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HANDBOOK OF HYDRAULIC METRIC CALCULATIONS

FOREWORD

Hydraulic calculations should be done directly in the International System (SI) metric units. Calculations in SI metric units could not be simpler. Data, however, are not always in these units, and conversions are necessary. Numerical examples are needed to follow as a guide for avoiding confusion.

Revision A enlarges the scope to include thermal effects and updated the hydraulic fluid data.

In this document, terminology is defined in each section where required. This is done to avoid any misunderstanding of dual uses of symbols such as N = Newton or N = rpm.

TABLE OF CONTENTS

1. SCOPE:

Metric (SI and CGS) and English units related to aerospace hydraulics are summarized. Conversion of units is provided as required. Fundamental fluid properties and physical laws governing fluid motion, pressure and other significant aspects are described in SI metric units. Examples of application to typical aerospace hydraulic system components are demonstrated.

2. REFERENCES:

2.1 SAE Publications:

Available from SAE, 400 Commonwealth Drive, Warrendale, PA 15096-0001.

AS1241 Fire Resistant Phosphate Ester Fluid for Aircraft
AIR1362 Physical Properties of Hydraulic Fluids
TSB 003 Rules for SAE Use of SI (Metric) Units

2.2 ASTM Publications:

Available from ASTM, 1916 Race Street, Philadelphia, PA 19103-1187.

ASTM E 380-79 Standard for Metric Practice

2.3 NAS Publications:

Available from Aerospace Industries Association, 1250 Eye Street NW, Washington, DC 20035.

NAS 10001 Preferred Metric Units for Aerospace

2.4 NFPA Publications:

Available from National Fluid Power Association, Inc., 3333 N. Mayfair Road, Milwaukee, WI 53222-3219.

NFPA/T2.10.1 Metric Units for Fluid Power Applications

2.5 ISO Publications:

Available from ANSI, 11 West 42nd Street, New York, NY 10036-8002.

ISO/R 31 Basic Quantities and Units of SI
ISO 10090 SI Units and Recommendations for Use of Multiples

2.6 Other Publications:

2.6.1 Lubrication Engineering, Vol. 44, 4, 324-329, May 1987, Pump Evaluation of Polyalphaolefin Candidates for a -54 C to 135 C Fire-Resistant Aircraft Hydraulic Fluid, L.J. Gschwender, C.E. Snyder & S.K. Sharma

2.6.2 Pennsylvania State University, Effect of Temperature on Bulk Modulus, AMCL 706-123, April 1971

2.7 U.S. Government Publications:

Available from DODSSP, Subscription Services Desk, Building 4D, 700 Robbins Avenue, Philadelphia, PA 19111-5094.

MIL-H-5606 Hydraulic Fluid, Petroleum Base; Aircraft, Missile and Ordinance
MIL-H-6083 Hydraulic Fluid, Petroleum Base, for Preservation and Testing
MIL-H-46170 Hydraulic Fluid, Rust Inhibited, Fire Resistant, Synthetic Hydrocarbon Base, NATO Code H-515
MIL-H-53119 Hydraulic Fluid, Non Flammable, Chloro-trifluoro-ethylene Base
MIL-H-83282 Hydraulic Fluid, Fire Resistant, Synthetic Hydrocarbon Base, Aircraft, NATO Code H-537
MIL-H-87257 Hydraulic Fluid, Fire Resistant, Low Temperature, Synthetic Hydrocarbon Base, Aircraft and Missile

3. TECHNICAL REQUIREMENTS:

3.1 Metric Units:

The SI system is coherent because force, pressure, and power are expressed in terms of basic quantities of mass, length, and time without empirical constants. Kilogram (kg), meter (m), and second (s) are the basic units. Unity (1) is the coefficient of the basic units in their definition. Multiples of the basic units are defined by powers of 10^3 (see Table 1).

TABLE 1 - Recommended Multiples of SI Units

10^9	1 000 000 000	giga	G
10^6	1 000 000	mega	M
10^3	1 000	kilo	k
1	1	basic unit	
10^{-3}	0.001	milli	m
10^{-6}	0.000 001	micro	μ
10^{-9}	0.000 000 001	nano	n

3.1 (Continued):

These multiples provide usefulness and convenience for practical purposes. Multiples other than powers of 10^3 are obsolete (see Table 2).

TABLE 2 - Not Recommended Multiples of Units

10^2	100	hecto	h
10^1	10	deka	da
1	1	basic unit	
10^{-1}	0.1	deci	d
10^{-2}	0.01	centi	c

The SI metric system was established by the International Organization for Standardization (ISO) and supersedes the CGS metric system, where centimeter (cm), and gram (g) and second (s) were the basic units, and the earlier metric system where kilogram force was the basic unit.

Table 3 provides conversion of metric units. NAS 10001, NFPA/T2.10.1 and ANSI/ASTM E 380, and other publications are available for conversion of metric and English units.

Table 4 refers to English units, which may be useful during the transition period.

3.2 Hydrostatics:

3.2.1 Static Head:

3.2.1.1 Primary Units: The hydrostatic pressure (P) is by definition:

$$P = \rho\ g\ h \quad \text{(Eq.1)}$$
$$\text{(i.e., } ML^{-1}T^{-2} = ML^{-3} \times LT^{-2} \times L\text{)}$$

where:

P = pressure (Pa) - ($ML^{-1}T^{-2}$)
ρ = mass density (kg/m^3) - (ML^{-3})
g = gravitational acceleration (m/s^2) - (LT^{-2})
g = 9.81 m/s^2
h = head (m) - (L)

3.2.1.2 Practical Units: By the definition of liter (L):

$$1\ L = 1\ m^3/1000 \quad \text{(Eq.2)}$$

Therefore:

$$\rho = \text{mass density (kg/L)} \quad \text{(Eq.3)}$$
$$1\ kg/L = 1000\ kg/m^3$$

TABLE 3 - Metric Units

Physical Quantity	Symbol	Dimension	SI Metric Unit	Other Metric Unit	Conversion to SI Units
Mass	M	M	kg, kilogram	g, gram	10^{-3} kg
Length	l	L	m, meter	cm, centimeter	10^{-2} m
Time	t	T	s, second	minute hour	60 s 3600 s
Angle	α	-	rad, radian	rev, revolution	2π rad
Absolute Temperature	T	-	K, kelvin		
Temperature	T	-	C, celsius	centigrade	K - 273.2
Frequency	f	T^{-1}	Hz, hertz	cps, cycle/s	1 Hz
Area	A	L^2	m^2	mm^2 cm^2	10^{-6} m^2 10^{-4} m^2
Volume	V	L^3	m^3	L, liter mL cm^3, cc mm^3, μL	10^{-3} m^3 10^{-6} m^3 10^{-6} m^3 10^{-9} m^3
Density	ρ	ML^{-3}	kg/m^3	kg/L g/cm^3	10^{-3} kg/m^3 10^{-3} kg/m^3
Moment of Inertia	J	ML^2	kg m^2		
Velocity	v	LT^{-1}	m/s	km/s km/h	10^3 m/s 10^3/3600 m/s
Acceleration	a	LT^{-2}	m/s^2	gN	9.81 m/s^2
Force	F	MLT^{-2}	N, Newton	daN, deka N kg_f (force) kN, kilo N dyne (CGS)	10 N 9.81 N 10^3 N 10^{-5} N
Work, Energy, Heat	F x l	ML^2T^{-2}	J, joule	W h kW h N.m	3600 J 3600 kJ 1 J
Pressure	P	$ML^{-1}T^{-2}$	Pa, pascal	N/m^2 kPa Mpa bar mm of Hg	1 Pa 10^3 Pa 10^6 Pa 10^5 Pa 133.3 Pa
Flow, Volumetric	Q_v	L^3T^{-1}	m^3/s	L/s L/min	10^{-3} m^3/s 10^{-3}/60 m^3/s
Torque	T	ML^2T^{-2}	Nm, Newton-meter	kN m	10^3 Nm
Speed of Rotation	ω	T^{-1}	rad/s	rpm, rev/minute	2π/60 rad/s
Power, Heat Flow	W Q_h	ML^2T^{-3}	W, watt	kW mW	10^3 W 10^{-3} W
Spring Rate	F/l	MT^{-2}	N/m	kN/m kN/mm	10^3 N/m 10^6 N/m

TABLE 3 (Continued)

Physical Quantity	Symbol	Dimension	SI Metric Unit	Other Metric Unit	Conversion to SI Units
Surface Tension	γ	MT^{-2}	N/m	mN/m dyne/cm	10^{-3} N/m 10^{-3} N/m
Bulk Modulus	B	$ML^{-1}T^{-2}$	Pa, pascal	kPa MPa GPa	10^{3} Pa 10^{6} Pa 10^{9} Pa
Dynamic Viscosity	m	$ML^{-1}T^{-1}$	Pa s	mPa s centipoise	10^{-3} Pa s 10^{-3} Pa s
Kinematic Viscosity	n	$L^{2}T^{-1}$	m^2/s	mm^2/s centistoke	10^{-6} m^2/s 10^{-6} m^2/s
Specific Heat	Cp	$L^{2}T^{-2}$	J/(kg K)	kJ/(kg K)	10^{3} J/(kg K)
Thermal Conductivity	k	MLT^{-3}	W/(m K)		
Surface Conductance	h_c	MT^{-3}	W/(m^2 K)	kW/(m^2 K)	10^{3} W/(m^2 K)
Heat Capacity Rate	C	$ML^{2}T^{-3}$	W/K watt/K	kW/K mW/K	10^{3} W/K 10^{-3} W/K

TABLE 4 - English Units

Physical Quantity	Symbol	Dimension	English Unit	SI Metric Unit	English Conversion to Metric
Mass	M	M	lb_m slug	kg kg	0.4536 kg 14.59 kg
Length	l	L	ft, foot in, inch μ, micron μin, microinch	m mm μm μm	0.3048 m 25.4 mm 1 μm 25.4 μm
Time	t	T	s, second minute h, hour	s	1 s 60 s 3600 s
Angle	α	-	degree of arc	rad, radian	π/180 rad
Absolute Temperature	T	-	R, rankine	K, kelvin	K/1.8
Temperature	T	-	F, fahrenheit	C, celsius	C/1.8 +32
Frequency	f	T^{-1}	cps, cycle/s	Hz, hertz	1 Hz
Area	A	L^2	ft^2 ft^2 in^2 cm^2	m^2 cm^2 mm^2 mm^2	0.0929 m^2 929 cm^2 654 mm^2 100 mm^2
Volume	V	L^3	ft^3 U.S. gallon Imperial gallon in^3 drop	L, liter L L mL mm^3	28.32 L 3.785 L 4.546 L 16.39 mL 50 mm^3
Density	p	ML^{-3}	lb/gal lb/ft^3	kg/L kg/m^3	0.1198 kg/L 16.02 kg/m^3
Moment of Inertia	J	ML^2	lb ft^2 lb-in^2 lb-in-s^2	kg m^2 kg mm^2 kg in^2	0.04214 kg m^2 292.6 kg mm^2 0.113 kg m^2
Velocity	v	LT^{-1}	ft/s in/s mile/hour knot	m/s mm/s km/h m/s	0.3048 m/s 25.4 m/s 1.609 km/h 0.5144 m/s
Acceleration	a	LT^{-2}	ft/s^2 gN	m/s^2 ft/s^2	0.3048 m/s^2 32.17 ft/s^2
Force	F	MLT^{-2}	lb_f kip	N, Newton kN	4.448 N 4.448 kN
Work, Energy, Heat	F x l	ML^2T^{-2}	ft lb BTU	J kJ	1.356 J 1.055 kJ
Pressure	P	$ML^{-1}T^{-2}$	lb/in^2, psi ksi in of Hg	kPa MPa kPa	6.895 kPa 6.895 MPa 3.386 kPa
Flow, Volumetric	Q_v	L^3T^{-1}	gal/min, gpm in^3/s, cis	L/min L/min	3.785 L/min 983.4 mL/min
Torque	T	ML^2T^{-2}	lb ft lb in	N m N m	1.356 N m 0.1130 N m

TABLE 4 (Continued)

Physical Quantity	Symbol	Dimension	English Unit	SI Metric Unit	English Conversion to Metric
Speed of Rotation	ω	T^{-1}	rpm, rev/minute	rad/s	2π/60 rad/s
Power, Heat Flow	W Q_h	ML^2T^{-3}	ft lb/s hp kBTU/h	W kW kW	1.356 W 0.7457 kW 0.293 kW
Spring Rate	F/l	MT^{-2}	lb_f/in kip/in	N/m kN/m	175.1 N/m 175.1 kN/m
Surface Tension	γ	MT^{-2}	lb_f/in $grain_f$/in $grain_f$/in	N/m mN/m dyne/cm	175.1 N/m 25 mN/m 25 dyne/cm
Bulk Modulus	B	$ML^{-1}T^{-2}$	kip/in^2, ksi	MPa	6.895 Mpa
Dynamic Viscosity	m	$ML^{-1}T^{-1}$	lb_f s/ft^2 10^{-3} lb s/ft^2	Pa s mPa s, centipose	47.88 Pa s 47.88 mPa s 47.88 cpoise
Kinematic Viscosity	n	L^2T^{-1}	ft^2/s ft^2/s	m^2/s mm^2/s	0.929 10^{-3} m^2/s 929 mm^2/s
Specific Heat	Cp	L^2T^{-2}	BTU/(lb F)	kJ/(kg K)	4.186 kJ/(kg K)
Thermal Conductivity	k	MLT^{-3}	BTU/(h ft F)	W/(m K)	1.731 W/(m K)
Surface Conductance	h_c	MT^{-3}	BTU/(h ft^2 F)	kW/(m^2 K)	5.679 W/(m^2 K)
Heat Capacity Rate	C	ML^2T^{-3}	BTU/(h F)	W/K	0.527 W/K

3.2.1.2 (Continued):

Compare the specific gravity and the density of water:

$$\text{specific gravity} = 1 \quad \text{(Eq.4)}$$
$$1\ \text{kg/L} = 1\ \text{g/cm}^3$$

The hydrostatic pressure in practical units is:

$$P = \rho\ g\ h \quad \text{(Eq.5)}$$

where:

P = pressure (kPa) - (1 kPa = 1000 Pa)
ρ = mass density (kg/L) - (1 kg/L = 1000 kg/m^3)
g = gravitational acceleration (m/s^2)
g = 9.81 m/s^2 at sea level, 45 deg latitude
h = head (m)

3.2.1.3 Application - Suction Pressure:

Given: MIL-H-5606 hydraulic fluid at 20 °C.
The hydraulic reservoir is at 2.27 m above baseline.
An actuator is at 5.32 m above baseline.

The hydraulic reservoir open to sea level atmosphere during maintenance. Calculate:

a. The suction the actuator seals must be able to withstand without air ingestion.

b. The absolute pressure at the actuator.

Density of MIL-H-5606 at 20 °C and 0 MPa per Figure 1 is:

$$\rho = 0.859\ \text{kg/L} \quad \text{(Eq.6)}$$

The difference in elevation is:

$$h = 5.32 - 2.27 = 3.05\ \text{m} \quad \text{(Eq.7)}$$

The suction pressure (below atmospheric) in kilopascals (kPa):

$$P = 0.859 \times 9.81 \times 3.05 \quad \text{(Eq.8)}$$
$$P = 25.7\ \text{kPa}$$

The standard sea level atmospheric pressure is 101.3 kPa per ANSI/ASTM 380.

The absolute pressure at the actuator is:

$$101.3 - 25.7 = 75.6\ \text{kPa} \quad \text{(Eq.9)}$$

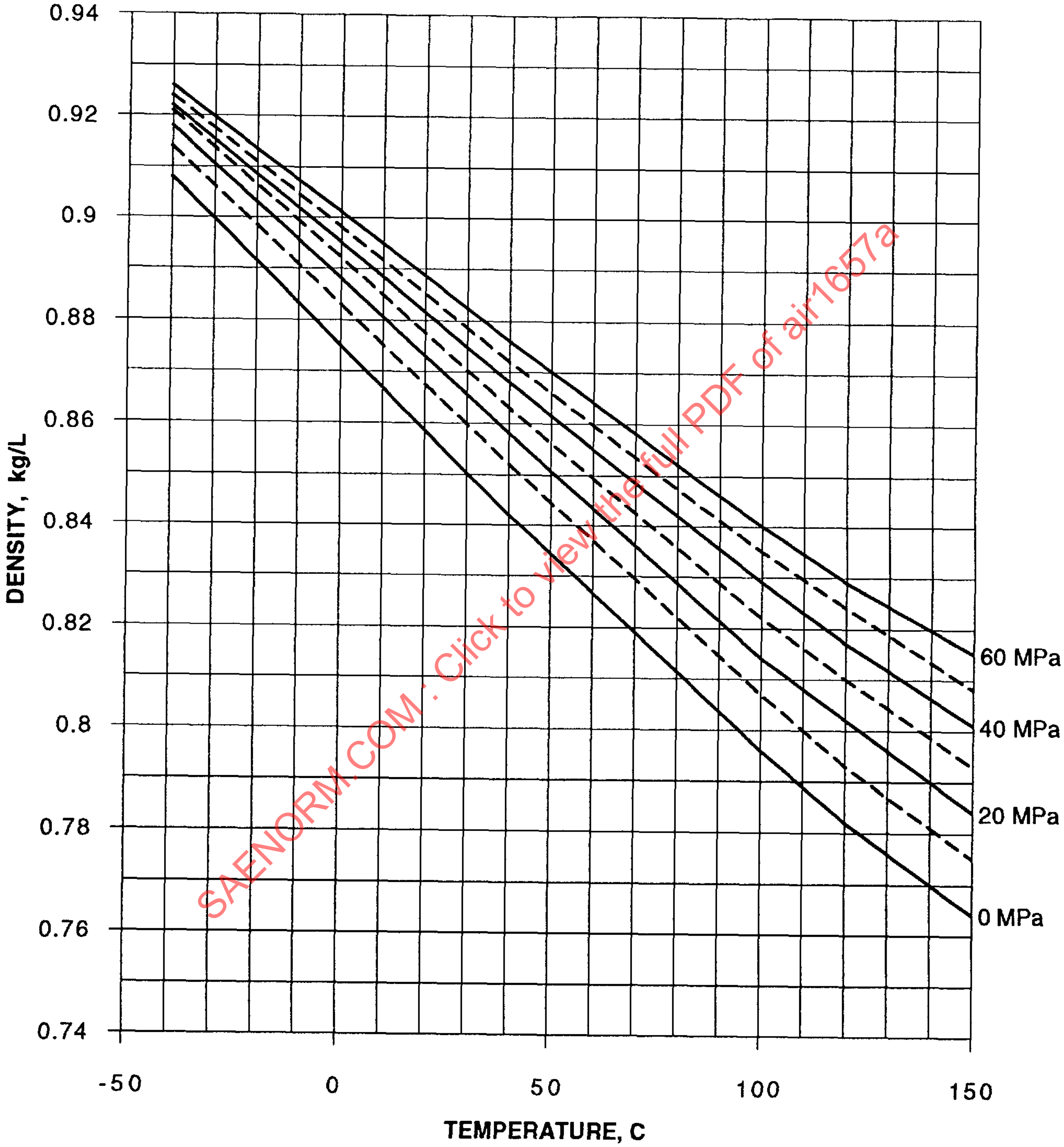

FIGURE 1 - Density Versus Temperature, MIL-H-5606

3.2.2 Axial Force:

3.2.2.1 Primary Units: The axial force is:

$$F = A\ P \qquad \text{(Eq.10)}$$
$$(MLT^{-2} = L^2 \times ML^{-1}T^{-2})$$

where:

F = force (Newton, N) - (MLT^{-2})
A = area (m^2) - (L^2)
P = pressure (pascal, Pa) - ($ML^{-1}T^{-2}$)

3.2.2.2 Practical Units: The axial force is:

$$F = A\ P \qquad \text{(Eq.11)}$$

where:

F = force (Newton, N)
A = area (mm^2) - (1 mm^2 = 10^{-6} m^2)
P = pressure (megapascal, MPa) - (1 MPa = 10^6 Pa)

3.2.2.3 Application - Hydrostatic Test: A hydraulic cylinder of 50.8 mm inside diameter is proof pressure tested to 1.5 factor of rated at 20.7 MPa pressure.

Calculate the axial force on the cylinder cap.

The test pressure is:

$$P = 1.5 \times 20.7\ \text{MPa} \qquad \text{(Eq.12)}$$
$$P = 31\ \text{MPa}$$

The cap area is:

$$A = \pi\ (50.8)^2/4 \qquad \text{(Eq.13)}$$
$$A = 2\ 027\ \text{mm}^2$$

The force is:

$$F = 31\ \text{E+6 Pa} \times 2\ 027\ \text{E-6 m}^2 \qquad \text{(Eq.14)}$$
$$F = 62\ 900\ \text{N}$$
$$F = 62.9\ \text{kN}$$

3.3 Hydrodynamics:

3.3.1 Bernoulli's Theorem: The total head in steady state flow of an incompressible fluid is:

$$H = h + v^2/2g + z \qquad \text{(Eq.15)}$$
$$(L = L + (LT^{-1})^2/LT^{-2} + L)$$

where:

h = static head (m) - (L)
v = velocity of low (m/s) - (LT^{-1})
g = gravitational acceleration (m/s^2) - (LT^{-2})
z = elevation of section above datum (m) - (L)

Bernoulli's theorem states that total head (H) along a streamline is constant, provided that the head losses (DH) between sections are accounted for:

$$h_1 + v_1^2/2g + z_1 = h_2 + v_2^2/2g + z_2 + DH \qquad \text{(Eq.16)}$$
$$H_1 = H_2 + DH$$

where:

subscripts 1 and 2 refer to sections
DH = loss in total head, (m)

3.3.1.1 Velocity Head: By definition, the velocity head is:

$$h_v = v^2/2g \qquad \text{(Eq.17)}$$
$$(L = (LT^{-1})^2/LT^{-2})$$

3.3.1.2 Application - Liquid Column Head: MIL-H-5606 fluid at 50 °C and 20 MPa pressure is flowing in a tube at 3 m/s velocity. What is the velocity head in terms of:

a. Liquid column height
b. Pressure?

In terms of liquid column height:

$$h_v = v^2/2g \qquad \text{(Eq.18)}$$

where:

v = 3.0 m/s
g = 9.81 m/s^2

Then:

$$h_v = 3.0^2/(2 \times 9.81) \qquad \text{(Eq.19)}$$
$$h_v = 0.46 \text{ m}$$

3.3.1.2 (Continued):

In terms of pressure head (dynamic pressure):

$$P_v = \rho\ g\ h_v - (ML^{-1}T^{-2} = ML^{-3}\ LT^{-2}\ L) \quad \text{(Eq.20)}$$
$$P_v = (\rho\ g)\ x\ (v^2/2g)$$
$$P_v = \rho\ v^2/2 - (ML^{-1}T^{-2} = ML^{-3}\ (LT^{-1})^2)$$

where:

ρ = 0.85 kg/L - (ML^{-3})
v = 3 m/s - (LT^{-1})

Then:

$$P_v = 0.85\ x\ 3^2/2 \quad \text{(Eq.21)}$$
$$P_v = 3.83\ \text{kPa}$$

3.3.2 Viscosity:

3.3.2.1 Dynamic (Absolute) Viscosity: It is by definition, the shear force per unit area due to a unit velocity gradient normal to the flow:

$$\mu = F/A/(dV/dy) \quad \text{(Eq.22)}$$
$$(ML^{-1}T^{-1} = MLT^{-2}/L^2/(LT^{-1}/L))$$

In primary units:

μ = dynamic viscosity, Pa . s - ($ML^{-1}T^{-1}$)
F = force, Newton - (MLT^{-2})
A = area, m^2 - (L^2)
dV = variation of velocity, m/s - (LT^{-1})
dy = normal distance, m - (L)

In practical units:

m = dynamic viscosity, mPa . s - (E-3 Pa . s $ML^{-1}T^{-1}$)
F = force, Newton - (MLT^{-2})
A = area, mm^2 - (E-6 m2 L^{2})
dV = variation of velocity, m/s - (LT^{-1})
dy = normal distance, mm - (E-3 L L)

Refer to Table 3 where 1 mPa . s = 1 cst.

3.3.2.2 Kinematic Viscosity: It is by definition, the shear force per unit area and per unit mass density due to a unit velocity gradient normal to the flow:

$$\nu = F/(A\ r)/(dV/dy) \quad \text{(Eq.23)}$$
$$(L^2T^{-1} = MLT^{-2}/(L^2\ x\ /(LT^{-1}/L))$$

3.3.2.2 (Continued):

That is:

$$\nu = \mu/\rho$$
$$(L^2T^{-1} = ML^{-1}T^{-1}/ML^{-3})$$ (Eq.24)

In basic units:

ν = kinematic viscosity, $m^2 . s$ - (L^2T^{-1})
F = force, Newton - (MLT^{-2})
A = area, m^2 - (L^2)
ρ = mass density, kg/m^3 - (ML^{-3})
dV = variation of velocity, m/s - (LT^{-1})
dy = normal distance, m - (L)

In practical units:

ν = kinematic viscosity, $mm^2 . s$ - (E-6 $m^2 . s$ L^2T^{-1})
F = force, Newton - (MLT^{-2})
A = area, mm^2 - (E-6 m^2 L^2)
ρ = mass density, kg/liter - (E-3 kg/m^3 ML^{-3})
dV = variation of velocity, m/s - (LT^{-1})
dy = normal distance, mm - (E-3 L L)

Refer to Table 3 where 1 mm^2/s = 1 centipoise.

3.3.2.3 Example: Calculate the absolute viscosity (μ) of MIL-H-83282 hydraulic fluid at -30 °C and 10 MPa by solving Equation 24:

$$\mu = \nu \ \rho$$ (Eq.25)

Viscosity ν = 1400 mm^2/s (centistoke) obtained from Figures 2 and 3
Density ρ = 0.88 kg/L obtained from Figure 4

In primary units:

ν = 1400 m^2 E-6 x 880 kg/m^3 = 1.23 Pa . s

In practical units:

ν = 1400 mm^2/s x 0.88 kg/L = 1230 mPa

In the usual (CGS) units:

ν = 1400 cst x 0.88 g/cc = 1230 centipoise
μ = 1230 mPa

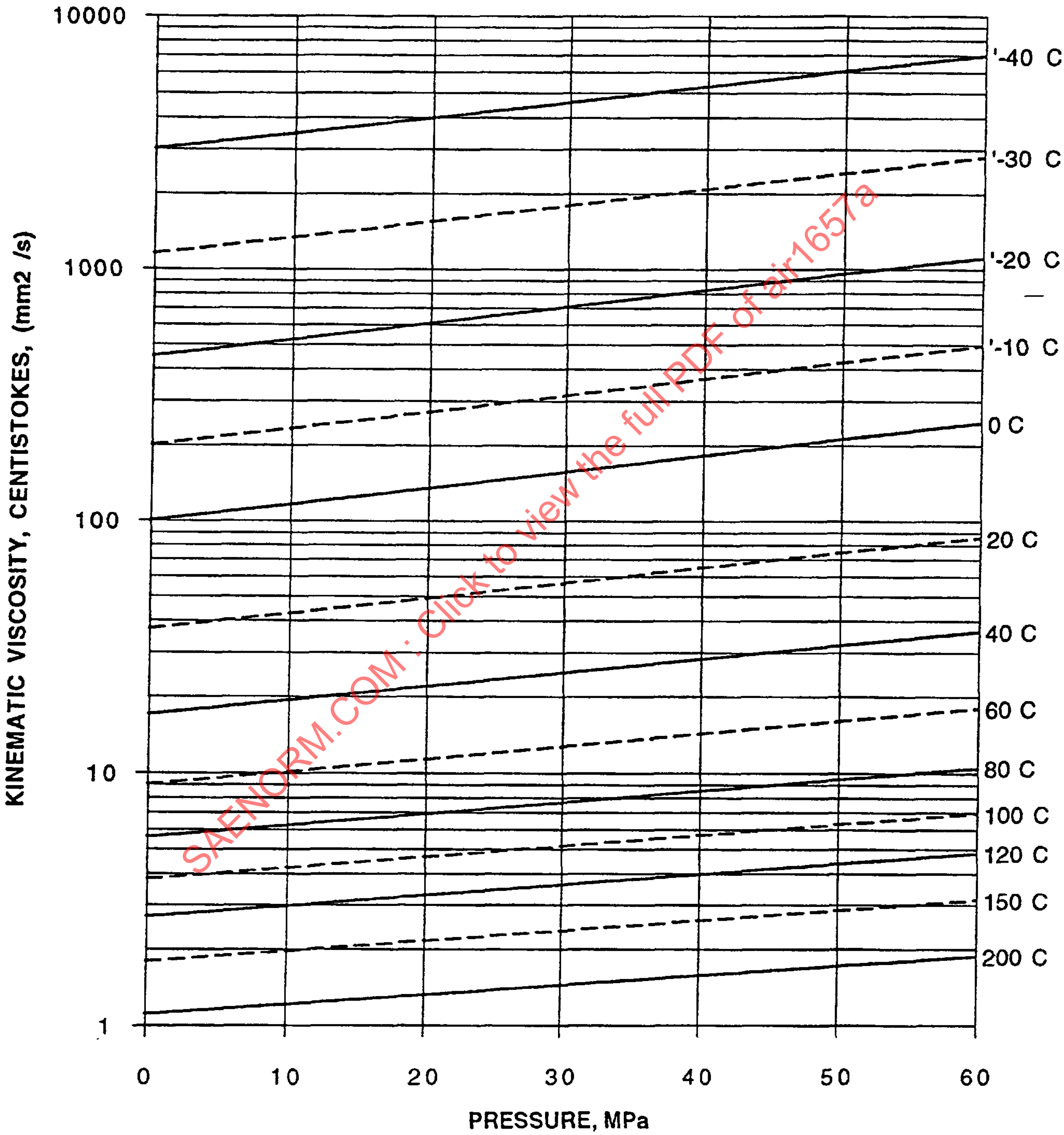

FIGURE 2 - Viscosity Versus Pressure, MIL-H-83282

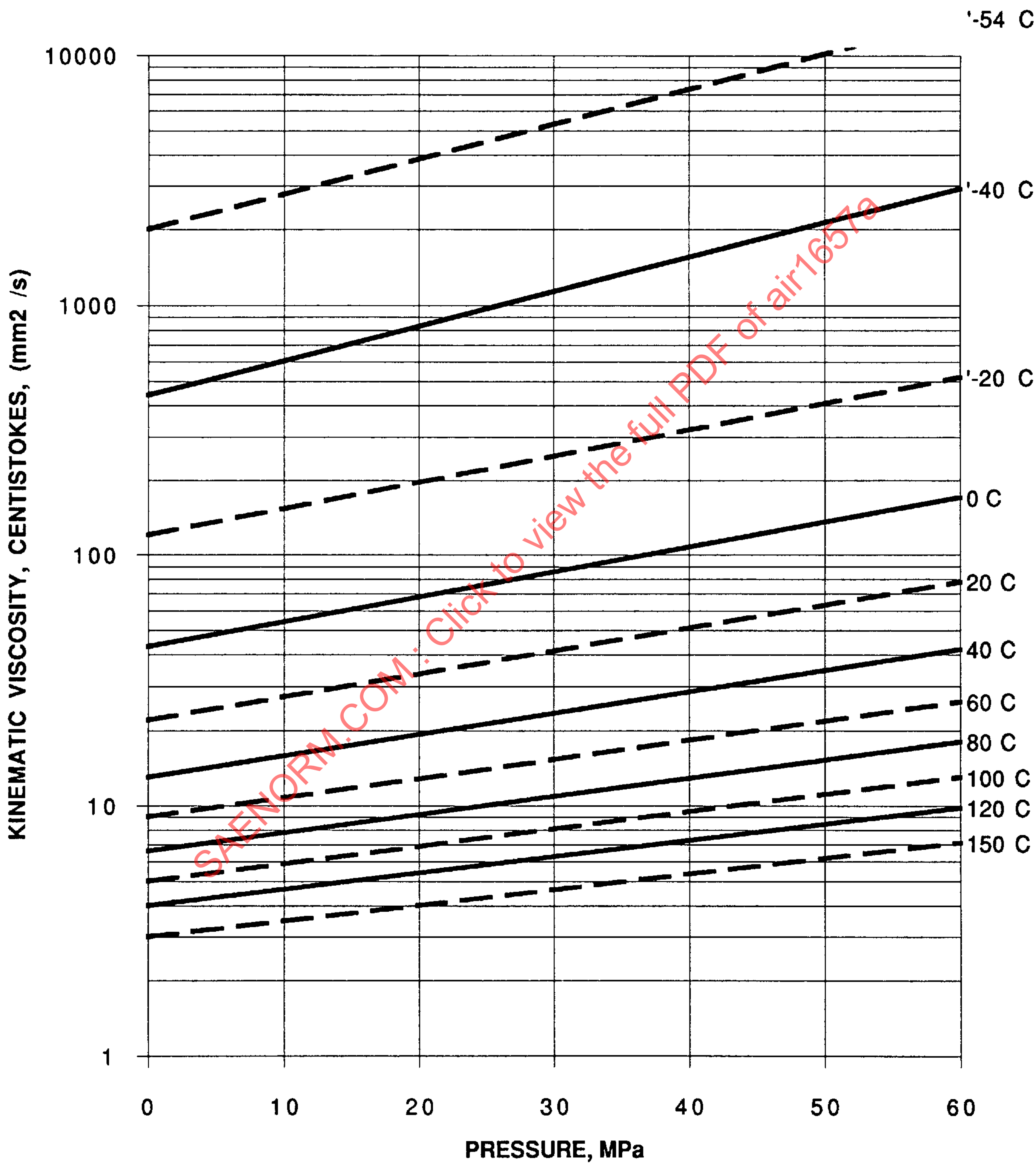

FIGURE 3 - Viscosity Versus Pressure, MIL-H-5606

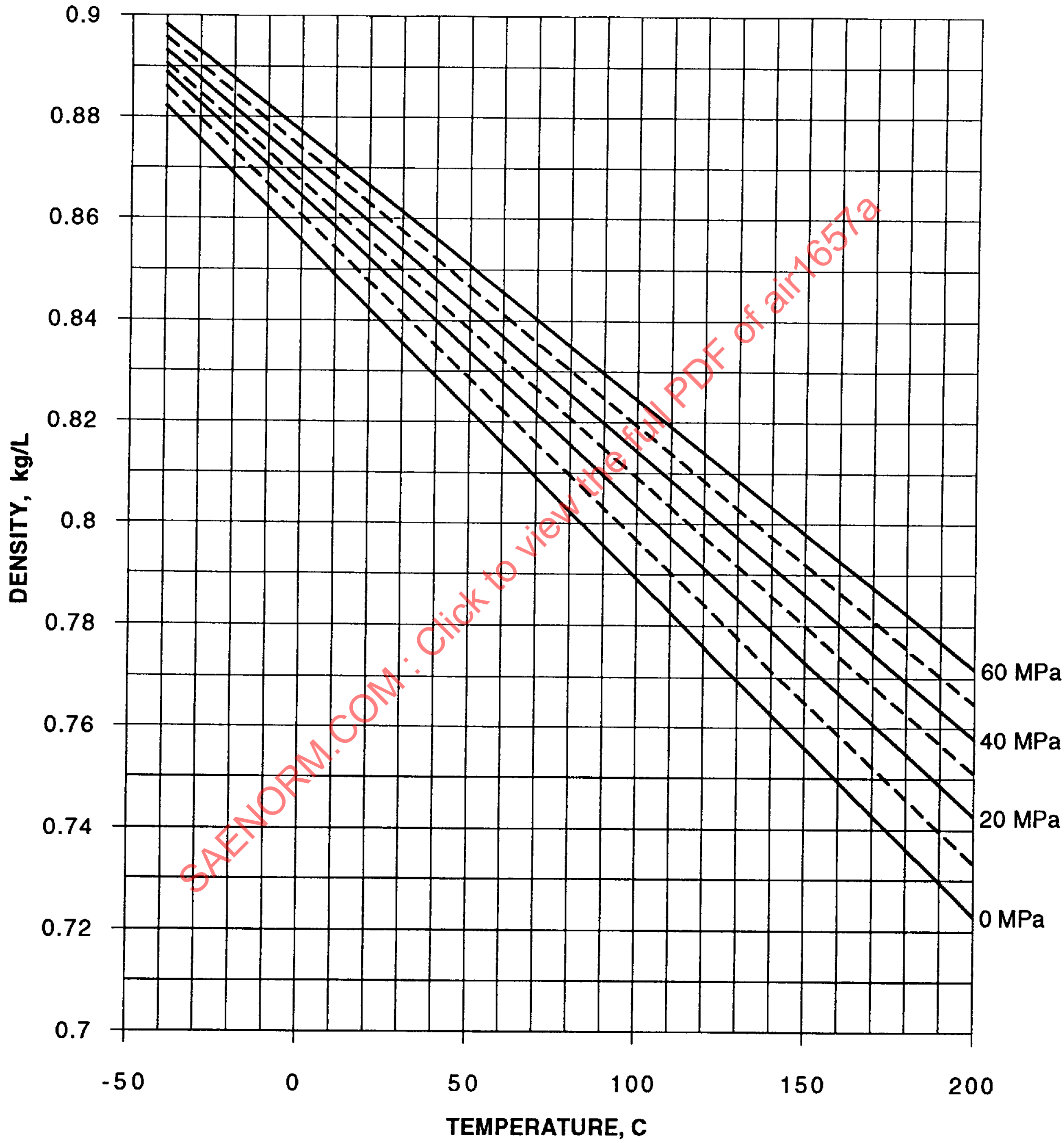

FIGURE 4 - Density Versus Temperature, MIL-H-83282

3.3.2.4 Application - Direct Drive Valve: Calculate the fluid shear stress (τ) between a valve slide and sleeve.

The slide is stroked 0.6 mm in 8 ms.
The diametral clearance is 3.8 μm (150 μin).
The slide is assumed to be centered.

Calculate the velocity (v) and the distance (δ) between valve slide and sleeve:

$$v = 0.6\ E\text{-}3\ m/8\ E\text{-}3\ s = 0.075\ m/s = 75\ mm/s \qquad (Eq.26)$$
$$\delta = 3.8\ \mu m/2 = 1.9\ \mu m$$

Use Equation 27:

$$\tau = \mu\ v/\delta \qquad (Eq.27)$$
$$(ML^{-1}T^{-2} = ML^{-1}T^{-1} \times LT^{-1}/L)$$

where:

τ = shear stress, (Pa) - ($ML^{-1}T^{-2}$)
μ = absolute viscosity (528 mPa . s) - ($ML^{-1}T^{-1}$)
v = velocity (60 mm/s) - (LT^{-1})
δ = radial clearance (1.9 μm) - (L)

In primary units:

τ = 1.23 Pa . s x 0.075 m/s/1.9 E-6 m = 48.6 E+3 Pa

In practical units:

τ = 1230 mPa . s x 0.075 m/s/1.9 μm = 48.6 kPa
τ = 48.6 kPa

Calculate the axial force and the power required to overcome viscous shear of a valve slide of 6 mm diameter by 20 mm total combined lap length.

Calculate the shear area:

$$A = \pi \times 6\ mm \times 20\ mm = 377\ mm^2 \qquad (Eq.28)$$

The axial force (F) is:

$$F = \tau\ A \qquad (Eq.29)$$
$$(MLT^{-2} = ML^{-1}T^{-2} \times L^2)$$

3.3.2.4 (Continued):

where:

τ = 48.6 kPa - $(ML^{-1}T^{-2})$
A = 377 mm^2 - (L^2)
F = 48.6 E+3 Pa x 377 E-6 m^2 = 18.3 N

The power (W) is:

$$W = F\ v \qquad (Eq.30)$$
$$(ML^2T^{-3} = MLT^{-2} \times LT^{-1})$$

where:

F = 18.3 N - (MLT^{-2})
v = 0.075 m/s - (LT^{-1})
W = 18.3 N x 0.075 m/s
W = 1.37 W

3.3.3 Reynolds Number: A dimensionless parameter characterizing laminar or turbulent flow:

$$Re = \rho\ v\ D/\mu \qquad (Eq.31)$$
$$(ML^{-3} \times LT^{-1} \times L/ML^{-1}T^{-1})$$

where:

ρ = mass density (kg/m^3) - (ML^{-3})
v = velocity of flow (m/s) - (LT^{-1})
D = inside diameter of tubing (m) - (L)
μ = dynamic viscosity (Pa . s) - $(ML^{-1}T^{-1})$

Using the kinematic viscosity ν, Reynolds Number is simply:

$$Re = v\ D/\nu \qquad (Eq.32)$$
$$(LT^{-1} \times L/L^2T^{-1})$$

where:

v = velocity of flow (m/s) - (LT^{-1})
D = inside diameter of tubing (m) - (L)
ν = kinematic viscosity (m^2/s) - (L^2T^{-1})

Flow is generally laminar if Re is less than 2000.

Flow is always turbulent if Re is more than 4000.

Upstream conditions determine the flow in between. Conservative design assumes the highest pressure drop condition, with transition at Re = 1190.

3.3.3.1 Practical Units: The tubing inside diameter is usually given in millimeters (mm).

The kinematic viscosity is usually given in centistokes.

According to Table 3:

$$1 \text{ cst} = 10^{-6} \text{ m}^2/\text{s} \qquad \text{(Eq.33)}$$
$$1 \text{ cst} = 1 \text{ mm}^2/\text{s}$$

Let the velocity also be in millimeter per second (mm/s) units for consistency.

Then:

$$Re = v\ D/\nu \qquad \text{(Eq.34)}$$

If the velocity is in m/s, instead of mm/s, then:

$$Re = 1000\ v\ D/\nu \qquad \text{(Eq.35)}$$

where:

v = velocity (m/s)
D = diameter (mm)
ν = centistoke (mm^2/s)

3.3.3.2 Application - Laminar, Turbulent Flow: A pump has an 18.85 mL/rev displacement and rotates at 6000 rpm speed. The pump is supplied with MIL-H-5606 fluid through a suction line with 31.75 mm OD and 0.89 mm wall thickness.

What is the Reynolds number at -40 and at 100 °C?

When is the flow laminar or turbulent?

The pump suction flow rate is:

$$Q = 18.85 \text{ mL/rev} \times 6000 \text{ rpm}/60 \text{ s/min} \qquad \text{(Eq.36)}$$
$$Q = 1885 \text{ mL/s}$$
$$Q = 1885 \text{ mL/s} \times (60 \text{ s/min})/(1000 \text{ mL/L})$$
$$Q = 113 \text{ L/min}$$

The inside diameter of the tube is:

$$D = OD - 2t \qquad \text{(Eq.37)}$$
$$D = 31.75 \text{ mm} - 1.78 \text{ mm}$$
$$D = 30.0 \text{ mm}$$

3.3.3.2 (Continued):

The open cross sectional area is:

$$A = \pi/4 \times (D)^{0.5} \qquad \text{(Eq.38)}$$
$$A = 705.5 \text{ mm}^2$$

The velocity of flow is:

$$v = Q/A \qquad \text{(Eq.39)}$$

where all the data are in consistent mm units:

v = velocity (mm/s)
Q = flow (mm^3/s)
A = mm^2

Remarking that:

$$1 \text{ mL} = 10^3 \text{ mm}^3 \qquad \text{(Eq.40)}$$

Then v in mm/s units is:

$$v = Q/A \times 10^3 \qquad \text{(Eq.41)}$$
v = velocity (mm/s)
Q = flow (mL/s)
A = area (mm^2)

Then:

$$v = 1\,885 \text{ mL/s}/705.5 \text{ mm}^2 \times 10^3 \qquad \text{(Eq.42)}$$
$$v = 2672 \text{ mm/s}$$
$$v = 2672 \text{ mm/s}/(1000 \text{ mm/m})$$
$$v = 2.67 \text{ m/s}$$

The kinematic viscosity of MIL-H-5606 fluid, is read from Figure 5:

$$\nu = 440 \text{ mm}^2/\text{s at } -40 \text{ °C} \qquad \text{(Eq.43)}$$
$$\nu = 4.9 \text{ mm}^2/\text{s at } 100 \text{ °C}$$

At -40 °C:

$$Re = 2672 \text{ mm/s} \times 30 \text{ mm}/440 \text{ mm}^2/\text{s} \qquad \text{(Eq.44)}$$
Re = 182, and the flow is laminar

At 100 °C:

$$Re = 2672 \text{ mm/s} \times 30 \text{ mm}/4.9 \text{ mm}^2/\text{s} \qquad \text{(Eq.45)}$$
$Re = 16.3 \times 10^3$, and the flow is turbulent

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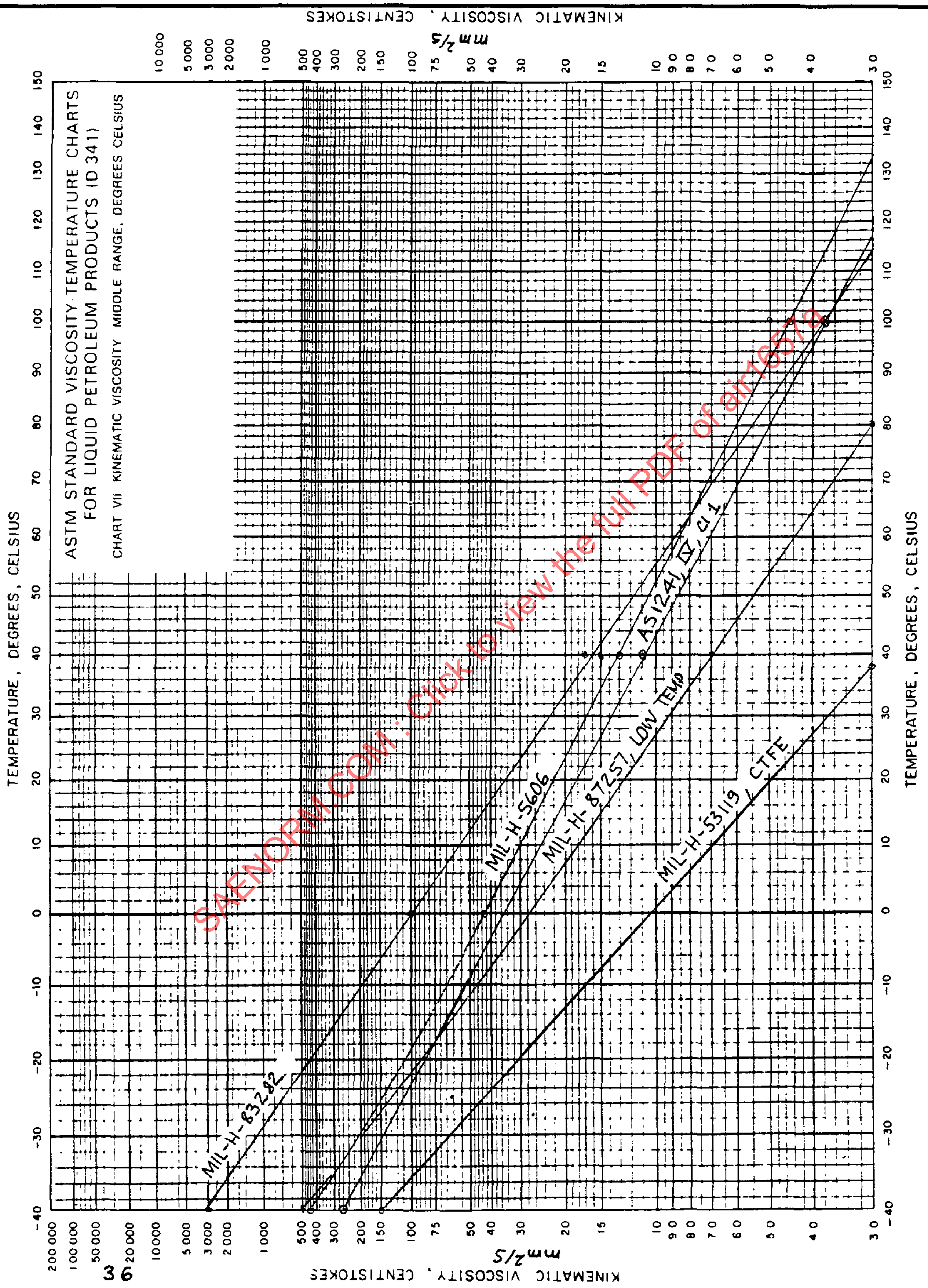

FIGURE 5 - Kinematic Viscosity of Hydraulic Fluids

3.3.4 Friction Loss:

3.3.4.1 Darcey's Equation: Head loss in a tube through wall friction is:

$$\Delta H = f\ (L/D)\ v^2/2g \qquad \text{(Eq.46)}$$
$$(L = (L/L)\ X\ (LT^{-1})^2/LT^{-2})$$

where:

ΔH = head of fluid column (m) - (L)
f = friction factor (dimensionless)
L/D = length to diameter ratio (dimensionless) - (L/L)
$v^2/2g$ = velocity head (m) - (L)

3.3.4.2 Pressure Drop: Pressure drop is given by:

$$\Delta P = f\ (L/D)\ \rho\ v^2/2 \qquad \text{(Eq.47)}$$
$$(ML^{-1}T^{-2} = (L/L)\ x\ (ML^{-3})\ x\ (LT^{-1})^2)$$

In primary units:

ΔP = total pressure drop (Pa)
L = length (m)
D = inside diameter (m)
ρ = density (kg/m^3)
v = velocity (m/s)

By definition:

$$1\ kPa = 10^3\ Pa \text{ and } 1\ kg/L = 10^3\ kg/m^3 \qquad \text{(Eq.48)}$$

In practical units:

ΔP = total pressure drop (kPa)
L = length (mm)
D = inside diameter (mm)
ρ = density (kg/L)
v = velocity (m/s)

3.3.4.3 Friction Factor - Laminar and Turbulent Flow: Friction factor in laminar flow is given by:

$$f = 64/Re \qquad \text{(Eq.49)}$$

Friction factor for turbulent flow in smooth tubing is approximately:

$$f = 0.316/Re^{0.25} \qquad \text{(Eq.50)}$$

For the transition region between laminar and turbulent flow, the friction factor is approximately 0.042. However, a more conservative position is usually to use the friction factor for turbulent flow.

3.3.4.4 Application - Head Loss: What is the head loss in a 4.57 m long, 30 mm ID tube carrying 113 L/min of MIL-H-6506 fluid at -40 °C and 100 °C temperatures?

$$L/D = 4570 \text{ mm}/30 \text{ mm} = 152.3 \qquad \text{(Eq.51)}$$

From Figure 1, density is obtained:

$$\rho = 0.908 \text{ kg/L at } -40\ °C \qquad \text{(Eq.52)}$$
$$\rho = 0.796 \text{ kg/L at } 100\ °C$$

From the 3.3.2.1 example, velocity is:

$$v = 2.67 \text{ m/s} \qquad \text{(Eq.53)}$$

Dynamic pressure:

$$\rho\, v^2/2 = 3.24 \text{ kPa at } -40\ °C \qquad \text{(Eq.54)}$$
$$\rho\, v^2/2 = 2.84 \text{ kPa at } 100\ °C$$

From the 3.5.2.1 example, Reynolds Number:

$$Re = 182 \text{ at } -40\ °C \text{ (Laminar)} \qquad \text{(Eq.55)}$$
$$Re = 16300 \text{ at } 100\ °C \text{ (Turbulent)}$$

Friction factor, using the applicable equation:

a. -40 °C, laminar flow:

$$f = 64/182 \qquad \text{(Eq.56)}$$
$$f = 0.352$$

b. 100 °C, turbulent flow

$$f = 0.316/(16300)^{0.25} \qquad \text{(Eq.57)}$$
$$f = 0.028$$

Pressure drop:

a. -40 °C:

$$\Delta P = 0.352 \times 152.3 \times 3.24 \text{ kPa} \qquad \text{(Eq.58)}$$
$$\Delta P = 174 \text{ kPa}$$

b. 100 °C:

$$\Delta P = 0.028 \times 152.3 \times 2.84 \text{ kPa} \qquad \text{(Eq.59)}$$
$$\Delta P = 12 \text{ kPa}$$

3.3.5 Momentum: By definition, momentum is:

a. $M\ v - (MLT^{-1})$

where:

M = mass (kg) - (M)
v = velocity (m/s) - (LT^{-1})

The force required to change the momentum is, by Newton's law:

$$F = d(Mv)/dt \quad (MLT^{-2} = MLT^{-1}/T) \qquad \text{(Eq.60)}$$

where:

F = external force (Newton) - (MLT^{-2})
$d(Mv)/dt$ = rate of change of momentum - (MLT^{-1}/T)

The momentum change of a column of fluid flowing though a tube:

$$d(Mv)/dt = M\ dv/dt + v\ dM/dt \qquad \text{(Eq.61)}$$

where:

dv/dt = acceleration of the fluid (m/s²) - (LT^{-1}/T)
dM/dt = mass flow rate (kg/s) - (MT^{-1})

In steady state flow:

$$dv/dt = 0 \quad d(Mv)/dt = v\ dM/dt \qquad \text{(Eq.62)}$$

In an incompressible fluid:

$$dM/dt = \rho\ Q \quad (MT^{-1} = ML^{-3} \times L^3T^{-1}) \qquad \text{(Eq.63)}$$

where:

ρ = density (kg/m³) - (ML^{-3})
Q = discharge (m³/s) - (L^3T^{-1})

3.3.5.1 Practical Units:

$$F = \rho\ Q\ v \qquad \text{(Eq.64)}$$

where:

F = force (Newton)
ρ = density (kg/L)
Q = discharge (L/s)
ρ Q = mass flow (kg/s)
v = velocity (m/s)

3.3.5.2 Application - Servovalve Nozzle: A nozzle of 0.20 mm inside diameter is supplied with MIL-H-5606 fluid at 50 °C. The pressure upstream of the nozzle is 10 MPa.

Calculate the jet velocity, discharge, mass flow and momentum flux.

Bernoulli's Equation with negligible losses through the nozzle yields:

$$\rho\ v^2/2 = P \qquad \text{(Eq.65)}$$
$$(ML^{-3} \times (LT^{-1})^2 = ML^{-1}T^{-2})$$

Solving for velocity:

$$v = (2P/\rho)^{0.5} \qquad \text{(Eq.66)}$$
$$(LT^{-1} = (ML^{-1}T^{-2}/ML^{-3})^{0.5})$$

where:

ρ = 0.843 kg/L per Figure 1 - (ML^{-3})
P = 10 MPa = 10^4 kPa - $(ML^{-1}T^{-2})$

Therefore:

$$v = (2 \times 10^4/0.843)^{0.5} \qquad \text{(Eq.67)}$$
$$(LT^{-1})$$
$$v = 154\ \text{m/s}$$

Flow is:

$$Q = A\ v \qquad \text{(Eq.68)}$$
$$(L^3T^{-1} = L^2 \times LT^{-1})$$

where:

$A = \pi\ D^2/4$ - (L2)
$A = \pi\ (0.20\ \text{mm})^2/4$
$A = 0.0314\ \text{mm}^2$

3.3.5.2 (Continued):

Note that:

$$1 \text{ mL} = 10^3 \text{ mm}^3 \qquad \text{(Eq.69)}$$

Therefore:

$$Q = 0.0314 \text{ mm}^2 \times 154 \text{ m/s} \qquad \text{(Eq.70)}$$
$$Q = 4.84 \text{ mL/s}$$
$$L^3T^{-1}$$

Common units:

$$Q = 4.84 \times 10^{-3} \text{ L/mL} \times 60 \text{ s/min} \qquad \text{(Eq.71)}$$
$$Q = 0.29 \text{ L/min}$$

Mass flow:

$$dM/dt = \rho\ Q \qquad \text{(Eq.72)}$$
$$(MT^{-1} = ML^{-3} \times L^3T^{-1})$$
$$dM/dt = 0.843 \text{ kg/L} \times (4.84 \times 10^{-3} \text{ L/s})$$
$$dM/dt = 4.08 \times 10^{-3} \text{ kg/s}$$
$$dM/dt = 4.08 \text{ g/s}$$

Momentum:

$$v\ dM/dt = \rho\ Q\ v \qquad \text{(Eq.73)}$$
$$(LT^{-1} \times MT^{-1} = ML^{-3} \times L^3T^{-1} \times LT^{-1})$$
$$v\ dM/dt = 154 \text{ m/s} \times 4.08 \text{ g/s}$$
$$v\ dM/dt = 628 \text{ mN (millinewton)}$$

3.3.6 Pressure Momentum Equation: A free body in steady state flow is in balance under the combined effect of momentum and pressure forces at the entry (1) and exit (2) sections:

$$(\rho\ Q\ v_2 - \rho\ Q\ v_1) + (A_1\ P_1 - A_2\ P_2) = \Sigma F \qquad \text{(Eq.74)}$$
$$(L^2 \times ML^{-1}T^{-2} = MLT^{-2})$$

where:

A = area (m^2) - (L^2)
P = pressure (Pa) - ($ML^{-1}T^{-2}$)
ΣF = sum of forces (N) - (MLT^{-2})

3.3.6.1 Practical Units:

$$1\ mm^2 = 10^{-6}\ m^2$$
$$1\ MPa = 10^6\ N/m^2 \qquad (Eq.75)$$

Therefore:

$$1\ mm^2 \times 1\ MPa = 1\ N \qquad (Eq.76)$$

3.3.6.2 Application - Sudden Expansion, Borda's Equation: The jet from the 0.20 mm diameter nozzle of example 3.3.5.2 enters into a duct of 0.37 mm inside diameter. The flow expands to fill the larger section through turbulent mixing.

a. What are steady state velocity, momentum flux and pressure at the expanded section? What is the loss in total pressure?

b. Check the result by Borda's equation for total pressure loss in a sudden expansion.

Neglecting elevation head and taking the entry pressure for datum, the pressure momentum equation becomes:

$$(\rho\ Q\ v_2 - \rho\ Q\ v_1) + A_2\ P_2 = 0 \qquad (Eq.77)$$
$$(ML^{-3} \times L^3T^{-1} \times LT^{-1} + L^2 \times ML^{-1}\ T^{-2} = 0)$$

where:

$\rho\ Q\ v_1$ = 628 mN jet momentum flux per 3.3.5.2
$\rho\ Q\ v_2$ = duct momentum (mN) - (MLT^{-2})
A_2 = duct area (mm^2) - (L^2)
P_2 = duct pressure (kPa) - $(ML^{-1}\ T^{-2})$

The duct area is:

$$A = \pi\ D^2/4 \qquad (Eq.78)$$
$$A = \pi\ (0.37)^2/4$$
$$A = 0.1075\ mm^2$$

At the exit section (2), where flow expands to the entire duct area, velocity is:

$$v_2 = Q/A_2 \qquad (Eq.79)$$

The flow is:

$$Q = 4.84\ mL/s\ \text{per 3.3.5.2} \qquad (Eq.80)$$

Therefore:

$$v_2 = 4.84\ mL/s/(0.1075\ mm^2) \qquad (Eq.81)$$
$$v_2 = 45\ m/s$$

3.3.6.2 (Continued):

Mass flow rate:

$$dM/dt = 4.08 \text{ g/s per } 3.3.5.2 \quad \text{(Eq.82)}$$

Momentum flux:

$$dM/dt\ v_2 = 4.08 \text{ g/s} \times 45 \text{ m/s} \quad \text{(Eq.83)}$$
$$dM/dt\ v_2 = 183 \text{ mN}$$
$$\rho\ Q\ v_2 = 183 \text{ mN}$$

The change of momentum is:

$$\rho\ Q\ v_2 - \rho\ Q\ v_1 = 628 \text{ mN} - 183 \text{ mN} \quad \text{(Eq.84)}$$
$$\rho\ Q\ v_2 - \rho\ Q\ v_1 = 445 \text{ mN}$$

The pressure forces on the duct equal the change of momentum:

$$A_2\ P_2 = \rho\ Q\ v_2 - \rho\ Q\ v_1 \quad \text{(Eq.85)}$$

The pressure:

$$P_2 = (\rho\ Q\ v_2 - \rho\ Q\ v_1)/A_2 \quad \text{(Eq.86)}$$
$$P_2 = 445 \text{ mN}/0.1075 \text{ mm}^2$$
$$P_2 = 4.14 \times 10^3 \text{ kPa}$$
$$P_2 = 4.14 \text{ MPa}$$

The total pressure of the jet is 10 MPa at the duct inlet (3.3.5.2). As the jet diffuses and fills the duct through turbulent mixing, the total pressure becomes:

$$P_2 + \rho\ v_2^2/2 \quad \text{(Eq.87)}$$

where:

ρ = 0.843 kg/L
v_2 = 45 m/s, uniform velocity in the duct

Velocity pressure:

$$\rho\ v_2^2/2 = 0.843 \text{ kg/L} \times (45 \text{ m/s})^2/2 \quad \text{(Eq.88)}$$
$$\rho\ v_2^2/2 = 854 \text{ kPa}$$
$$\rho\ v_2^2/2 = 0.854 \text{ MPa}$$

Total pressure:

$$P_2 + \rho\ v_2^2/2 = 4.14 \text{ MPa} + 0.854 \text{ MPa} \quad \text{(Eq.89)}$$
$$P_2 + \rho\ v_2^2/2 = 4.99 \text{ MPa}$$

3.3.6.2 (Continued):

Drop in total pressure:

$$\Delta P = 10 \text{ MPa} - 4.99 \text{ MPa} \quad \text{(Eq.90)}$$
$$\Delta P = 5.01 \text{ MPa}$$

Borda's equation for total pressure loss in a sudden expansion is:

$$\Delta P = \rho \, (v_1 - v_2)^2/2 \quad \text{(Eq.91)}$$
$$(ML^{-1}T^{-2} = ML^{-3} \times (LT^{-1})^2)$$

where:

ρ = 0.843 kg/L
v_1 = 154 m/s
v_2 = 45 m/s

Yielding:

$$\Delta P = 0.843 \text{ kg/L} \times (154 \text{ m/s to } 45 \text{ m/s})^2/2 \quad \text{(Eq.92)}$$
$$\Delta P = 5008 \text{ kPa}$$
$$\Delta P = 5.01 \text{ MPa, verifying the preceding results}$$

3.3.7 Pump Efficiency:

3.3.7.1 Overall Efficiency: The overall efficiency of a hydraulic pump is the hydraulic power output of the pump divided by the drive shaft power input:

$$\eta = P \, Q/T \, \omega \quad \text{(Eq.93)}$$
$$(ML^{-1}T^{-2} \times L^3T^{-1} = ML^2T^{-2} \times T^{-1})$$

where:

P = pressure (kPa) - $(ML^{-1}T^{-2})$
Q = dischargee (L/s) - (L^3T^{-1})
T = torque (N.m) - (ML^2T^{-2})
ω = speed of rotation (rad/s) - (T^{-1})

3.3.7.2 Volumetric Efficiency: The volumetric efficiency of a hydraulic pump is the actual discharge (L/min) divided by the calculated discharge:

$$\eta = \text{L/min}/(\delta/N) \quad \text{(Eq.94)}$$
$$(L^3T^{-1}/(L^3 \times T^{-1}))$$

where:

δ = displacement per revolution (L/rev) - (L^3)
N = speed, revolution per minute (rpm) - (T^{-1})

3.3.7.2 (Continued):

NOTE:

$$\omega(\text{rad/s}) = 2\pi \text{ N (rev/min)/60 s/min} \quad (T^{-1}) \qquad \text{(Eq.95)}$$

3.3.7.3 Application - Pump Performance: A 7.2 mL/rev pump running at 4000 rpm discharges into a 19.7 MPa hydraulic system. The volumetric efficiency is 96%. The drive shaft torque measured 24.9 Nm.

Calculate the input power, the delivery, the output power, and the overall pump efficiency.

$$\text{Input Power} = \text{Torque X Speed} \quad (ML^2T^{-3} = ML^2T^{-2} \times T^{-1}) \qquad \text{(Eq.96)}$$

where:

Input Power = 24.9 Nm x 2p x 4000 rpm/60 s/min
Input Power = 10 430 W
Input Power = 10.43 kW

Discharge:

$$Q = \eta \text{ d N} \quad (L^3T^{-1} = L^3 \times T^{-1}) \qquad \text{(Eq.97)}$$

Calculate:

$$\eta\ \delta\ N = 0.96 \times 7.2 \text{ mL/rev} \times 4000 \text{ rpm} \qquad \text{(Eq.98)}$$
$$= 27\ 650 \text{ mL/min}$$
$$= 27.6 \text{ L/min}$$

Convert:

$$\eta\ \delta\ N = 27.6 \text{ L/min/60 s/min} \qquad \text{(Eq.99)}$$

Discharge:

$$Q = 0.46 \text{ L/s} \qquad \text{(Eq.100)}$$

$$\text{Output Power} = \text{Pressure x Discharge} - (ML^2T^{-3} = ML^{-1}T^{-2} \times L^3T^{-1}) \qquad \text{(Eq.101)}$$
$$= 19.7 \text{ MPa} \times 0.46 \text{ L/s}$$
$$= 9.08 \text{ kW}$$

$$\text{Overall Efficiency} = \text{Output Power/Input Power} \qquad \text{(Eq.102)}$$
$$\eta = 9.08 \text{ kW}/10.43 \text{ kW}$$
$$\eta = 0.87$$
$$\eta = 87\%$$

3.4 Thermodynamics:

3.4.1 Warm-up of Hydraulic Fluid:

3.4.1.1 Power and Heat Parameters:

TABLE 5 - Symbols, Physical Quantities, and Units Used for Warm-up

Symbol	Dimensions	Description	Basic Unit	Practical Unit
W	ML^2T^{-3}	Power consumption	W	kW
Q_v	L^3T^{-1}	Volumetric flow	m^3/s	L/s
ΔP	$ML^{-1}T^{-2}$	Pressure drop	Pa	MPa
Q_h	ML^2T^{-3}	Heat flow	W	kW
ρ	ML^{-3}	Density	kg/m^3	kg/L
C_p	L^2T^{-2}	Specific heat	J/kg	kJ/kg
ΔT		Temperature rise	K	C
t	T	Elapsed warm-up time	s	s
dT/dt	T^{-1}	Rate of temperature change	K/s	C/s
η		Efficiency		
V	L^3	Volume of hydraulic system	m^3	L

3.4.1.2 Power Conversion Equations: The power (W) consumed by a flow Q_v under a pressure drop ΔP is:

$$W = Q_v \, \Delta P \qquad (ML^2T^{-3} = L^3T^{-1} \times ML^{-1}T^{-2}) \qquad \text{(Eq.104)}$$

The heat flow (Q_h) of the fluid of density ρ and specific heat C_p at a temperature rise ΔT:

$$Q_h = \rho \, C_p \, Q_v \, \Delta T \qquad (ML^2T^{-3} = ML^{-3} \times L^2T^{-2} \times L^3T^{-1}) \qquad \text{(Eq.105)}$$

Equating the heat flow generated by throttling to the power consumed:

$$\rho \, C_p \, Q_v \, \Delta T = Q_v \, \Delta P \qquad (ML^{-3} \times L^2T^{-2} \times L^3T^{-1} = L^3T^{-1} \times ML^{-1}T^{-2}) \qquad \text{(Eq.106)}$$

Cancelling the volume flow on each side of Equation 106:

$$\rho \, C_p \, Q_v \, \Delta T = Q_v \, \Delta P \qquad \text{(Eq.107)}$$

becomes:

$$\rho \, C_p \, \Delta T = \Delta P \qquad \text{(Eq.108)}$$

3.4.1.2 (Continued):

The temperature rise across the flow restrictor is directly proportional to the pressure drop:

$$\Delta T = \Delta P/(\rho \ C_p) \quad (ML^{-1}T^{-2}/(ML^{-3} \times L^2T^{-2})) \qquad \text{(Eq.109)}$$

The number of passes required:

$$n = (T_1 - T_0)/\Delta T \qquad \text{(Eq.110)}$$

Duration of each pass:

$$\Delta t = V/Q_v \quad (T = L^3/L^3T^{-1}) \qquad \text{(Eq.111)}$$

Time required for the warm-up without heat transfer to the environment:

$$\tau = n \ \Delta t \qquad \text{(Eq.112)}$$

As the hydraulic fluid temperature rises above the ambient, heat will flow to the environment. The heat loss slows down the temperature rise. A thermal equilibrium temperature (ΔT_f) is approached asymptotically. The temperature rise ΔTx versus the elapsed time tx follows approximately the exponential law:

$$\Delta Tx = \Delta T_f(1 - e^{-tx/\tau}) \qquad \text{(Eq.113)}$$

where:

ΔT_f = final equilibrium temperature rise
τ = the time constant determined in Equation 111

3.4.1.3 Application - Warm-up by Throttling: MIL-H-83282 hydraulic fluid is warmed up from a -20 °C cold start to a desired final thermal equilibrium temperature of 40 °C.

The fluid flow is 120 L/min.

The pressure drop through the throttle valve is 17 MPa.

The fluid volume in the hydraulic system is 100 L.

Calculate:

a. Hydraulic power consumption
b. Temperature rise across the throttle valve
c. Number of passes required assuming no heat losses
d. Duration of one pass
e. Length of time required for the warm-up at the initial rate

3.4.1.3 (Continued):

f. Percentage of the equilibrium temperature achieved in 15 min
g. The temperature at the end of 15 min

The MIL-H-83282 hydraulic fluid properties are:

$$\rho = 0.84 \text{ kg/L, fluid density, Figure 4} \qquad \text{(Eq.114)}$$
$$C_p = 2.0 \text{ kJ/kg, fluid specific heat, Figure 6}$$

Unit conversion:

$$120 \text{ L/min} = (120 \text{ L/min})/(60 \text{ s/min}) \qquad \text{(Eq.115)}$$
$$120 \text{ L/min} = 2.0 \text{ L/s}$$

a. The power consumption is by Equation 104:

$$W = 2.0 \text{ L/s} \times 17 \text{ MPa} \qquad \text{(Eq.116)}$$
$$W = 34 \text{ kW}$$

b. The temperature rise is by Equation 109:

$$\Delta T = 17 \text{ MPa}/(0.84 \text{ kg/L} \times 2.0 \text{ kJ/kg}) \qquad \text{(Eq.117)}$$
$$\Delta T = 10 \text{ °C}$$

c. The duration of one pass is by Equation 111:

$$\Delta t = 100 \text{ L}/(2.0 \text{ L/s}) \qquad \text{(Eq.118)}$$
$$\Delta t = 50 \text{ s each pass}$$

d. The final temperature rise to the equilibrium temperature is obtained as:

$$\Delta T_f = 40 \text{ °C} - (-20 \text{ °C}) \qquad \text{(Eq.119)}$$
$$\Delta T_f = 60 \text{ °C}$$

The number of passes required without heat loss is given by Equation 110:

$$n = 60 \text{ °C}/10 \text{ °C} \qquad \text{(Eq.120)}$$
$$n = 6 \text{ passes}$$

e. The length of time required for the warm-up at the initial rate by Equation 112:

$$\tau = 6 \times 50 \text{ s} \qquad \text{(Eq.121)}$$
$$\tau = 300 \text{ s}$$

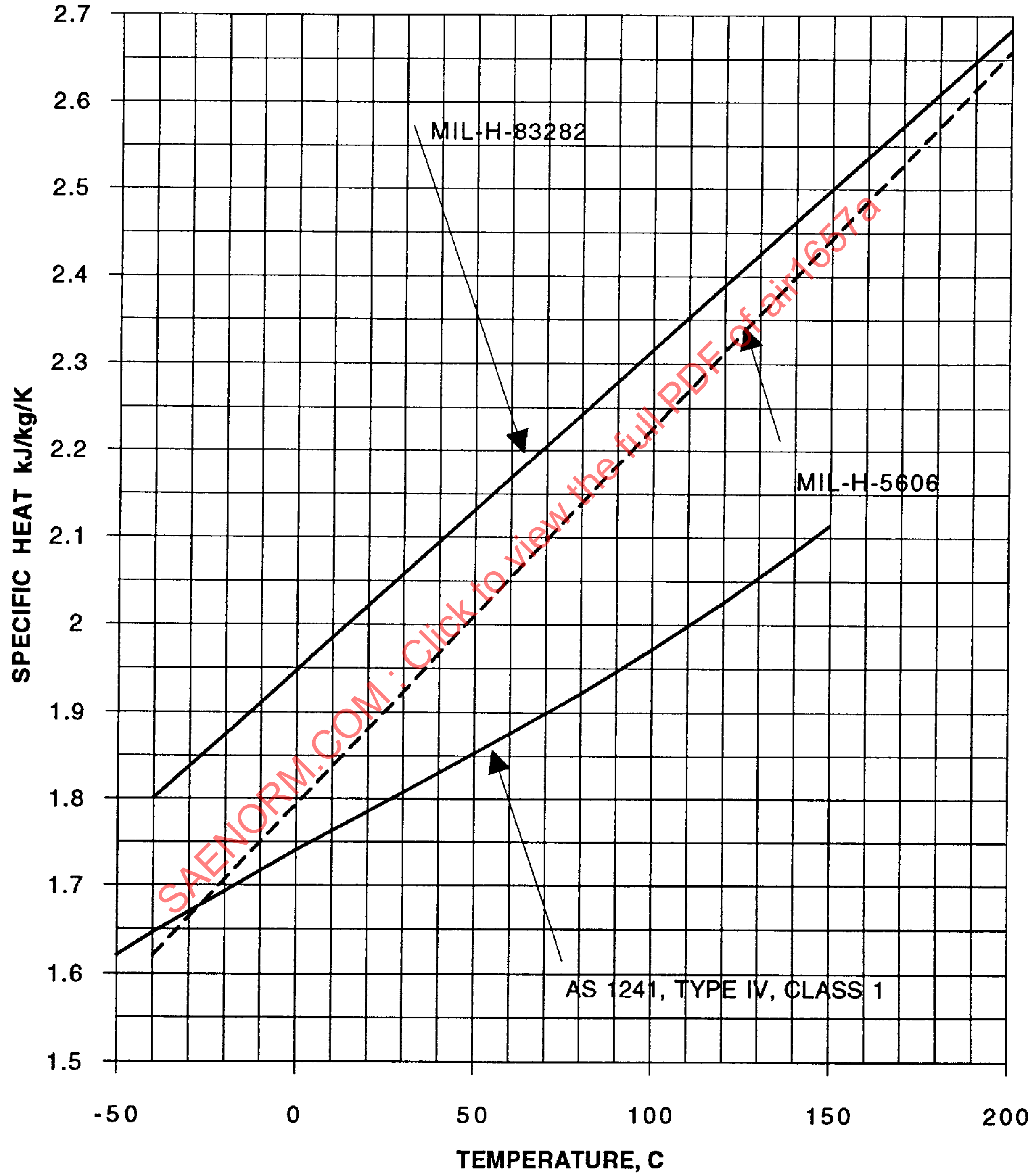

FIGURE 6 - Specific Heat of Hydraulic Fluids

3.4.1.3 (Continued):

Conversion:

$$\tau = 300\ s/(60\ s/min)$$ (Eq.122)
$$\tau = 5\ min$$

where t defines the warm-up time constant of the hydraulic system:

t = 5 min

f. The percentage of the final temperature rise achieved in 15 min with heat loss to the environment is given by Equation 113:

$$\Delta Tx/\Delta T_f = (1 - e^{-tx/\tau})$$ (Eq.123)

where:

tx = 15 min
τ = 5 min

Calculate:

$$(1 - e^{-tx/\tau}) = 1 - e^{(-15/5)}$$ (Eq.124)
$$\Delta Tx/\Delta T_f = 95\%$$

The final temperature rise is:

$$\Delta Tx = 0.95 \times 60\ °C$$ (Eq.125)
$$\Delta Tx = 57\ °C$$

g. The temperature at the end of 15 min is 37 °C.

3.4.2 Thermal Expansion:

3.4.2.1 Thermal Expansion Coefficient: By definition, C is the relative (percentage) volume expansion obtained by a unit rise in temperature:

$$C = \Delta V/V\ \Delta T$$ (Eq.126)

TABLE 6 - Symbols and Units Used for Thermal Expansion

Symbol	Dimensions	Description	Basic Unit	Practical Unit
C	L^3/L^3	coefficient of thermal expansion	$m^3/(m^3\ K)$	%/100 °C
V	L^3	initial volume of fluid	m^3	L
ΔV	L^3	variation of fluid volume	m^3	L
ΔT		variation of temperature	K	100 °C
ρ	M/L^3	density	kg/m^3	kg/L
1/ρ	L^3/M	specific volume	m^3/kg	L/kg

TABLE 7 - Thermal Expansion Coefficients at Sea Level Pressure

Fluid	Primary Unit $m^3/(m^3K)$	Practical Unit %/100 °C	Source of Data
MIL-H-83282	8.6 E-4	8.6	Figure 4
AS1241, Type IV, Class 1	9.6 E-4	9.6	Figure 7
MIL-H-5606	9.8 E-4	9.8	Figure 1

3.4.2.1 (Continued):

The density versus temperature charts Figure 1, 4, and 7 provide data on the fluid density (ρ) at pressures from 0 to 60 MPa. Specific volume is obtained as the inverse of density ($1/\rho$). The variation of specific volume [$\Delta(1/\rho)$] is then obtained for the various fluid temperature and pressure conditions.

3.4.3 Adiabatic Tangent Bulk Modulus (Figure 8):

3.4.3.1 Definitions - Secant/Tangent, Isothermal/Adiabatic: The bulk modulus of compressibility of hydraulic fluids is defined with reference to the curve of the pressure rise versus the relative volume variation.

The secant bulk modulus is the slope of the secant to the curve, generally drawn through the origin, defined by the equation:

$$B = V_0(P_1-P_0)/(V_0-V_1) \quad \text{(Eq.127)}$$
$$(ML^{-1}T^{-2} = L^3 \times ML^{-1}T^{-2}/L^3)$$

where:

P_1 = initial pressure, usually zero gauge - ($ML^{-1}T^{-2}$)
P_1 = final pressure - ($ML^{-1}T^{-2}$)
V_0 = initial fluid volume - (L^3)
V_1 = final fluid volume - (L^3)

Let:

$$\Delta P = P_1 - P_0 = \text{finite variation in pressure } (ML^{-1}T^{-2}) \quad \text{(Eq.128)}$$
$$\Delta V = V_0 - V_1 = \text{finite variation in volume } (ML^{-1}T^{-2})$$

$$B = V_0(\Delta P)/(\Delta V) \quad \text{(Eq.129)}$$

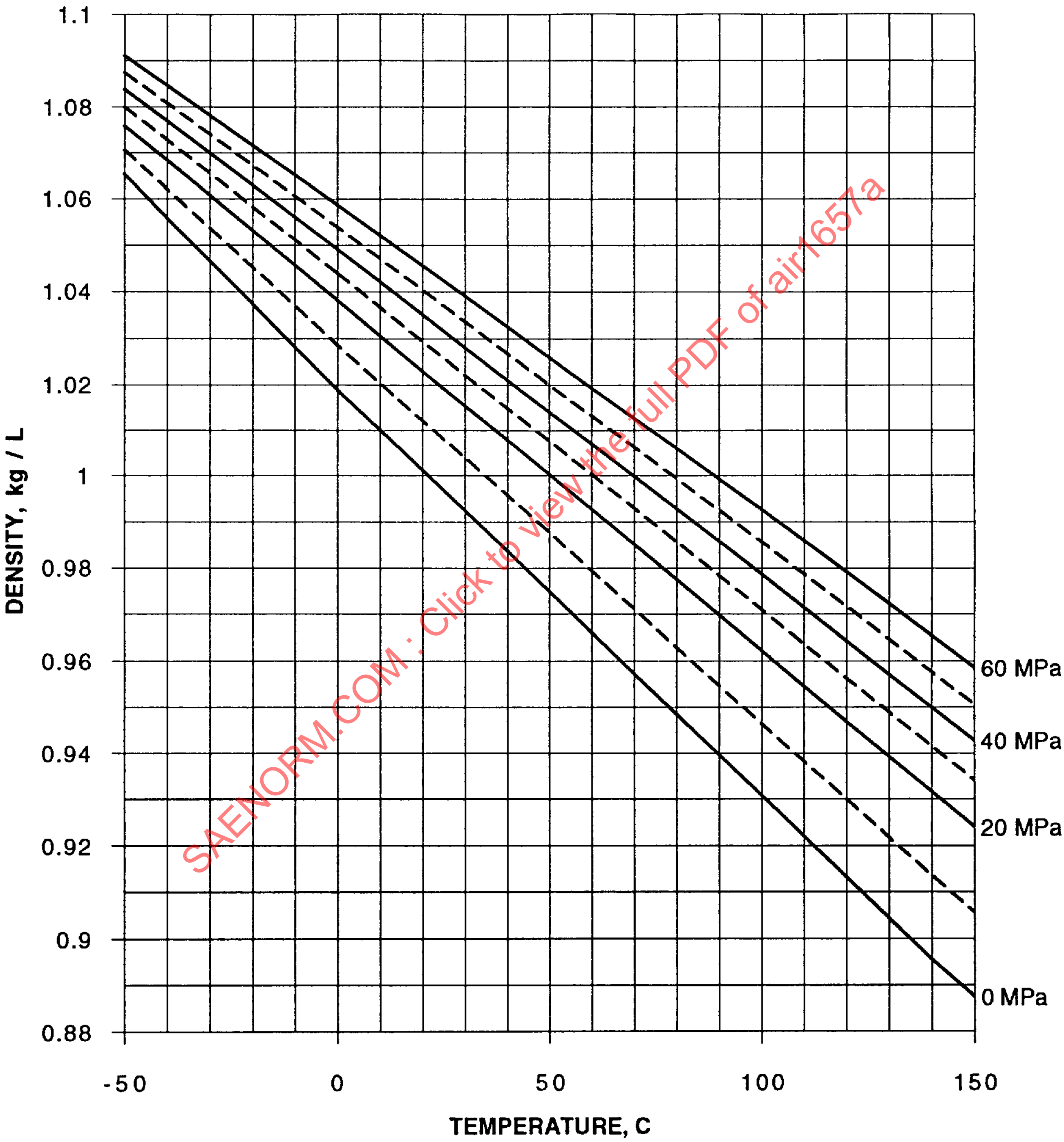

FIGURE 7 - Density, AS1241, Type IV, Class 1

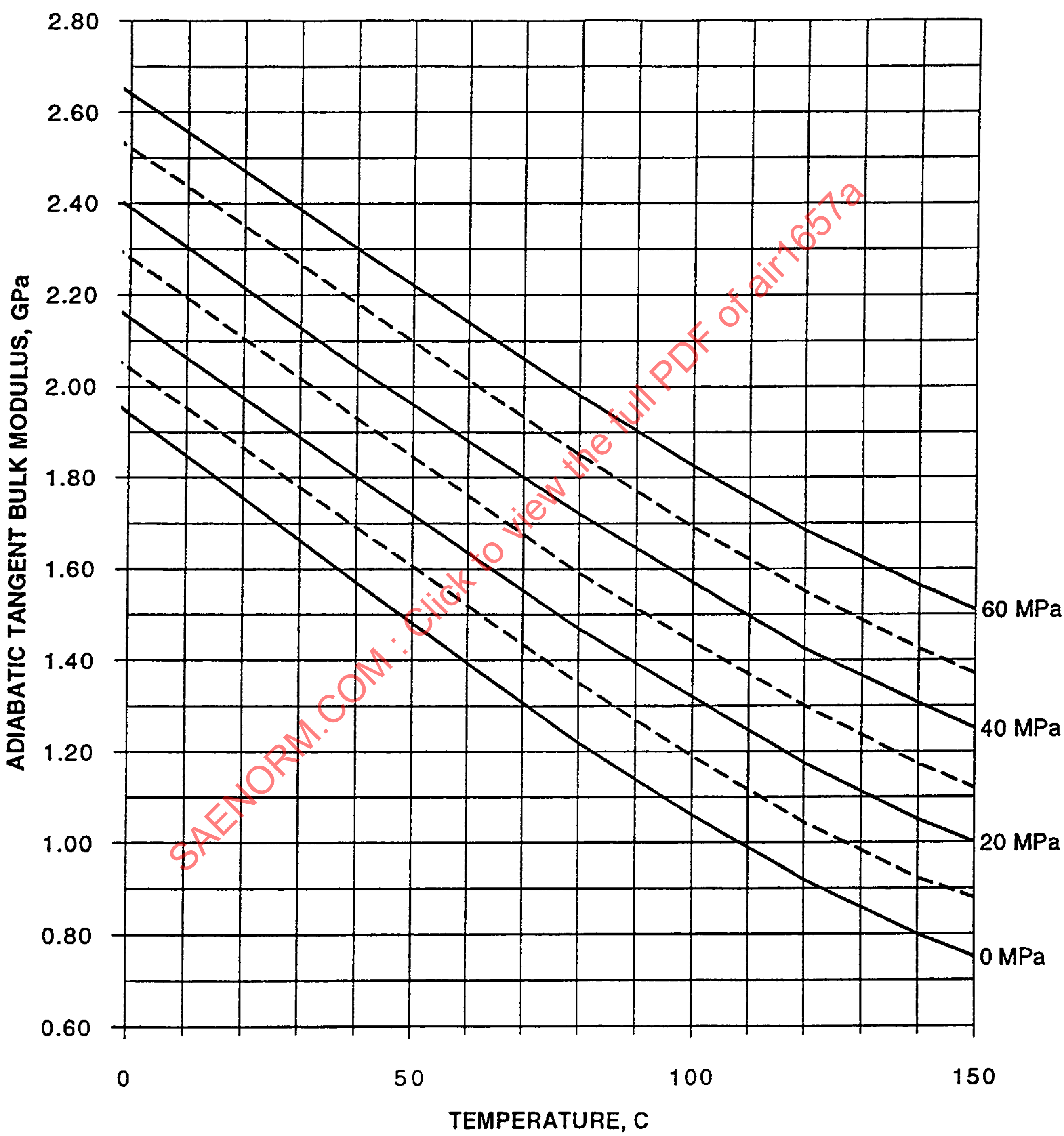

FIGURE 8 - Tangent Bulk Modulus, MIL-H-5606

3.4.3.1 (Continued):

The tangent bulk modulus is the slope of the tangent to the curve, which is the limit of the secant bulk modulus when the size of the variation is reduced to zero. It is defined by:

$$B = V_0(\delta P/\delta V) \qquad \text{(Eq.130)}$$
$$(ML^{-1}T^{-2} = L^3 \times ML^{-1}T^{-2}/L^3)$$

where:

δP = infinitely small variation in pressure
δV = infinitely small variation in volume

The conditions usually considered for bulk modulus determination are the isothermal process (at constant temperature) and the adiabatic process (with no heat transfer).

Compression of the hydraulic fluid produces heat. In an isothermal process the heat of compression is dissipated to the environment. In an adiabatic process the heat of compression remains in the fluid. The internal heat results in thermal expansion of the fluid, which increases the resistance to the compression. As a result, the adiabatic bulk modulus of compressibility is larger than the isothermal bulk modulus.

The ratio of specific heat at constant pressure (cp) to the specific heat at constant volume (cv) is the ratio of the adiabatic to the isothermal bulk modulus for a perfect gas.

For air, and other diatomic gases, cp/cv = 1.4. The isothermal tangent bulk modulus B_{iso} of air at the absolute pressure P is B_{iso} = P, and the adiabatic tangent bulk modulus is B_{adi} = 1.4 P. For aerospace hydraulic fluids the specific heat ratio, which equals the ratio of adiabatic to isothermal bulk modulus, is shown in Figure 9.

The isothermal bulk modulus applies to processes that are sufficiently slow for removal of the heat of compression to the environment.

The adiabatic tangent bulk modulus applies to small amplitude oscillations about a given pressure level:

a. Propagation of sound in the hydraulic fluid
b. Small oscillations of a hydraulic actuator under load

3.4.3.2 Speed of Sound and Water Hammer: The speed of sound in the hydraulic fluid in a rigid tube with no entrained air is:

$$C = (B/\rho)^{0.5} \qquad \text{(Eq.131)}$$
$$(LT^{-1} = (ML^{-1}T^{-2}/ML^{-3})^{0.5})$$

where:

B = adiabatic tangent bulk modulus - $(ML^{-1}T^{-2})$
ρ = mass density - (ML^{-3})

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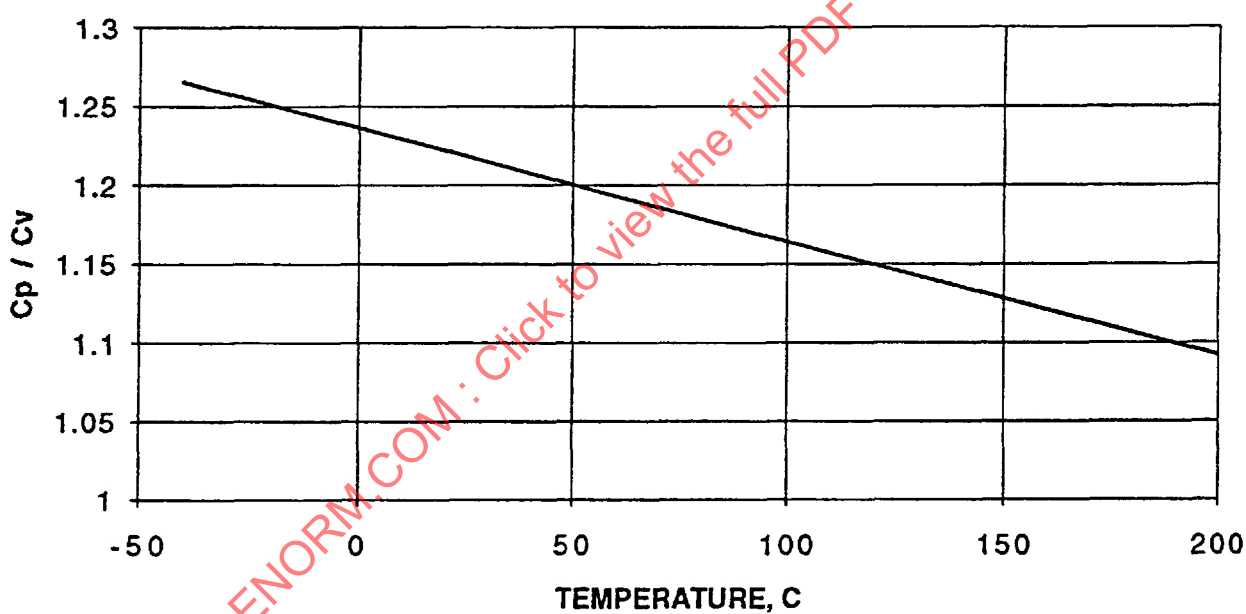

FIGURE 9 - Specific Heat Ratio of Hydraulic Fluids

3.4.3.2 (Continued):

The adiabatic bulk modulus (B) is replaced by the effective bulk modulus (B_e) in a conduit with elastic walls, and in the presence of entrained air.

In determining the effective bulk modulus, dissolved gas has little or no effect on the bulk modulus of the hydraulic fluid. However, entrained air has a significant effect, especially at low operating pressures, such as occur at the pump suction. For example, 1% entrained air by volume in the fluid reduces the effective bulk modulus to approximately 5% of the fluid without any entrained air.

The compliance effect of the transmission line can usually be ignored for tubing and piping but must be considered for flexible hose.

The effective bulk modulus of hose and fluid is in the 200 to 400 MPa (29 000 to 58 000 psi) range.

Numerical example, MIL-H-83282 at 55 MPa pressure and 100 °C:

TABLE 8

Source of Data	Primary Unit	Practical Unit
Figure 10	$B = 1.42 \times 10^9$ Pa	B = 1.42 GPa
Figure 4	$\rho = 837$ kg/m^3	$\rho = 0.837$ kg/L
	$C = (1.42 \times 10^9/837)^{0.5}$	
	$C = 10^3\ (1.42/0.837)^{0.5}$	$C = (1.42\ \text{GPa}/0.837\ \text{kg/L})^{0.5}$
	C = 1300 m/s	C = 1.3 km/s

Water hammer pressure resulting from sudden velocity reduction (Δv) of a long fluid column in a rigid tube:

$$\Delta p = \rho\ C\ \Delta v \qquad (ML^{-1}T^{-2} = ML^{-3} \times LT^{-1} \times LT^{-1}) \tag{Eq.132}$$

TABLE 9

Primary Unit	Practical Unit
$\rho = 837$ kg/m^3	$\rho = 0.837$ kg/L
C = 1300 m/s	C = 1.3 km/s
$\Delta v = 5$ m/s	$\Delta v = 5$ m/s
$\Delta p = 837 \times 1300 \times 5$	$\Delta p = 0.837 \times 1.3 \times 5$
$\Delta p = 5.44 \times 10^6$ Pa	$\Delta p = 5.44$ MPa

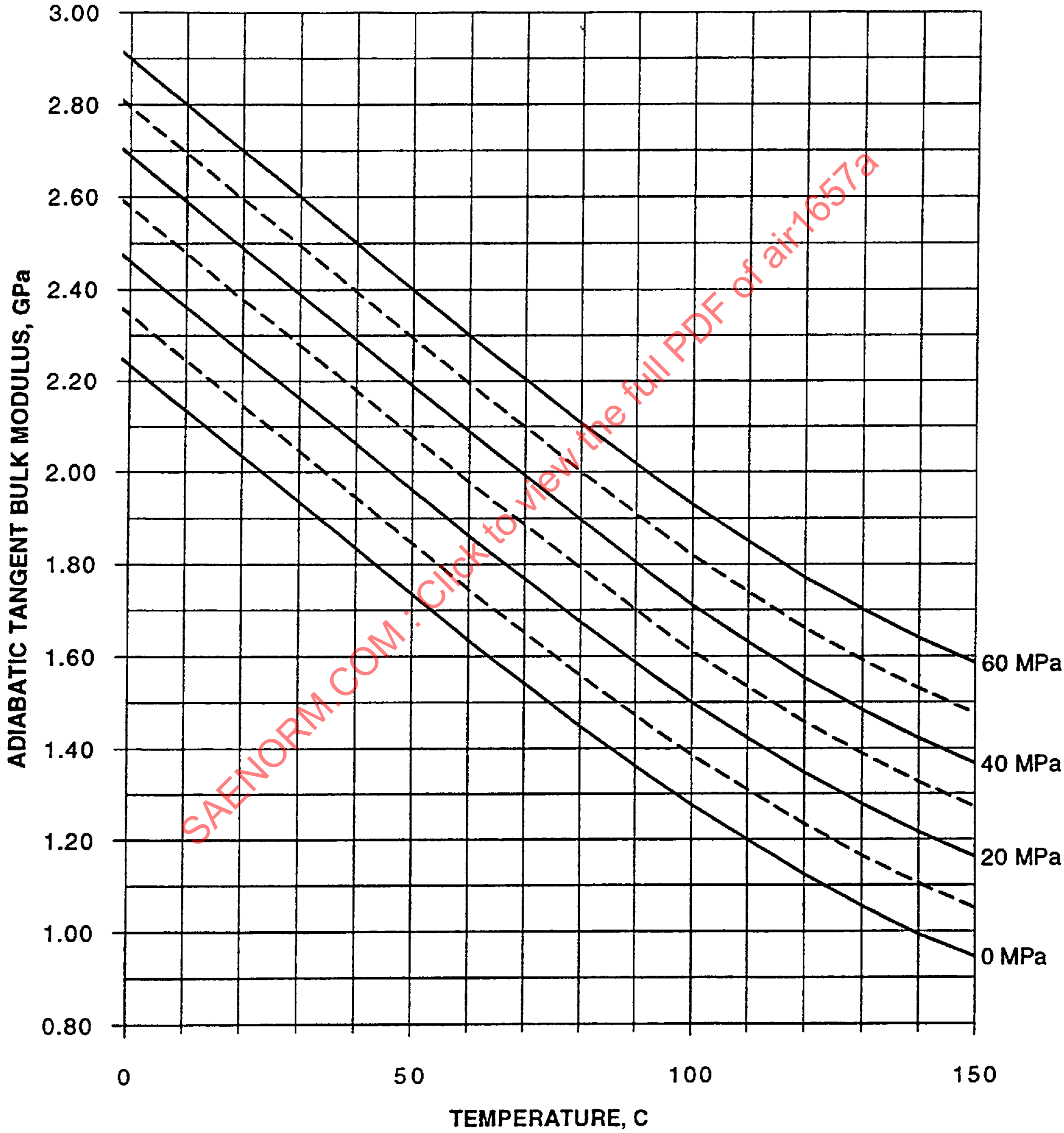

FIGURE 10 - Tangent Bulk Modulus, MIL-H-83282

3.4.3.3 Organ Pipe Resonance: Standing waves can be established in some hydraulic lines by pulsating pressures. Organ pipe resonance occurs when the length of a straight hydraulic line run equals one quarter of the pipe ripple wavelength:

$$L = 0.25\ \lambda \qquad \text{(Eq.133)}$$

The wavelength is:

$$\lambda = C/f \qquad \text{(Eq.134)}$$
$$(L = LT^{-1}/T^{-1})$$

where:

f = frequency of the pulsations, Hz

Numerical example: The pulsation frequency of a 9-piston hydraulic pump running at 3600 rpm is:

$$f = 9 \times 3600/60 \qquad \text{(Eq.135)}$$
$$f = 540\ \text{Hz}$$

Let:

$$C = 1300\ \text{m/s} \qquad \text{(Eq.136)}$$

Then:

$$\lambda = 1300/540 \qquad \text{(Eq.137)}$$
$$\lambda = 2.41\ \text{m}$$
$$L = 0.25 \times 2.41\ \text{m}$$
$$L = 0.60\ \text{m}$$

3.4.4 Hydraulic Actuation System Oscillations:

3.4.4.1 Stiffness Parameters:

TABLE 10 - Actuator Stiffness Parameters

Symbol	Dimension	Name	Primary Unit	Practical Unit
A	L^2	Piston Area, Net	m^2	mm^2
B	$ML^{-1}T^{-2}$	Bulk Modulus, Adiabatic Tangent	Pa	GPa
c	LT^{-1}	Crank Lever Arm	m	mm
D	L	Cylinder Bore Diameter	m	mm
d	L	Piston Rod Diameter	m	mm
f	T^{-1}	Natural Frequency	radian/s	Hz
J	ML^2	Moment of Inertia	kg m^2	kg m^2
K_1	MT^{-2}	Linear Spring Stiffness	N/m	MN/m
K_t	ML^2T^{-2}	Torsional Spring Stiffness	Nm/rad	kN m/rad
S	L	Length of Piston Stroke	m	mm
M	M	Mass of Control Surface	kg	kg
r_g	L	Radius of Gyration	m	mm

3.4.4.2 Spring/Mass Oscillation Equations: The net piston area of a balanced area double acting linear actuator is:

$$A = \pi/4\ (D^2 - d^2) \qquad (L^2 = L^2) \tag{Eq.138}$$

The linear spring stiffness at midstroke, with blocked valves, and with negligible trapped volume in fluid passages be is:

$$K_1 = 4\ \ B\ \ A/S \qquad (MT^{-2} = ML^{-1}T^{-2} \times L^2/L) \tag{Eq.139}$$

The torsional spring stiffness of the actuation system is:

$$K_t = K_1\ c^2 \qquad (ML^2T^{-2} = MT^{-2} \times L^2) \tag{Eq.140}$$

3.4.4.2 (Continued):

The polar moment of inertia of the control surface about the hinge line is:

$$J = M\, r_g^{\,2} \qquad (ML^2 = M \times L^2) \tag{Eq.141}$$

The natural frequency of the control surface due to actuator stiffness is:

$$f = 1/2\pi\ (K_t/J)^{0.5} \qquad (T^{-1} = (ML^2T^{-2}/ML^2)^{0.5}) \tag{Eq.142}$$

3.4.4.3 Application - Natural Frequency: Calculate the natural frequency of small oscillations of a hydraulic actuation system.

The hydraulic cylinder inside diameter (D) and the rod diameter are:

$$D = 50\ \text{mm} \qquad d = 25\ \text{mm} \tag{Eq.143}$$

By Equation 138:

$$A = 1473\ \text{mm}^2 \tag{Eq.144}$$

Assume a hydraulic system pressure of 35 MPa.

The average piston pressure, with blocked valves, at light load, is approximately one half of the maximum pressure, that is 18 MPa.

The fluid is phosphate ester per AS1241, Type IV, Class 1 specification.

The adiabatic tangent bulk modulus in Figure 11 at 120 °C and 18 MPa pressure is:

$$B = 1.1\ \text{GPa} \tag{Eq.144}$$

Let the stroke be:

$$S = 75\ \text{mm} \tag{Eq.145}$$

By Equation 139:

$$K_1 = 4 \times 1.1\ \text{GPa} \times 1473\ \text{mm}^2/75\ \text{mm} \qquad K_1 = 86.4\ \text{MN/m} \tag{Eq.146}$$

Let the crank lever arm be:

$$c = 75\ \text{mm, i.e., } 0.075\ \text{m} \tag{Eq.147}$$