error components, S and B, are necessary to: (1) indicate corrective action if the uncertainty is unacceptably large before the test, (2) to propagate the uncertainty to more complex parameters, and (3) to substantiate the uncertainty limit.

The influence coefficients for each measurement are provided to document the calculation of the thrust error components (using equations 2-10 and 2-11). The test condition (or range of conditions) must be identified to qualify both the error components and influence coefficients. The flight condition (actual or simulated) and engine power level must be identified in a manner that completely defines the engine operating point.

ELEMENTAL ERROR SOURCES REPORT

The elemental error sources report records all the elemental errors of each basic measurement. This report would normally be maintained by the test center and be available for review.

The elemental contributions are required to confirm measurement uncertainty estimates and to support any corrective action needed to reduce the uncertainty or to identify data validity problems. This list of elemental error sources should be reviewed to ensure against potential missing sources.

Table 2.5 provides a suggested reporting format and illustrates the data required to be recorded.

Test Condition: e.g., Altitude, Mach-No., Ram Recovery, and Power Setting or Engine Inlet Total Pressure and Temperature, Thrust Level and Ambient Press

Basic Measurement	Range	Precision Index S_k	Bias Limit B_k	Influence Coefficient θ_k	Uncertainty Limit U	Supporting Elemental Error Sources Report Number
Basic Measurement 1	$M_1 - M_u$	S_1	B_1	θ_1	$U_1 = B_1+2S_1$	XXX1
Basic Measurement 2	$M_1 - M_u$	S_2	B_2	θ_2	$U_2 = B_2+2S_2$	XXX2
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
Thrust	$M_1 - M_u$	$S = \sqrt{\sum_k (\theta_k S_k)^2}$	$B = \sqrt{\sum_k (\theta_k B_k)^2}$		$U = B+2S$	
Additional Performance Parameters of Interest						

TABLE 2.4 Recommended Uncertainty Summary Report Format

Basic Measurement: Type and Station e.g., PT2, N1	Type of Sensor: Basic description e.g., Thermocouple	Measurement Range: Expected range for test condition
Test Condition(s): e.g., Altitude, Mach number, recovery, power lever angle or Engine Inlet Total Pressure and Temperature, Thrust Level and Ambient Pressure		

Sub-Script	Elemental Error Source	Precision Index S_{ij}	Bias Limit B_{ij}	Substantiation Source and Notes
	Calibration			
11 21 31 41		S_{11} S_{21} . .	B_{11} B_{21} . .	Identify substantiating sources for error estimates. e.g., latest calibration on file, source specification, etc.
	Data Acquisition			
12 22 32 42 52 62 72 . . .		S_{12} S_{22} . . .	B_{12} B_{22} . . .	
	Data Reduction			
13 23 . . .		S_{13} S_{23} . .	B_{13} B_{23} . .	
	Resultant Measurement Error Components	$S = \sqrt{\sum\sum S_{ij}^{2}}$	$B = \sqrt{\sum\sum B_{ij}^{2}}$	Elemental Error Sources Report Number XXXX

TABLE 2.5 Recommended Elemental Error Sources Report Format

Note: A separate sheet is needed for each basic measurement.

3. IN-FLIGHT THRUST MEASUREMENT PROCESSES:

The in-flight thrust measurement process comprises:

- Selection of the in-flight thrust determination methods and definition of appropriate math models (see SAE AIR 1703).
- Calibration of the math models by facility testing.
- Calculation of thrust using in-flight measurements and the calibrated math models.

During the ground test phase, errors are introduced in measuring test facility thrust, in calibrating the math model and from the instrumentation used to determine test facility flight conditions. In flight test, errors result from the instrumentation required to determine the actual flight condition. An accurate assessment of thrust uncertainty is dependent on both the precise definition of all error elements as well as a thorough understanding of how the ground test errors are classified, combined and propagated to the flight measured result.

The initial decision on classifying elemental errors may be changed after the in-flight measurement process has been defined and the instrumentation system has been considered. The error is classified as precision if it increases the scatter in the test data; otherwise, it is a bias error. The decision logic for error classification, based on the actual measurement process, is discussed in Appendix C. Reclassification of initial precision error(s) to a bias error(s) is marked with an asterisk (*) to indicate that the precision error has become a fossilized or fixed error.

Two sets of measurement are identified; the facility measurement and the engine measurement. The former consists of those measurements required to produce the facility-measured thrust, while the latter consists of those measurements, including simulated flight condition, that are expected to be made in flight to calculate thrust. Both sets of measurements contribute errors from calibration, data acquisition, and data reduction processes. These errors are propagated through their respective thrust calculation procedures, as outlined in Section 2, to define the calculated thrust error components. The facility-measured thrust error components are referred to as S_{gt} and B_{gt}. The instrumentation errors for the in-flight thrust calculation have components S_{gi} and B_{gi}.

The following paragraphs discuss ground test and flight test phases of the thrust measurement process. The associated error components are defined for each phase and the combination and propagation of errors are discussed.

3.1 Ground Test Measurement Process: The purpose of the ground test measurement process is to develop calibrated math models and define their associated precision and bias error. The process is illustrated in Figure 3.1. For each methodology selected, two math models are required. One model is used to calculate thrust for test day conditions. This math model is commonly referred to as the "in-flight thrust deck." In this report its error components are defined as S_{mm} and B_{mm}. A second model is used to correct test day thrust to some reference condition. This is normally referred to as a Lapse Rate Model and it has error components S_{cm} and B_{cm}.

The difference between the thrust value calculated using the math model and the facility-measured value is termed the "Observed Residual Error." β_r is the average observed residual error. S_r is the precision index of the observed residual error. Conceptually, the observed residual error components S_r and β_r are used to adjust, i.e. calibrate, the math model to minimize the differences throughout the engine operating envelope. Both bias and precision may be a function of the value of the independent variables used in the math model.

The observed residual error is combined with the facility measurement and thrust calculation error components to estimate the in-flight thrust decks error components. The bias limit is defined by:

$$B_{mm} = \sqrt{B_{gt}^2 + B_{gi}^2 + \left(\frac{2S_r^*}{\sqrt{n}}\right)^2} \qquad (3.1)$$

The precision index is bounded between zero (0) and the observed residual error precision index (S_r).

The lapse rate correction error components are estimated from flight test instrumentation only.

3.2 Flight Test Measurement Process: Flight tests provide calculated thrust using the math model or in-flight thrust deck. The general thrust measurement process is illustrated in Figure 3.2. The flight condition and engine basic measurement errors are combined to define the flight test instrumentation error components, S_{fi} and B_{fi}. These are then combined with the thrust deck model error components, S_{mm} and B_{mm}, to determine the as-tested in-flight thrust error components, S_t and B_t. The as-tested flight data may be corrected to reference conditions by means of the lapse rate model. The error components for the instrumentation used to determine the flight test conditions are similarly combined to define the lapse rate model error components, S_{cm} and B_{cm}. Multiple data points may be averaged to improve the thrust estimate.

The in-flight calculated thrust, corrected to the reference flight condition has error components S_c and B_c. These are related to instrumentation, thrust and lapse rate model error components by the relations:

$$S_c = \sqrt{S_{fi}^2 + S_{mm}^2 + S_{cm}^2} \qquad (3.2)$$

$$B_c = \sqrt{B_{fi}^2 + B_{mm}^2 + B_{cm}^2} \qquad (3.3)$$

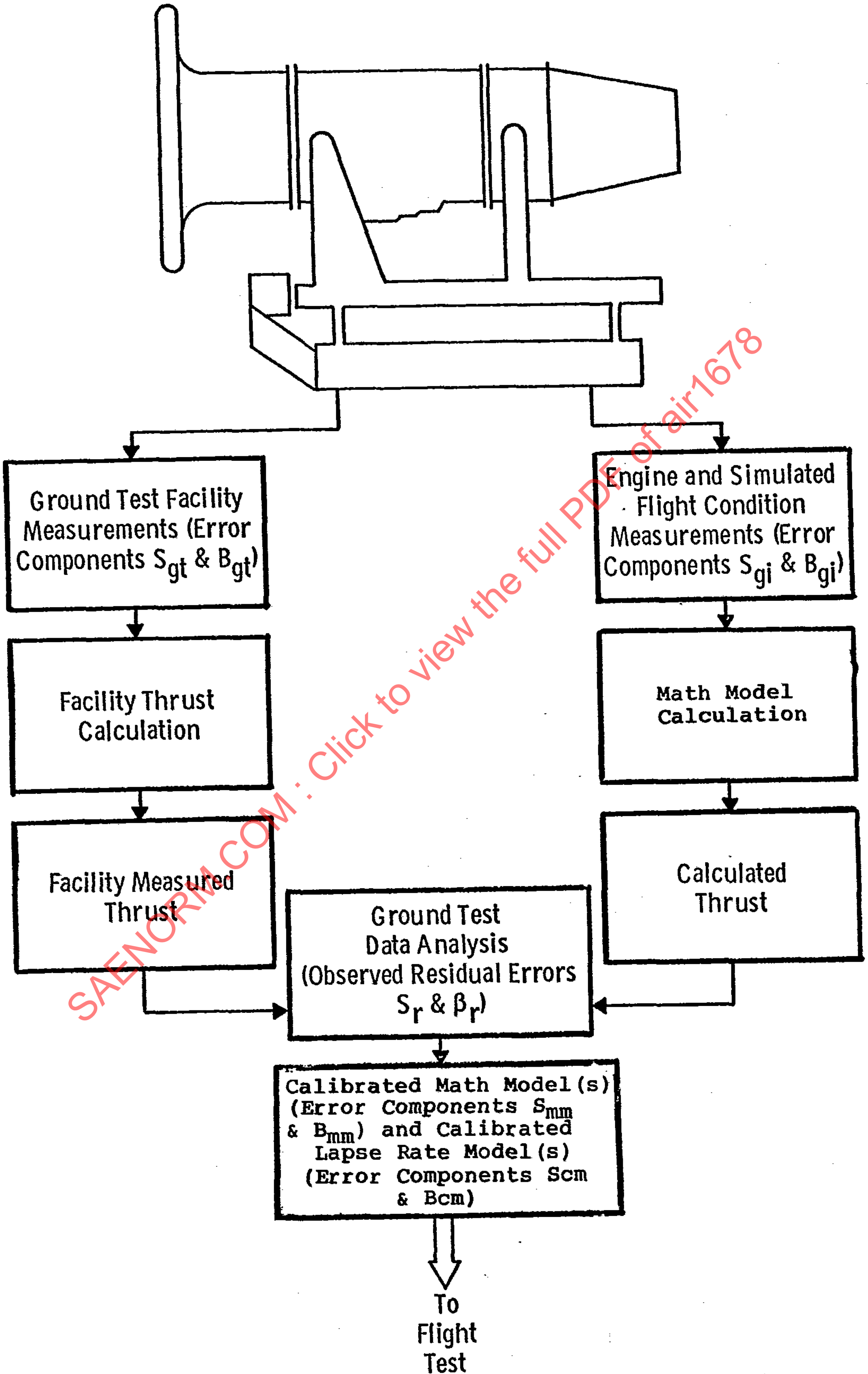


Figure 3.1 - Ground Test Thrust Measurement Process Including Definition/Calibration of Math Model(s).

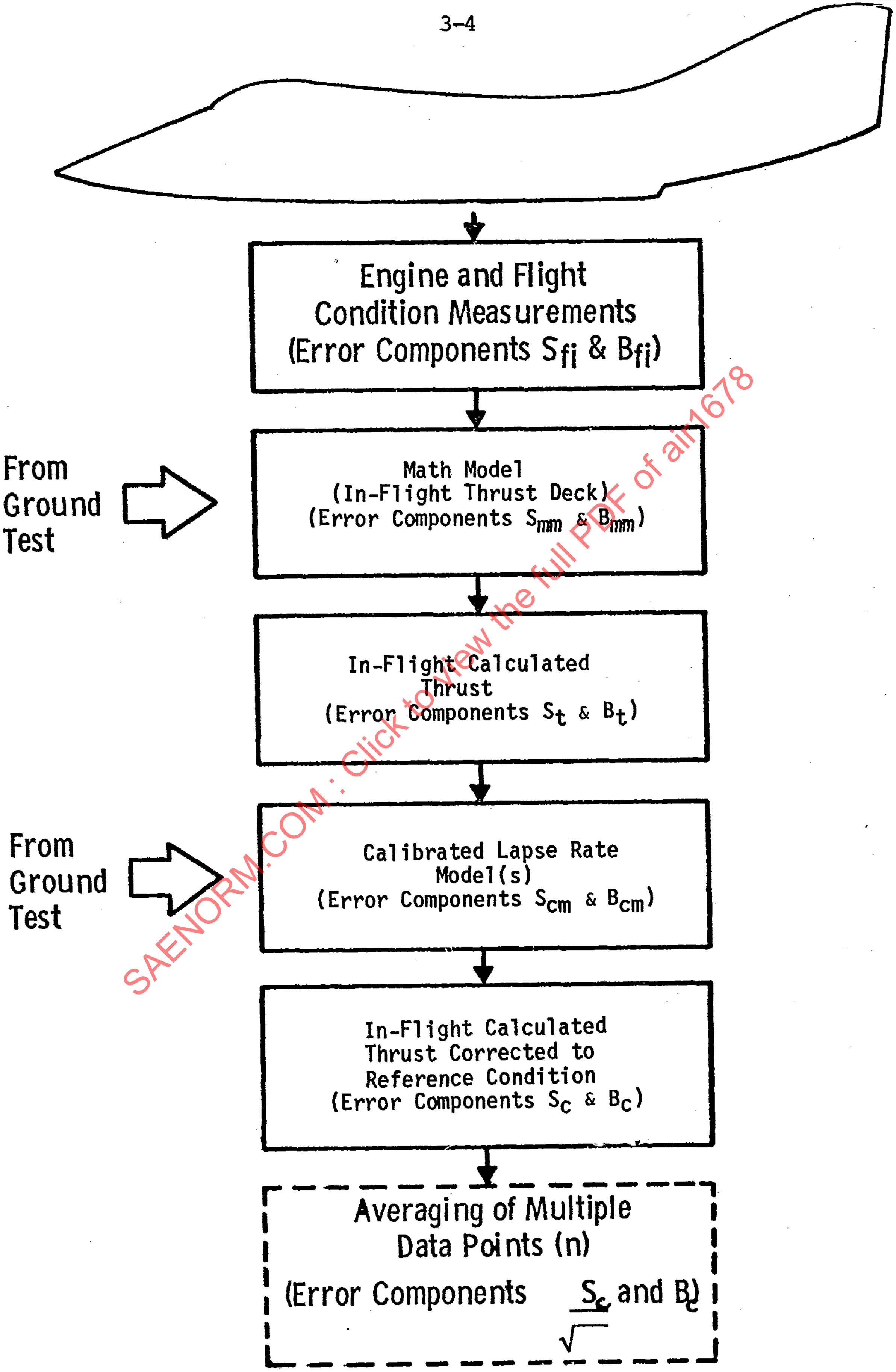


Figure 3.2 - General Flight Test Thrust Measurement Process.

4. APPLICATION OF UNCERTAINTY METHODOLOGY TO THE IN-FLIGHT THRUST PROGRAM:

Uncertainty analyses should be carried out throughout the test program. The program can be organized into four phases: Program Definition and Planning, Ground Test, Flight Test, and Results Analyses. Table 4.1 shows the activities in each of these phases.

The overall process of uncertainty estimation is iterative. The closed loop consists of estimating the error components (pretest) then re-evaluating them in light of actual test data (posttest).

Careful application of the described methodology should lead to both an improvement in the actual estimate of the in-flight thrust through better control of the elemental errors and an improvement in the interpretation and understanding of the final flight test results.

4.1 Program Definition and Planning: The first task is to establish the measurement uncertainty requirement based on the objectives of the program. The second is to consider a number of in-flight thrust measurement options and conduct pretest uncertainty estimates. The number of options are reduced with the aid of these estimates. Several options should be used to provide redundancy.

4.1.1 In-Flight Thrust Measurement Options: The identification of potential options to provide the desired accuracy is required. Net thrust has three separate components, gross thrust, airflow, and aircraft velocity. Each of these components may have a number of evaluation options available. Cost/accuracy trades are considered.

Table 4.2 illustrates a number of options, taken from AEDC-TR-81-2, "ALCM Preflight-Thrust Uncertainty Analysis" (4.1). The matrix would be augmented by considering additional options for airflow and/or aircraft velocity.

The measurements required are identified for each thrust option. These include the facility and flight test thrust measurements. Data from previous testing is used unless some new instrumentation is utilized.

The error components for each measurement are estimated. Table 4.3 shows an example format that can be used to tabulate the measurement error components. Ground and flight test measurements are identified separately.

4.1.2 Initial Survey of Thrust Measurement Options: The survey results will direct attention to the measurements which are most critical to the success of the test program. The approach to the survey is to determine the influence that each measurement has on the output net thrust, F_N.

Program Phases	Related Activities	Relevant Section
Program Definition and Planning	o Establish Program In-Flight Thrust Uncertainty Requirements	4.1
	o Identify Potential Thrust Measurement Math Model Options	4.1.1
	o Conduct Math Model Options Survey and Select Options for Use in the Program	4.1.2 4.1.3
Ground Test	Pretest	4.2.1
	o Estimate Engine and Facility Measurement Errors (S_{gi}, B_{gi}) (S_{gt}, B_{gt})	
	o Estimate Selected Math Model(s) Errors (S_{mm}, B_{mm})	
	Posttest	4.2.2
	o Confirm Pretest Estimates, Adjust as Necessary	
	o Calibrate Math Model(s) and Derive Errors (S_{mm}, B_{mm})	
Flight Test	Pretest	4.3.1
	o Estimate Flight Measurement Errors (S_{fi}, B_{fi})	
	o Estimate Selected Math Model(s) Errors (obtained from ground test estimates) (S_{mm}, B_{mm})	
	o Estimate the In-Flight Thrust Errors (S_c, B_c)	
	Posttest	4.3.2
	o Confirm Pretest Estimates, Adjust as Necessary	
	o Calculate In-Flight Thrust Measurement Uncertainty	
Results Analyses	o Examine Consistency of Ground and Flight Test Data	4.4
	o Compare Thrust Options Results with Program Predictions	4.4.1
	o Resolve Test Data and Performance Discrepancy Problems	4.4.1
	o Report Test Results and Uncertainties	4.4.3

TABLE 4.1 In-Flight Thrust Measurement Uncertainty Activities

OPTION	1	2	3	4
Airflow - WA	Fan Map Characteristics →	→	→	→
Velocity - VO	Airstream Measurements →	→	→	→
Gross Thrust FG	Gross Thrust Parameter $FG = \left[(FGP)\ \frac{P2}{PS1} - 1\right] X$ $(A8)\ (PS1)^*$	Mass Weighted Velocity Coeff Dual Stream $FG = (CV_1)(M_1,\ ideal^*)$	Mass Weighted Velocity Coeff Mixed Stream $FG = (CV_2)(M_2,\ ideal^*)$	Area Weighted Velocity Coeff Mixed Stream $FG = (CV_3)(M_3,\ ideal^*)$
Basic Measurements Required	Low Rotor Speed High Rotor Speed Inlet Static Press Inlet Delta Press Inlet Total Temp	Low Rotor Speed → Inlet Static Press → Inlet Delta Press → Inlet Total Temp → Turbine Exit Press → Duct Disch Press → Turbine Exit Temp → Duct Disch Temp → Fuel Flow → Burner Press →	→ → → → → → → → → →	→ → → → → → → → →

Note: The Terminology in this Table may be inconsistent with AIR 1703, but is used to be consistent with AEDC-TR-81-2, the example source.

$^*M_i$, ideal is the ideal nozzle exit momentum.

TABLE 4.2 Example Net Thrust Options

Test Condition: Altitude/Mach Number/Engine Power Setting

Basic Measurements	Facility Measurements		Thrust Calculation Measurements				Influence Coefficient
			Ground		Flight		
	Sgt	Bgt	Sgi	Bgi	Sfi	Bfi	θ
Gross Thrust							
Facility Measurement Load Cell	$S_{gt,\ 1}$	$B_{gt,\ 1}$	-	-	-	-	$\theta_{gt,\ 1}$
.	$S_{gt,\ 2}$	$B_{gt,\ 2}$	-	-	-	-	$\theta_{gt,\ 2}$
	.	.	-	-	-	-	.
.	$S_{gt,\ n}$	$B_{gt,\ n}$	-	-	-	-	$\theta_{gt,\ n}$
Thrust Calculation Exhaust Area (A)	-	-	$S_{gi,\ 1}$	$B_{gi,\ 1}$	$S_{fi,\ 1}$	$B_{fi,\ 1}$	$\theta_{gi,\ 1}$ and $\theta_{fi,\ 1}$
Exhaust Press (P)	-	-	$S_{gi,\ 2}$	$B_{gi,\ 2}$	$S_{fi,\ 2}$	$B_{fi,\ 2}$	$\theta_{gi,\ 2}$ and $\theta_{fi,\ 2}$
.	-	-	.	.	.	.	.
.	-	-	$S_{gi,\ n}$	$B_{gi,\ n}$	$S_{fi,\ n}$	$B_{fi,\ n}$	$\theta_{gi,\ n}$ and $\theta_{fi,\ n}$
Airflow Facility Measurement Venturi Δp	$S_{gt,\ n+1}$	$B_{gt,\ n+1}$	-	-	-	-	$\theta_{gt,\ n+\ 1}$
.	$S_{gt,\ n+2}$	$B_{gt,\ n+2}$	-	-	-	-	$\theta_{gt,\ n+\ 2}$
.	.	.	-	-	-	-	.
Thrust Calculation Fan Speed (N)	-	-	$S_{gi,\ n+1}$	$B_{gi,\ n+1}$	$S_{fi,\ n+1}$	$B_{fi,\ n+1}$	$\theta_{gi,\ n+1}$ and $\theta_{fi,\ n+1}$
.	-	-	$S_{gi,\ n+2}$	$B_{gi,\ n+2}$	$S_{fi,\ n+2}$	$B_{fi,\ n+2}$	$\theta_{gi,\ n+2}$ and $\theta_{fi,\ n+2}$
	-	-	.	.	.	.	.
.	-	-	$S_{gi,\ m}$	$B_{gi,\ m}$	$S_{fi,\ m}$	$B_{fi,\ m}$	.
Velocity							
.							
.							

TABLE 4.3 Example Preliminary Performance Survey For Net Thrust Option 1

The table of influence coefficients, θ, are established (Table 4.3). The influence coefficient is the ratio of the change in F_N for each measurement change, expressed in units of the measurement. The influence coefficients are obtained readily by programming the thrust measurement equations on a computer and observing the F_N output variations with successively small input perturbations, as illustrated in Appendix E. The values of θ will vary with flight condition, engine power setting and thrust option and should be established for every option under consideration. Each error component is propagated to define the precision index and bias limit for the ground instrumentation (S_{gi} and B_{gi}), the flight instrumentation (S_{fi} and B_{fi}), and the facility measured performance parameter (S_{gt} and B_{gt}). Thus:

$$S_{gt} = \sqrt{\sum_k (\theta_{gt,k} S_{gt,k})^2} \qquad (4.1)$$

$$B_{gt} = \sqrt{\sum_k (\theta_{gt,k} B_{gt,k})^2} \qquad (4.2)$$

$$S_{gi} = \sqrt{\sum_k (\theta_{gi,k} S_{gi,k})^2} \qquad (4.3)$$

$$B_{gi} = \sqrt{\sum_k (\theta_{gi,k} B_{gi,k})^2} \qquad (4.4)$$

$$S_{fi} = \sqrt{\sum_k (\theta_{fi,k} S_{fi,k})^2} \qquad (4.5)$$

$$B_{fi} = \sqrt{\sum_k (\theta_{fi,k} B_{fi,k})^2} \qquad (4.6)$$

where there are k measurements contributing to each bias and precision summation.

An estimate of the in-flight thrust calculation precision index, S_c, and bias limit, B_c, are determined using Equations 3.2 and 3.3. The procedure is repeated for each test condition and thrust option. The final results for each can be combined to define the single value, U, representing uncertainty. This procedure will rank the candidate options and may identify different options as being best for different flight conditions. It will also identify critical areas to be closely monitored and areas for improvement. An example is provided in Appendix F.

When the survey is completed the options for use in the test program are selected. The selection will involve partiction by aircraft, engine, flight and ground test, instrumentation, and performance analysis experts.

4.2 Ground Test: The uncertainty evaluation activities include: (1) pretest estimating of engine and facility measurement errors and math model errors, and (2) posttest confirmation of pretest error estimates and definition of the calibrated math model including model error determination. The end product is the calibrated math model which will satisfy the in-flight thrust measurement accuracy requirement.

4.2.1 Ground PreTest Error Analysis: The objective of the pretest error analysis is to ensure that the candidate measurement systems will satisfy the program accuracy requirement. A logic flow chart is shown in Figure 4.1.

The total measurement system, including calibration, data acquisition, and data reduction processes, is identified using the selected thrust options. A detailed pretest error analysis, including elemental error estimates for each error category, is made following the methodology of Section 2.0. The elemental error sources and estimates may be tabulated using the format shown in Table 2.5. An example is shown for a typical pressure measurement in Table 4.4.

The results of the pretest error analysis of the measurement parameters should be reported in the Measurement Uncertainty Summary Report format shown in Table 2.4. The measurement errors should be propagated to the desired performance parameters. These results would also be reported in the summary report. Performance parameter uncertainties may vary widely with test conditions. To report representative uncertainties it may be necessary to provide a separate summary report for each expected test condition, or to provide a number of summary reports each of which cover a range of test conditions. Table 4.5 illustrates a typical summary report format. The measurement error component data for this example are taken from AEDC-TR-81-2, "ALCM Preflight-Test Thrust Uncertainty Analysis." (4.1) The influence coefficients and net thrust error components were calculated using one thrust model from the same report. A summary of this is contained in Appendix E. The net thrust error components illustrated represent the ground instrumentation error components, S_{gi} and B_{gi}. Similar data would be required for the facility instrumentation error components, S_{gt} and B_{gt}.

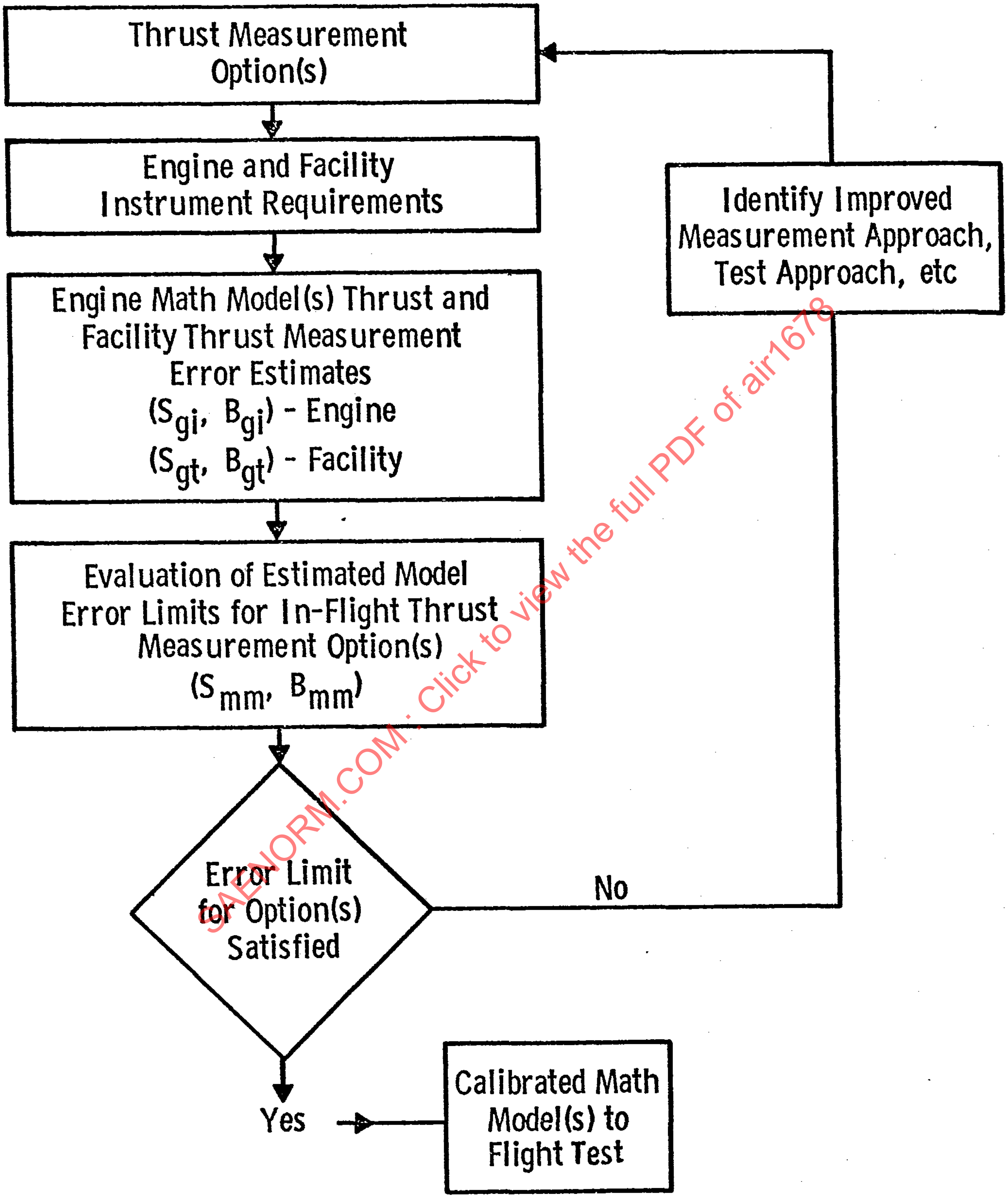


Figure 4.1 - Ground Test Measurement Process Evaluation Cycle

<table>
<tr><td colspan="2">Basic Measurement
Inlet Static Pressure PS1</td><td colspan="2">Type Sensor
Strain Gage</td><td>Measurement Range
0 to 2000 PSF</td></tr>
<tr><td colspan="5">Test Condition(s)</td></tr>
<tr><td>Sub-Script</td><td>Elemental Error Source</td><td>Precision Index Sij</td><td>Bias Limit Bij</td><td>Substantiation Source and Notes</td></tr>
<tr><td></td><td>Calibration</td><td></td><td></td><td></td></tr>
<tr><td>11</td><td>SL - ILS</td><td>0.5</td><td>1.0</td><td>Calibration Sheet Dated 3/3/73</td></tr>
<tr><td>21</td><td>ILS - TS</td><td>0.5</td><td>1.0</td><td>Calibration Sheet Dated 6/6/73</td></tr>
<tr><td>31</td><td>TS - WS</td><td>0.6</td><td>1.5</td><td>Calibration Sheet Dated 11/26/73</td></tr>
<tr><td>41</td><td>WS - MI</td><td>0.8</td><td>2.0</td><td>Calibration Sheet Dated 12/01/73</td></tr>
<tr><td></td><td>Data Acquisition</td><td></td><td></td><td></td></tr>
<tr><td>12</td><td>Excitation Voltage</td><td>0.4</td><td>3.0</td><td>Special Test Sheet Dated 6/17/74</td></tr>
<tr><td>22</td><td>Electrical Simulation</td><td>0.4</td><td>3.0</td><td>Special Test Sheet Dated 6/17/74</td></tr>
<tr><td>32</td><td>Signal Conditioning</td><td>0.2</td><td>2.2</td><td>Special Test Sheet Dated 6/17/74</td></tr>
<tr><td>42</td><td>Recording Device</td><td>0.12</td><td>0.25</td><td>Special Test Sheet Dated 6/17/74</td></tr>
<tr><td>52</td><td>Pressure Transducer</td><td>0.8</td><td>9.0</td><td>Manufactured Spec</td></tr>
<tr><td>62</td><td>Probe Errors</td><td>0.8</td><td>10.0</td><td>Special Test Sheet Dated 6/17/74</td></tr>
<tr><td>72</td><td>Environmental Effect</td><td>0.2</td><td>5.0</td><td>Special Test Sheet Dated 6/17/74</td></tr>
<tr><td></td><td>Data Reduction</td><td></td><td></td><td></td></tr>
<tr><td>13</td><td>Curve Fit</td><td>0.15</td><td>0.0</td><td>Curve Fit Sheet Dated 4/17/74</td></tr>
<tr><td>23</td><td>Computer Resolution</td><td>0.15</td><td>0.0</td><td>Round Off Error Analysis</td></tr>
<tr><td></td><td>Resultant Measurement Error Components</td><td>1.80 PSF</td><td>15.40 PSF</td><td>Elemental Error Source Report Number 0003</td></tr>
</table>

TABLE 4.4 Example Elemental Error Sources Report

Test Condition: 1000'/Mach 0.65/Cruise Power

Basic Measurement Identification	Units	Range	Precision Index	Bias Limit	Influence Coefficient	Uncertainty Limit (U_{ADD})	Supporting Elemental Error Sources Report Number
Low Rotor Speed N1	(RPM)	17 to 3000	5.7	3.0	-.0083	-	0001
High Rotor Speed N2	(RPM)	94 to 64000	31.3	3.2	-.1046	-	0002
Inlet Static Press PS1	(psf)	0 to 2000	1.8	15.4	9.6450	-	0003
Inlet Delta Press DELPO	(psf)	0 to 1000	1.0	8.4	10.7613	-	0004
Inlet Total Temp T2	(Deg F)	-110 to 220	0.35	2.7	1.2151	-	0005
Net Thrust		1826	3.46	12.3	-	19.21	-

Note 1: Elemental Error Source Report 0003 is documented in Table 4.4

TABLE 4.5 Example Measurement Uncertainty Summary Report

Only one flight condition is shown in this example. When examining a range of test conditions, the error components for each test condition should be propagated and the results combined in an appropriate fashion. A conservative estimate would use the largest value of uncertainty for the performance parameter over the range of test conditions.

4.2.2 Ground Posttest Analysis: The objectives of posttest data analysis are (1) to confirm the pretest measurement error estimates and (2) to provide a calibrated engine thrust math model(s) and the lapse rate correction model, including error components.

CONFIRMATION OF PRETEST ESTIMATES

The precision index for each measurement can be verified from repeat data points. The bias limit can only be checked in a gross sense in that thrust can be checked against predicted values and the final results can be tested for consistency, and deviations from predicted performance. If multiple thrust options are available they may be compared to provide additional assurance that bias limits have been properly assessed. This procedure guards against large unknown biases going undetected.

CALIBRATION OF THE MATH MODEL AND ESTIMATION OF ERROR COMPONENTS

The procedure for the derivation of the calibrated math model will depend on the specific form of the math model itself. Appendix G provides details. Once the calibrated math model is defined, the math model error components, illustrated in Figure 4.2, can be defined in a four-step process.

First, the ground test facility instrumentation error components are propagated to define the error components B_{gt} and S_{gt} of the facility performance measurement.

Second, the error components of the engine ground test instrumentation are propagated through the calibrated math model to define the ground instrumentation error components B_{gi}, and S_{gi}.

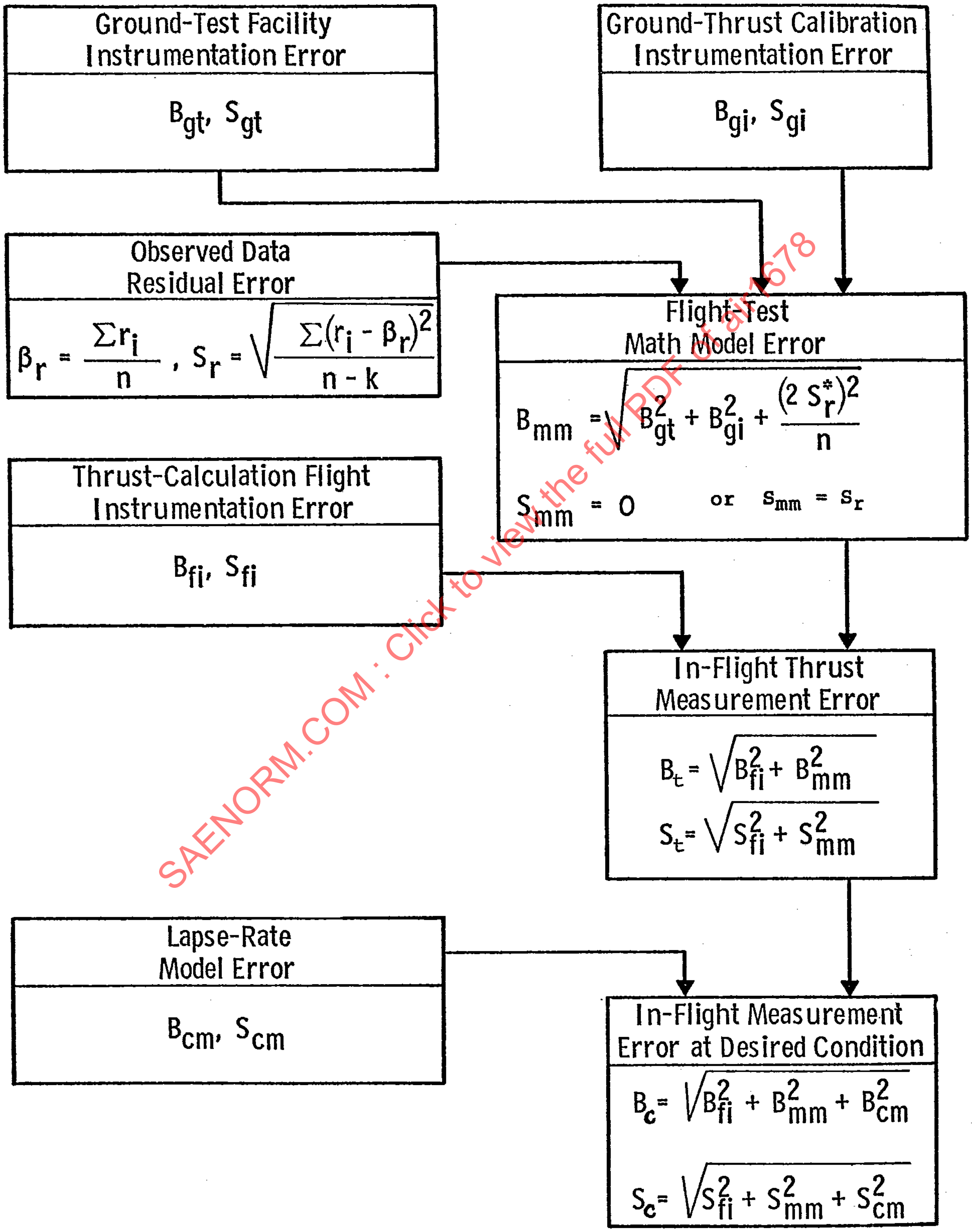


Figure 4.2 - In-Flight Thrust Measurement Process Error Propagation

Third, the thrust is calculated using the calibrated math model for each ground test data point, comparing this calculated thrust (F_{calc}) to the facility measured thrust (F_m) to define a residual error (r). This could be done as an absolute difference;

$$r = F_{calc} - F_m \qquad (4.7)$$

or as a fractional value;

$$r' = \frac{F_{calc} - F_m}{F_m} \qquad (4.8)$$

The fractional value which is not necessarily constant through the range of F_m values, is more advantageous when examining a range of test conditions. The definition used will affect the numerical value of the error estimates.

Fourth, the information of the first three steps is combined to identify the math model error. The math model will always have a bias error. The limit of this error can be estimated from the information above. Initially, the observed bias error (β_r) is defined. This is the average residual error for the flight condition or range of conditions of interest (Figure 4.3). Care should be taken in defining the range of test conditions for which data can be combined. Appendix G provides some guidelines.

$$\beta_r = \frac{\sum_i r_i}{n} \qquad (4.9)$$

The precision index of the observed residual error (S_r) must also be calculated (Figure 4.4).

$$S_r = \sqrt{\frac{\sum_i (r_i - \beta_r)^2}{n-k}} \qquad (4.10)$$

where k is the order of the curve fit.

For any reasonable calibration process the residual bias error should be essentially zero.

Two cases are considered to define the math model bias limit. In the simplest case the flight test engine and its instrumentation are assumed to be those used in the ground test. If the total flight test data acquisition and reduction process is replicated then the math model bias limit is defined by the ground facility measurement bias limit (B_{gt}) and the fossilized precision element from the calibration process; $\frac{2S^*_r}{\sqrt{n}}$ for large sample sizes ($n \geq 30$). The math model bias limit is defined by root-sum-squaring these two components.

$$B_{mm} = \sqrt{B_{gt}^2 + \left(\frac{2S^*_r}{\sqrt{n}}\right)^2} \qquad (4.11)$$

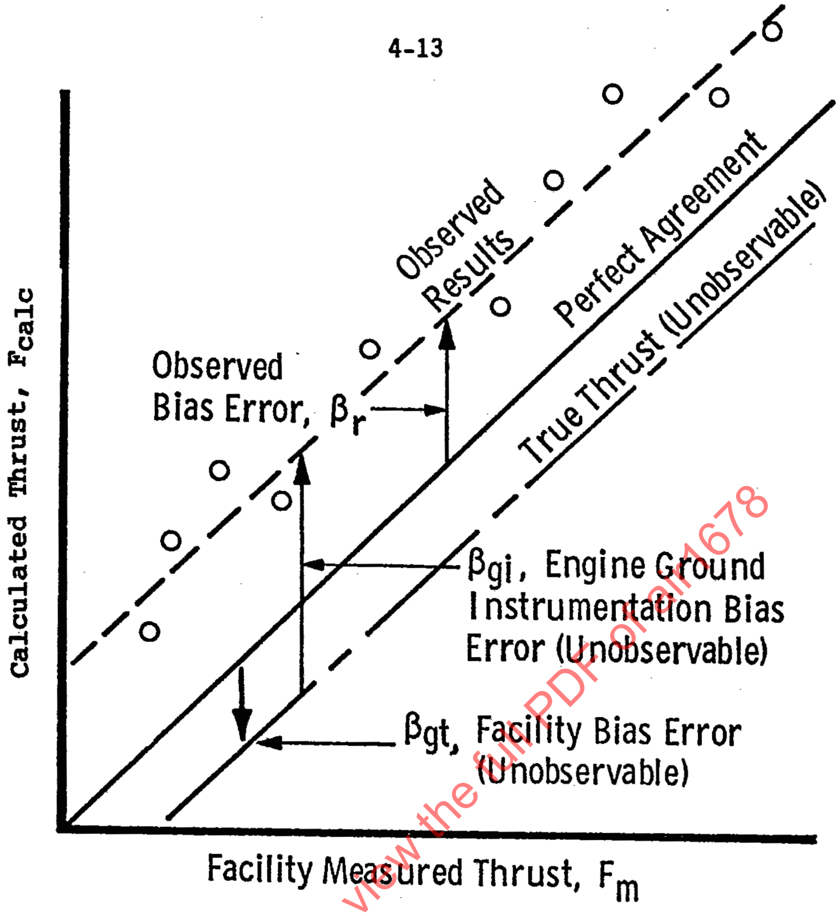


Figure 4.3 - Illustration of Math Model Bias Error

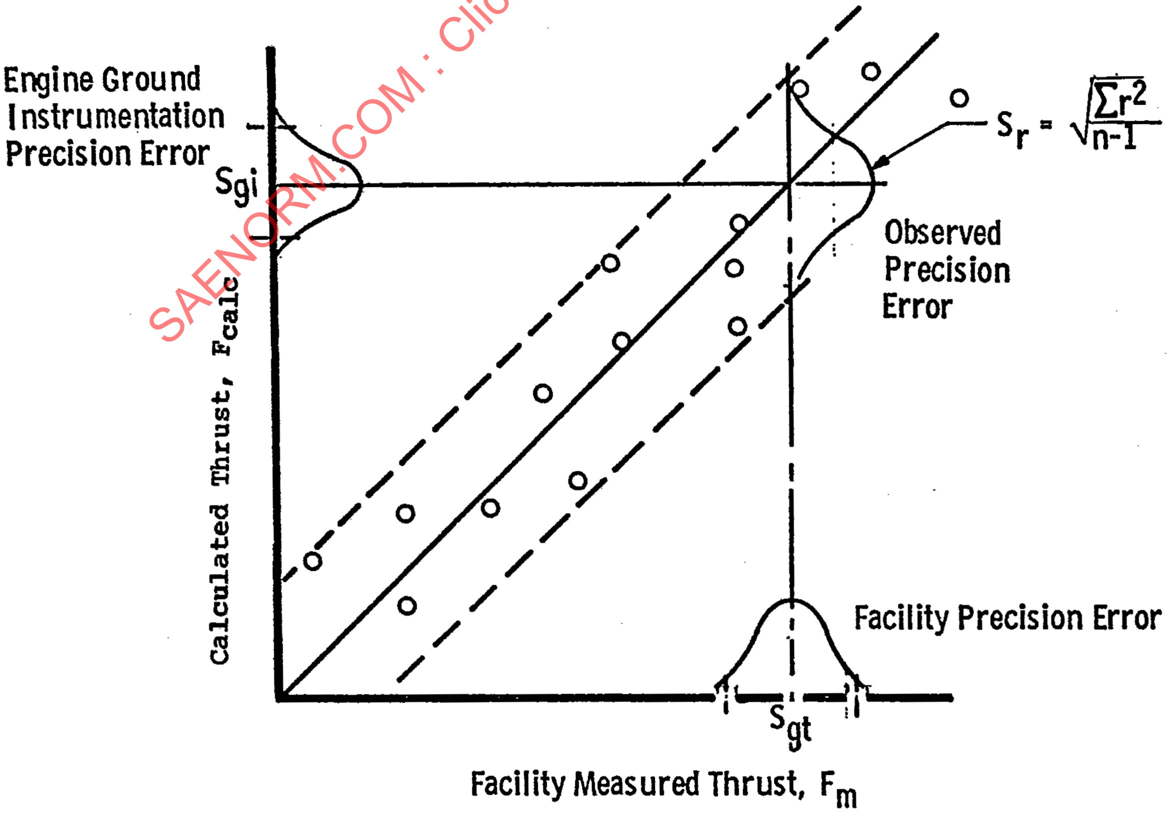


Figure 4.4 - Illustration of Math Model Precision Error

For the case where any portion of the flight test measurement process is not identical to the ground test measurement process the bias limit of the ground instrumentation, B_{gi}, (or at least that portion which is not carried over into flight test) must be considered. The math model bias limit is then defined by:

$$B_{mm} = \sqrt{B_{gt}^2 + B_{gi}^2 + \left(\frac{2S_r^*}{\sqrt{n}}\right)^2} \quad (4.12)$$

In addition to the bias error, the precision error should be investigated. The thrust calculation procedure is not normally expected to introduce any random errors. An estimate of the precision index of the residual error ($\hat{S}_r$) is calculable from the root-sum-square of the precision components of the two sets of measurement errors, S_{gt} and S_{gi}.

$$\hat{S}_r = \sqrt{S_{gt}^2 + S_{gi}^2} \quad (4.13)$$

The actual precision index (S_r) is calculated from the observed data using equation 4.10. The two are compared using an F test as defined in Appendix G.4 If they are essentially the same it can be assumed that the ground test process is well controlled and understood and that the math modeling process does not introduce any random error. In this event the Math Model precision error is zero as shown in Eq. 4.14.

$$S_{mm} = 0 \quad (4.14)$$

If a significant difference exists then the Math Model precision error is equal to S_r.

$$S_{mm} = S_r \quad (4.15)$$

Care must be taken to ensure that the derived error from the resultant calibrated math model does not excessively misstate the uncertainty. The evaluation could assess only the worst case condition and provide a conservative estimate (maximum value) for the desired test envelope, or could assess data throughout the envelope to provide an average estimate. Since it may not be feasible to identify the worst case condition prior to completion of the data analysis, the latter option may be the most practical for evaluations which require broad flight envelope testing. The assumptions made in combining flight test conditions can be tested by subdividing the final combined conditions and showing that the math model results remain essentially constant.

DERIVATION OF THE CALIBRATED LAPSE RATE CORRECTION MODEL AND ITS ERROR COMPONENTS

An additional task which may be part of the posttest analysis is that of defining the procedure to correct actual flight test data to desired standard conditions. This also requires an evaluation of the errors introduced by the correction process, B_{cm} and S_{cm}.

The procedures are discussed in Appendix G.3. The lapse rate correction will have a number of similarities to the math model. The lapse rate correction error components can be shown to be dominated by the flight test instrumentation error components. The flight test instrumentation error components are required since the correction is from a measured test condition, which contains error, to an absolute desired condition. These flight test instrumentation error components are estimated using the same techniques required for the other instrumentation sets.

4.3 Flight Test: The flight test activities include pretest estimating of flight measurement errors and in-flight thrust errors, and posttest confirmation of pretest error estimates. A logic flow chart is shown in Figure 4.5.

4.3.1 Flight Pretest Error Analysis: The total measurement system, including calibration, data acquisition, and data reduction processes, is identified from the selected thrust options. A detailed pretest error analysis is made as for ground test. Tables similar to 4.4 and 4.5 are completed. This analysis provides the flight instrumentation error components S_{fi} and B_{fi}. The results are combined with the math model error components (S_{mm} and B_{mm}) and, where appropriate, the lapse rate correction model error components (S_{cm} and B_{cm}) to define the flight test thrust measurement error components (S_c and B_c):

$$S_c = \sqrt{S_{fi}^2 + S_{mm}^2 + S_{cm}^2} \qquad (4.16)$$

$$B_c = \sqrt{B_{fi}^2 + B_{mm}^2 + B_{cm}^2} \qquad (4.17)$$

For as-tested conditions, the lapse rate error components would be deleted.

4.3.2 Flight Posttest Analysis: The primary purposes of the posttest data analysis are to confirm the pretest measurement error estimates and to calculate the in-flight thrust measurement uncertainty for the selected math model options. The same procedure described in Section 4.2.2 is used.

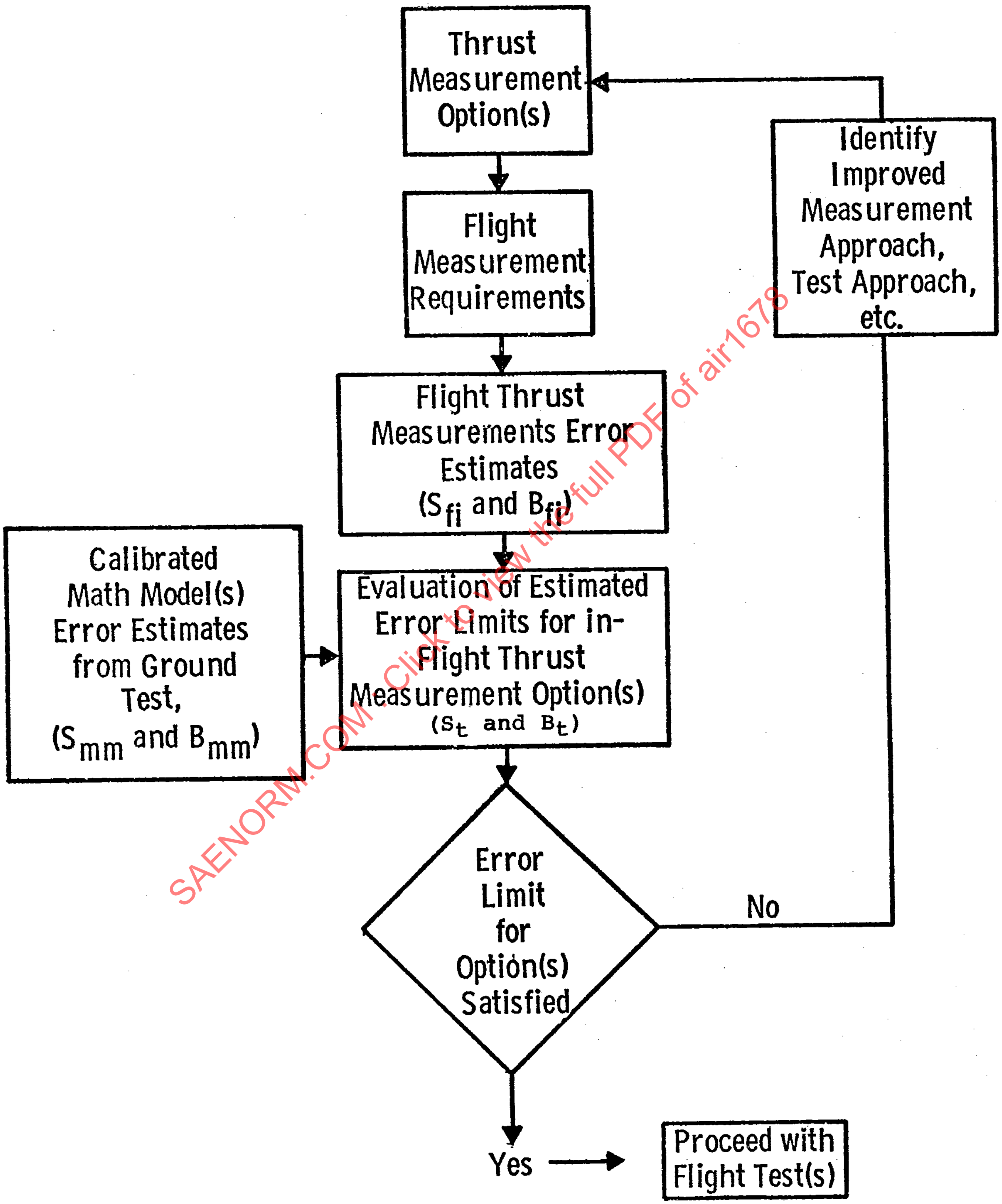


Figure 4.5- Flight Test Measurement Process Evaluation Cycle

4.4 Results Analyses: The objective of this effort is to ensure that systematic unknown bias errors do not go undetected in the various program processes. The program results review consists of examining the consistency of the program test results (ground and flight tests) with pretest predictions and with vehicle performance characteristics. Gross checks should reveal specific areas requiring detailed test data analysis. Problem areas should be resolved and, as a result, the uncertainty intervals for the in-flight thrust measurement options may require adjustment.

4.4.1 Review of Ground and Flight Test Results: The prime objective is to ensure that the ground test calibrated math model(s) is viable and corresponding error components are applicable to the actual flight tests. In the review, factors that may void or alter the applicability of the math model should be identified, such as instrumentation failures (and possibly subsequent substitutions), differences in the instrumentation type and probe location between ground and flight test, differences between the simulated flight conditions and the actual flight conditions, and differences in engines or engine configurations between ground and flight test. Adjustments to the final flight thrust results are made on the basis of this review.

Techniques for testing data validity using aircraft performance data consistency (e.g., drag polar analysis, Ref. AIR 1703, Section 7) may be employed. These techniques are useful for identifying problems.

4.4.2 Test Data Averaging:

It may be advantageous to combine data points to improve the performance estimation. The precision index of an average is reduced by the inverse ratio of the square root of the number of data points in the average compared to the precision index of single points. This results in an uncertainty for the average ($U_{\bar{X}}$) defined by:

$$U_{\bar{X}} = B_c + \frac{2S_c}{\sqrt{n}} \qquad (4.18)$$

Since repeat data points are generally not totally achievable, it is corrected test data which would normally be averaged. Therefore, the error components would include the lapse rate model error.

If more than one independent thrust measurement option is available for a given flight test condition it may be possible to combine data to improve the estimate. However, defining the uncertainty of such combined data is very complex and beyond the scope of this document, (4.2).

4.4.3 Program Data Uncertainty Reporting: The flight test performance results should be reported following the general requirements of Section 2.3. The uncertainty interval and its components for each thrust option should be reported and should include supportive error information: each option's measurements error components and the corresponding elemental error sources with their error estimates. It may be useful to report intermediate performance results.

For combined data points, it is necessary to report the number of data points represented by the average value in addition to the S_C and B_C components.

5. REFERENCES:

1.1 G. R. Adams, et al, "Uncertainty Methodology for In-Flight Thrust Determinations", SAE Technical Paper 831438 Presented at the Aerospace Congress and Exposition, Long Beach, California, October 3-6, 1983.

1.2 G. R. Adams, et al, "Application of In-Flight Thrust Determination Uncertainty", SAE Technical Paper 831439 Presented at the Aerospace Congress and Exposition, Long Beach, California, October 3-6, 1983,

2.1 P. J. Campion, J. E. Burns, and A. Williams, Section 5 Recommendations, Her Majesty's National Physical Laboratory Report "A Code of Practice for the Detailed Statement of Accuracy", 1973,.

2.2 Dr. R. B. Abernethy and J. W. Thompson, Jr. "Handbook Uncertainty in Gas Turbine Measurements", AEDC-TR-73-5, February, 1973.

2.3 A. T. J. Hayward in "Repeatability and Accuracy", Mechanical Engineering Publications, Ltd.

4.1 B. D. Couch, W. O. Boals, and B. M. Bishop, "ALCM Preflight-Test Thrust Uncertainty Analysis", AEDC-TR-81-2, July, 1981.

4.2 G. R. Adams, "Improvement of Parameter Estimation Through Uncertainty Analysis", ASME Paper 80-GT-9 Presented at the Gas Turbine Conference and Products Show, New Orleans, La., March 10-13, 1980.

APPENDIX A

DEFINITIONS

Accuracy	The closeness or agreement between a measured value and the true value.
Average value ($\overline{X}$)	The arithmetic mean of n readings.
Bias (β)	The difference between the average of the measurement population and the true value.
Bias Limit (B)	The expected upper limit of the true bias error, β, often determined by judgment.
Calibration	The process of comparing the response of an instrument to a standard instrument over the measurement range. Corrections may be applied in an attempt to reduce the instrument bias.
Calibration Hierarchy	The chain of calibrations which links or traces a measuring instrument to a primary standard.
Confidence Level	The frequency in percent with which the calculated statistic is correct. (See also coverage).
Coverage	The frequency with which an interval estimate of a parameter may be expected to contain the true value. A ninety five percent uncertainty interval provides 95% coverage of the true value. That is, in repeated sampling, when a 95% uncertainty interval is constructed for each sample, over the long run, the intervals will contain the true value 95% of the time.
Degrees of Freedom (ν)	A sample of n values is said to have n degrees of freedom, and the first statistic calculated from it is also said to have n degrees of freedom. One degree of freedom is lost for each previously calculated statistic used to calculate a new statistic.
Elemental Error	The bias and/or precision error associated with a single error source.
Estimate	A value calculated from a sample of data as a substitute for an unknown population constant. For example, the precision index (S) is an estimate of the population standard deviation (σ).
Fossilization	Sometimes the precision error from the calibration process becomes part of the fixed bias error. This conversion from random to bias error should be called fossilization, and is indicated by an asterisk superscript.

Influence Coefficient, (θ)	For a given functional relationship, the effect a small change in an input parameter has on the output.
Laboratory Standard	An instrument which is calibrated periodically against a primary standard. The laboratory standard may also be called an interlab standard.
Measurement Error	The difference between the true value and the measured value. It includes both bias and precision errors.
Precision	The closeness of agreement between repeated measurements.
Precision Error	The random error based on a set of repeated measurements. Also known as repeatability error or sampling error. The product of the Student t value and the precision index is an estimate of the precision error uncertainty.
Precision Index (S_X)	The estimated standard deviation based on N measurements.
Precision Index of Mean ($S_{\bar{X}}$)	Calculated as follows for n measurements: $S_{\bar{x}} = \frac{S_x}{\sqrt{n}}$
Primary Standard	A primary physical standard or one maintained at a National Standards Laboratory
Repeatability	See precision error.
Sample Size (n)	The number of independent measurements of a parameter in a sample.
Sampling Error	See precision error.
Standard Deviation (σ)	The true measure of dispersion of a population.
Traceability	The ability to trace the calibration of a measuring device through a chain of calibrations to a primary standard. Traceability does not guarantee accuracy but helps to assure it by the discipline of a documented calibration hierarchy.

Transfer Standard	A laboratory instrument which is used to calibrate working standards and which is periodically calibrated against the laboratory standard.
True Value	The actual value of the parameter being measured.
Uncertainty Interval	The interval expected to contain the true value: ± U for symmetrical bias limits and $+U^+$ and $-U^-$ for nonsymmetrical bias limits. See Appendix D for a discussion on confidence level or coverage of the true value by the interval.
Uncertainty Limit (U)	The expected error limit of a result for a given coverage.
Working Standard	An instrument which is calibrated in a laboratory against an interlab or transfer standard and is used as a standard in calibrating measuring instruments.

APPENDIX B

SMALL SAMPLE STATISTICAL METHODS

With a large sample arbitrarily defined as > 30, the following small sample methods are not required as the value of student's t may be assumed to be 2.0.

B.1 Student's "t"

The uncertainty limit, defined either as:

$$U_{ADD} = (B + t_{95}S) \text{ or } U_{RSS} = \sqrt{B^2 + \left(t_{95}S\right)^2} \qquad \text{(B.1)}$$

where B is the bias limit, S is the precision index, and t_{95} is the 95th percentile point for the two-tailed Student's "t" distribution (Table B-1). The t value is a function of the number of degrees of freedom (ν) used in calculating S. For small samples, t will be large, and for larger samples t will be smaller, approaching 1.96 as a lower limit. The use of t inflates the limit U to reduce the risk of underestimating S when a small sample is used. Since 30 degrees of freedom yield a t of 2.04 and infinite degrees of freedom yield a t of 1.96, an arbitrary selection of t=2 for values of ν from 30 to infinity is made for simplicity.

B.2 Degrees of Freedom (ν): In a sample, the number of degrees of freedom is the size of the sample. When a statistic is calculated from the sample the degrees of freedom associated with the statistic are reduced by one for every estimated parameter used in calculating the statistic. For example, from a sample of size n, $\overline{X}$ is calculated:

$$X = \left[\frac{\sum_{i=1}^{n} X_i}{n}\right] \qquad \text{(B.2)}$$

which has ν = n degrees of freedom, and

$$S = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \overline{X})^2}{n-1}} \qquad \text{(B.3)}$$

which has ν=n-1 degrees of freedom because $\overline{X}$ (based on the same sample of data) is used to calculate S. In calculating other statistics, more than one degree of freedom may be lost. For example, in calculating the standard error of a curve fit, the number of degrees of freedom which are lost is equal to the number of estimated coefficients used to define the curve.

Each precision index (S) with its associated degrees of freedom (ν) are used to find the corresponding value of Student's "t".

B.3 Propagation of Degrees of Freedom: When precision indices of elemental error sources are combined (root sum square), the degrees of freedom of the result must be determined if any of the elemental degrees of freedom are less than 30. The Welch-Satterthwaite formula is used for this purpose. It is a function of the degrees of freedom and the magnitude of the elemental precision indices.

If $S_1 = \sqrt{S_{11}^2 + S_{21}^2 + S_{31}^2}$ with degrees of freedom ν_{11}, ν_{21}, ν_{31}, (B.4)

then, $$\nu_1 = \frac{\left[S_{11}^2 + S_{21}^2 + S_{31}^2\right]^2}{\frac{S_{11}^4}{\nu_{11}} + \frac{S_{21}^4}{\nu_{21}} + \frac{S_{31}^4}{\nu_{31}}} \qquad \text{(B.5)}$$

The general form being:

$$\nu_R = \frac{\left[\sum_i S_i^2\right]^2}{\sum_i \frac{S_i^4}{\nu_i}} \qquad \text{(B.6)}$$

When precision indices for measurement are propagated to the final test result with influence coefficients, the precision index of the result will be obtained from an equation like:

$$S_R = \sqrt{\left(\frac{\partial R}{\partial X} S_x\right)^2 + \left(\frac{\partial R}{\partial Y} S_y\right)^2 + \left(\frac{\partial R}{\partial Z} S_z\right)^2} \qquad \text{(B.7)}$$

then

$$\nu_R \cong \frac{\left[\left(\frac{\partial R}{\partial X} Sx\right)^2 + \left(\frac{\partial R}{\partial Y} Sy\right)^2 + \left(\frac{\partial R}{\partial Z} Sz\right)^2\right]^2}{\frac{\left(\frac{\partial R}{\partial X} Sx\right)^4}{\nu_x} + \frac{\left(\frac{\partial R}{\partial Y} Sy\right)^4}{\nu_y} + \frac{\left(\frac{\partial R}{\partial Y} Sz\right)^4}{\nu_z}} \qquad \text{(B.8)}$$

In this manner, the degrees of freedom are propagated to the final test result and used to determine the t_{95} value for the uncertainty interval. Note that it is conservative to round ν_R down if the ν_R is not a whole number.

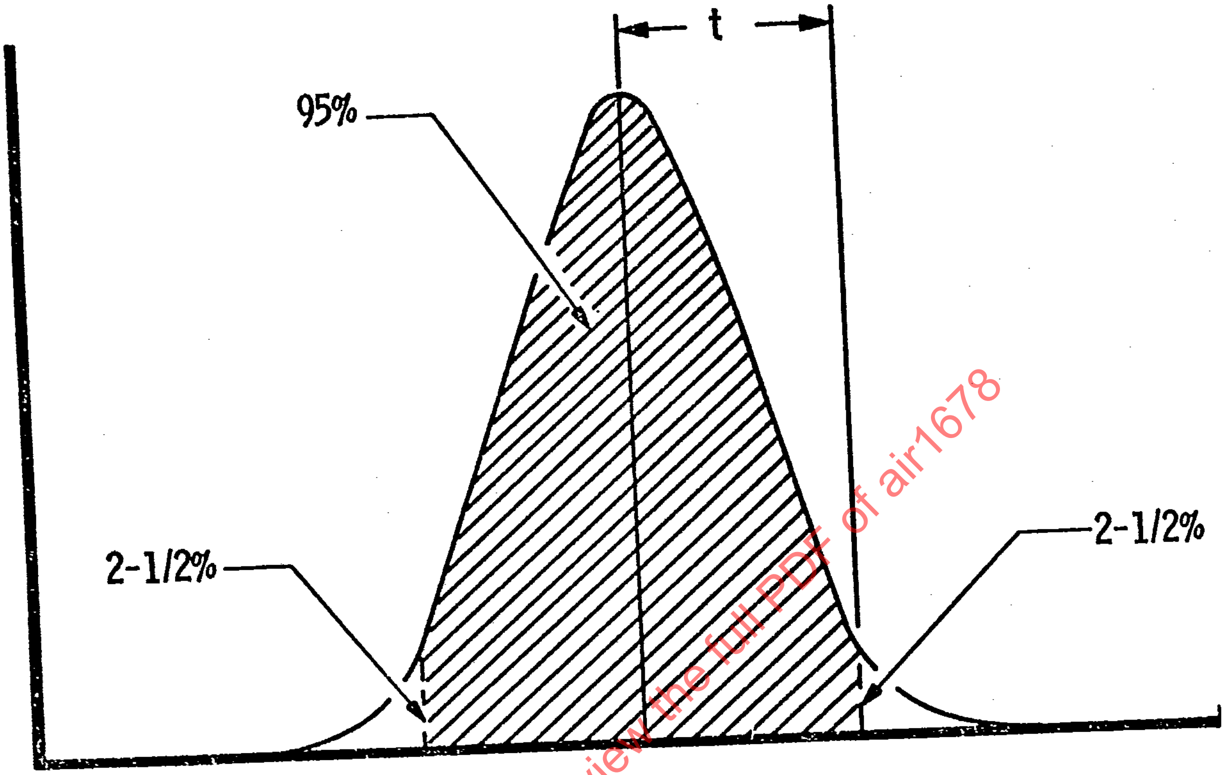
B.4 Students "t" Table: The table of Student's "t" distribution (Table B.1) presents the two-tailed 95% "t" values for the degrees of freedom from one to 30. Above 30, the value is rounded to 2.0.

The table is used to provide an interval estimate of the true value about an observed value. The interval is the measurement plus and minus the standard deviation of the observed value times the "t" value (for the degrees of freedom of that standard deviation):

$$\text{interval} = \text{measurement} \pm t_{95}S$$

The 95 Student's "t" value for 17 deg of freedom is 2.110. With a standard deviation of 50 the interval is:

$$\text{measurement} \pm \left(2.11 \times 50\right) = \text{measurement} \pm 105.50$$



Degrees of Freedom	"t"$_{95}$	Degrees of Freedom	"t"$_{95}$
1	12.706	17	2.110
2	4.303	18	2.101
3	3.182	19	2.093
4	2.776	20	2.086
5	2.571	21	2.080
6	2.447	22	2.074
7	2.365	23	2.069
8	2.306	24	2.064
9	2.262	25	2.060
10	2.228	26	2.056
11	2.201	27	2.052
12	2.179	28	2.048
13	2.160	29	2.045
14	2.145		
15	2.131		
16	2.120	30 or more use 2.0	

Table B.1 - Two-tailed Student's "t" Table

APPENDIX C

IMPACT OF MEASUREMENT PROCESS ON ERROR CLASSIFICATION AND ESTIMATION

Measurement errors are initially put into one of two classes, precision or bias, depending on how the error is derived. The precision error, S, is derived by a statistical analysis of repeated measurements while the bias limit, B, is at least partially estimated by non-statistical methods. This approach[1] makes a very complex situation manageable and keeps the statistical estimate and the judgment estimate error components separate until the appropriate time to combine them.

The initial decision on classifying elemental errors may be changed after considering the defined in-flight measurement process and the instrumentation system.

Calibration errors are the most common initial error(s) which may or may not be reclassified as a result of the defined measurement process. For example, considering the measurement of air flow for a gas turbine engine at a test facility over a long period, the uncertainty of this measurement will contain errors due to variations between calibrations, test stands, and measurement instruments. The initial classification of errors would likely remain unchanged. However, if the same measurement were from a single test stand involving one calibration, then the initial precision in the calibration process would manifest itself as a bias for this process. Reclassification of initial precision error(s) to a bias error(s) is marked with an asterisk (*) to indicate that it is a fossilized or fixed error for this process.

The uncertainty analysis for the above examples will be different from the uncertainty analysis for a comparative, back-to-back test to measure air flow on a single test stand for a single engine, which is a different measurement process. Calibration errors (bias and precision) which propagate as a bias into the measurement process may be ignored in comparative testing when the same instrumentation and equipment are used for all testing, and calibration errors do not affect the comparison of one test with another (the test objective being to determine if a design change is beneficial). In these examples, the defined measurement process may include the same engine, instrumentation, and test stand; however, there is still a difference in uncertainty due to the difference in test objectives and test duration.

The planned instrumentation, type and number, is also part of the definition of the measurement process. If the end measurement is an average of (1) a series of individual repeat points or (2) a number of simultaneous readings, or (3) a combination of both, this must also be specified, as the precision index depends on this information. Significant reductions in precision error can be obtained

[1] Her Majesty's National Physical Laboratory Report "A Code of Practice for the Detailed Statement of Accuracy", 1973, P. J. Campion, J. E. Burns, and A. Williams, Section 5 Recommendations.

if averaging can be used. (Averaging can be used (1) with repeated single measurements if the measured variable is constant, or (2) if redundant instruments can be recorded simultaneously). The decision logic for error classification based on the actual measurement process is shown in Table C.1.

In summary, errors from the calibration process merit special consideration. There are four cases to consider:

1. If the test period is long enough that instrumentation may be calibrated more than once and/or several test stands are involved, the precision errors in the calibration hierarchy should be treated as contributing to the overall precision index.

2. For a single set of instrumentation, calibrated only once during the test, all the calibration error is frozen or fossilized into bias. The uncertainty of the calibration process is all bias. An asterisk is used to designate calibration precision error that is fossilized, i.e. S*.

3. For comparative, back-to-back, development tests where the test objective is the difference between two successive tests, all the calibration error (bias plus precision) is a constant in both tests and is cancelled by taking the difference. Trending errors are an exception as described next.

4. Elemental errors that trend with time merit special attention. For example, considering a flow meter with a calibration history as shown in Figure C.1, the data show some trending characteristics. Every effort should be made to remove or reduce the trending. If the test process is long, like "B", including many calibrations, this trending error is a precision error. On the other hand, if the test is short, like "A", an argument can be made that this error is fixed, a bias.

   Therefore, trending errors should always be included in an uncertainty estimate. A trending error is the exception to both paragraphs 2 and 3 above and will always contribute to the uncertainty estimate. In comparative back-to-back testing, trending errors should be carefully evaluated as they may introduce large errors.

MEASUREMENT PROCESS DEFINED AS:	ERRORS CLASSIFIED AS:
Many Tests over Long Periods with Many Calibrations	$B_1 = B_{CAL}$, $S_1 = S_{CAL}$ S_2, S_3, B_2, B_3 Unaffected
Single Test, One Calibration	All error from calibration process is fixed or fossilized into bias $B_1 = \sqrt{B_{CAL}^2 + \left(\frac{2S^*_{CAL}}{n}\right)^2}$ where n = number of samples $S_1 = 0$ S_2, S_3, B_2, B_3 = Unaffected
Comparative or Back-to-Back Test, One Calibration, Ground Test or Flight Test	All biases (B_1, B_2, B_3) cancel out when first test result is subtracted from second result. Calibration precision is fossilized into $S_1^* = 0$. $S_1^* = B_1 = B_2 = B_3 = 0$ S_2, S_3 Unaffected, $S = \sqrt{S_2^2 + S_3^2}$ U_{ADD} or $U_{RSS} = \pm (2S)$, a two-sided statistical confidence bound
Flight Test uses Same Engine Instrumentation as Ground Test Without Recalibration	Ground test calibration of thrust model will eliminate engine instrumentation bias error.
Pressure Differential ($P_1 - P_2$) uses Same Transducer for Both P_1 and P_2	Bias limit for transducer cancels out.
Trending Error	Cannot be dropped out by any of the above.
Subscript Notation:	1 = Calibration; 2 = Data Acquisition; 3 = Data Reduction

TABLE C.1 DECISION LOGIC DIAGRAM TO SUMMARIZE RULES OF ERROR CLASSIFICATION

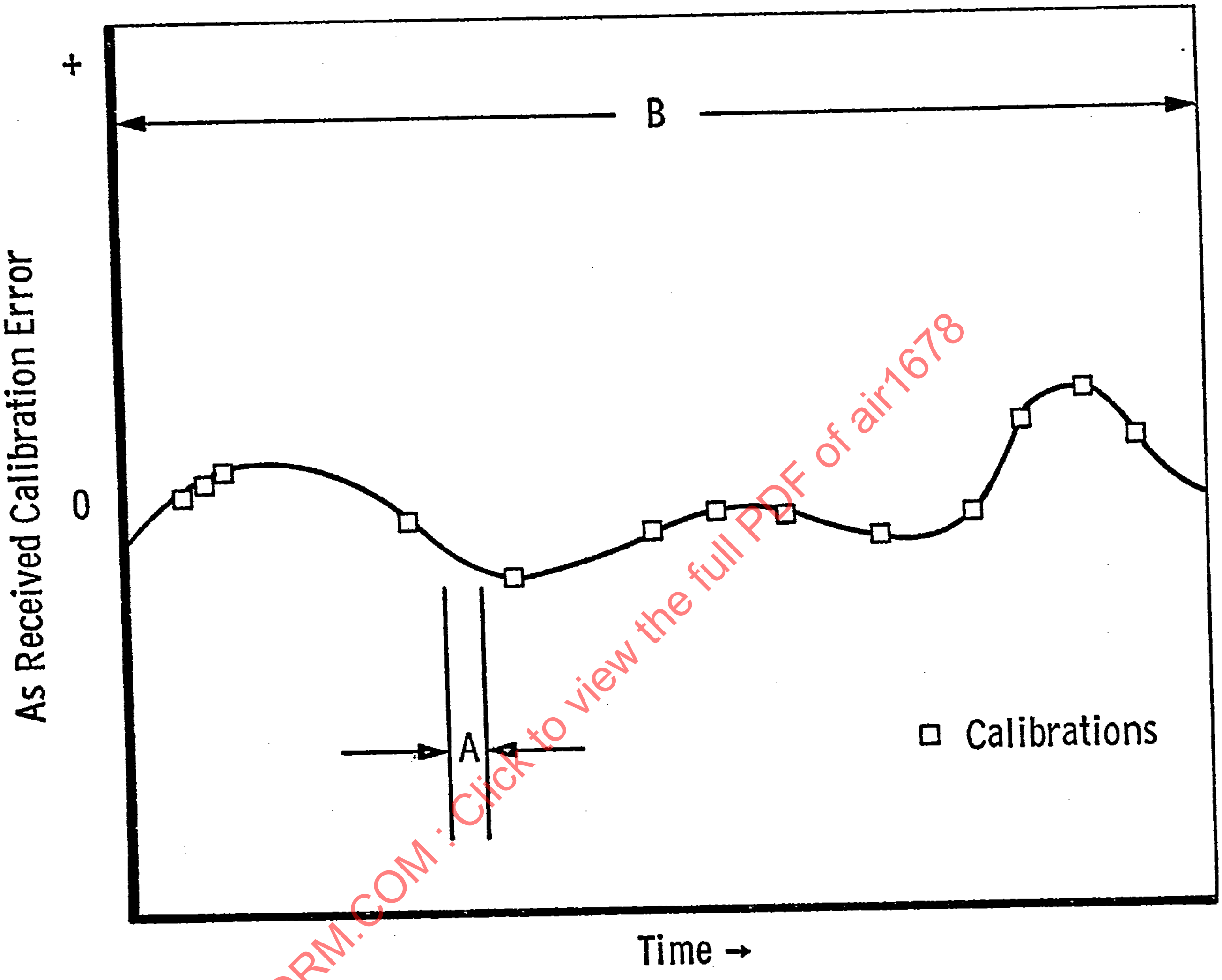


Figure C.1 - Trending Error Calibration History--Treat as Precision

APPENDIX D

MONTE CARLO COMPARISON OF ALTERNATIVE UNCERTAINTY METHODS

If the bias and precision error estimates are propagated separately to the test end result and the equation used to combine them into uncertainty is stated, either U_{ADD} or U_{RSS} can be used. Monte Carlo simulation was used to compare the additive and root-sum-squared values.

UNCERTAINTY INTERVAL COVERAGE

A rigorous calculation of confidence level or coverage of the true value by the interval is not possible because the distributions of bias errors and limits, based on judgment, cannot be determined. Monte Carlo simulation of the intervals can provide approximate coverage assuming various bias limit distributions.

For large samples (N > 30):

$$U_{ADD} = B + 2S \qquad (D.1)$$

$$U_{RSS} = \sqrt{B^2 + (2S)^2} \qquad (D.2)$$

For small samples:

$$U_{ADD} = B + t_{95} S \qquad (D.3)$$

$$U_{RSS} = \sqrt{B^2 + (t_{95}S)^2} \qquad (D.4)$$

where t_{95} is determined from the degrees of freedom calculated from the Welch-Satterthwaite approximation.

As the actual bias error and bias limit distributions will probably never be known, the simulation studies were based on a range of assumptions.

SIMULATION CASES CONSIDERED (102 cases)

A number (102) of simulation cases were first considered, These included:

- From 3 to 5 error sources, both bias and precision
- Bias errors distributed both normally and rectangularly
- Precision distributed normally
- Bias limits at both 95% and 99.7% for both the normal and the rectangular distribution

- Precision indices based on sample sizes from 3 to 30
- Ratio of precision to bias errors at 1/2, 1.0 and 2.0

  Another 102 cases were simulated with errors from up to 19 sources with similar results.

The result of the studies comparing the two intervals was:

- U_{ADD} averaged 99.1% coverage while U_{RSS} provided 95% based on bias limits assumed to be 95% (2σ for normally distributed biases and 1.645σ for rectangularly distributed biases.) (Figure D.1.)
- U_{ADD} averaged approximately 99.7% coverage and U_{RSS} coverage was 97.5% if the bias limits were assumed 99.7% (3σ for normal and 1.732σ for rectangular bias limit.) (Figure D.2.).
- Because of these coverages, U_{ADD} is sometimes called U_{99} and U_{RSS} is called U_{95}.
- If the bias error is negligible, both intervals provide a 95% statistical coverage.
- If the precision error is negligible, both intervals provide 95% to 99.7% depending on the assumed bias limit size.
- When the interval coverages were compared, U_{ADD} provides a more precise estimate of the interval size (range of 98% to 100%) as opposed to 93% to 100% for U_{RSS} (Figures D.1 to D.4).
- U_{ADD} was larger than U_{RSS}. The average ratio of the U_{ADD} interval size to U_{RSS} interval size was 1.35:1.

THE SIMULATION PROCEDURE

1. Measured values, $X = b_1 + . . . +b_N +S_1 +. . . +S_N$ were generated. The b_i's are randomly generated bias errors generated from N distributions (rectangular or normal) and the S_i's are randomly generated precision errors from the normal distribution. The means of the distributions were all zero. The precision indices were input as above.

2. N random samples of size $n_1, . . ., n_N$ were drawn from normal distributions, i=1, . . ., N.

3. The standard deviations of the N distributions, the Welch-Satterthwaite degrees of freedom and the U_{ADD} and U_{RSS} uncertainty intervals were calculated.

4. It was determined whether

$$(X - U_{RSS}) \leq 0 \leq (X + U_{RSS})$$

$$(X - U_{ADD}) \leq 0 \leq (X + U_{ADD})$$

The percent of times the interval covered 0 was calculated for 500 cases. The results are shown on Figures D.1, 2, 3 and 4.

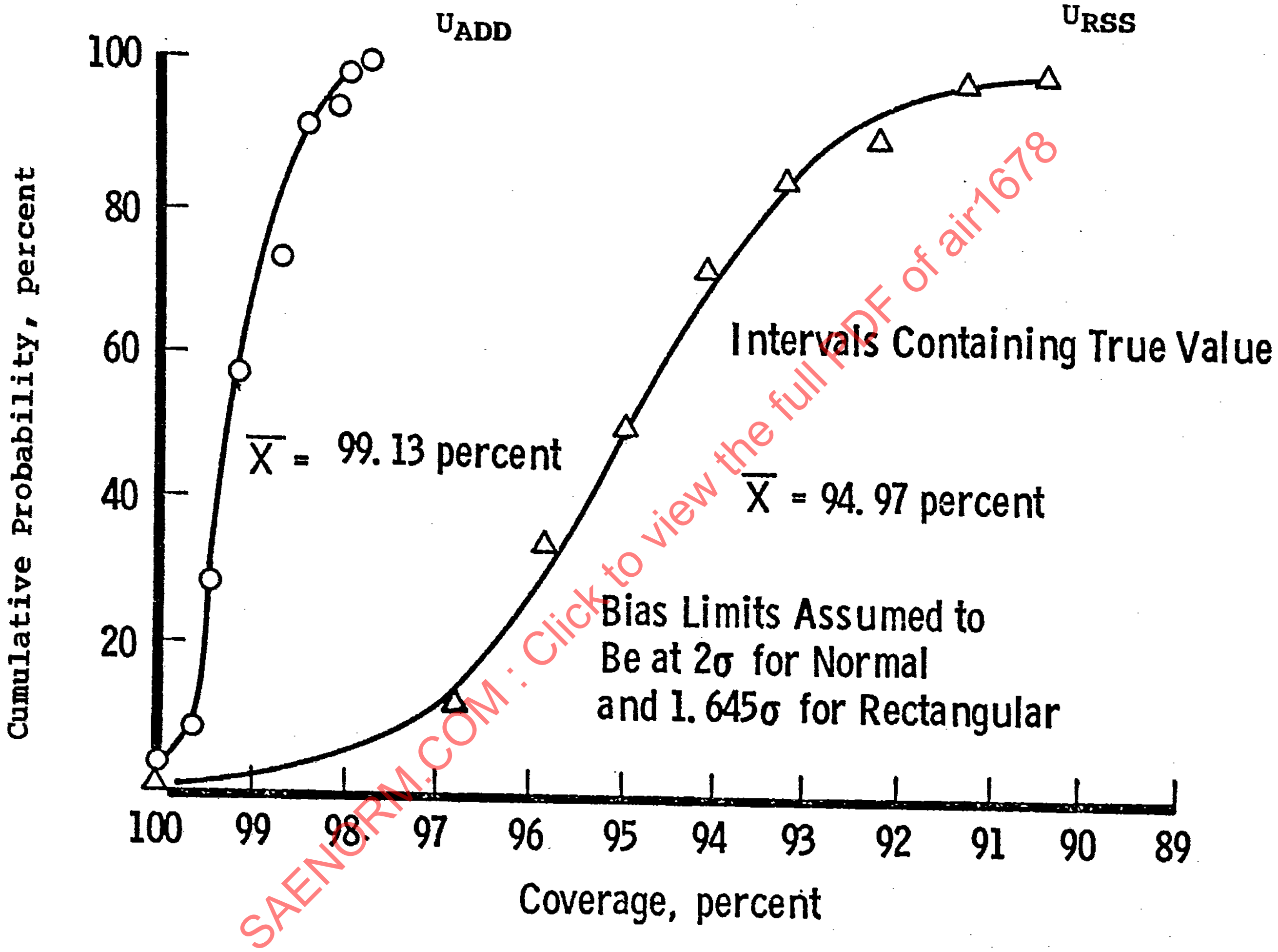


Figure D.1 - Coverage Distribution for 95% Bias Limits

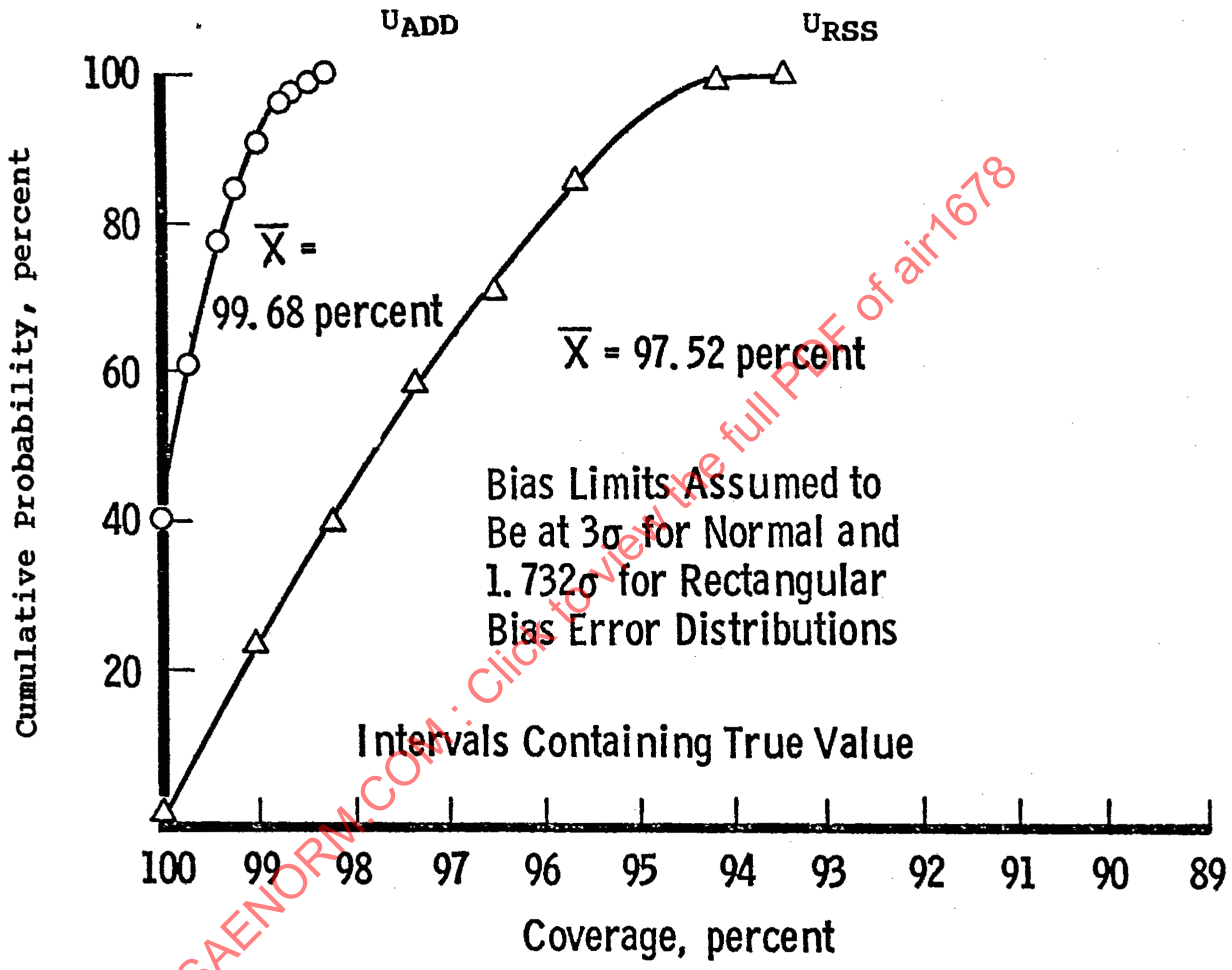


Figure D.2 - Coverage Distribution for 99.7% Bias Limits

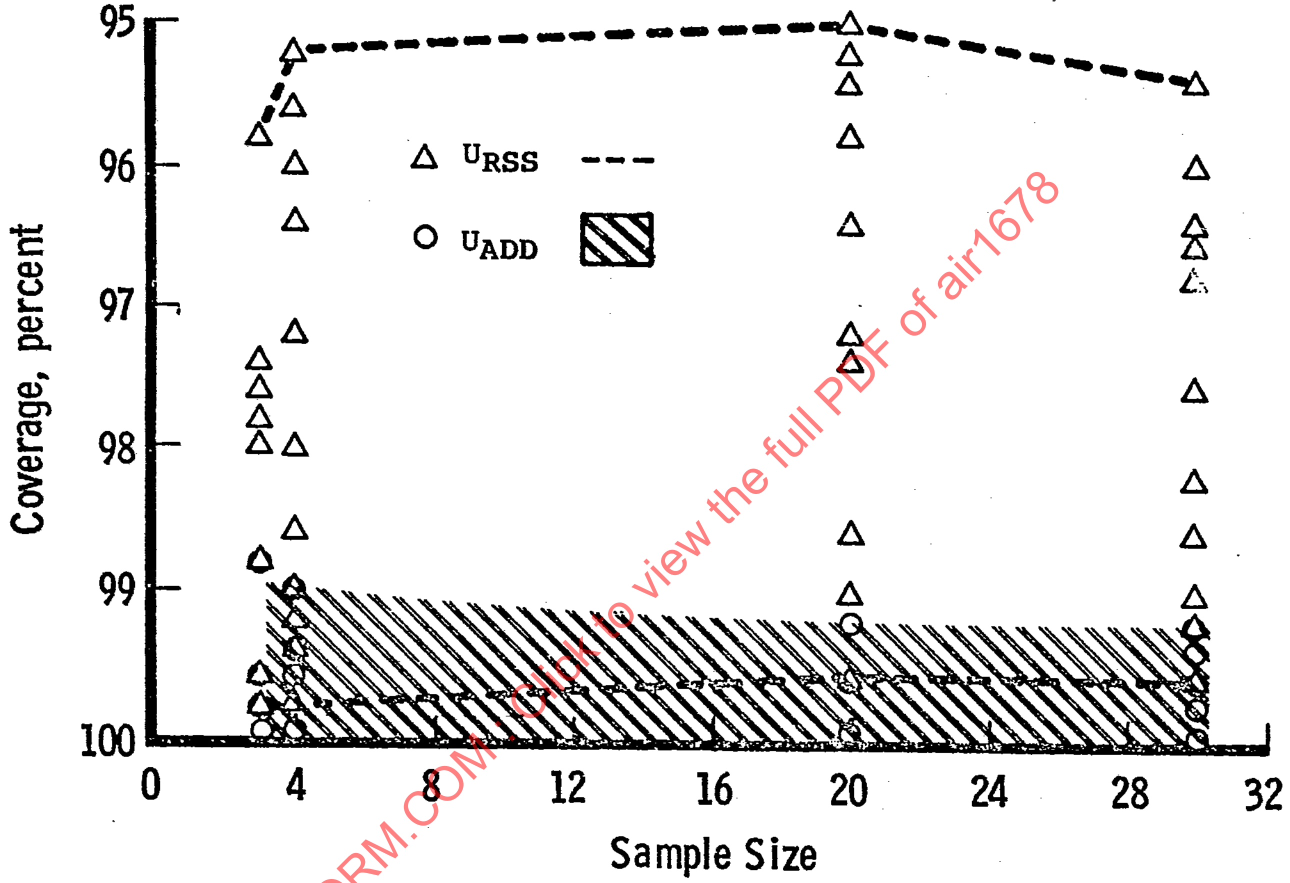


Figure D.3 - U_{ADD} and U_{RSS} Sensitivity to Variation in Sample Size

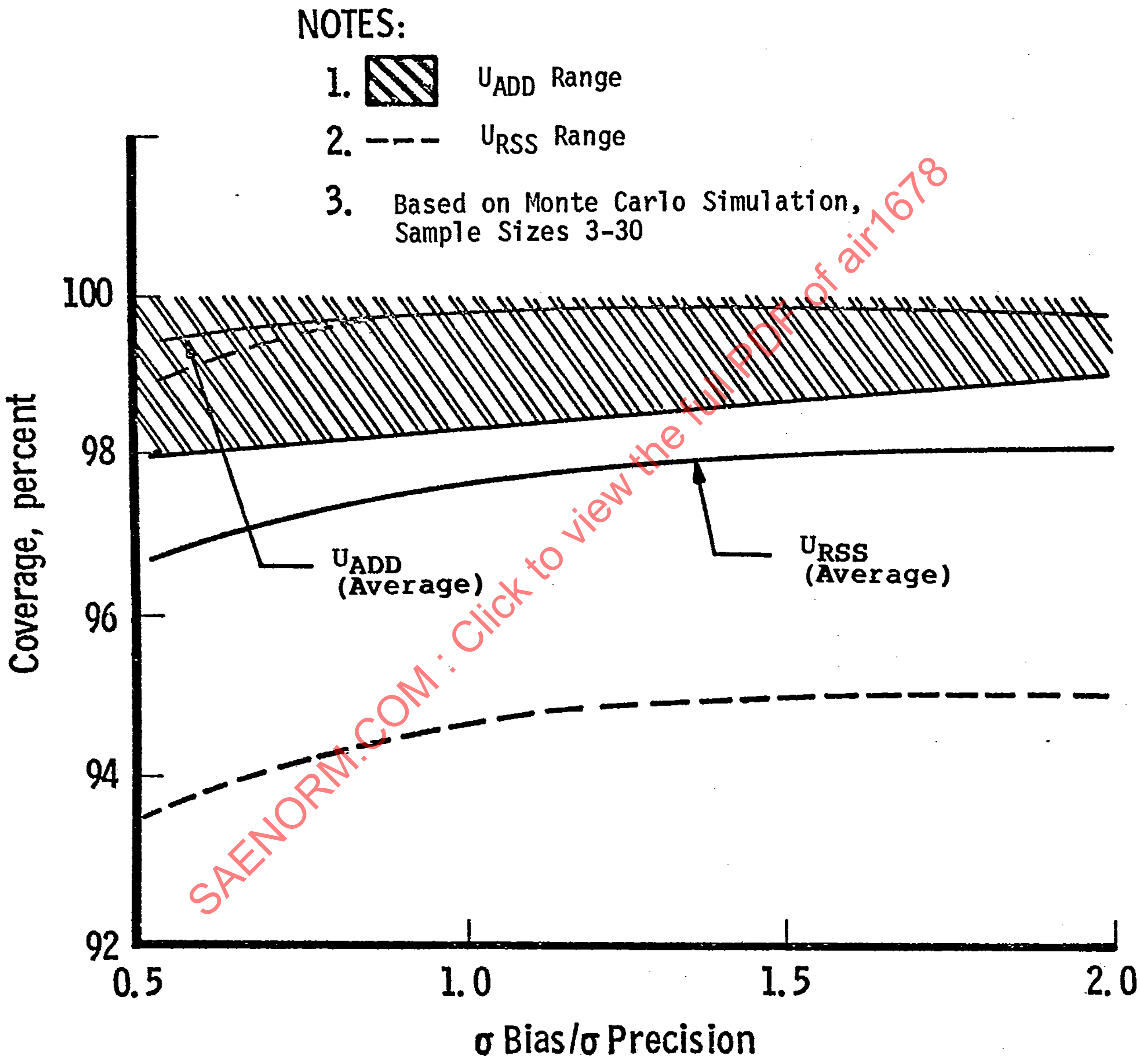


Figure D.4 - U_{ADD} and U_{RSS} Sensitivity to Bias to Precision Error Ratio

APPENDIX E

EXAMPLE INFLUENCE COEFFICIENT CALCULATIONS

The basis of this example is found in report AEDC-TR-81-2, ALCM Preflight-Thrust Uncertainty Analysis. The format has been changed slightly and additional information provided as necessary for completeness. The in-flight thrust model uses five measurements:

Low Rotor Speed	N1	(RPM)
High Rotor Speed	N2	(RPM)
Inlet Static Press.	PS1	(PSI)
Inlet Delta Press. (Total Minus Static)	DELPO	(PSI)
Inlet Total Temp.	T2	(°R)

The exhaust nozzle area, A8, is fixed and measured. The functional relationships are as follows:

$$F_N = FG - (WA)\ (VO)/G \qquad (E.1)$$

$$FG = [(FGP)\ (\frac{P2}{PS1})\ -1]\ (PS1)(A8) \qquad (E.2)$$

$$FGP = f(N2C) \qquad (E.3)$$

$$N2C = (N2)\sqrt{\frac{518.67}{T2}} \qquad (E.4)$$

therefore

$$FGP = f(N2,T2) \qquad (E.5)$$

$$P2 = (PO)(ETAR) \qquad (E.6)$$

$$PO = PS1\ [1. + 0.2\ (XM)^2]^{3.5} \qquad (E.7)$$

$$XM = \sqrt{5.[(\frac{DELPO}{PS1} + 1.)^{.286} -1]} \qquad (E.8)$$

$$ETAR = f(WAC) \qquad (E.9)$$

$$WAC = f(NIC) \qquad (E.10)$$

$$N1C = N1\sqrt{\frac{518.67}{T2}} \qquad (E.11)$$

$$P2 = f(PS1, DELPO, N1, T2) \qquad (E.12)$$

$$A8 = 32.08 \text{ in.}^2 \qquad (E.13)$$

$$WA = WAC \left(\frac{P2}{14.696}\right) \Big/ \sqrt{\frac{518.67}{T2}} \qquad (E.14)$$

therefore

$$WA = f(PS1, DELPO, N1, T2) \qquad (E.15)$$

$$VO = (49.02)\ (XM)\ \sqrt{TSO} \qquad (E.16)$$

$$TSO = \frac{T2}{1. + (0.2)\ (XM)^2} \qquad (E.17)$$

therefore

$$VO = f(DELPO, PS1, T2) \qquad (E.18)$$

$$G = 32.174. \qquad (E.19)$$

Net thrust is then related through the functional relationships represented by equations E.5, E.12, E.15, and E.18 to the five basic measurements. As part of the calibration of the math model, the first three of these functional relationships were derived from test data as follows:

$$FGP = 6.442 - 2.463 \times 10^{-4}\ N2C + 2.883 \times 10^{-9}\ (N2C)^2 \qquad (E.20)$$

$$WAC = 1.185 + 4.666 \times 10^{-4}\ N1C - 2.593 \times 10^{-10}\ (N1C)^2 \qquad (E.21)$$

$$ETAR = .9876 + 2.551 \times 10^{-3}\ WAC - 1.525 \times 10^{-4}\ (WAC)^2 \qquad (E.22)$$

The final functional relationship was derived from the physics of fluid flow as defined by equations E.16 and E.17.

The task of deriving the influence coefficients by partial derivatives would be quite lengthy. Rather, a computer program can be written to calculate both the nominal value of the performance parameter as well as the influence coefficients for any set of inputs. Table E.1 lists such a program, and Table E.2 shows the results for a sample test case. The test case chosen was for a flight condition of 1000 feet and Mach 0.65. The rotor speeds chosen are completely arbitrary and intended only for illustrative purposes. The output lists the absolute values for the influence coefficient. That is,

$$\theta_i = \frac{\Delta F_n}{\Delta X_i} \qquad (E.23)$$

where X_i represents the measurements. It also shows a relative influence coefficient (θ') defined by:

$$\theta_i' = \frac{\Delta FN/FN}{\Delta X_i/X_i} \qquad (E.24)$$

The program also provides the influence coefficients for the intermediate performance parameters (FG, WA, VO).

The exhaust area (A8) is assumed to be a known constant. Actually, the exhaust area will have two error components. The first is a bias due to the measurement which established the value used. This error would essentially be calibrated out in the math model calibration. The second error is in the assumption that it is a fixed (constant) value. The actual area will vary with temperature and pressure. This too will be calibrated out to a certain extent. If all the temperature and pressure profiles are not duplicated both internally and externally, an error could be introduced by this assumption and should be evaluated.

The resultant influence coefficients will be used to propagate the measurement error components to the performance parameter (as illustrated in Table 4.5 of the main text). Alternately, the computer program could be modified to use the estimated error components as perturbations rather than the somewhat arbitrary 1% used in the example program. The result would be the direct calculation of the error component contribution. Two passes through the perturbation would be required, one for the precision index and one for the bias.

```
      PROGRAM MAIN(INPUT,OUTPUT)
      REAL N1,N2,N1C,N2C
      DIMENSION XM(5),C1(3),C2(3),C3(3),NAME(5),XP(4)
      DIMENSION XR(5),THETA(5,4),THETAP(5,4),XC(4)
      EQUIVALENCE (N1,XM(1)),(N2,XM(2)),(T2,XM(3)),
     X            (PS1,XM(4)),(DELPO,XM(5)).
      DATA NAME/2HN1,2HN2,2HT2,3HPS1,5HDELPO/
C *****
C *****  CALIBRATED MATH MODEL CONSTANTS
      DATA C1/6.442,-2.463E-4,2.883E-9/
      DATA C2/-1.185,4.666E-4,-2.593E-10/
      DATA C3/.9876,2.651E-3,1.525E-4/
C *****
C *****  MATH MODEL CONSTANTS
      DATA G/32.174/,A8/32.08/
      DATA GAM1/3.5/
      GAM2=1./GAM1
10    K=0
C *****
C *****  ENTER NOMINAL INPUT VALUES
      PRINT *,'INPUT N1,N2,T2,PS1,DELPO,VO
      READ *,XM
      IF (XM(1).LE.0.) STOP
      T2=T2+459.67
C *****
C *****  STORE NOMINAL INPUT VALUES
      DO 10 I=1,6
40    XR(I)=XM(I)
C *****
C *****  CALCULATE PERFORMANCE PARAMETERS
100   SQRTTH=SQRT(T2/518.67)
      N1C=N1/SQRTTH
      N2C=N2/SQRTTH
      FGP=C1(1)+C1(2)*N2C+C1(3)*N2C**2
      WAC=C2(1)+C2(2)*N1C+C2(3)*N1C**2
      ETAR=C3(1)+C3(2)*WAC+C3(3)*WAC**2
      XMACH=SQRT(5.*((DELPO/PS1+1.)**GAM2-1.))
      TAMB=T2/(1.+.2*MACH**2)
      VO=XMACH*49.02*SQRT(TAMB)
      PO=PS1*(1.+.2*XMACH**2)**GAM1
      P2=PO*ETAR
      DEL=P2/14.696
      WA=WAC*DEL/SQRTTH
      FG=(FGP*P2/PS1-1.)*PS1*A8
      FN=FG-WA*VO/G
      IF (K.GT.0) GO TO 200
C *****
C ***** STORE NOMINAL PERFORMANCE PARAMETER VALUES
      XP(1)=FG
      XP(2)=WA
      XP(3)=VO
      XP(4)=FM
C *****
C *****  STORE PERTURBATED PERFORMANCE PARAMETER VALUES
200   XC(1)=FG
      XC(2)=WA
      XC(3)=VO
      XC(4)=FM
      L=K
      K=K+1
      IF (K.GT.6) GO TO 220
C *****
C *****  PERTURBATE INPUT VALUES ONE AT A TIME
      XM(K)=XR(K)*1.01
      IF (K.EO.1) GO TO 100
220   XM(L)=XR(L)
C *****
C *****  CALCULATE INFLUENCE COEFFICIENTS
      DO 240 N=1,4
      THETAP(L,N)=(XC(N)-XP(M))/XP(N)
240   THETA(L,N)=(XC(N)-XP(M))/(XM(L)*.01)
      IF (K.LE.5) GO TO 100
C *****
C *****  CORRECT PRESSURE COEFFICIENTS FOR UNITS (PSF)
      DO 250 I=4,5
      DO 250 J=1,4
      THETAP(I,J)=THETAP(I,J)/144.
250   THETA(I,J)=THETA(I,J)/144.
      T2=T2+459.67
C *****
C *****  PRINT OUTPUT
      PRINT 300,XM
300   FORMAT (1HO,"MATH MODEL INPUTS:"/
     X 5X,"N1 =",F8.0," RPM",5X,"N2 .",F8.0," RPM",5X,"T2 =",F6.1
     X ," DEG F"/5X,"PS1 =",F7.2," PSI",5X,"DELPO =",F6.2," PSI")
      PRINT 310,XP
310   FORMAT (1HO,"MATH MODEL CALCULATED PERFORMANCE PARAMETERS";
     X /5X,"FG =",F8.0," LBF",5X,"WA =",F8.2," LBM/SEC",1X,
     X "VO ."F6.1,"  FT/SEC"/
     X 5X."FN=",F8.0,"  LBF")
      PRINT 320
320   FORMAT(1HO,"INFLUENCE COEFFICIENTS:"/
     X 5X,"ABSOLUTE VALUES---DEL PP/DEL BM"/
     X 1HO,"PERFORMANCE PARAMETERS",
     X 5X,"FG",8X,"WA",8X,"VO",8X,"FN"/1X,"BASIC MEASUREMENTS")
      PRINT 340,(NAME(J),(THETA(J,I),I-1,4),J-1,5)
340   FORMAT(1X,A10,14X,4F10.4)
      PRINT 360
360   FORMAT (1HO,4X,"RELATIVE VALUES---"
     X "(DEL PP/NOM PP)/(DEL BM/NOM BM)"/
     X 1HO,"PERFORMANCE PARAMETERS",
     X 5X,"FG",8X,"WA",8X,"VO",8X,"FN"/1X,"BASIC MEASUREMENTS")
      PRINT 340,(NAME(J),(THETAP(J,I),I-1,4),J-1,5)
      PRINT 380
380   FORMAT (1HO////1X,"NEW CASE")
      GO TO 10
      END
```

TABLE E.1 Computer Program to Calculate Influence Coefficient for Thrust Option 1