TECHNICAL REPORT 8389

Published 1984-10-01

Building construction — Modular coordination — System of preferred numbers defining multimodular sizes

Construction immobilière — Coordination modulaire — Système de nombres préférentiels définissant les dimensions multimodulaires

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— type 1, when the necessary support within the technical committee cannot be obtained for the publication of an International Standard, despite repeated efforts ;

— type 2, when the subject is still under technical development requiring wider exposure ;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard (''state of the art'', for example).

Technical reports are accepted for publication directly by ISO Council. Technical reports types 1 and 2 are subject to review within three years of publication, to decide if they can be transformed into International Standards. Technical reports type 3 do not necessarily have to be reviewed until the data they provide is considered no longer valid or useful.

ISO/TR 8389 was prepared by Technical Committee ISO/TC 59, *Building construction*.

The reasons which led to the decision to publish this document in the form of a technical report type 2 are explained in the Introduction.

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0 Introduction

An attempt to develop a system of preferred modular sizes in building has been discussed since 1970.

The very idea of this subject has been challenged as unrealistic, since nearly every ISO member body involved in the discussions has its own theory of preferred modular sizes.

Real progress was first achieved when it was decided to draw up two draft documents:

a) a purely theoretical one, dealing only with numbers arranged in theoretical orders in an unlimited system (this document), and

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b)   a draft International Standard containing a limited series of multimodular sizes arranged in series based on an earlier choice of multimodule.

At the ISO/TC 59/SC 1 meeting in May 1977 it was decided that the numerical system should be presented in a technical report.

The reasons for this decision were

a)   Some committee members found that the theoretical approach had no practical purpose, for which reason they want it to be guidelines to other standardizers only.

b)   Some committee members found that the time was not ripe for introducing such an advanced system of guidance.

c)   Looking upon the system as a practical means of control in decentralized development, some members whose countries are highly industrialized in this field would have liked it to become a standard.

The final compromise of these conflicting viewpoints was to issue the document as a technical report in order to inform interested specialists of its content.

The rationale for the choice of this system, together with the reasons for which it has not been based on the series of preferred numbers given in ISO 3, *Preferred numbers — Series of preferred numbers*, is explained in the annex.

1   Scope

This technical report establishes a system of preference for numbers for use as the numerical factor in multimodular sizes especially for horizontal dimensions in building, and guidance for its use.

2   Field of application

This technical report is applicable to buildings of all types designed in accordance with the principles and rules of modular coordination as laid down in ISO 2848.

As the system is based on the number 3 and its multiples and as the flexibility in relation to halving and doubling of sizes is the main principle of the system, it is primarily suitable for horizontal dimensions.

3   References

ISO 1040, *Building construction — Modular coordination — Multimodules for horizontal coordinating dimensions*.

ISO 1791, *Building construction — Modular coordination — Vocabulary*.

ISO 2848, *Modular coordination — Principles and rules*.

4   Definitions

For the purposes of this technical report, the definitions given in ISO 1791 are applicable.

5   System

5.1   Values

The entire system includes all multiples of 3 arranged in doubling series. The doubling series are arranged as follows:

Series 1:	Numbers of the form $3 \times 1 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 3:	Numbers of the form $3 \times 3 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 5:	Numbers of the form $3 \times 5 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 7:	Numbers of the form $3 \times 7 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 9:	Numbers of the form $3 \times 9 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 11:	Numbers of the form $3 \times 11 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 13:	Numbers of the form $3 \times 13 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 15:	Numbers of the form $3 \times 15 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 17:	Numbers of the form $3 \times 17 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 19:	Numbers of the form $3 \times 19 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 21:	Numbers of the form $3 \times 21 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 23:	Numbers of the form $3 \times 23 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		
Series 25:	Numbers of the form $3 \times 25 \times 2^n$	(n	$= 0, 1, 2, 3 \ldots$)		

...

Series $2m + 1$ Numbers of the form $3 \times (2m + 1) \times 2^n$ (m and n $= 0, 1, 2, 3 \ldots$)

The second factor is the odd numbers which characterize the series.

The numbers of the system are given schematically in the table.

5.2 Characteristics

The system is based only on the mathematical properties of the numbers.

The table is arranged (with series of numbers in columns) in such a way that numbers in a lower series are more flexible in relation to halving than numbers of the same order of size in higher series.

This indicates that if many possibilities of halving are important, series 1 should be preferred to series 3, series 3 to series 5, etc.

Other considerations can however be of equal importance. Figures from series 3, 9, 15 and 21 contain supplementary factors of 3 and can therefore be divided again[1] by three still giving multimodular sizes; numbers in series 5, 15 and 25 contain the factor of 5 and can be divided by 5 in multimodular sizes.

Furthermore, considerations of a technical or functional nature can indicate that numbers from a higher series should have preference.

6 Application

This system of preferred numbers — which are all multiples of 3 — is developed as a tool for quick judgement of the mathematical properties of numbers which can be used for horizontal multimodular sizes in building.

The system can also be used for systematic arrangement of series of multimodular sizes by telescoping a limited number of series in the system.

The system can be limited by choosing upper limits.

The development of the system and further information on its practical application are included in the annex which is not intended to form an integral part of any future International Standard which might be based on this technical report.

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1) As all the numbers are multiples of 3, all sizes based on them can be divided into three equal modular, but not necessarily multimodular, sizes.
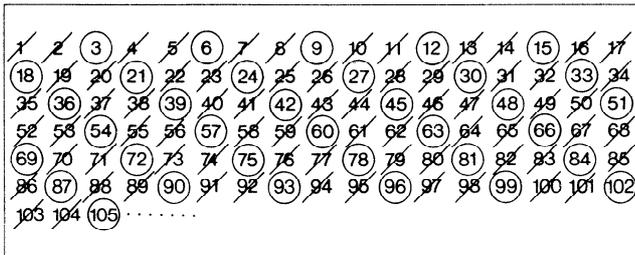
Table

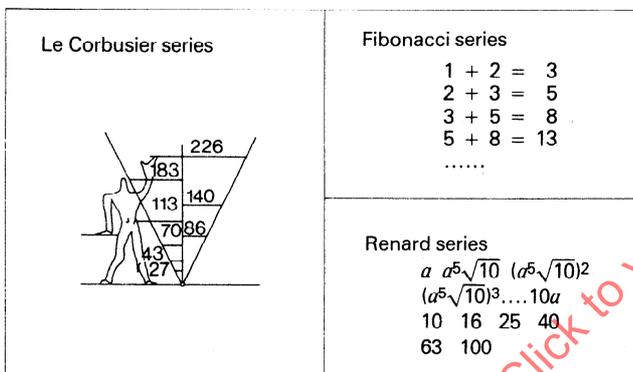| Multiples of 3	Doubling series														
	1	3	5	7	9	11	13	15	17	19	21	23	25	27	etc.
3	3														
6	6														
9		9													
12	12														
15			15												
18		18													
21				21											
24	24														
27					27										
30			30												
33						33									
36		36													
39							39								
42				42											
45								45							
48	48														
51									51						
54					54										
57										57					
60			60												
63											63				
66						66									
69												69			
72		72													
75													75		
78							78								
81														81	
84				84											
87															
90								90							
93															
96	96														
etc.															

Annex

Systematic arrangement of selected numbers

(This annex does not form an integral part of the technical report.)

The system is based upon a version of the normal series of numbers where all numbers which are not multiples of the figure 3 have been excluded.

In Modular Coordination (M-C) in building, the remaining numbers may often be especially applicable in connection with the designation of horizontal dimensions relating to structural subsystems, building envelopes, etc.

Le Corbusier series	Fibonacci series
	$1 + 2 = 3$
	$2 + 3 = 5$
	$3 + 5 = 8$
	$5 + 8 = 13$
	......

Renard series

$a \quad a^5\sqrt{10} \quad (a^5\sqrt{10})^2$
$(a^5\sqrt{10})^3 ....10a$
10 16 25 40
63 100

The desire to establish some kind of simple order in building — dimensionally — leads to a need for a systematic arrangement of the numbers and thus the dimensions to be used. Throughout the ages various such systems have been known. Among later systems may be mentioned the Fibonacci series, the le Corbusier series, and the Renard series which is used in engineering science.
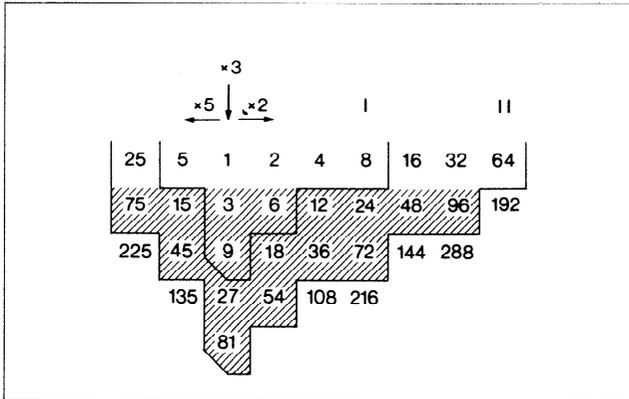
EPA system (1956)
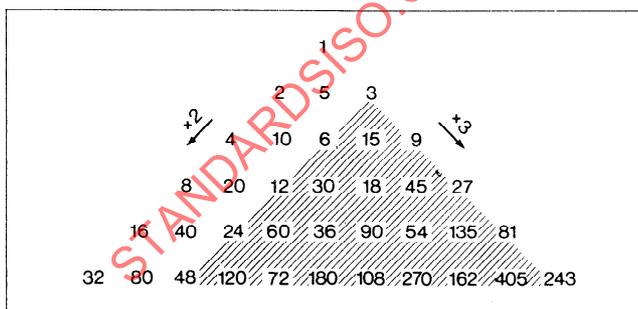
This system was developed by the European Productivity Agency (EPA) in the midfifties and formed the pattern for many early systems. It is based on the numbers 2, 3, and 5. Multiples of 3 are shaded.

Comecon system (1960)

Multiple of multimodules	Within the range of up to
3	36 /72/
6	72
12	72
15	120
30	180/240/
60	

The 1960 Comecon system includes arithmetical progression of sizes that are multiples to multimodules of 3, 6, 12, 15, 30, 60 (and accordingly to the same multiples of 2, 3, 5 as the sizes of the EPA system) having certain limits.

Japanese system (1966)

The Japanese system is a development of the EPA system, but incorporates a more complete series of sizes approximating to 3, 6, 12, 15, 30 and 60. This system allows also for multiplication of all sizes by 10.
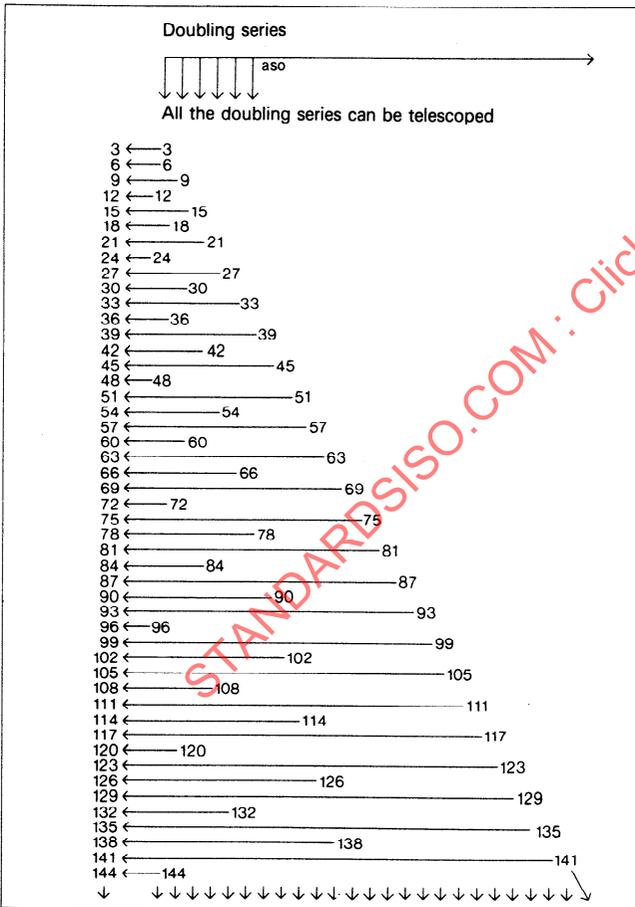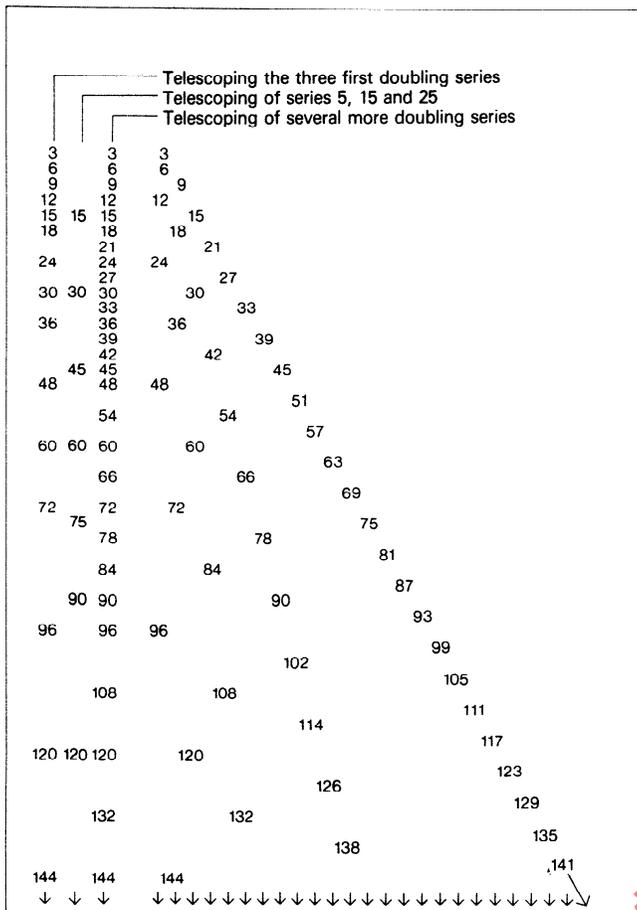
Lately one more system has been proposed for a systematic arrangement of numbers or dimensions. The basic unit is the figure 3 corresponding to the size 3M (1M = 100 mm).

The system starts with 3 and is infinite towards the right and downwards. The system consists of an infinite series of simple doubling series.

This is the system reflected in this Technical Report.

Due to the way the system is constructed, the doubling series can be telescoped into each other without any overlapping of numbers. If all the doubling series are telescoped to form just one series, this will naturally be identical with the original series of numbers, i.e. all multiples of 3.

Telescoping the three first doubling series
Telescoping of series 5, 15 and 25
Telescoping of several more doubling series

```
3     3     3
6     6     6
9     9     9
12    12    12
15 15 15    15
18    18    18
            21
24    24 24
         27
30 30 30    27
         33
36    36 36
         39
         42
45 45    42
48 45 48 45
48    48    51
54       54
            57
60 60 60 60
66       66
            63
            69
72 75 72 72
75 78    78    75
         84    81
90 90    90    87
96    96 96   93
         102   99
108      108   105
         114   111
120 120 120 120   117
         126   123
132      132   129
         138   135
144 144 144     141
↓   ↓   ↓   ↓↓↓↓↓↓↓↓↓↓↓↓↓↓↓↓↓↓↓↓↓
```

Increments

```
        3
        6
        9
        12
3       15
        18
        21
        24
                15

        30              30
6       36

        42              3
        48
                                24
                                27
                                30
                                33
                                36
                                39
                                42
                                45
                                48

                                54
        60      60      6       60
                                66

12      72              72
                                72
        84
                60     12       84
        96      60      96
                                96

(Beam spans
as example only)

24      120     120     120
        ↓

144
↓
```

Systematic telescoping of a limited number of doubling series will lead to a new series, but any such series will have the same properties as the simple doubling series.

It has been acknowledged internationally that the following properties are desirable:

a) All numbers are multiples of 3 (~ 3M).

b) All numbers can be divided by the largest possible number of smaller numbers.

c) All sizes are interrelated. Larger sizes can be obtained through either multiplication or addition of smaller sizes.

d) Small sizes are more closely spaced than larger sizes.

Telescoping of doubling series has been used as a basis for national standards and for draft International Standards.

The number of doubling series which are telescoped may vary.

In general, only a few series are telescoped if a limited selection of sizes is deemed to be sufficient, i.e. in connection with industrial building.

More series are usually telescoped when it is desired to have more sizes to choose between, i.e. in connection with traditional housing.
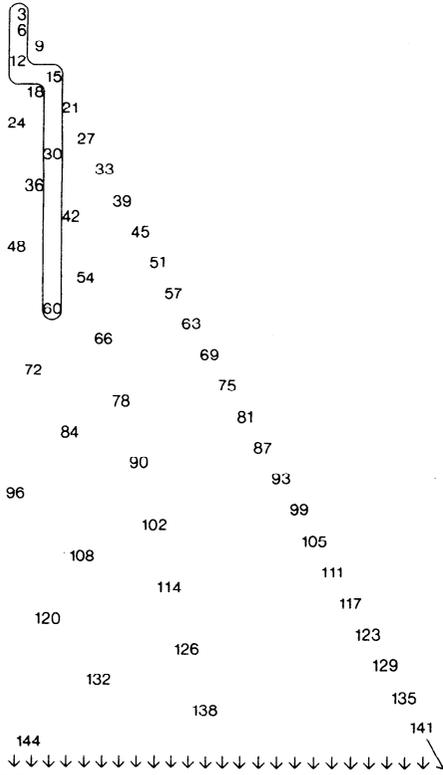
Not only consecutive series could be telescoped. As an example of this, the telescoping column 5, 15 and 25 is given.
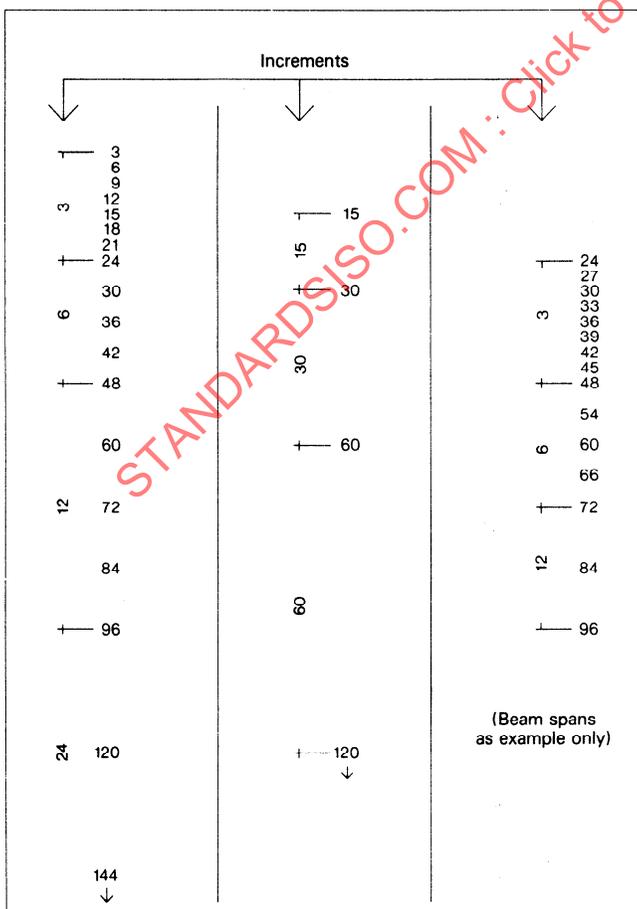
However, series are selected or created through telescoping, and regardless of how such series are presented, they will also be characterized by a certain grouping of increment steps.
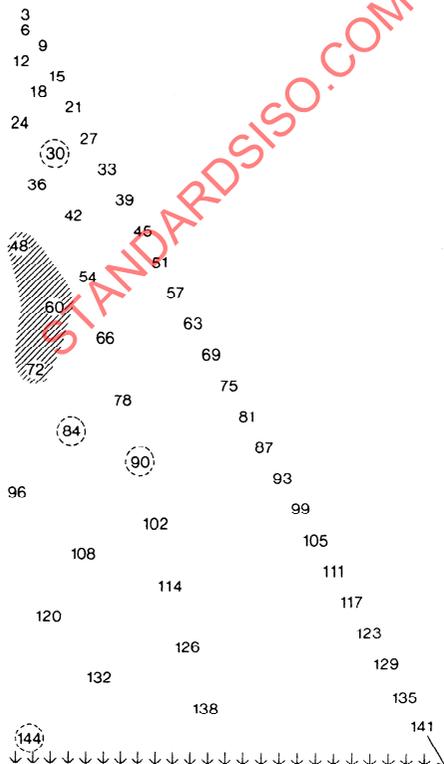
Some telescoped series, as for example series of dimensions for spans, do not contain the very smallest sizes and are not infinite. The simple reason is naturally that there are limits to how short it is reasonable to make a beam and how long a span can be.

Two limited doubling series for special purposes

3
6
9
12
15
18
21
24
27
30
33
36
39
42
45
48
51
54
57
60
63
66
69
72
75
78
81
84
87
90
93
96
99
102
105
108
111
114
117
120
123
126
129
132
135
138
141
144

The multimodules recommended in ISO 1040 also represent a selected, short series of numbers contained in the system presented here.

Cluster
Single number

3
6
9
12
15
18
21
24
27
30
33
36
39
42
45
48
51
54
57
60
63
66
69
72
75
78
81
84
87
90
93
96
99
102
105
108
111
114
117
120
123
126
129
132
135
138
141
144

It is also possible in the system to select a so-called cluster of sizes. As an example, production equipment which can manufacture beams economically within certain limits of length can be taken. A relevant cluster of sizes for spans can in this case be chosen in the system.

Single numbers may also be chosen in the system. In this connection the larger numbers will often be of special interest, to be used for the overall dimensions of buildings.

In a normal design procedure it is not uncommon first to lay out the overall pattern of a building, by deciding spans or the spacing of columns or walls.

Later the spans are then "filled out" with components such as windows, doors, partition components etc.

The more different ways especially the larger sizes used for spans, etc. can be divided up, the more different ways they can be filled out with regular modular components. The various numbers in the system have different properties in this respect. Some can be filled out in many different ways, others can only be filled out with a single component or with **3M** components.

The judgement of the flexibility of a number depends on the number of divisors i.e. the dimensions of components which can fill out.

Examples of two sets of divisors are given in the scheme

6, 9, 12, 15, 18 and

2, 3, 4, 5, 6, 8, 9, 10, 12, 15

and the numbers of possibilities for each example. Another type of flexibility is related to possible addition of components of different sizes, but it is not possible to give a simple indication of this.

In the same way that the larger sizes have different properties of subdivision, smaller sizes are different when it is checked how many of the larger sizes they can fill out when used additively.

Very small component sizes, as for example 6M, are naturally very flexible in use. But where telescoped series of sizes in the system are considered, a surprising result is that a size like **12M** will not only be more flexible in use than the larger sizes **15M** and **18M**, and so on, but also more flexible in use than the smaller size **9M**.

This mathematical fact is simple proof of why so many manufacturers have made the right choice in choosing **12M** rather than other neighbouring dimensions.

(Example only: Eight doubling series telescoped)