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Thermoplastics pipes for the transport of fluids — Methods of extrapolation of hydrostatic stress rupture data to determine the long-term hydrostatic strength of thermoplastics pipe materials

Tubes thermoplastiques pour le transport des fluides — Méthodes d'extrapolation des essais de rupture sous pression, en vue de la détermination de la résistance à long terme des matières thermoplastiques pour les tubes

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Foreword

ISO (the International Organization for Standardization) is a world-wide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of ISO technical committees is to prepare International Standards. In exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

- type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

- type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard.

- type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art"), for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 9080, which is a Technical Report of type 2, was prepared by Technical Committee ISO/TC138 "Plastics pipes, fittings and valves for the transport of fluids".

This Technical Report is the result of considerable discussion within task group 10 of working group 5 of technical committee 138 of the International Organization for Standardization (ISO) (referred to hereafter as ISO/TC138/WG5/TG10) which was entrusted with generation of the Report and is an agreed compromise which incorporates features of several accepted National procedures.

Furthermore it is emphasized that these standard extrapolation methods (SEM) are not intended to be used to disqualify existing procedures to arrive at design stresses or allowable pressures for pipelines from plastics materials nor to disqualify pipelines from materials proven by such procedures which have shown to be satisfactory over long years of experience. The SEM are generally meant to be used to qualify a material for pipes by the introduction of such a material on the market.

At present the SEM are considered to be at a stage where they need to be tested with real data. Therefore the publication as a Technical Report, type 2, was considered to be justified.

The Committee wishes to have comments which will be based on analysis of pipe stress rupture data using one or both of the procedures. By generating constructive criticism the SEM can be improved and, if necessary, modified.

Comments of a general nature, for example, pertaining to the theoretical basis for the concept of the SEM, are unlikely, at this stage, to be useful. Many such submissions have already been considered within the study. The need is for pragmatic appraisal of the proposals.

This document is being issued in the type 2 Technical Report series of publications (according to subclause G.6.2.2 of part 1 of the IEC/ISO Directives) as a "prospective standard for provisional application" in the field of thermoplastics pipes for the transport of fluids because there is an urgent need for guidance on how standards in this field should be used to meet an identified need.

This document is not to be regarded as an "International Standard". It is proposed for provisional application so that information and experience of its use in practice may be gathered. Comments on the content of this document should be sent to the ISO Central Secretariat.

A review of this type 2 Technical Report will be carried out not later than two years after its publication with the options of: extensions for another two years; conversion into an International Standard; or withdrawal.

Annexes A to F form an integral part of this Technical Report. Annex G is for information only.

Introduction

0.1 General principles

The suitability for use of a plastics pressure pipe is in the first instance determined by the performance under stress of its material of construction taking into account the envisaged service conditions (e.g. temperature).

It is conventional to express this by means of the hoop stress to which a plastics pipe made of the material under consideration is expected to be able to withstand fifty years at an ambient temperature of 20 °C, using water as the test environment.

In certain cases it is necessary to estimate the value for this hoop stress at either shorter life times or higher temperatures, or on occasion both.

The methods given in this report are designed to meet the needs for both estimations. The result obtained will generally indicate the average expected value of hoop stress which can cause failure in the stated time at a stated temperature (the ultimate hoop stress).

In most cases it is necessary to obtain a design value rather than the ultimate value, therefore it is necessary to define an appropriate factor taking into account other relevant material properties as well as aspects of the specific application envisaged.

This technical report provides a definitive procedure incorporating extrapolation by using test data at different temperatures, analysed by curve fitting techniques in conjunction with linear regression analysis.

Those curve fitting techniques are of mathematical character, but the formula used is based on the Eyring deviation of so-called rate processes (see annex G).

In order to assess the predictive value of the model used, it has been considered necessary to make use of the estimated 97,5 % lower confidence limit where the 97,5 % lower confidence limit is equivalent with the lower confidence limit of the 95 % confidence interval. This convention is used in the mathematical calculations to be consistent with literature. This aspect has necessitated the use of statistical techniques. It is recognized that such procedures previously have not been specifically quantified and it is presumed this will be accounted for within the consideration of the choice of the value of the factor to be used to convert the ultimate circumferential stress to a design stress.

The methods can provide a systematic basis for the interpolation of stress rupture characteristics at working life temperature conditions different from the conventional 50 years at 20 °C.

These methods are not applicable if any chemical attack or degradation effect, such as oxidation or consummation of additives such as stabilisers or anti-oxidants, has been found to occur during the pipe testing programme.

It is essential that the medium used for pressurizing the pipe has no other effect. In general water is considered to be such a medium. The effect of chemical attack on plastics pressure pipes is a subject of study by ISO/TC138/SC3.

The study necessary to prepare a 'standard extrapolation method' (SEM) has been undertaken by members of ISO/TC138/WG5/TG10, which was first convened as an ad hoc group of WG5 in March 1976. Membership included invited individual experts from France, Germany, the Netherlands, Switzerland, the United Kingdom and the United States of America. In the course of the deliberations it has studied the relevance of procedures identified by Larson Miller and the Goldfein derivation for plastics, National Standards, published technological papers from different countries including the United States of America, the United Kingdom, Canada, Germany, Sweden, the Netherlands, France, and Switzerland. In addition specific statistical tasks have been commissioned from experts in France, the Netherlands, the United Kingdom, the United States of America and Finland. A total of over 200 working documents, the majority of which have been highly technological, have been examined in the course of this study.

Long consideration has been given to resolve which variable should be taken to be the independent variable to calculate the long-term hydrostatic stress. The choice was between time and stress. The basic question to which the method has to give an answer can be formulated in two ways as follows.

a) What is the maximum stress (or pressure) that a given pipe system can withstand at a given temperature for a defined time?

This question is answered if time is chosen to be the independent variable to calculate the long-term hydrostatic strength (σ_{LTHS}).

b) How long will a pipe system last when subjected to a defined stress (pressure) at a given temperature?

This question is answered if the stress (pressure) is chosen as the independent variable to calculate the long-term hydrostatic strength (σ_{LTHS}).

Both questions may be asked by users of existing pipe systems and by intending users, and of new systems. Both questions have equal validity.

When the test data on the pipe under study do not show any scatter and when the pipe material can be described perfectly by the chosen empirical model, the regression with either time independent or stress independent will be identical. This is never so, because testing circumstances are never ideal and the material will not be 100 % homogeneous, therefore the observations will show scatter. Moreover, the model is an idealisation. The calculated regressions will not be identical and the difference between the calculated values will increase as the scatter increases.

It can be shown that the regression of log time on log stress always gives a lower, more conservative, result than the regression of log stress on log time, caused by the scatter of the data. The choice between the two methods of carrying out the regression analysis (time-independent or stress independent) for the SEM should not allow a possibly unjustified optimistic value.

In order to achieve this all SEM calculations are made using stress as the independent variable.

0.2 Use of the methods

0.2.1 These SEM methods are designed to meet basically two requirements. These are:

a) to estimate the mean hoop stress which a pipe made of the material under consideration is able to withstand for 50 years at an ambient temperature of 20 °C using water as the test environment;

b) to estimate the value for the mean hoop stress at either shorter life times, or higher temperatures, or on occasion both.

0.2.2 There are several extrapolation models in existence, which have different degrees of freedom or a different number of variables, as indicated in figure 0.1. It was decided that the SEM will only consider the models QI and QII, RI and RII as shown in figure 0.1.

In models RI and RII a fourth coefficient has been added. The addition of this fourth coefficient inevitably leads to a better correlation coefficient and lack-of-fit values, because of the additional degree of freedom. It was necessary to add this fourth coefficient, because it has been shown that for certain materials (PVC, PVC-C) this leads to better fit.

For other materials (PE and PP) however, good fit is already reached with the model with three coefficients. To add more coefficients will improve the correlation, but leads to more uncertain extrapolation and has therefore not adopted.

0.2.3 Method I of the SEM describes a method to estimate such a mean hoop stress whether a knee is found or not, in accordance with models QI, QII, RI and RII of figure 0.1.

This method is intended for new materials or for materials not previously evaluated for pipe production.

0.2.4 Method II of the SEM describes a method for estimation of the hoop stress at 50 years at a chosen temperature for pipe materials and variations thereof already widely used and under consideration in ISO/TC138 for water and/or gas transport and for industrial pressure pipe applications. This method can only be used if model QI has shown to be applicable.

0.2.5 The materials have to be tested in pipeform to enable the method to be applied.

0.2.6 The end result of the SEM for a specific material is the value for the long-term hydrostatic strength (σ_{LTHS}) and lower confidence limit (σ_{LCL}).

0.2.7 Methods for use of the σ_{LTHS} and/or σ_{LCL} to arrive at allowable design stresses still have to be considered.

Service factors or safety factors have to be introduced.

An estimate for a safety factor taking into account the effects of testing and estimating stress by extrapolation could be the ratio of the 97,5 % lower confidence level (l.c.l.) and the extrapolated mean hoop stress.

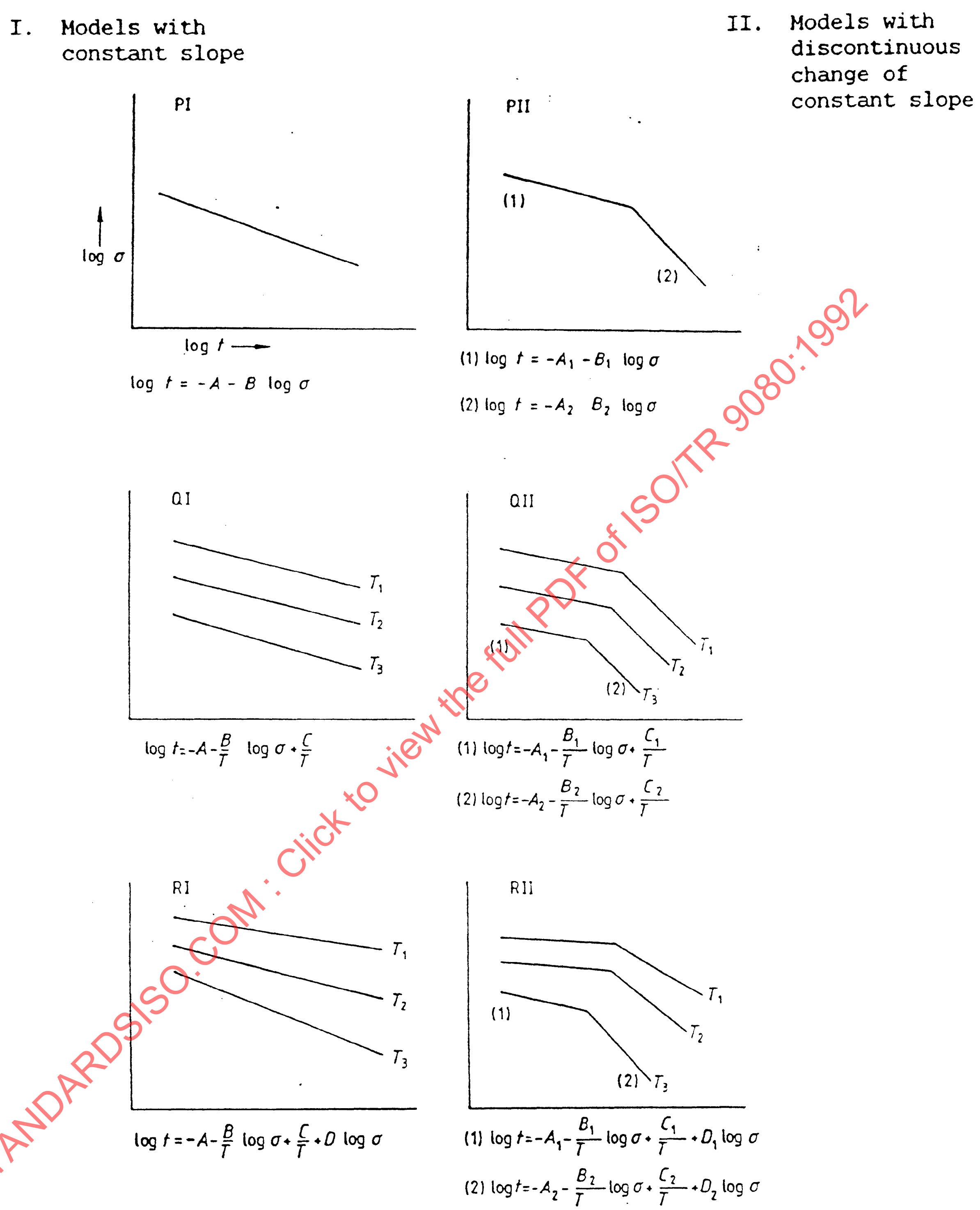

Figure 0.1: Scheme of material behaviour models

Thermoplastics pipes for the transport of fluids — Methods of extrapolation of hydrostatic stress rupture data to determine the long-term hydrostatic strength of thermoplastics pipe materials

1 Scope

This Technical Report describes methods for estimation of the long-term hydrostatic strength of thermoplastics materials.

The methods are applicable to all known types of thermoplastics and cross-linked thermoplastics pipes at any temperature and to any practicable test medium. The methods were developed on the basis of test data from pipes of relatively small sizes. The pipe sizes to be tested are specified in the relevant product standard.

These methods do not cover effects which are caused by oxidation, hydrolysis, or exhaustion of additives such as anti-oxidants within or outside the testing times. If such effects occur, then other test methods may be appropriate.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of ISO/TR 9080. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on ISO/TR 9080 are encouraged to investigate the possibility of applying the most recent editions of the standards listed below. Members of IEC and ISO maintain registers of currently valid international standards.

ISO 1167 - 1973: Plastics pipes for the transport of fluids - Determination of the resistance to internal pressure

ISO 3126 - 1974: Plastics pipes - Measurement of dimensions

3 Definitions, symbols and abbreviations

For the purposes of this Technical Report, the following definitions apply:

3.1 internal pressure (P): Force per unit area exerted by the medium in the pipe.

3.2 stress (σ): Force per unit area in the wall of the pipe in the hoop direction due to internal pressure. The stress is denoted as σ and derived from the internal pressure using the following equation:

$$\sigma = \frac{P\,(D_{m,max.} - e)}{2e_{min.}}$$

where

$D_{m,max.}$ is the maximum mean outside diameter;

$e_{min.}$ is the minimum wall thickness.

3.3 test temperature (T): The temperature at which stress rupture data have been determined.

3.4 service temperature (T_S): The temperature at which the pipe will be used.

3.5 failure: Physical breakdown of pipe as manifested by ductile bursting, brittle cracking, splitting or weeping (seepage of liquid through the pipe wall) during testing.

3.6 time-temperature dependent hydrostatic strength (σ_{tTHS}): A quantity with the dimensions of stress which can be considered as a a property of the material under consideration. It is denoted as:

$$\sigma_{tTHS} = \sigma(T, \log t, \alpha)$$

where

T is a temperature, in kelvins;

t is a time, in hours;

α is a factor related to the chance that a pipe, made from the material under study, will survive without failure during a time t when submitted to a stress σ, at a constant temperature.

NOTE - The σ_{tTHS} is derived from the available test data by using one of the methods I or II in accordance with clause 5.

3.7 long-term hydrostatic strength (σ_{LTHS}): A quantity with the dimensions of stress which represents the 50 % lower confidence limit (LCL) for the long-term hydrostatic strength and can be considered as a property of the material under consideration. It equals the mean (average) strength or predicted (average) mean strength at a temperature T and a time t when the factor α has a value of 0,5 (see 3.6). It is denoted as:

$$\sigma_{LTHS} = \sigma(T, \log t, 0{,}5)$$

3.8 lower confidence limit long-term hydrostatic strength (σ_{LCL}): A quantity with the dimensions of stress which represents the 97,5 % lower confidence limit of the long-term hydrostatic strength and can be considered as a property of the material under consideration. It equals the mean (average) strength or predicted mean (average) strength at a temperature *T* and a time *t* when the factor α has a value of 0,975 (see 3.6). It is denoted as:

$$\sigma_{LCL} = \sigma(T, \log t, 0{,}975)$$

3.9 knee: The transition point between two probably different modes of failure represented by a change of slope on a log stress versus log time plot of hydrostatic stress rupture data.

4 Acquisition of test data

4.1 Test conditions

The pipe stress rupture data shall be determined using the procedure described in ISO 1167 - 1973 except that in the case of conflict between that standard and the provisions of this SEM, the provisions of this SEM shall apply.

The pressure medium inside the pipe shall be water or any other liquid. The outside environment shall be air, water or any other liquid. The inside medium and the outside environment shall be mentioned in the test report. The inside and the outside environment shall be maintained within ± 1 °C, but preferably within ± 0,5 °C of the test temperature during a conditioning period, extending from 15 min before the beginning of the test, and during the test period.

The mean outside diameter and minimum wall thickness of each pipe test piece shall be determined in accordance with ISO 3126.

4.2 Distribution of pressure levels

For each selected temperature a minimum of 25 failure stress-time points above 10 h shall be obtained, spread over at least 5 pressure levels and such that at each pressure level at least one failure point is recorded. (For statistical reasons it is recommended that more failure points are recorded at each pressure level.) If possible, the pressure levels shall be selected so that at least eight failures will occur between 10 h and 100 h, at least eight failures between 100 h and 1000 h and at least nine failures above 1000 h; however, one shall have at least four of these points above 7000 h and at least one of these points above 9000 h.

Test pieces which have not failed at the lowest pressure levels shall be used in the calculations as failure points if they increase the value of σ_{LTHS}, if not then they shall be deleted.

NOTE - To be able to take advantage of advanced statistical methods it is recommended that the differences between successive pressure levels is arranged to follow the relationship : (delta) log (stress) is constant.

5 Procedure

5.1 Selection of method for data gathering and analysis

Select and perform a method of determination of a long-term hydrostatic strength in the light of the following information.

Two methods for obtaining the long-term hydrostatic strength, σ_{LTHS}, are presented. They are designated as method I, detailed in 5.2 and for which a flow chart is given in annex A, and method II, detailed in 5.3.

Both methods are based on linear regression and supported as applicable by reference to appropriate calculation details given in annex B and a validation test for linearity given in annex C. The choice of method may be specified by a relevant product standard in reference to this Technical Report, and depends on the indication of a knee in the data pattern, for which a statistical test is provided in annex D, and/or on the purpose of the evaluation, as follows.

- **Method I** is the most complete and decisive method for determining burst pressure characteristics. It requires observations at several temperatures and times over one year or longer and is applicable whether or not indications are found in the test data of the presence of a knee. The consequent procedural alternatives are selected as indicated in annex A. The risk that in the extrapolation range the real behaviour will deviate from the predicted behaviour, e.g. because of another change of slope in the burst pressure characteristics, is considered to be minimal, although not zero. Method I is the appropriate method for determining the burst pressure characteristics of new materials.

- **Method II** is a method in which the required range of experiments is more restricted in the number of temperature levels than in Method I and the observations shall not show any sign of a knee. This method is more suitable for testing new varieties of well-known materials where the polymer itself has not changed.

5.2 Method I

(See also annex A.)

5.2.1 Required test data

Obtain test data in accordance with clause 4 and using at least three temperatures T_1, T_2, T_3, ...; where $T_1 < T_2 < T_3 < \ldots$ and the following conditions also apply:

a) each pair of adjacent temperatures shall be separated by at least 10 K;

b) the highest test temperature, $T_{max.}$, shall not exceed the glass transition temperature minus 20 K in amorphous or predominantly amorphous polymers, or the melting temperature minus 15 K in crystalline or semi-crystalline polymers;

c) the number of observations and the distribution of pressure levels per temperature shall comply with 4.2;

d) the maximum test temperature, $T_{max.}$, shall be selected taking into account for each material the maximum temperature at which the material can be used and the highest possible test temperature;

e) the number of observations of class 2 n_2, as given in B.5.2 of annex B, shall at least be 20.

NOTE - To obtain an optimal estimation of a σ_{LTHS} value, it is recommended that the range in test temperatures is selected in such a way that it covers the service temperature or range in service temperatures.

5.2.2 Detection of a knee; validation of data and model

5.2.2.1 Apply linear regression to the observations at every test temperature separately and determine at every test temperature the slope of the regression line and the stress at which 50 % and 2,5 % failure is predicted after 50 years.

For the procedure to calculate these values, see B.3 of annex B.

5.2.2.2 If at one or more temperatures the slope of the regression line b is positive, consider the test data at that temperature or at those temperatures unsuitable.

5.2.2.3 Apply the test for a knee in accordance with annex D. If the presence of a knee is confirmed, apply 5.2.4. Otherwise apply 5.2.2.4.

5.2.2.4 If the ratio of the slope of the regression line at the lowest temperature, $b_{T\ min.}$ and the slope of the regression line at the highest test temperature, $b_{T\ max.}$, exceeds the value of three, i.e.:

$$\frac{b_{T\ min.}}{b_{T\ max.}} > 3$$

consider this as the possible indication of a knee within the experimental range and apply 5.2.4.

5.2.2.5 If the ratio of the 97,5 % lower confidence limit long-term hydrostatic strength, $\sigma_{(T,\ \log t,\ 0,975)}$ at a time, t_1, of one increment in log time, in hours, beyond the longest testing time and the long-term hydrostatic strength, $\sigma_{(T,\ \log t,\ 0,5)}$ at that time, both at the highest temperature, has a value equal to or below 0,85 i.e.:

$$\frac{\sigma(T_{max.}, \log t, 0{,}975)}{\sigma(T_{max.}, \log t, 0{,}5)} \leq 0{,}85$$

consider this as a possible indication of a knee within the experimental range of the highest test temperature data and apply 5.2.4.

5.2.2.6 Inspect, visually the time-to-failure/stress data points when plotted on a log (time)-log (stress) basis. If a knee appears to be present, apply 5.2.4.

5.2.2.7 If the conditions given in 5.2.2.4, 5.2.2.5 and 5.2.2.6 were not fulfilled, assume that there is no indication of the presence of a knee within the experimental range and apply 5.2.3.

5.2.3 Calculation of log σ_{LTHS} when no knee is found

5.2.3.1 In accordance with the relevant referring product specification or otherwise based on the information given in 5.1 and annex E, apply multiple linear regression to find the coefficients *A*, *B*, *C* and *D* in one of the following equations where the procedure to calculate these values for equation (1) is outlined in B.4 and comparable calculations can be made by applying appropriate mathematics to formula (1a).

$$\log t = -A - \frac{B}{T} \log \sigma_{LTHS} + \frac{C}{T} \qquad \ldots (1)$$

$$\log t = -A - \frac{B}{T} \log \sigma_{LTHS} + \frac{C}{T} + D \log \sigma_{LTHS} \qquad \ldots (1a)$$

If the values of A, B and C are not positive, consider the data unsuitable.

5.2.3.2 Calculate the mean strength, $\sigma_{(T, \log t_e, 0,5)}$, and the 97,5 %

lower confidence limit (LCL) of the mean strength, $\sigma_{(T, \log t_e, 0,975)}$, at a life time t_e and a temperature *T*; and as outlined in B.4, and equation 1a providing that extrapolation time limits given in 5.2.5 are complied with.

5.2.3.3 Apply the lack-of-fit test in accordance with annex C. If the hypothesis of linearity is rejected, reject the model used.

Reject a model if *F* > 20, where *F* is the coefficient of lack-of-fit of the model to the data.

If the linear model (models) is (are) not rejected, apply 5.2.3.4.

5.2.3.4 Choose the model with the lowest value for *F* (lack-of-fit) to describe the experimental data.

With this model calculate the values for the σ_{LTHS} and σ_{LCL} at various times and temperatures.

5.2.3.5 If the ratio of σ_{LCL} (see B.3) and the mean strength, σ_{LTHS}, at service temperature T_s and time t_e is below 0,85, that is, if

$$\frac{\sigma(T_s, \log t_e, 0{,}975)}{\sigma(T_s, \log t_e, 0{,}5)} < 0{,}85$$

consider the mean strength σ_{LTHS} to be equal to

$$\frac{1}{0{,}85} \times \sigma(T_s, \log t_e, 0{,}975)^{*}$$

5.2.4 Calculation of σ_{LTHS} when a knee is found

5.2.4.1 For the equation (based on equation (1) of 5.2.3: see annex G)

$$\log t = \tfrac{1}{2} \left\{ -(A_1 + A_2) - \left(\frac{B_1 + B_2}{T}\right) \log \sigma_{LTHS} + \left(\frac{C_1 + C_2}{T}\right) - \left| -(A_1 - A_2) - \left(\frac{B_1 - B_2}{T}\right) \log \sigma_{LTHS} + \left(\frac{C_1 - C_2}{T}\right) \right| \right\} \quad \ldots(2)$$

use the procedures given in annexes E and F, or equivalent, to choose the coefficients A_1, A_2, B_1, B_2, C_1, C_2 in such a way that the equation describes the observations optimally.

If the values of A_1 to C_2 are not all positive, consider the data usuitable.

Alternatively, for equivalent equations derived from equation (1a) of 5.2.3.1, perform analogous calculations by applying appropriate mathematics to equivalent equations with the fourth term, i.e. including also D_1 and D_2. If the values of A_1 to C_2 are not all positive, consider the data unsuitable.

5.2.4.2 When a knee is found and the observations are divided into two groups of data on each side of the knee, verify that the class 2 group of data, which contains the failure times obtained at hoop stresses below the knee-hoop stress for the temperatures, has sufficient data, i.e.:

- the total number of class 2 data at a maximum test temperature (T_3) and the test temperature one step below (T_2) shall be equal or greater than 20;

- the number of class 2 points at T_2 shall be equal or greater than 2.

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* This 'derating' of the mean strength results in a pessimistic prediction when data are showing a large scatter. By 'derating' the mean stress, valuable information can still be used.

If these conditions are not both fulfilled, consider the data unsuitable for evaluation.

5.2.4.3 Apply the lack-of-fit test according to annex C on each class of databody. If the hypothesis of linearity is rejected, reject the model used. Reject a model if $F > 20$ (lack-of-fit).

If the linear model (models) is (are) not rejected, than apply 5.2.4.4.

5.2.4.4 Choose the models that show the lowest F (lack-of-fit) to describe the experimental data.

Calculate the values for σ_{LTHS} and other stresses at various times and temperatures using this model.

If at temperatures above 20 °C and time shorter than 50 years stresses have to be calculated, use the relevant time and temperature in the evaluation.

If such calculations are made with the single parts of equation 2, given in 5.2.4.1, then choose the lowest value for the stresses.

5.2.4.5 Calculate the mean strength, $\sigma_{(T, \log t_e, 0,5)}$, and the 97,5 % lower confidence limit of the mean strength, $\sigma_{(T, \log t_e, 0,975)}$, at a life time t_e and a temperature T as outlined in B.5 of annex B provided that the extrapolation time limits as stated in 5.2.5 are complied with.

5.2.4.6 If the ratio of σ_{LCL} and σ_{LTHS} at service temperature T_S and time T_e is below 0,85, that is if:

$$\frac{\sigma_{(T_S, \log t_e, 0,975)}}{\sigma_{(T_S, \log t_e, 0,5)}} < 0,85$$

consider the mean strength to be equal to

$$\frac{1}{0,85} \times \sigma_{(T_S, \log t_e, 0,975)}{}^{*}$$

* This 'derating' of the mean strength results in a pessimistic prediction when data are showing a large scatter. By 'derating' the mean stress, valuable information can still be used.

5.2.4.7 If the ratio of the σ_{LCL} and the mean strength at service temperature T_s and time t_e is less than 0,5 then consider the data unsuitable for extrapolation.

5.2.5 Extrapolation time limits

Determine the extrapolation time limits using the following information and procedures.

The time limits t_e, for which extrapolation is allowed are bound to temperature-dependent values. The time t_e, includes the testing time. Table 1 gives the extrapolation-time factor, K_e, as a function of (delta)T based on the following equation

$$(\text{delta})T = T_{max.} - T_s,$$

where

$T_{max.}$ is the maximum test temperature;

T_s is the service temperature.

The extrapolation time t_e can be calculated using the following equation:

$$t_e = K_e t_{max.}$$

Obtain the maximum test time $t_{max.}$ by averaging the 5 longest failure times, where test pieces which have not yet failed may be considered as "failures" for this purpose.

Table 1 - Relation between (delta)T (= $T_{max.}$ - T_s) and K_e

delta T (K) >	delta T (K) ≤	K_e
0	10	1
10	15	3
15	20	5
20	25	9
25	30	16
30	35	28
35	40	50

In the case when $t_{max.}$ is equal to 8760 h (1 year), K_e indicates the maximum allowed extrapolation time t_e in years.

Table 2 indicates the extrapolation time limit, t_e, in years as a function of the maximum test temperature, $T_{max.}$, and the service temperature, T_s, from 20 °C inclusive up to 25 °C (not included), from 25 °C inclusive up to 30 °C (not included) and so on, provided the maximum test time, $t_{max.}$ at $T_{max.}$, is at least 8760 h.

Table 2 - Extrapolation time limit as a function of the maximum test temperature

Time in years

$T_{max.}$ °C	T_s, in degrees Celsius												
	20	25	30	35	40	45	50	55	60	65	70	75	etc.
50	16	9	5	3	1	1							
55	28	16	9	5	3	1	1						
60	50	28	16	9	5	3	1	1					
65		50	28	16	9	5	3	1	1				
70			50	28	16	9	5	3	1	1			
75				50	28	16	9	5	3	1	1		
80					50	28	16	9	5	3	1	1	
85						50	28	16	9	5	3	1	
90							50	28	16	9	5	3	
95								50	28	16	9	5	
100									50	28	16	9	etc
105										50	28	16	
110											50	28	

5.2.6 Visual verification

Plot the observed failure points, the σ_{LTHS} linear regression lines and the σ_{LCL} curves in a diagram on a (log σ/log (time)) scale.

Verify the fit of the regression lines to the data by visual inspection.

5.3 Method II

5.3.1 Required test data

Obtain test data in accordance with 4.1 and using two temperatures, T_1 and T_2 where temperature T_2 shall be at least 40 K higher than T_1 and the number of observations and the distribution of pressure levels at both temperature T_1 and T_2 are in accordance with 4.2.

NOTE - To obtain an optimal estimation of the σ_{LTHS} regression lines, it is recommended to select the maximum test temperature $T_{max.}$ (which is T_2) in accordance with table 2 using the required extrapolation time and service temperature.

5.3.2 Validation of data and model

5.3.2.1 Apply linear regression to the test data at temperature T_1 and determine the slope of the regression line, b_1; the predicted average strength at 50 years, $\sigma(T_1, 5{,}65, 0{,}5)$; and the 97,5 % lower confidence limit (LCL) at 50 years, $\sigma(T_1, 5{,}65, 0{,}975)$. For the procedure to calculate these values see B.3 of annex B.

5.3.2.2 Apply the linear regression to the test data at temperature T_2 and determine the slope of the regression line, b_2; the predicted average strength at 10 000 h, $\sigma_{(T_2,\ 4,\ 0{,}5)}$, and the 97,5 % LCL at 10 000 h, $\sigma_{(T_2,\ 4,\ 0{,}975)}$. For the procedure to calculate these values, see B.3 of annex B.

5.3.2.3 If the slopes of the regression lines, b_1 and b_2 are not negative, consider the data unsuitable.

5.3.2.4 Apply the test for a knee, in accordance with annex D. If the presence of a knee is confirmed, do not use this method but, if possible, obtain a third set of data, at a different test temperature and in accordance with 5.2.1, and apply method I to the dataset (see 5.2.4).

5.3.2.5 If the ratio of the 97,5 % LCL at 50 years at temperature T_1, $\sigma_{(T_1,\ 5{,}65,\ 0{,}975)}$, and the average strength at 50 years at temperature T_1, $\sigma_{(T_1,\ 5{,}65,\ 0{,}5)}$, is not greater than 0,85 i.e. unless

$$\frac{\sigma_{(T_1,\ 5.65,\ 0{,}975)}}{\sigma_{(T_1,\ 5{,}65,\ 0{,}5)}} > 0{,}85$$

consider the data unsuitable.

5.3.2.6 If the ratio of the 97,5 % LCL at 10 000 h at temperature T_2, $\sigma_{(T_2,\ 4,\ 0{,}975)}$, and the average strength, σ_{LTHS}, at 10 000 h at temperature T_2, $\sigma_{(T\ ,\ 4,\ 0{,}5)}$, is not greater than 0,85 i.e. unless

$$\frac{\sigma_{(T_2,\ 4,\ 0{,}975)}}{\sigma_{(T_2,\ 4,\ 0{,}5)}} > 0{,}85$$

consider the data unsuitable.

5.3.2.7 Confirm that the slopes of the regression lines, b_1 and b_2, at temperatures T_1 and T_2, fulfil the relationship.

$$\frac{b_1}{b_2} < 3$$

If not, consider the data unsuitable.

5.3.2.8 Apply the multiple linear regression, the lack-of-fit test and the correlation test in accordance with 5.2.3 but with only data sets for two test temperatures.

5.3.3 Extrapolation time limits

Ensure that the time limits t_e, for which extrapolation is applied comply with 5.2.5.

5.3.4 Visual verification

Plot the observed failure points, the σ_{LTHS} lines and the σ_{LCL} curves in a diagram on a log (stress)/log (time) scale.

Verify the fit of the regression lines to the data by visual inspection.

6 Test report

The test report shall include the following.

a) Complete identification of the sample, including manufacturer, material type, code number, source and previous significant history, if any.

b) Dimensions of the pipes used for testing.

c) Outside test environment and pressure medium inside the pipes.

d) A table of the observations, including for each observation: the test temperature, pressure level (in bars), stress (in megapascals), time of failure (in h), date of the test and other observations which could be relevant.

e) The method and the model used to estimate the long-term hydrostatic strength, σ_{LTHS}.

f) If method II has been used the σ_{LTHS} and σ_{LCL} at 50 years as $\sigma(T, 5{,}65, 0{,}5)$ and $\sigma(T, 5{,}65, 0{,}975)$ in megapascals, or if method I has been used to establish the σ_{LTHS}, the $\sigma(T, 5{,}65, 0{,}5)$ and the $\sigma(T, 5{,}65, 0{,}975)$ at the test temperatures and either

i) the three (four) coefficients A, B, C (and D) when no knee was found; or

ii) the six (eight) coefficients A_1, A_2, B_2, C_1, C_2, (D_1 and D_2) when a knee was found in the experimental range.

g) The diagram representing observed failure points, σ_{LTHS} multiple linear regression lines and σ_{LCL} curves.

h) Any unusual behaviour observed in the tests.

i) Name of the laboratory and person, responsible for carrying out the testing procedures.

j) Date of the report.

Annex A (normative)

Flow sheet for method I

A.1 Figure A.1 illustrates the flow sheet for the procedure for method I of the SEM.

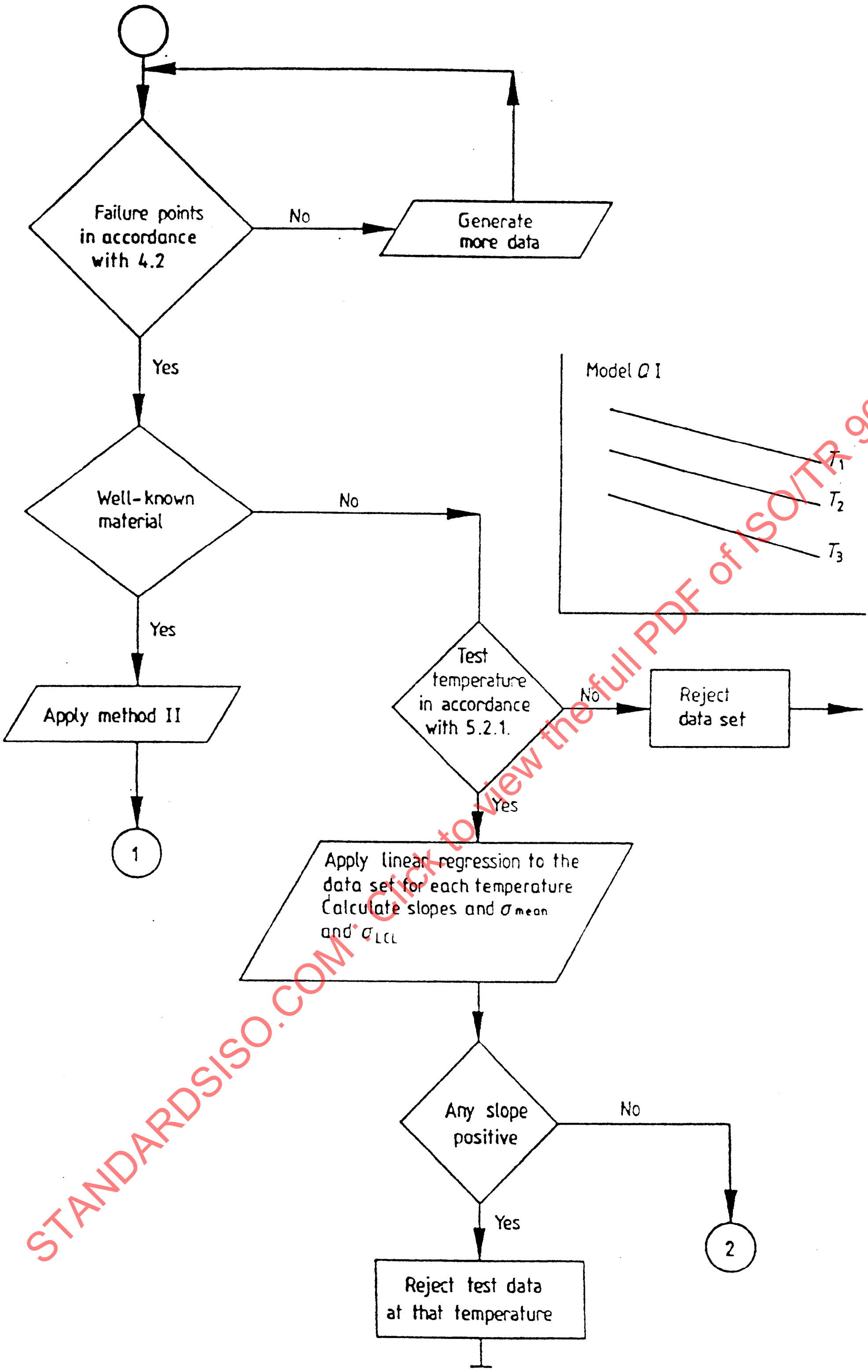

Figure A.1: Flow sheet for method I of the SEM

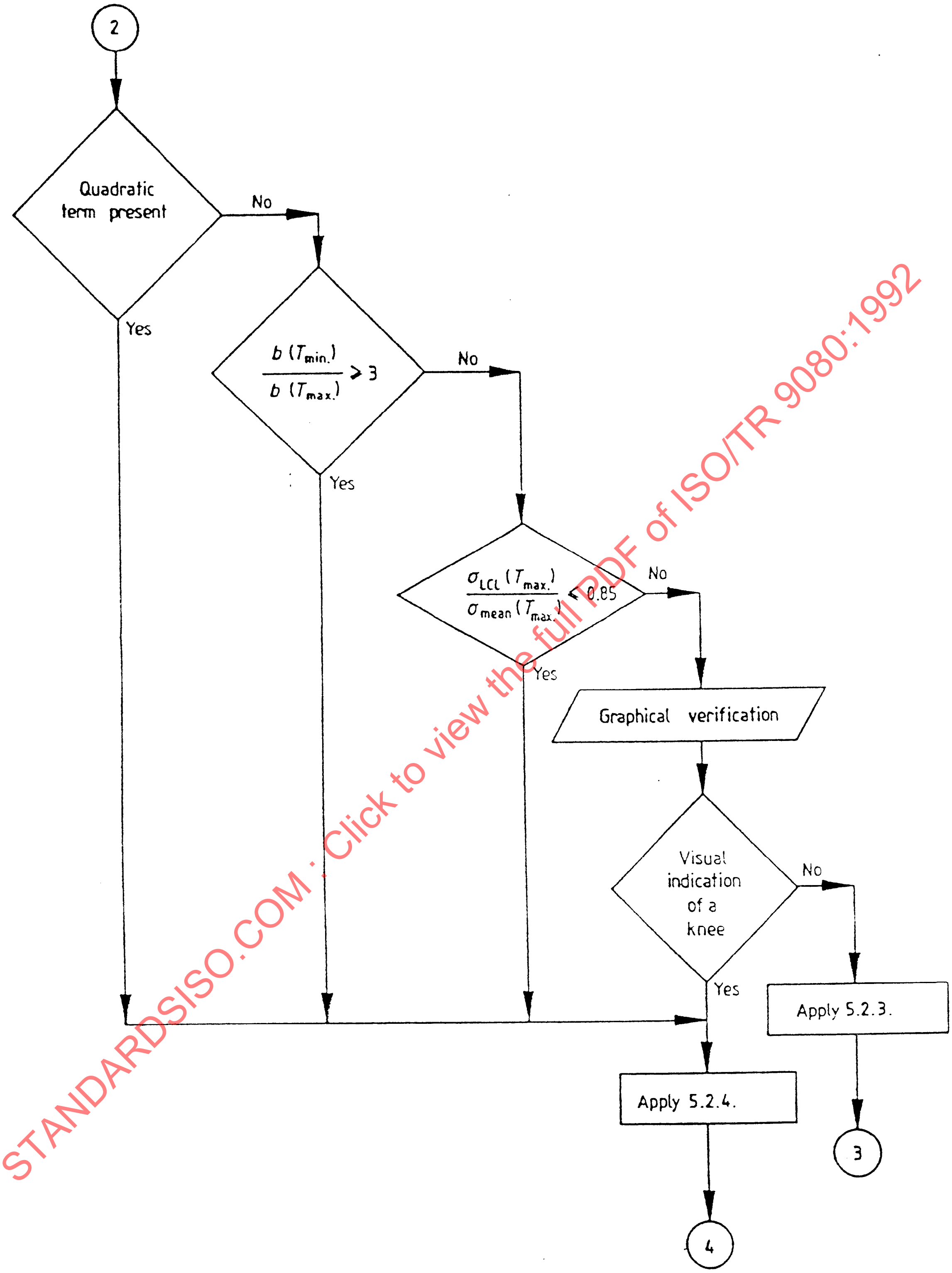

Figure A.1 (continued)

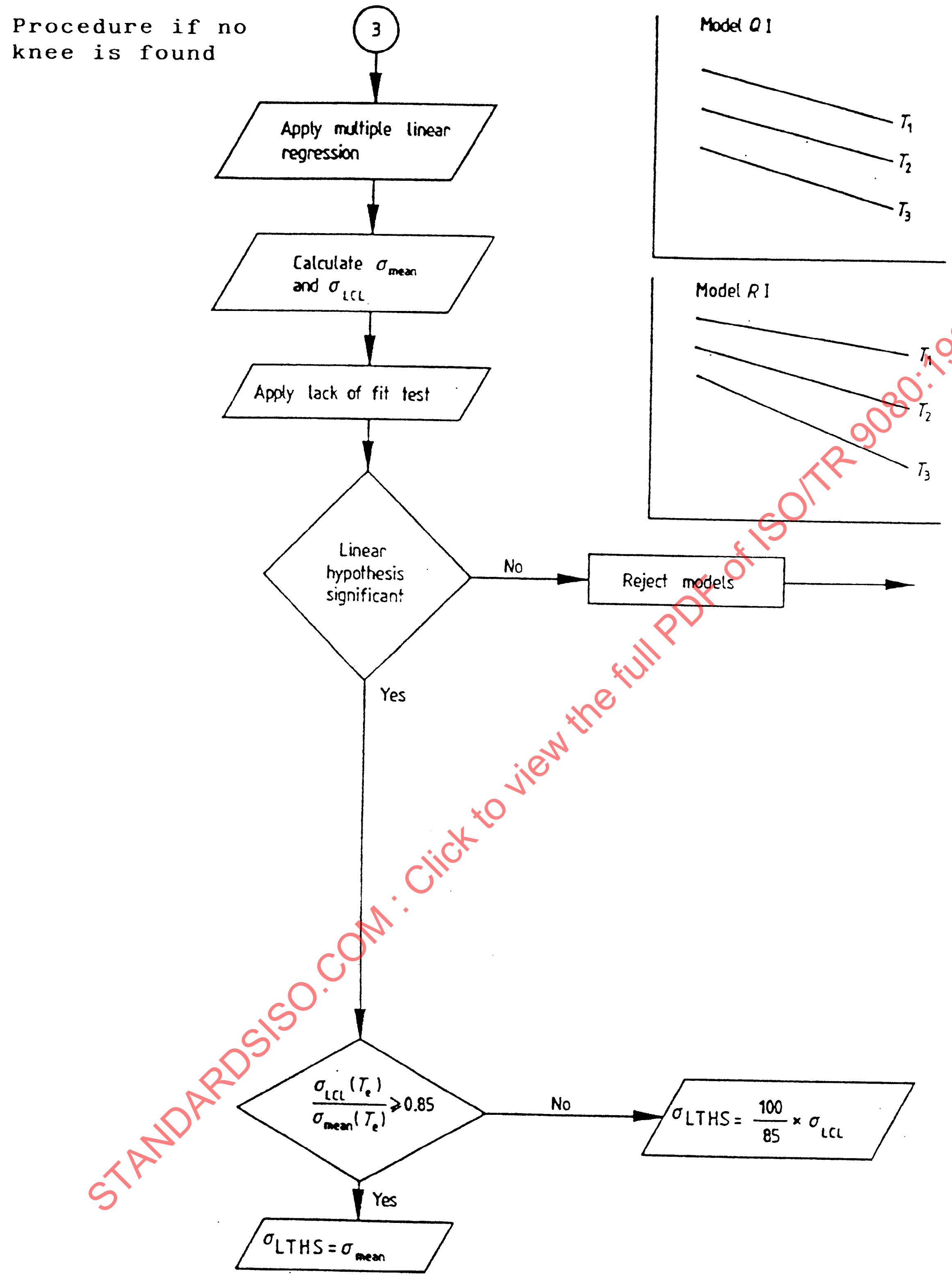

Figure A.1 (continued)

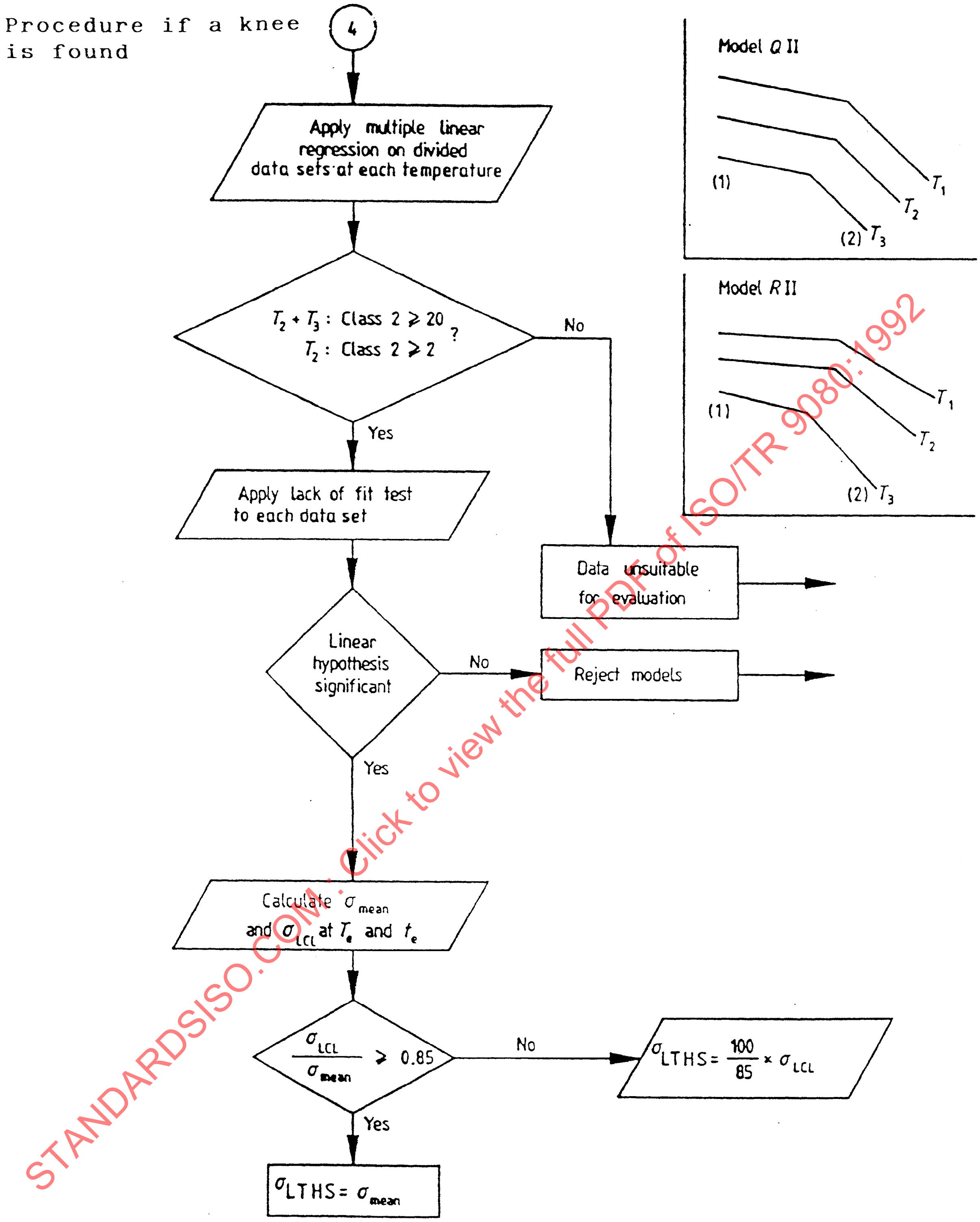

Figure A.1 (concluded)

Annex B (normative)

Calculation of linear regression

B.1 General

This annex contains the outlines of all the calculations needed for the Report.

In B.3 the equations are given to apply for linear regression with one independent variable. These are used in all cases where a set of observations at one single temperature has to be analysed and yield the regression equation, which is in this case a single straight line in a log (time) versus log (stress) plot σ_{LTHS}, σ_{LCL} $(\sigma(T_1, t_e, 0{,}975))$.

B.4 gives the equations to apply a linear regression with two independent variables. These are used when observations at several temperatures have to be taken into account and yield the regression equation, in this case a family of lines and the σ_{LTHS}, σ_{LCL} $(\sigma(T_e, t_e, 0.975))$.

B.5 gives the procedure for calculating the 97,5 % LCL when a family of curves has been established to describe the observations. In fact B.5 applies the equations as given in B.4, but only to a part of the observations. The conditions for selecting this procedure are given in B.5.

Derivation of the equations and theoretical backgrounds of linear regression analysis can be found in many fundamental text books on statistics, e.g. "Applied Regression Analysis" by N R Draper and H Smith; Wiley, New York (1966).

B.2 Symbols

The following symbols are used:

n is the number of observations;

f_i is the logarithm of stress (in megapascals) of observations i; $i = 1, \ldots, n$;

h_i is the logarithm of failure time (in hours) of observation i; $i = 1, \ldots, n$;

T_i is the temperature (in Kelvin) of observation i; $i = 1; \ldots, n$;

$t_{(M,\alpha)}$ is the Student's t factor for N degrees of freedom and at 100 α % one-sided confidence level.

B.3 Linear regression with one independent variable

B.3.1 We define:

$$\bar{f} = \frac{1}{n} \sum_{i=1}^{n} f_i$$

$$\bar{h} = \frac{1}{n} \sum_{i=1}^{n} h_i$$

$$S_{ff} = \frac{1}{n-1} \Sigma \left(f_i^2 - n\bar{f}^2 \right)$$

$$S_{hh} = \frac{1}{n-1} \Sigma \left(h_i^2 - n\bar{h}^2 \right)$$

$$S_{fh} = \frac{1}{n-1} \Sigma \left(f_i h_i - n\bar{f}\bar{h} \right)$$

B.3.2 The regression equation of log (time) on log (stress) is:

log (time) = $a + b$log (stress)

where

$$b = \frac{S_{fh}}{S_{ff}} ;$$

$$a = \bar{h} - b\bar{f}.$$

The slope of the regression line, b, shall be negative, otherwise the data are unsuitable.

B.3.3 If s^2 is the residual variance about the regression, then

$$s^2 = \frac{n-1}{n-2} \left(S_{hh} - \frac{S_{fh}^2}{S_{ff}} \right)$$

The 97,5 % lower confidence limit (σ_{LCL}) for one future observation at a given stress σ is derived from the following equation:

$$\log t_e = a + b\log \sigma - tS\sqrt{\left(1 + \frac{1}{n} + \frac{(\log \sigma - \bar{f})^2}{(n - 1)S_{ff}}\right)} \quad \ldots \text{(B.1)}$$

where

$$t = t_{(n-2,\ 0,975)}.$$

In order to find a stress σ_{LCL} at which for at least 97,5 % of the pipes will survive for at least the time t_{e1}, equation (B.1) can be solved to give:

$$\log \sigma_{LCL} = \frac{-\beta \pm \sqrt{(\beta^2 - 4\alpha\Gamma)}}{2\alpha}$$

where

$$\alpha = \frac{t^2 S^2}{(n - 1)\ S_{ff}} - b^2$$

$$\beta = 2b(\log t_e - a) - \frac{2 f_{\bar{t}}{}^2 S^2}{(n - 1)\ S_{ff}}$$

$$\Gamma = t^2\ S^2\ \left(1 + \frac{1}{n} + \frac{\bar{f}^{\,2}}{(n - 1)\ S_{ff}}\right) - (\log t_e - a)^2$$

So if $\alpha > 0$ then

$$\log \sigma_{LCL} = \frac{-\beta - \sqrt{(\beta^2 - 4\alpha\Gamma)}}{2\alpha}$$

and if $\alpha < 0$ then

$$\log \sigma_{LCL} = \frac{-\beta + \sqrt{(\beta^2 - 4\alpha\Gamma)}}{2\alpha}$$

B.4 Linear regression with two independent variables

B.4.1 We define

$$X = \begin{bmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ . & . & . \\ . & . & . \\ 1 & x_n & y_n \end{bmatrix} \quad \text{where } x_i = \frac{f_i}{T_i} \quad \text{and} \quad y_i = \frac{1}{T_i}$$

X^T is the transpose of X.

$$Z = \begin{bmatrix} z_1 \\ z_2 \\ . \\ . \\ z_n \end{bmatrix} \quad \text{where} \quad z_i = h_i$$

Let $K = (X^T X)^{-1}$, so that the components of K are:

$$K_{11} = \frac{1}{D}\{(\Sigma\ y^2)(\Sigma\ x^2) - (\Sigma\ xy)^2\}$$

$$K_{12} = \frac{1}{D}\{(\Sigma\ y)(\Sigma\ xy) - (\Sigma\ x)(\Sigma\ y^2)\}$$

$$K_{13} = \frac{1}{D}\{(\Sigma\ x)(\Sigma\ xy) - (\Sigma\ x^2)(\Sigma\ y)\}$$

$$K_{22} = \frac{1}{D}\{n(\Sigma\ y^2) - (\Sigma\ y)^2\}$$

$$K_{23} = \frac{1}{D}\{(\Sigma\ x)(\Sigma\ y) - n(\Sigma\ xy)\}$$

$$K_{33} = \frac{1}{D}\{n(\Sigma\ x^2) - (\Sigma\ x)^2\}$$

$$K_{21} = K_{12},\quad K_{31} = K_{13},\quad K_{32} = K_{23}$$

where

$$D = n\{(\Sigma\ x^2)(\Sigma\ y^2) - (\Sigma\ xy)^2\} - (\Sigma\ x)\{(\Sigma\ x)(\Sigma\ y^2) - (\Sigma\ y)(\Sigma\ xy)\} + (\Sigma\ y)\{(\Sigma\ x)(\Sigma\ xy) - (\Sigma\ y)(\Sigma\ x^2)\}$$

B.4.2 The regression equation is as follows:

$$\log\ (\text{time}) = -A - B\ \frac{\log\ \sigma}{T} + \frac{C}{T}$$

where

$$\sigma = \sigma_{\text{LTHS}}$$

The least squares estimates (A, B, C) are given by:

$$\begin{bmatrix} (-A) \\ (-B) \\ (+C) \end{bmatrix} = CX^TZ$$

i.e.

$$-A = K_{11}(\Sigma\ z) + K_{12}(\Sigma\ xz) + K_{13}(\Sigma\ yz)$$

$$-B = K_{21}(\Sigma\ z) + K_{22}(\Sigma\ xz) + K_{23}(\Sigma\ yz)$$

$$C = K_{31}(\Sigma\ z) + K_{32}(\Sigma\ xz) + K_{33}(\Sigma\ yz)$$

The coefficients shall satisfy the conditions $A > 0$, $B > 0$ and $C > 0$, otherwise the data shall be considered unsuitable.

B.4.3 The residual variance about the regression, S^2, is given, on n-3 degrees of freedom, by the following equation:

$$S^2 = \frac{1}{n-3}\left\{ Z^TZ - Z^TX \begin{bmatrix} -A \\ -B \\ +C \end{bmatrix} \right\}$$

where

$$z^T z = \Sigma\ z^2;$$

$$z^T X \begin{bmatrix} (-A) \\ (-B) \\ (+C) \end{bmatrix} = -A(\Sigma\ z) - B(\Sigma\ xz) + C(\Sigma\ yz)$$

The 97,5 % lower confidence limit for the predicted value of log (time) at a given stress σ and temperature T can be calculated using the following equation:

$$\log\ t = -A - B\frac{\log\ \sigma}{T} + \frac{C}{T} - tS\sqrt{(1 + x^T{}_0 C_0\ x_0)} \qquad \text{... (B.2)}$$

where

$$t = t_{(n-3,\ 0{,}975)};$$

$$x_0 = \begin{bmatrix} 1 \\ (\log\ \sigma)/T \\ 1/T \end{bmatrix} \text{ and } x^T_0 \text{ is } x_0 \text{ transposed.}$$

The matrix expression may be expanded as:

$$x^T{}_0 K x_0 = (K_{11} + \frac{2K_{13}}{T} + \frac{K_{33}}{T^2}) + 2(K_{12} + \frac{K_{23}}{T})(\frac{\log\ \sigma}{T}) + K_{22}(\frac{\log\ \sigma}{T})^2$$

In order to find the stress for which the lower confidence limit has a certain value t_e the equation (B.2) can be solved for $(\log\ \sigma)/T$ to give

$$\frac{\log\ \sigma}{T} = \frac{-\beta \pm \sqrt{(\beta^2 - 4\alpha\Gamma)}}{2\alpha}$$

where

$$\alpha = S^2 t^2 K_{22} - B^2$$

$$\beta = 2S^2 t^2 (K_{12} + \frac{K_{23}}{T}) - 2(\log\ t_e + A - \frac{C}{T})B$$

$$\Gamma = S^2 t^2 (1 + K_{11} + \frac{2K_{13}}{T} + \frac{K_{33}}{T^2}) - (\log\ t_e + A - \frac{C}{T})^2$$

So if $\alpha > 0$ then

$$\frac{\log \sigma_{LCL}}{T} = \frac{-\beta - \sqrt{(\beta^2 - 4\alpha\Gamma)}}{2\alpha}$$

and if $\alpha < 0$ then

$$\frac{\log \sigma_{LCL}}{T} = \frac{-\beta + \sqrt{(\beta^2 - 4\alpha\Gamma)}}{2\alpha}$$

B.5 Calculation of the lower confidence limit when a knee has to be taken into account

B.5.1 General

As outlined in annex G, it is assumed that two failure mechanisms may be active, each in its own range of temperatures and failure times. The standard deviations of each mechanism must be calculated independently. In order to do this, the available test data have to be sub-divided into two groups; in each group one of the mechanisms is assumed to be active.

In each group, the lower confidence limit can now be calculated with standard methods, provided sufficient data are available and their distribution over the temperature range is suitable.

The procedure, outlined in this clause is divided into two steps as follows:

a) the observations are classified in two classes and it is necessary to verify whether each group contains sufficient data and that their temperature distribution is suitable;

b) it is established which class has to be used to calculate the lower confidence limit at a temperature T and a life time t_e. This calculation is carried out in accordance with B.4.3.

B.5.2 Classification of the observations

A specific observation (T_i, t_{fi}, σ_i), belongs to class 1 when the following relationship holds:

$$-A_1 - B_1\frac{\log \sigma}{T} + \frac{C_1}{T} \leq -A_2 - B_2\frac{\log \sigma}{T} + \frac{C_2}{T}$$

where

A_1, A_2, B_1, B_2, C_1 and C_2 are the coefficients of the regression equation, in accordance with 5.2.4.

When this relation is not obeyed the observation belongs to class 2.

If n_1 is the number of observations in class 1 and n_2 is the number of observations in class 2, then if either n_1 or n_2 is less than 20 the data are not sufficient and are unsuitable for further evaluation.

Each class shall contain at least two observations of which the temperatures are 10 K or more apart, otherwise if this is not the case and one of the classes contains observations at only one temperature, then the observation with the longest failure time at the next lower temperature shall be taken as belonging to this class.

B.5.3 Procedure for calculation of lower confidence limit at temperature T and time t_e

At temperature T and time t_e:

$$\sigma_{LCL} = \sigma(T, \log t_e, 0{,}975)$$

when:

$$-A_1 - B_1\frac{\log \sigma}{T} + \frac{C_1}{T} \leq -A_2 - B_2\frac{\log \sigma}{T} + \frac{C_2}{T}$$

use the observations in class 1 (see B.5.2) to calculate the σ_{LCL}: when this relationship is not obeyed, use the observations in class 2.

Calculate the lower confidence limit in accordance with B.4.3 with

$n = n_1$, $A = A_1$, $B = B_1$ and $C = C_1$

when the observations in class 1 have to be used, and with

$n = n_2$, $A = A_2$, $B = B_2$ and $C = C_2$

when the observations in class 2 have to be used.

Take the summations as given in B.4.3 only over the observations in the class under consideration.

Calculate the components of the matrix C as given in B.4.3, taking into account only the observations in the class under consideration.

ANNEX C (normative)

Test for lack-of-fit to validate linearity

C.1 General

We assume that at a certain temperature, using the common units, it is valid to write

$$\log(\text{time}) = a + b\log(\text{stress}).$$

Clearly it is important to check the validity of this hypothesis of linearity. Let it be clear that it is not to be shown that the regression curve of log (time) on log (stress) really is a straight line, but only that the experimental facts do not justify rejection of this hypothesis of linearity.

It is more or less evident that in reality such a curve is not strictly straight: there is no reason in general, why it should be so simple. What we want to know, however, is whether the hypothesis of linearity (which has the advantage of simplifying the solution considerably) can be accepted as a reasonable approximation, that is to say, if nothing is against it.

C.2 Test of linearity[1)]

It is always possible to form the equation of a linear regression line.

The question remains, whether this straight line takes into account correctly the variations of the average response as a function of log (stress), or in other words: to verify the linearity of the regression.

This verification can be done either by visual inspection of the graph by carrying out a test on linearity.

The principle of this test is to study whether the average response $\bar{y}_i$ (= log (time)) for each of the x_i (= log (stress)) values does not deviate too much from the estimated regression line, taking into account their precision.

To do so, one calculates the weighted sum of squares of the "vertical" distances from the average responses to the regression line.

We can write the regression line in the form:

$$\log(\text{time}) = \overline{\log(\text{time})} + b\,(\log(\text{stress}) - \overline{\log(\text{stress})})$$

or

$$y_i = \bar{y}_c + b(x_i - \bar{x}_i)$$

1) In this annex the symbol σ is not used for stress, but σ^2 is used for the variance y (independent of x).

The weighted sum of squares (E) is given by the equation:

$$E = \Sigma_i [W_i \bar{y}_i - \bar{y} - b(x_i - \bar{x})^2$$

where

W is a weighting factor

with

$$SWxy = \Sigma_i W_i (x_i - \bar{x})(\bar{y}_i - \bar{y})$$

$$SWx^2 = \Sigma_i W_i (x_i - \bar{x})^2$$

$$SWy^2 = \Sigma_i W_i (\bar{y}_i - \bar{y})^2$$

and

$$b = \frac{SWxy}{SWx^2}$$

it is easy to show that:

$$E = SWy^2 - \frac{(SWxy)^2}{SWx^2}$$

To test for linearity implies a check on whether the expression for E is sufficiently small for the hypothesis of linearity to be accepted.

The quantity E measures the weighted (by W) distances between the average responses at each of the L stress levels and the regression line.

The weighting factor W equals $n_L / \sigma_y{}^2$, in which $\sigma_y{}^2$ denotes the variance of y (independent of x) and n_L is the number of data points at a stress level. The expression for E can now be written as:

$$E = \frac{1}{\sigma_y{}^2} \{ Sn_L y^2 - \frac{(Sn_L xy)^2}{Sn_L x^2} \}$$

It can be shown that, assuming linearity, the part between brackets of this expression is the numerator of an estimate of $\sigma_y{}^2$, based on $(L - 2)$ degrees of freedom.

If the hypothesis of linearity is not correct, the value of that numerator tends to increase. An estimate of the denominator of E is obtained by measuring the residual variance of log (time), when the effect of log (stress) is eliminated. This variance s^2 with $(N_d - L)$ degrees of freedom, where N_d is the total number of data points, results from an analysis of variance for the L values of log (stress).

The variance s^2 is independent of the validity of the hypothesis of linearity.

Finally, the test on linearity is effected by calculating the coefficient of lack-of-fit of the model to the data, *F*, using the following equation.

$$F = \frac{Sn_L y^2 - \dfrac{(Sn_L xy)^2}{Sn_L x^2}}{s^2 \, (L - 2)}$$

F is the ratio of two independent estimates of σ_y^2. It follows a Snedecor distribution with $(L - 2)$ and $(N - L)$ degrees of freedom.

If the calculated *F* is lower than the value in the table[1]) at the appropriate level of probability (i.e. 0,05), then the hypothesis of linearity is accepted.

1) This table can be found in textbooks.

ANNEX D (normative)

Detection of the presence of a knee

D.1 General

This annex gives a method for testing the significance of the quadratic term in the regression

$$\log(\text{time}) = a^* + b^* (\log(\text{stress})) + c^* (\log(\text{stress}))^2$$

D.2 Procedure

D.2.1 Using n, f_i and h_i where

n is the number of observations;

f_i is the logarithm of stress (in megapascals) of observation i, $i = 1, \ldots, n$;

h_i is the logarithm of failure time (in hours) of observations i, $i = 1, \ldots, n$;

Calculate the values of the following quantities:

$$A^* = \sum_{i=1}^{n} f_i^2$$

$$B^* = \sum f_i^3$$

$$C^* = \sum f_i^4$$

$$D^* = \sum f_i h_i$$

$$E^* = \sum f_i^2 h_i$$

$$\bar{f} = \frac{1}{n} \sum f_i$$

$$G^* = \sum h_i^2$$

$$\bar{h} = \frac{1}{n} \sum h_i$$

D.2.2 Calculate the following additional quantities:

$$K^* = D^* - n\bar{f}\bar{h}$$

$$L^* = E^* - A^*\bar{h}$$

$$M^* = B^* - A^*\bar{f}$$

$$U^* = G^* - n\bar{h}^2$$

$$V^* = A^* - n\bar{f}^2$$

$$W^* = C - \frac{(A^*)^2}{n}$$

Since the least squares equations are:

$$V^*b + M^*c^* = K^*$$

$$M^*b^* + W^*c^* = L^*$$

$$a^* + \bar{f}b^* + \frac{A^*c^*}{n} = \bar{h}$$

use these quantities in the following equations to calculate b^* and c^* respectively:

$$b^* = \frac{W^*K^* - M^*L^*}{V^*W^* - (M^*)^2} \qquad c^* = \frac{M^*K^* - L^*V^*}{(M^*)^2 - V^*W^*}$$

D.2.3 Calculate the following sums of squares using the applicable equation as follows.

SS_1 (the sum of squares due to quadratic regression), using

$$SS_1 = b^*K^* + c^*L^*$$

SS_2 (the residual sums of squares about the quadratic regression), using

$$SS_2 = U^* - b^*K^* - c^*L^*$$

SS_3 (the sum of squares due to linear regression), using

$$SS_3 = \frac{(K^*)^2}{V^*}$$

SS_4 (the sum of squares due to the quadratic term), using

$$SS_4 = b^* K^* + c^* L^* - \frac{(K^*)^2}{V^*}$$

D.2.4 Calculate the value of the following expression:

$$\frac{SS_4}{SS_2} (n - 3)$$

and compare this with values of Fisher's F-statistic on 1, $(n - 3)$ degrees of freedom at the 5 % significance level.

If a significant result is obtained, conclude that the quadratic term has a significant effect in the regression.

Furthermore, if $c^* < 0$, then this significant non-linear effect is consistent with the presence of a knee.

ANNEX E (normative)

Estimation of the equation coefficients for method I by curve fitting

E.1 General

There are several ways to find a reasonable fit of the equation as mentioned in method I of this report, with the help of non-linear estimation methods. Some examples are:

a) Taylor series method;

b) Steepest descent method;

c) Marquardt's method.

A review of such procedures is given in "Applied Regression Analysis", N.R. Draper and H. Smith; published by John Wiley and Sons Inc., New York (1966).

All these methods are based on minimization of the sum of squares.

However, in general the sum of squares, expressed as a function of the coefficients, shows numerous local minima and the methods mentioned above will in general yield a set of coefficients, which correlates with one of these local minima.

A systematic procedure to find the absolute minimum does not exist, but experience teaches that the methods mentioned above generally lead to a satisfactory fit, especially when reasonable realistic values of the coefficients are used as starting points for these iterative procedures. Such realistic values can be obtained by visual judgement of the data.

E.2 Example

As an example a set of data on a certain type of polyethylene has been chosen.

Data for various aspects of an analysis in accordance with method I and presented in tables E.1 to E.4 as appropriate are derived from the full set of data listed in table E.5 and represented in figure E.1. Hence an example procedure for an analysis in accordance with method I is executed as follows.

a) Detection of a knee (see 5.2.2)

As visual inspection of figure 2, in accordance with 5.2.2.6, clearly indicates the presence of a knee and all validation arguments from 5.2.2 indicate the presence of a knee, conclude therefore that 5.2.4 shall apply.

b) Calculation of σ_{LTHS} when a knee is found (see 5.2.4)

Since the table of coefficients in accordance with 5.2.4.1 gives

A_1 = 54,9231 A_2 = 14,2182
B_1 = 11019,4 B_2 = 1082,27
C_1 = 29402,5 C_2 = 6621,23

and since the number of class 2 data (behind the knee) in accordance with 5.2.4.2 is as follows:

52 at 80 °C;
28 at 60 °C;
and 21 at 40 °C;

conclude that the data are suitable for evaluation.

c) Lack-of-fit test (see 5.2.4.3)

Since

i) the lack-of-fit test for class 1 data gives F = 4,724; and

ii) the lack-of-fit test for class 2 data gives F = 1,260;

conclude that the model is accepted.

d) Table of stresses at various times and temperatures (see 5.2.4.4 and 5.2.4.5)

Derive a table such as table E.1 and indicate any observed differences in modes of failure.

Table E.1 - Stresses at various times and temperatures

Stresses in megapascals

T (°C)	20		40		60		80	
t (h)	σ_{LTHS}	σ_{LCL}	σ_{LTHS}	σ_{LCL}	σ_{LTHS}	σ_{LCL}	σ_{LTHS}	σ_{LCL}
1	16,11	14,99	12,81	11,87	10,18	9,39	8,09	7,42
10	15,15	14,11	12,00	11,12	9,50	8,76	7,52	6,89
100	14,25	13,27	11,24	10,41	8,86	8,15	6,70	5,02
1 000	13,41	12,46	10,52	9,72	6,57	5,01	3,10	2,36
10 000	12,61	11,68	7,02	5,41	3,24	2,45		
100 000	8,17	6,32	3,61	2,74	1,59	1,18		
50 years	5,48	4,20	2,35	1,77				

The σ_{LTHS} and σ_{LCL} above the lines at the various temperatures give failures of ductile nature, the values below the line give failures of brittle nature.

e) Detection of a knee; validation (see 5.2.2)

For the relationships for linear regression at every temperature separately (see 5.2.2.1), i.e.

$$\log t = A + \frac{B}{T} \log \sigma$$

the coefficients A and B; the σ_{LTHS} and the σ_{LCL} at 50 years; and the Student's factor, t^*, on the quadratic term are given in table E.2.

Table E.2 - Coefficients, σ_{LTHS}, σ_{LCL}, t^* for linear regression at specific temperatures

T (°C)	A	B	σ_{LTHS} (MPa)	σ_{LCL}(97,5) (MPa)	t^* (h)
20	36,2848	-8676,81	10,85	9,65	1,21
40	15,9028	-4151,75	5,94	4,03	5,89
60	7,44795	-1917,43	2,06	1,06	3,60
80	5,04187	-1483,96	0,72	0,41	6,25

f) Slope of the regression line (see 5.2.2.2)

Since at all temperatures the slope of the regression line is negative, conclude that the requirement is fulfilled.

g) The presence of a knee (see 5.2.2.3)

Since when $t^* > 5$ a quadratic term cannot be excluded and this is the situation at 40 °C and 80 °C, therefore apply 5.2.2.5.

h) Ratio of slope of regression line (see 5.2.2.4).

Since the ratio of the slope of the regression line at 20 °C and 60 °C is 5,14 and at 20 °C and 80 °C is 7,04, and therefore in both cases an indication of the possibility of a knee is present, apply 5.2.4.

i) Ratio of σ_{LCL} and σ_{LTHS} (see 5.2.2.5)

Since the longest testing time is 16 000 h and the ratio between the σ_{LTHS} and σ_{LCL} at 60 °C and 80 °C at 100 000 h and at 50 years are all below 0,85, conclude that 5.2.4. shall apply.

j) Extrapolation time limit (see 5.2.5)

Abstract data for consideration of extrapolation time limits for compliance with 5.2.5, for example as shown in tables E.3 and E.4.

Table E.3 – Extrapolation time limits

Times in hours

T (°C)	delta T (°C)	K_e	$t_{1,max}$ 1)	$t_{2,max}$ 2)	$t_{3,max}$ 3)
20	> 40	> 50	5 137	4 343	3 379
40	40	50	16 920	14 240	13 281
60	20	5	15 911	13 077	10 483
80	0	1	15 136	9 397	7 586

1) $t_{1,max}$ is the single maximum failure time found;
2) $t_{2,max}$ is the average value of the largest three failure times;
3) $t_{3,max}$ is the average value of the largest five failure times.

Table E.4 – Calculation extrapolation time in hours for the possibilities 1, 2, and 3 (see table E.3)

Data in hours

T °C	1 data	3 data	5 data
20	> 256 850	> 217 150	> 168 950
40	846 000	712 000	664 050
60	79 555	65 385	52 414
80	15 136	9 397	7 586

Since table E.4 indicates that with averaging over five longest failure times at 40 °C extrapolation to 75 years is allowed, conclude that therefore extrapolation at 20 °C up to 50 years is also allowed, even though the calculated time with five longest failure times is only longer-than-20 years.

k) Ratio of σ_{LCL} and σ_{LTHS} at T_S and at 50 years (see 5.2.4.6)

Since at 50 years and 20 °C the extrapolated σ_{LTHS} is 5,48 MPa and the σ_{LCL} is 4,2 MPa but the ratio between these extrapolated values is 0,77 and lower than 0,85, consider therefore the σ_{LTHS} equal to

$$\frac{100}{85} \times 4{,}2 \text{ MPa}$$

i.e. 4,94 MPa.

l) Ratio of σ_{LCL} and σ_{LTHS} at T_S and t_S (see 5.2.4.7)

Since this ratio is less than 0,5, conclude that the data is suitable for extrapolation.

m) re. clause 6 Report

The data used for this example and presented in table E.5 were handed over by a manufacturer of pipe resin and most probably derived from pipes of 25 mm diameter and 2 mm wall thickness extruded from the same batch of resin. The test was executed with water inside and outside of the pipe test pieces. The calculations were executed using a computer programme designed for the purpose by a pipe manufacturer.

Table E.5 - List of observations on polyethylene

Temp. (°C)	Stress (Mpa)	Time (h)	Log (stress) (MPa)	Log (time) (h)
20	16,0	11	1,2038	1,0298
20	15,9	9	1,2011	0,9547
20	15,0	58	1,1761	1,7819
20	15,0	44	1,1781	1,6474
20	14,9	21	1,1735	1,3151
20	14,5	25	1,1617	1,3909
20	14,5	24	1,1617	1,3874
20	14,3	46	1,1559	1,6818
20	14,1	111	1,1498	1,0455
20	14,0	201	1,1467	2,3042
20	14,0	260	1,1467	2,4146
20	14,0	201	1,1467	2,3040
20	13,9	13	1,1440	2,1072
20	13,7	392	1,1377	1,5933
20	13,7	440	1,1377	2,6435
20	13,7	512	1,1377	2,7093
20	13,7	464	1,1377	2,6665
20	13,7	536	1,1377	2,7292
20	13,6	680	1,1345	2,8325
20	13,5	411	1,1313	2,6138
20	13,5	412	1,1313	2,6154
20	13,5	3368	1,1313	3,5274
20	13,5	865	1,1313	2,9370
20	13,5	946	1,1313	2,9759
20	13,5	4524	1,1313	3,6555
20	13,4	122	1,1284	2,0864
20	13,4	5137	1,1284	3,7107
20	13,3	1112	1,1252	3,0461
20	13,3	2108	1,1252	3,3239
20	13,2	1651	1,1219	3,2177
20	13,2	1760	1,1219	3,2455
20	12,8	837	1,1089	2,9227
40	12,1	1	1,0820	0,0800
40	12,2	2	1,0850	0,2000
40	11,9	2	1,0750	0,2300
40	11,6	3	1,0630	0,4800
40	11,5	3	1,0590	0,5000
40	11,5	5	1,0600	0,7000
40	11,2	9	1,0500	0,9400
40	11,1	10	1,0440	0,9900