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PERFORMANCE OF LOW PRESSURE RATIO EJECTORS FOR ENGINE NACELLE COOLING

1. PURPOSE

In typical helicopter gas turbine engine installations, the engine is enclosed within a nacelle. Within the nacelle, heat is rejected from the engine skin and from other sources such as the engine oil cooler, generator, and airframe accessories. Therefore, it becomes necessary to provide a flow of ventilating air through the nacelle to maintain the ambient temperature surrounding the engine at an acceptable level.

One posssible means of providing this ventilating air is to utilize the kinetic energy of the engine exhaust gas in an ejector to induce an airflow through the enclosure. This device is also commonly called an eductor, an aspirator, or a jet pump.

A straightforward method of defining the ejector geometry to provide the required cooling flow for a given application is needed.

2. SCOPE

2.1 Method: A general method for the preliminary design of a single, straight-sided, low subsonic ejector is presented. The method is based on the information presented in References 1, 2, 3, and 4, and utilizes analytical and empirical data for the sizing of the ejector mixing duct diameter and flow length. The low subsonic restriction applies because compressibility effects were not included in the development of the basic design equations. The equations are restricted to applications where Mach numbers within the ejector primary or secondary flow paths are equal to or less than 0.3.

2.2 Procedure: A recommended step-by-step procedure is shown.

2.3 Equations: The equations used in the procedure, as well as their derivations, are given.

2.4 Sample Calculation: A sample calculation is presented to illustrate the use of the basic method.

3. METHOD

3.1 Description: An ejector is a device that utilizes the kinetic energy of a relatively high velocity gas stream to induce the flow of another, lower velocity, stream into a common duct by depressing the static pressure at the point where the high velocity stream enters the mixing duct.

3.1.1 Figures 1 and 2 illustrate schematically the two most generally used types of gas turbine-ejector systems. Figure 1 represents an installation where the compartment and gas turbine engine have separate inlets. This is the most common type of installation encountered. Figure 2 represents an installation where the gas turbine engine receives its airflow from the compartment. Refer to Table I for nomenclature.

3.1.2 The main components of the ejector are as follows:

(a) The ejector primary nozzle, which is also the exit of the engine-mounted tail pipe. This is represented schematically by Station 6. The relatively high velocity exhaust gas enters the ejector at this point.

3.1.2 Continued

(b) The ejector mixing duct, extending from Station 6 to Station 10. In the mixing duct, the kinetic energy of the exhaust gas is partly transferred to the ventilating airflow.

(c) The ejector secondary nozzle, which is the annular area between the engine tail pipe or primary nozzle and the ejector mixing duct at the point where the ventilating airflow enters the ejector. The secondary nozzle is represented by Station 9.

3.1.3 Another component sometimes present in an ejector system is an exhaust diffuser. This device is shown schematically in Fig. 1 and 2 as extending from Station 10 to Station 11. Use of this device will reduce the static pressure at Station 10.

3.2 Flow Path Assumptions:

3.2.1 With the separately ducted configuration illustrated in Figure 1, the engine air flow is assumed to enter the engine inlet duct at Station 1 at a specified pressure and a specified temperature. The nacelle internal ventilating air is assumed to enter at a specified temperature and a specified pressure through a separate entrance at Station 7, from which it flows through the nacelle with a resultant rise in temperature due to heat transfer from the engine and its accessories and a drop in pressure due to flow losses and velocity increases. The engine exhaust leaves the turbine tailpipe (ejector primary nozzle) with a velocity V_6, at a temperature T_6, and with a static pressure P_6. The ventilating-air flow enters the ejector at Station 9 at a temperature equal to T_9, at a velocity, V_9, and at a pressure, P_9, which is assumed to be equal to P_6. In the mixing zone of the ejector, the kinetic energy of the exhaust gas is partly transferred to the ventilating-air flow, so that the exhaust gas velocity decreases and the ventilating-air velocity increases. The mixed stream leaves the ejector at Station 10 or Station 11, as applicable. In most cases, because of the usual physical limitations on mixing zone length, the two streams are not completely mixed. This deviation from perfect mixed conditions is taken into consideration in the basic equations by the use of an empirical mixing constant M. The mixed stream is discharged at a mean temperature T_{10}, at a mean velocity V_{10}, and at a pressure P_{10} or P_{11}, as applicable, which is usually equal to ambient pressure.

3.2.2 With the configuration shown in Fig. 2, the engine and ventilating air enter the nacelle through a common inlet at Station 1. Part of this air enters the engine and the balance flows through the nacelle around the engine and into the ejector. Other assumptions are the same as above for the separate inlet configuration.

3.3 Ejector Performance Requirements: The quantity of nacelle cooling-airflow required for a given engine installation will depend on the amount of heat to be removed in each engine zone and on the maximum allowable temperature of each zone. This may be seen by examining the following equation for the transfer of heat to the ventilating airflow.

$$q = W_c C_p \ (T_b - T_a) \ x \ 60$$

where q = total heat transferred in a given zone, Btu per hour

W_c = nacelle ventilating airflow, lbs per min

T_b = applicable zone ventilating air discharge temperature, °F

T_a = applicable zone ventilating air inlet temperature, °F

C_p = mean specific heat of ventilating air, Btu per (lb-°F)

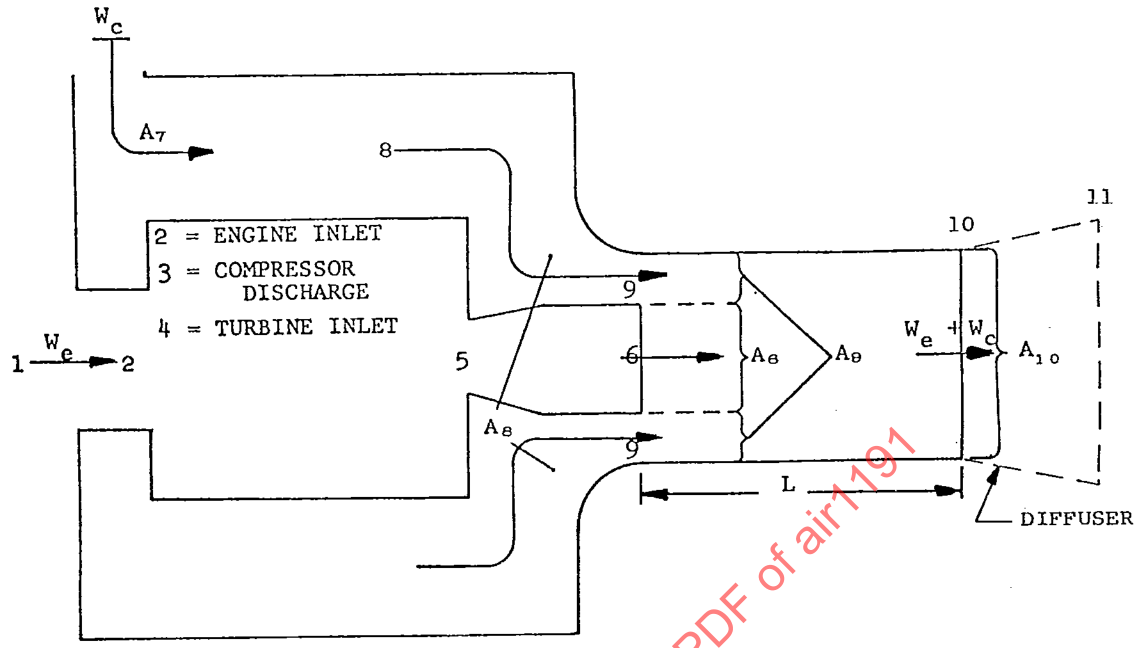


FIGURE 1

FLOW SCHEMATIC AND STATION IDENTIFICATION
FOR GAS TURBINE-EJECTOR INSTALLATION WHERE
ENGINE AND NACELLE HAVE SEPARATE INLETS

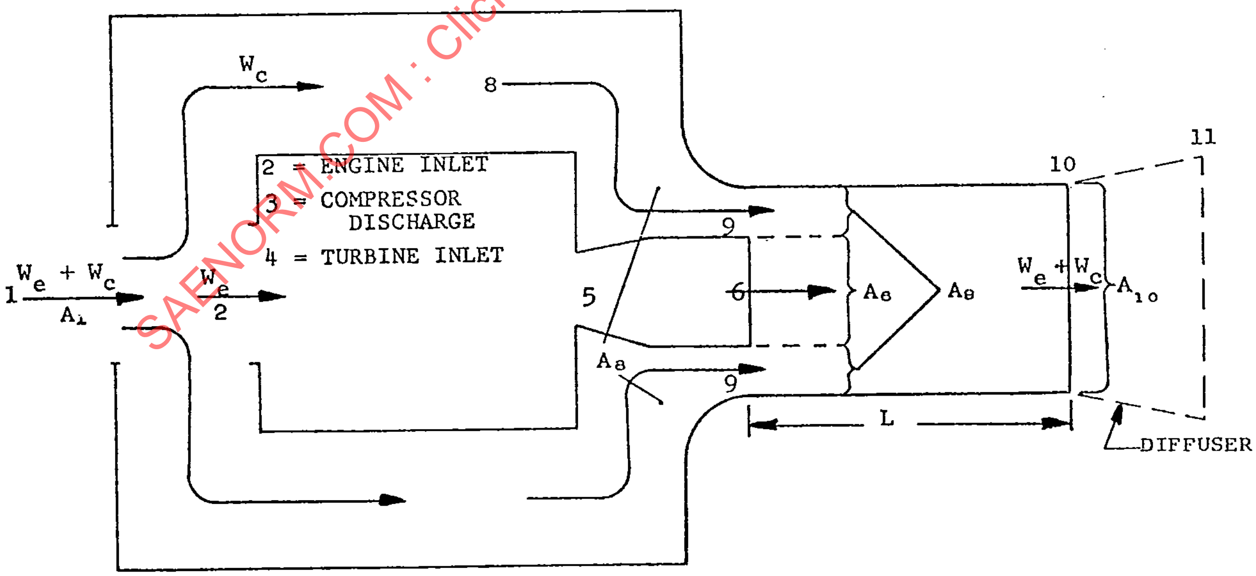


FIGURE 2

FLOW SCHEMATIC AND STATION IDENTIFICATION
FOR GAS TURBINE-EJECTOR INSTALLATION WHERE
ENGINE AIR IS TAKEN FROM THE NACELLE

TABLE I

NOMENCLATURE

A	Cross-sectional area or equivalent orifice area	sq ft
C	Flow coefficient	Dimensionless
D	Diameter	ft or inches
E	Ventilation characteristic	Dimensionless
f	Fanning friction factor	Dimensionless
g	Gravitational constant (32.174)	$ft/sec.^2$
h	Enthalpy	Btu/lb
L	Mixing zone length	ft or inches
M	Mixing constant	Dimensionless
P	Absolute pressure	$lb/in.^2$
ΔP	Differential pressure	in. H_2O
Q	Volume flow	$ft^3/sec.$
R	Gas constant (53.32 for air)	$ft/^oR$
Re	Reynolds number	Dimensionless
T	Absolute temperature	^oR
V	Velocity	ft/sec.
W	Weight flow	lb/sec.
ρ	Density	lb/ft^3
η	Efficiency	Dimensionless
μ	Viscosity	lb/ft-sec.

TABLE I (Continued)

Subscripts

1	Refers to entrance to compartment or engine inlet duct
2	Refers to engine inlet
3	Refers to compressor discharge
4	Refers to engine discharge
5	Refers to engine discharge
6	Refers to exit of engine discharge tail pipe (ejector primary nozzle)
7	Refers to compartment inlet
8	Refers to compartment flow path
9	Refers to ejector secondary nozzle
10	Refers to exit of ejector mixing section
11	Refers to exit of diffuser section
am	Refers to ambient
c	Refers to compartment
d	Refers to diffuser
e	Refers to engine
f	Refers to fan
m	Refers to mixing zone
vp	Refers to velocity pressure

3.3.1 Once the following are known, it is a simple matter to solve the above equation for the required nacelle cooling airflow:

The total heat to be removed in each zone, q.

The inlet temperature of each zone (the discharge temperature of a given zone will be the inlet temperature of the next zone).

The maximum allowable nacelle cooling-air temperature of each zone (this is usually specified in the engine installation manual).

3.3.2 Typical sources of heat release to the nacelle cooling airflow in a gas turbine installation include the following:

Heat transfer from the engine external skin, including the accessory gearbox, the tail pipe, and the external skin of the engine oil cooler.

Heat rejection to the generator cooling air.

Heat rejection to the oil-cooler airflow.

Heat rejection from engine-driven airframe accessories.

3.3.3 The values of heat rejection from engine external surfaces and to the engine lubricating oil are normally supplied by the engine manufacturer. Both of these types of heat rejection vary as a function of engine load and ambient temperature surrounding the engine. The heat rejection from engine external surfaces also typically varies as a function of the cooling air velocity around the engine. Heat rejection to generator cooling air or to oil cooler airflow need to be included in the nacelle cooling load only if these airflows are discharged into the nacelle.

3.4 Recommended Design Procedure:

3.4.1 General Guidelines: The following general guidelines are recommended in the design of gas turbine exhaust ejectors:

(a) The mixing duct should be made adequately long to assure good mixing of the two flow streams. If space permits, a flow length equal to several mixing duct diameters is recommended. The design procedure described herein, however, enables one to design a shorter ejector if the resulting cooling airflow is adequate for the installation.

(b) The ejector should be designed, if possible, with use of a primary nozzle area equal to the engine exhaust area upon which the engine estimated performance is based. If, after investigating various combinations of mixing-duct diameter and length, the nacelle cooling airflow obtained with this tail-pipe configuration is still inadequate, the cooling airflow can be increased by use of a diffuser at the end of the mixing duct to reduce the back pressure or by decreasing the tail-pipe discharge flow area to increase the velocity of the primary flow. It should be recognized that reducing the tail-pipe discharge area will result in a loss in available engine performance.

(c) If turning of the exhaust gas is necessary, this turning should be made as gradually as possible. Abrupt turns and bends in the exhaust duct should be avoided.

(d) The effect of mixing-duct pressure losses (caused by friction and turning of the exhaust gas in the mixing duct) on ejector performance may be treated in the calculation procedure by use of an equivalent frictional loss coefficient, $4fL/D_{10}$.

(e) The ejector primary and secondary nozzles should be as concentric as possible and the center lines of the secondary nozzle and the mixing-duct inlet section should be as parallel as possible in order to promote uniform distribution of the airflow through the secondary nozzle.

(f) The design procedure presented herein is for the primary nozzle throat located as shown in Figure 3(a). However, location (c) or (d) where the primary nozzle throat is located approximately one diameter outside the mixing duct throat gives better performance and location (b) gives poorer performance.

3.4.2 Design Procedure: The design method consists of the solution of the basic equations given in Table II with the use of known and estimated data. The derivation of the equations of Table II is given in Section 5.

The following inputs are required to solve the basic equations:

1. Engine and nacelle inlet temperatures, T_1 and T_7
2. Engine and nacelle inlet pressures, P_1 and P_7
3. Engine discharge temperature, T_6
4. Engine air flow rate, W_e
5. Engine tail-pipe or ejector primary nozzle discharge area, A_6
6. Required compartment cooling-air flow rate, W_c
7. Compartment cooling-air temperature rise, T_9-T_7 or T_9-T_{11} as applicable
8. Compartment inlet "pressure drop", ΔP_7
9. Compartment flow path pressure drop, ΔP_8
10. Ejector discharge pressure, P_{11}
11. Mixing duct discharge diffuser area ratio and efficiency, if applicable

Items 1, 2, 3, and 10 are usually given operating conditions for the gas turbine engine. Item 4 may be read from basic gas turbine performance curves as a function of items 1, 2, and 3. Item 5 is known or specified. Items 6 and 7 are interrelated, since the heat transferred to the ventilating air is equal to these two quantities times the mean specific heat of the ventilating air. If the heat release to the compartment and the maximum allowable compartment temperature are known, the required W_c can be calculated from these quantities. Items 8 and 9 can be estimated from the geometry of the compartment and the required ventilating flow or can be specified as design requirements for the compartment if the enclosure configuration has not yet been specified.

3.4.2.1 Engine and Nacelle Separately Ducted: Once the above inputs have been defined, the procedure is as follows for the separately ducted configuration of Figure 1:

1. The equivalent orifice areas, A_7 and A_8 of the compartment flow path are calculated with equation (17a)*. As a word of explanation, since the flow area of the compartment is usually large compared with the compartment inlet and exit (ejector inlet) areas, the ventilating air pressure drops may be considered to be caused by flow through throttling areas A_7 and A_8. This is further explained in Section 5.

2. The ventilating-air characteristic is calculated with the use of equation (22)*.

*See Table II

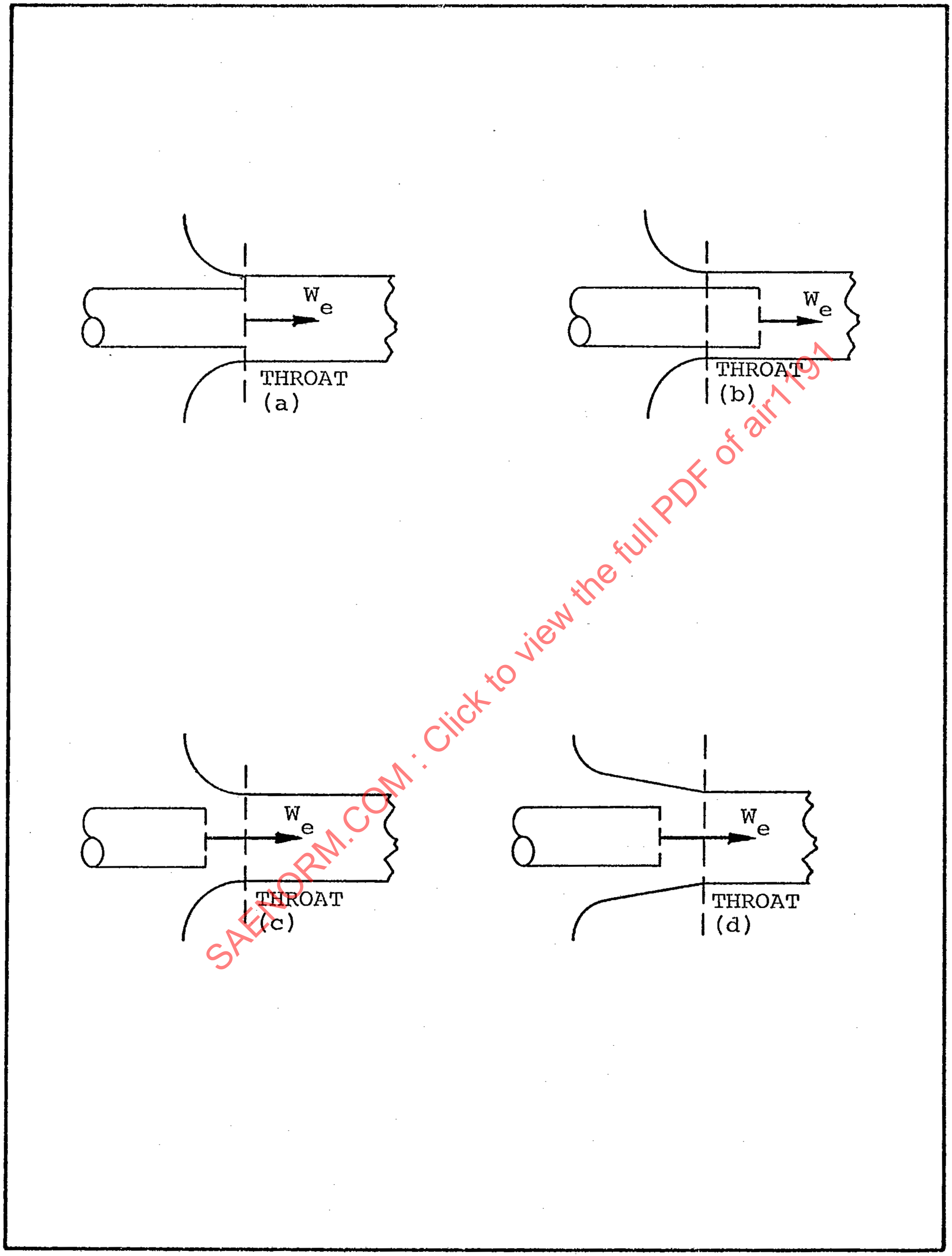


FIGURE 3. ALTERNATE THROAT LOCATIONS

TABLE II

BASIC EQUATIONS

Equivalent Orifice Areas

$$A = 0.0333\ W \sqrt{\frac{T}{P \Delta P}} \qquad (17a)$$

where the units are as follows:

$T = {}^{o}R$

$P = \text{psia}$

$\Delta P = \text{inches } H_2O$

$W = \text{lb/sec.}$

For engine and nacelle with separate inlets

Ventilation Characteristic

$$E = \frac{T_7}{T_9}\left(\frac{A_6}{A_7}\right)^2 + \frac{T_8}{T_9}\left(\frac{A_6}{A_8}\right)^2 \qquad (22)$$

Basic Design Equation

$$\frac{W_c}{W_e} = x \qquad (27)$$

where x = positive root of $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (31)

$$\text{where } a = \frac{T_9}{T_6}\left\{\frac{1}{2M}\left[E\left(\frac{A_{10}}{A_6}\right)^2 + \left(\frac{A_{10}}{A_9}\right)\left(\frac{A_{10}}{A_9} - 2\right)\right] + y\right\} \qquad (28)$$

$$b = y\left(1 + \frac{T_9}{T_6}\right) \qquad (29)$$

$$c = y - \frac{1}{M}\left(\frac{A_{10}}{A_6}\right)\left[1 + \frac{1}{2}\left(\frac{A_{10}}{A_6}\right)\left(\frac{P_7 - P_{11}}{P_{vp_6}}\right)\right] \qquad (30)$$

$$y = 1 + \frac{\left(\frac{4fL}{D_{10}}\right)}{2M}\ \frac{\eta_D}{2M}\left[1 - \frac{A_{10}}{A_{11}}\right]^2 \qquad (24a)$$

Values of M may be read from Figures 5a through 5d as a function of

$$\frac{L}{D_{10}},\ \left(\frac{W_c}{W_e}\right)\left(\frac{T_9}{T_6}\right)^{0.5}, \text{ and } \frac{A_{10}}{A_6}$$

TABLE II (Continued)

Primary Nozzle Discharge Conditions

$$P_{vp_6} = \frac{RT_6 \; W_e^{\,2}}{2g \; P_6 \; A_6^{\,2}} \tag{9}$$

$$\frac{P_{11} - P_6}{P_{vp_6}} = \frac{T_9}{T_6}\left(\frac{W_c}{W_e}\right)^2 \left[E + \frac{A_6}{A_9}^{\,2}\right] - \left(\frac{P_7 - P_{11}}{P_{vp_6}}\right) \tag{23a}$$

For engine and nacelle with a common inlet

Ventilation Characteristic

$$E = \left(\frac{T_1}{T_9}\right)\left(\frac{A_6}{A_1}\right)^2 + \left(\frac{T_8}{T_9}\right)\left(\frac{A_6}{A_8}\right)^2 \tag{38}$$

Basic Design Equation

$$\frac{W_c}{W_e} = x \tag{46}$$

where x = positive root of $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\text{where } a = \frac{T_9}{T_6}\left\{\frac{1}{2M}\left[E\left(\frac{A_{10}}{A_6}\right)^2 + \left(\frac{A_{10}}{A_9}\right)\left(\frac{A_{10}}{A_9} - 2\right)\right] + Y\right\} \tag{47}$$

$$b = y\left(1 + \frac{T_9}{T_6}\right) + \frac{1}{M}\left(\frac{A_{10}}{A_6}\right)\left(\frac{T_1}{T_6}\right)\left(\frac{A_6}{A_1}\right)\left(\frac{A_{10}}{A_1}\right) \tag{48}$$

$$c = y - \frac{1}{M}\left(\frac{A_{10}}{A_6}\right)\left\{1 - \frac{1}{2}\left[\left(\frac{T_1}{T_6}\right)\left(\frac{A_{10}}{A_1}\right)\left(\frac{A_6}{A_1}\right) - \left(\frac{A_{10}}{A_6}\right)\left(\frac{P_1 - P_{11}}{P_{vp_6}}\right)\right]\right\} \tag{49}$$

$$y = 1 + \frac{4fL/D_{10}}{2M} - \frac{\eta_D}{2M}\left[1 - \left(\frac{A_{10}}{A_{11}}\right)^2\right] \tag{24a}$$

Values of M may be read from Figures 5a through 5d as a function of

$$\frac{L}{D_{10}}, \left(\frac{W_c}{W_e}\right)\left(\frac{T_9}{T_6}\right)^{0.5}, \text{ and } \frac{A_{10}}{A_6}$$

TABLE II (Continued)

Primary Nozzle Discharge Conditions

$$P_{vp_6} = \frac{R\,T_6\,W_e^2}{2g\,P_6\,A_6^2} \qquad (9)$$

$$\frac{P_{11} - P_6}{P_{vp_6}} = \left(\frac{T_9}{T_6}\right)\left(\frac{W_c}{W_e}\right)^2 \left[E + \left(\frac{A_6}{A_9}\right)^2\right] + \left(\frac{T_1}{T_6}\right)\left(\frac{A_6}{A_1}\right)^2 \left[1 + 2\,\frac{W_c}{W_e}\right] - \left(\frac{P_1 - P_{11}}{P_{vp_6}}\right) \qquad (41)$$

3.4.2.1 Continued

3. A value of mixing length to mixing diameter ratio, L/D_{10}, is assumed.

4. Using the inputs listed above, the information calculated in steps 1 and 2, and the assumption in step 3, the flow ratio, W_c/W_e, is calculated with use of Equations (27)* through (31)* by iterating on the ejector area ratio, A_{10}/A_6, until the specified value of W_c/W_e is satisfied.

5. The required value of Fanning friction factor, f, shown in the equations may be estimated from data available in the literature for pipe flow as a function of the average Reynolds number, $(W_e + W_c) D_{10}/(A_{10}\ \eta_{10})$, of the flow in the mixing duct. On the other hand, a 3:1 variation in the magnitude of f does not significantly influence the ejector performance and an estimated of f = 0.003 is a reasonable approximation to use in most design calculations.

6. Figure 4 may be used to obtain a preliminary estimate of A_{10}/A_6 for the first iteration. This curve presents values of (W_c/W_e) as a function of ventilation characteristic E, ejector area ratio, A_{10}/A_6, and temperature ratio, T_9/T_6 for the case where the mixing constant M equals 1.17.

   For each iteration, the mixing constant M is determined as a function of A_{10}/A_6 and the required

   value of $\frac{W_c}{W_e}\left(\frac{T_9}{T_6}\right)^{0.5}$ with the use of Figures 5a through 5d.

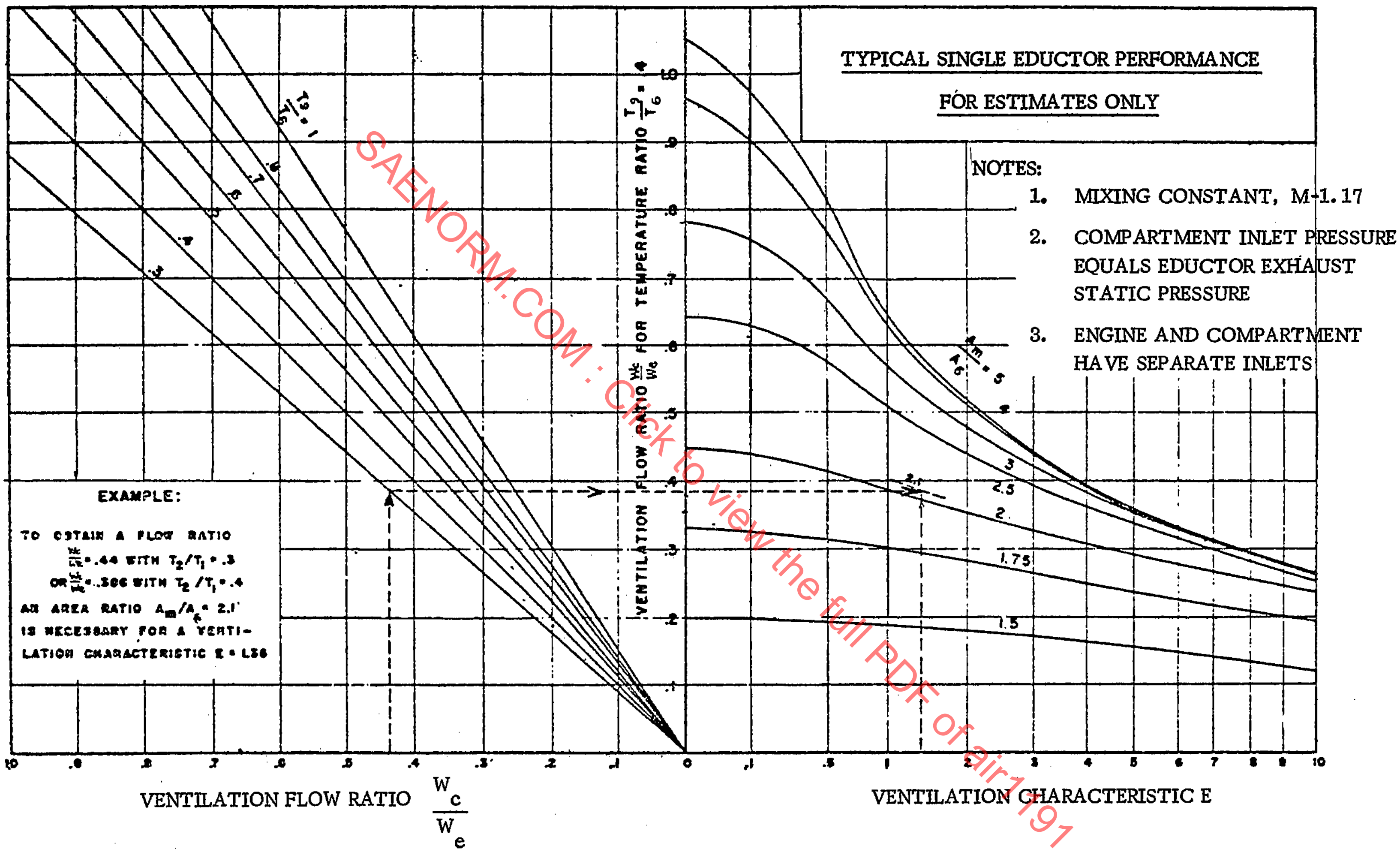
7. If, for the assumed values of A_6 and L, the value of A_{10}/A_6 satisfying the required value of W_c/W_e is unacceptable, or no solution is found at all, it may be necessary to incorporate an exhaust diffuser, increase the value of L, or decrease the value of A_6 in order to obtain a satisfactory solution.

3.4.2.2 Engine and Nacelle with a Common Inlet: The procedure to be followed in the case of an engine-ejector system with a common inlet is the same as that presented above except that:

1. The equivalent orifice area A_1 is calculated instead of A_7.

2. The equations listed in Table II for the configuration with a common inlet for engine and nacelle are used in lieu of those referenced above for the separately ducted configuration.

*See Table II



APPROXIMATE VENTILATION FLOW RATIOS AS FUNCTION OF VENTILATION CHARACTERISTICS AND TEMPERATURE RATIOS FOR VARIOUS AREA RATIOS A_{10}/A_6

FIGURE 4

REF: PENNSYLVANIA STATE COLLEGE
EDUCTOR DESIGN MANUAL
INDEX NO. NS622-078

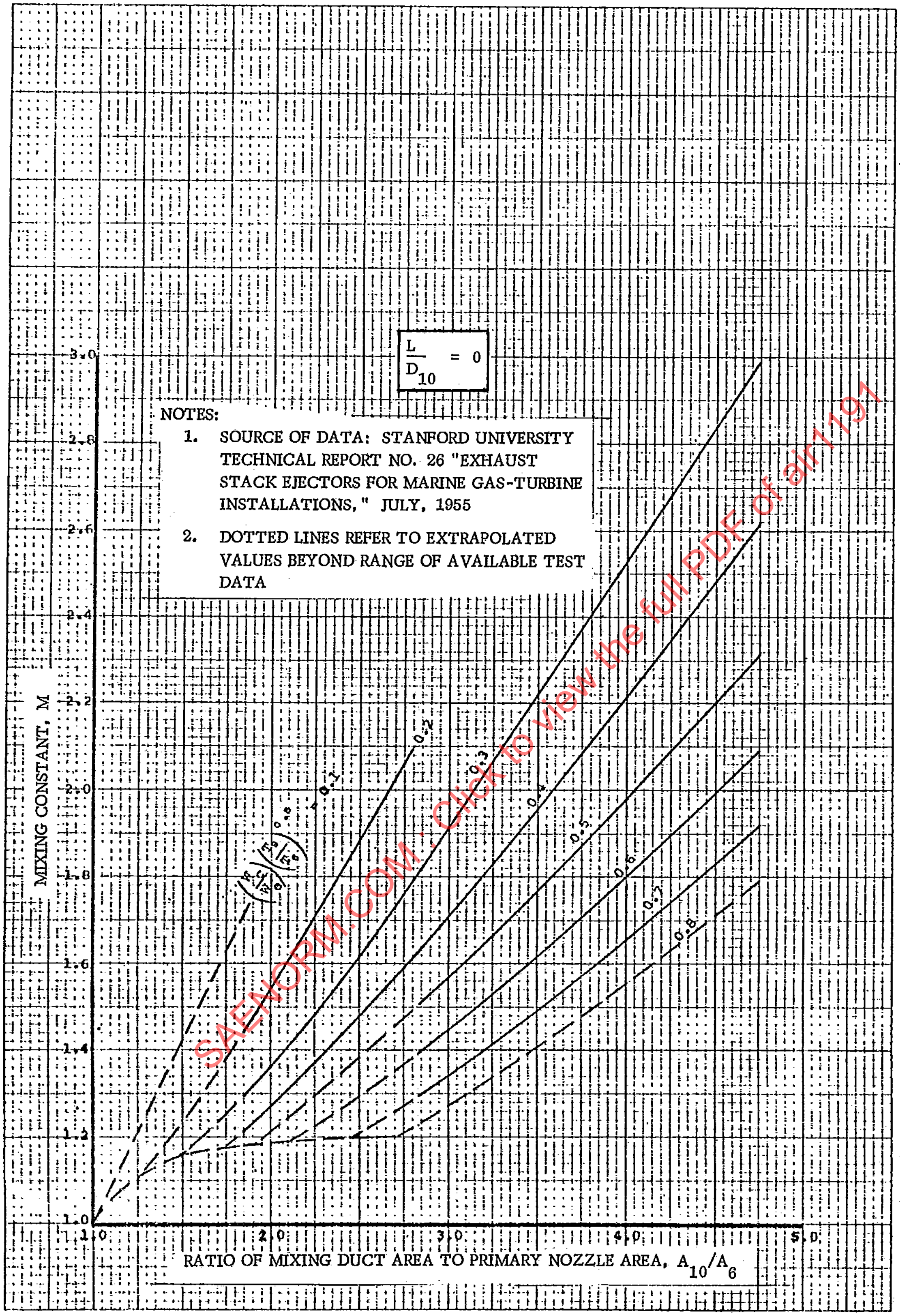


FIGURE 5a. MIXING CONSTANT DATA FOR $L/D_{10} = 0$

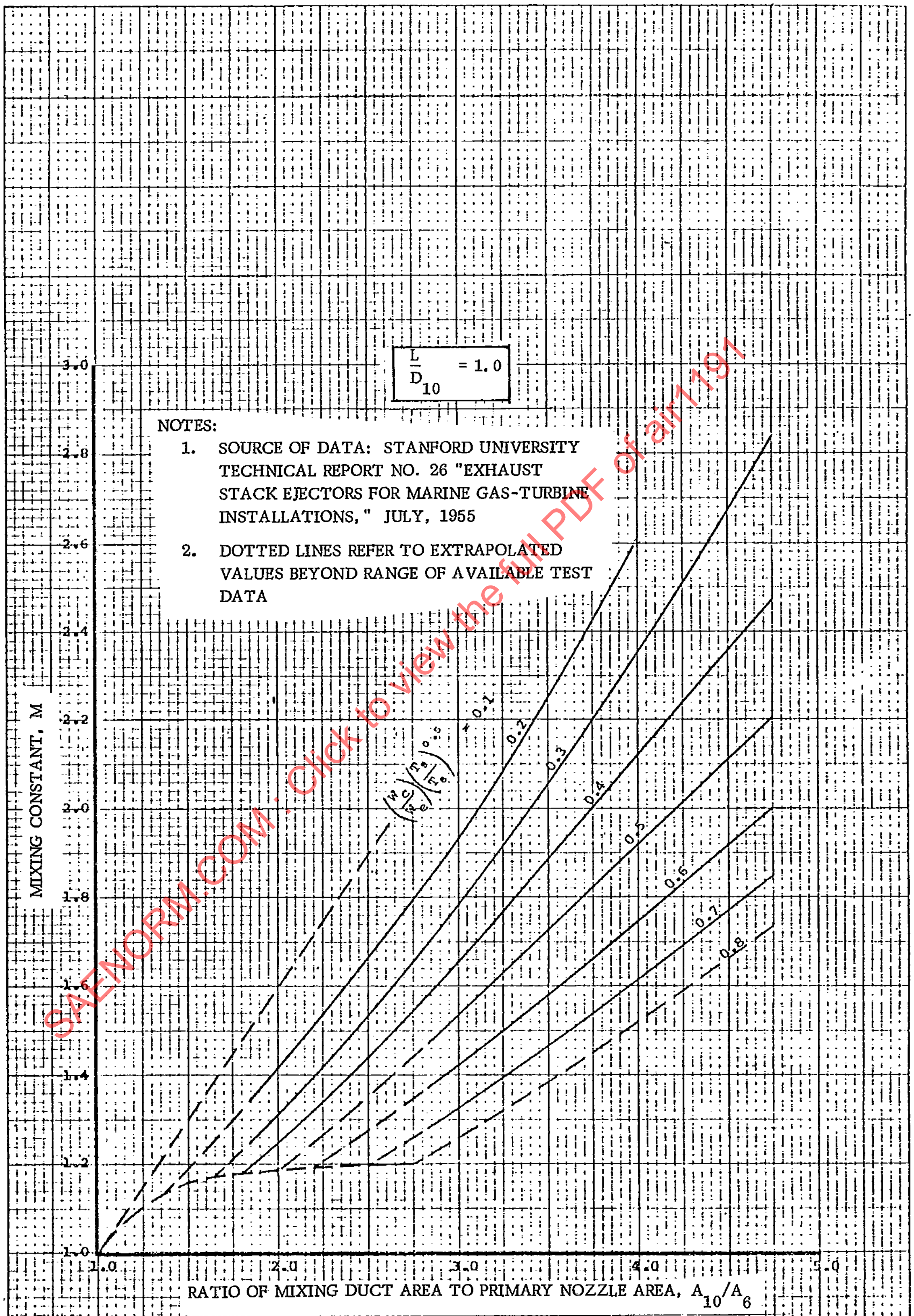


FIGURE 5b. MIXING CONSTANT DATA FOR $L/D_{10} = 1.0$

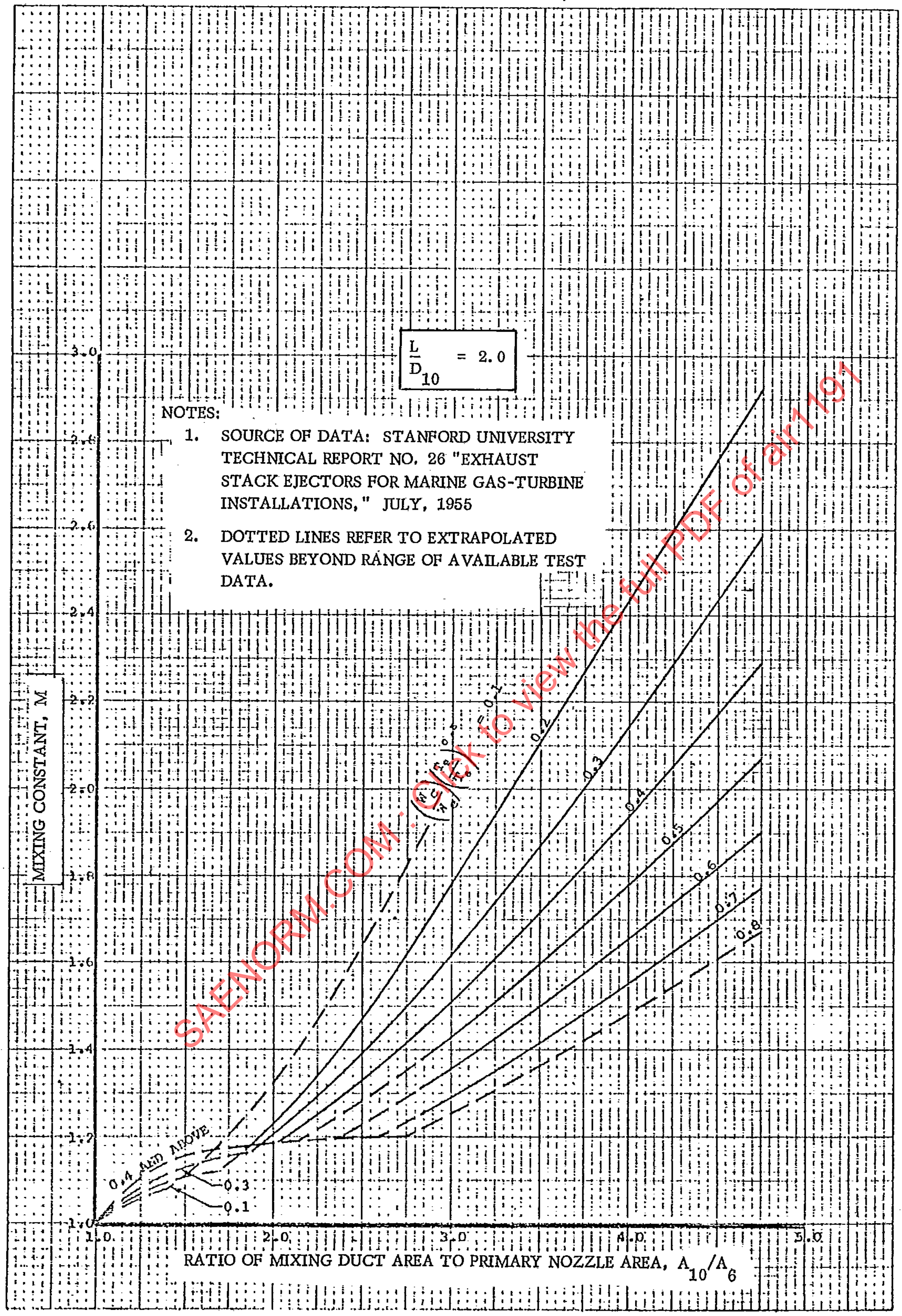


FIGURE 5c. MIXING CONSTANT DATA FOR $L/D_{10} = 2.0$

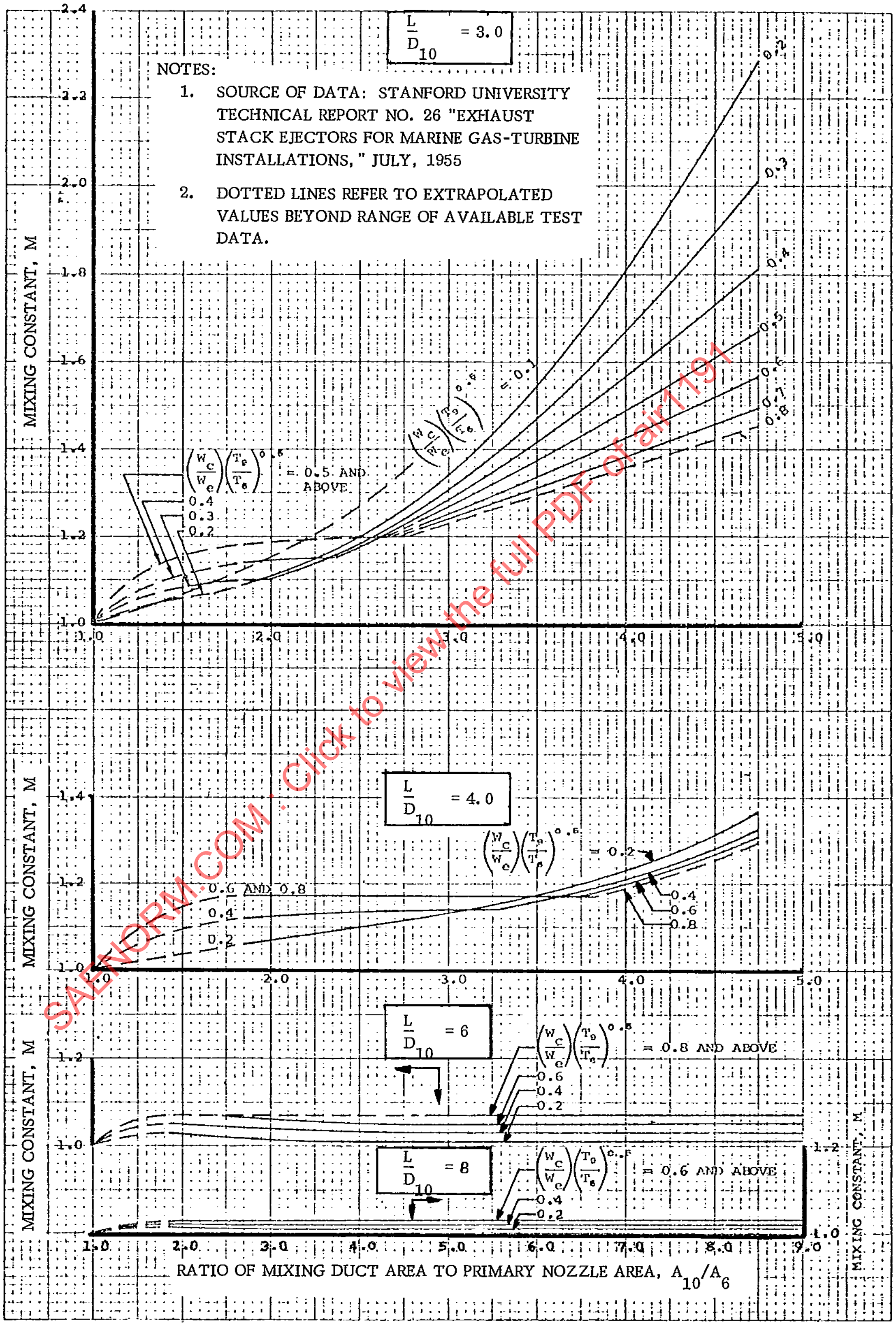


FIGURE 5d. MIXING CONSTANT DATA FOR L/D_{10} = 3, 4, 6, and 8

3.4.2.3 Effect of a Fan Operating in Series with Ejector: A fan operating within the enclosure will have an effect on the ejector performance. Since there is a multitude of combinations that could exist for this case, no general case is presented in Table II. However, the following general rules may be followed in certain specific instances:

1. If the fan operates in series with the ejector and all the cooling air flow, W_c, passes through the fan, the pressure rise through the fan, ΔP_f, will have the same effect on cooling airflow rate as an increase in compartment inlet pressure of an equal amount. Therefore, for a separately ducted inlet configuration, replace the term $(P_7 - P_{11}) / P_{vp_6}$ with the term $\left[(P_7 - P_{11}) + \frac{\Delta P_f}{27.7} \right] \Big/ P_{vp_6}$ in the applicable equations. For a configuration with common engine and compartment inlet, replace the term $(P_1 - P_{11}) / P_{vp_6}$ with the term $\left[(P_1 - P_{11}) + \frac{\Delta P_f}{27.7} \right] \Big/ P_{vp_6}$ in the applicable equations. The term ΔP_f is input in units of inches H_2O and is divided by 27.7 to convert it to psi units to be compatible with P_1, P_7, and P_{11}. The final value of fan ΔP_f and cooling flow would be obtained by iteration to satisfy both the fan and ejector characteristics, as illustrated in Figure 6.

2. If the fan operates in parallel with another compartment inlet, the fan will assist the ejector only from the standpoint of decreasing the "pressure drop" ΔP_7 or increasing the effective orifice area, A_7, for a given total compartment flow rate. For this case, the compartment flow minus the fan flow is used in calculating the effective orifice area, A_7. The total compartment flow is used in calculating the effective orifice area, A_8. The final match point must satisfy both the fan and compartment inlet pressure drop characteristics, as illustrated in Figure 7. If the ejector is not capable of handling all of the fan airflow, part of the fan airflow will be discharged through the compartment inlet. The equations and their derivation for this case are given in Section 5.

3.4.2.4 Alternate Method: An alternate method of solving the ejector problem is to graphically match the flow-pressure rise characteristics of the ejector and the flow-pressure drop characteristics of the system being powered by the ejector.

The flow-pressure rise characteristics of the ejector may be calculated for a specified geometry with the use of Equation (15b) of Section 5 by assuming a range of values of W_c/W_e. Figure 8 illustrates the matching of the ejector and system characteristics. Note that any difference in pressures P_7 and P_{11} and the pressure drop due to the velocity increase of the nacelle cooling air in entering the ejector secondary nozzle at Station 9 must be included in the system pressure drop.

This method may be used to match the ejector to any system.

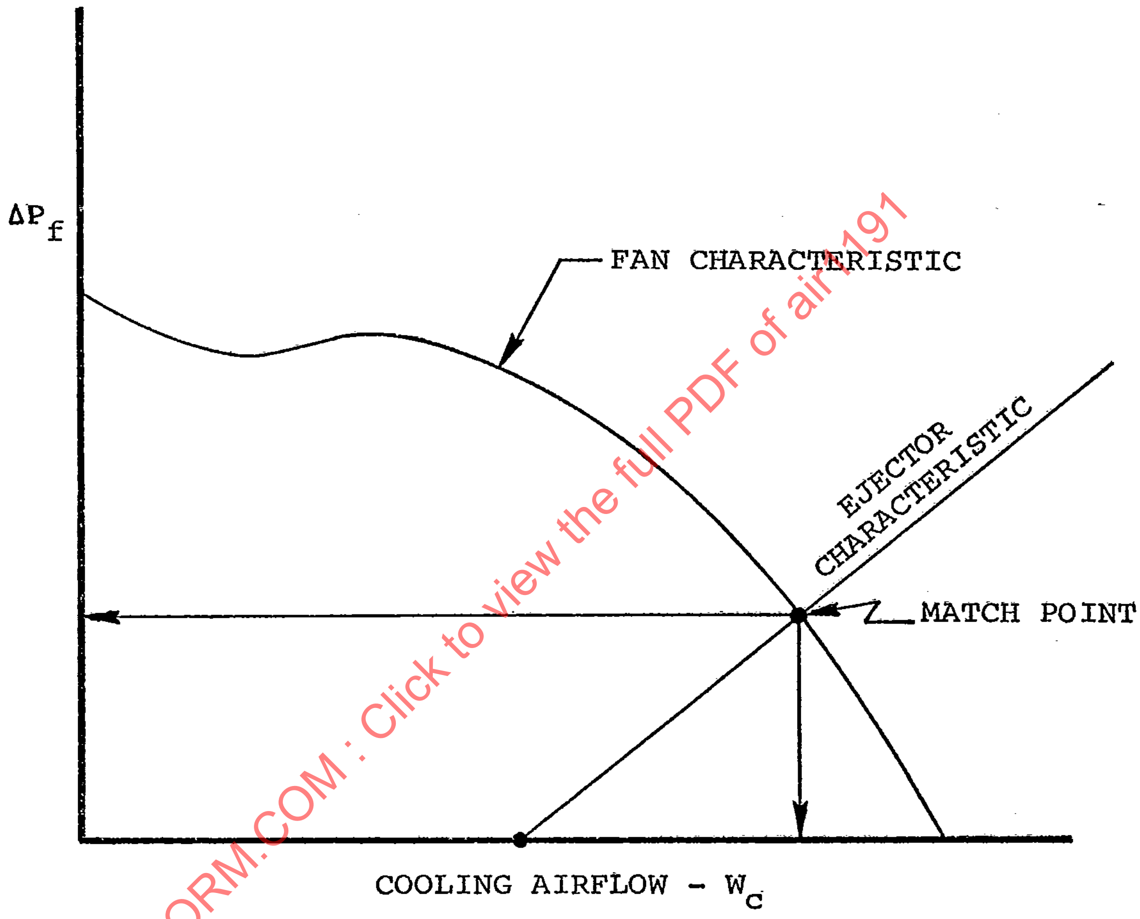


FIGURE 6. ILLUSTRATION OF MATCHING OF FAN AND EJECTOR CHARACTERISTIC WITH FAN OPERATING IN SERIES WITH COMPARTMENT INLET; ENGINE AND NACELLE SEPARATELY DUCTED

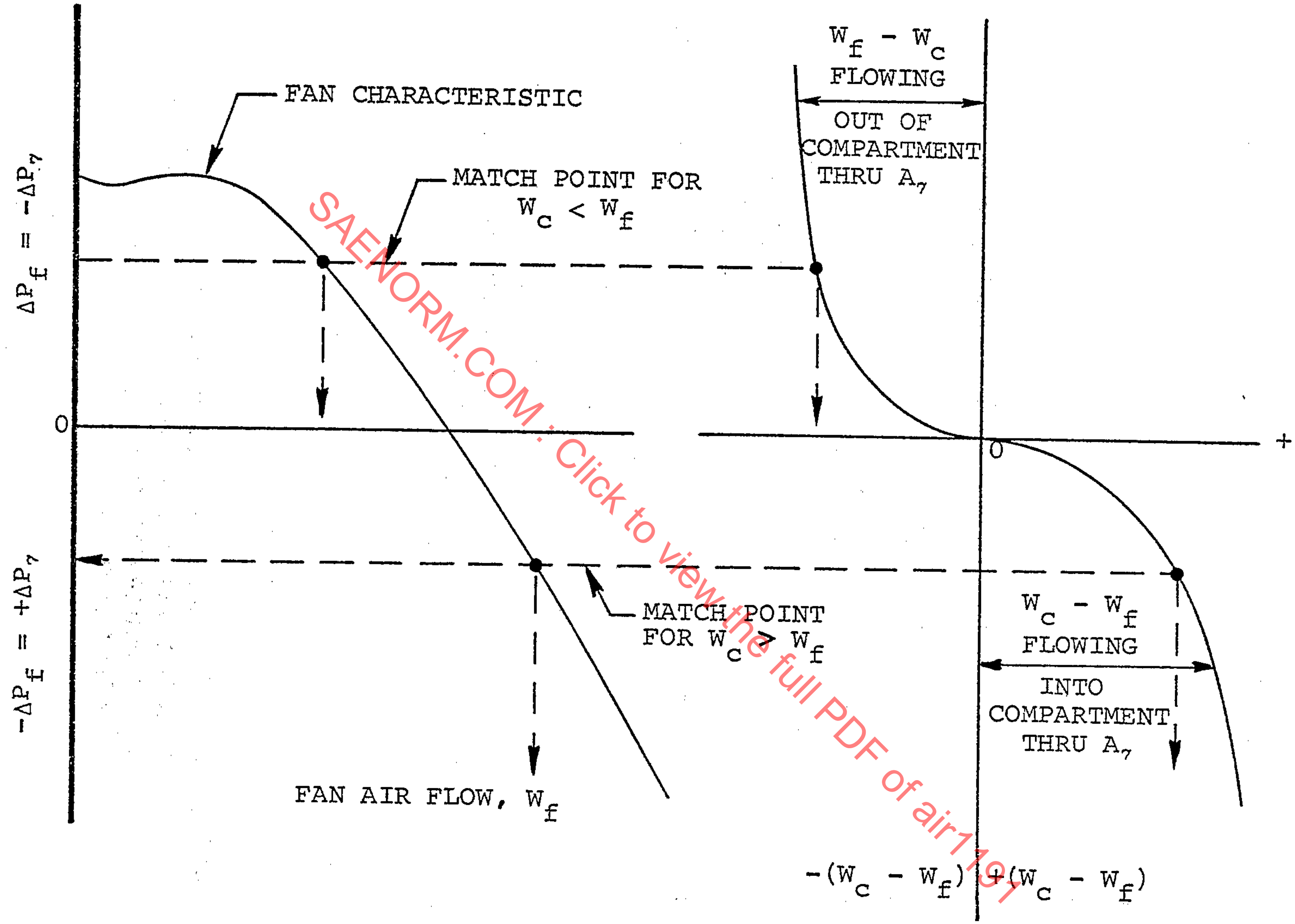


FIGURE 7. ILLUSTRATION OF MATCHING OF FAN AND INLET A_7 CHARACTERISTICS WITH FAN OPERATING IN PARALLEL WITH COMPARTMENT INLET; ENGINE AND NACELLE SEPARATELY DUCTED

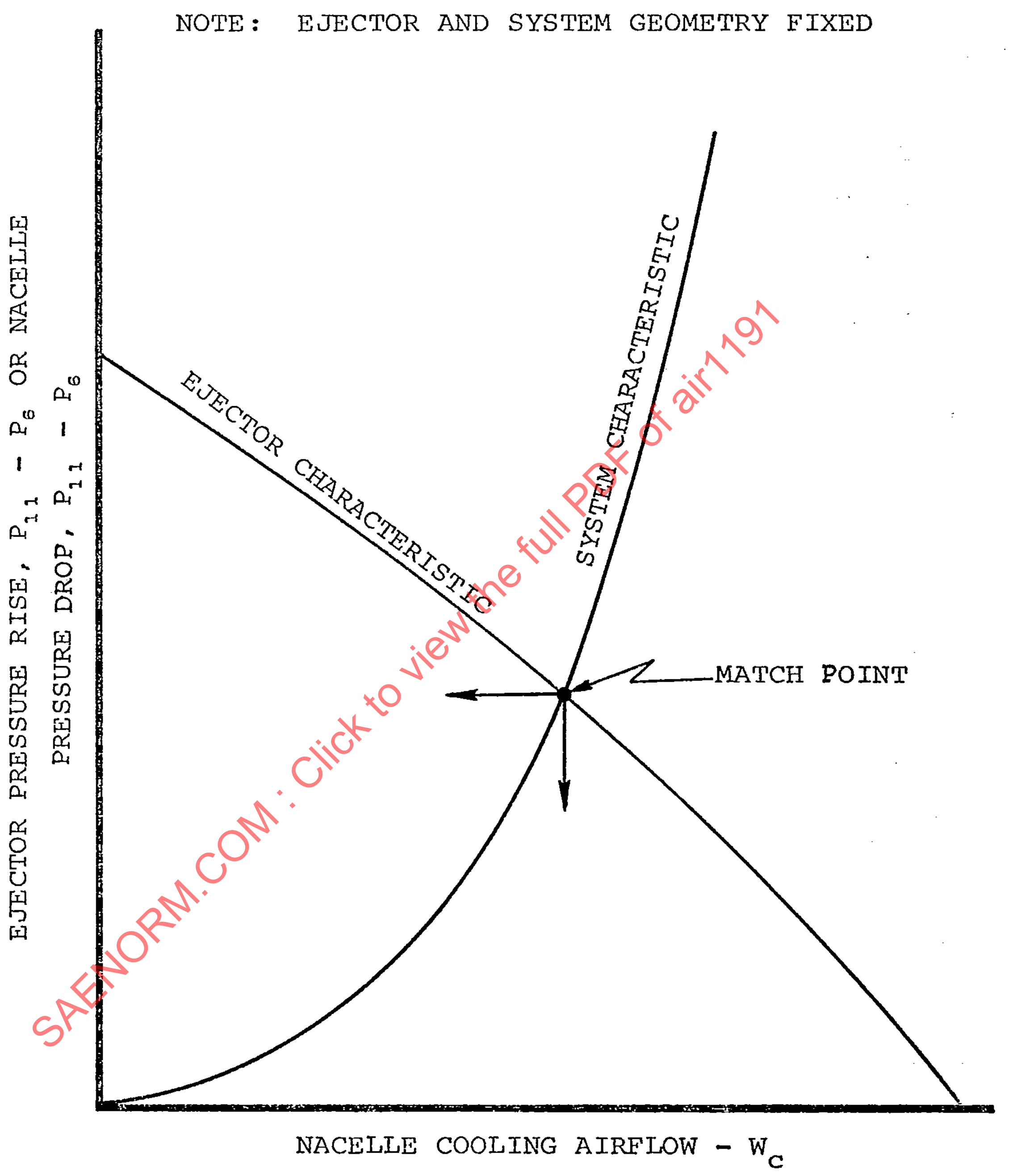


FIGURE 8. ILLUSTRATION OF GRAPHICAL MATCHING OF EJECTOR PRESSURE RISE CHARACTERISTIC WITH SYSTEM PRESSURE DROP CHARACTERISTIC

4. SAMPLE CALCULATION

4.1 Input Quantities:

(1)	Engine and nacelle inlet duct arrangement	Separate
(2)	Nacelle inlet temperature, T_7, ^oR	560
(3)	Nacelle inlet pressure, P_7, psia	14.7
(4)	Turbine discharge temperature, T_6, ^oR	1700
(5)	Engine air flow, W_e, lb/sec.	3.83
(6)	Turbine tail-pipe (primary nozzle) discharge flow area, A_6, sq ft	0.60
(7)	Compartment cooling-air flow required, W_c, lb/sec.	0.66
(8)	Compartment cooling-air temperature rise, ^oF	35
(9)	Compartment inlet pressure drop ΔP_7, in. H_2O	0.2
(10)	Compartment flow path pressure drop ΔP_8, in. H_2O	0.2
(11)	Ejector discharge pressure, P_{11}, psia	14.7
(12)	Diffuser area ratio, A_{11}/A_{10}	1.00
(13)	Diffuser efficiency, dimensionless	--
(14)	Estimated friction factor	0.003

4.2 Compartment and Ventilating-Air Characteristics:

4.2.1 Equivalent Orifice Area:

(15)	$A_7 = 0.0333 \; (7) \sqrt{(2) / ((3) \times (9))}$, sq ft	0.303
(16)	$A_8 = 0.0333 \; (7) \sqrt{((2) + (8)) / ((3) \times (10))}$, sq ft	0.313

4.2.2 Ventilating Characteristic (Equation 22):

(17)	$T_9 = (2) + (8)$, ^oR	595
(18)	$T_7/T_9 = (2) / (17)$	0.941
(19)	$T_8/T_9 = 1.0$	1.00
(20)	$(A_6/A_7)^2 = [(6) \div (15)]^2$	3.92
(21)	$(A_6/A_8)^2 = [(6) \div (16)]^2$	3.67
(22)	$(T_7/T_9)(A_6/A_7)^2 = (18) \times (20)$	3.69
(23)	$(T_8/T_9)(A_6/A_8)^2 = (19) \times (21)$	3.67
(24)	$E = (22) + (23)$	7.36

4.2.2 Continued:

(25) T_9/T_6 = (17) / (4) 0.35

4.2.3 Ejector Design Solution:

Item	Expression	Trial No. 1	Trial No. 2	Trial No. 3
(26)	A_{10}/A_6 assume (refer to Figure 4)*	2.0	1.5	1.40
(27)	A_{10} = (26) x (6), sq ft	1.20	0.900	0.840
(28)	A_9 = (27) - (6), sq ft	0.600	0.300	0.240
(29)	$D_{10} = \sqrt{(27) \div 0.7854}$, ft	1.236	1.070	1.034
(30)	(L/D_{10}) (assume)	3.0	3.0	3.0
(31)	$\left(\frac{W_c}{W_e}\right)\left(\frac{T_9}{T_6}\right)^{0.5} = \frac{(7)}{(5)}\sqrt{(25)}$	0.102	0.102	0.102
(32)	M enter Figures 5a through 5f with (26), (30) and (31)	1.15	1.07	1.05
(33)	L = (30) x (29), feet	3.70	3.21	3.10
(34)	$(A_{10}/A_6)^2$ = $(26)^2$	4.00	2.25	1.96
(35)	A_{10}/A_9 = (27) ÷ (28)	2.000	3.00	3.50
(36)	$4fL/D_{10}$ = 4 x (14) x (30)	0.036	0.036	0.036
(37)	$y = 1 + \frac{(36)}{2\,(32)} - \frac{(13)}{2\,(32)}\left[1 - \frac{1}{(12)^2}\right]$	1.0156	1.0156	1.0156
(38)	$E\ (A_{10}/A_6)^2$ = (24) x (34)	29.44	16.56	14.43
(39)	$\left[(A_{10}/A_9)\right] -2$ = (35) -2	0.0	1.000	1.500
(40)	$(A_{10}/A_9)\left[(A_{10}/A_9) - 2\right]$ = (35) x (39)	0.0	3.000	5.25
(41)	$1/2M \left\{E\ (A_{10}/A_6)^2 + (A_{10}/A_9)\left[(A_{10}/A_9) - 2\right]\right\}$ = [(38) + (40)] ÷ [2 x (32)]	12.80	9.140	9.371
(42)	a, Eq. (28) = [(41) + (37)] x (25)	4.835	3.554	3.635
(43)	b, Eq. (29) = (1 + (25)) x (37)	1.371	1.371	1.371
(44)	$(W_e/A_6)^2 = \left[(5) / (6)\right]^2$	40.75	40.75	40.75

* The first assumption here is intentionally selected as a value not agreeing with Figure 4 to demonstrate the rapid convergence of the final answer.

4.2.3 <u>Continued:</u>

		Trial No. 1	Trial No. 2	Trial No. 3
(45)	$R\ T_6/(2g\ P_6) = \dfrac{[53.32\ (4)]}{2 \times 32.174 \times (11) \times 144}$	0.6656	0.665	0.665
(46)	$P_{vp_6} = [(44) \times (45)]\ /144$, psi	0.1883	0.1883	0.1883
(47)	$c = (37) - \dfrac{(26)}{(32)}\left[1 + \dfrac{1}{2}(26)\dfrac{(3) - (11)}{(46)}\right]$	-0.7235	-0.3863	-0.3177
(48)	$b^2 = (43)^2$	1.8796	1.8796	1.8796
(49)	$4ac = 4 \times (42) \times (47)$	-13.992	-5.492	-4.619
(50)	$b^a - 4ac = (48) - (49)$	15.871	7.372	6.499
(51)	$\sqrt{b^2 - 4ac} = \sqrt{(50)}$	3.984	2.715	2.549
(52)	$-b + \sqrt{b^2 - 4ac} = (51) - (43)$	2.613	1.344	1.178
(53)	$W_c/W_e = (52) \div [2\ (42)]$	0.2902	0.1890	0.1620
(54)	Req'd $W_c/W_e = (7) \div (5)$	0.1724	0.1714	0.1724

If (53) does not agree with (54) within a reasonable tolerance go back to step (26) and make a new assumption on A_{10}/A_6.

		Trial No. 1	Trial No. 2	Trial No. 3
(55)	(A_{10}/A_6) final (by cross-plotting)			1.435
(56)	(A_{10}) final $= (55) \times (6)$, sq. ft.			0.861
(57)	(D_{10}) final $= \sqrt{(56) \div 0.7854}$, feet			1.047
(58)	(L) final $= (30) \times (57)$, feet			3.14

4.2.4 <u>Turbine Tail-Pipe (Primary Nozzle) Discharge Conditions</u>:

(59)	$(T_9/T_6)\ (W_c/W_e)^2 = (25) \times (54)^2$	0.0104
(60)	$A_9 = (56) - (6)$, sq ft	0.261
(61)	$(A_6/A_9)^2 = [(6) \div (60)]^2$	5.285
(62)	$E + (A_6/A_9)^2 = (24) + (61)$	12.64
(63)	$(P_{11} - P_6)\ /\ P_{vps} = (59) \times (62) - ((3) - (11))/ (46)$	0.1314
(64)	$P_{11} - P_6 = (63) \times (46) \times 27.7$, in. H_2O	0.68

4.2.5 Check on Assumed Friction Factor:

*(65)	$W_c / (W_c + W_e)$ = (7)/((5) + (7))	0.147
(66)	$W_e/(W_c + W_e)$ = (5)/((5) + (7))	0.853
(67)	T_{10} = (65) x (17) + (66) x (4), ^oR (Eq. 14)	1538
(68)	Viscosity, μ, from Reference 5 as a function of (67), lb/ft-sec	2.57×10^{-6}
(69)	Reynolds number, Re = $\frac{((5) + (7)) \times (57)}{(56) \times (68)}$	2.124×10^6
*(70)	Fanning friction factor, f, from Reference 6 for a smooth pipe, as a function of (69)	0.004

Since a 3:1 variation in the magnitude of f does not significantly affect ejector performance, the initial estimate of f = 0.003 is satisfactory.

5. DERIVATION OF BASIC EQUATIONS

5.1 Equations for the Case Where the Engine and Compartment have Separate Inlets:

5.1.1 Momentum Equation for the Ejector Mixing Duct: The relationships between pressure, flow, and velocity for the simple case of an ejector with a mixing zone of constant cross-sectional area may be conveniently expressed by the momentum equation, which states that:

$$(\text{Momentum})_{in} - (\text{Momentum})_{out} = \left[(P_s)_{out} - (P_s)_{in} \right] \times A_{10} + \text{Wall Friction} \qquad (1)$$

where momentum equals mass flow rate x velocity.

The wall friction is a numerically small item but can be estimated from available friction factor data.

5.1.1.1 In most cases, because of the usual physical limitations on mixing zone length, the two flow streams are not completely mixed, since a mixing zone length of the order of 10 mixing duct diameters would be required for complete mixing. As a result, the outgoing momentum is, in general, different from that for a homogeneous mean velocity. To take this effect into account, a "mixing constant" M is defined by the following equation:

$$(\text{Momentum})_{actual} = M \times (\text{Momentum})_{mean} \qquad (2)$$

The factor M is equal to unity for complete mixing, but for incomplete mixing, its value increases. Measured values[4] of M, which are used in the design method presented herein, are shown in Fig. 5a through 5d of Section 3.

[4] See References, Section 6

* Steps (65) through (70) may be eliminated in most cases without serious loss of accuracy.

5.1.1.2 Referring to Fig. 1 and Table I of Section 3 for nomenclature and station identification, Equation (1) for this case may be written as:

$$\frac{W_c V_9}{g} + \frac{W_e V_6}{g} - \frac{(W_c + W_e)}{g} V_{10}M = (P_{10} - P_6) A_{10} + \frac{4fL}{D_{10}} A_{10} \rho_{10} \frac{V_{10}^2}{2g} \tag{3}$$

from continuity:

$$W = \rho AV \tag{4}$$

Substituting from (4) into (3):

$$(P_{10} - P_6) A_{10} = \frac{(\rho_9 A_9 V_9) V_9}{g} + \frac{(\rho_6 A_6 V_6) V_6}{g} - \frac{(\rho_{10} A_{10} V_{10}) V_{10} M}{g} - \frac{4fL}{D_{10}} \rho_{10} A_{10} \frac{(V_{10})^2}{2g} \tag{5}$$

Dividing both sides of equation (5) by $\frac{\rho_6 V_6^2}{2g}$ gives:

$$\frac{(P_{10} - P_6) A_{10}}{\frac{\rho_6 V_6^2}{2g}} = 2A_6 + 2\frac{\rho_9}{\rho_6} A_9 \left(\frac{V_9}{V_6}\right)^2 - 2M \frac{\rho_{10}}{\rho_6} A_{10} \left(\frac{V_{10}}{V_6}\right)^2 - \frac{4fL}{D_{10}} \left(\frac{\rho_{10}}{\rho_6}\right) A_{10} \left(\frac{V_{10}}{V_6}\right)^2 \tag{6}$$

For a perfect gas:

$$\frac{P}{\rho} = RT \tag{7}$$

$$\text{or} \quad \rho = \frac{P}{RT} \tag{7a}$$

Solving equation (4) for V:

$$V = \frac{W}{\rho A} \tag{4a}$$

Substituting the term P_{vp_6} for $\frac{\rho_6 V_6^2}{2g}$ (turbine velocity pressure)

$$P_{vp_6} = \frac{\rho_6 V_6^2}{2g} \tag{8}$$

Substituting from (4a) into (8):

$$P_{vp6} = \frac{\rho_6 W_6^2}{2g \rho_6 A_6^2} = \frac{W_e^2}{2g \rho_6 A_6^2} \tag{8a}$$

Substituting from (7a) into (8a)

$$P_{vp6} = \frac{RT_6 W_e^2}{2g\ P_6 A_6^2} = \frac{RT_6}{2g\ P_6} \frac{W_e^2}{A_6^2} \qquad (9)$$

From equation (7a), it may be seen that for a constant gas constant R:

$$\frac{\rho_b}{\rho_a} = \frac{P_b}{P_a} \frac{T_a}{T_b} \qquad (10)$$

5.1.1.3 With the small pressure differences involved in the normal ejector system, the effect of changes in pressure on density changes may also be ignored ($P_a/P_b = 1.0$), and so in this case, equation (10) becomes:

$$\frac{\rho_b}{\rho_a} = \frac{T_a}{T_b} \qquad (10a)$$

From equations (10a) and (4a)

$$\frac{\rho_b}{\rho_a}\left(\frac{V_b}{V_a}\right)^2 = \left(\frac{T_a}{T_b}\right)\left(\frac{W_b}{W_a}\right)^2\left(\frac{T_b}{T_a}\right)^2\left(\frac{A_a}{A_b}\right)^2$$

$$= \left(\frac{W_b}{W_a}\right)^2\left(\frac{T_b}{T_a}\right)\left(\frac{A_a}{A_b}\right)^2 \qquad (11)$$

Substituting from equations (8) and (11) into equation (6) and dividing both sides of the equation by A_{10}:

$$\frac{(P_{10} - P_6)}{P_{vp6}} = 2\frac{A_6}{A_{10}} + 2\frac{A_9}{A_{10}}\left(\frac{W_c}{W_e}\right)^2\left(\frac{A_6}{A_9}\right)^2\left(\frac{T_9}{T_6}\right) - 2M\left(\frac{W_c+W_e}{W_e}\right)^2\left(\frac{A_6}{A_{10}}\right)^2\left(\frac{T_{10}}{T_6}\right)$$

$$- \left(\frac{4fL}{D_{10}}\right)\left(\frac{W_c + W_e}{W_e}\right)^2\left(\frac{A_6}{A_{10}}\right)^2\left(\frac{T_{10}}{T_6}\right) \qquad (12)$$

Assuming perfect mixing, the mixed mean temperature may be calculated from an enthalpy balance:

$$W_c\ h_9 + W_e\ h_6 = (W_c + W_e)\ h_{10} \qquad (13)$$

Or, assuming constant specific heats:

$$W_c\ T_9 + W_e\ T_6 = (W_e + W_c)\ T_{10} \qquad (13a)$$

Solving for T_{10}:

$$T_{10} = \left(\frac{W_e}{W_e + W_c}\right) T_6 + \left(\frac{W_c}{W_e + W_c}\right) T_9 \qquad (14)$$

Substituting from (14) into (12) and consolidating terms:

$$\frac{(P_{10} - P_6)}{P_{vp6}} = 2\left(\frac{A_6}{A_{10}}\right) + 2\left(\frac{T_9}{T_6}\right)\left(\frac{A_6}{A_{10}}\right)\left(\frac{A_6}{A_9}\right)\left(\frac{W_c}{W_e}\right)^2$$

$$- \left(2M + \frac{4fL}{D_{10}}\right)\left(\frac{A_6}{A_{10}}\right)^2\left(\frac{W_c + W_e}{W_e}\right)^2\left[\frac{W_e + W_c\left(\frac{T_9}{T_6}\right)}{W_e + W_c}\right]$$

$$\frac{(P_{10} - P_6)}{P_{vp6}} = 2\left(\frac{A_6}{A_{10}}\right) + 2\left(\frac{T_9}{T_6}\right)\left(\frac{A_6}{A_{10}}\right)\left(\frac{A_6}{A_9}\right)\left(\frac{W_c}{W_e}\right)^2$$

$$- \left(2M + \frac{4fL}{D_{10}}\right)\left(\frac{A_6}{A_{10}}\right)^2\left(1 + \frac{W_c}{W_e}\right)\left[1 + \frac{W_c}{W_e}\left(\frac{T_9}{T_6}\right)\right] \qquad (15)$$

Equation (15) establishes a relationship between nozzle static pressure depression divided by nozzle dynamic pressure and the various area, flow, and the temperature ratios, when no exhaust diffuser is used.

5.1.1.4 When a diffuser is present, an additional pressure rise $P_{11} - P_{10}$ is produced which is given by equation (15a)

$$P_{11} - P_{10} = \eta_D \frac{RT_{10}\,(W_c + W_e)^2}{2g\,P_{10}\,(A_{10})^2}\left[1 - \left(\frac{A_{10}}{A_{11}}\right)^2\right] \qquad (15a)$$

Combining equations (14), (15), and (15a) and solving for $\frac{P_{11} - P_6}{P_{vp}}$:

$$\frac{P_{11} - P_6}{P_{vp6}} = 2\left(\frac{A_6}{A_{10}}\right) + 2\left(\frac{T_9}{T_6}\right)\left(\frac{A_6}{A_{10}}\right)\left(\frac{A_6}{A_9}\right)\left(\frac{W_c}{W_c}\right)^2$$

$$- \left\{2M + \frac{4fL}{D_{10}} - \eta_D\left[1 - \left(\frac{A_{10}}{A_{11}}\right)^2\right]\right\}\left(\frac{A_6}{A_{10}}\right)^2\left(1 + \frac{W_c}{W_e}\right)\left[1 + \frac{W_c}{W_e}\left(\frac{T_9}{T_6}\right)\right] \qquad (15b)$$

5.1.2 <u>Energy Equation for the Ventilating Air</u>: A second equation is provided by determining the flow, pressure, and temperature relationships of the ventilating air.

5.1.2.1 The pressure loss analysis of the compartment air flow is based on the concept of equivalent orifices. Consider, for example, a ventilation duct in which a volume flow, Q, at a density ρ, produces a pressure drop, ΔP. The flow-pressure drop characteristics of this duct can be simulated by an equivalent orifice having the same characteristics. The area of an ideal equivalent orifice (with contraction and velocity coefficients equal to 1.0) may be calculated from the following equations:

$$\Delta P = \frac{\rho V^2}{2g} = \frac{\rho}{2g}\left(\frac{Q}{A}\right)^2 \qquad (16)$$

Substituting from equations (4) and (7) into equation (16):

$$\Delta P = \frac{\rho W^2}{2g \rho^2 A^2} = \frac{RTW^2}{2g\ PA^2} \quad (16a)$$

Equation (16a) may be solved for equivalent orifice area as follows:

$$A = W \sqrt{\frac{RT}{2g\ P\ \Delta P}} \quad (17)$$

where all units are in consistent ft, lb, sec., and R units.

With P in psia and Δ P in inches H_2O units, equation (17) becomes

$$A = W \sqrt{\frac{53.32 \times T \times 27.7}{2 \times 32.174 \times P \times 144 \times \Delta P \times 144}}$$

$$A = 0.0333\ W \sqrt{\frac{T^oR}{P_{(psia)}\ \Delta P_{(in.\ H_2O)}}} \quad (17a)$$

5.1.2.2 Referring to Fig. 1 of Section 2, the pressure drops in the compartment are considered to consist of a pressure loss in the compartment inlet ducting represented by an equivalent orifice A_7 and a loss between the compartment and the eductor nozzle, represented by an equivalent orifice A_8. A further reduction in static pressure occurs in the conversion of static pressure to velocity pressure when the ventilating air enters the annular area surrounding the primary eductor nozzle. In equation form:

$$(P_7 - P_9) = (P_7 - P_6) = \Delta P_7 + \Delta P_8 + \frac{W_c^2}{2g\ \rho_9\ A_9^2} \quad (18)$$

Substituting from equation (16a):

$$(P_7 - P_6) = \frac{R\ T_7\ W_c^2}{2g\ P_7\ A_7^2} + \frac{R\ T_8\ W_c^2}{2g\ P_8\ A_8^2} + \frac{R\ T_9\ W_c^2}{2g\ P_9\ A_9^2} \quad (19)$$

Dividing both sides of equation (19) by P_{vp6} given by equation (9):

$$\frac{P_7 - P_6}{P_{vp6}} = \frac{T_7}{T_6}\left(\frac{P_6}{P_7}\right)\left(\frac{W_c}{W_e}\right)^2\left(\frac{A_6}{A_7}\right)^2 + \frac{T_8}{T_6}\left(\frac{P_6}{P_8}\right)\left(\frac{W_c}{W_e}\right)^2\left(\frac{A_6}{A_8}\right)^2 + \frac{T_9}{T_6}\left(\frac{P_6}{P_9}\right)\left(\frac{W_c}{W_e}\right)^2\left(\frac{A_6}{A_9}\right)^2 \quad (20)$$

As in the previous case, the pressure ratios are all approximately equal to 1.0. Setting the pressure ratios equal to 1.0 and factoring out $\left(\frac{W_c}{W_e}\right)^2 \left(\frac{T_9}{T_6}\right)$, equation (20) becomes:

$$\frac{P_7 - P_6}{P_{vp6}} = \left(\frac{T_9}{T_6}\right)\left(\frac{W_c}{W_e}\right)^2\left[\frac{T_7}{T_9}\left(\frac{A_6}{A_7}\right)^2 + \frac{T_8}{T_9}\left(\frac{A_6}{A_8}\right)^2 + \left(\frac{A_6}{A_9}\right)^2\right] \quad (21)$$