
**Guide for the selection of an acceptance
sampling system, scheme or plan for
inspection of discrete items in lots**

*Guide pour la sélection d'un système d'échantillonnage pour acceptation,
d'un schéma ou d'un plan pour le contrôle d'individus discrets dans un lot*



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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

- type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;
- type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;
- type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 8550, which is a Technical Report of type 3, was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

The primary purpose of this Technical Report is to give guidance in the selection of an acceptance sampling system, scheme or plan. It does this principally in the context of existing or draft ISO Standards. It reviews the available systems and shows ways in which they can be compared in order to assess their suitability for an intended application. The guide also indicates how prior knowledge of the manufacturing or service delivery process and quality performance could influence the choice of the sampling system, scheme or plan, and likewise how the particular needs of the customer affect the selection. Some specific circumstances encountered in practice are described and the method of choosing a plan is explained. Some check lists or pointers and tables are provided to assist users in selecting an appropriate system, scheme or plan for their purpose. Two charts are included to illustrate the procedures followed in the selection process.

Introduction

For the inspection of discrete items submitted in lots this International Technical Report gives guidance in the selection of an appropriate acceptance sampling scheme from those described in the relevant ISO standards.

There are many situations where products (materials, parts, components, assemblies and systems) are transferred from one organization to another, where the organizations may be different companies or parts of a single company or even different shops within a plant. In these situations both the supplier and the customer may use acceptance sampling procedures to satisfy themselves that the product is of acceptable quality. The supplier will be seeking to maintain a reputation for good quality and to reduce the likelihood of claims under warranty, but without incurring unnecessary production and supply costs. On the other hand, the customer will require adequate evidence, at minimum cost to himself, that the product he receives conforms to specification. Compared with, say, 100% inspection, suitable sampling methods will often be beneficial in achieving these aims. Sometimes acceptance sampling methods may be the only practical procedure, especially when the tests for conformance are destructive.

Several types of sampling systems, schemes and plans are available for these purposes. They are presented in a number of ISO Standards which explain how they are to be used. However, it is often difficult to decide on the most appropriate procedure for use in a particular situation. The purpose of this Technical Report is to assist in that decision.

The choice of which sampling system, scheme or plan to use depends on a number of conditions and the circumstances prevailing. In any supply situation the first essential is that the supplier and the customer understand, and have agreed, the requirements and the basis for release and acceptance of the product, including any acceptance sampling methods to be used.

The parties should agree on the following:

- (a) the specification to which the discrete items of product are to conform; this is necessary because in all dealings between the parties there has to be agreement on what constitutes a conforming item and what constitutes a nonconforming item;
- (b) whether the acceptance of the product is to be determined by the acceptance of individual items or collectively by the acceptance of inspection lots of items. Acceptance of individual items precludes sampling.

When the acceptance is to be on a lot basis, the agreement between supplier and recipient needs to include not only the criteria for item conformance but also the criteria for lot acceptance, the criteria for non-acceptance of the lot and the acceptance sampling system, scheme or plan to be used. The latter should be based on risk factors that are mutually acceptable between producer and customer.

Having agreed on the acceptance sampling system, scheme or plan to be used, the supplier knows, at various quality levels, the probability that his supply lots will be accepted. Likewise the customer understands the protection that the sampling system, scheme or plan gives him to prevent acceptance of poor quality product.

Lots that are not acceptable cause difficulties for both supplier and customer. The supplier incurs additional costs in rework, scrap, increased inspection, damage to reputation and he may suffer loss of sales. Delays in delivery and reinspection costs are a burden to the customer. For these reasons it is usually considered essential for the supplier to provide lots that have a very high probability of being accepted - 95% or more. The supplier has to ensure that quality control of the production or delivery process provides lots of a quality sufficient to meet this objective. A basic principle of some acceptance sampling inspection schemes is to promote the *production* of lots of acceptable quality. The primary purpose in these schemes is not to discriminate between acceptable and non-acceptable lots, i.e. to sort, but to keep production under control to yield an acceptable process average quality. Although all acceptance sampling plans are discriminatory to some degree, the process average quality (expressed in terms of percent nonconforming or number of nonconformities) should not be greater than half the acceptable quality level in order to ensure a very high probability of acceptance.

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Guide for the selection of an acceptance sampling system, scheme or plan for inspection of discrete items in lots

1 Scope

The primary purpose of this Technical Report is to give guidance in the selection of an acceptance sampling system, scheme or plan. It does this principally in the context of existing ISO standards.

The guidance in this Technical Report is confined to acceptance sampling of products that are supplied in lots and that can be classified as consisting of discrete items (discrete articles of product). Each item in a lot can be identified and segregated from the other items in the lot and has an equal chance of being included in the sample. Each item of product is countable and has specific characteristics that are measurable or classifiable as being conforming or nonconforming (to a given specification).

The ISO Standards on acceptance sampling, and hence this Technical Report, are applicable to a wide variety of inspection situations. These include, but are not limited to, the following:

- (a) end items, such as complete products or sub-assemblies;
- (b) components and raw materials;
- (c) services;
- (d) materials in process;
- (e) supplies in storage;
- (f) maintenance operations;
- (g) data or records;
- (h) administration procedures.

Although the Technical Report is written principally in terms of manufacture and production, this should be interpreted liberally as it is applicable to the selection of sampling systems, schemes and plans for all types of product and processes as defined in ISO 8402.

2 References

- ISO 2854:1976 *Statistical interpretation of data - Techniques of estimation and tests relating to means and variances*
- ISO 2859-0¹⁾ *Sampling procedures for inspection by attributes*
Part 0: *Introduction to the ISO 2859 attribute sampling system*
- ISO 2859-1:1989 *Sampling procedures for inspection by attributes*
Part 1: *Sampling plans indexed by acceptable quality level (AQL) for lot-by-lot inspection*
- ISO 2859-2:1985 *Sampling procedures for inspection by attributes*
Part 2: *Sampling plans indexed by limiting quality (LQ) for isolated lot inspection*
- ISO 2859-3:1991 *Sampling procedures for inspection by attributes*
Part 3: *Skip-lot sampling procedures*
- ISO 3534-1:1993 *Statistics - Vocabulary and symbols*
Part 1: *Probability and general statistical terms*
- ISO 3534-2:1993 *Statistics - Vocabulary and symbols*
Part 2: *Statistical quality control*
- ISO 3951:1989 *Sampling procedures and charts for inspection by variables for percent nonconforming*
- ISO 8402:1994²⁾ *Quality management and quality assurance - Vocabulary*
- ISO 8422:1991 *Sequential sampling plans for inspection by attributes*
- ISO 8423:1991 *Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)*

¹⁾ In preparation

²⁾ Publication imminent

3 The selection process

This Technical Report reviews the salient features of a number of sampling systems, schemes and plans and the specific applications for which they were designed. The task of selecting a suitable system, scheme or plan is influenced by production and marketing conditions. In addition, the economics of the sampling system, the resources of the inspection organization and other aspects need to be considered. Therefore, the selection process becomes complex and rarely is there one method of acceptance sampling that fits all situations even though they may appear to be similar.

In this Technical Report, tables 3, 4A and 4B, together with figures 8 and 9, illustrate the process for selecting a sampling system, scheme or plan. The tables provide 'candidate' sampling systems, schemes and plans to fit given inspection situations, production conditions and market conditions, respectively. The method suggested is to review tables 3, 4A and 4B and to select as many 'candidate plans' as fit the situation. These candidates should then be reviewed through figure 8 or 9 so that the user finally arrives at a system, scheme or plan that is most feasible and economic for the situation.

The initial selection decisions are for long and short runs, isolated lots, attribute or variables sampling; these decisions will lead to one or more possible ISO standards.

Figures 8 and 9 are not intended as flow charts. These figures show, in summary form, the procedures contained within each ISO Standard for arriving at a sampling system, scheme or plan. The summaries are listed side by side, which allows direct comparison between them. The solid connecting lines indicate the primary course to be followed in choosing and operating a standard, the broken lines indicate alternatives. These alternatives are applicable only under certain conditions. Text references and notes to the figures are given for guidance in following these procedures. The standards and procedures given below the horizontal line in each of the two figures are particularly for use when economy of average sample size is an important consideration.

The procedure presented in figure 8 is followed when production is continuous and there are more than 10 lots of product undergoing inspection. ISO 2859-1, ISO 8422, ISO 3951 and ISO 8423 are included here as potential 'candidate' systems.

Figure 9 is to be used when other conditions prevail, e.g. when there is no continuity of production, when the number of lots is 10 or fewer, when the warranty does not survive acceptance and/or when the presence of a small number of nonconforming items can result in a large loss.

A final selection should be based on both the requirements of the situation and the resources of the inspection organization.

In the process of selecting a system, scheme or plan, it is possible to find that the selection has indicated a system, scheme or plan that has one or more insurmountable deficiencies. This is a signal that the process needs to be repeated. The cases in which there is only one appropriate method are few. Iterative investigations will usually identify two or more methods. The most economic and appropriate one should be chosen.

4 Acceptance sampling systems, schemes and plans

An acceptance *sampling plan* is a set of rules by which a lot is to be inspected and 'sentenced'. The plan stipulates the number of items (units) in the sample, to be drawn randomly from a lot for inspection against the product specification. The lot is then sentenced as 'acceptable' or 'not acceptable' according to how the inspection results compare with the criteria of the acceptance sampling plan.

Sometimes, when a long series of lots is being inspected, a sampling procedure may call for a shift from one sampling plan to another, depending on the current and previous sample results. Sampling procedures that call for switching from one sampling plan to another, and possibly back again, are called *sampling schemes*. A sampling scheme may also call for discontinuation of inspection if product quality appears to remain poor. The customer may then shift to another supplier, if available, or initiate 100% screening until the supplier can upgrade the production process so as to produce acceptable product.

In the case of destructive testing, the customer may cease to accept product until the supplier has demonstrated that the production problems that were giving rise to the previous low quality have been overcome.

A collection of sampling plans and related sampling schemes constitute a *sampling system*. The system will be indexed in some way, e.g. by lot size, inspection level and acceptable quality level (cf. ISO 2859-1).

The current ISO Standards present plans for single, double, multiple or sequential sampling. Procedures for skip-lot sampling for inspection by attributes are given in ISO 2859-3. A comparison of the various sampling methods and the principles on which they are based will assist in assessing their suitability for a particular application and will enable an appropriate selection to be made.

5 ISO Standards for 'acceptance sampling' of lots

5.1 General

This clause presents in summary the salient features of each of the current ISO Standards concerned with acceptance sampling methods. The summaries of the scope and application of the available ISO Standards should enable a user to select those Standards which are most likely to suit a given purpose.

The comparisons between the various ISO Standard acceptance sampling systems in these summaries are not enough to allow a final selection of a system, scheme or plan to be made in a particular situation. Before this can be done a number of factors need to be understood and considered. These factors are reviewed in 6 and 7.

5.2 Sampling for inspection by attributes

Within this category the following ISO Standards are relevant:

- (a) ISO 2859-0 *Sampling procedures for inspection by attributes*
Part 0: *Introduction to the ISO 2859 attribute sampling system*

This is a companion document to this Technical Report and can be read in conjunction with it, if desired, but this is not essential. It is not a source of sampling schemes or plans.

ISO 2859-0 consists of two sections. Section 1 *General introduction to acceptance sampling* is essentially an introduction to the sampling schemes employed in ISO 2859 and ISO 8422 but it treats the subject in a general way. It contains explanations of terms, gives practical advice on sampling inspection and discusses some underlying concepts. Section 2 *The ISO 2859-1 system* extends Section 1 and amplifies the introductory text and instructions contained in ISO 2859-1, by giving detailed comments and examples to assist in using the procedures and tables that make up the ISO 2859-1 system.

- (b) ISO 2859-1 *Sampling procedures for inspection by attributes*
 Part 1: Sampling plans indexed by acceptable quality
 level (AQL) for lot-by-lot inspection

This Part of ISO 2859 presents a sampling system indexed by lot-size ranges, inspection levels and AQLs and specifies sampling plans and procedures for inspection by attributes of discrete items. It contains sampling plans for single, double and multiple sampling indexed by percent nonconforming and nonconformities per 100 items.

ISO 2859-1 is intended to be used as a system employing tightened, normal and reduced inspection on a continuing series of lots to achieve customer protection while assuring the producer that, if quality is better than the AQL, acceptance will occur most of the time.

The objective in ISO 2859-1 is to induce a supplier, through the economic and psychological pressure of potential non-acceptance, to maintain a process average quality at least as good as the specified AQL, while at the same time providing an upper limit for the risk to the consumer of accepting the occasional lot of poor quality.

The continuing series of lots should be of sufficient duration to allow the switching rules to be applied. These rules provide for the following:

- an automatic protection to the customer (by means of a switch to tightened inspection or to discontinuation of inspection) in the event that an apparent deterioration in quality is detected;
- an incentive to reduce inspection costs (by means of a switch to reduced inspection - at the discretion of a Responsible Authority) if consistently good quality is being achieved.

The plans in ISO 2859-1 may also be used for the inspection of lots in isolation, but in this case the user is strongly advised to consult the operating characteristic curves to find a plan which will yield the desired protection. A much simpler procedure to follow in this type of situation is presented in ISO 2859-2.

- (c) ISO 2859-2 *Sampling procedures for inspection by attributes*
Part 2: *Sampling plans indexed by limiting quality (LQ)*
for isolated lot inspection

This Part of ISO 2859 establishes sampling plans indexed by limiting quality and procedures that can be used when the switching rules of ISO 2859-1 cannot be applied. The LQ is used to indicate the customer protection. The plans are primarily intended for use with single lots (procedure A), or lots isolated from a series (procedure B) where the switching rules are precluded. Both procedures treat the limiting quality as an indicator of the actual percentage nonconforming in the lots submitted. They can also be used to cover cases where quality is expressed in non-conformities per 100 items.

The two procedures provide for situations often met in practice. Procedure A is used when both the supplier and the customer wish to regard the lot in isolation and it is also used as the default procedure (i.e. it is used unless there is a specific instruction to use procedure B).

Procedure B is used when the supplier regards the lot as one of a continuing series, but the customer considers the lot received in isolation. The plans employed permit a producer to maintain consistent procedures for his customers, irrespective of whether the customers receive individual lots or a continuing series of lots. The manufacturer is concerned with all of the production but the individual customer only with the particular lot received.

For procedure A plans are identified by their lot size and LQ; for procedure B they are identified by lot size, LQ and inspection level.

Procedure A includes plans with an acceptance number of zero, whereas procedure B does not. Double and multiple sampling plans can be used as alternatives to single sampling plans in procedure B and for the non-zero acceptance number plans in procedure A.

- (d) ISO 2859-3 *Sampling procedures for inspection by attributes*
Part 3: *Skip-lot sampling procedures*

This Part of ISO 2859 presents a sampling system that extends the procedures contained in ISO 2859-1. It provides a procedure for reducing the inspection effort on products submitted by suppliers who have demonstrated their ability to control, in an effective manner, all facets of product quality and to produce superior quality material consistently. However, this procedure is proscribed for the inspection of product characteristics that involve the safety of personnel.

The skip-lot programme uses the acceptance sampling plans described in ISO 2859-1 and is intended only for a continuing series of lots. It is totally inappropriate for isolated lots. All lots in the series are expected to be of a similar quality and the customer should have no reason to believe that lots not inspected are of a poorer quality than the ones inspected.

In a skip-lot sampling procedure some lots in a series are accepted without inspection when the sampling results for a stated number of immediately preceding lots meet stated criteria. The lots to be inspected are chosen randomly with a stated frequency, called the 'skip-lot frequency'.

(e) ISO 8422 *Sequential sampling plans for inspection by attributes*

This Standard presents a sampling system that provides a wide range of sequential sampling plans indexed in terms of the consumer's risk point (CRP) and the producer's risk point (PRP). It also contains a sequential sampling system indexed by lot size ranges, inspection levels and AQLs to supplement the system in ISO 2859-1, including switching rules. (For the relationship between AQL, LQ, CRP and PRP see figures 1 and 2, page 23.)

In sequential sampling the sample is formed by taking items randomly one after another until a decision point is reached. The decision that 'sentences' the lot acceptable or not acceptable can occur at almost any stage and for sequential sampling by attributes depends on the number of items inspected and the cumulative number of nonconforming items or nonconformities found up to that point.

This Standard provides procedures based on a sequential assessment of inspection results that may be used to induce the supplier - through the economic and psychological pressure of non-acceptance of lots of inferior quality - to supply lots of a quality with a high probability of acceptance while maintaining an upper limit for the risk to the consumer of accepting lots of poor quality.

The plans are intended primarily for use in inspection of a continuing series of lots from the same production run. Subject to certain provisions the plans may also be used for the inspection of lots in isolation.

In terms of the average number of items inspected per lot, this Standard offers plans that are more economic than those in ISO 2859-1, albeit at the expense of an increase in administrative complication.

5.3 Sampling for inspection by variables

In sampling for inspection by variables, the product characteristic of each item in the sample is measured. The criterion for lot acceptance is based on an assessment of the percentage nonconforming determined from the average and variability of the measurements. Within this category the following ISO Standards are relevant:

(a) ISO 3951 *Sampling procedures and charts for inspection by variables for percent nonconforming*

This Standard presents a sampling system indexed by lot size ranges, inspection levels and AQLs and is complementary to ISO 2859-1. The two Standards share a common philosophy and purpose. ISO 3951 is intended primarily for the inspection of a continuing series of lots from one source of sufficient duration to allow the switching rules to operate, but, like ISO 2859-1, it can also be used for lots of an isolated nature.

It is only applicable where a single product characteristic, measurable on a continuous scale, is considered. The product characteristic should be distributed according to a normal distribution or to a distribution closely approximating normality ¹⁾. A lot is judged as unacceptable when the

¹⁾ Often a simple mathematical transformation, such as taking the logarithm or square root, will convert a set of measurements from a non-normal to a normal (or near normal) distribution.

distribution of the product characteristic fails to indicate an average and variability which meets the sampling criteria for the single or double specification limits prescribed. A choice is available between numerical and graphical acceptance criteria. Procedures are given for the case where the process standard deviation is known and also for the case where it is unknown. Guidance is given on how these procedures can be used in combination with sampling for inspection by attributes, the most important suitable product characteristic being sampled by variables.

ISO 3951 is intended primarily for the inspection of a continuing series of lots from one source of sufficient duration to allow the switching rules to operate, but, like ISO 2859-1, it can also be used for lots of an isolated nature. However, inspection carried out on an isolated lot will provide little evidence about the normality of the distribution of the product characteristic and about the standard deviation of the process. In practice, therefore, ISO 3951 does not apply to the inspection of isolated lots.

There are no double or multiple sampling plans in the current edition (1989) of this ISO Standard.

(b) ISO 8423 *Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)*

This Standard presents a sampling system providing a wide range of sequential sampling plans indexed in terms of the consumer's risk point (CRP) and the producer's risk point (PRP). It also contains a sequential sampling system indexed by lot size ranges, inspection levels and AQLs to supplement the system in ISO 3951, including switching rules. Like ISO 3951, ISO 8423 requires that the product characteristic should be distributed normally (see footnote on p.11).

The procedures in ISO 8423 are based on a sequential assessment of inspection results and may be used to induce a supplier - through the economic and psychological pressure of non-acceptance of lots of inferior quality - to supply lots of a quality with a high probability of acceptance while maintaining an upper limit for the risk to a consumer of accepting lots of poor quality.

The plans are intended primarily for use in the inspection of a continuing series of lots from the same production run. In theory, the Standard could be applied to an isolated lot. However, inspection carried out on an isolated lot will provide little evidence about the normality of the distribution of the product characteristic and about the standard deviation of the process. Therefore, in practice, the Standard does not apply to the inspection of isolated lots.

5.4 Advantages of specifying ISO standard sampling plans

To those concerned with the writing of specifications it is of benefit that statistically sound sampling procedures be provided. Because of the prime need to ensure that a sample is representative, most sampling schemes presented in the ISO Standards reviewed above relate sample size to lot size. Apart from providing control over the methods of selection of the sample, these Standards should normally be invoked because they provide clauses that control the treatment of nonconformities found during inspection and the treatment of lots resubmitted after initial non-acceptance. Furthermore, the AQL indexed systems contain built-in switching rules (from 'normal' to 'tightened' or to 'reduced' inspection) to adjust the sampling plan in the event of deterioration or improvement in quality. Use of these basic reference standards can save much time often wasted in subjective discussion, and reduce the large areas of

discretion often contained in non-standard sampling schemes that have only limited value, particularly for international trade.

More notes on the practical and economic advantages of using the above Standards are given in 6.1.

6 Some general considerations influencing a selection

6.1 Practical and economic advantages of using the ISO standard sampling plans

Sampling involves risk and quite naturally all parties concerned will attempt to minimize their share. Theoretically these risks are functions of the sampling plan and the quality level specified, without relation to the industry or the product. In practice these risks can be reduced by controlling the production process and improving the level of quality.

These risks cannot be eliminated completely, but they can be precisely calculated and economically assessed by the use of modern statistical techniques. Consequently, it is of benefit to all parties that statistically sound acceptance criteria be specified in product/process specifications and that, wherever possible, the generally applicable basic reference Standards on sampling, such as ISO 2859 and ISO 3951, be utilized.

Generally speaking, in arriving at the optimum performance of an acceptance sampling plan or scheme, prevention costs should be balanced against the probabilities of failure in service. Providing that various assumptions can be made with regard to the sample size to lot size ratio (n/N) and to the appropriate distribution theory, it is a relatively straightforward matter to formulate sampling plans from statistical theory. It should be noted that, while ISO 3951 is only applicable to a product characteristic that has a normal distribution, ISO 2859 is not dependent on the distributional shape of the product characteristic.

It is a more difficult matter to establish practical sampling schemes or Standards that take account of the many and varied situations met in practice to such an extent that they are likely to be adopted for general use by industry world-wide, as is the case with the established AQL indexed procedures given in ISO 2859 and ISO 3951. There are undeniable advantages in having relatively few standard schemes, as this leads to greater uniformity of action and simplifies the administrative procedures across organizational and national boundaries.

The motivation for acceptance sampling is primarily economic: inspection of a sample from a lot is the (usually small) price paid to achieve desirable quality in the accepted lots. This quality is achieved by two pressures, i.e. the purely statistical pressure of different probabilities of acceptance of good and bad quality lots and, secondly, when sequences of lots are purchased, the commercial pressure of frequent non-acceptance of lots and the switch to tightened inspection when quality is poor.

The problem associated with acceptance sampling inspection relates to defining unambiguously the criteria used to judge discrete individual items supplied in quantity, the criterion for acceptance of the lot, the quality level expected from the manufacturing process, the discrimination afforded by the plans and the rules to be followed when a lot is not accepted. Above all, however, it is necessary to design the sampling scheme so that it may be invoked easily in a purchasing contract. The plans in the set of related ISO Standards discussed in 5 enable this to be done efficiently.

6.2 Long and short production runs

ISO 2859-1, ISO 2859-3, ISO 8422, ISO 3951 and ISO 8423 are all intended primarily to be used for a continuing series of lots of sufficient duration to allow the switching rules to be applied. This implies a 'long' production run.

The Limiting Quality (LQ) plans of ISO 2859-2 can be used where the switching rules of ISO 2859-1 are not applicable. They are primarily intended for use with single lots or lots of an 'isolated nature'. By implication this embraces a 'short' series of inspection lots - or a 'short' production run.

ISO 8422 and ISO 8423 can provide sequential plans which match the other Standards and in many cases are thus similarly applicable to long or short runs.

In order for a production run to qualify as 'long', one criterion is clearly that the switching rules have a reasonable chance of coming into effect if "the quality is unsatisfactory". It is equally clear that this alone raises a number of supplementary issues (as indicated by the quotation marks) depending on the requirements and circumstances prevailing in each case considered. It is impossible to stipulate simply and precisely what constitutes a short run (number of lots) in the context of sampling inspection.

In the absence of any other guide, or evidence, on which to base a judgement, anything up to 10 consecutive inspection lots should be considered as a 'short run' and the plans in ISO 2859-2 should be used. However, lots should not be subdivided arbitrarily in order to create a 'long run' artificially. To do so simply introduces other disadvantages. It is always preferable to have large homogeneous lots because they allow a smaller sample size to lot size ratio, and provide better representation by the sample, sharper discrimination and more economical inspection.

In a long production run there will be continuity and stability, so production will settle down to a long term stable process average. Nevertheless, the quality of individual lots will vary about this process average. On the other hand, at the start of production, after a significant break or change in production, or for a short production run, the lot quality may well be somewhat different and more variable, even markedly so. The practical factor to consider is whether there is evidence that a stable process average has been established and exists.

6.3 Nonconformity and nonconforming item

6.3.1 Failure to conform

For the purpose of ISO 2859 and ISO 8422, any failure to conform with a specified characteristic, dimension, attribute or performance requirement represents a nonconformity. A nonconforming item may have one or more nonconformities.

For example, suppose that a ball point pen fails to write. The failure to write is a nonconformity; the pen is nonconforming. The same pen could also have failed to meet its specification in a number of other ways, e.g. colour, dimensions, etc. Though it exhibited several nonconformities it would be counted as one nonconforming item.

The qualification 'nonconformity' does not necessarily imply that the unit of product cannot be used for the purpose intended. For example, a brick with one of its dimensions outside the prescribed tolerance interval, though nonconforming, can still be used for building.

The distinction between nonconformity and nonconforming item is of no importance if the items have no more than one nonconformity, but becomes essential when multiple nonconformities can occur.

The quality of a given quantity of product may be expressed either as percent nonconforming or as the number of nonconformities per hundred items, but these are not usually interchangeable.

Sampling plans are available for either percent nonconforming or the number of nonconformities per hundred items.

Example 1: In counting pinholes in metal foil the number of pinholes per square metre may be of interest. Here we would count all the pin holes in each square metre (item) examined and express the quality in pinholes/100 square metres.

Example 2: Suppose there is a lot of 500 articles. Of these, 480 conform and are acceptable, 15 have one nonconformity each, 4 have two nonconformities each, and 1 has three nonconformities.

The lot percent nonconforming is given by the formula:

$$\begin{aligned} \text{Percent nonconforming} &= \frac{\text{number of nonconforming items}}{\text{total number of items}} \times 100 \\ &= (20/500) \times 100 = 4 \end{aligned}$$

That is, the lot is 4% nonconforming.

The number of nonconformities per hundred items in the lot is given by the formula:

$$\begin{aligned} \text{Nonconformities per 100 items} &= \frac{\text{number of nonconformities}}{\text{total number of items}} \times 100 \\ &= (26/500) \times 100 = 5,2 \end{aligned}$$

That is, the lot has 5,2 nonconformities per hundred items.

Whether percent nonconforming or nonconformities per hundred items is to be used is a matter for individual consideration in each particular case. The important thing is that it has to be considered, specified, and agreed beforehand - not left until a sample has been inspected and then considered.

Factors to be taken into account in deciding whether to use percent nonconforming or nonconformities per hundred items are as follows.

a) Inspection for percent nonconforming assumes that if an item contains one or more nonconformities, the item is nonconforming and is not acceptable.

It also presupposes that the number of different ways in which an item can be nonconforming is limited and known, e.g. there are only 5 ways in which each particular item could be nonconforming (see also b) below).

Under the conditions of inspection for percent nonconforming, a record should be kept of all nonconformities found in each of the nonacceptable items, so that corrective action can be taken for each type of nonconformity. No differentiation need be made in the count. An item with one nonconformity or an item with several is counted as a nonconforming item.

b) Inspection for nonconformities per hundred items counts each nonconformity found. Three nonconformities found in one item count as three, and are given the same weight as three items each with one nonconformity.

A special case arises when a nonconformity can occur an unknown and almost unlimited number of times in items, e.g. surface blemishes or pinholes can occur in any number and it is not known how many times they do not occur, so percent nonconforming for this feature is meaningless. In such cases nonconformities per hundred items should be used (see example 1).

NOTE - Percent nonconforming implies a binomial distribution. For nonconformities per hundred items a Poisson distribution is appropriate. See 6.4 for information on the operating characteristic curves of sampling plans.

c) Two properties will be dependent if nonconformities in an item arise, in part or wholly, through some common cause, or if one property affects the other. Detailed knowledge of the production process is thus needed to decide that properties are independent. In mathematical terms, if two characteristics, say length and diameter, are independent, it means that if all the units produced were taken and sorted into two groups according to whether the length was nonconforming or not, then the percentage nonconforming for diameter would be found to be essentially the same in each of these two groups; or, alternatively, if they were sorted into two groups according to whether the diameter was nonconforming or not, then the percentage nonconforming for length would be essentially the same in the two groups. It can be shown mathematically that these two procedures are equivalent.

If two nonconformities are not independent, then they are said to be related, or dependent. It should be agreed that the occurrence of both in one item is to count as only one nonconformity, not as two. Occasionally the correlation between two related nonconformities is low. Under these conditions the two may be considered to be independent. Inspection for percent nonconforming avoids this difficulty.

d) If the percentage of nonconformities in the lot is less than 2,5%, then the probability distributions of nonconforming items and nonconformities will be almost identical. In the range 2,5% to 10% some difference will be apparent, a nonconformities per hundred items plan being rather more severe than the equivalent percent nonconforming plan.

e) At an inspection station, and where admissible, it may be simpler and better practice to use one method rather than to change frequently from one method to the other, e.g. nonconforming items rather than nonconformities/hundred items.

f) From the point of view of keeping records that will be useful for improving quality, nonconformities per hundred items is preferable, as the records will then automatically contain information on all nonconformities, whereas some nonconformities may escape the record if the percent nonconforming approach is adopted.

6.3.2 Nomenclature

The discussion in the remainder of this guide will be in terms of inspection for nonconforming items. When appropriate it may be read in terms of inspection for nonconformities, by replacing 'nonconforming items' by 'nonconformities', and 'percent nonconforming' by 'nonconformities per hundred items.'

6.3.3 Classification of nonconformities

The discussion so far has assumed that, if an article can be nonconforming in more than one way, the different possible nonconformities are all of equal importance. It is then possible to sentence by counting the nonconforming items. For example, if there are three dimensions to be checked and, in a sample, three articles are nonconforming in the first dimension alone, three articles in the second dimension alone, one article in the third dimension alone, and one article in both the first and second dimensions, this gives a total of eight nonconforming items, which is the number to compare with the acceptance and rejection numbers.

The procedure of adding nonconforming items of different types is reasonable only if the nonconformities are of equal, or nearly equal, importance. Where this is not so, it is necessary to classify the possible nonconformities into groups so that nonconformities in different groups are of different orders of importance but all nonconformities within a group are of approximately the same order of importance. Different AQLs are then used for the different groups.

For many purposes, two groups are sufficient, namely major nonconformities of class A which are of greatest concern and nonconformities of class B which are of the next greatest concern. Sometimes it is necessary to introduce further classes or sub-classes within these classes. The most important class of all contains the critical nonconformities which render the articles hazardous, potentially hazardous, or adversely affect usage.

Critical nonconformities are a special case and are discussed in more detail in 6.3.4. For the moment, the discussion will be restricted to the major and minor classes. It should be realized that these classes refer to the relative importance of different nonconformities within any given product, and as products themselves vary in importance, the classes do not correspond to any absolute standards. Therefore, there is no particular AQL that normally goes with any class.

The classifying of nonconformities should be done properly. It is clear that care has to be taken not to 'under-classify' (for example, to classify as a class B nonconformity a feature that should be in class A), as this will lead to the allowance of more nonconformities of this class in the plan for the feature concerned than is really required. However, often it is not realized that it is also very important not to 'over-classify'.

When the system of classification of nonconformities is adopted, it is necessary to allocate a different AQL to each class to ensure that the more important, class A, nonconformities are more tightly controlled than the class B nonconformities.

If an article has more than one nonconformity and the nonconformities come within different classes, it counts as a nonconforming item of the more serious class. (However, if inspection is in terms of nonconformities rather than in terms of nonconforming items, each nonconformity in the sample is counted in its appropriate class.)

6.3.4 Critical nonconformities

By definition, critical nonconformities present a hazard and/or adversely affect usage or safety. These nonconformities form a special category. It is impossible to choose any value of percent nonconforming for these nonconformities and say, "... this percentage of critical nonconformities is tolerable."

Where non-destructive inspection is involved, the solution generally adopted is to require that critical characteristics are to be inspected using a sample size equal to the lot size and an acceptance number of zero. This is 100% inspection, but it should be noted that it is not the traditional 100% sorting. There is no attempt here to sort the articles into the good and the bad but an attempt to check that there are no bad ones. If a critical nonconformity is found, this does not merely mean that it is put into a different box and the inspection continues; it means that the whole lot is not accepted (although non-acceptance does not necessarily mean scrapping). Whenever possible, it should also mean that production is stopped while a thorough investigation takes place to attempt to discover how the nonconformity arose and to devise methods to prevent another occurrence. The reason for this procedure is to try to prevent the production of items with serious nonconformities and to avoid giving the manufacturer the impression that it will not matter too much if some are produced as the inspector will sort them out. Even the best inspector may occasionally fail to notice a nonconformity, so it is only by preventing critical nonconformities from being made that it can be ensured that none will get through to the customer.

If it is ever thought that any particular critical nonconformity does not warrant this procedure, then serious consideration should be given to having it reclassified as a major nonconformity. Critical nonconformities really have to be critical; then no amount of effort is too great.

Where the only possible inspection for critical nonconformities is destructive, the search for ways of preventing them from ever being made at all is even more important. In this case, we cannot have a sample which is 100% of the lot, and it is necessary to decide what sample should be taken. This can be done using a simple formula relating (a) the number of nonconformities/nonconforming items for which, if they were present, we would wish to be almost certain of finding at least one nonconformity/nonconforming item in the sample, (b) the lot size, (c) the sample size, and (d) the risk we are prepared to take of failing to find a nonconformity/nonconforming item.

Obtain the sample size (n) from the following formula and then round up to the nearest integer¹⁾. The lot is acceptable if no critical nonconformities are found in the sample.

$$n = (N - d/2)(1 - \beta^{1/(d+1)}) \quad (1)$$

where

N : lot size;

β : the specified probability of failing to find at least one critical nonconformity;

d : the maximum number of critically nonconforming items 'allowed' in the lot. If p is the maximum fraction nonconforming specified for the lot, then

$$d = Np \text{ rounded down to the nearest integer } ^2).$$

Example 3: Suppose there is a lot of 3 454 items. A probability, β , of 0,001 and a maximum percentage of 0,2% critically nonconforming items are specified.

Then $p = 0,2/100 = 0,002$ and $Np = 3\,454 \times 0,002 = 6,908$ which is rounded down to give $d = 6$.

Thus, $(N - d/2)(1 - \beta^{1/(d+1)}) = (3\,454 - 3)(1 - 0,001^{1/7}) = 3\,451 \times 0,627\,24 = 2\,164,61$ which is rounded up to give $n = 2165$.

The sampling plan is:

- sample size	$n = 2\,165$;
- acceptance number	$A_c = 0$ nonconforming items;
- rejection number	$R_e = 1$ nonconforming item.

1) This approximation is accurate enough for most practical purposes in acceptance sampling. In rare cases it will give a result which is one unit larger than necessary.

2) Only small values of percent nonconforming should be considered tolerable, as the nonconformities are critical.

Example 4: To find the lot size, N , needed to yield a specified number of items, L , after destruction of the sample of n items under test, assuming no nonconforming items are found, then for given values of the probability β and the number of nonconforming items in the lot, the lot size is

$$N = (L - d/2)/\beta^{1/(d+1)} + d/2 \quad (2)$$

rounded up.

Now, suppose that 1 500 items are required after testing the sample, using $\beta = 0,001$ and $d = 6$ as in example 3. Then L is 1 500 and the lot size is $(1\,500 - 6/2)/0,001^{1/7} + 6/2 = 1497/0,372\,76 + 3 = 4\,018,99$ which is rounded up to give $N = 4\,019$.

(It follows that $n = N - L = 4\,019 - 1\,500 = 2\,519$. This value of n is also obtained using equation (1) with a lot size of 4 019.)

If the initial calculation yields an unacceptable sample or lot size, then the risk (probability) and/or the possible number of nonconformities/nonconforming items in the lot need(s) to be reassessed and new criteria established.

An alternative plan for critical nonconformities, where the critical characteristic is something that can be measured rather than a pure attribute, is to sample with a safety margin. Thus, if the minimum allowable breaking load for some component were 2 000 kg, it might be possible, instead of saying that the limit was 2 000 kg and the nonconformity was critical, to say that the limit was 2 500 kg and the nonconformity was major. Just where the limits should be set, and what plan is allowable, depends upon some past knowledge of the amount of variability observed in the strength of the components in question. When this approach is possible, it can give much more satisfactory results for all concerned than does 100% inspection. In this case, there is the possibility of sampling by variables (ISO 3951) which will allow over-stress testing and yield information on the average and the variability of the characteristic.

6.4 The operating characteristic (OC) curve

The operating characteristic curve is a curve that shows what any particular sampling plan can be expected to do in terms of accepting and not accepting lots; that is, it is a sort of 'efficiency curve'. An OC curve refers to a particular sampling plan. Each possible plan has its own curve.

In acceptance sampling there are two 'types' of OC curve, known as Type A and Type B. Taking first the general case of a long production run with a stable process average quality (100

percent nonconforming, where p lies in the range 0 to 1), the quality of the lots taken from the run will vary about this process average in accordance with a binomial distribution. For each variation in lot quality the corresponding ordinate of the OC curve gives the proportion of lots (of that particular quality) that, on average, will be accepted by the sampling plan on which the OC curve is based. The OC curve in this case is said to be of Type B and describes how a user would view the operating characteristic of a sampling plan in respect of a steady supply of product from a given source.

In the case of isolated or individual lots, the ordinates of the OC curve cannot be interpreted in terms of long-run average proportions of accepted lots. However, such an interpretation is possible if we consider a fictitious process producing a series of *identical* lots, i.e. lots that are all of exactly the same size and quality (100p percent nonconforming). The ordinate of the OC curve is then, again, the proportion of those identical lots that will be accepted by the given sampling plan. However, in this case we are not sampling from a process with random variations in quality but from a finite number of items making up one lot. The ordinates of the OC curve indicate probabilities of acceptance (rather than average proportions of lots accepted) which are given by the hypergeometric distribution and depend on the lot size. The OC curve is said to be of Type A and describes how a user would view the operating characteristic in the case of isolated or individual lots.

Although the two types of OC curves are determined by different probability distributions, the Type B curve serves both purposes because it can fortunately be taken as a good approximation of the Type A curve when the lot size is sufficiently large, say, 10 or more times the size of the sample, although it should be kept in mind that the quality is that of the isolated lot and not that of the production. If the sample size comprises a greater proportion of the lot and acceptance numbers are positive integers (as opposed to zero), the Type B curve (as an approximation of Type A) gives a pessimistic indication of the producer's and consumer's risks, i.e. it errs 'on the safe side'. In the limit (i.e. for large lots) the Type A and Type B curves are identical. Thus, for practical purposes Type B curves can be used for both types of sampling without significant error in most cases. The OC curves for acceptance sampling plans given in the ISO Standards are of Type B.

ISO 2859-1 presents operating characteristic curves of sampling inspection plans for percentage of items nonconforming and for nonconformities per hundred items. These operating characteristic (OC) curves show the average percentage of lots accepted as an ordinate plotted against the percentage of nonconforming items or the number of nonconformities per hundred items in the process quality as the abscissa. For percent nonconforming they have been calculated based on the binomial distribution when the single sample size is equal to or less than 80. For nonconformities per 100 items the Poisson distribution is appropriate and has been used when calculating the OC curves for these plans.

The Poisson distribution is based on the assumption that nonconformities occur independently with constant expectation. This assumption holds in many cases. Any substantial departure from this assumption yields distributions with greater variance than that of the Poisson distribution. In these cases the consumer's protection is somewhat better than that indicated by the operating characteristic curves.

In ISO 2859-2 the tables for procedure A (lot in isolation) are based on random sampling from finite lots for both consumer's and producer's risk. However, for procedure B, the tables are based on random sampling from a finite lot for the consumer's risk at the LQ, but on random sampling from a process for the producer's risk and the OC curves. The actual operating characteristic will accept with greater probability when the indicated probability is greater than or equal to 0,90 and it will accept with smaller probability when the indicated probability is less than 0,10.

For sampling by variables the operating characteristic curves are matched to those for the attribute sampling plans for similar lot sizes and quality levels. The acceptability decisions are based on an assessment of the percentage nonconforming determined from the average and variability of the data obtained by measuring the product characteristic on all items in the sample.

6.5 Sampling risks

6.5.1 Risks when sampling: PR and CR

Because samples constitute only a small part of the whole of an inspection lot or consignment, sampling involves risks for both the producer and the consumer. Occasionally a 'good' lot may not be accepted because the sample inspected, though randomly selected, does not reflect the true quality of the lot. The risk of this happening is known as 'producer's risk' (PR). Conversely, a 'poor quality' lot may pass inspection because of the limited data available in the sample. This eventuality is known as 'consumer's risk' (CR).

In 6.1 it was stated that the risks associated with sampling can be calculated and assessed. Using its operating characteristic curve we can, for each sampling plan, read off the proportion of lots that will be accepted for a given input (or process) quality, i.e. the probability of acceptance for a stated quality.

The producer requires a high probability of acceptance if the quality is good while the customer would want a low probability of acceptance if the quality is poor. Conventionally these probabilities have been set at 0,95 and 0,10, respectively. This gives a PR of non-acceptance of 0,05, or 5%, and a CR of accepting poor quality of 0,10 or 10%. It is becoming increasingly common practice to make both the PR and the CR equal to 5%. For predetermined PR and CR percentages the corresponding producer's risk quality (PRQ) and consumer's risk quality (CRQ) can be read off the OC curve (see figure 1). Conversely, for a given OC curve the AQL and the limiting quality level (LQL) determine the PR and the CR, respectively (see figure 2). Alternatively, the sampling plan and its OC curve may be specially designed to 'fit' the preselected points (AQL, 1,0 - PR) *) and (LQL, CR).

*) Alternatively expressed as (AQL, 100% - PR).

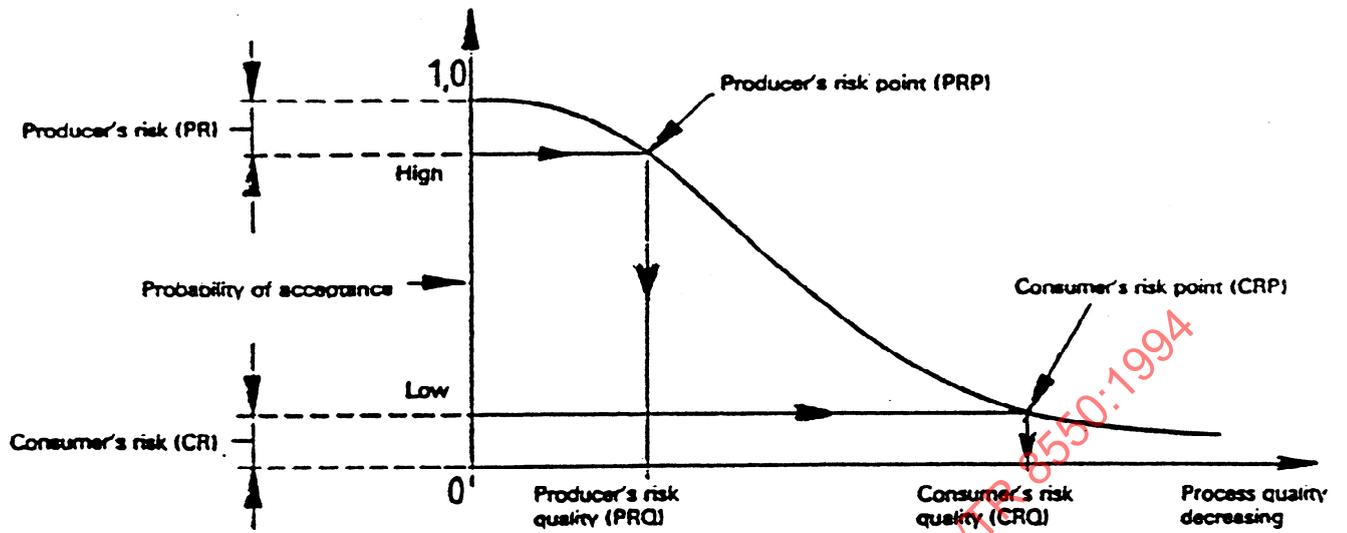


Figure 1 - Operating characteristic curve defined by producer's risk (PR) and consumer's risk (CR)

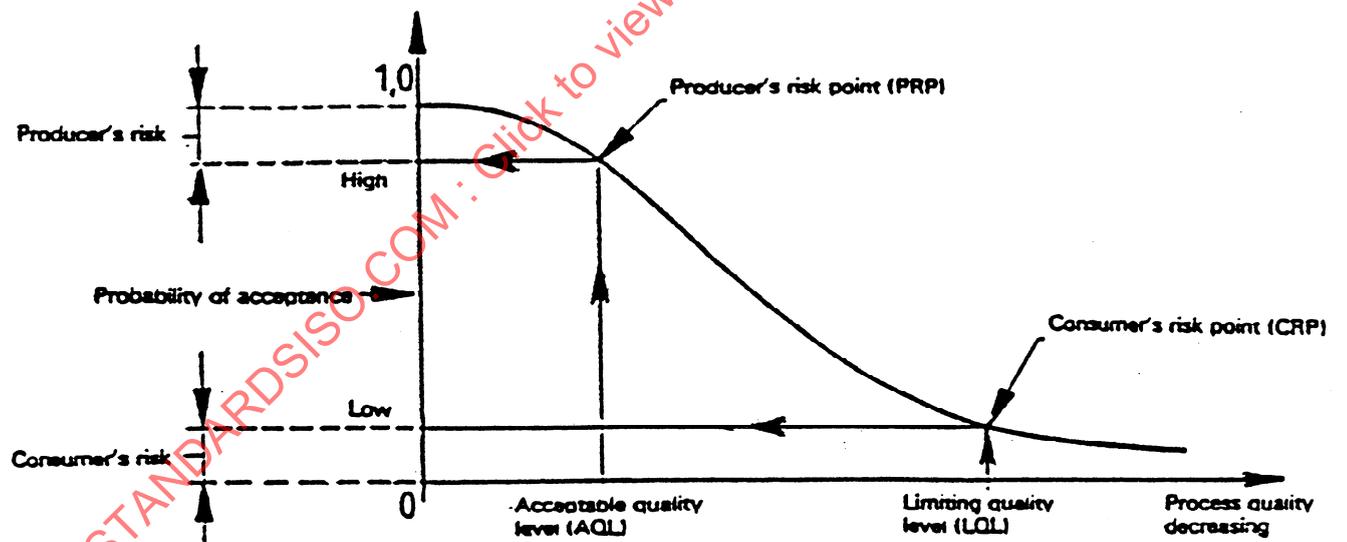


Figure 2 - Operating characteristic curve defined by acceptable quality level (AQL) and limiting quality (LQ)

An examination of the OC curves given in ISO 2859-1 for the sampling plans indexed by AQL at normal inspection will show that at the designated AQL the probability of acceptance varies from approximately 0,87 to 0,99 (i.e. the PR varies from 13% to 1%). This is a feature of these AQL sampling plans and, of course, of any plans designed to have characteristics 'matching' these AQL-designated single sampling plans. The term 'AQL' should not be used without reference to one or other of the ISO Standards. The OC charts and the tables also show the effect of moving to tightened inspection: the PR increases for the same AQL whereas the CR decreases for the same LQ.

When a sampling system is operated, the switching rules are an important factor in considering the risks due to sampling. For example, in ISO 2859-1 the OC curves show what to expect under normal inspection. They show that for all the plans in that Standard the percentage of lots likely to be accepted if the process quality is running at twice the AQL is less than 80%. Before long, such an acceptance rate will lead to tightened inspection if the rules are followed. The rate of acceptance at the AQL under tightened inspection will be only of the order of 80% and at twice the AQL it will drop to something like 50% and much less in a number of cases. These low acceptance rates under tightened inspection should prompt investigation into the cause of the inferior quality. The rule for discontinuation of sampling inspection ultimately makes this investigation a necessity. The remedial action taken will result in a return to the previous quality level or, as happens often, to an improved quality.

CAUTION - Although OC curves are a very useful concept, not only in risk analysis, it should be recognized that in practice lots in a series are rarely, if ever, identical and that operating processes are rarely strictly random. While the curves indicate what to expect under the stated conditions, they cannot accurately describe what happens in a period when conditions are constantly changing. Therefore, one has to be wary of making dogmatic assertions.

6.5.2 Methods for reducing the risks

Risks in sampling inspection, in both the acceptance of bad lots and the non-acceptance of good lots, are unavoidable but these risks should be tolerable, provided that the AQL and inspection level have been well chosen.

If either the manufacturer or the customer should consider in a particular instance that the risk he is taking is too high, it would be well to check that the AQL and the inspection level have been well chosen, but for the remainder of this subclause it will be assumed that they are appropriate and were properly selected.

The manufacturer will be interested in reducing risks when quality is better than the AQL - he is not entitled to any reduction of risk otherwise. The customer will be particularly interested in the risks when quality is worse than the AQL, as, if quality is better than the AQL, he is getting the quality required.

There are three methods that can be used to reduce the risks for both parties.

The first method is to improve the quality of production. This may seem too obvious to be worth saying, but it is surprisingly easy in discussions on sampling plans, OC curves, switching rules, etc., to forget the simple rule that a low percentage nonconforming in the production gives the customer what he wants and ensures a high proportion of acceptance to the manufacturer.

The second method applies only in a particular case, but it is the case which is most likely to cause anxiety, namely, where the acceptance number is 0. Plans with a zero acceptance number have such shallow OC curves that big risks are unavoidable.

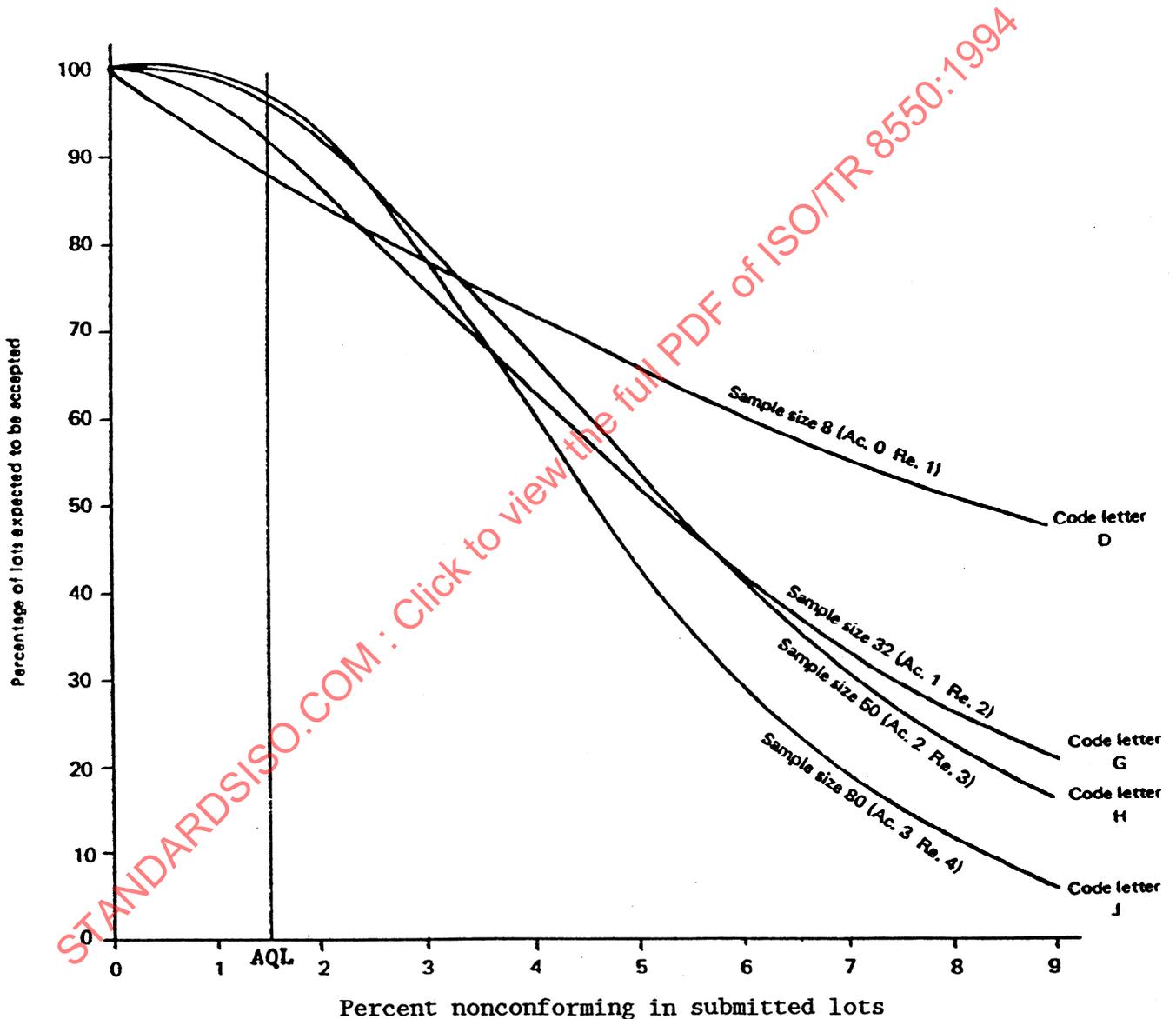


Figure 3 - OC curves: Four sampling plans for AQL 1,5% nonconforming, normal inspection, single sampling

For this reason, ISO 2859-1 allows the use of an alternative when the tables lead to a zero acceptance number (provided the responsible authority approves). This alternative is to use the plan for the same AQL, with an acceptance number of 1, instead of 0. There is a price to be paid, in that a sample size about four times as big is required, but the risks for both parties are so much reduced that it is often well worthwhile.

The cost may be reduced somewhat by adopting double or multiple sampling (see 7.3 and ISO 2859-0 clauses 1.21 and 1.22). These alternatives become available when the acceptance number is 1 or more. Sequential sampling is a further possibility, but it is outside the scope of this section.

The third method is to consider the possibility of increasing the lot size. If the lot size can be increased far enough to lead to a change of code letter and an increase of sample size, this will reduce the risks for both parties, as the larger sample size leads to a steeper OC curve, and the tables are so arranged that this curve will be higher than the old curve at most points where quality is better than the AQL, and lower at most points where quality is worse than the AQL.

Unfortunately, it is not possible to arrange the tables so that these features are always as desired, without losing other desirable features. Figure 3 shows, as an example, four of the normal inspection plans associated with an AQL of 1,5%. For quality better than the AQL, it is seen that the larger the sample the higher is the proportion of lots accepted, whereas for quality worse than twice the AQL, the largest sample does not accept most and the smallest sample does not accept least (and it is desired that the plan as often as possible does not accept when quality is worse than the AQL). The crossing point of the curves for sample sizes 32 and 50 is not so satisfactory.

The idea of increasing lot sizes, to gain better protection in the sampling, may be objected to as it is not always easy, or sensible, to change lot sizes. Lot sizes have to be fixed according to such things as continuity of production, the quantity of production that can be handled at one time, transport problems, stock control problems, and so on. This is all true, but it is nevertheless worth remembering that, other things being equal, an increased lot size can be helpful from the sampling inspection point of view.

In examining the height of the curves in figure 3 at twice, three times and four times the AQL, it has to be remembered that the curves show only part of the picture - the normal inspection part. For nearly all the normal inspection plans in ISO 2859-1, the percentage of lots accepted, if quality is twice the AQL, is less than 80%. Such an acceptance rate will always lead to tightened inspection before long.

In some circumstances, it may be decided that the compromise necessarily involved in using a complete sampling scheme is not worthwhile. The parties involved may then negotiate to choose a plan direct from the OC curves, but where such an approach is adopted, the parties need to be knowledgeable if a satisfactory choice is to result.

6.6 Selecting the AQL, PRQ, LQ and CRQ values

6.6.1 The AQL and PRQ

For the purpose of this consideration the AQL and the PRQ can be deemed synonymous. They are both indices of what quality can be tolerated for the purposes of sampling inspection, the difference being that the PRQ is associated with a specified small PR whereas the AQL denotes a quality level for which the (unspecified) PR will be small.

In setting an AQL, it has to be remembered that the AQL provides an indication of the quality that is required in production. The supplier is being asked to produce lots of an average quality better than the AQL. On the one hand, this quality has to be reasonably attainable, whilst on the other hand it has to be a reasonable quality from the customer's^{*)} point of view. Frequently this will mean a compromise between the quality the customer would like and the quality he can afford, for the tighter the requirement the more difficult it may be for the production to meet it, and the more expensive may be the inspection to ensure that it is met.

The primary consideration has to be the customer's requirement but it is necessary to make sure that the customer is being realistic and is not demanding something tighter than is really needed. It is necessary to take into account how the items in question are to be used and the consequences of a failure. If the items are to be available in large numbers and the failure is simply a failure to assemble so that the nonconforming item can be put aside and another used in its place, a relatively generous AQL may be tolerable. If, on the other hand, a failure is going to cause a failure to function of an expensive and important piece of equipment at a time and place where a replacement of the nonconforming item cannot be made, a tighter AQL will be required.

It is also necessary to consider how many components will be contained in the eventual equipment. If, for example, it is decided that the quality of a piece of equipment containing three different but equally important components should be not more than 10% nonconforming, then each of the three components could be 3,5% nonconforming and the requirement would be met, whereas if it were to contain ten components, these would have to be no worse than 1% nonconforming.

It should be remembered that even if components are sampled to one AQL, they may and should come from a source or sources with a smaller process average to increase the probability of lot acceptance.

Provided that the components conform or do not conform independently, the formula here is that if k is the number of components in the assembly, X is the AQL of the assembly, and x is the AQL of the components, then from the multiplication law of probabilities, it follows that

$$\frac{X}{100} = 1 - \left\{ \frac{100 - x}{100} \right\}^k$$

^{*)} The "customer" could be the next process in production, not necessarily the end user.

The AQL is the limit of a satisfactory process average. It is to be expected that the supplier's quality would be better than the AQL.

The value of X , however, does not take into account nonconformities which may arise through a faulty assembly process.

The simple approach above has assumed that all components are equally important to the successful operation of the equipment. This is seldom true, so their application will have to be graded and the equipment rate subdivided accordingly. The appropriate rate for the supply of components (if identical) would then be that for the most important component application. Another approach would be to require different actual product standards - a design matter. In determining quality levels the designer should always be consulted for advice, if possible.

In these circumstances the producer would probably wish to choose what would seem to be a suitable AQL for each component and then calculate what quality can be expected in the overall equipment, whereas the consumer would wish to specify an AQL for the overall equipment and then calculate what has to be the quality for the components. In general, the second of these approaches is probably the more reasonable in that it is the performance of the overall equipment that really matters, but it is also the more expensive approach because it almost always leads to tighter AQLs. However, it has to be accepted that good quality in a complicated article is inevitably more expensive than equally good quality in a simple one.

The question of what quality level can reasonably be expected, at the price the customer is prepared to pay, and with the methods of production envisaged, can often be answered by an examination of what quality level has been produced and tolerated in the past. Where the article is a new one, and there has been no past production, there will often be other similar articles from which relevant information can be obtained. Past process average calculations may be particularly helpful. This idea of looking at the quality obtained in the past should not be taken as meaning that past quality levels are sacrosanct and are always good enough. This is because the production cost of a nonconforming item is nearly equal to that of a conforming item and reduction of percentage nonconforming frequently means reduction of production cost. It is simply one of the factors to be taken into account in assessing what is a reasonable AQL to set.

Although it is a common belief that better quality costs more, the reverse is often true. Effective process control can provide extremely good quality, even to the extent of parts per million. Quality achieved in this manner is much less costly than the poorer quality that results when process control is non-existent. The main reason for this is that there is virtually no scrap, rework or returned material.

Attempting to achieve excellent quality by inspection and sorting is much more expensive and, due to the inefficiency of inspection, not highly effective. Therefore, it becomes necessary that attempts to achieve the quality of fractional AQLs or parts per million be accompanied by a review of process control procedures and the design of the product, rather than attempting to achieve such results by inspection.

It has to be remembered that the mere setting of an AQL does not give the customer a guarantee that lots of a worse quality will not be accepted. In the first place, the AQL refers to the average. Some lots may be worse than the AQL while the average is better than the AQL. In the second place, if the average quality being offered is a little worse than the AQL, a number of lots will probably be accepted before a switch to tightened inspection is called for, and even after the switch there is likely still to be some acceptance. In general, however, it can be expected that the customer will get a product with an average which is better than the AQL, as sampling schemes have a built-in economic incentive in that a manufacturer cannot afford to have more than a small proportion of lots not accepted and will take steps to improve the quality of production if this proportion is exceeded.

It might be thought that this is not very satisfactory from the customer's point of view, relying as it does upon what is likely to happen rather than upon what will certainly happen. But in practice most manufacturers take steps to see that their process average does not exceed the AQL, if only because relatively frequent lot non-acceptance makes life difficult. In any case, the customer's protection depends upon the lower end of the OC curve as well as upon the upper end with which the AQL is concerned, and this lower end can be adjusted by considering the LQ values of any suggested plan.

If it is decided that this approach is not adequate for any particular product, and more positive customer protection is needed, then this can always be attained by specifying a tighter AQL, bearing in mind that this is likely to increase the cost of the product. It cannot be denied that this extra cost will sometimes be worthwhile. Most frequently a proper approach to improved quality and fewer nonconformities will result in lower, not higher, costs.

It is not necessary that the AQL should always be the primary choice from which all else is derived. It is always possible, when circumstances so require, to enter sampling tables "through the back door", choosing a plan by some other criterion and then finding the AQL specified to get the desired result. In this case, the AQL is a convenient index to enable the standard tables to be used, and is also valuable as an answer to the question in which the manufacturer is primarily interested - at what quality does he have to manufacture to get most lots accepted?

If such a "back-door" method is used, the primary choice may be either a low point on the curve, where this is thought particularly important, or some economic criterion. Probably the simplest economic criterion that has been suggested is to make an estimate of the break-even point, i.e. the lot quality such that, if the lot were accepted, the cost of the damage done by the accepted nonconforming items would be exactly the same as the cost of failing to accept the lot.

If this break-even quality can be estimated, it would be well to choose a plan for which this quality gives 50% of lots expected to be accepted, not because a 50% chance of acceptance with this quality is particularly wanted (by definition, if this particular quality is offered, it is not of much interest what the inspection plan does with it), but because this ensures a greater than 50% chance of acceptance for better quality than break-even quality, and a greater than 50% chance of nonacceptance for quality worse than break-even quality.

Sometimes it is suggested that there could be a 'trade-off' between AQL and tolerances. For example, a dimension is specified with a tolerance of $\pm 0,6$ mm and an AQL of 0,1% selected. On the (not unreasonable) assumption that the measurement of this dimension is normally distributed, a tolerance of $\pm 0,3$ mm with an AQL of 10 % will give much the same result. This is based on the relative areas in the tails of the distribution against the measurement scale. Adoption of this modification has the merit that 10% nonconforming is much easier to detect, leading to a big reduction in sample size. However, there are penalties associated with these extremes.

To illustrate this, using ISO 2859-1 and a moderate inspection level, an AQL of 10% implies that the lot size can range from 30 to 30 000, but for an AQL of 0,1% the lot size has to exceed 10 000. A reduction in tolerance imposes unreal pressure on the inspection function as well. Inspection will be required to record measurements actually conforming to the product specification tolerance as unacceptable. Worse still is the situation in which a lot has been sentenced as 'not acceptable' and has to be screened when, in fact, most of the product is within specification. Under these circumstances it would be tempting, but blatantly wrong, to reassess the offending sample measurements on the basis of the product specification tolerance. These uses of compressed or narrowed 'tolerance' limits to gauging conformance are more appropriate for process control or internal acceptance practices. They are not appropriate for specifications.

This example points up the need for setting realistic specifications and quality levels for sampling purposes, which brings us back to the nonconformity classification and the question of just how serious are departures from conformity. The specifications should be realistic and the quality levels should properly reflect the classification and its relative importance and be consistent.

Among the most common bases for setting the AQL are the following:

- (a) Historical data: Analysis of past data to estimate process average. The level is then set at, or close to, the estimate obtained.
- (b) Empirical judgement: The level is set relative to a known satisfactory level for a similar item.
- (c) Engineering judgement: The level is based on 'engineering' estimates of the quality requirements for function, performance, life, interchangeability, etc.
- (d) Experimental: Levels are set tentatively and subsequently adjusted by performance and experience.
- (e) Minimum total cost: The level is based on an analysis of the 'cost of quality' versus the 'cost of not having quality'.
- (f) Knowledge of the product and of the supplier: The level is based on 'experience'.

These bases may be used singly or in combination.

Finally, having taken all these factors into account, it is desirable to choose one of the AQL values given in the tables to be used if possible, as the tables are otherwise inapplicable and a special plan would have to be designed. The preferred AQLs values approximately form a geometric progression with a common ratio of about 1,6 so it should be a rare occurrence to find none of them suitable.

6.6.2 The LQ and CRQ

In analogy with the AQL and the PRQ, the LQ and the CRQ can be considered to be equivalent indices whose stipulated values express, for the purpose of sampling, a level of 'objectionable' quality which has only a small chance of acceptance.

The process of setting these quality levels is similar to that for the AQL, except that we are now considering an intolerable level that, if it were to exist, would cause operational problems, additional costs, etc. The levels will be associated with the chosen level of consumer's risk and should be decided accordingly.

6.7 Inspection level (IL) - Sample size/lot size relation

6.7.1 The inspection level

The inspection level is an index of the relative amount of inspection for a sampling scheme and relates the sample size to the lot size and hence to the discrimination afforded between 'good' and 'poor' quality. As an example, table 1 of ISO 2859-1 provides seven inspection levels, ISO 3951 provides five.

Having set the AQL (or PRQ), the 'inspection level' is determined in table 1 of ISO 2859-1 or ISO 3951, or, more generally, by considering what quality should have only a small chance of acceptance if an occasional lot of that quality is submitted for inspection, i.e. the LQ or the CRQ. The sampling plan OC curves in the basic ISO Standards (ISO 2859 and ISO 3951) are then surveyed to ascertain which plan most nearly meets the joint requirement and the sample size code letter is noted. Table 1 of the standard gives the range of lot sizes appropriate to this code letter for different inspection levels and it indicates the preferred lot size range to be used. Inspection level II is considered to be appropriate for many applications.

NOTE 1. ISO 8422 and ISO 8423 both provide plans indexed by PRP and CRP (Table 1) where, therefore, inspection level does not apply. However, in Annex A these standards also provide plans corresponding to the basic standards (ISO 2859-1 and ISO 3951, respectively).

NOTE 2. It is always possible to deal with 'special' cases by assigning a constant code letter irrespective of the lot size, for example if a definite OC curve were required. The design of 'special plans' for a particular case is outside the scope of this guide but it should be possible to avoid this recourse in the vast majority of cases.

NOTE 3. It is possible that a 'low' inspection level, such as S1, may have to be used for economic reasons or because the tests are destructive. In these cases the discrimination may suffer. However, if records are kept for a continuing series of lots the cumulative sample may show that the CR is more acceptable.

NOTE 4. Procedure A in ISO 2859-2 does not index the plans by inspection level because this procedure is for use with isolated lots where both the producer and the consumer are concerned with the LQ.

If no OC curve can be found that meets the required AQL and LQ values, the requirement should be questioned as to whether it is essential. If the CRQ (or LQ) is not relaxed, then the only course is to tighten the AQL, with a consequent increase in producer's risk.

If, after having established a sample size code letter, it is found that none of the inspection levels includes that code letter for the expected lot sizes, it will be necessary to review and amend the requirement or, failing this, to specify the sample size code letter without an inspection level. If the latter results in a sample size exceeding the lot size, then 100% inspection is called for.

6.7.2 Comment on sample size/lot size relation

There is no simple mathematical relationship between the sample size and the lot size. The reason most sampling schemes relate sample size to lot size is the requirement that the sample should be representative of the lot. Provided the sample size is small compared with the lot size, the lot size can, for practical purposes, be ignored in developing the OC curve for a sampling plan. The choice of a sampling plan and the sample size depends on the homogeneity of the lot. Larger samples are needed when there is a lack of homogeneity. AQL plans require that lots are homogeneous. LQ plans require larger samples. When the plans of ISO 2859-1 are used, the lots should originate from essentially similar conditions and should not be 'mixed'. Although the effect of lot size is greater for small lots, it is the absolute sample size that is more important than its proportion of the lot.

From the point of view of sampling inspection there is an advantage in large lots, provided homogeneity is maintained, as it is then possible and economic to take a large sample while maintaining a large lot-to-sample ratio, thereby achieving better discrimination. Furthermore, for a given efficiency the sample size will not increase as rapidly as the lot size and will not increase at all after a certain lot size. However, there are a number of reasons for limiting the lot size, as follows:

- (a) the formation of larger lots may result in the inclusion of a widely varying quality;
- (b) the production or supply rate may be too low to permit the formation of large lots;
- (c) storage and handling may preclude large lots;
- (d) accessibility for drawing random samples is difficult with large lots;
- (e) the economic consequence of non-acceptance of a large lot in terms of scrap, rework and further inspection may be large.

6.8 Rectifying inspection for lot-by-lot sampling - AOQL

A lot acceptance sampling plan has no direct effect on the quality of the lots that are being inspected except for the replacement of the nonconforming items found in the sample. If 3% of the product is nonconforming prior to submission to sampling inspection, approximately 3% will be nonconforming after sampling inspection. The primary effects of a sampling plan on product quality are indirect and stem from the effects that the non-acceptance of lots has on subsequent production, e.g. the attitude of the operators and the steps taken by management to improve the process.

Rectifying inspection for lot-by-lot sampling is a combination of sampling inspection and 100% inspection (or 'screening') that seeks directly to control the quality of product that passes through inspection. All lots are sample inspected but lots that are not accepted under sampling inspection are inspected 100%. Any nonconforming items that are found are discarded and may be replaced by conforming items. The average outgoing quality (AOQ) that results from this rectifying inspection will be better than the incoming quality. It is to be noted that the AOQ is an average over many lots and does not pertain to individual lots. The AOQ and the average outgoing quality limit (AOQL) are described in ISO 2859-0 : Section 1. (The AOQL is the maximum value of the AOQ.)

While rectifying inspection undoubtedly puts some pressure on the supplier to improve the quality (assuming he is carrying out the inspection), the requirements for assessing the effect are somewhat idealistic. The calculation of the AOQ assumes perfect 100% sorting and the replacement of nonconforming product with good product (new or reworked). As perfection is unlikely in practice, the results of the screening process and the calculation should be viewed with some caution as the calculated result could be quite optimistic. Tables giving AOQL factors are included in ISO 2859-1.

A major problem with rectifying inspection is that the supplier is liable to rely on the rectification, especially if it is carried out by the purchaser, and may not diligently attempt to improve the process quality.

Another problem associated with this kind of inspection is the pressure and cost imposed on the inspection department. Whenever a lot is sentenced as unacceptable, the increased workload can be considerable and demanding in manpower and equipment resources such that 'standards' and throughput can be difficult to maintain. This may adversely affect the 100% screening efficiency and it may also very likely affect the quality of 'original' inspection. There should be awareness of the danger that this situation could possibly create a conscious or subconscious temptation to cut corners, with a consequent degeneration of quality. Notwithstanding, the AOQL can be a useful concept in assessing the effect of rectifying inspection on outgoing quality, as well as providing an incentive to produce good quality in the first place.

The AOQL can also be used as a basis for comparing and selecting a plan for an application. Where AOQL factors are not given in a standard, the AOQL can be calculated from AOQ values using the approximation:

$$AOQ = P_a \times 100p$$

where P_a = the probability of acceptance
 p = fraction nonconforming

from which AOQ, and subsequently AOQL, are obtained in percent nonconforming. In fact, this approximation could be a better indication than the more detailed calculation when rectification is not applied efficiently.

7 Making a comparison of the methods for sampling inspection

7.1 Attributes vs variables

Provided certain assumptions are true, the variables method has the advantage of requiring a smaller sample size than the attributes method to attain a given degree of protection against incorrect decisions. Also it provides more information as to whether quality is being adversely affected by process mean, process variability or both. The attributes method has the advantage that it is more robust (not subject to assumptions of distributional shape) and that it is simpler to use. The larger sample sizes and the increased costs associated with using attribute sampling methods may be justifiable for these reasons. Furthermore, an attribute scheme may be understood and accepted more readily by inspection personnel. With 'short runs' or lots of an 'isolated' nature, sampling by attributes is recommended even to the extent of converting measurements to attributes. This avoids the assumptions of normality and the attendant inability or difficulty in checking for it. Sampling by variables has a substantial advantage when inspection is expensive, e.g. when testing is destructive. It becomes less suitable as the number of measurements on one item increases because each quality characteristic has to be considered separately. ISO 3951 and ISO 8423 deal with sampling inspection by variables but these standards only apply to inspection of a single variable. Therefore, it could be advantageous to apply attribute sampling to the majority of characteristics, reserving sampling inspection by variables to one, or at most two, of the more important requirements. Sampling inspection by variables (ISO 3951 and ISO 8423) is dependent on the assumption that the distribution of measurements is normal (but see the footnote on p.11).

Table 1 gives a comparison of the sample sizes for inspection by attributes and by variables for a few code letters when using normal single sampling at inspection level II. Similar advantages exist when comparing inspection by variables and by attributes in sequential sampling.

Sample size code letter	Sample sizes	
	Inspection by attributes	Inspection by variables
C	5	4
F	20	10
H	50	20
K	125	50
N	500	150

Under ' σ known' conditions even smaller samples can be used for inspection by variables (see ISO 3951 table 1-B).

7.2 Relationship between form of distribution of product characteristic and percent nonconforming

A key aspect of sampling by variables is the form of the distribution of the quality characteristic. If this characteristic is normally distributed and if an upper specification limit is located at the mean plus two standard deviations, the percentage nonconforming will be about 2,5%. If the specification limit is located at the mean plus three standard deviations the percentage nonconforming will be about 0,1%. If the distribution of the product characteristic is not normal and has a large positive skewness (a long tail to the right) an upper specification limit located at the mean plus three standard deviations might yield a percentage nonconforming approaching 10% instead of equal to 0,1 % (see figures 4 and 5).

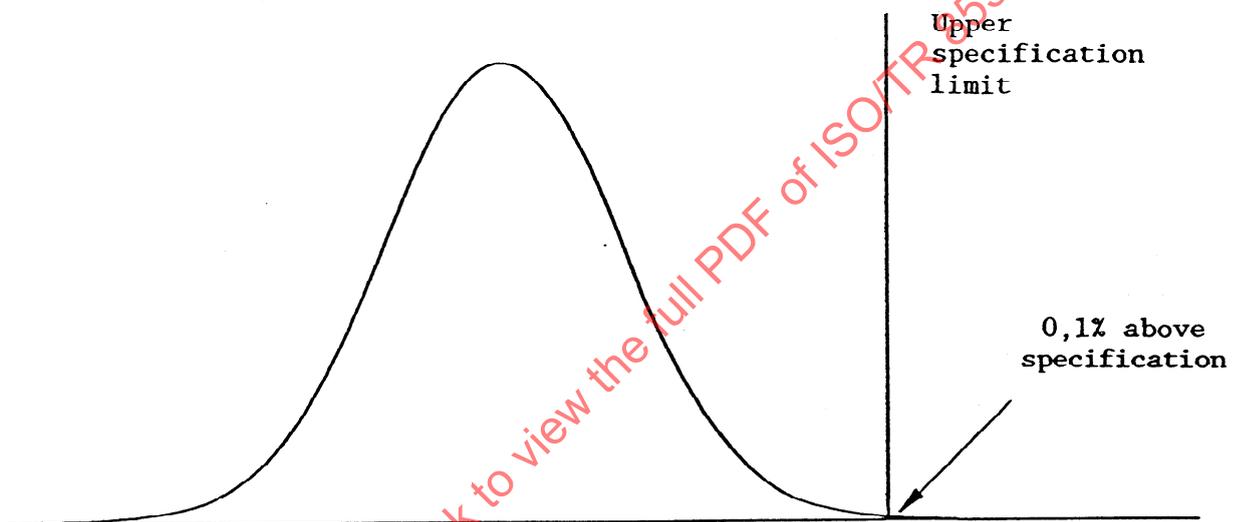


Figure 4 Normal distribution



Figure 5 Distribution with large positive skewness

Therefore, whenever a sampling plan for inspection by variables for percent nonconforming is to be employed, it is highly desirable to check any assumptions about the shape of the distribution, especially in the tails of the distribution. If the AQL is very small, for example 0,1%, a study should be based on several thousand items. In addition, a test of distributional form should be applied. Details of such tests can be found in Section 2 of ISO 2854 *Statistical interpretation of data - Techniques of estimation and tests relating to means and variances* or in ISO 5479 *Statistical interpretation of data - Tests for departure from normality* (in preparation). Attention is also drawn to the footnote on p.11.

7.3 Use of OC curves for comparing sampling plans

OC curves provide an excellent basis for making direct comparisons between sampling plans because each sampling plan has its own OC curve. Alternatively, some comparison can be made by using factors derived from the OC curve, e.g. the AOQL (see 6.8) or the discrimination ratio (see 7.4 below).

7.4 Discrimination ratio (DR)

If there were a perfect sampling inspection plan, this would give absolute discrimination at a percentage nonconforming equal to the quality level. The OC curve would be a straight vertical line at the nonconforming quality level. For qualities of a lower percentage nonconforming than the AQL there would be a region of total acceptance and for qualities of a higher percentage nonconforming than the AQL there would be a region of total non-acceptance. This ideal is unattainable (see ISO 2859-0 or ISO 2859-1).

Once a method of sampling inspection is introduced, the ability to discriminate absolutely between 'acceptable' and 'unacceptable' quality is lost. The penalty for not inspecting every item is reflected in the slope of the OC curve, in particular the slope of that part of the curve lying between the PRP and the CRP which is known as the discrimination ratio (DR). The more vertical this slope, the greater is the discrimination of the plan. Thus, the comparison of the slope over this part of the OC curves provides a direct comparison of the effectiveness of the sampling plans in terms of discrimination.

When OC curves are on different pages in a book or Standard, it is not so easy to compare them directly and the following numerical alternative can then be useful. This method utilizes the slope of the straight line joining the PRP and the CRP. The straight line is a satisfactory approximation of that part of the OC curve.

The factor $(CRQ)/(PRQ)$ gives this slope as a numerical index of the discrimination ratio. The larger this number, the less discriminatory is the sampling plan. The index can be used for plan selection as well as for comparison, or for any other purpose based on the OC curves.

However, sampling plans with the same discrimination ratio are not necessarily equivalent. For example, using ISO 2859-1, the OC curve for the plan with AQL 1% and code letter L is similar to the curve for the plan with AQL 0,25% and code letter M but the CR and PR points are quite different. In the most frequent application of this method, curves with similar PR points and quality level are compared for their degree of discrimination against poor quality, i.e. the variation in consumer's risk.

7.5 Comparison of single, double, multiple and sequential sampling

7.5.1 Equivalent plans

If the acceptance number in a single sampling plan by attributes is greater than zero, it is possible to find a double, multiple or sequential sampling plan with an operating characteristic curve close to that of the single sampling plan. Hence, except for those single plans with acceptance number zero, there is no reason to choose between single, double, multiple or sequential sampling on the basis of the operating characteristic curve. Neither is there reason to prefer one to another for all possible situations. The balance of advantages and disadvantages sometimes favours one, sometimes another of the sampling procedures. The characteristics that should be taken into account are as follows:

a) **Simplicity.** Single sampling is the easiest to describe and administer. Double sampling requires more administration to arrange for the second sample to be made available when required. Multiple and sequential sampling are obviously even more complicated. Sometimes the attraction of simplicity is the major consideration in the selection of the sampling plan. There will be other occasions when the psychological attraction of being able to take a second sample in apparently marginal cases will favour double sampling plans.

b) **Variability in the amount of sampling inspection.** In single sampling the sample size is fixed and the amount of inspection effort required to reach a decision is known in advance. For the other types of sampling, the number of items tested varies according to the results from the early samples. It is possible to calculate an average amount of sampling inspection and the average cost of inspection for any given input quality. This varies with the quality, being least for both very good and very poor quality. In addition to the uncertainty associated with the unknown input quality, but also even when the input quality is known, there is the uncertainty due to the variation of the amount of sampling inspection about this average. This uncertainty can lead to problems in arranging for sufficient resources to be made available for the inspection required. If insufficient resources are available the result is delayed. In the contrary case there will be inefficient use of resources. In some situations the variable inspection load will often be considered a small price to pay for the significant reduction in the average total inspection cost.

(c) **Ease of drawing sample items.** Sometimes it is easy to draw a second sample and to draw two samples is no more trouble than to draw one sample of the combined size. At other times, however, the situation arises where the drawing of sample items forms a large part of the inspection task and here, having disturbed the lot to draw one sample, it is hardly feasible to disturb it again to draw another sample. In these cases, single sampling is usually the best plan. There is, of course, the alternative possibility of drawing a sample of the maximum size that could be needed and then inspecting according to the preselected double, multiple or sequential plan. This may give little cost saving compared with the single plan due to problems in returning uninspected items to the lot.

(d) **Duration of test.** If a test is of long duration and it is possible to apply it to a number of items simultaneously, it will usually be better to do so rather than to risk finding that at the end of the test of a first sample the result is inconclusive and that a second sample, or even more, is needed, therefore at least doubling the time taken. This is another case where single sampling is usually the best, provided that the whole of the single sample size can be tested at once. However, if only one or two articles can be tested at one time, multiple (or sequential) sampling may be preferable.

Example 3: Tinned meat is to be tested for keeping qualities by storing a number of tins for 3 weeks under certain atmospheric conditions.

To achieve a desired OC curve, the choice might perhaps lie between a single sample of 80 tins, a double plan with samples each of 50 tins, and a seven-stage multiple plan with samples each of 20 tins. If single sampling is used, the answer will be available 3 weeks after the test is started; under double sampling, the result might be available in 3 weeks, but might require 6 weeks instead; under multiple sampling, nearly 5 months might be required in an unlucky event.

Single sampling will probably be chosen in these circumstances.

Example 4: A destructive inspection is to be performed. All the articles in the lot are available at the testing station and the testing apparatus can take only one article at a time. As the principal cost of the test is the destruction of the article, it is desirable to destroy as few as possible consistent with the desired OC curve.

As the articles in the sample have to be tested one at a time, the use of sequential rather than single sampling will probably save time as well as cut down the average sample size and would be well worth considering.

(e) **Multiple nonconformities.** The more complicated the product in terms of the number of possible nonconformities and the number of classes of nonconformities, the more involved double or multiple sampling becomes. Efficient use of labour and inspection equipment is difficult if the first sample has to be inspected for all features, a second sample only for some features, and possibly a third sample only for some of those. In general, it can be said that a complicated inspection favours a simple sampling plan, whereas, where the inspection is simpler, a more complicated sampling plan may pay rich dividends.

The operating characteristic curve for the single sampling plan with sample size 200, acceptance number 3, and rejection number 4, and the equivalent double and multiple plans are shown in figure 6. The match is not exact, but is good enough for most practical purposes. The equivalent sequential plan is also matched to the single sampling plan OC curve but is not shown in order to avoid overcrowding. The operating characteristic curves of the sequential and single sampling plans are virtually indistinguishable from each other.

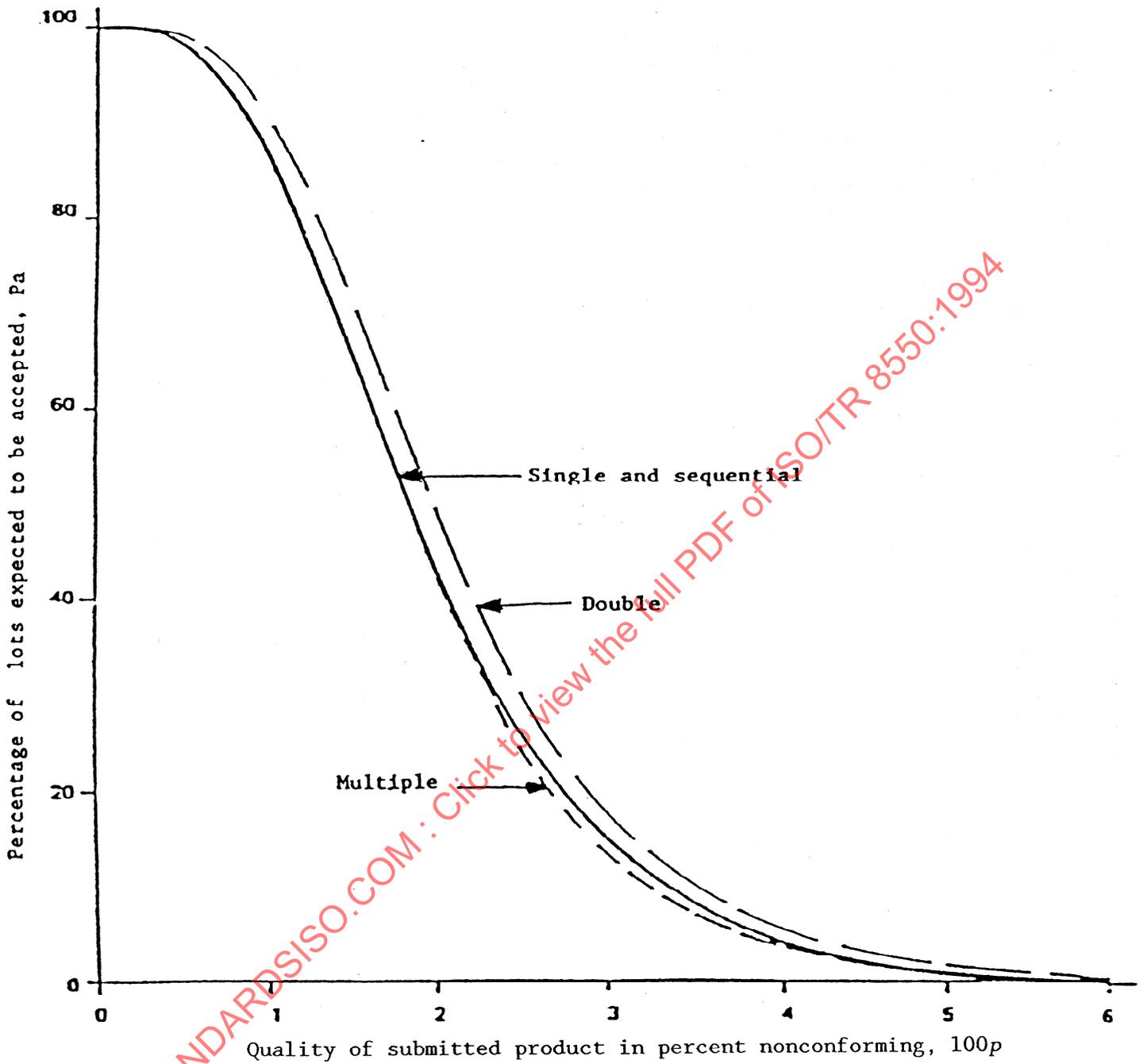


Figure 6 Comparison of operating characteristic curves for single, double, multiple and sequential sampling plans (code letter L, AQL 0,65%)

NOTE. The OC curve of the sequential plan is virtually indistinguishable from that of the single plan.

7.5.2 Average sample size

In order to compare single, double, multiple and sequential sampling it is helpful to consider the average sample size that would be needed in a long run of sampling at different average product quality. This produces an average sample size curve which is indicative of the relative efficiency of the several sampling systems. These curves indicate the number of items to be examined on average before arriving at a decision to accept or reject. Figure 7 shows the average sample sizes for the set of equivalent single, double, multiple and sequential sampling plans given in table 2 and featured in figure 6.

Type of sampling plan	Sample size(s)	Acceptance/rejection numbers
Single	Sample $n = 200$	Ac = 3 Re = 4
Double	First sample $n = 125$ Combined 1st and 2nd sample $n = 250$	Ac = 1 Re = 4 Ac = 4 Re = 5
Multiple (# acceptance not permitted at this sample size)	First sample $n = 50$ Cumulative sample size $n = 100$ $n = 150$ $n = 200$ $n = 250$ $n = 300$ $n = 350$	Ac = # Re = 3 0 3 1 4 2 5 3 6 4 6 6 7
Sequential	See ISO 8422	

On average the number of items to be examined before reaching a decision is largest when single sampling is used. The greatest reduction in sample size when using double, multiple or sequential sampling occurs when lots are of very good quality or very bad quality.

For good or bad quality the average saving in inspection can be substantial, but the actual number of items to be inspected for a particular lot when using a double, multiple or sequential sampling plan may exceed that for the corresponding single plan. This is most likely to occur when quality is at an intermediate value, e.g. 2 or 3 times the AQL.

It is for these reasons that single sampling may be preferred in some instances, for example, when the test duration is long and all items can be tested at the same time. On the other hand, when the tests can only be done one at a time, or are destructive, double, multiple or sequential sampling can offer a substantial advantage (see examples 3 and 4).

For double and multiple plans there is an upper limit to the number of items to be inspected. For sequential plans there is generally no such limit unless the truncation rule has been invoked to restrict the potential number of items inspected. ISO 8422 and ISO 8423 provide for curtailment of sample size.

Double, multiple and sequential sampling offer the opportunity for significant savings in sample size, but they require more administrative control. When apparatus for semi-automatic use is available, automated sequential sampling offers an opportunity for increased efficiency and economy, particularly when destructive tests are performed.

Average sample size curves for double and multiple plans are given in ISO 2859-1. For sequential sampling plans by attributes the average sample sizes are tabulated in ISO 8422. For sequential sampling plans by variables for known process standard deviation corresponding to the single sampling plans of ISO 3591, the average sample sizes are given in ISO 8423. ISO 3951 does not contain double, multiple or sequential sampling plans.

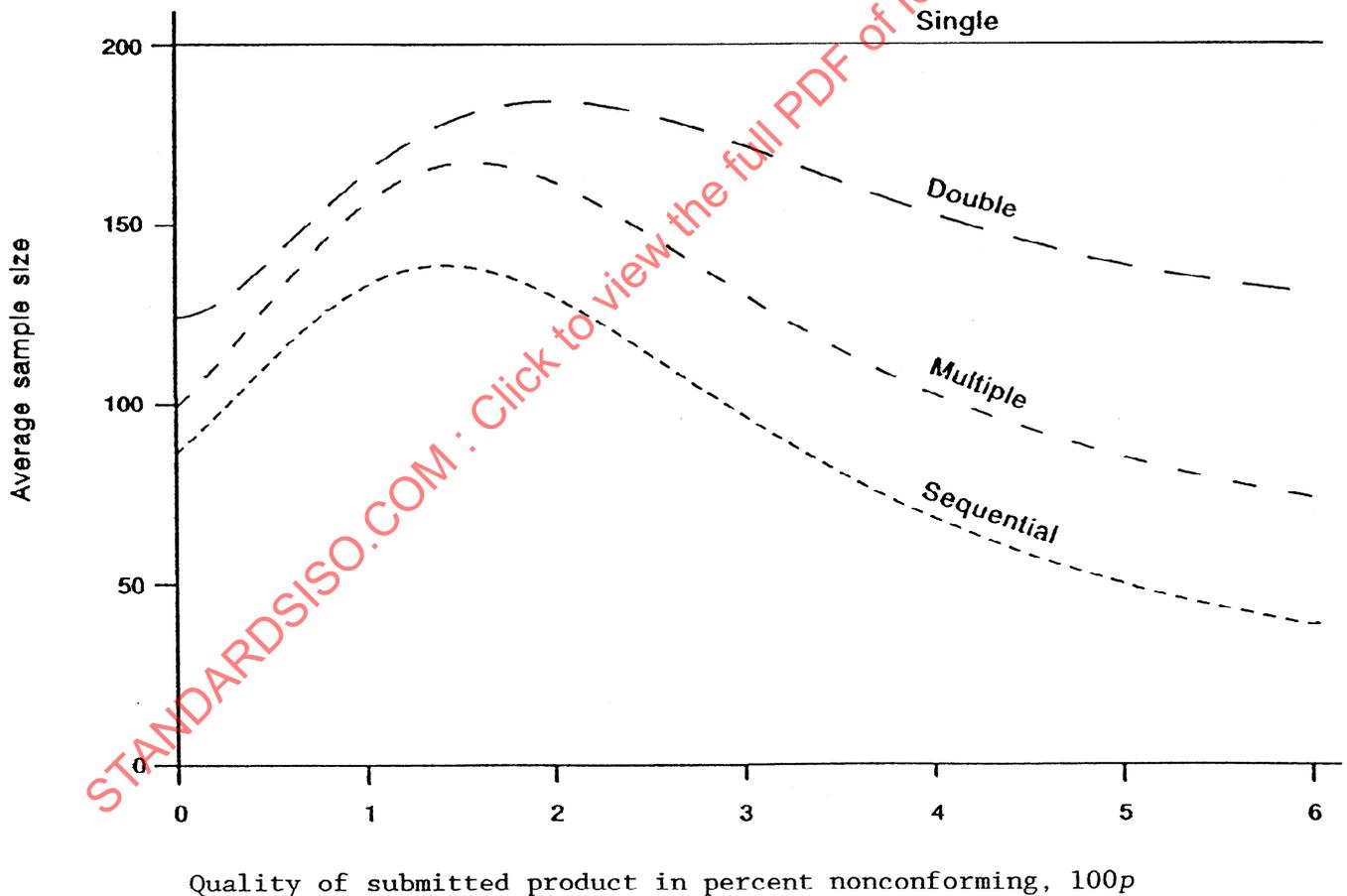


Figure 7 Comparison of average sample sizes for single, double, multiple and sequential sampling (for acceptance plans of figure 6)

8 Other methods sometimes adopted in practice

8.1 100 % inspection

Many modern plants incorporate automatic methods of product quality checking on each item produced. Similar practices are adopted to remove nonconforming items from lots accepted by sampling. However, some independent monitoring may be necessary to insure against malfunctions of the test system that result in the continued production and 'acceptance' of nonconforming product. In some cases the actual process control, or some other part of the quality system, can provide this monitoring.

8.2 Grab samples

The practice of grab sampling, a technique of sampling according to no fixed plan, is generally deprecated because it effects no statistical control and is thus unreliable, uneconomic and ineffective for acceptance and control purposes. However, it is sometimes used in a 'feed forward' system where a nonconforming item causes no real inconvenience at a subsequent stage. For these reasons this practice is normally confined to 'in-house' operations.

A grab sample of between 2 and 20 items (see ISO 2859-1 sampling plans with code letters A through F at inspection level II) can be used to establish that the shipment contains items similar to those ordered, particularly when all the items in the small grab sample conform to the item description and specification. A grab sample has none of the statistical characteristics of a proper random sample and a sampling plan with acceptance criteria. Grab sampling should only be used when the history of the vendor and the item have been good, or when conditions are such that the recipient cannot contribute to controlling or correcting the supplier's process and when the nonconformity is of such a nature that it will be easily recognized and nonconforming items will be replaced in subsequent assembly or test. Grab sampling is inappropriate for expensive items whose warranty does not survive lot acceptance.

A grab sample that is found to contain any nonconforming items is a signal to revert to the use of a standard sampling plan and to follow the normal decision process that such a sample indicates.

8.3 One-of-a-kind lots

An unusual situation exists when one specially produced, unique or 'one-off' lot fails to be accepted by the sampling plan. Resubmission of the same lot, which even after screening and/or correction contains many more nonconforming items than desired, or submission of a substitute lot, is likely to lead to an eventual acceptance due to cumulative sampling risk. Sampling plans have been designed to reduce the probability of the acceptance of a lot containing an excessive number of nonconformities, whether the lot be submitted once or resubmitted several times.*¹ This procedure changes the relationship between sample size and acceptance number for each successive resubmission of a rejected lot and limits the number of resubmissions allowed.

*¹ These sampling plans are not yet available as an ISO Standard. Defence Standard 05-58 of the UK Ministry of Defence contains plans for inspection (by attributes) of isolated lots. (DEF STAN 05-58 utilizes a later development of the principles presented in ISO Document ISO/TC 69/SC 5 N 47.)