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Explanatory notes on ISO 281/1-1977

Notes explicatives sur l'ISO 281/1-1977

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— type 1, when the necessary support within the technical committee cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development requiring wider exposure;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical reports are accepted for publication directly by ISO Council. Technical reports types 1 and 2 are subject to review within three years of publication, to decide if they can be transformed into International Standards. Technical reports type 3 do not necessarily have to be reviewed until the data they provide is considered no longer valid or useful.

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The reasons which led to the decision to publish this document in the form of a technical report type 3 are explained in the Introduction.

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1. Introduction

This Technical Report gives supplementary background information regarding the derivation of formulae and factors given in ISO 281/I, Rolling bearings - Dynamic load ratings and rating life - Part I : Calculation methods.

2. Brief History

2.1 ISO/R281-1962

A first discussion on an international level of the question of standardizing calculation methods for load ratings of rolling bearings took place at the 1934 conference of the International Federation of the National Standardizing Associations (ISA). When ISA held its last conference in 1939 no progress had been made. However, in its 1945 report on the state of rolling bearing standardization, the ISA 4 Secretariat included proposals for definition of concepts being fundamental for load rating and life calculation standards. This report was distributed in 1949 as document ISO/TC 4 (Secretariat-1)1, and the definitions it contained are in essence those given in ISO 281/I for the concepts "life" and "basic dynamic load rating".

In 1946, on the initiative of the Anti-Friction Bearing Manufacturers Association (AFBMA), New York, discussions of load rating and life calculation standards were started between bearing industries in U.S.A. and Sweden. Chiefly on the basis of results of scientific investigations by G. Lundberg and A. Palmgren, published in 1947 [1]*, an AFBMA Standard "Method of Evaluating Load Ratings of Annular Ball Bearings" was worked out and published in 1949. On the same basis, the member body of Sweden presented in Feb., 1950 a first proposal to ISO, "Load Rating of Ball Bearings", doc. ISO/TC 4/SC 1 (Sweden-1)1.

* figures in brackets indicate literature references in References.

In view of results of further research, of a modification of the AFBMA Standard in 1950, and of the interest also in roller bearing rating standards, the member body of Sweden submitted in 1951 a modified proposal for rating of ball bearings, doc. ISO/TC 4/SC 1 (Sweden-6) 20, as well as a proposal for rating of roller bearings, doc. ISO/TC 4/SC 1 (Sweden-7) 21.

Load rating and life calculation methods were then studied by ISO/TC 4, ISO/TC 4/SC 1 and ISO/TC 4/WG 3 at eleven different meetings during 1951 - 1959. An additional paper by Lundberg-Plamgren published in 1952 [2] was of considerable use, serving as a major basis for the sections regarding roller bearing rating.

The framework for the Recommendation was settled at TC 4/WG 3 meeting in 1956. At the time, deliberation of the draft for revision of AFBMA Standards was concluded in U.S.A. and ASA B3 approved the revised standard. It was proposed to the meeting by U.S.A. and discussed in detail, together with the Secretariat's proposal. At the meeting, WG3 proposal was prepared which adopted many parts of the U.S.A. proposal.

In 1957, Draft Proposal (doc. TC 4 N145) based on the WG proposal was issued. At the next year's WG3 meeting, this Draft Proposal was investigated in detail, and at the following TC4 meeting, the adoption of TC 4 N145, with some minor ammendments, was concluded. Then, Draft ISO Recommendation No. 278 as TC 4 N188 was issued in 1959, and ISO/R281 was accepted by ISO Council in 1962.

2.2 ISO 281/I-1977

In 1964 the member body of Sweden suggested that, in view of the development of improved bearing steels, the time had come to review R281 and submitted a proposal, ISO/TC 4/WG3 (Sweden-1) 9. However, at this time WG3 was not in favour of a revision.

In 1969, on the other hand, TC 4 followed a suggestion by the member body of Japan (doc. TC 4 N627) and reconstituted its WG3, giving it the task of revising R281. The AFBMA load rating working group had at this time started to work on a revised standard, and the member body of U.S.A. submitted the Draft AFBMA Standard "Load ratings and fatigue life for ball bearings" for consideration, ISO/TC 4/WG3 (USA-1) 11, in 1970 and "Load ratings and fatigue life for roller bearings", ISO/TC4/WG3 (USA-3) 19, in 1971.

In 1972, TC 4/WG3 was reorganized and became TC 4/SC8. This proposal was investigated in detail at the five meetings during 1971 - 1974. The final proposal, Third Draft Proposal (doc. TC 4/SC 8 N23), with some amendments, was circulated as Draft International Standard in 1976 and ISO 281/I was accepted by ISO Council in 1977.

The major part of this International Standard constitutes a re-edition of R281, the substance of which was only very slightly modified. However, based mainly on American investigations during the 1960's, a new clause was added, dealing with adjustment of rating life for reliability other than 90% and for material and operating conditions.

Furthermore, supplementary background information regarding the derivation of formulae and factors given in ISO 281/I was to be published, preliminarily as ISO 281/II Explanatory Notes In 1979, however, TC 4/SC 8 and TC 4 decided to publish it as Technical Report.

3. Basic Dynamic Load Rating

The background of basic dynamic load ratings according to the standard ISO 281 of rolling bearings is in the Lundberg and Palmgren papers listed as references [1] and [2].

The formulae for calculation of basic dynamic load ratings of rolling bearings develop from a power equation that can be written as follows:

$$\ell n \frac{1}{S} \sim \frac{\tau_o^c N^e V}{z_o^h} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}1)$$

where
- S = probability of survival
- τ_o = maximum orthogonal subsurface shear stress
- N = number of stress applications to a point on the raceway
- V = volume representative of the stress concentration
- z_o = depth of the maximum orthogonal subsurface shear stress
- c, h = exponents determined experimentally
- e = measure of life scatter, i.e. Weibull slope determined experimentally.

For "point" contact conditions (ball bearings) it is assumed that the volume (V) representative of the stress concentration in equation (3-1) is proportional to major axis of the projected contact ellipse (2a), the circumference of the raceway (ℓ) and the depth (z_o) of the maximum orthogonal subsurface shear stress (τ_o):

$$V \sim a z_o \ell . \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}2)$$

Substituting (3-2) into relationship (3-1):

$$\ell n \frac{1}{S} \sim \frac{\tau_o^c N^e a \ell}{z_o^{h-1}} , \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}3)$$

"Line" contact was considered by Lundberg and Palmgren to be approached under conditions where the major axis of the calculated Hertz contact ellipse is 1,5 times the effective roller contact length:

$$2a = 1{,}5\ L_{we} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}4)$$

In addition, b/a should be small enough to permit the introduction of the limit value of ab^2 for b/a approaching 0:

$$ab^2 = \frac{2}{\pi} \cdot \frac{3Q}{E_o \Sigma\rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}5)$$

(for notation see 3.1).

3.1 Basic dynamic radial load rating C_r for radial ball bearings

From the Hertz's theory, the maximum orthogonal subsurface shear stress τ_o and the depth z_o can be expressed in terms of a radial load Fr, i.e. a maximum rolling element load Q_{max} or a maximum contact stress σ_{max} and dimensions for the contact area between a rolling element and the raceways. The relationships are given as follows:

$$\tau_o = T\sigma_{max},$$

$$z_o = \zeta\ b,$$

$$T = \frac{(2t - 1)^{1/2}}{2t(t + 1)},$$

$$\zeta = \frac{1}{(t + 1)(2t - 1)^{1/2}},$$

$$a = \mu\left[\frac{3Q}{E_o \Sigma\rho}\right]^{1/3},$$

$$b = \nu\left[\frac{3Q}{E_o \Sigma\rho}\right]^{1/3}$$

where σ_{max} = maximum contact stress

t = auxiliary parameter

a = semimajor axis of the projected contact ellipse

b = semiminor axis of the projected contact ellipse

Q = normal force between a rolling element and the raceways

E_o = modulus of elasticity

$\Sigma\rho$ = curvature sum

μ, ν = auxiliary quantities introduced by Hertz.

Consequently, for a given rolling bearing τ_o, a, ℓ and z_o can be expressed in terms of bearing geometry, load and revolutions. The relationship (3-3) is changed to an equation by inserting a constant of proportionality. Inserting a specific number of revolutions (e.g. 10^6) and a specific reliability (e.g. 0,9), the equation is solved for a rolling element load for basic dynamic load rating which is designated to point contact rolling bearings introducing a constant of proportionality A_1:

$$Q_c = \frac{1,3}{4^{\frac{2c+h-2}{c-h+2}}\, 0,5^{\frac{3e}{c-h+2}}}\, A_1 \left[\frac{2r}{2r-D_w}\right]^{0,41} \frac{(1\mp\gamma)^{\frac{1,59c+1,41h-5,82}{c-h+2}}}{(1\pm\gamma)^{\frac{3e}{c-h+2}}}$$

$$\times \left(\frac{\gamma}{\cos\alpha}\right)^{\frac{3}{c-h+2}} D_w^{\frac{2c+h-5}{c-h+2}}\, Z^{-\frac{3e}{c-h+2}} \quad \ldots\ldots\ldots\ldots\ldots \quad (3\text{-}6)$$

where Q_c = rolling element load for the basic dynamic load rating of the bearing

D_w = ball diameter

γ = $D_w \cos\alpha / D_{pw}$

D_{pw} = pitch diameter of ball set

α = nominal contact angle

Z = number of balls per row.

(x is used as multiplication symbol.)

The basic dynamic radial load rating C_1 of a rotating ring is given as follows:

$$C_1 = Q_{c1} Z \cos\alpha \frac{J_r}{J_1} = 0{,}407 Q_{c1} Z \cos\alpha \quad \ldots\ldots\ldots \quad (3\text{-}7)$$

The basic dynamic radial load rating C_2 of a stationary ring is given as follows:

$$C_2 = Q_{c2} Z \cos\alpha \frac{J_r}{J_2} = 0{,}389 Q_{c2} Z \cos\alpha \quad \ldots\ldots\ldots \quad (3\text{-}8)$$

where Q_{c1} = rolling element load for the basic dynamic load rating of a ring rotating relative to the applied load

Q_{c2} = rolling element load for the basic dynamic load rating of a ring stationary relative to the applied load

$J_r = J_r(0,5)$ = radial load integral (see table 4-1)

$J_1 = J_1(0,5)$ = factor relating mean equivalent load on a rotating ring to Q_{max} (see table 4-1)

$J_2 = J_2(0,5)$ = factor relating mean equivalent load on a stationary ring to Q_{max} (see table 4-1).

The relationship among C_r for an entire radial ball bearing, C_1 and C_2 is expressed in terms of the product law of probability as follows:

$$C_r = C_1 \left[1 + \left(\frac{C_1}{C_2}\right)^{\frac{c-h+2}{3}} \right]^{-\frac{3}{c-h+2}} \quad \ldots\ldots\ldots \quad (3\text{-}9)$$

Substituting equations (3-7), (3-8) and (3-6) into equation (3-9), the basic dynamic radial load rating C_r for an entire ball bearing is expressed as follows:

$$C_r = 0,41 \frac{1,3}{4^{\frac{2c+h-2}{c-h+2}}\, 0,5^{\frac{3e}{c-h+2}}} A_1 \left[\frac{2r_i}{2r_i-D_w}\right]^{0,41} \frac{(1-\gamma)^{\frac{1,59c+1,41h-5,82}{c-h+2}}}{(1+\gamma)^{\frac{3e}{c-h+2}}}$$

$$\times\ \gamma^{\frac{3}{c-h+2}}$$

$$\times \left[1+\left\{1,04\left(\frac{r_i}{r_e}\times\frac{2r_e-D_w}{2r_i-D_w}\right)^{0,41}\left(\frac{1-\gamma}{1+\gamma}\right)^{\frac{1,59c+1,41h+3e-5,82}{c-h+2}}\right\}^{\frac{c-h+2}{3}}\right]^{-\frac{3}{c-h+2}}$$

$$\times\ (i\cos\alpha)^{\frac{c-h-1}{c-h+2}}\ Z^{\frac{c-h-3e+2}{c-h+2}}\ D_w^{\frac{2c+h-5}{c-h+2}} \quad \ldots\ldots\ldots\ldots\ (3\text{-}10)$$

where A_1 = proportionality constant determined experimentally
r_i = cross-sectional raceway groove radius of inner ring
r_e = cross-sectional raceway groove radius of outer ring
i = number of rows of balls.

Here, a contact angle α, number of rolling elements (balls) Z and the diameter D_w depend on bearing design. On the other hand, the ratios of raceway groove radii r_i and r_e to a half diameter of a rolling element (ball) $D_w/2$ and $\gamma = D_w\cos\alpha/D_{pw}$ are not dimensional, therefore it is convenient in practice that the value for the first three lines in the right side of equation (3-10) is designated as a factor fc.

Consequently,

$$C_r = f_c (i\cos\alpha)^{\frac{c-h-1}{c-h+2}}\ Z^{\frac{c-h-3e+2}{c-h+2}}\ D_w^{\frac{2c+h-5}{c-h+2}} \quad \ldots\ldots\ (3\text{-}11)$$

With radial ball bearings we must consider the faults in bearings resulting from the manufacturing, and a reduction factor λ is introduced to reduce the value for a basic dynamic radial load rating for radial ball bearings from its theoretical value, and it is convenient to contain the factor λ in the factor f_c. The value for the factor λ is determined experimentally.

Consequently the factor f_c is given as follows:

$$f_c = 0{,}41\lambda \frac{1{,}3}{4^{\frac{2c+h-2}{c-h+2}} \, 0{,}5^{\frac{3e}{c-h+2}}} A_1 \left[\frac{2r_i}{2r_i - D_w}\right]^{0{,}41} \frac{(1-\gamma)^{\frac{1{,}59c+1{,}41h-5{,}82}{c-h+2}}}{(1+\gamma)^{\frac{3e}{c-h+2}}}$$

$$\times \gamma^{\frac{3}{c-h+2}}$$

$$\times \left[1 + \left\{1.04\left(\frac{r_i}{r_e} \times \frac{2r_e - D_w}{2r_i - D_w}\right)^{0{,}41} \left(\frac{1-\gamma}{1+\gamma}\right)^{\frac{1{,}59c+1{,}41h+3e-5{,}82}{c-h+2}}\right\}^{\frac{c-h+2}{3}}\right]^{-\frac{3}{c-h+2}} \qquad \text{................ (3-12)}$$

Based on the original experimental work by Lundberg and Palmgren with ball bearings the following values were assigned to the experimental constants in the load rating equations:

$e = 10/9$
$c = 31/3$
$h = 7/3$.

Substituting the numerical values into equation (3-11) gives the following, however, a sufficient number of test results are only available for small balls, i.e. up to a diameter of about 25 mm, and these show that the load rating may be taken as being proportional to $D_w^{1,8}$. In the case of larger balls the load rating appears to increase even more slowly in relation to the ball diameter, and $D_w^{1,4}$ can be assumed where $D_w > 25,4$ mm:

$$C_r = f_c (i \cos\alpha)^{0,7} Z^{2/3} D_w^{1,8} \quad (D_w \leqslant 25,4 \text{ mm}), \quad \ldots (3\text{-}13)$$

$$C_r = 3,647 f_c (i \cos\alpha)^{0,7} Z^{2/3} D_w^{1,4} \quad (D_w > 25,4 \text{ mm}), \quad \ldots (3\text{-}14)$$

$$f_c = 0,089 A_1 \times 0,41 \lambda \left[\frac{2r_i}{2r_i - D_w}\right]^{0,41} \frac{\gamma^{0,3}(1-\gamma)^{1,39}}{(1+\gamma)^{1/3}} \times \left[1 + \left\{1,04 \left(\frac{1-\gamma}{1+\gamma}\right)^{1,72} \left(\frac{r_i}{r_e} \times \frac{2r_e - D_w}{2r_i - D_w}\right)^{0,41}\right\}^{10/3}\right]^{-3/10} \quad \ldots\ldots\ldots\ldots (3\text{-}15)$$

Values for f_c on table 1 in ISO 281/I are calculated from substituting raceway groove radius and reduction factor which are given in table 3-1 into equation (3-15).

The value for $0,089 A_1$ is 98,0665 to calculate C_r in Newtons.

3.2 Basic dynamic axial load rating Ca for single row thrust ball bearings

3.2.1 Thrust ball bearing with contact angle $\alpha \neq 90°$

Similarly according to 3.1, for thrust ball bearings with contact angle $\alpha \neq 90°$:

$$C_a = f_c (\cos\alpha)^{\frac{c-h-1}{c-h+2}} \tan\alpha \; Z^{\frac{c-h-3e+2}{c-h+2}} \; D_w^{\frac{2c+h-5}{c-h+2}} . \quad \ldots\ldots (3\text{-}16)$$

For most thrust ball bearings the theoretical value of a basic dynamic axial load rating must be reduced on the basis of unequal distribution of load among the rolling elements in addition to the reduction factor λ which is introduced in radial ball bearing load ratings. This reduction factor is designated as η.

Consequently, the factor f_c is given as follows:

$$f_c = \lambda\eta \frac{1,3}{4^{\frac{2c+h-2}{c-h+2}} \; 0,5^{\frac{3e}{c-h+2}}} A_1 \left[\frac{2r_i}{2r_i-D_w}\right]^{0,41} \frac{(1-\gamma)^{\frac{1,59c+1,41h-5,82}{c-h+2}}}{(1+\gamma)^{\frac{3e}{c-h+2}}} \gamma^{\frac{3}{c-h+2}}$$

$$\times \left[1+\left\{\left(\frac{r_i}{r_e}\times\frac{2r_e-D_w}{2r_i-D_w}\right)^{0,41} \left(\frac{1-\gamma}{1+\gamma}\right)^{\frac{1,59c+1,41h+3e-5,82}{c-h+2}}\right\}^{\frac{c-h+2}{3}}\right]^{-\frac{3}{c-h+2}}$$

.... (3-17)

Substituting experimental constants $e = 10/9$, $c = 31/3$ and $h = 7/3$ into equations (3-16) and (3-17), however, considering the effect of ball size similarly,

$$Ca = f_c(\cos\alpha)^{0,7}\tan\alpha \; Z^{2/3} D_w^{1,8} \qquad (D_w \leqslant 25,4\text{mm}) \quad ...(3-18)$$

$$Ca = 3,647 f_c(\cos\alpha)^{0,7}\tan\alpha \; Z^{2/3} D_w^{1,4} \qquad (D_w > 25,4\text{mm}) \quad ...(3-19)$$

$$f_c = 0,089 A_1 \lambda\eta \left(\frac{2r_i}{2r_i-D_w}\right)^{0,41} \frac{\gamma^{0,3}(1-\gamma)^{1,39}}{(1+\gamma)^{1/3}}$$

$$\times \left[1+\left\{\left(\frac{r_i}{r_e}\times\frac{2r_e-D_w}{2r_i-D_w}\right)^{0,41} \left(\frac{1-\gamma}{1+\gamma}\right)^{1,72}\right\}^{10/3}\right]^{-3/10}.$$

.... (3-20)

The value for $0{,}089A_1$ is 98,0665 to calculate C_a in Newtons. Values for f_c on the right column of table 3 in ISO 281/I are calculated from substituting raceway groove radius and reduction factor which are given in table 3-1 into equation (3-20).

3.2.2 Thrust ball bearings with contact angle α = 90°

Similarly according to 3.1, for thrust ball bearings with contact angle α = 90°:

$$C_a = f_c Z^{\frac{c-h-3e+2}{c-h+2}} D_w^{\frac{2c+h-5}{c-h+2}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3\text{-}21)$$

$$f_c = \lambda\eta \frac{1{,}3}{4^{\frac{2c+h-2}{c-h+2}} 0{,}5^{\frac{3e}{c-h+2}}} A_1 \left[\frac{2r_i}{2r_i-D_w}\right]^{0{,}41} \gamma^{\frac{3}{c-h+2}}$$

$$\times \left[1+\left\{\left(\frac{r_i}{r_e}\,\frac{2r_e-D_w}{2r_i-D_w}\right)^{0{,}41}\right\}^{\frac{c-h+2}{3}}\right]^{-\frac{3}{c-h+2}} \quad \ldots\ldots (3\text{-}22)$$

in which $\gamma = D_w/D_{pw}$.

Substituting experimental constants $e = 10/9$, $c = 31/3$ and $h = 7/3$ into equations (3-21) and (3-22), however, considering the effect of ball size similarly:

$$C_a = f_c Z^{2/3} D_w^{1{,}8} \quad (D \leqslant 25{,}4\text{mm}), \quad \ldots\ldots\ldots (3\text{-}23)$$

$$C_a = 3{,}647 f_c Z^{2/3} D_w^{1{,}4} \quad (D > 25{,}4\text{mm}), \quad \ldots\ldots\ldots (3\text{-}24)$$

$$f_c = 0{,}089 A_1 \lambda\eta \left(\frac{2r_i}{2r_i-D_w}\right)^{0{,}41} \gamma^{0{,}3}$$

$$\times \left[1+\left\{\left(\frac{r_i}{r_e}\times\frac{2r_e-D_w}{2r_i-D_w}\right)^{0{,}41}\right\}^{10/3}\right]^{-3/10} \quad \ldots\ldots (3\text{-}25)$$

The value for $0,089A_1$ is 98,0665 to calculate C_a in Newtons. Values for f_c on the left column of table 3 in ISO 281/I are calculated from substituting raceway groove radius and reduction factor which are given in table 3-1 into equation (3-25).

3.3 Basic dynamic axial load rating Ca for thrust ball bearings with two or more rows of balls

According to the product law of probability, relationships between the basic axial load rating of an entire thrust ball bearing and of both the rotating and stationary washers are given as follows:

$$C_{ak} = \left[C_{a1k}^{-\frac{c-h+2}{3}} + C_{a2k}^{-\frac{c-h+2}{3}} \right]^{-\frac{3}{c-h+2}}, \quad \ldots\ldots \quad (3\text{-}26)$$

$$\left. \begin{aligned} C_{a1k} &= Q_{c1} \sin\alpha \; Z_k , \\ C_{a2k} &= Q_{c2} \sin\alpha \; Z_k . \end{aligned} \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}27)$$

$$C_a = \left[C_{a1}^{-\frac{c-h+2}{3}} + C_{a2}^{-\frac{c-h+2}{3}} \right]^{-\frac{3}{c-h+2}}, \quad \ldots\ldots\ldots \quad (3\text{-}28)$$

$$\left. \begin{aligned} C_{a1} &= Q_{c1} \sin\alpha \sum_{K=1}^{n} Z_k , \\ C_{a2} &= Q_{c2} \sin\alpha \sum_{K=1}^{n} Z_k . \end{aligned} \right\} \quad \ldots\ldots\ldots\ldots \quad (3\text{-}29)$$

where C_{ak} = basic dynamic axial load rating as a row k of an entire thrust ball bearing

C_{a1k} = basic dynamic axial load rating as a row k of the rotating washer of an entire thrust ball bearing

C_{a2k} = basic dynamic axial load rating as a row k of stationary washer of an entire thrust ball bearing

C_a = basic dynamic axial load rating of an entire thrust ball bearing

C_{a1} = basic dynamic axial load rating of the rotating washer of an entire thrust ball bearing

C_{a2} = basic dynamic axial load rating of the stationary washer of an entire thrust ball bearing

Z_k = number of balls as a row k.

Substituting equations (3-29), (3-27) and (3-26) into equation (3-28), and rearrangement of equation (3-28) gives:

$$C_a = \sum_{k=1}^{n} Z_k \left[\frac{\left(Q_{c1}\sin\alpha \sum_{k=1}^{n} Z_k\right)^{-\frac{c-h+2}{3}} + \left(Q_{c2}\sin\alpha \sum_{k=1}^{n} Z_k\right)^{-\frac{c-h+2}{3}}}{\left(\sum_{k=1}^{n} Z_k\right)^{-\frac{c-h+2}{3}}} \right]^{-\frac{3}{c-h+2}}$$

$$= \sum_{k=1}^{n} Z_k \left[\sum_{k=1}^{n} \frac{\left[\left\{(Q_{c1}\sin\alpha Z_k)^{-\frac{c-h+2}{3}} + (Q_{c2}\sin\alpha Z_k)^{-\frac{c-h+2}{3}}\right\}^{-\frac{3}{c-h+2}}\right]^{-\frac{c-h+2}{3}}}{Z_k^{-\frac{c-h+2}{3}}} \right]^{-\frac{3}{c-h+2}}$$

$$= \sum_{k=1}^{n} Z_k \left[\sum_{k=1}^{n} \left(\frac{Z_k}{C_{ak}}\right)^{\frac{c-h+2}{3}} \right]^{-\frac{3}{c-h+2}} .$$

Substituting experimental constants c = 31/3 and h = 7/3,

$$C_a = (Z_1 + Z_2 + Z_3 + \ldots\ldots\ldots\ldots\ldots + Z_n) \times \left[\left(\frac{Z_1}{C_{a1}}\right)^{10/3} + \left(\frac{Z_2}{C_{a2}}\right)^{10/3} + \left(\frac{Z_3}{C_{a3}}\right)^{10/3} + \ldots + \left(\frac{Z_n}{C_{an}}\right)^{10/3} \right]^{-3/10} .$$

.... (3-30)

The load ratings C_{a1}, C_{a2}, C_{a3}, ... C_{an} for the rows with z_1, z_2, z_3, ... z_n balls are calculated from the appropriate single row thrust ball bearing formula in 3.2.

3.4 Basic dynamic radial load rating C_r for radial roller bearings

By a procedure similar to that used to obtain equation (3-10) for point contact in 3.1, but applying (3-4) and (3-5), the basic dynamic radial load rating of radial roller bearings (line contact) is obtained :

$$C_r = 0{,}377 \frac{1}{2^{\frac{c+h-1}{c-h+1}}\, 0{,}5^{\frac{2e}{c-h+1}}}\, B_1 \frac{(1-\gamma)^{\frac{c+h-3}{c-h+1}}}{(1+\gamma)^{\frac{2e}{c-h+1}}}\, \gamma^{\frac{2}{c-h+1}}$$

$$\times \left[1 + \left\{1{,}04 \left(\frac{1-\gamma}{1+\gamma}\right)^{\frac{c+h+2e-3}{c-h+1}}\right\}^{\frac{c-h+1}{2}}\right]^{-\frac{2}{c-h+1}}$$

$$\times (i L_{we} \cos\alpha)^{\frac{c-h-1}{c-h+1}}\, Z^{\frac{c-h-2e+1}{c-h+1}}\, D_{we}^{\frac{c+h-3}{c-h+1}} \quad \text{............ (3-31)}$$

where B_1 = proportional constant determined experimentally

γ = $D_{we} \cos\alpha / D_{pw}$

D_{we} = mean roller diameter

α = nominal contact angle

D_{pw} = pitch diameter of roller set

L_{we} = effective contact length of roller

i = number of rows of rollers

Z = number of rollers per row.

Here, a contact angle α, number of rollers Z, the mean diameter D_{we}, and the effective contact length L_{we} depend on bearing design. On the other hand, $\gamma = D_{we}\cos\alpha / D_{pw}$ is not dimensional, therefore it is convenient in practice that the value for the first two lines in the right side of equation (3-31) is designated as a factor fc.

Consequently,

$$C_r = f_c (i L_{we} \cos\alpha)^{\frac{c-h-1}{c-h+1}} Z^{\frac{c-h-2e+1}{c-h+1}} D_{we}^{\frac{c+h-3}{c-h+1}} \quad \ldots (3-32)$$

For the basic dynamic radial load rating for radial roller bearings adjustments are made to take account of stress concentration (e.g. edge loading) and of the use of a constant instead of a varying life formula exponent (see clause 5). Adjustment for stress concentration is a reduction factor λ and for exponent variation a factor ν.
It is convenient to contain both factors - which are determined experimentally - in the factor f_c, which consequently is given as follows:

$$f_c = 0{,}377\lambda\nu \frac{1}{2^{\frac{c+h-1}{c-h+1}} 0{,}5^{\frac{2e}{c-h+1}}} B_1 \frac{(1-\gamma)^{\frac{c+h-3}{c-h+1}}}{(1+\gamma)^{\frac{2e}{c-h+1}}} \gamma^{\frac{2}{c-h+1}}$$

$$\times \left[1 + \left\{ 1{,}04 \left(\frac{1-\gamma}{1+\gamma} \right)^{\frac{c+h+2e-3}{c-h+1}} \right\}^{\frac{c-h+1}{2}} \right]^{-\frac{2}{c-h+1}} \quad \ldots (3-33)$$

The Weibull slope e, the constants c and h are determined experimentally. Based on the original experimental work by Lundberg and Palmgren with ball bearings and subsequent verification tests with spherical, cylindrical and tapered roller bearings the following values were assigned to the experimental constants in the rating equations:

$e = 9/8$
$c = 31/3$
$h = 7/3$.

Substituting experimental constants $e = 9/8$, $c = 31/3$ and $h = 7/3$ into equations (3-32) and (3-33),

$$C_r = f_c (iL_{we}\cos\alpha)^{7/9} Z^{3/4} D_{we}^{29/27}, \qquad \ldots\ldots\ldots\ldots\ldots \text{(3-34)}$$

$$f_c = 0{,}483B_1 \times 0{,}377\lambda\nu \frac{\gamma^{2/9}(1-\gamma)^{29/27}}{(1+\gamma)^{1/4}}$$

$$\times \left[1+\left\{1{,}04\left(\frac{1-\gamma}{1+\gamma}\right)^{143/108}\right\}^{9/2}\right]^{-2/9}. \qquad \ldots\ldots \text{(3-35)}$$

The value for $0{,}483B_1$ is 551,13373 to calculate C_r in Newtons. Values for f_c on table 5 in ISO 281/I are calculated from substituting reduction factor which is given in table 3-2 into equation (3-35).

3.5 Basic dynamic axial load rating Ca for single row thrust roller bearings

3.5.1 Thrust roller bearings with contact angle $\alpha \neq 90°$

Extension of 3.1 gives:

$$C_a = f_c (L_{we}\cos\alpha)^{\frac{c-h-1}{c-h+1}} \tan\alpha \; Z^{\frac{c-h-2e+1}{c-h+1}} D_{we}^{\frac{c+h-3}{c-h+1}}.$$

$$\ldots\ldots\ldots\ldots\ldots \text{(3-36)}$$

For thrust roller bearings the theoretical value of a basic dynamic axial load rating must be reduced on the basis of unequal distribution of load among the rolling elements in addition to the reduction factor λ which is introduced in radial roller bearing load ratings. This reduction factor is designated as η.

Consequently the factor f_c is given as follows:

$$f_c = \lambda\nu\eta \frac{1}{2^{\frac{c+h-1}{c-h+1}} 0{,}5^{\frac{2e}{c-h+1}}} B_1 \frac{(1-\gamma)^{\frac{c+h-3}{c-h+1}}}{(1+\gamma)^{\frac{2e}{c-h+1}}} \gamma^{\frac{2}{c-h+1}}$$

$$\times \left[1 + \left\{ \left(\frac{1-\gamma}{1+\gamma}\right)^{\frac{c+h+2e-3}{c-h+1}} \right\}^{\frac{c-h+1}{2}} \right]^{-\frac{2}{c-h+1}} \quad \ldots\ldots \quad (3\text{-}37)$$

Substituting experimental constants $e = 9/8$, $c = 31/3$ and $h = 7/3$,

$$C_a = f_c (L_{we} \cos\alpha)^{7/9} \tan\alpha \; Z^{3/4} \; D_{we}^{29/27}, \quad \ldots\ldots\ldots \quad (3\text{-}38)$$

$$f_c = 0{,}483 B_1 \lambda\nu\eta \frac{\gamma^{2/9}(1-\gamma)^{29/27}}{(1+\gamma)^{1/4}} \left[1 + \left\{ \left(\frac{1-\gamma}{1+\gamma}\right)^{143/108} \right\}^{9/2} \right]^{-2/9}$$

$$\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}39)$$

The value for $0{,}483B_1$ is 551,13373 to calculate C_a in Newtons. Values for f_c on the right column of table 7 in ISO 281/I are calculated from substituting reduction factors which are given in table 3-2 into equation (3-39).

3.5.2 Thrust roller bearings with contact angle $\alpha = 90°$

Extension of 3.1 gives:

$$C_a = f_c L_{we}^{\frac{c-h-1}{c-h+1}} Z^{\frac{c-h-2e+1}{c-h+1}} D_{we}^{\frac{c+h-3}{c-h+1}}, \quad \ldots\ldots\ldots\ldots \quad (3\text{-}40)$$

$$f_c = \lambda\nu\eta \frac{1}{2^{\frac{c+h-1}{c-h+1}} 0{,}5^{\frac{2e}{c-h+1}}} B_1 \gamma^{\frac{2}{c-h+1}} 2^{-\frac{2}{c-h+1}}. \quad \ldots\ldots \quad (3\text{-}41)$$

Substituting experimental constants $e = 9/8$, $c = 31/3$ and $h = 7/3$,

$$C_a = f_c L_{we}^{7/9} Z^{3/4} D_{we}^{29/27}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}42)$$

$$f_c = 0{,}41 B_1 \lambda\nu\eta\gamma^{2/9}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3\text{-}43)$$

The value for $0{,}41B_1$ is 472,45388 to calculate C_a in Newtons. Values for f_c on the left column of table 7 in ISO 281/I are calculated from substituting reduction factors which are given in table 3-2 into equation (3-43).

3.6 Basic dynamic axial load rating Ca for thrust roller bearings with two or more rows of rollers

According to the product law of probability, relationships between the basic dynamic axial load rating of an entire thrust roller bearing and of both the rotating and stationary washers are given as follows:

$$C_{ak} = \left[C_{a1k}^{-\frac{c-h+1}{2}} + C_{a2k}^{-\frac{c-h+1}{2}} \right]^{-\frac{2}{c-h+1}}, \quad \ldots\ldots\ldots \quad (3\text{-}44)$$

$$\left. \begin{aligned} C_{a1k} &= Q_{c1} \sin\alpha Z_k L_{wek}, \\ C_{a2k} &= Q_{c2} \sin\alpha Z_k L_{wek}, \end{aligned} \right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (3\text{-}45)$$

$$C_a = \left[C_{a1}^{-\frac{c-h+1}{2}} + C_{a2}^{-\frac{c-h+1}{2}} \right]^{-\frac{2}{c-h+1}}, \quad \ldots\ldots\ldots\ldots (3\text{-}46)$$

$$\left. \begin{aligned} C_{a1} &= Q_{c1} \sin\alpha \sum_{k=1}^{n} Z_k L_{wek}, \\ C_{a2} &= Q_{c2} \sin\alpha \sum_{k=1}^{n} Z_k L_{wek} \end{aligned} \right\} \quad \ldots\ldots\ldots\ldots (3\text{-}47)$$

where C_{ak} = basic dynamic axial load rating as a row k of an entire thrust roller bearing

C_{a1k} = basic dynamic axial load rating as a row k of the rotating washer of an entire thrust roller bearing

C_{a2k} = basic dynamic axial load rating as a row k of the stationary washer of an entire thrust roller bearing

C_a = basic dynamic axial load rating of an entire thrust roller bearing

C_{a1} = basic dynamic axial load rating of the rotating washer of an entire thrust roller bearing

C_{a2} = basic dynamic axial load rating of the stationary washer of an entire thrust roller bearing

Z_k = number of rollers as a row k.

Substituting equations (3-47), (3-45) and (3-44) into equation (3-46), and rearrangement of equation (3-46) gives:

$$C_a = \sum_{k=1}^{n} Z_k L_{wek} \left[\frac{\left(Q_{c1} \sin\alpha \sum_{k=1}^{n} Z_k L_{wek}\right)^{-\frac{c-h+1}{2}} + \left(Q_{c2} \sin\alpha \sum_{k=1}^{n} Z_k L_{wek}\right)^{-\frac{c-h+1}{2}}}{\left(\sum_{k=1}^{n} Z_k L_{wek}\right)^{-\frac{c-h+1}{2}}} \right]^{-\frac{2}{c-h+1}}$$

$$= \sum_{k=1}^{n} Z_k L_{wek} \left[\sum_{k=1}^{n} \frac{\left[\left\{\left(Q_{c1} \sin\alpha \, Z_k L_{wek}\right)^{-\frac{c-h+1}{2}} + \left(Q_{c2} \sin\alpha \, Z_k L_{wek}\right)^{-\frac{c-h+1}{2}}\right\}^{-\frac{2}{c-h+1}}\right]^{-\frac{c-h+1}{2}}}{\left(Z_k L_{wek}\right)^{-\frac{c-h+1}{2}}} \right]^{-\frac{2}{c-h+1}}$$

$$= \sum_{k=1}^{n} Z_k L_{wek} \left[\sum_{k=1}^{n} \left(\frac{Z_k L_{wek}}{C_{ak}}\right)^{\frac{c-h+1}{2}} \right]^{-\frac{2}{c-h+1}}.$$

Substituting experimental constants $c = 31/3$ and $h = 7/3$,

$$C_a = (Z_1 L_{we1} + Z_2 L_{we2} + Z_3 L_{we3} + \dots + Z_n L_{wen}) \times \left[\left(\frac{Z_1 L_{we1}}{C_{a1}}\right)^{9/2} + \left(\frac{Z_2 L_{we2}}{C_{a2}}\right)^{9/2} + \left(\frac{Z_3 L_{we3}}{C_{a3}}\right)^{9/2} + \dots + \left(\frac{Z_n L_{wen}}{C_{an}}\right)^{9/2} \right]^{-2/9}.$$

.... (3-48)

The load ratings, C_{a1}, C_{a2}, C_{a3}, ..., C_{an} for the rows with $Z1$, $Z2$, $Z3$, ..., Z_n rollers of lengths L_{we1}, L_{we2}, L_{we3}, ..., L_{wen} are calculated from the appropriate single row thrust roller bearing formula in 3.3.

TABLE 3-1 — Raceway groove radius and reduction factor for ball bearings

TABLE No. in ISO 281/I	Bearing type	Raceway groove radius		Reduction factor	
		r_i	r_e	λ	η
TABLE 1	Single row radial contact groove ball bearings Single and double row angular contact groove ball bearings	$0,52D_w$		0,95	-
	Double row radial contact groove ball bearings	$0,52D_w$		0,90	-
	Single and double row self-aligning ball bearings	$0,53D_w$	$0,5(\frac{1}{\gamma}+1)D_w$	1	-
	Single row radial contact separable ball bearings (magneto bearings)	$0,52D_w$	∞	0,95	-
TABLE 3	Thrust ball bearings	$0,535D_w$		0,90	$1-\frac{\sin\alpha}{3}$

NOTE — Values for f_c on tables 1 and 3 in ISO 281/I are calculated from substituting raceway groove radius and reduction factor in the above table 3-1 into equations (3-15), (3-20) and (3-25) respectively.

TABLE 3-2 — Reduction factor for roller bearings

TABLE No. in ISO 281/I	Bearing type	Reduction factor	
		λ_ν	η
TABLE 5	Radial roller bearings	0,83	-
TABLE 7	Thrust roller bearings	0,73	$1-0,15\sin\alpha$

NOTE — Values for f_c on tbales 5 and 7 in ISO 281/I are calculated from substituting reduction factor in the above table 3-2 into equations (3-35), (3-39) and (4-3) respectively.

4. Dynamic Equivalent Load

4.1 Formulae of dynamic equivalent load

4.1.1 Theoretical dynamic equivalent radial load Pr for single row radial bearings

If the indices 1 and 2 are assigned to the ring which rotates relative to the direction of load and the stationary ring respectively, then the mean values of the rolling element loads which are decisive for a single row radial bearing ring's life are given by equations:

$$\left.\begin{aligned} Q_{c1} &= Q_{max}\, J_1 = \frac{F_r}{Z\cos\alpha}\,\frac{J_1}{J_r} = \frac{F_a}{Z\sin\alpha}\,\frac{J_1}{J_a}, \\ Q_{c2} &= Q_{max}\, J_2 = \frac{F_r}{Z\cos\alpha}\,\frac{J_2}{J_r} = \frac{F_a}{Z\sin\alpha}\,\frac{J_2}{J_a} \end{aligned}\right\} \quad \cdots\cdots\cdot \quad (4\text{-}1)$$

where Q_{max} = maximum rolling element load
J_1 = factor relating Q_{c1} to Q_{max}
J_2 = factor relating Q_{c2} to Q_{max}
F_r = radial load
F_a = axial load
J_r = radial load integral
J_a = axial load integral
Z = number of rolling elements
α = nominal contact angle.

Radial and axial load integrals are given by following equations:

$$\left.\begin{aligned} J_r &= J_r(\varepsilon) = \frac{1}{2\pi}\int_{-\varphi_0}^{+\varphi_0} \left[1-\frac{1}{2\varepsilon}(1-\cos\varphi)\right]^t \cos\varphi\, d\varphi, \\ J_a &= J_a(\varepsilon) = \frac{1}{2\pi}\int_{-\varphi_0}^{+\varphi_0} \left[1-\frac{1}{2\varepsilon}(1-\cos\varphi)\right]^t d\varphi \end{aligned}\right\} \quad \cdots\cdot \quad (4\text{-}2)$$

where t = 3/2 for point contact
= 1,1 for line contact
φ_o = one half of the loaded arc
ε = parameter indicating the width of the loaded zone in the bearing.

Introducing the notation

$$J(t;s) = [\frac{1}{2\pi}\int_{-\varphi_o}^{+\varphi_o}[1 - \frac{1}{2\varepsilon}(1 - \cos\varphi)]^t d\varphi]^{\frac{1}{s}}, \quad \ldots\ldots\ldots \quad (4\text{-}3)$$

$$\left.\begin{array}{ll} J_1 = J_1(\varepsilon) = J(9/2;3), & J_2 = J_2(\varepsilon) = J(5;10/3), \\ J_1 = J_1(\varepsilon) \simeq J(9/2;4), & J_2 = J_2(\varepsilon) \simeq J(5;9/2\ ) \end{array}\right\} \ldots \quad (4\text{-}4)$$

for point and line contact respectively.

If P_{r1} and P_{r2} are the dynamic equivalent radial loads for the respective rings, then with radial displacement of the rings (ε=0.5)

$$Q_{c1} = \frac{P_{r1}}{Z\ \cos\alpha}\ \frac{J_1(0,5)}{J_r(0,5)}, \quad Q_{c2} = \frac{P_{r2}}{Z\ \cos\alpha}\ \frac{J_2(0,5)}{J_r(0,5)} \quad \ldots\ldots \quad (4\text{-}5)$$

where the values $J_1(0,5)$, $J_2(0,5)$ and $J_r(0,5)$ are given in table 4-1.

From equations (4-1), (4-5) and

$$(\frac{P_r}{C_r})^w = (\frac{P_{r1}}{C_1})^w + (\frac{P_{r2}}{C_2})^w$$

is obtained

$$\left.\begin{array}{l} \frac{P_r}{F_r} = \left[(\frac{C_r}{C_1}\ \frac{J_r(0,5)}{J_1(0,5)}\ \frac{J_1}{J_r})^w + (\frac{C_r}{C_2}\ \frac{J_r(0,5)}{J_2(0,5)}\ \frac{J_2}{J_r})^w \right]^{\frac{1}{w}}, \\ \frac{P_r}{F_a\ \cot\alpha} = \left[(\frac{C_r}{C_1}\ \frac{J_1}{J_1(0,5)})^w + (\frac{C_r}{C_2}\ \frac{J_2}{J_2(0,5)})^w \right]^{\frac{1}{w}} \frac{J_r(0,5)}{J_a} \end{array}\right\} \ldots \quad (4\text{-}6)$$

where C_r = basic dynamic radial load rating
C_1 = basic dynamic radial load rating of a rotating ring
C_2 = basic dynamic radial load rating of a stationary ring
w = pe (p = exponent in life formula,
e = Weibull slope)

TABLE 4-1 — Values for $J_r(0,5)$, $J_a(0,5)$, $J_1(0,5)$ $J_2(0,5)$ and w

	Point contact		Line contact		Point and line contact	
	Single row bearing	Double row bearing	Single row bearing	Double row bearing	Single row bearing	Double row bearing
$J_r(0,5)$	0,2288	0,4577	0,2453	0,4906	0,2369	0,4739
$J_a(0,5)$	0,2782	0	0,3090	0	0,2932	0
$J_1(0,5)$	0,5625	0,6925	0,6495	0,7577	0,6044	0,7244
$J_2(0,5)$	0,5875	0,7233	0,6744	0,7867	0,6295	0,7543
$J_r(0,5)/J_a(0,5)$	0,822	–	0,794	–	0,808	–
$J_r(0,5)/J_1(0,5)$	0,407	0,661	0,378	0,648	0,392	0,654
$J_r(0,5)/J_2(0,5)$	0,389	0,633	0,364	0,623	0,376	0,628
$J_2(0,5)/J_1(0,5)$	1,044		1,038		1,041	
$\frac{J_r(0,5)}{\sqrt{J_1(0,5) \cdot J_2(0,5)}}$	0,398 (≈0,40)	0,647 (≈0.65)	0,371	0,635	0,384	0,641
w	10/3		9/2		180/47	
$2^{1-1/w}$	1,625		1,714		1,669	

For radial displacement of the bearing rings (ε=0.5) and fixed outer ring load ($C_1=C_i$, basic dynamic load rating for inner ring, $C_2=C_e$, basic dynamic load rating for outer ring) from equation (4-6) is found

$$P_r = F_r = \frac{J_r(0,5)}{J_a(0,5)} F_a \cot\alpha$$

$$= (0,822 \cdots 0,794)\, F_a \cot\alpha \quad \ldots\ldots\ldots\ldots \quad (4\text{-}7)$$

for point and line contact respectively.

For ε=0,5 and fixed inner ring load ($C_1=vC_e$, $C_2=C_i/v$), it is found

$$P_r = VF_r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4\text{-}8)$$

where V is the rotation factor.

The factor V varies between 1±0,044 and 1±0,038 for point and line contact respectively. In ISO 281/I, the rotation factor V has been deleted.

NOTE — The value of 1,2 for the rotation factor V was given in ISO/R281 for radial bearings except self-aligning ball bearings as safety factor.

For axial displacement of the bearing rings ($\varepsilon=\infty$) and fixed outer ring load ($C_1=C_i$, $C_2=C_e$),

$$P_r = YF_a, \qquad Y = f_1\left(\frac{C_i}{C_e}\right) \frac{J_r(0,5)}{J_1(0,5)} \cot\alpha. \quad \ldots\ldots\ldots \quad (4\text{-}9)$$

The factor $f_1(\frac{C_i}{C_e})$ varies between 1 and $1/v = J_1(0,5)/J_2(0,5)$. Introducing as a good approximation the geometrical mean value $1/\sqrt{v}$ between these two values (see table 4-1),

$$Y = \frac{J_r(0,5)}{\sqrt{J_1(0,5) \cdot J_2(0,5)}} \cot\alpha. \qquad \text{.................... (4-10)}$$

For non self-aligning bearings consideration must be given to the effect of the manufacturing precision on the factor Y.

The value of Y given in equation (4-10) is corrected by the reduction factor η.

$$Y_1 = Y/\eta. \qquad \text{.............................. (4-11)}$$

For combined loads, equation (4-6) gives related values of F_r/P_r and $F_a \cot\alpha/P_r$ corresponding to the curves given in figure 4-1 for the limiting cases $C_1/C_2 \approx 0$ and $C_2/C_1 \approx 0$.

The points A represent ε=0,5, that is radial displacement of the bearing rings. For these points,

$$F_a = (1,22 \cdot\cdot\cdot\cdot\cdot 1,26)\ F_r \tan\alpha \qquad \text{................. (4-12)}$$

for point and line contact respectively.

4.1.2 Theoretical dynamic equivalent radial load Pr for double row radial bearings

For double row radial bearings, the indices I and II are assigned to the respective rows. The determining factors for life of the rotating and stationary rings are the mean values

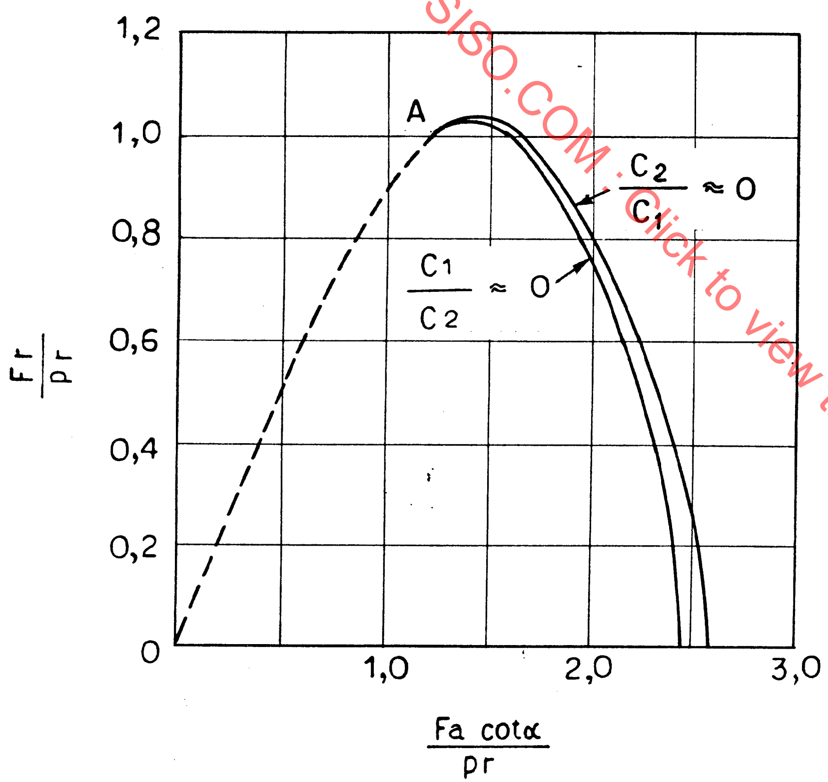
$$Q_{C1} = J_1\ Q_{max\,I}, \qquad Q_{C2} = J_2\ Q_{max\,II} \qquad \text{............. (4-13)}$$

where

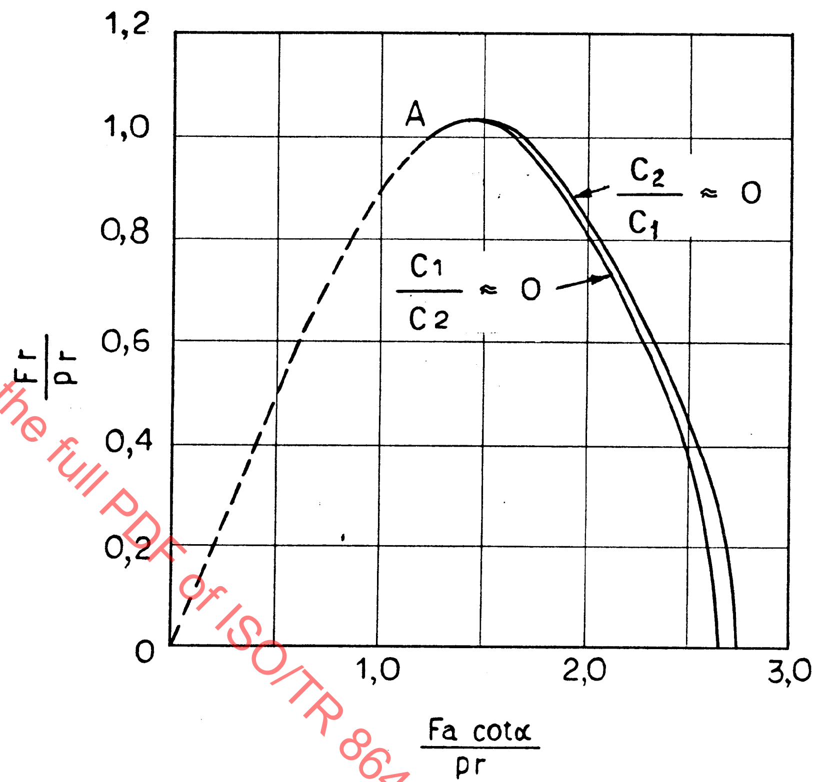
$$\left.\begin{aligned} J_1 &= \left[J_1(\varepsilon_I)^w + \left(\frac{Q_{max II}}{Q_{max I}}\right)^w J_1(\varepsilon_{II})^w \right]^{\frac{1}{w}}, \\ J_2 &= \left[J_2(\varepsilon_I)^w + \left(\frac{Q_{max II}}{Q_{max I}}\right)^w J_2(\varepsilon_{II})^w \right]^{\frac{1}{w}}. \end{aligned}\right\} \quad \text{.........} \quad (4\text{-}14)$$

For a bearing without internal clearances,

$$\left.\begin{aligned} \varepsilon_I + \varepsilon_{II} &= 1 \quad \text{for} \quad \varepsilon_I \leqslant 1, \\ \varepsilon_{II} &= 0 \quad \text{for} \quad \varepsilon_I \geqslant 1. \end{aligned}\right\} \quad \text{..................} \quad (4\text{-}15)$$



(a) Point contact



(b) Line contact

FIGURE 4-1 — Dynamic equivalent load Pr for single row radial bearings with constant contact angle α

If we introduce the values of Jr, Ja, J_1 and J_2 for double row bearings, then the equivalent bearing load is obtained from the same equation (4-6) for single row bearings. Jr(0,5), Ja(0,5), J_1(0,5) are here the values valid for $\varepsilon_I = \varepsilon_{II} = 0{,}5$ (see table 4-1).
The bent curves given in figure 4-2 are found for the limiting cases $C_1/C_2 \approx 0$ and $C_2/C_1 \approx 0$.

Both rows are loaded if $\varepsilon_I < 1$, that is if

$$Fa < (1{,}67 \cdots 1{,}91)\ Fr\ \tan\alpha \qquad \text{(4-16)}$$

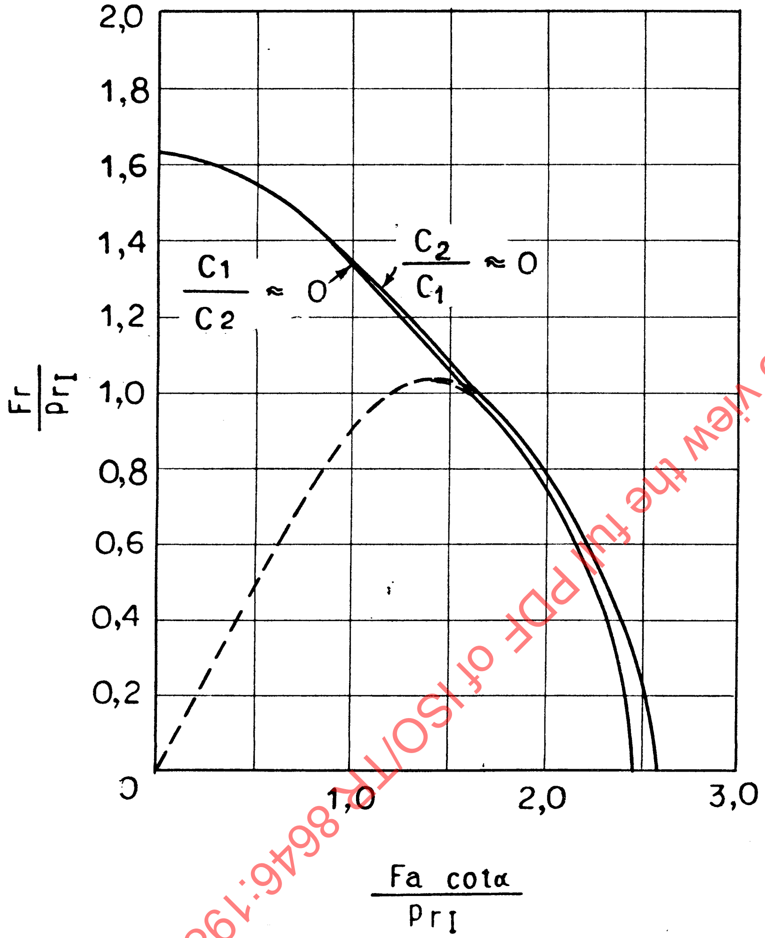
for point and line contact respectively.

Only one row is loaded if Fa is greater than that value. In that case the life for double row bearings can be calculated from the theory of single row bearings as well as from the theory of double row bearings.

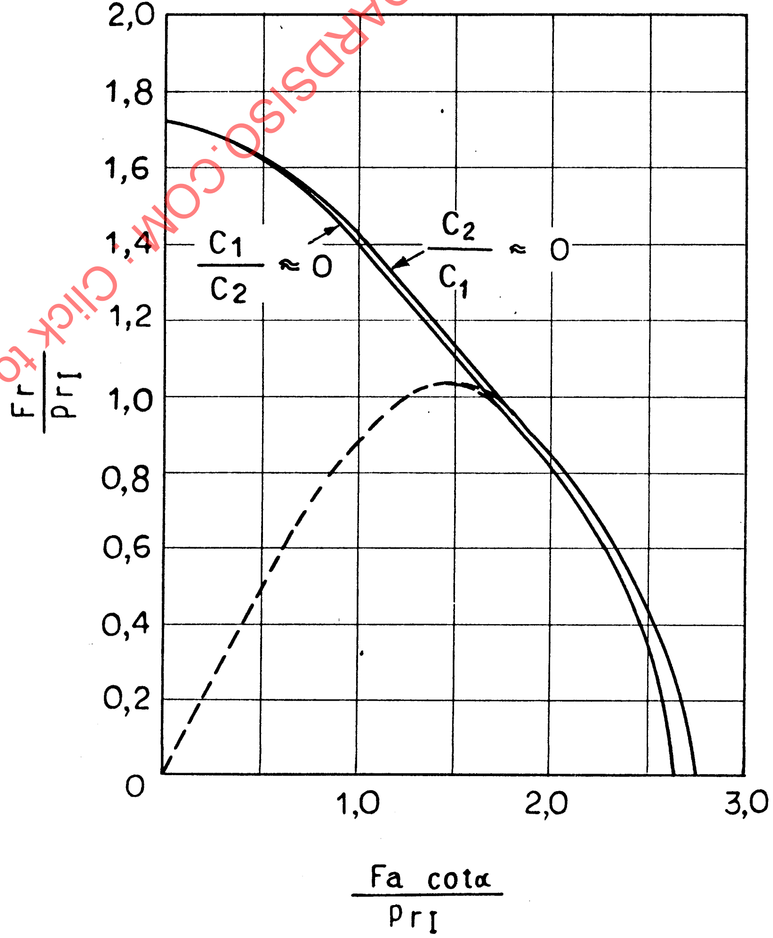
If Pr_I is the equivalent radial load for the loaded row considered as a single row bearing and Pr is the equivalent load for the double row bearing,

$$\frac{Pr}{Pr_I} = \frac{C_r}{C_I} = 2^{1-1/w}. \qquad \text{(4-17)}$$

Figures 4-1 and 4-2 are calculated on the assumption of a constant contact angle. Figures 4-1(a) and 4-2(a) are also approximately applicable to angular contact groove ball bearings, if cotα' is determined from the following formula.



(a) Point contact



(b) Line contact

FIGURE 4-2 — Dynamic equivalent load for double row bearings with constant contact angle α

$$\left(\frac{\cos\alpha}{\cos\alpha'} - 1\right)^{3/2}\sin\alpha' = \left(\frac{c}{2r/D_w - 1}\right)^{3/2} \frac{F_a}{Z\ D_w^2}. \quad \ldots\ldots \quad (4\text{-}18)$$

"c" is a compression constant, which depends on the modulus of elasticity and the conformity 2r/Dw, where r is a cross-sectional raceway groove radius and Dw is ball diameter.

4.1.3 Theoretical dynamic equivalent radial load Pr for radial contact groove ball bearings

Figure 4-3 is applicable to radial contact groove ball bearings. The curve AC has been determined from equation (4-6) and approximate formula

$$\tan\alpha' \approx \left(\frac{2c}{2r/D_w-1}\right)^{3/8}\left(1 - \frac{1}{2\varepsilon}\right)^{3/8}\left(\frac{F_a}{J_a\, i Z D_w^2}\right)^{1/4} \quad \ldots\ldots \quad (4\text{-}19)$$

and gives the functional relationship between Fr/Pr and Fa cotα'/Pr, where α' is the contact angle calculated from the following formula [1]

$$\tan\alpha' \approx \left(\frac{2c}{2r/D_w-1}\right)^{3/8}\left(\frac{F_a}{i Z D_w^2}\right)^{1/4}. \quad \ldots\ldots\ldots\ldots\ldots \quad (4\text{-}20)$$

This formula is obtained from formula (4-19) for a centric axial load F_a and F_r=0, i.e. $\varepsilon = \infty$ and J_a= 1.

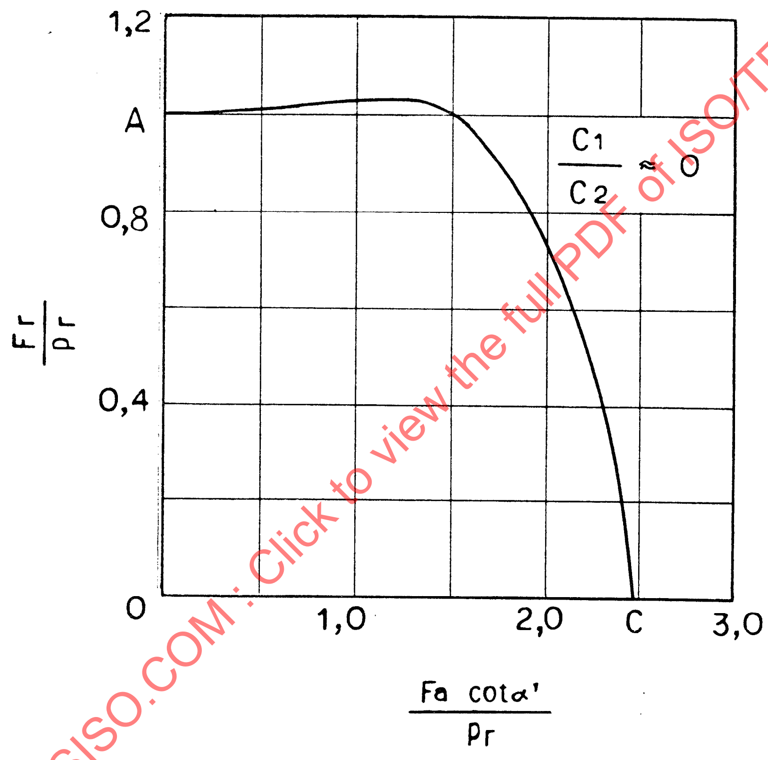


FIGURE 4-3 — Dynamic equivalent load Pr for radial contact groove ball bearings

4.1.4 Practical formulae of dynamic equivalent radial load Pr for radial bearings with constant contact angle

From a practical standpoint it is preferable to replace the theoretical curves in figures 4-1 and 4-2 by broken lines A_1BC for single row bearings and ABC for double row bearings in figure 4-4.

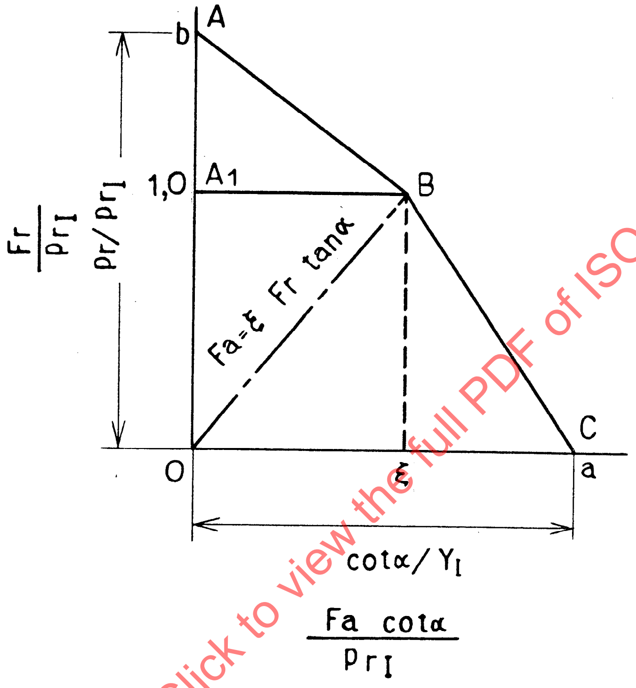


FIGURE 4-4 — Dynamic equivalent radial load for radial bearings with constant contact angle α

The equation for the straight line A_1B in figure 4-4 is

$F_r/P_{rI} = 1.$

Therefore, for $F_a/F_r \leqslant \xi\tan\alpha$, we have

$$P_{rI} = F_r \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (4\text{-}21)$$

and the straight line passing through the points B $(\xi, 1)$ and C (a, o) is given by

$$\frac{Fr/Pr_I - 1}{Fa\ \cot\alpha/Pr_I - \xi} = \frac{-1}{a - \xi} .$$

From this equation, for $Fa/Fr > \xi\tan\alpha$, we have

$$\left.\begin{aligned}
Pr_I &= (1 - \frac{\xi}{a})\ Fr + \frac{1}{a} \cot\alpha\ Fa \\
&\equiv X_1 Fr + Y_1 Fa \\
\text{where} & \\
X_1 &= 1 - \frac{\xi}{a} = 1 - \xi Y_1 \tan\alpha . \\
\text{Therefore, from} & \text{ the equation (4-11)} \\
X_1 &= 1 - \frac{J_r(0,5)}{\sqrt{J_1(0,5)\ J_2(0,5)}} \frac{\xi}{\eta} , \\
Y_1 &= \frac{J_r(0,5)\cot\alpha}{\sqrt{J_1(0,5)\ J_2(0,5)}} \frac{1}{\eta} .
\end{aligned}\right\} \qquad \text{(4-22)}$$

For the double row bearings, the equation for the straight line AB is

$$\frac{Fr/Pr_I - b}{Fa\ \cot\alpha/Pr_I} = \frac{1 - b}{\xi} .$$

From this,

$$Pr_I = \frac{Fr}{b} + (\frac{b - 1}{b})\ \frac{Fa\ \cot\alpha}{\xi} .$$

Therefore, for $Fa/Fr \leq \xi\tan\alpha$ we find

$$\left.\begin{aligned} Pr = 2^{1-1/w}Pr_I &= Fr + (2^{1-1/w}-1)\frac{\cot\alpha}{\xi}Fa \\ &\equiv X_3Fr + Y_3Fa \\ \text{where} \\ X_3 = 1, \quad Y_3 &= (2^{1-1/w}-1)\frac{1}{\xi}\cot\alpha. \end{aligned}\right\} \quad \ldots\ldots (4\text{-}23)$$

Further, from the equation (4-22) which represents straight line BC, we find for $Fa/Fr > \xi\tan\alpha$

$$\left.\begin{aligned} Pr = 2^{1-1/w}Pr_I &= 2^{1-1/w}X_1Fr + 2^{1-1/w}Y_1Fa \\ &\equiv X_2Fr + Y_2Fa \\ \text{where} \\ X_2 = 2^{1-1/w}X_1, \quad Y_2 &= 2^{1-1/w}Y_1. \end{aligned}\right\} \quad \ldots\ldots (4\text{-}24)$$

Integrating the above, table 4-2 shows the formulae of dynamic equivalent load Pr for radial bearing with constant contact angle and of factors X and Y.

TABLE 4-2 — Formulae of dynamic equivalent load and of factors for radial bearings with constant contact angle α

		Single row bearings	Double row bearings
Formulae	$F_a/F_r \leqslant e$	$P_r = F_r$	$P_r = X_3 F_r + Y_3 F_a$
	$F_a/F_r > e$	$P_r = X_1 F_r + Y_1 F_a$	$P_r = X_2 F_r + Y_2 F_a$
Radial load factor X and axial load factor Y		$X_1 = 1 - \frac{J_r(0,5)}{\sqrt{J_1(0,5) \cdot J_2(0,5)}} \frac{\xi}{\eta}$ $Y_1 = \frac{J_r(0,5)\cot\alpha}{\sqrt{J_1(0,5) \cdot J_2(0,5)}} \frac{1}{\eta}$	$X_2/X_1 = Y_2/Y_1 = 2^{1-1/w}$ $X_3 = 1$ $Y_3 = \frac{1}{\xi}(2^{1-1/w}-1)\cot\alpha$
e		$e = \xi\tan\alpha$	

4.1.5 Practical formulae of dynamic equivalent radial load Pr for radial ball bearings

Generally, the contact angle of radial ball bearings varies with the load, but table 4-2 can be approximately applicable to angular contact groove ball bearings, if α is replaced by contact angle α' under the axial load Fa given by the equation (4-18).

Therefore, according table 4-1,

$$\left.\begin{aligned} X_1 &= 1 - 0,4\,\frac{\xi}{\eta}, & Y_1 &= \frac{0,4}{\eta}\cot\alpha', \\ X_2 &= 1,625\, X_1, & Y_2 &= 1,625\, Y_1, \\ X_3 &= 1, & Y_3 &= \frac{0,625}{\xi}\cot\alpha'. \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (4\text{-}25)$$

For single row and double row radial contact groove ball bearings, the theoretical curve in figure 4-3 is replaced by the broken line A_1BC in figure 4-5.

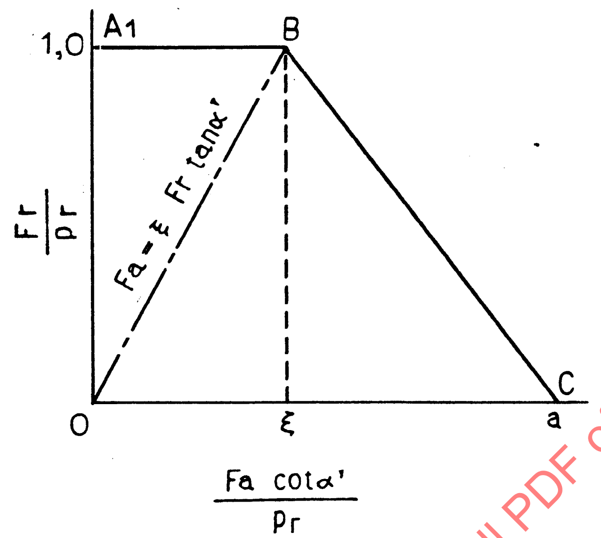


FIGURE 4-5 — Dynamic equivalent radial load for radial contact groove ball bearings

For this type of bearings,

$$\left.\begin{aligned} X_1 &= X_2 = 1 - 0,4\frac{\xi}{\eta}, \\ Y_1 &= Y_2 = 0,4\,\frac{\cot\alpha'}{\eta}, \\ X_3 &= 1, \quad Y_3 = 0. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4\text{-}26)$$

For self-aligning ball bearings, the contact angle can be considered as independent of the load ($\alpha' = \alpha$), and also $\eta = 1$.

4.1.6 Practical formulae of dynamic equivalent axial load Pa for thrust bearings

The radial and axial load factors X_a and Y_a for single and double direction bearings with $\alpha \neq 90°$ are obtained on the basis of the formulae of dynamic equivalent radial load Pr for single row and double row radial bearings, respectively.

That is, for single direction bearings, when $F_a/F_r > \xi \tan\alpha$,

$$Y_1 P_a = P_r = X_1 F_r + Y_1 F_a .$$

So

$$\left.\begin{aligned} P_a &= \frac{X_1}{Y_1} F_r + F_a \\ &\equiv X_{a1} F_r + Y_{a1} F_a \\ \text{where} & \\ X_{a1} &= X_1/Y_1, \quad Y_{a1} = 1 \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \quad (4\text{-}27)$$

and for double direction bearings, when Fa/Fr > $\xi \tan\alpha$, also then

$$\left.\begin{aligned} P_a &= \frac{X_2}{Y_2} F_r + F_r \\ &\equiv X_{a2} F_r + Y_{a2} F_a \\ \text{where} & \\ X_{a2} &= X_2/Y_2, \quad Y_{a2} = 1. \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots \quad (4\text{-}28)$$

Further, when Fa/Fr $\leqslant \xi \tan\alpha$, approximately

$$Y_2 P_a = P_r = X_3 F_r + Y_3 F_a ,$$

therefore

$$\left.\begin{aligned} P_a &= \frac{X_3}{Y_2} F_r + \frac{Y_3}{Y_2} F_a \\ &= X_{a3} F_r + Y_{a3} F_a \\ \text{where} & \\ X_{a3} &= X_3/Y_2, \quad Y_{a3} = Y_3/Y_2. \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4-29)$$

Integrating the above, table 4-3 shows the formulae of dynamic equivalent load Pa for thrust bearings and of factors X_a and Y_a.

TABLE 4-3 — Formulae of dynamic equivalent load and of factors for thrust bearings

		Single direction bearings	Double direction bearings
Formulae	$F_a/F_r \leq e$	——	$P_a = X_{a3}F_r + Y_{a3}F_a$
	$F_a/F_r > e$	$P_a = X_{a1}F_r + Y_{a1}F_a$	$P_a = X_{a2}F_r + Y_{a2}F_a$
Radial load factor X_a and axial load factor Y_a		$X_{a1} = X_1/Y_1$ $Y_{a1} = 1$	$X_{a2} = X_2/Y_2$ $Y_{a2} = 1$ $X_{a3} = X_3/Y_2$ $Y_{a3} = Y_3/Y_2$
e		$e = \xi \tan\alpha$	

4.2 Factors X, Y and e

4.2.1 Radial ball bearings

a) Values of ξ

For single row radial contact groove ball bearings, G. Lundberg and A. Palmgren [1] gave a value of ξ = 1,2 based on the results of tests, and for other bearings ξ = 1,5 which are close to the theoretical curves. However, based on later tests, ISO/R281 takes values of ξ = 1,05 for radial contact groove ball bearings and single row angular contact groove types with α = 5°; ξ = 1,25 for other angular contact groove types, and ξ = 1,5 for self-aligning types [3].

b) Values of η

The reduction factor η depends on the contact angle α and is given by

$$\eta = 1 - k \sin\alpha \qquad (4\text{-}30)$$

Based on the experience and preliminary tests, k=0,4 [1] and k=0,15 to 0,33 [2] are given by G. Lundberg and A. Palmgren. In ISO/R281, k=0,4(=1/2,5) is used for radial contact groove bearings (α=5°) and angular contact groove bearings with α = 5°, 10° and 15° and k=1/2,75 is used for angular contact groove bearings with α = 20° to 45° [3].

NOTE — ISO/R281 does not include factors for bearings with α = 45°. It is additionally specified in ISO 281/I.

c) Values of contact angle α'

For radial contact groove ball bearings as well as angular contact groove bearings with nominal contact angle $\alpha \leqslant 15°$ the real contact angle varies considerably with the load. Consequently table 2 of ISO 281/I gives all factors as functions of the relative axial load.

The values of contact angle α' under an axial load Fa can be calculated from

$$\left(\frac{\cos 5°}{\cos\alpha'} - 1\right)^{3/2}\sin\alpha' = \left(\frac{c}{2r/D_w-1}\right)^{3/2}\frac{Fa}{iZD_w^2} \quad \ldots\ldots (4\text{-}31)$$

for radial contact groove ball bearings (considering them as angular contact groove bearings with a nominal contact angle $\alpha = 5°$), and from equation (4-18) for angular contact groove bearings with a nominal contact angle α.

For $2r/Dw = 1{,}035$, $c = 0{,}00043871$ is given, with units in N and mm.

Table 4-4 shows the values of contact angle α' calculated from equations (4-18) and (4-31) for $2r/Dw = 1{,}035$.

For angular contact groove ball bearings with $\alpha \geqslant 20°$, the influence of axial load on the contact angle is comparatively small and therefore the ISO 281/I table has only one set of X, Y and e factors for each α. With regard to the calculation rules applied to these bearings, see 4.2.2 c).

TABLE 4-4 — Values of contact angle α' for radial and angular contact groove ball bearings (α = 5°, 10° and 15°)

F_a/ZD_w^2 *		α = 5°	α = 10°	α = 15°
lbf/in²	N/mm²	α'		
25	0,17237	10,230°	12,953°	16,781°
50	0,34474	11,811°	14,177°	17,652°
100	0,68948	13,734°	15,768°	18,866°
150	1,03421	15,037°	16,893°	19,767°
200	1,37895	16,048°	17,786°	20,503°
300	2,0684	17,607°	19,187°	21,688°
500	3,4474	19,809°	21,207°	23,448°
750	5,1711	21,761°	23,028°	25,075°
1000	6,8948	23,263°	24,444°	26,360°

* For radial contact groove bearings F_a/iZD_w^2.

4.2.2 Values of X, Y and e for each type of radial ball bearing

Integrating the above, methods of calculating values of X, Y and e are as follows (see tables 4-6 and 4-7).

a) Radial contact groove ball bearings

$$X_1 = X_2 = 1 - \frac{0,4 \times 1,05}{1 - 0,4 \sin 5°} = 0,5648 \approx 0,56,$$

$$Y_1 = Y_2 = \frac{0,4 \cot\alpha'}{1 - 0,4 \sin 5°} = 0,41445 \cot\alpha',$$

$$e = 1,05 \tan\alpha'.$$

The calculated Y_1 value of 0,9641 ≈ 0,96 for Fa/iZD_w^2 = 6,89 N/mm^2 is adjusted to 1,00 in consideration of the relationship with the value of Y_1 for angular contact groove type with $\alpha \geq 20°$ (see figure 4-6): namely, the calculated contact angle α' of 23,263° is adjusted to 22,512° ($\alpha' = \tan^{-1}$ 0,41445). Therefore, the calculated e value of 0,45142 ≈ 0,45 becomes 0,4352 ≈ 0,44 [e = 1,05 tan 22,512° or 0,4 x 1,05/(1-0,4 sin5°)].

b) Angular contact groove ball bearings with $\alpha \leq 15°$

For single row bearings with α = 5°, the values of X_1, Y_1 and e are same as those for the radial contact groove type above.

For double row bearings with α = 5°

X_2 = 1,625 x 0,48 = 0,78, because

$$X_1 = 1 - \frac{0,4 \times 1,25}{1-0,4\ \sin 5°} = 0,4819 \approx 0,48,$$

$$Y_2 = 1,625 Y_1, \text{ where } Y_1 = \frac{0,4\ \cot\alpha'}{1-0,4\ \sin 5°} = 0,41445\ \cot\alpha',$$

$$Y_3 = \frac{0,625}{1,25}\cot\alpha' = 0,5\ \cot\alpha', \quad e = 1,25\ \tan\alpha'.$$

For Fa/ZD_w^2 = 6,89 N/mm^2, the contact angle α' of 22,512° is used. Therefore, Y_2 = 1,625 x 1 = 1,625 ≈ 1,63, Y_3 = 0,5 x cot 22,512° or 1,5625 (1 - 0,4 sin5°) = 1,2064 ≈ 1,21 and e = 1,25 tan 22,512° or (0.4 x 1,25)/(1 - 0,4 sin5°) = 0,5181 ≈ 0,52.

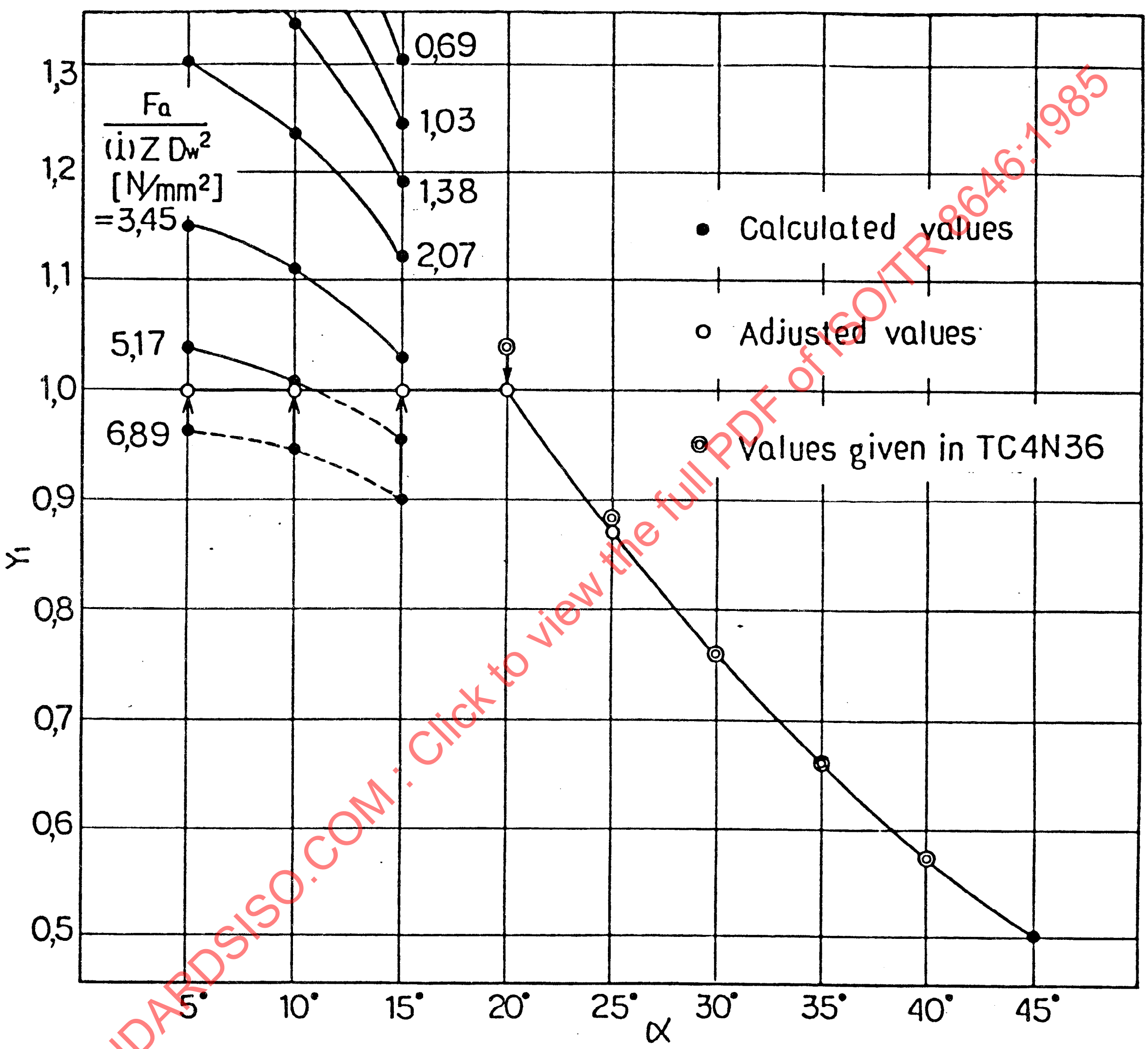


FIGURE 4-6 — Adjustment of Y_1 values for radial and angular contact groove ball bearings

For bearings with α = 10° and 15°,

$$X_1 = 1 - \frac{0,4 \times 1,25}{1 - 0,4 \sin\alpha}, \qquad X_2 = 1,625\ X_1,$$

$$Y_1 = \frac{0,4 \cot\alpha'}{1 - 0,4 \sin\alpha}, \qquad Y_2 = 1,625\ Y_1,$$

$$Y_3 = \frac{0,625}{1,25} \cot\alpha' = 0,5 \cot\alpha', \qquad e = 1,25 \tan\alpha'.$$

Namely, for α = 10°, X_1 = 0,4627 ≈ 0,46, Y_1 = 0,42986 x cotα', and for α = 15°, X_1 = 0,4423 ≈ 0,44, Y_1 = 0,44619 x cotα'.

For the above-stated reason, in the case of the calculated value of Y_1 being less than 1, we must take Y_1 to be Y_1 = 1,00 (see figure 4-6). Therefore, we have Y_2 = 1,625 ≈ 1,63, Y_3 = 0,5 cot 23,261° or 1,25 (1−0,4 sin10°) = 1,1632 ≈ 1,16 and e = 1,25 tan 23,261° or (0,4 x 1,25)/(1-0,4 sin10°) = 0,5373 ≈ 0,54 for Fa/ZD_w^2 = 6,89 N/mm², and also Y_2 = 1,625 x 1 = 1,625 ≈ 1,63, Y_3 = 0,5 cot24,046° or 1,25 (1-0,4 sin15°) = 1,1206 ≈ 1,12 and e = 1,25 x tan24,046° or (0,4 x 1,25)/(1-0,4 sin15°) = 0,5577 ≈ 0,56 for Fa/ZD_w^2 = 5,17 and 6.89 N/mm².

c) Angular contact groove ball bearings with α=20° to 45°

$$X_1 = 1 - \frac{0,4 \times 1,25}{1 - \frac{1}{2,75}\sin\alpha}, \quad \text{(see table 4-5)}$$

$$X_2 = 1,625\ X_1.$$

TABLE 4-5 — Values of X_1 for bearings with α = 20° to 45°

α	X_1	α	X_1
20°	0,4290 ≈ 0,43	35°	0,3682 ≈ 0,37
25°	0,4092 ≈ 0,41	40°	0,3475 ≈ 0,35
30°	0,3889 ≈ 0,39	45°	0,3269 ≈ 0,33

For values of Y_1, in principle we use the following values given in document ISO/TC4 N36 (=TC4N56 = TC4N110) (see NOTE),

α	20°	25°	30°	35°	40°	(45°)
Y_1	1,04	0,89	0,76	0,66	0,57	(0,50)

where the first and second values of 1,04 and 0,89 are adjusted to 1,00 and 0,87 respectively in consideration of the relationship with the values of Y_1 for $\alpha \leqslant 15°$ (see figure 4-6).

And then values of Y_2, e and Y_3 are calculated from the following equations which are obtained from equation (4-25):

$$Y_2 = 1{,}625\,Y_1, \quad e = \frac{1 - X_1}{Y_1}, \quad Y_3 = \frac{0{,}625}{e} \quad \ldots\ldots\ldots \ (4\text{-}32)$$

NOTE — The values of Y_1 are obtained from the following equation.

$$Y_1 = \frac{0{,}4}{\eta} \cot\alpha' = \frac{0{,}4}{1 - \frac{1}{3}\sin\alpha'} \cot\alpha', \quad \ldots\ldots\ldots\ldots \ (4\text{-}33)$$

where the values of contact angle α' are determined by the equation

$$\cos\alpha' = \cos\alpha \times 0{,}972402,$$

as follows:

α =	20°	25°	30°	35°	40°	(45°)
α' =	23,97°	28,20°	32,63°	37,20°	41,85°	(46,56°)

The above equation is obtained from equation (4-18) for $2r/D_w = ri/D_w + r_e/D_w = 0{,}5175 + 0{,}53 = 1{,}0475$, $c = 0{,}00045835$ and the specific rolling body load $Fa/ZD_w^2 \sin\alpha' = 4{,}9033$ N/mm² (=0,5 Kgf/mm²).

Moreover, for bearings with $\alpha = 45°$, which were not included in ISO/R281, Y_1 value of $0{,}4986 \approx 0.50$ is determined from the same equation (4-33), and is specified in ISO281/I together with the values of remaining factors Y_2, Y_3 and e, which are obtained from the equations (4-32).

d) Self-aligning ball bearings

Taking $\alpha' = \alpha$, $\eta = 1$ and $\xi = 1{,}5$,

$$X_1 = 1-0{,}4 \times 1{,}5 = 0{,}4,\quad X_2 = 1{,}625 \times 0{,}4 = 0{,}65,$$

$$Y_1 = 0{,}4 \cot\alpha,\quad Y_2 = 1{,}625\ Y_1 = 0{,}65 \cot\alpha,$$

$$Y_3 = \frac{0{,}625}{1{,}5}\cot\alpha = 0{,}4167 \cot\alpha \approx 0{,}42 \cot\alpha,\quad e = 1{,}5 \tan\alpha.$$

4.2.3 Summarized table of factors X, Y and e for radial ball bearings

Table 4-6 shows the summary for the basic formulae for calculating the factors X, Y and e and α', ξ and η values for each type of radial ball bearing.

TABLE 4-6 — Summarized table of factors X, Y and e for radial ball bearings

Bearing type		Single row bearings $F_a/F_r > e$ X_1	Single row bearings $F_a/F_r > e$ Y_1	Double row bearings $F_a/F_r \leqslant e$ X_3	Double row bearings $F_a/F_r \leqslant e$ Y_3	Double row bearings $F_a/F_r > e$ X_2	Double row bearings $F_a/F_r > e$ Y_2	e	α'	ξ	η
Radial contact groove bearings		$1-\frac{0,4\xi}{\eta}$	$\frac{0,4}{\eta}\cot\alpha'$ $\geqslant 1.00$	1	0	$1-\frac{0,4\xi}{\eta}$	$\frac{0,4}{\eta}\cot\alpha'$ $\geqslant 1.00$	$\xi\tan\alpha'$ $\leqslant 0,4\frac{\xi}{\eta}$	Determined from formula (1)*	1,05	$1-\frac{\sin 5°}{2,5}$
Angular contact groove bearings	α =5°				$\frac{0,625}{\xi}\cot\alpha'$ $\geqslant 1,5625\frac{\eta}{\xi}$	$1,625X_1$	$1,625Y_1$		Determined from formula (2)**	1,05 (for single row) 1,25 (for double row)	
	10°									1,25	$1-\frac{\sin\alpha}{2,5}$
	15°										
	20°		1,00***		$\frac{0,625}{e}$			$\frac{1-X_1}{Y_1}$	—		$1-\frac{\sin\alpha}{2,75}$
	25°		0,87***								
	30°		0,76***								
	35°		0,66***								
	40°		0,57***								
	45°		0,50***								
Self-aligning bearings			$\frac{0,4}{\eta}\cot\alpha'$		$\frac{0,625}{\xi}\cot\alpha'$			$\xi\tan\alpha'$	α	1,5	1

* Formula (1) $\cos 5°/\cos\alpha' = 1+0,012534\ (F_a/iZD_w^2 \sin\alpha')^{2/3}$

** Formula (2) $\cos\alpha/\cos\alpha' = 1+0,012534\ (F_a/iZD_w^2 \sin\alpha')^{2/3}$

*** Determined by the formula $Y_1 = \frac{0,4\cot\alpha'}{1-\frac{1}{3}\sin\alpha'}$, where α' is determined from the formula $\cos\alpha' = \cos\alpha \times 0,9724$.

The Y_1-values 1,04 and 0,89 obtained for α = 20° and 25° are adjusted to 1,00 and 0,87, respectively, to fit the data for α < 20° smoothly.

4.2.4 Calculated values Y and e different from Standard

Table 4-7 shows the Y and e values, which are calculated by contact angle α' given in table 4-4 and differ from the values given in table 2 of ISO 281/I. The maximum discrepancy is within ±0,02.

This slight difference could be explained by following reasons. The values of factors Y and e are related to contact angle α'. However, α' cannot be calculated directly for the given values of F_a/ZD_w^2 (or F_a/iZD_w^2) from equations (4-18) and (4-31).

Therefore, the discrepancies are thought to be due to the inaccuracy of the calculated value of contact angles α'.

TABLE 4-7 — Calculated values different from Standard (α ⩽ 15°)

F_a/ZD_w^2* [N/mm²]			0,172	0,345	0,689	1,03	1,38	2,07	3,45	5,17	6,89
Radial contact groove bearings		Y_1		1,98	1,70	1,54	1,44				**
Radial contact groove bearings		e						0,33			**
Angular contact groove bearings	α = 5°, double row bearings	Y_2		3,22	2,76	2,50	2,34				**
Angular contact groove bearings	α = 5°, double row bearings	Y_3	2,77	2,39	2,05	1,86	1,74			1,25	**
Angular contact groove bearings	α = 5°, double row bearings	e			0,31						**
Angular contact groove bearings	α = 10°	Y_1	1,87	1,70		1,42		1,24	1,11		**
Angular contact groove bearings	α = 10°	Y_2	3,04	2,76		2,31		2,02	1,80		**
Angular contact groove bearings	α = 10°	Y_3	2,17		1,77	1,65	1,56	1,44	1,29	1,18	**
Angular contact groove bearings	α = 10°	e			0,35			0,43	0,48	0,53	**
Angular contact groove bearings	α = 15°	Y_1	1,48		1,31	1,24			1,03	**	**
Angular contact groove bearings	α = 15°	Y_2	2,41		2,13	2,02			1,67	**	**
Angular contact groove bearings	α = 15°	Y_3	1,66			1,39			1,15	**	**
Angular contact groove bearings	α = 15°	e				0,45			0,54	**	**

* For radial contact groove bearings F_a/iZD_w^2

** Adjusted values