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Plain bearings — Hydrodynamic plain journal bearings under steady-state conditions —

Part 1:
Calculation of multi-lobed and tilting pad journal bearings

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 123, *Plain bearings*, Subcommittee SC 8, *Calculation methods for plain bearings and their applications*.

A list of all parts in the ISO/TS 31657 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The aim of this document is the operationally-safe design of plain journal bearings for medium or high journal circumferential velocities, U_J, up to approximately 90 m/s by applying a calculation method for oil-lubricated hydrodynamic plain bearings with complete separation of journal and bearing sliding surfaces by a lubricating film.

For low circumferential velocities up to approximately 30 m/s usually circular cylindrical bearings are applied. For these bearings a similar calculation method is given in ISO 7902-1, ISO 7902-2 and ISO 7902-3.

Based on practical experience the calculation procedure is usable for application cases where specific bearing load times circumferential speed, $\overline{p} \cdot U_J$, does not exceed approximately 200 MPa·m/s.

This document discusses multi-lobed journal bearings with two, three and four equal, symmetrical sliding surfaces, which are separated by laterally-closed lubrication pockets, and symmetrically-loaded tilting-pad journal bearings with four and five pads. Here, the curvature radii, R_B, of the sliding surfaces are usually chosen larger than half the bearing diameter, D, so that an increased bearing clearance results at the pad ends.

The calculation method described here can also be used for other gap forms, for example asymmetrical multi-lobed journal bearings like offset-halves bearings, pressure-dam bearings or other tilting-pad journal bearing designs, if the numerical solutions of the basic formulas are available for these designs.

Plain bearings — Hydrodynamic plain journal bearings under steady-state conditions —

Part 1: Calculation of multi-lobed and tilting pad journal bearings

1 Scope

This document specifies the general principles, assumptions and preconditions for the calculation of multi-lobed and tilting-pad journal bearings by means of an easy-to-use calculation procedure based on numerous simplifying assumptions. For a reliable evaluation of the results of this calculation method, it is indispensable to consider the physical implications of these assumptions as well as practical experiences for instance from temperature measurements carried out on real machinery under typical operating conditions. Applied in this sense, this document presents a simple way to predict the approximate performance of plain journal bearings for those unable to access more complex and accurate calculation techniques.

The calculation method serves for the design and optimisation of plain bearings, for example in turbines, compressors, generators, electric motors, gears and pumps. It is restricted to steady-state operation, i.e. in continuous operating states the load according to size and direction and the angular velocity of the rotor are constant.

Unsteady operating states are not recorded. The stiffness and damping coefficients of the plain journal bearings required for the linear vibration and stability investigations are indicated in ISO/TS 31657-2 and ISO/TS 31657-3.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

No terms and definitions are listed in this document.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

4 Symbols and units

Table 1 contains the symbols used in the ISO 31657 series.

Table 1 — Symbols and units

Symbol	Description	Unit
B	Bearing width	m
B^*	Relative bearing width, width ratio as given by: $B^* = \frac{B}{D}$	1

Table 1 *(continued)*

Symbol	Description	Unit
b_P	Width of lubricant pocket	m
b_P^*	Relative width of lubricant pocket, as given by: $b_P^* = \frac{b_P}{B}$	1
C_R	Bearing radial clearance, as given by: $C_R = R - R_J$	m
$C_{R,eff}$	Effective radial bearing clearance	m
c_{ik}	Stiffness coefficient of lubricant film (i,k = 1,2)	N/m
c_{ik}^*	Non-dimensional stiffness coefficient of lubricant film, as given by: $c_{ik}^* = \frac{\psi_{eff}^3}{2 \cdot B \cdot \eta_{eff} \cdot \omega} \cdot c_{ik}\ (i,k=1,2)$	1
c_p	Specific heat capacity (p = constant)	J/(kg K)
D	Nominal bearing diameter (inside diameter of journal bearing)	m
D_{max}	Maximum value of D	m
D_{min}	Minimum value of D	m
D_J	Journal diameter (diameter of the shaft section located inside of a journal bearing)	m
$D_{J,max}$	Maximum value of D_J	m
$D_{J,min}$	Minimum value of D_J	m
d_{ik}	Damping coefficient of lubricant film (i,k = 1,2)	N s/m
d_{ik}^*	Non-dimensional damping coefficient of lubricant film, as given by: $d_{ik}^* = \frac{\psi_{eff}^3}{2 \cdot B \cdot \eta_{eff} \cdot \omega} \cdot \omega \cdot d_{ik}\ (i,k=1,2)$	1
e	Eccentricity (distance between journal and bearing axis)	m
e_B	Eccentricity of the bearing sliding surfaces (pads) of a multi-lobed or tilting-pad journal bearing	m
f	Bearing force, bearing load, nominal bearing load, load-carrying capacity	N
ΔF_x	Component of additional dynamic force in x-direction	N
ΔF_y	Component of additional dynamic force in y-direction	N
ΔF_x^*	Component of additional dynamic force parameter in x-direction, as given by: $\Delta F_x^* = \frac{\Delta F_x \cdot \psi_{eff}^2}{B \cdot D \cdot \eta_{eff} \cdot \omega}$	1
ΔF_y^*	Component of additional dynamic force parameter in y-direction, as given by: $\Delta F_y^* = \frac{\Delta F_y \cdot \psi_{eff}^2}{B \cdot D \cdot \eta_{eff} \cdot \omega}$	1
F_f	Friction force, as given by: $F_f = f \cdot F$	N
F_f^*	Friction force parameter, as given by: $F_f^* = \frac{f}{\psi_{eff}} \cdot So$	1
F_{tr}	Bearing force at transition to mixed friction	N
F	Coefficient of friction	1
f_J	Journal deflection	m
$h(\varphi)$	Local lubricant film thickness	m

Table 1 *(continued)*

Symbol	Description	Unit
$h^*(\varphi)$	Relative local lubricant film thickness, as given by: $h^*(\varphi)=\frac{h(\varphi)}{C_R}$	1
$h_{lim,tr}$	Minimum admissible lubricant film thickness at transition to mixed friction	m
$h^*_{lim,tr}$	Minimum admissible relative lubricant film thickness at transition to mixed friction, as given by: $h^*_{lim,tr}=\frac{h_{lim,tr}}{C_{R,eff}}$	1
h_{min}	Minimum lubricant film thickness, minimum gap	m
h^*_{min}	Minimum relative lubricant film thickness, minimum relative gap, as given by: $h^*_{min}=\frac{h_{min}}{C_{R,eff}}$	1
$h_{min,tr}$	Minimum lubricant film thickness at transition to mixed friction	m
$h^*_{min,tr}$	Minimum relative lubricant film thickness at transition to mixed friction, as given by: $h^*_{min,tr}=\frac{h_{min,tr}}{C_{R,eff}}$	1
$h_0(\varphi)$	Local gap at $\varepsilon=0$, gap function	m
$h^*_0(\varphi)$	Relative local gap at $\varepsilon=0$, profile function, as given by: $h^*_0(\varphi)=\frac{h_0(\varphi)}{C_R}$	1
$h_{0,max}$	Maximum gap at $\varepsilon=0$	m
$h^*_{0,max}$	Maximum relative gap at $\varepsilon=0$, gap ratio, as given by: $h^*_{0,max}=\frac{h_{0,max}}{C_R}$	1
K_P	Profile factor (relative difference between lobe or pad bore radius and journal radius), as given by: $K_P=\frac{\Delta R_B}{C_R}=\frac{1}{1-m}$	1
$K_{P,eff}$	Effective profile factor	1
$K_{P,20}$	Profile factor at 20 °C	1
M	Mixing factor	1
m	Preload factor, preload of bearing or pad sliding surface	1
N	Rotational speed (rotational frequency) of the rotor (revolutions per time unit)	s^{-1}
N_{cr}	Critical speed (critical rotational frequency)	s^{-1}
N_{lim}	Rotational speed (rotational frequency) at the stability speed limit of the rotor supported by plain bearings	s^{-1}
N_{rsn}	Resonance speed (resonance rotational frequency) of the rotor supported by plain bearings	s^{-1}
N_{tr}	Rotational speed (rotational frequency) at transition to mixed friction, transition rotational speed, transition rotational frequency	s^{-1}
O_B	Centreline of plain bearing	1
O_i	Centreline of sliding surface No. *i*	1
O_J	Centreline of journal	1
P_f	Frictional power, as given by: $P_f=F_f\cdot U_J$	W
$P_{th,L}$	Heat flow via the lubricant	W
p	Lubricant film pressure, local lubricant film pressure	Pa

Table 1 *(continued)*

Symbol	Description	Unit
$\bar{p}$	Specific bearing load, as given by: $\bar{p}=\frac{F}{B\cdot D}$	Pa
p_{en}	Lubricant supply pressure	Pa
p_{en}^*	Lubricant supply pressure parameter, as given by: $p_{en}^*=\frac{p_{en}\cdot\psi_{eff}^2}{\eta_{eff}\cdot\omega}$	1
p_{lim}	Maximum admissible lubricant film pressure	Pa
$\bar{p}_{lim,tr}$	Maximum admissible specific bearing load at transition to mixed friction	Pa
p_{max}	Maximum lubricant film pressure	Pa
p_{max}^*	Maximum lubricant film pressure parameter, as given by: $p_{max}^*=\frac{p_{max}}{\bar{p}}$	1
$\bar{p}_{tr}$	Specific bearing load at transition to mixed friction, as given by: $\bar{p}_{tr}=\frac{F_{tr}}{B\cdot D}$	Pa
Q	Lubricant flow rate, as given by: $Q=Q_3+Q_p$	m^3/s
Q_{lim}	Minimum admissible lubricant flow rate	m^3/s
Q_p	Lubricant flow rate due to supply pressure	m^3/s
Q_p^*	Lubricant flow rate parameter due to supply pressure, as given by: $Q_p^*=\frac{Q_p}{p_{en}^*\cdot Q_0}$	1
Q_0	Reference value of Q, as given by: $Q_0=R^3\cdot\omega\cdot\psi_{eff}$	m^3/s
Q_1	Lubricant flow rate at the entrance into the lubrication gap (circumferential direction)	m^3/s
Q_2	Lubricant flow rate at the exit of the lubrication gap (circumferential direction), as given by: $Q_2=Q_1-Q_3$	m^3/s
Q_2^*	Lubricant flow rate parameter at the exit of the lubrication gap (circumferential direction), as given by: $Q_2^*=\frac{Q_2}{Q_0}$	1
Q_3	Lubricant flow rate due to hydrodynamic pressure build-up (side flow rate)	m^3/s
Q_3^*	Lubricant flow rate parameter due to hydrodynamic pressure build-up (side flow parameter), as given by: $Q_3^*=\frac{Q_3}{Q_0}$	1
R	Journal bearing inside radius, as given by: $R=\frac{D}{2}$	m
R_B	Lobe or pad bore radius of a multi-lobed or tilting-pad journal bearing	m
ΔR_B	Difference between lobe or pad bore radius and journal radius, as given by: $\Delta R_B=R_B-R_J$	m
R_J	Journal radius (radius of the shaft section located inside of a journal bearing), as given by: $R_J=\frac{D_J}{2}$	m
$R_{z,B}$	Surface finish ten-point average of bearing sliding surface	m
$R_{z,J}$	Surface finish ten-point average of journal sliding surface	m
Re	Reynolds number, as given by: $Re=\frac{\rho\cdot\omega\cdot R\cdot C_{R,eff}}{\eta_{eff}}$	1
Re_{cr}	Critical Reynolds number	1

Table 1 *(continued)*

Symbol	Description	Unit
So	Sommerfeld number, as given by: $So=\frac{F\cdot\psi_{eff}^2}{B\cdot D\cdot\eta_{eff}\cdot\omega}$	1
So_{tr}	Sommerfeld number at transition to mixed friction	1
S	Displacement amplitude of the rotor (mechanical oscillation)	m
T	Temperature	°C
ΔT	Heating of lubricant between bearing entrance and exit, as given by: $\Delta T=T_{ex}-T_{en}$	K
ΔT_{lim}	Maximum admissible heating of lubricant between bearing entrance and exit	K
T_B	Bearing temperature	°C
T_{eff}	Effective temperature of lubricant film	°C
T_{en}	Lubricant temperature at the bearing entrance	°C
T_{ex}	Lubricant temperature at the bearing exit	°C
T_J	Journal temperature	°C
T_{lim}	Maximum admissible bearing temperature	°C
T_{max}	Maximum temperature of lubricant film	°C
ΔT_{max}	Difference between maximum temperature of lubricant film and lubricant temperature in the lubricant pocket, as given by: $\Delta T_{max}=T_{max}-T_1$	K
ΔT_{max}^*	Non-dimensional difference between maximum temperature of lubricant film and lubricant temperature in the lubricant pocket, as given by: $\Delta T_{max}^*=\frac{\rho\cdot c_p\cdot\Psi_{eff}}{\bar{p}\cdot f}\cdot\Delta T_{max}$	1
T_1	Lubricant temperature at the entrance into the lubrication gap (circumferential direction)	°C
ΔT_1	Difference between lubricant temperature at the entrance into the lubrication gap and lubricant temperature at the bearing entrance, as given by: $\Delta T_1=T_1-T_{en}$	K
T_2	Lubricant temperature at pressure profile trailing edge (circumferential direction)	°C
ΔT_2	Difference between lubricant temperature at pressure profile trailing edge and lubricant temperature at the entrance into the lubrication gap, as given by: $\Delta T_2=T_2-T_1$	K
t	Time	s
U_J	Circumferential speed of the journal, sliding velocity $U_J=\omega\cdot R_J$	m/s
U_{tr}	Circumferential speed at transition to mixed friction	m/s
$U_{lim,\,tr}$	Minimum admissible circumferential speed at transition to mixed friction	m/s
u	Velocity component in the φ-direction	m/s
$\bar{u}$	Average velocity component in the φ-direction	m/s
w	Velocity component in the z- direction	m/s
$\bar{w}$	Average velocity component in the z-direction	m/s
x	Coordinate of journal radial motion, normal to direction of load	m
x^*	Relative coordinate of journal radial motion, normal to direction of load, as given by: $x^*=\frac{x}{C_R}$	1
y	Coordinate normal to sliding surface (across the lubricant film, in the radial direction); coordinate of journal radial motion, in direction of load	m
y^*	Relative coordinate of journal radial motion, in direction of load, as given by: $y^*=\frac{y}{C_R}$	1

Table 1 *(continued)*

Symbol	Description	Unit
y_h	Coordinate normal to sliding surface (across the lubricant film)	m
Z	Number of sliding surfaces (pads), number of pockets per bearing	1
z	Coordinate parallel to the sliding surface, normal to direction of motion (normal to circumferential direction, in the axial direction)	m
$\alpha_{l,B}$	Linear thermal expansion coefficient of bearing material	K^{-1}
$\alpha_{l,J}$	Linear thermal expansion coefficient of journal material	K^{-1}
β	Attitude angle (angular position of journal eccentricity related to the direction of load)	°
$\beta_{h,\min}$	Angle between direction of load and position of minimum lubricant film thickness	°
δ_J	Journal misalignment angle (angular deviation of journal)	°
ε	Relative eccentricity: $\varepsilon = \frac{e}{C_{R,\mathrm{eff}}}$	1
η	Dynamic viscosity of the lubricant	Pa s
η_{eff}	Effective dynamic viscosity in the lubricant film	Pa s
ρ	Density of the lubricant	kg/m^3
φ	Angular coordinate in circumferential direction	°
φ_F	Angular coordinate of pivot position of pad (tilting-pad bearing)	°
φ_P	Angular coordinate of lubricant pocket centreline	°
φ_0	Angular coordinate of bearing sliding surface (segment or pad) centreline at multi-lobed or tilting-pad journal bearings (with non-tilted pads), see Figure 1, a)	°
φ_1	Angular coordinate at the entrance into the gap	°
φ_2	Angular coordinate at the end of the hydrodynamic pressure build-up	°
φ_3	Angular coordinate at the exit of the gap	°
ψ	Relative bearing clearance, as given by: $\psi = \frac{C_R}{R}$	‰
$\Delta\psi$	Tolerance of ψ, as given by: $\Delta\psi = \psi_{\max} - \psi_{\min}$	‰
ψ_{eff}	Effective relative bearing clearance	‰
$\psi_{\max}$	Maximum value of ψ	‰
$\psi_{\min}$	Minimum value of ψ	‰
$\Delta\psi_{\mathrm{th}}$	Thermal change of ψ	‰
ψ_{20}	Relative bearing clearance at 20 °C	‰
Ω	Angular span of bearing sliding surface (segment or pad), as given by: $\Omega = \varphi_3 - \varphi_1$	°
Ω_F	Angular distance between leading edge and pivot position of pad (tilting-pad bearing), as given by: $\Omega_F = \varphi_F - \varphi_1$	°
Ω_F^*	Relative angular distance between leading edge and pivot position of pad (tilting-pad bearing), as given by: $\Omega_F^* = \Omega_F / \Omega$	1
Ω_P	Angular span of lubricant pocket, as given by: $\Omega_P = \frac{360°}{Z} - \Omega$	°
ω	Angular speed of the rotor, as given by: $\omega = 2 \cdot \pi \cdot N$	s^{-1}
ω_{tr}	Angular speed at transition to mixed friction	s^{-1}

5 General principles, assumptions and preconditions

The bearing bore form of multi-lobed journal bearings [see Figure 1, a)] and tilting-pad journal bearings [with non-tilted pads according to Figure 1, b)] is described by the profile function $h_0^*(\varphi)=\frac{h_0(\varphi)}{C_R}$ in the case of a centric journal position $\varepsilon=\frac{e}{C_R}=0$. The angle φ is counted, starting from the load direction, in the journal rotational direction.

Formula (1) applies to the shell segment or pad *i* with the angular length $\Omega_i=\varphi_{3,i}-\varphi_{1,i}$:

$$h_{0,i}^*(\varphi)=\frac{\Delta R_B}{C_R}+\left(\frac{\Delta R_B}{C_R}-1\right)\cdot\cos(\varphi-\varphi_{0,i}),i=1,...,Z \quad (1)$$

with the profile factor

$$K_P=\frac{\Delta R_B}{C_R}=\frac{R_B-R_J}{C_R}=1+\frac{e_B}{C_R},$$

minimum clearance

$$C_R=R-R_J=\frac{D-D_J}{2}$$

and the lubricant film thickness ratio as given by Formula (2):

$$h_0^*(\varphi_{P,i})=h_{0,\max}^*=\frac{h_{0,\max}}{C_R} \quad (2)$$

Here the position of the sliding surface (segment or pad) axis (curvature centre "point") of the shell segment or pad *i* is uniquely described by the sliding surface eccentricity e_B and the associated angle coordinate $\varphi_{0,i}$.

In the case of cylindrical bearings, K_P = 1 and $h_0^*(\varphi)=1$.

NOTE Instead of the profile factor, K_P, the "preload factor", *m*, is frequently used internationally; the following relation exists between both variables:

$$K_P=\frac{1}{1-m}$$

In the case of an eccentric position of the journal (ε, β), Formula (3) applies to the lubricant film thickness, $h(\varphi)$, of the multi-lobed journal bearings [(see Figure 1, c)]:

$$h(\varphi)=C_R\cdot h^*(\varphi)=C_R\cdot[h_0^*(\varphi)-\varepsilon\cdot\cos(\varphi-\beta)] \quad (3)$$

In the case of tilting-pad journal bearings [see Figure 1, d)], the individual pads automatically adjust themselves (optimally) so that the lubricant film force F_i passes through the supporting pad pivot, respectively[9]. For a more precise calculation of tilting-pad journal bearings, the elasticities in the pad support and the elastic and thermal deformations of the pads shall be considered.

The pressure formation in the lubrication gaps is basically calculated with the numerical solutions of the Reynolds differential equation for a finite bearing width:

$$\frac{1}{R_J^2}\cdot\frac{\partial}{\partial\varphi}\left(h^3\cdot\frac{\partial p}{\partial\varphi}\right)+\frac{\partial}{\partial z}\left(h^3\cdot\frac{\partial p}{\partial z}\right)=6\cdot\eta\cdot\omega\cdot\frac{\partial h}{\partial\varphi} \quad (4)$$

with $\omega=2\cdot\pi\cdot N$ angular speed of the rotor.

For derivation of the Reynolds differential equation, reference is made to Reference [8], for the numerical solution to Reference [9].

When solving Formula (4), the following idealising assumptions and preconditions are made, whose permissibility shall be estimated according to Clause 6, if necessary[10].

a) The lubricant corresponds to a Newtonian fluid.

b) All flow processes of the lubricant are laminar.

c) The lubricant adheres fully to the sliding surfaces.

d) The lubricant is incompressible.

e) At the leading edge of the segment or pad, the lubrication gap is completely filled with lubricant.

f) Inertia effects, gravitation and magnetic forces of the lubricant are negligible.

g) The components forming the lubrication gap are rigid or their deformation is negligible; the surfaces of the journal and bearing bore are ideal circular cylinders or cylindrical segments.

h) The curvature radii of the surfaces moving relative to one another are large in comparison to the lubricant film thicknesses.

i) The lubricant film thickness in an axial direction (z coordinate) is constant.

j) Pressure changes in the lubricant film normal to the sliding surfaces (in the lubricant film thickness direction) are negligible.

k) A movement normal to the sliding surfaces (in the lubricant film thickness direction) is not considered here, in contrast to 6.10.

l) The lubricant film is isoviscous in the entire lubrication gap.

m) The lubricant is supplied at the leading edge of the segments or pads respectively; the level of the supply pressure is negligible compared to the lubricant film pressures themselves.

The boundary conditions for the lubricant film pressure build-up satisfy the continuity condition.

The following applies respectively to the individual segments or pads (see Figures 2 and 4):

— at the lateral bearing edge $p(\varphi,z=\pm B/2)=0$;

— in the lubrication pocket and on the sealing land $p(\varphi,z)=0$;

— at the pressure profile trailing edge $p[\varphi_2(z),z]=\frac{\partial p}{\partial\varphi}[\varphi_2(z),z]=0$;

— at the beginning of cavitation area $p[\varphi(z),z]=\frac{\partial p}{\partial\varphi}[\varphi(z),z]=\frac{\partial p}{\partial z}[\varphi(z),z]=0$;

— at the end of cavitation area $p[\varphi(z),z]=0$.

The cavitation theory according to Jakobsson, Floberg and Olsson[15][16] is used in the cavitation area and on its edge for fulfilment of the continuity condition.

The numerical integration of the Reynolds differential equation is done using the transformation of the pressure proposed in Reference [9] by conversion into a difference formula, which is applied to a grid of nodal points and which leads to a system of linear formulas.

After specifying the boundary conditions, the integration yields the pressure profile in the circumferential and axial direction.

The maximum lubricant film temperature is calculated using the numerical solution of the energy equation averaged by integration with respect to the lubricant film thickness, h

$$\frac{\bar{u}}{R_\mathrm{J}}\cdot\frac{\partial T}{\partial \varphi}+\bar{w}\cdot\frac{\partial T}{\partial z}=\frac{\eta}{\rho\cdot c_p}\cdot\frac{1}{h}\cdot\int_0^h\left[\left(\frac{\partial u}{\partial y_h}\right)^2+\left(\frac{\partial w}{\partial y_h}\right)^2\right]\cdot dy_h \tag{5}$$

for the two-dimensional temperature distribution $T(\varphi, z)$[9][13][14].

This includes

$$\bar{u}=\frac{1}{h}\cdot\int_0^h u\cdot dy_h\,,\bar{w}=\frac{1}{h}\cdot\int_0^h w\cdot dy_h$$

the flow rates averaged over the lubricant film thickness h in the circumferential and axial direction.

When deriving the energy equation, Formula (5), it is also assumed besides the above preconditions that no heat is dissipated from the lubrication gap by thermal conduction (adiabatic calculation).

When solving Formula (5) the following boundary conditions apply (see Figure 4):

— at the entrance gap $T(\varphi_1, z)=T_1$;

— in the axial bearing centre $\frac{\partial T}{\partial z}(\varphi, z=0)=0$.

The numerical integration of Formula (5) is carried out similar to the solution of the Reynolds differential equation, Formula (4), using a suitable difference formula and yields for the specified boundary conditions the temperature distribution in the circumferential and axial direction.

The application of the similarity principle in the hydrodynamic plain bearing theory leads to dimensionless similarity variables for the interesting characteristic values (such as load-carrying capacity, frictional power, lubricant flow rate and relative bearing width). Use of the similarity variables reduces the number of necessary numerical solutions of the Reynolds differential equation, Formula (4), and the energy equation, Formula (5), which are summarised in ISO/TS 31657-2 and ISO/TS 31657-3.

As a rule, other solutions can also be used, insofar as they satisfy the conditions indicated in this document and a corresponding numerical accuracy.

ISO/TS 31657-4 contains operational guide values for checking the calculation results, in order to ensure the functionality of the plain bearings.

In special use cases, operational guide values different from ISO/TS 31657-4 can be agreed.

6 Calculation method

6.1 General

Calculation refers to the mathematical determination of the functional capability based on operational characteristic values (see Figure 5), which are to be compared with permissible operational parameter values. The operational characteristic values determined in different operating states shall be permissible with their permissible operational parameter values. All continuous operating states shall be examined for this.

The safety against wear is ensured when a complete separation of the sliding partners is attained by the lubricant. Continuous operation in the mixed friction area leads to premature functional incapability. Short-term operation in the mixed friction area, for example when starting up and running down of machines with slide bearings, is unavoidable and does not usually lead to bearing damage. At high loads, a hydrostatic jacking can be required during slow start-up or run-down. Running-in and adapting wear for compensating the surface form deviations from the ideal form are permissible as long as these occur with local and time restrictions and without signs of overload.

The limits of the mechanical load are given by the strength of the bearing material. Minor plastic deformations are permissible as long as they do not impair the functional capability of the plain bearing.

The limits of the thermal load result from the high-temperature strength of the bearing material, but also from the viscosity temperature dependence and the tendency of the lubricant to age.

The calculation of the functional capability of plain bearings presupposes that the operating conditions are known for all continuous operating states. In practice, however, additional disturbing influences frequently occur, which are still unknown during the design and which are also not always accessible to a mathematical approach. It is therefore recommended to work with a corresponding safety interval between the operational characteristic values and the permissible operating parameter values. Disturbing influences are for example:

— disturbing forces (e.g. imbalances, vibrations);

— form deviations from the ideal geometry (e.g. operational deformations, production tolerances, assembly deviations);

— lubricant impurities due to solid, liquid and gaseous foreign bodies;

— corrosion, electro-erosion.

Information on some further influencing variables is given in 6.9.

The applicability of this document, in which a laminar flow in the lubrication gap is presupposed, shall be checked by the Reynolds number[11][12]:

$$Re = \frac{\rho \cdot \omega \cdot R \cdot C_{R,\text{eff}}}{\eta_{\text{eff}}} \leq Re_{\text{cr}} \approx \frac{41{,}3}{\sqrt{K_P \cdot \psi_{\text{eff}}}} \quad (6)$$

with $\psi_{\text{eff}} = \frac{C_{R,\text{eff}}}{R}$ effective relative bearing clearance.

In the case of plain bearings with $Re > Re_{\text{cr}}$ (e.g. due to high circumferential velocities), higher power losses shall be expected. The load carrying capacity can rise[10][11][12].

Bearings with turbulent flow can only be calculated approximately according to this document.

The plain bearing calculation grasps the following, based on the known bearing dimensions and operating data:

— the relation between bearing load carrying capacity and lubricant film thickness;

— the frictional power;

— the lubricant flow rate;

— the heat balance;

— the maximum lubricant film temperature and the maximum lubricant film pressure;

all these interacting with one another. The solution happens in an iterative process whose sequence is summarised in the calculation flow chart according to Figure 5.

A parameter variation can be performed for the optimisation of individual parameters. It is possible to modify the calculation procedure

6.2 Load carrying capacity

Characteristic for the load carrying capacity is the (non-dimensional) Sommerfeld number

$$So = \frac{F \cdot \psi_{\text{eff}}^2}{B \cdot D \cdot \eta_{\text{eff}} \cdot \omega} \tag{7}$$

whose dependence on the relative eccentricity $\varepsilon = \frac{e}{C_{R,\text{eff}}}$, the relative bearing width $B^* = \frac{B}{D}$ and the profile function $h_0^*(\varphi)$ is indicated in ISO/TS 31657-2 and ISO/TS 31657-3. The state variables η_{eff}, ψ_{eff} consider thermal influences (see 6.5 and 6.9).

From this, with the attitude angle β [So, B^*, $h_0^*(\varphi)$] according to ISO/TS 31657-2 and ISO/TS 31657-3 the components of the static bearing flexibility $\varepsilon \cos\beta$, $\varepsilon \sin\beta$ depending on the static load parameter So can be determined.

In the case of multi-lobed and tilting-pad journal bearings, the static displacement e and minimum lubricant film thickness h_{min} add up vectorially to the radial journal mobility (see Figure 3). The dependence indicated in ISO/TS 31657-2 and ISO/TS 31657-3 $h_{\text{min}}^*\left[So, B^*, h_0^*(\varphi)\right] = \frac{h_{\text{min}}}{C_{R,\text{eff}}}$ yields through comparison with the permissible operational parameter value $h_{\text{lim,tr}}^* = \frac{h_{\text{lim,tr}}}{C_{R,\text{eff}}}$ the load carrying capacity at the transition to mixed friction.

6.3 Frictional power

The losses due to frictional power in a hydrodynamic plain bearing are determined by the dimensionless friction force, F_f^*, (or the friction coefficient, f)

$$F_f^* = \frac{f}{\psi_{\text{eff}}} \cdot So \tag{8}$$

whose dependence on So, B^* and $h_0^*(\varphi)$ is indicated in ISO/TS 31657-2 and ISO/TS 31657-3. Here it is assumed that the lubricant supply pressure, p_{en}, remains very low and, in cavitation areas, the friction force has a linear dependence on calculated degree of filling[15][16].

The friction power in the bearing or the heat flow caused by it is

$$P_f = F_f \cdot U_J \tag{9}$$

with $F_f = f \cdot F$ friction force and $U_J = \omega \cdot R_J$ circumferential speed of the journal.

NOTE Particularly in the lubricant pockets of multi-lobed journal bearings and between the pads of tilting-pad journal bearings already at moderate circumferential speeds, turbulent flow can occur, leading to increasing power losses in these areas not taken into account in the calculation procedure described in this document. In addition, depending on the geometrical design of the lubricant supply and lubricant scraper elements (if applicable), churning losses can arise in the cavities between the tilting-pads. The extent of these power losses not considered in this calculation method depend also on the filling degree of the cavities (see 6.5).

6.4 Lubricant flow rate

The lubricant supplied to the bearing via lubrication pockets forms a load-carrying lubricant film for separating the sliding surfaces. The pressure formation in the lubricant film forces the lubricant out from the sides of the bearing.

This is the fraction, Q_3, of lubricant flow rate due to inherent pressure formation[9][13]:

$$Q_3 = Q_0 \cdot Q_3^* \tag{10}$$

with $Q_0 = U_\mathrm{J} \cdot C_{R,\mathrm{eff}} \cdot \frac{D}{2} = R^3 \cdot \omega \cdot \psi_\mathrm{eff}$ reference lubricant flow rate

and $Q_3^* = f\,[So,\, B^*,\, h_0^*(\varphi)\,]$ according to ISO/TS 31657-2 and ISO/TS 31657-3.

The lubricant supply pressure, p_en, at the inlet into the bearing also forces lubricant out from the sides of the plain bearing. This is the fraction, Q_p, of lubricant flow rate due to supply pressure[13]:

$$Q_p = Q_0 \cdot p_\mathrm{en}^* \cdot Q_p^* \tag{11}$$

with $Q_p^* = f\,[So,\, B^*,\, h_0^*(\varphi)\,]$ according to ISO/TS 31657-2

and $p_\mathrm{en}^* = \frac{p_\mathrm{en} \cdot \psi_\mathrm{eff}^2}{\eta_\mathrm{eff} \cdot \omega}$ as lubricant supply pressure parameter.

The lubricant supply pressure, p_en, is normally between 0,05 and 0,2 MPa (above the ambient pressure).

In the case of tilting-pad journal bearings, the lubricant flow rate Q_p is normally set by throttling the supply or discharge flow, or via corresponding nozzles (in case of injection lubrication).

The total lubricant flow rate is:

$$Q = Q_3 + Q_p \tag{12}$$

For the (later) calculation of the lubricant temperature in the pockets, the lubricant flow rate, Q_2, which enters in the circumference direction through the narrowest lubrication gap into the divergent gap, is also required:

$$Q_2 = Q_1 - Q_3 = Q_0 \cdot Q_2^* \tag{13}$$

with $Q_2^* = f\,[So,\, B^*,\, h_0^*(\varphi)\,]$ according to ISO/TS 31657-2 and ISO/TS 31657-3.

Lubrication pockets as defined in this document are design elements for the distribution of the lubricant over the bearing width. The recesses machined into the sliding surfaces of the bearing extend in the axial direction and should be as short as possible in the circumferential direction.

Relative pocket widths should be $b_p^* = b_p / B \leq 0{,}8$.

Although greater values increase the oil flow rate, the oil escaping at the narrow throttling lands at the sides does not take part in the heat dissipation. This applies more so if the side lands have axial grooves.

For the calculation of the flow rate fraction Q_p a relative pocket width of b_p^* = 0,8 is presupposed in this document. The effect of the lubricant inertia forces is not considered here.

The depth of the lubrication pockets is significantly greater than the bearing clearance.

6.5 Heat balance

The thermal state of the plain bearing results from the heat balance.

The heat flow resulting from the frictional power, P_f, in the bearing is dissipated to the surroundings via the bearing housing and via the lubricant escaping from the bearing.

Pressure-lubricated multi-lobed and tilting-pad journal bearings (forced lubrication) primarily dissipate the heat via the lubricant (recooling):

$$P_f = P_{\text{th},L} \tag{14}$$

By neglecting the convective heat dissipation via the bearing housing, an additional safety results with the design. Formula (15) applies for the heat dissipation by the lubricant:

$$P_{\text{th},L} = \rho \cdot c_p \cdot Q \cdot (T_{\text{ex}} - T_{\text{en}}) = \rho \cdot c_p \cdot Q \cdot \Delta T \tag{15}$$

In the case of mineral lubricants, the volume-specific heat capacity is:

$$\rho \cdot c_p = (1{,}7 \ldots 1{,}8) \cdot 10^6 \text{ J/(m}^3 \cdot \text{K)}$$

In practice, with regard to the lubricant service life and/or the available cooling capacity of the lubricating system, the heating of lubricant, ΔT, frequently has to be limited to a certain extent, ΔT_{lim}, (e.g. 20 ... 25 K) by increasing the total lubricant flow rate, Q, appropriately as shown in Formula (16):

$$Q_{\text{lim}} = \frac{P_f}{\rho \cdot c_p \cdot \Delta T_{\text{lim}}} \tag{16}$$

Mixing processes in the lubrication pockets:

As a multi-lobed and tilting-pad journal bearing comprises several pads, it is necessary to consider not only the lubricant flow rate of an individual pad, but that of the complete bearing and hence also the reciprocal effect of the individual lubricant flow rate fractions. The lubricant escaping at the end of the segments or pads is mixed with freshly supplied lubricant in the following oil pocket. This means that the lubricant temperature, T_1, at the entrance of the lubrication gap is higher than that of lubricant freshly supplied with the temperature, T_{en} (see Figure 4).

To simplify, the same temperature, T_1, and, when calculating it, for all segments or pads, an averaged oil heating to the temperature, T_2, is presupposed for all oil pockets. When determining the temperature difference as shown in Formula (17)

$$\Delta T_1 = T_1 - T_{\text{en}} \tag{17}$$

an empirical factor must be introduced, as a purely theoretical treatment of this mixing problem has not yet led to satisfactory results.

For adaptation to the experience gained up to the present (see Reference [14]), it is possible via a heat balance at the lubrication pockets (see Figure 4) to introduce a mixing factor, M, as follows:

$$\Delta T_1 = \frac{Q_2}{M \cdot Q + (1 - M) \cdot Q_3} \cdot \Delta T_2 \tag{18}$$

The limiting values are considered for explanation of the mixing factor. A mixing factor $M = 0$ means no mixing in the lubrication pockets, i.e., the lubricant flow rate leaving the lubrication gaps Q_2 fully enters into the following lubrication gaps. As a result, a high lubricant flow rate, Q, would be ineffective, as the majority of this freshly supplied lubricant would flow axially out of the lubrication pockets, without affecting the operational characteristic values. A mixing factor $M = 1$ means “complete” mixing in the lubrication pockets. $M = 0{,}4$ to $0{,}6$ can be used as an empirical value. It is influenced by the engineering design to an extent that cannot be indicated more precisely. Particularly in the case of tilting-pad bearings, it is possible to increase the mixing factor (e.g. up to 0,75) by using directed lubrication methods combined with unrestricted drain instead of a flooded design with fixed or floating seals on the bearing sides. In addition, such designs are suitable to reduce the churning losses in the cavities between the tilting-pads (see 6.3).

The averaged oil heating ΔT_2 at the pressure profile trailing edge, given in Formula (19)

$$\Delta T_2 = T_2 - T_1 \tag{19}$$

can be calculated from the frictional power P_f in the lubrication gap via Formula (14):

$$\Delta T_2 = \frac{P_f}{\rho \cdot c_p \cdot \left(Q_2 + \frac{Q_3}{2} \right)} \tag{20}$$

As a result, the average pocket temperature T_1 and the effective lubricant film temperature T_{eff} can be determined:

$$T_1 = T_{\text{en}} + \Delta T_1 ,$$
$$T_{\text{eff}} = \frac{T_1 + T_2}{2} = T_{\text{en}} + \Delta T_1 + \frac{\Delta T_2}{2} \tag{21}$$

In the calculation sequence, only the lubricant film supply temperature T_{en} is initially known, but not the effective lubricant film temperature, T_{eff}, which is required at the beginning of the calculation. The temperatures resulting from the heat balance are improved iteratively by averaging with the temperatures previously forming the basis until the difference between the results of two iteration steps in succession becomes negligibly small, e.g. <1 K. The state then attained corresponds to the steady state. The iteration usually converges rapidly (see Annex A).

Additionally, in the case of tilting-pad journal bearings, the lubricant flow rate, Q_p, to be set for ensuring the functional capability (see 6.4) is generally not initially known in the calculation sequence. In such cases, it is therefore recommended setting in a first step $Q_p = 0$ and checking the functional capability of the bearing (i.e. in particular compliance with the permissible operational parameter values for the minimum lubricant film thickness, h_{min}, and the maximum lubricant film temperature, T_{max}) for this. If necessary, the lubricant flow rate, Q_p, can be determined in a second step by targeted specification of a value $Q_p > 0$ so that the functional capability of the bearing is ensured (see A.2).

6.6 Maximum lubricant film temperature

The now known average pocket temperature, T_1, allows the maximum lubricant film temperature, T_{max}, of the bearing to be determined from the energy equation, Formula (5) , (see Figure 4), as shown in Formula (22):

$$T_{\text{max}} = T_1 + \Delta T_{\text{max}} \tag{22}$$

with

$$\Delta T_{\text{max}} = \frac{\bar{p} \cdot f}{\rho \cdot c_p \cdot \Psi_{\text{eff}}} \cdot \Delta T^*_{\text{max}} = \frac{\bar{p}}{\rho \cdot c_p} \cdot \frac{F^*_f}{So} \cdot \Delta T^*_{\text{max}} , \tag{23}$$

$\Delta T^*_{\text{max}} = f\,[So, B^*, h^*_0(\varphi)\,]$ according to ISO/TS 31657-2 and ISO/TS 31657-3, and the specific bearing load:

$$\overline{p}=\frac{F}{B\cdot D} \tag{24}$$

The maximum lubricant film temperature, T_{max}, shall be checked with the empirical limiting values, T_{lim}, according to ISO/TS 31657-4 with respect to its permissibility.

6.7 Maximum lubricant film pressure

The specific bearing load, $\overline{p}$, as defined by Formula (24) allows the maximum lubricant film pressure, p_{max}, to be determined (according to ISO/TS 31657-2 and ISO/TS 31657-3) which shall be checked with the operational limiting values, $p_{lim,}$ (according to ISO/TS 31657-4) with respect to its permissibility.

$$p_{max}^{*}=\frac{p_{max}}{\overline{p}}=f[So,B^{*},h_0^{*}(\phi)] \tag{25}$$

6.8 Operating states

If the plain bearing is to be operated in several different operating states over a longer time, the operating states shall be checked, among which p_{max}, h_{min} and T_{max} are the least favourable.

If an operating state with thermally high load (low dynamic lubricant viscosity) is directly followed by another with high specific bearing load and low rotational speed, this new operating state should be examined while retaining the thermal state from the previous operating point.

The transition to mixed friction occurs upon contact by the roughness peaks of the journal and bearing corresponding to the criterion for $h_{min,tr}=h_{min,tr}^{*}\cdot C_{R,eff}$ in ISO/TS 31657-4, whereby deformations shall also be considered. With the specific bearing load at transition to mixed friction, $\overline{p}_{tr}=\frac{F_{tr}}{B\cdot D}$, a transition Sommerfeld number, according to ISO/TS 31657-2 and ISO/TS 31657-3, can be assigned to this value as given below.

$$So_{tr}=\frac{\overline{p}_{tr}\cdot\psi_{eff}^{2}}{\eta_{eff}\cdot\omega_{tr}}=f\left[h_{min,tr}^{*},B^{*},h_0^{*}(\varphi)\right] \tag{26}$$

The individual transition conditions (load, viscosity, rotational speed) can be determined from this. The transition state can therefore only be described by these three coupled data indications. To be able to determine one of these, the two others shall be used in the manner appropriate to this state. At rapid run-down of the machine, the thermal state usually corresponds to the previously continuous operating state of high thermal load. If the cooling immediately fails when shutting down the machine, this can result in a heat accumulation in the bearing, so that for η_{eff} even a less favourable value has to be chosen. If the machine runs down slowly, a reduction of the lubricant film or bearing temperature is also to be expected.

6.9 Further influencing parameters

The dynamic viscosity is highly dependent on the temperature. It is therefore necessary to know this temperature dependency of the lubricant and its specification, see ISO 3448.

The effective dynamic viscosity, η_{eff}, is determined at the effective lubricant film temperature, T_{eff}, i.e. η_{eff} results from the averaging of the temperatures T_1 and T_2.

The dynamic viscosity also depends on the pressure to a slight extent. In the case of stationary bearings and normal specific bearing loads, $\bar{p}$, the pressure dependency is however negligible. The neglect leads to an additional design safety.

In the case of highly shear thinning oils, reversible and irreversible viscosity changes occur depending on the shear stress in the lubrication gap and the duration of use. These effects have so far only been examined for a few lubricants and are not considered in this document.

Crucial for the calculation is the effective relative bearing clearance, ψ_{eff}, at the effective lubricant film temperature, T_{eff}.

Insofar as the coefficients of thermal expansion and temperatures of the shaft $\alpha_{l,\text{J}}$, T_{J} and bearing $\alpha_{l,B}$, T_B do not differ, the warm clearance, $\psi_{\text{eff},}$ equals the cold clearance, ψ_{20} (average fabricated clearance at 20 °C).

If the shaft and bearing (bearing shell with housing) exhibit, because of external influences, different temperatures (T_{J}, T_B) and different coefficients of thermal expansion ($\alpha_{l,J}$, $\alpha_{l,B}$), this shall be considered as a first approximation according to Formula (27). The thermal expansion of the thin bearing sliding layer is negligible.

At different coefficients of thermal expansion of shaft and bearing, the thermal change of the relative bearing clearance is

$$\Delta\psi_{\text{th}} = \alpha_{l,B} \cdot (T_B - 20\,°\text{C}) - \alpha_{l,\text{J}} \cdot (T_{\text{J}} - 20\,°\text{C}) \tag{27}$$

with $T_{\text{J}} \approx T_{\text{eff}}$ and $T_B \leq T_{\text{eff}}$ depending on the installation conditions and the cooling of the bearing shell.

Formula (28) describes the yield in the general case for the average fabricated clearance (at 20 °C):

$$\psi_{20} = \psi_{\text{eff}} - \Delta\psi_{\text{th}} \tag{28}$$

Here, ΔR_B shall also be increased so that $K_{P,20} = K_{P,\text{eff}}$ remains. The bearing clearance tolerance $\Delta\psi = \psi_{\max} - \psi_{\min}$ to be selected as small as possible yields then the upper and lower limiting value for the relative fabricated clearance, as shown in Formula (29):

$$\begin{aligned} \psi_{\max} &= \psi_{20} + \frac{\Delta\psi}{2} = \frac{D_{\max} - D_{\text{J},\min}}{D} \\ \psi_{\min} &= \psi_{20} - \frac{\Delta\psi}{2} = \frac{D_{\min} - D_{\text{J},\max}}{D} \end{aligned} \tag{29}$$

Permissible operational values for the effective bearing clearance, ψ_{eff}, are contained in ISO/TS 31657-4.

At high effective lubricant film temperatures, T_{eff} , of more than approximately 100 °C or at high specific bearing loads, $\bar{p}$, greater than approximately 2 MPa the sliding surfaces in particular of tilting-pad journal bearings can show increasing thermal or mechanical elastic deformations which can change the static and dynamic bearing characteristics considerably. This is often the case at bearing nominal diameters, D, greater than approximately 300 mm. Since these deformations are not taken into account, the results of the simplified calculation procedure given in this document should be checked extra critically by an experienced person in such cases.

6.10 Stiffness and damping coefficients

Due to the (generally anisotropic) flexibility and damping of the radial bearing lubricant films, the critical bending speed, N_{cr}, of a rigidly supported shaft is reduced and frequently split into two resonance speeds, $N_{\text{rsn},1}$ and $N_{\text{rsn},2}$ (see Figure 6); the associated resonance amplitudes are limited to finite values.

Owing to the particular stiffness properties of the bearing lubricant films, hazardous self-excited vibrations can also occur above a stability speed limit, N_{lim}.

To simplify vibration calculations, it is advisable to approximate the non-linear anisotropic flexibility and damping properties of the lubricant film through stiffness and damping coefficients.

In the case of small radial movements (x, y) of the journal around its static equilibrium position (e, β), see Figure 7, it is possible, for the components ΔF_x, ΔF_y of the dynamic additional force, to write as a first approximation as shown in Formula (30)[17]:

$$\begin{aligned} \Delta F_x &= c_{11} \cdot x + c_{12} \cdot y + d_{11} \cdot \dot{x} + d_{12} \cdot \dot{y} \\ \Delta F_y &= c_{21} \cdot x + c_{22} \cdot y + d_{21} \cdot \dot{x} + d_{22} \cdot \dot{y} \end{aligned} \quad (30)$$

This includes $c_{i,k}$, the lubricant film stiffness coefficients of journal bearing, and $d_{i,k}$, the lubricant film damping coefficients, of journal bearing (i,k = 1,2), whereby the indices i and k (in the ISO 31657 series) indicate the directions of the dynamic additional force and the associated radial movement of the journal.

These coefficients can be calculated from the theory of the dynamically loaded plain journal bearing by developing the components F_x^*, F_y^* of the dimensionless bearing force with respect to all arguments (ε, β, ε', β') or (x^*, y^* , $x^{*\prime}$, $y^{*\prime}$) up to the first derivation in a Taylor series. For the dimensionless dynamic additional force, Formulae (31) and (32) apply respectively as a first approximation[9][12]:

$$\begin{aligned} \Delta F_x^* &= \frac{\Delta F_x \cdot \psi_{\text{eff}}^2}{B \cdot D \cdot \eta_{\text{eff}} \cdot \omega} = c_{11}^* \cdot x^* + c_{12}^* \cdot y^* + d_{11}^* \cdot x^{*\prime} + d_{12}^* \cdot y^{*\prime} \\ \Delta F_y^* &= \frac{\Delta F_y \cdot \psi_{\text{eff}}^2}{B \cdot D \cdot \eta_{\text{eff}} \cdot \omega} = c_{21}^* \cdot x^* + c_{22}^* \cdot y^* + d_{21}^* \cdot x^{*\prime} + d_{22}^* \cdot y^{*\prime} \end{aligned} \quad (31)$$

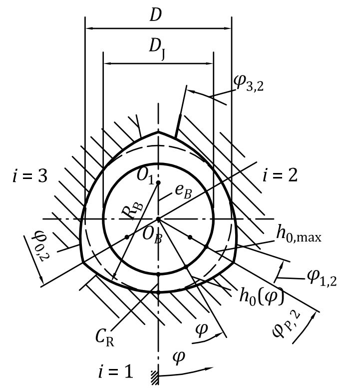
with $x^* = \frac{x}{C_R}, x^{*\prime} = \frac{dx^*}{d(\omega \cdot t)}, \ldots$

and the non-dimensional stiffness and damping coefficients

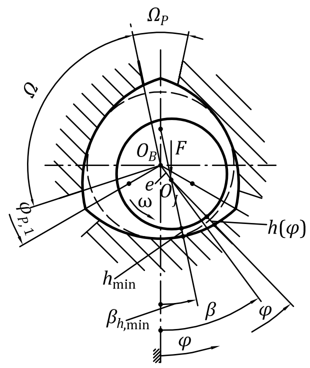
$$\left. \begin{matrix} c_{i,k}^* \\ d_{i,k}^* \end{matrix} \right\} = \frac{\psi_{\text{eff}}^3}{2 \cdot B \cdot \eta_{\text{eff}} \cdot \omega} \left\{ \begin{matrix} c_{i,k} \\ \omega \cdot d_{i,k} \end{matrix} \right\} = So \cdot \frac{C_{R,\text{eff}}}{F} \left\{ \begin{matrix} c_{i,k} \\ \omega \cdot d_{i,k} \end{matrix} \right\} (i,k=1,2) \quad (32)$$

These are summarised in the tabular forms in ISO/TS 31657-2 and ISO/TS 31657-3, depending on the Sommerfeld number, So, the relative bearing width B^* and the profile function $h_0^*(\varphi)$.

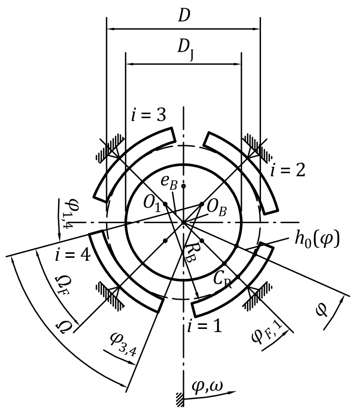
7 Figures



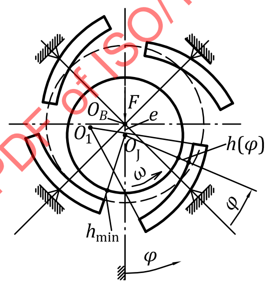
a) Multi-lobed journal bearing



c) Lubrication gap geometry of the multi-lobed journal bearing in eccentric position of the journal



b) Tilting-pad journal bearing (with non-tilting pads)



d) Lubrication gap geometry of the tilting-pad journal bearing (with non-tilting pads) in eccentric position of the journal

$$C_R = \frac{D - D_j}{2} = R - R_j$$

$$\psi = \frac{C_R}{\frac{D}{2}} = \frac{C_R}{R}$$

$$h_0(\varphi) = C_R \times h_0^*(\varphi)$$

$$\Delta R_B = R_B - R_j$$

$$K_p = \frac{R_B - R_j}{C_R}$$

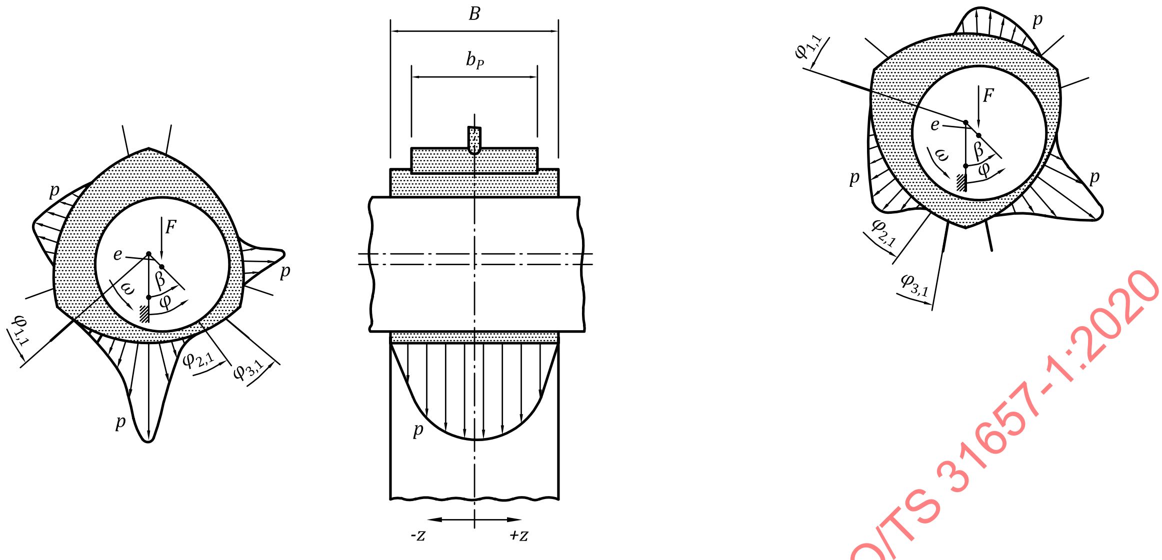
$$h_{0,\max} = C_R \times h_{0,\max}^*$$

$$e_B = R_B - \frac{D}{2} = R_B - R$$

$$\varepsilon = \frac{e}{C_R}$$

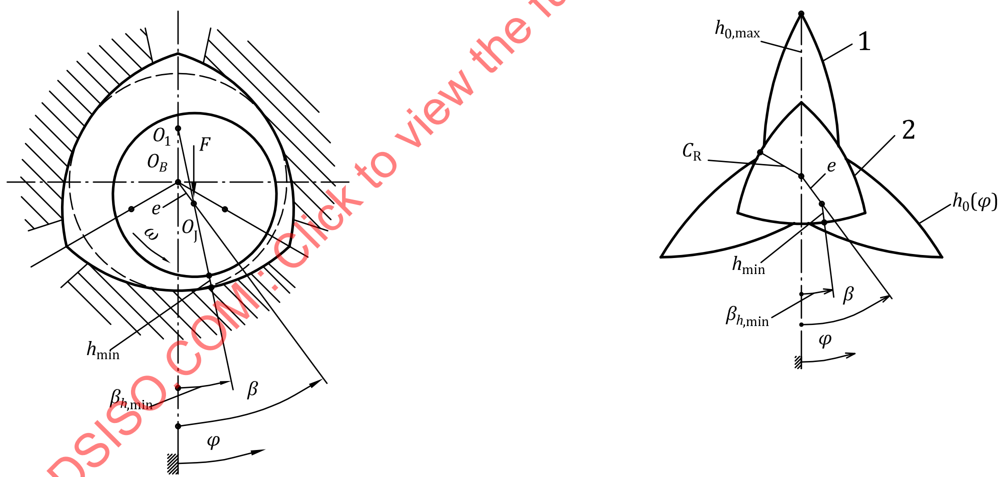
$$h(\varphi) = C_R \times h^*(\varphi)$$

Figure 1 — Bearing bore shape of a) multi-lobed journal bearing and b) tilting-pad journal bearing (with non-tilting pads) and c) and d) lubrication gap geometry of the same bearings in eccentric position of the journal



a) Segment centre **b) Lubrication pocket centre**

Figure 2 — Pressure distribution $p(\varphi, z)$ in the lubricating gap of a multi-lobed journal bearing loaded on the a) segment centre and the b) lubrication pocket centre



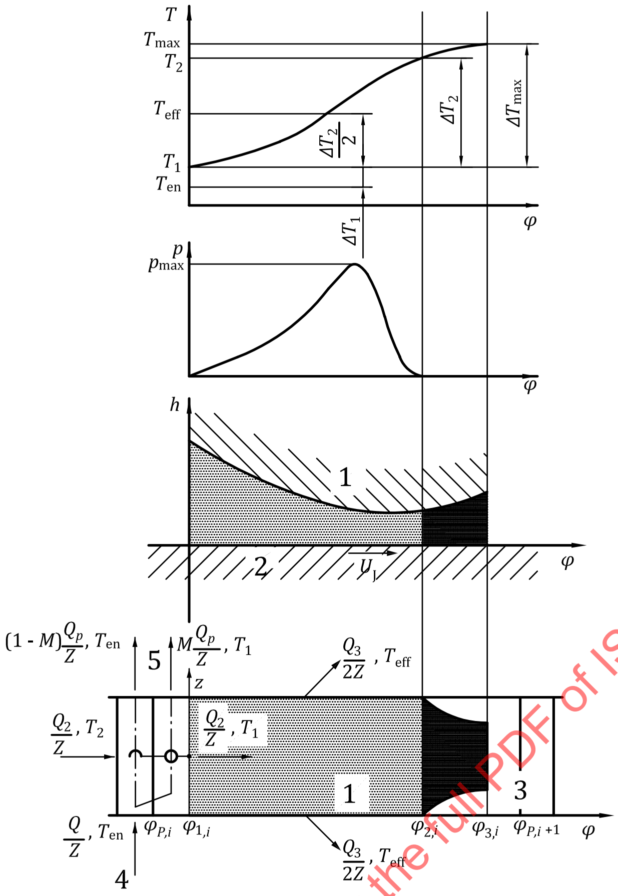
a) Lubrication gap geometry of a multi-lobed journal bearing at eccentric position of the journal **b) Associated gap function and radial journal mobility**

Key

1 gap function

2 radial journal mobility

Figure 3 — a) Lubrication gap geometry of a multi-lobed journal bearing at eccentric position of the journal and b) associated gap function and radial journal mobility



Key

1 segment or pad

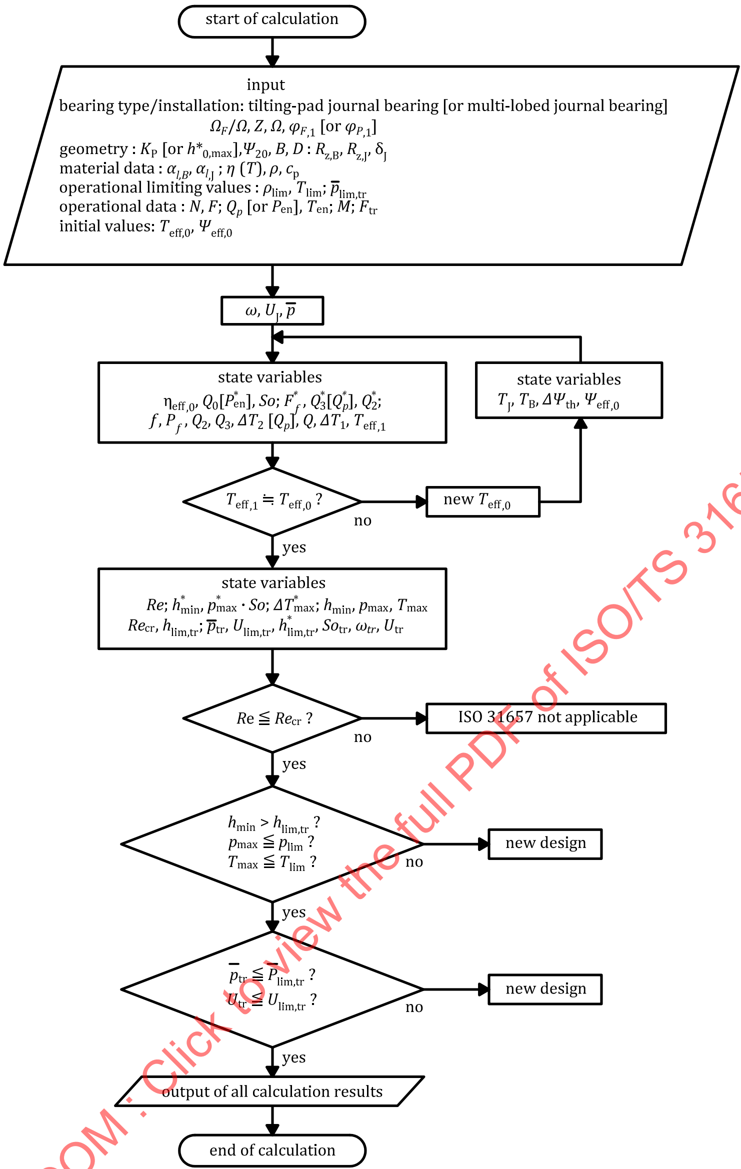
2 journal

3 pocket

4 in

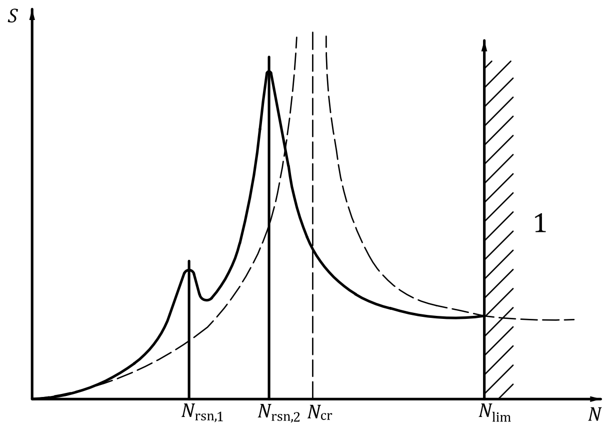
5 out

Figure 4 — Distribution of lubricant film temperature, lubricant film pressure and lubricant film thickness in bearing centre (z = 0) **as well as lubricant flow rate and heat balance in the lubrication pocket and lubrication gap (schematic diagram)**



NOTE Data in square brackets only applies to multi-lobed journal bearings.

Figure 5 — Calculation flow chart (schematic diagram)



Key

_ _ _ _	with rigid bearings
———	with plain journal bearings
1	self-excited vibrations

Figure 6 — Vibration amplitude $S(N)$ of a symmetrical single-mass rotor supported by two bearings

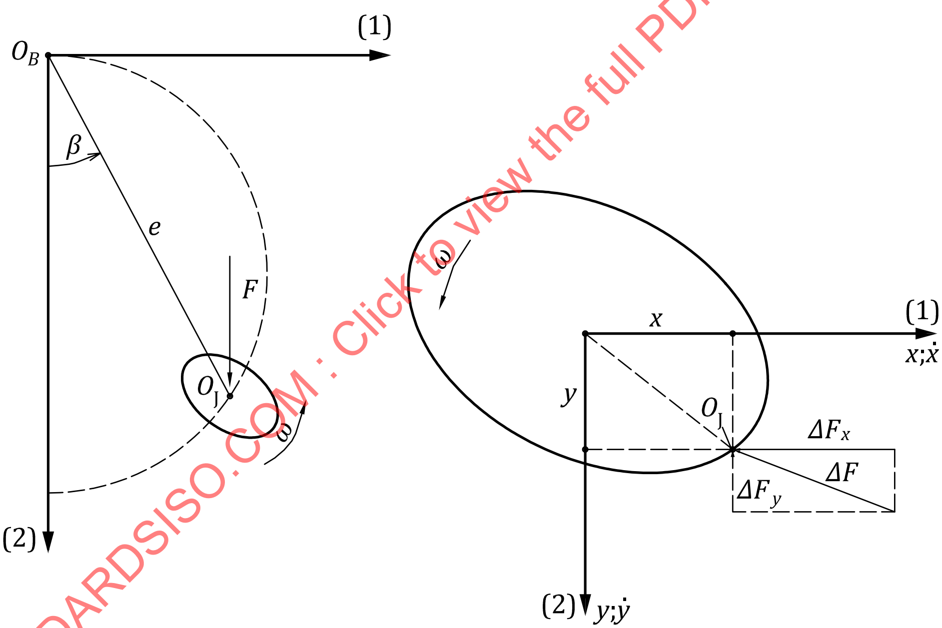


Figure 7 — For definition of the stiffness and damping coefficients

Annex A
(informative)

Calculation examples

A.1 Calculation of a four-lobe bearing

For a four-lobe bearing statically loaded on the segment centre (angular span of bearing sliding surface $\Omega = 70°$, gap ratio $h^*_{0,\max} = 3$), the operational safety is to be verified for a specific speed.

The bearing has a nominal width of $B = 180$ mm, a nominal diameter of $D = 180$ mm and in the operating state a relative bearing clearance of $\psi = 1{,}7$ ‰. At a speed of $N = 6\,852$ min^{-1} the nominal bearing load is $F = 78\,618$ N. The bearing is supplied by forced lubrication with oil of the type ISO VG 46 (supply temperature Ten = 50 °C, supply pressure pen = 0,15 MPa, maximum admissible lubricant heating $\Delta T_{\lim} = 20$ K).

The lubricant viscosity has the temperature dependence shown in Table A.1.

Given data:

Number of sliding surfaces	$Z = 4$
Angular span of bearing sliding surface	$\Omega = \frac{360°}{Z} - \Omega_P = 70°$
Angular coordinate of lubricant pocket centreline in front of the (in direction of rotation)first segment	$\varphi_{P,1} = 315°$
gap ratio	$h^*_{0,\max} = 3$
Effective relative bearing clearance	$\Psi_{\text{eff}} = 0{,}001\,7$
Bearing nominal width	$B = 180$ mm
Bearing nominal diameter	$D = 180$ mm
Viscosity class of the lubricant	ISO VG 46
Lubricant density	$\rho = 900$ kg/m^3
Specific heat capacity of the lubricant	$C_p = 2\,000$ J/(kg K)

Table A.1 — Temperature dependence of lubricant viscosity

Effective lubricant film temperature, T_{eff} °C	Effective dynamic lubricant viscosity, η_{eff} mPa·s
40	40
50	24,67
60	16,62
70	11,91
80	8,92
90	6,91
100	5,5

Nominal load	F = 78 618 N
Rotational speed	N = 6 852 min^{-1}
Lubricant supply pressure	p_{en} = 0,15 MPa
Lubricant supply temperature	T_{en} = 50 °C
Mixing factor	M = 0,5
Minimum admissible lubricant film thickness	$h_{lim,tr}$ = 6 µm
Maximum admissible lubricant film pressure	p_{lim} = 14 MPa
Maximum admissible bearing temperature	T_{lim} = 125 °C
Initial value for the effective lubricant film temperature	$T_{eff,0}$ = 70 °C

Calculation based on the flow chart according to Figure 5:

First the angular velocity of the rotor ω, the circumferential speed of the journal U_J and the specific bearing load $\bar{p}$ are calculated:

$$\omega = 2 \cdot \pi \cdot N = \frac{2 \cdot \pi \cdot 6\,852}{60} = 717{,}5 \ \ \mathrm{s}^{-1}$$

$$U_J = \omega \cdot R_J \approx \omega \cdot D/2 = \frac{717{,}5 \cdot 0{,}18}{2} = 64{,}58 \ \ \mathrm{m/s}$$

$$\bar{p} = \frac{F}{B \cdot D} = \frac{78\,618}{0{,}18 \cdot 0{,}18} = 2{,}426 \ \ \mathrm{MPa}$$

The reference lubricant flow rate Q_0 yields:

$$Q_0 = R^3 \cdot \omega \cdot \psi_{eff} = \left(\frac{D}{2}\right)^3 \cdot \omega \cdot \psi_{eff} = \left(\frac{0{,}18}{2}\right)^3 \cdot 717{,}5 \cdot 0{,}0017 \ \ \mathrm{m}^3/\mathrm{s} = 0{,}889 \ \ \mathrm{l/s}$$

The dimensionless operating characteristic values required for the iterative calculation below can be taken from ISO/TS 31657-2:2020, Table 41 by (e.g. linear) interpolation.

First calculation step:

The effective lubricant film temperature $T_{eff,0}$ = 70 °C yields the dynamic lubricant viscosity:

η_{eff} = 11,91 mPa·s

and hence the dimensionless lubricant supply pressure:

$$p_{en}^* = \frac{p_{en} \cdot \psi_{eff}^2}{\eta_{eff} \cdot \omega} = \frac{0{,}15 \cdot 10^6 \cdot (0{,}0017)^2}{0{,}01191 \cdot 717{,}5} = 0{,}0507$$

and according to Formula (7), the Sommerfeld number:

$$So = \frac{F \cdot \psi_{eff}^2}{B \cdot D \cdot \eta_{eff} \cdot \omega} = \frac{78\,618 \cdot (0{,}0017)^2}{0{,}18 \cdot 0{,}18 \cdot 0{,}01191 \cdot 717{,}5} = 0{,}8206$$

Hence as ISO/TS 31657-2:2020, Table 41 provides, upon linear interpolation, the following values for the friction force parameter F_f^* and the lubricant flow rate parameters Q_3^*, Q_p^* and Q_2^*:

$F_f^* = 3{,}24$

$Q_3^* = 0{,}771$

$Q_p^* = 40{,}699$

$Q_2^* = 4{,}243$

This yields according to Formulae (8) and (9) the coefficient of friction:

$$f = \frac{F_f^* \cdot \psi_{\mathrm{eff}}}{So} = \frac{3{,}24 \cdot 0{,}0017}{0{,}8206} = 6{,}712 \cdot 10^{-3}$$

and the frictional power:

$$P_f = F_f \cdot U_J = 6{,}712 \cdot 10^{-3} \cdot 78\,618 \cdot 64{,}58 \text{ W} = 34{,}08 \text{ kW}$$

Furthermore, according to Formulae (10) to (13) the lubricant flow rates due to inherent pressure formation:

$$Q_3 = Q_0 \cdot Q_3^* = 0{,}889 \cdot 0{,}771 \text{ l/s} = 0{,}685 \text{ l/s}$$

due to supply pressure:

$$Q_p = Q_0 \cdot p_{\mathrm{en}}^* \cdot Q_p^* = 0{,}889 \cdot 0{,}0507 \cdot 40{,}699 \text{ l/s} = 1{,}834 \text{ l/s}$$

in total:

$$Q = Q_3 + Q_p = (0{,}685 + 1{,}834) \text{ l/s} = 2{,}519 \text{ l/s}$$

and at the exit of the lubrication gap (pressure profile trailing edge):

$$Q_2 = Q_0 \cdot Q_2^* = 0{,}889 \cdot 4{,}243 \text{ l/s} = 3{,}772 \text{ l/s}$$

Hence Formulae (18), (20) and (21) yield the averaged oil heating ΔT_1 in the lubrication pockets and ΔT_2 in the lubrication gaps and finally a new value for the effective lubricant film temperature T_{eff}:

$$\Delta T_2 = \frac{P_f}{\rho \cdot c_p \cdot (Q_2 + Q_3/2)} = \frac{34\,080}{900 \cdot 2\,000 \cdot (3{,}772 + 0{,}685/2) \cdot 10^{-3}} = 4{,}6 \text{ K}$$

$$\Delta T_1 = \frac{Q_2}{M \cdot Q + (1-M) \cdot Q_3} \cdot \Delta T_2 = \frac{3{,}772}{0{,}5 \cdot 2{,}519 + (1-0{,}5) \cdot 0{,}685} \cdot 4{,}6 = 10{,}83 \text{ K}$$

$$T_{eff} = T_{en} + \Delta T_1 + \Delta T_2 / 2 = (50 + 10{,}83 + 4{,}6/2)\,°C = 63{,}13\,°C$$

As on account of $|T_{eff} - T_{eff,0}|$ = |63,13 − 70| K = 6,87 K > 1 K, for example, the effective lubricant film temperature, T_{eff}, is not yet determined with sufficient accuracy, a further calculation step shall be attached; as a new initial value for this example the arithmetic mean

$$T_{eff,0} = \frac{70 + 63{,}13}{2}\,°C = 66{,}57\,°C$$

can be set.

Second calculation step:

As in the first calculation step, the following values now result:

$$\eta_{eff} = 13{,}27\ \text{mPa·s}$$

$$p_{en}^* = \frac{0{,}15 \cdot 10^6 \cdot (0{,}0017)^2}{0{,}01327 \cdot 717{,}5} = 0{,}0455$$

$$So = \frac{78\,618 \cdot (0{,}0017)^2}{0{,}18 \cdot 0{,}18 \cdot 0{,}01327 \cdot 717{,}5} = 0{,}7365$$

$$F_f^* = 3{,}135$$

$$Q_3^* = 0{,}7704$$

$$Q_p^* = 40{,}488$$

$$Q_2^* = 4{,}248$$

$$f = \frac{3{,}135 \cdot 0{,}0017}{0{,}7365} = 7{,}236 \cdot 10^{-3}$$

$$P_f = 7{,}236 \cdot 10^{-3} \cdot 78\,618 \cdot 64{,}58\ \text{W} = 36{,}74\ \text{kW}$$

$$Q_3 = 0{,}889 \cdot 0{,}7704\ \text{l/s} = 0{,}685\ \text{l/s}$$

$$Q_p = 0{,}889 \cdot 0{,}0455 \cdot 40{,}488\ \text{l/s} = 1{,}638\ \text{l/s}$$

$$Q = (0{,}685 + 1{,}638)\ \text{l/s} = 2{,}323\ \text{l/s}$$

$$Q_2 = 0{,}889 \cdot 4{,}248\ \text{l/s} = 3{,}776\ \text{l/s}$$

$$\Delta T_2 = \frac{36\,740}{900 \cdot 2\,000 \cdot (3{,}776 + 0{,}685/2) \cdot 10^{-3}} = 4{,}96\ \text{K}$$

$$\Delta T_1 = \frac{3{,}776}{0{,}5 \cdot 2{,}323 + (1 - 0{,}5) \cdot 0{,}685} \cdot 4{,}96 = 12{,}45\ \text{K}$$

$$T_{eff} = (50 + 12{,}45 + 4{,}96/2)\,°C = 64{,}93\,°C$$

Owing to $|T_{eff} - T_{eff,0}|$ = |64,93 – 66,57| K = 1,64 K > 1 K the result shall be further improved with the new start value:

$$T_{eff,0} = \frac{66{,}57 + 64{,}93}{2} = 65{,}75\,°C$$

Third calculation step:

$$\eta_{eff} = 13{,}64\,\text{mPa·s}$$

$$p_{en}^{*} = \frac{0{,}15 \cdot 10^{6} \cdot (0{,}0017)^{2}}{0{,}01364 \cdot 717{,}5} = 0{,}0443$$

$$So = \frac{78\,618 \cdot (0{,}0017)^{2}}{0{,}18 \cdot 0{,}18 \cdot 0{,}01364 \cdot 717{,}5} = 0{,}7165$$

$$F_{f}^{*} = 3{,}11$$

$$Q_{3}^{*} = 0{,}7704$$

$$Q_{p}^{*} = 40{,}438$$

$$Q_{2}^{*} = 4{,}249$$

$$f = \frac{3{,}11 \cdot 0{,}0017}{0{,}7165} = 7{,}379 \cdot 10^{-3}$$

$$P_{f} = 7{,}379 \cdot 10^{-3} \cdot 78\,618 \cdot 64{,}58\,\text{W} = 37{,}464\,\text{kW}$$

$$Q_{3} = 0{,}889 \cdot 0{,}7704 = 0{,}685\,\text{l/s}$$

$$Q_{p} = 0{,}889 \cdot 0{,}0443 \cdot 40{,}438 = 1{,}593\,\text{l/s}$$

$$Q = (0{,}685 + 1{,}593) = 2{,}278\ \ \text{l/s}$$

$$Q_{2} = 0{,}889 \cdot 4{,}249 = 3{,}777\ \ \text{l/s}$$

$$\Delta T_{2} = \frac{37\,464}{900 \cdot 2\,000 \cdot (3{,}777 + 0{,}685/2) \cdot 10^{-3}} = 5{,}05\,\text{K}$$

$$\Delta T_{1} = \frac{3{,}777}{0{,}5 \cdot 2{,}278 + (1 - 0{,}5) \cdot 0{,}685} \cdot 5{,}05\,K = 12{,}87\,\text{K}$$

$$T_{eff} = (50 + 12{,}87 + 5{,}05/2)\,°C = 65{,}4\,°C$$

With $|T_{eff} - T_{eff,0}|$ = |65,4 – 65,75| K = 0,35 K < 1 K the effective lubricant film temperature is now calculated with sufficient accuracy.

The given maximum admissible lubricant heating, ΔT_{lim}, leads to the minimum admissible lubricant flow rate

$$Q_{\text{lim}} = \frac{37\,464}{900 \cdot 2\,000 \cdot 20} \frac{\text{m}^3}{\text{s}} = 1{,}041 \frac{\text{l}}{\text{s}} < 2{,}278 \frac{\text{l}}{\text{s}} = Q\,,$$

so that the lubricant flow rate, Q, has not to be further increased.

For the profile factor, K_P follows according to Formula (1) from ISO/TS 31657-2:

$$K_P = \frac{h^*_{0,\max} - 1/\sqrt{2}}{1 - 1/\sqrt{2}} = \frac{3 - 1/\sqrt{2}}{1 - 1/\sqrt{2}} = 7{,}828$$

And for the critical Reynolds number, Re_{cr}, Formula (6) then yields:

$$Re_{\text{cr}} \approx \frac{41{,}3}{\sqrt{K_P \cdot \psi_{\text{eff}}}} = \frac{41{,}3}{\sqrt{7{,}828 \cdot 0{,}0017}} = 358{,}01$$

For the Reynolds number, Re, it then follows (also according to Formula (6)):

$$Re = \frac{\rho \cdot \omega \cdot R_J \cdot C_{R,\text{eff}}}{\eta_{\text{eff}}} \approx \frac{\rho \cdot \omega \cdot (D/2)^2 \cdot \psi_{\text{eff}}}{\eta_{\text{eff}}} = \frac{900 \cdot 717{,}5 \cdot (0{,}18/2)^2 \cdot 0{,}0017}{0{,}013\,64} = 651{,}9$$

$$Re \approx 651{,}9 > Re_{\text{cr}} \approx 358{,}01$$

Turbulent flow therefore occurs in the lubrication gap; this document is thus applicable for the design of this four-lobe bearing only approximately, see Formula (6).

A.2 Calculation of a tilting-pad journal bearing with five tilting pads

For a tilting-pad journal bearing with five pads supported centrally subject to a static load between pads (angular span of pad sliding surface Ω = 45° and profile factor K_P = 2), the operational safety is to be verified for a specific speed. The bearing has a nominal width of B = 50 mm, a nominal diameter of D = 100 mm and in the operating state a relative bearing clearance of ψ = 1,3 ‰. At a speed of N = 11 762 min^{-1} the nominal load is F = 7 400 N. The bearing is supplied by forced lubrication with oil of the type ISO VG 46 (supply temperature 65 °C, maximum admissible lubricant heating 15 K).

The lubricant viscosity has the temperature dependence shown in Table A.2.

Given data:

Relative angular distance between leading edge and pivot position of pad	$\Omega_F^* = \frac{\Omega_F}{\Omega} = \frac{\varphi_F - \varphi_1}{\varphi_3 - \varphi_1} = 0{,}5$
Number of pads	Z = 5
Angular span of bearing sliding surface	Ω = 45°
Angular coordinate of pivot position of the first (in direction of rotation) pad	$\varphi_{F,1}$ = 36°
Profile factor	K_P = 2
Effective relative bearing clearance	ψ_{eff} = 0,001 3
Nominal width	B = 50 mm

Nominal diameter	D = 100 mm
Viscosity class of the lubricant	ISO VG 46
Lubricant density	ρ = 900 kg/m^3
Specific heat capacity of the lubricant	c_p = 2 000 J/(kg K)

Table A.2 — Temperature dependence of lubricant viscosity

Effective lubricant film temperature T_{eff} °C	Effective dynamic lubricant viscosity η_{eff} mPa s
40	40
50	24,67
60	16,62
70	11,91
80	8,92
90	6,91
100	5,5

Nominal load	F = 7 400 N
Rotational speed	N = 11 762 min^{-1}
Lubricant supply temperature	T_{en} = 65 °C
Mixing factor	M = 0,5
Minimum admissible lubricant film thickness	$h_{lim,tr}$ = 6 µm
Maximum admissible lubricant film pressure	p_{lim} = 15 MPa
Maximum admissible bearing temperature	T_{lim} = 115 °C
Initial value for the effective lubricant film temperature	$T_{eff,0}$ = 90 °C

Calculation based on the flow chart according to Figure 5:

First the angular velocity ω of the rotor, the circumferential velocity of the journal U_J and the specific bearing load $\bar{p}$ are calculated:

$$\omega = 2 \cdot \pi \cdot N = \frac{2 \cdot \pi \cdot 11762}{60} = 1\,231{,}7\ \text{s}^{-1}$$

$$U_J = \omega \cdot R_J \approx \omega \cdot \frac{D}{2} = \frac{1\,231{,}7 \cdot 0{,}1}{2} = 61{,}59\ \text{m/s}$$