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ISO/TR 5168

First edition
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Measurement of fluid flow — Evaluation of uncertainties

Mesure de débit des fluides — Calcul de l'incertitude

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Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 5168, which is a Technical Report of type 1, was prepared by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits,* Subcommittee SC 9, *Uncertainties in flow measurement*.

This document is being issued as a type 1 Technical Report because no consensus could be reached between ISO TC 30/SC 9 and ISO/TAG 4, *Metrology,* concerning the harmonization of this document with the *Guide to the expression of uncertainty in measurement,* which is a basic document in the ISO/IEC Directives. A future revision of this Technical Report will align it with the *Guide*.

This first edition as a Technical Report cancels and replaces the first edition as an International Standard (ISO 5168:1978), which has been technically revised.

Annexes A and B form an integral part of this Technical Report. Annexes C, D, E and F are for information only.

Introduction

One of the first International Standards to specifically address the subject of uncertainty in measurement was ISO 5168, *Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement*, published in 1978. The extensive use of ISO 5168 in practical applications identified many improvements to its methods; these were incorporated into a draft revision of this International Standard, which in 1990 received an overwhelming vote in favour of its publication. However, this draft revision of ISO 5168 was withheld from publication for a number of years since, despite lengthy discussions, no consensus could be reached with the draft version of a document under development by a Working Group of ISO Technical Advisory Group 4, *Metrology* (ISO TAG 4/WG 3). The TAG 4 document, *Guide to the expression of uncertainty in measurement* (GUM), was published in late 1993 as a basic document in the ISO/IEC Directives. At a meeting of the ISO Management Board in May 1995 it was decided to publish the revision of ISO 5168, *Measurement of fluid flow — Evaluation of uncertainties,* as a Technical Report.

One of the major differences between ISO/TR 5168 and the GUM is in the definitions and terminology. In addition, a substantial difference exists with respect to the concepts to be used to define practical measurement processes. In this Technical Report a normal distribution of the measurement data is assumed and Student's t-factor is used to determine the uncertainty. The method used to propagate elemental uncertainties to the overall uncertainty is essentially identical to that used in the GUM.

This document is published as a type 1 Technical Report instead of an International Standard because it is not consistent with the GUM. A future revision of this Technical Report will align the two documents.

Measurement of fluid flow — Evaluation of uncertainties

1 Scope

Whenever a measurement of flowrate (discharge) is made, the value obtained from the experimental data is simply the best possible estimate of the true flowrate. In practice, the true flowrate may be slightly greater or less than this value.

1.1 This Technical Report details step-by-step procedures for the evaluation of uncertainties in individual flow measurements arising from both random and systematic error sources and for the propagation of component uncertainties into the uncertainty of the test results. These procedures enable the following processes to be carried out:

a) estimation of the accuracy of results derived from flowrate measurement;

b) selection of a proper measuring method and devices to achieve a required level of accuracy of flowrate measurement;

c) comparison of the results of measurement;

d) identification of the sources of errors contributing to a total uncertainty;

e) refinement of the results of measurement as data accumulate.

NOTE — It is assumed that the measurement process is carefully controlled and that all calibration corrections have been applied.

1.2 This Technical Report describes the calculations required in order to arrive at an estimate of the interval within which the true value of the flowrate may be expected to lie. The principle of these calculations is applicable to any flow measurement method, whether the flow is in an open channel or in a closed conduit.

NOTE — Although this Technical Report has been drafted taking mainly into account the sources of error due to the instrumentation, it should be emphasized that the errors due to the flow itself (velocity distribution, turbulence, etc.) and to its effect on the method and on the response of the instrument can be of great importance with certain methods of flow measurement (see 5.7). Where a particular device or technique is used, some simplifications may be possible or special reference may have to be made to specific sources of error not identified in this Technical Report. Therefore reference should be made to the "Uncertainty of measurement" clause of the appropriate International Standard dealing with that device or technique.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this Technical Report. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this Technical Report are encouraged to investigate the possibility of applying the most recent editions of the standards listed below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 5725-1:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 1: General principles and definitions.*

ISO 5725-2:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 2: Basic method for the determination of repeatability and reproducibility of a standard measurement method.*

ISO 5725-3:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 3: Intermediate measures of the precision of a standard measurement method.*

ISO 5725-4:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 4: Basic methods for the determination of the trueness of a standard measurement method.*

ISO 5725-6:1994, *Accuracy (trueness and precision) of measurement methods and results — Part 6: Use in practice of accuracy values.*

3 Definitions and symbols

For the purposes of this Technical Report, the following definitions and symbols apply.

3.1 Definitions

3.1.1 correction: Value which must be added algebraically to the indicated value to obtain the corrected result. It is numerically the same as a known error, but of opposite sign.

3.1.2 coverage: Percentage frequency at which an interval estimate of a parameter contains the true value. That is, in repeated sampling when the uncertainty interval provides 95 % coverage for each sample, over the long run the intervals will contain the true value 95 % of the time.

3.1.3 error: Result of a measurement minus the (conventional) true value of the measurement. See figure 1.

NOTE — The known parts of an error of measurement may be compensated by applying appropriate corrections. The error of the corrected result can be characterized by an uncertainty.

3.1.4 estimate: Value calculated from a sample of data as a substitute for an unknown population parameter.

For example, the experimental standard deviation (s) is the estimate which describes the population standard deviation (σ).

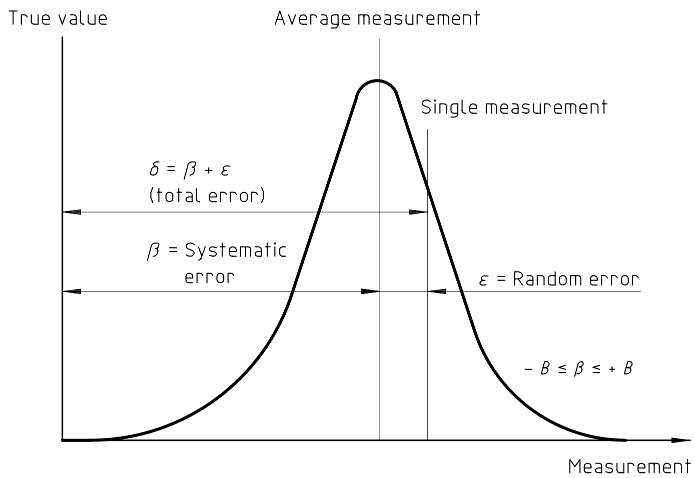

Figure 1 — Measurement error

3.1.5 fossilization: Creation of a fixed systematic error from a live random error when only a single calibration is relevant in the calibration process.

3.1.6 influence [sensitivity] coefficient: Uncertainty propagated to the result due to unit uncertainty of the measurement (see subclause 7.4).

3.1.7 observed value: Value of a characteristic determined as the result of an observation or test.

3.1.8 random error: See figure 2 and subclause 4.2.

3.1.9 random uncertainty: Component of the uncertainty associated with the random error. See figure 2.

3.1.10 statistical quality control chart: Chart on which limits are drawn and on which are plotted values of any statistic computed from successive samples of a population.

The statistics which are used (mean, range, percent defective, etc.) define the different kinds of control charts.

3.1.11 systematic error: See figures 2 and 3 and subclause 4.3.

3.1.12 systematic uncertainty: Component of the uncertainty associated with the systematic error. See figure 2.

3.1.13 Taylor's series: Power series to calculate the value of a function at a point in the neighbourhood of some reference point.

The series expresses the difference or differential between the new point and the reference point in terms of the successive derivatives of the function. Its form is:

$$f(x) - f(a) = \sum_{r=1}^{r=n-1} \frac{(x-a)^r}{r!} f^r(a) + R_n$$

where $f^r(a)$ denotes the value of the rth derivative of $f(x)$ at the reference point $x = a$. Commonly, if the series converges, the remainder R_n is made infinitesimal by selecting an arbitrary number of terms and usually only the first term is used.

3.1.14 uncertainty:

(1) Half the uncertainty interval, for a symmetrical uncertainty interval.

(2) The positive and negative components of a nonsymmetrical uncertainty interval, denoted by U^+ and U^- respectively.

3.1.15 uncertainty interval: Estimate characterizing the range of values within which the true value of a measurand is expected to lie.

3.1.16 Welch-Satterthwaite method: Method for estimating degrees of freedom of the result when combining experimental standard deviations with unequal degrees of freedom.

3.1.17 working standard: Standard, usually calibrated against a reference standard, which is used routinely to calibrate or check material measures or measuring instruments.

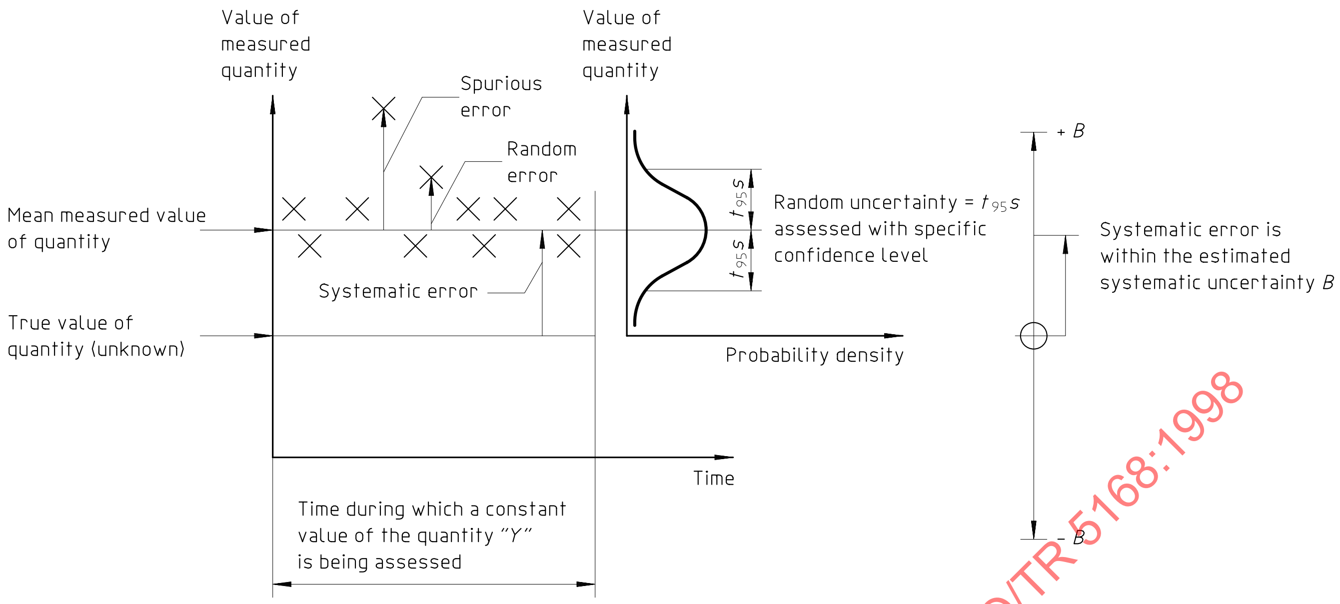

Figure 2 — Illustration of terms relating to errors and uncertainties

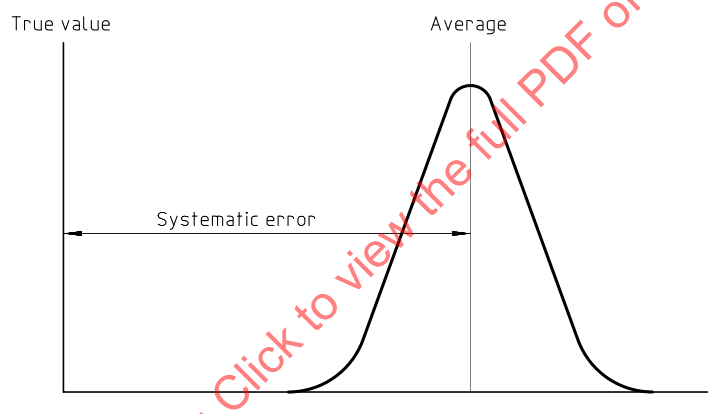

Figure 3 — Systematic error

3.2 Symbols

Symbol	Meaning
B	Systematic uncertainty of a symmetrical uncertainty interval. $B = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} B_{ij}^2}$
B_{ij}	Elemental systematic uncertainty. The j subscript indicates the category, i.e.: j = 1 calibration; = 2 data acquisition; = 3 data reduction; = 4 method; = 5 subjective or personal. The i subscript is the number assigned to a given elemental source of error. If i is more than a single digit, a comma is used between i and j.
B^+, B^-	Positive and negative systematic uncertainties of a nonsymmetrical uncertainty interval.

overbar (‾) Mean value (of a variable).

M Number of redundant instruments or tests.

N Sample size.

s^2 Unbiased estimate of the variance, σ^2.

s_{ij} Estimate of the experimental standard deviation from one elemental source. The subscripts are the same as the elemental systematic uncertainties in B_{ij}.

$$s = \sqrt{\sum_j \sum_i s_{ij}^2}$$

$s_{\bar{x}}$ Experimental standard deviation of the mean; equal to $\frac{s}{\sqrt{N}}$

$$s_{\text{pooled}} = \left[\frac{\sum_{i=1}^{M} \sum_{j=1}^{N} \left(x_{ij} - \bar{x}_i \right)^2}{M \left(N - 1 \right)} \right]^{1/2}$$

where

$\bar{x}_i$ is the arithmetic mean of all x_i at the jth datum point.

t_{95} Student's statistical parameter at the 95 % confidence level. The degrees of freedom, ν, of the sample estimate of the experimental standard deviation are needed to obtain the t values.

U^+, U^- Positive and negative uncertainties of a nonsymmetrical uncertainty interval.

U_{ADD} $= B + t_{95} s_{\bar{x}}$

U_{RSS} $= \sqrt{B^2 + \left(t_{95} s_{\bar{x}} \right)^2}$

$\bar{x}$ Arithmetic mean of the data values; x_i.

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

x_i Value of x at the ith datum point.

x_{ij} Value of x_i at the jth datum point.

$\bar{Y}$ Arithmetic mean of the n measurements of the variable Y.

Y_i A basic measurement.

β Systematic error, the fixed, or constant component of the total error, δ.

Δ Difference between measurements.

δ Total error.

ε Random error.

Θ_i Influence coefficient $\partial R / \partial Y_i$.

μ Population mean.

σ^2 Variance, the square of the standard deviation.

Subscripts

ADD Additive model.

RSS Root-sum-square model.

NOTE — In ISO 5168:1978 and in many standards for flowrate measurement, e is used to indicate absolute uncertainty and E is used for relative uncertainty. In this Technical Report U' is used for relative uncertainty.

4 General principles of measurement uncertainty analysis

4.1 Nature of errors

All measurements have errors even after all known corrections and calibrations have been applied. The errors may be positive or negative and may be of a variable magnitude. Many errors vary with time. Some have very short periods while others vary daily, weekly, seasonally or yearly. Those which remain constant or apparently constant during the test are called systematic errors. The actual errors are rarely known; however, upper bounds on the errors can be estimated. The objective is to construct an uncertainty interval (or sometimes referred to as range) within which the true value will lie with a stated probability.

Errors are the differences between the measurements and the true value which is always unknown. The total measurement error, δ, is divided into two components: β, a fixed systematic error and a random error, ε, as shown in figure 2. In some cases, the true value may be arbitrarily defined as the value that would be obtained by a specific metrology laboratory. Uncertainty is an estimate of the error which in most cases would not be exceeded. There are three types of error to be considered:

a) random errors — see 4.2;

b) systematic errors — see 4.3;

c) spurious errors or mistakes (assumed to be identified and rejected prior to statistical analysis) — see 4.4.

It is rarely possible to give an absolute upper limit to the value of the error. It is, therefore, more practicable to give an interval within which the true value of the measured quantity can be expected to lie with a suitably high probability. This "uncertainty interval" is shown as $\left[\bar{x} - U, \bar{x} + U\right]$ in figure 4 (the interval is twice the calculated uncertainty).

Since measurement systems are subject to two types of errors, systematic and random, it follows that an accurate measurement is one that has both small random and small systematic errors (see figure 5).

4.2 Random error

Random errors are caused by numerous, small, independent influences which prevent a measurement system from delivering the same reading when supplied with the same input value of the quantity being measured. The data points deviate from the mean in accordance with the laws of chance, such that the distribution usually approaches a normal distribution as the number of data points is increased. Random errors are sometimes referred to as precision errors. The standard deviation (σ) (see figure 6) is used as a measure of the random error, ε. A large standard deviation means large scatter in the measurements. The statistic (s) is calculated from a sample to estimate the standard deviation and is called the experimental standard deviation.

$$s = \sqrt{\frac{\sum_{i=1}^{N} \left(x_i - \bar{x}\right)^2}{N-1}} \qquad \text{. . . (1)}$$

where

N is the number of measurements;

$\bar{x}$ is the average value of individual measurements x.

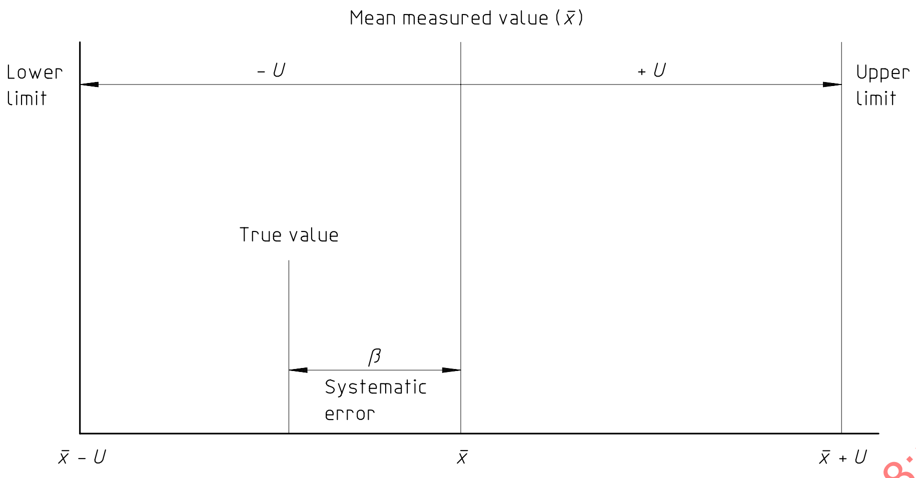

Figure 4 — Uncertainty interval $\bar{x} \pm U$ (see also figure 2)

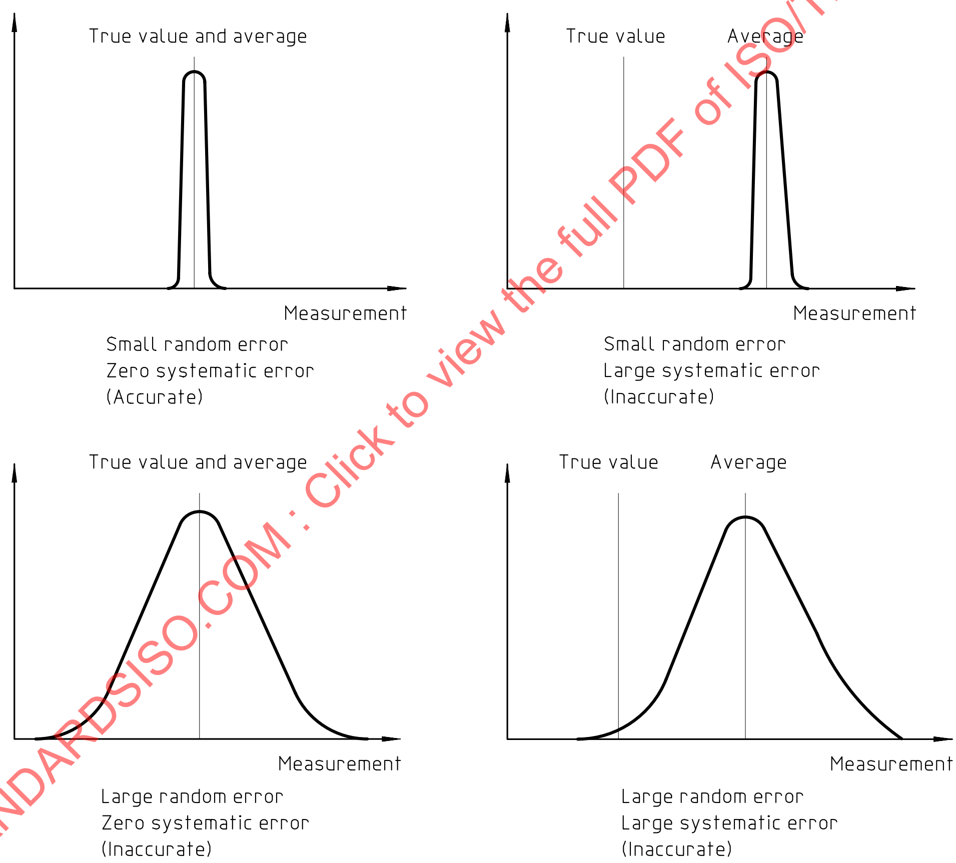

Figure 5 — Measurement error (systematic, random) and accuracy

For the normal distribution, the interval $\bar{x} \pm t_{95}\ s/\sqrt{N}$ will include the true mean, μ, approximately 95 % of the time. The random uncertainty of the mean is $t_{95}\ s/\sqrt{N}$. When the sample size is small, it is necessary to use the Student's t value at the 95 % level. For sample sizes equal to or greater than 30, two experimental standard deviations (2s) are used as an estimate of the random uncertainty in an individual measurement. This is explained in annex A.

The random uncertainty can be reduced by making as many measurements as possible and using the arithmetic mean value, since the standard deviation of the mean of N independent measurements is $\sqrt{N}$ times smaller than the standard deviation of the measurements themselves.

$$\sigma_{\text{average}} = \frac{\sigma_{\text{individual}}}{\sqrt{N}} \qquad \ldots (2)$$

and, analogously

$$s_{\bar{x}} = \frac{s}{\sqrt{N}} \qquad \ldots (3)$$

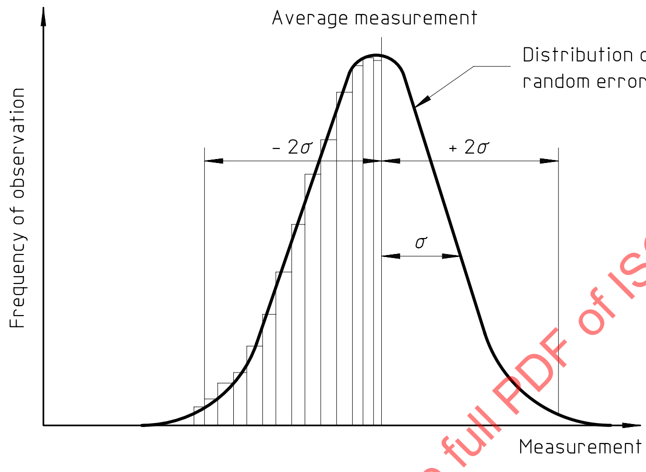

Figure 6 — Random error

4.3 Systematic error

The second component of the total error is the systematic error, β. At each flow level this error is constant for the duration of the test (figure 1). In repeated measurements of a given sample, each measurement has the same systematic error. The systematic error can be determined only when the measurements are compared with the true value of the quantity measured and this is rarely possible. Systematic errors are sometimes referred to as biases.

Every effort shall be made to identify and account for all significant systematic errors. These may arise from (1) imperfect calibration corrections, (2) imperfect instrumentation installation, (3) imperfect data reduction, and may include (4) method errors, and (5) human errors. As the true systematic error is never known, an upper limit, B, is used in the uncertainty analysis.

In most cases, the systematic error, β, is equally likely to be plus or minus about the measurement. That is, it is not known if the systematic error is positive or negative, and the systematic uncertainty reflects this as $\pm B$. The systematic uncertainty, B, is estimated as an upper limit of the systematic error, β.

4.4 Spurious errors

Spurious errors are errors, such as human mistakes or instrument malfunction, which invalidate a measurement; for example, the transposing of numbers in recording data or the presence of pockets of air in leads from a water line to a manometer. Such errors cannot be treated with statistical analysis and the measurement should be discarded. Every effort should be made to eliminate spurious errors to properly control the measurement process.

To ensure control, all measurements should be monitored with statistical quality control charts. Drifts, trends and movements leading to out-of-control situations should be identified and investigated. Histories of data from calibrations are required for effective control. It is assumed herein that these precautions are observed and that the measurement process is under control; if not, the methods described are invalid.

After all obvious mistakes have been corrected or removed, there may remain a few observations which are suspicious solely because of their magnitude.

For errors of this nature, the statistical outlier tests given in annex B should be used. These tests assume the observations are normally distributed. It is necessary to recalculate the experimental standard deviation of the distribution of observations whenever a datum is discarded as a result of the outlier test. It should also be emphasized that outliers should not be discarded unless there is an independent technical reason for believing that spurious errors may exist: data should not lightly be thrown away.

4.5 Combining elemental uncertainties

The test objective, test duration and the number of calibrations related to the test affect the classification of uncertainties into systematic and random components. Guidelines are presented in clause 6.

After all elemental error sources have been identified and classified as calibration, data acquisition, data reduction, methodic and subjective error sources and elemental standard deviations and systematic uncertainties estimated for each error source, a method for combining these elemental components into the experimental standard deviation and systematic uncertainty of the measurement is needed. The root-sum-square or quadrature combination is recommended.

$$s = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} s_{ij}^2} \qquad \text{. . . (4)}$$

$$B = \sqrt{\sum_{\text{all } j} \sum_{\text{all } i} B_{ij}^2} \qquad \text{. . . (5)}$$

4.6 Uncertainty of measurements

The measurement uncertainty analysis will be completed when:

a) the systematic uncertainties and standard deviations of the measure have been propagated to uncertainty in the test result, keeping systematic and random components separate;

b) if small samples are involved, an estimate of the degrees of freedom of the experimental standard deviation of the test result has been calculated from the Welch-Satterthwaite formula (see annex A);

c) the random and systematic uncertainties are combined into a single number to express a reasonable value for the overall uncertainty.

For simplicity of presentation, a single number, U, is needed to express a reasonable limit of error. The single number, some combination of the systematic and random uncertainties, must have a simple interpretation (e.g. the largest error reasonably expected), and be useful without complex explanation. For example, the true value of the measurement is expected to lie within the interval

$$\left[\bar{x} - U, \bar{x} + U\right] \qquad \text{. . . (6)}$$

Since systematic uncertainties include those based on judgement and not on data, there is no way of combining systematic and random uncertainties to produce a single uncertainty figure with a statistically rigorous confidence level. However, since it is accepted that a single figure for the uncertainty of a measurement is often required, two alternative methods of combination are permitted:

1) linear addition:

$$U_{\text{ADD}} = B + t_{95} s_{\bar{x}} \qquad \text{. . . (7)}$$

2) root-sum-square combination:

$$U_{\text{RSS}} = \sqrt{B^2 + \left(t_{95} s_{\bar{x}}\right)^2} \qquad \text{. . . (8)}$$

where B is the systematic uncertainty from equation (5) and s_x is the experimental standard deviation of the mean [equations (4) and (3)]. If large samples ($N > 30$) are used to calculate s, the value 2,0 may be used for t_{95} for simplicity. If small samples ($N \leqslant 30$) are used to calculate s, the methods in annex A are required. There are two situations where it is possible to use a statistical confidence level for the uncertainty interval:

a) if the systematic uncertainty is based on interlaboratory comparisons (see ISO 5725); and

b) if the systematic uncertainty is judged to be negligible compared to the random uncertainty. Here the uncertainty interval is the test result $\pm t_{95} s_{\bar{x}}$, which is at the 95 % confidence level.

Typically U_{RSS} is considered to have coverage of approximately 95 %, and U_{ADD} is considered to have coverage between 95 % and 99 %.

4.7 Propagation of measurement uncertainties to test result uncertainties

If the test result is a function of several measurements, the experimental standard deviations and systematic uncertainties of the measurements must be combined or propagated to the test result using sensitivity factors, Θ, that relate the measurement to the test result (see 7.4). Small sample methods are given in annex A.

In general, for m measurements, the experimental standard deviation and systematic uncertainty of the test result are obtained as follows:

$$s_R = \sqrt{\sum_{\text{all } m} (\Theta_m s_m)^2} \qquad \text{. . . (9)}$$

$$B_R = \sqrt{\sum_{\text{all } m} (\Theta_m B_m)^2} \qquad \text{. . . (10)}$$

The overall uncertainty for the test result is formed in the same manner as described for the measurement in 4.6.

4.8 Uncertainty analysis before and after measurement

Uncertainty analysis before measurement allows corrective action to be taken prior to the test to reduce uncertainties when they are too large or when the difference to be detected in the test is the same size or smaller than the predicted uncertainty. Uncertainty analysis before the test identifies the most cost-effective corrective action and the most accurate measurement method.

The before-measurement uncertainty analysis is based on data and information that exists before the test, such as calibration histories, previous tests with similar instrumentation, prior measurement uncertainty analysis, expert opinions and, if necessary, special tests. With complex tests, there may be alternatives to evaluate prior to the test such as different test designs, instrumentation arrangements, alternative calculation procedures and concomitant variables. Corrective action resulting from this before-measurement analysis may include:

a) improvements to instrumentation if the uncertainties are unacceptably high;

b) selection of a different measurement or calibration method;

c) repeated testing and/or increased sample sizes if the random uncertainties are unacceptably high. The experimental standard deviation of the mean is reduced as the number of samples used to calculate the mean is increased;

d) instead of repeated testing, the test duration may be extended in order to average the output scatter (noise) of the flowmeter, resulting in a smaller random error per observation and hence a smaller random uncertainty;

NOTE — For example, ultrasonic and vortex shedding meters may have to be calibrated against a master meter, allowing longer test times than allowed by a compact prover.

e) rotating flowmeters normally generate an output showing a periodic cycle superimposed on an average meter factor. In this case the test duration shall be matched to an integer multiple of half or full periodic intervals in order to obtain the shortest test times.

After-measurement analysis is based on the actual measurement data. It is required to establish the final uncertainty. It is also used to confirm the before-measurement estimates and/or to identify data validity problems. When redundant instrumentation or calculation methods are available, the individual uncertainties should be compared for consistency with each other and with the before-measurement uncertainty analysis. If the uncertainty intervals do not overlap, a problem is indicated. The after-measurement random uncertainties should be compared with the before-measurement predictions.

5 Identification and classification of elemental measurement error sources

5.1 Summary of procedure

Make a complete list of every possible source of measurement error for all measurements that affect the end test result. For convenience, group them by some or all of the following categories:

a) calibration,

b) data acquisition,

c) data reduction,

d) errors of method and

e) subjective or personal.

Within each category, there may be systematic and/or random error.

5.2 Systematic versus random

Systematic errors are those which remain constant in the process of measurement for a given value of flowrate.

Typical examples of systematic errors of flowrate measurements are:

a) errors from a single flowmeter calibration;

b) errors of determination of the constants in the working formula of a measuring method;

c) errors due to truncating instead of rounding off the results of measurement.

Where the value and sign of a systematic error are known, it is assumed to be corrected (the correction being equal in value and opposite in sign to the systematic error). Inaccuracy of the correction results in a residual systematic uncertainty.

Random errors are those that produce variation (not predictable) in repeated measurements of the same quantity.

Typical random errors associated with flowrate measurement are those caused by inaccurate reading of the scale of a measuring instrument or by the scatter of the output signal of an instrument.

The effect of random errors on the random uncertainty may be reduced by averaging multiple results of the same value of the quantity.

The preliminary decision to determine if a given elemental source contributes to systematic uncertainty, random uncertainty or both, is made by adopting the following recommendation: the uncertainty of a measurement should be put into one of two categories depending on how the uncertainty is derived. The value of a random uncertainty is derived by a statistical analysis of repeated measurements, while that of a systematic uncertainty is estimated by nonstatistical methods.

This recommendation avoids a complex decision and keeps the statistical estimates separate from the judgement estimates as long as possible. The decision is preliminary and will be reviewed after consideration of the defined measurement process.

5.3 Categorization of elemental error sources

Possible error sources can be divided arbitrarily into three to five categories:

1) calibration (see 5.4);
2) data acquisition (see 5.5);
3) data reduction (see 5.6);
4) method-related (see 5.7);
5) subjective or personal (see 5.8).

The size and complexity of the measurement uncertainty analysis may lead to the use of any or all of these categories.

For example, metrological maintenance (calibration, verification, certification) of flowmeters, flowrate measurements and processing of the data are done by different personnel. To control the possible sources of error, it is advisable to relate them to the stages of preparation, measurement and processing of the data.

In such cases, it is advisable to classify error sources into:

a) calibration error sources (see 5.4);
b) errors of measurement or data acquisition error sources (see 5.5);
c) errors of processing the measurement data or data reduction error sources (see 5.6).

5.4 Calibration error sources

The major purpose of the calibration process is to determine systematic errors in order to eliminate them. Thus, the calibration process exchanges the large systematic uncertainty of an uncalibrated or poorly calibrated instrument for the smaller combination of the systematic uncertainty of the reference instrument and the random uncertainty of the comparison. This exchange of uncertainties is fundamental and is the basis of the notion that the uncertainty of the standard should be substantially less than that of the test instrument.

Figure 7 shows a typical transducer calibration hierarchy. Each calibration in this hierarchy constitutes an error source, with which is associated a pair of elemental uncertainties — the systematic uncertainty and the experimental standard deviation of the process. It should be noted that, from one step to another, these elemental uncertainties, as listed in table 1, may be cumulative or independent. For example, B_{21} may include B_{11}. The second digit of the subscript indicates the category, i.e. 1 indicates calibration.

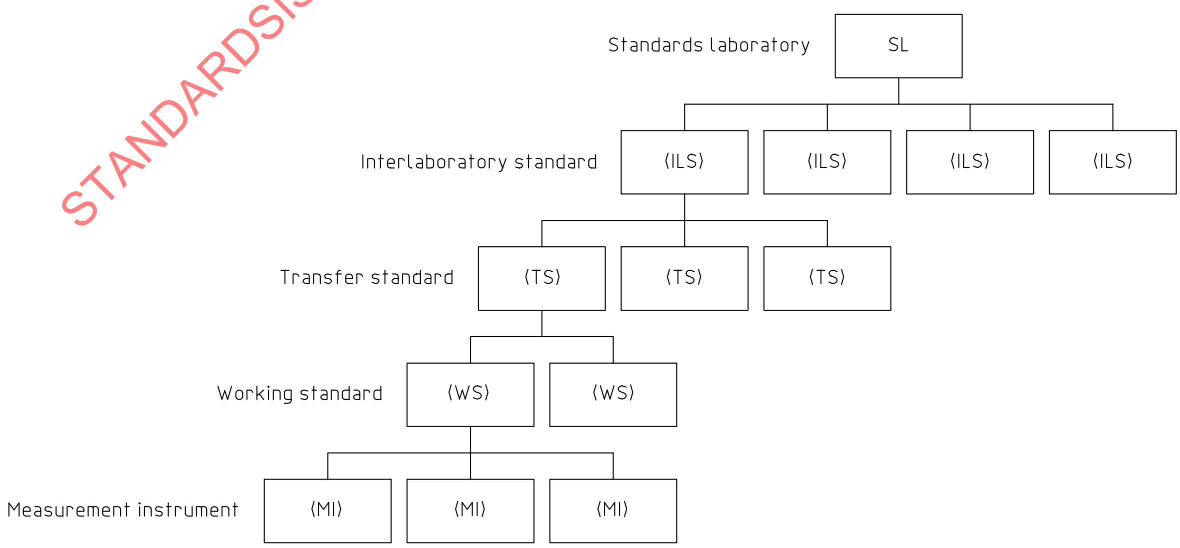

Figure 7 — Basic measurement calibration hierarchy

5.5 Data acquisition error sources

Figure 8 illustrates some of the error sources associated with a typical pressure data acquisition system. Data are acquired by measuring the electrical output resulting from pressure applied to a strain gauge type pressure-measurement instrument. Other error sources, such as probe errors, installation effects and environmental effects, also may be present. The effects of these error sources should be determined by performing overall system calibrations, comparing known applied pressures with measured values. However, should it not be possible to do this, then it is necessary to estimate each of the elemental uncertainties and combine them to determine the overall uncertainty.

Some of the data acquisition error sources are listed in table 2. Symbols for the elemental systematic uncertainties and the experimental standard deviations and for the degrees of freedom are shown. Note these elemental uncertainties are independent, not cumulative.

Table 1 — Calibration hierarchy error sources

Calibration	Systematic uncertainty	Experimental standard deviation	Degrees of freedom
SL — ILS	B_{11}	s_{11}	ν_{11}
ILS — TS	B_{21}	s_{21}	ν_{21}
TS — WS	B_{31}	s_{31}	ν_{31}
WS — MI	B_{41}	s_{41}	ν_{41}

Table 2 — Data acquisition error sources

Error source	Systematic uncertainty	Experimental standard deviation	Degrees of freedom
Excitation voltage	B_{12}	s_{12}	ν_{12}
Signal conditioning	B_{22}	s_{22}	ν_{22}
Recording device	B_{32}	s_{32}	ν_{32}
Pressure transducer	B_{42}	s_{42}	ν_{42}
Probe errors	B_{52}	s_{52}	ν_{52}
Environmental effects	B_{62}	s_{62}	ν_{62}
Spatial averaging	B_{72}	s_{72}	ν_{72}

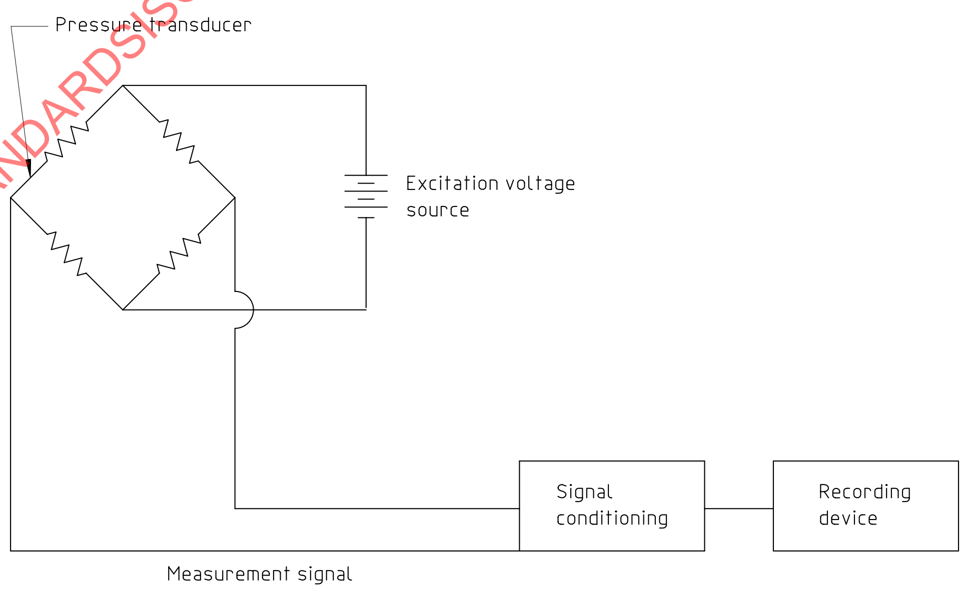

Figure 8 — Data acquisition system

5.6 Data reduction error sources

Computations on raw data produce output in the required engineering units. Typical errors in this process stem from curve fits and computational resolution. Uncertainties associated with these error sources are often negligible.

Symbols for the data reduction error sources are listed in table 3.

Table 3 — Data reduction error sources

Error sources	Systematic uncertainty	Experimental standard deviation	Degrees of freedom
Curve fit	B_{13}	s_{13}	v_{13}
Computational resolution	B_{23}	s_{23}	v_{23}

5.7 Method error sources

Errors of method are those associated with a particular measurement procedure (principles of use of instruments) and also with the uncertainty of constants used in calculations.

Some examples are errors from indirect methods of flowrate measurement associated with physical inaccuracy of the relationship between the measured quantity and flowrate, or with inaccuracy of the constants in the relationship. These inaccuracies may be due, for instance, to the fact that the flow conditions prevailing during the measurement are not identical to the conditions in which the calibration has been carried out or for which a standardized discharge coefficient has been established. In certain methods of flow measurement (differential pressure devices for instance), these sources of error arising from the flow conditions are covered by the uncertainty associated with the discharge coefficient, as far as the installation conditions prescribed in the standard are satisfied; if they are not, that standard does not apply. In other methods (velocity-area method for instance), the uncertainty arising from the flow conditions is identified as a component of the total uncertainty; it shall be evaluated by the user in each case and combined with the other elemental uncertainties.

5.8 Subjective error sources

Subjective error sources are caused by personal characteristics of the operators who calibrate flowmeters, perform measurements and process the data. These can include reading errors and miscalculations.

6 Estimation and presentation of elemental uncertainties

6.1 Summary of procedure

Obtain an estimate of each elemental uncertainty. If the data is available to estimate the experimental standard deviation, classify the uncertainty as a random uncertainty. Otherwise, classify it as a systematic uncertainty.

Review the test objective, test duration and number of calibrations that will affect the test result. Make the final classification of elemental uncertainties for each measurement. If an error increases the scatter in the measurement result in the defined test, it is a random error; otherwise, it is a systematic error.

6.2 Calculation of experimental standard deviation

There are many ways to calculate the experimental standard deviation.

a) If the parameter to be measured can be held constant, a number of repeated measurements can be used to evaluate equation (1), repeated here:

$$s = \sqrt{\frac{\sum_{i=1}^{N} \left(x_i - \bar{x}\right)^2}{N-1}} \qquad \ldots (11)$$

b) If there are M redundant instruments or M redundant measurements and the parameter to be measured can be held constant to take N repeat readings, the following pooled estimate of the experimental standard deviation for individual readings can be used:

$$s_{\text{pooled}} = \left[\frac{\sum_{i=1}^{M} \sum_{j=1}^{N} \left(x_{ij} - \bar{x}_i\right)^2}{M\left(N-1\right)}\right]^{1/2} \qquad \ldots (12)$$

NOTE — Here ij is used differently from elsewhere in the text.

For the experimental standard deviation of the average value of the parameter

$$s_{\bar{x}} = \frac{s_{\text{pooled}}}{\sqrt{MN}} \qquad \ldots (13)$$

c) If a pair of instruments (providing measurements x_{1i} and x_{2i} which have the same experimental standard deviation are used to estimate a parameter that is not constant with time, the difference between the readings, Δ, may be used to estimate the experimental standard deviation of the individual instruments as follows:

$$s = \left[\frac{\sum_{i=1}^{N} \left(\Delta_i - \bar{\Delta}\right)^2}{2\left(N-1\right)}\right]^{1/2} \qquad \ldots (14)$$

where

$$\bar{\Delta} = \frac{1}{N} \sum_{i=1}^{N} \Delta_i$$

$$\Delta_i = x_{1i} - x_{2i}$$

If the degrees of freedom are less than 30, the small sample methods shown in annex A are required.

6.3 Estimation of systematic uncertainty

In spite of applying all known corrections to overcome imperfections in calibration, data acquisition and data reduction processes, some systematic errors will probably remain. To determine the exact systematic error in a measurement, it would be necessary to compare the true value and the measurements. However, as the true value is unknown, it is necessary to carry out special tests or utilize existing data that will provide systematic uncertainty information. The following examples are given, in order of preference.

a) Interlaboratory or interfacility tests make it possible to obtain the distribution of systematic errors between facilities (see ISO 5725).

b) Comparisons of standards with instruments in the actual test environment may be used.

c) Comparison of independent measurements that depend on different principles can provide systematic uncertainty information. For example, in a gas turbine test, airflow can be measured with (1) an orifice, (2) a bellmouth nozzle, (3) compressor speedflow rig data, (4) turbine flow parameters and (5) jet nozzle calibrations.

d) When it is known that a systematic error results from a particular cause, calibrations may be performed allowing the cause to vary through its complete range to determine the range of systematic error.

e) If there is no source of data on which to estimate the systematic uncertainty, the estimate must be based on judgement. An estimate of an upper limit of the systematic error is needed. Instrumentation manufacturers' reports and other references may provide information. It is important to distinguish between the “estimate” of an upper limit on systematic error obtained by this method and the more reliable estimate of a random uncertainty arrived at by analysing data. There is a general tendency to underestimate systematic uncertainties when a subjective approach is used, partly through human optimism and partly through the possibility of overlooking the existence of some sources of systematic error. Great care is therefore necessary when quoting systematic uncertainties.

f) If the mean of results from redundant instruments or measurements differs by more than has been predicted by individual uncertainties, then a source of systematic uncertainty has been overlooked.

Sometimes the physics of the measurement system provide knowledge of the sign but not the magnitude of the systematic error. For example, hot thermocouples radiate and conduct thermal energy from the sensor to indicate lower temperatures. The systematic uncertainties interval in this case is nonsymmetrical, i.e. not of the form $\pm B$. It is of the form B^+ for the positive and B^- for the negative uncertainty. Thus, typical systematic uncertainties associated with a radiating thermocouple could be:

$B^+ = 0°$

$B^- = -10°$

For elemental uncertainties, the interval from B^+ to B^- shall include zero.

6.4 Final uncertainty classification based on the defined measurement process

Uncertainty statements must be related to a well-defined measurement process. The final classification of uncertainties into systematic and random depends on the definition of the measurement process. Some of these considerations are:

a) long versus short-term testing (see 6.4.1);

b) comparative versus absolute testing (see 6.4.2);

c) averaging to reduce random error (see 6.4.3).

6.4.1 Long versus short-term testing

The calibration histories accumulated before, during or after the testing period may influence the uncertainty analysis.

a) When the instrumentation is calibrated only once, all the calibration uncertainty is frozen into systematic uncertainty. The error in the calibration correction is a constant and cannot increase the scatter in a test result. Thus, the calibration uncertainty, made up in general of systematic and fossilized random uncertainties, is considered to be all systematic uncertainties in this case.

b) If the test period is long enough that instrumentation may be calibrated several times and/or several test stands are involved, the random error in the calibration hierarchy (see 5.4) should be treated as contributing to the overall experimental standard deviation. The experimental standard deviations may be derived from calibration data.

6.4.2 Comparative versus absolute testing

The objective of a comparative test is to determine, with the smallest measurement uncertainty possible, the net effect of a change. The first test is run with the standard or baseline configuration. The second test is run with the change. The difference between the results of these tests is an indication of the effect of the change. As long as only the difference or net effect between the two tests is considered, all systematic errors, being fixed, will cancel out. The measurement uncertainty will be composed of random errors only.

All errors in a comparative test arise from random errors in data acquisition and data reduction. Systematic errors are effectively zero. Since calibration random errors have been considered systematic errors, they also are effectively zero.

The test result is the difference in flowrate between two test results, r_1 and r_2.

$$\Delta r = r_1 - r_2 \qquad \ldots (15)$$

and

$$s_{\Delta r} = \sqrt{s_{r_1}^2 + s_{r_2}^2} = \sqrt{2} s_{r_1} \qquad \ldots (16)$$

where s_{r_1} is the experimental standard deviation of the random error of the first test, $s_{\Delta r}$ is the root-sum-square of the experimental standard deviations from data acquisition and data reduction, and s_{r_2} is assumed to equal s_{r_1}.

6.4.3 Averaging to reduce random error

Averaging test results is often used to improve the random uncertainty. Careful consideration should be given to designing the test series to average as many causes of variation as possible within cost constraints. The design should be tailored to the specific situation. For example, if experience indicates time-to-time and rig-to-rig variations are significant, a design that averages multiple test measurement results on one rig on one day may produce optimistic random uncertainty compared to testing several rigs, each mounted several times, over a period of weeks. The list of possibilities may include the above plus test stand-to-test stand, instrument-to-instrument, mount-to-mount and environmental, power supply and test crew variation. Historic data is invaluable for studying these effects.[1)] If the pretest uncertainty analysis identifies unacceptably large elemental uncertainties, special tests to measure the effects should be considered.

6.5 Example of uncertainty classification: a calibration constant

The final classification of elemental uncertainty depends on the defined measurement process. To illustrate, assume a test meter is to be compared or calibrated with a master meter at one flow level. The objective is to determine a correction, called a calibration constant, that will be added to the test meter observations when it is installed for test. This calibration constant correction will, over a limited time period, make the test meter "read like" the master meter. During the calibration, the master meter is used to set the flow level, as it is normally more accurate than the test meter. To reduce the calibration random uncertainty, N = 13 comparisons will be made and averaged. If the data were plotted, the data might look like figure 9.

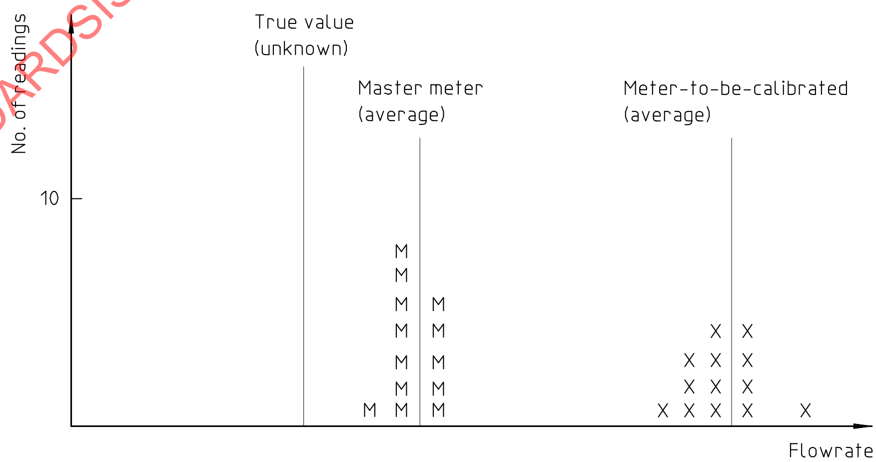

Figure 9 — Calibration should compensate for test meter systematic error

1) A statistical technique, analysis of variance (ANOVA) is useful for partitioning total variance by cause.

If the master meter systematic uncertainty derived from its own calibration is judged to be no larger than B_M, what will the test meter uncertainty be after calibration?

Define Δ_i = Master meter reading$_i$ – Test meter reading$_i$.

Calibration constant K equals the average

$$K = \bar{\Delta} = \frac{\sum \Delta_i}{13} \qquad \text{. . . (17)}$$

The experimental standard deviation of the calibration constant K is:

$$s_K = \frac{s_\Delta}{\sqrt{13}} = \left[\frac{\sum (\Delta_i - \bar{\Delta})^2}{13 \times 12} \right]^{1/2} \qquad \text{. . . (18)}$$

The test meter is later installed in a test stand. Each observation made on the test meter is corrected by adding K. By this process, the error in K from the calibration process is propagated to the corrected data from the test stand.

If the defined measurement process is short, involving a single calibration, K is constant and this error is a constant or systematic error. The uncertainty includes the systematic uncertainty in the master meter plus the random uncertainty in the calibration process. The random uncertainty is fossilized into systematic uncertainty. The fossilization is indicated by an asterisk. The systematic uncertainty may be estimated:

$$B_K = \sqrt{B_M^2 + (t_{95} s_K)^{*2}} \qquad \text{. . . (19)}$$

where

B_M is the systematic uncertainty of the master meter;

t_{95} = 2,179 for 12 degrees of freedom (see annex A).

This calibration systematic uncertainty should be combined with systematic uncertainties arising from other sources to obtain the systematic uncertainty of the measurement. There may also be random uncertainties arising from these other sources.

If the uncalibrated test meter had a systematic uncertainty judged to be B_T, the calibration process improved the test accuracy if B_K is less than B_T. Note that the calibration process does not change the test meter random uncertainty, which is included in the data acquisition random uncertainty. However, the test meter random uncertainty contributes to the calibration random uncertainty. This contribution is reduced by averaging the calibration data.

If the test process is long and involves several calibrations, the calibration error contributes both systematic uncertainty (B_M) and random uncertainty ($t_{95} s_K$) to the final test result.

If the test process is comparative (the difference between two tests with a single calibration), the calibration error is all systematic error and cancels out when one result is subtracted from the other.

7 Combination and propagation of uncertainties

7.1 Summary of procedure

For each measurement, combine separately the elemental systematic uncertainties and the elemental experimental standard deviations by the root-sum-square method. Propagate the measurement systematic uncertainty and the experimental standard deviation separately all the way to the final test result, either by sensitivity coefficients or by finitely incrementing the data reduction program. Work consistently in either absolute units or percentages.

7.2 Combining elemental experimental standard deviations

The experimental standard deviation (s) of the measurement is the root-sum-square of the elemental experimental standard deviations from all sources, that is:

$$s = \sqrt{\sum_{j=1}^{5} \sum_{i=1}^{k} s_{ij}^2} \quad \text{. . . (20)}$$

where j defines the category, such as (1) calibration, (2) data acquisition, (3) data reduction, (4) errors of method and (5) subjective or personal, and i defines the sources within the categories.

For example, the experimental standard deviation for the calibration process in table 1 is:

$$s_1 = s_{\text{calibration}} = \sqrt{\sum_{i=1}^{4} s_{i1}^2} = \sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2} \quad \text{. . . (21)}$$

The measurement experimental standard deviation is the root-sum-square of all the elemental experimental standard deviations in the measurement system:

$$s = s_{\text{measurement}} = \sqrt{\sum_{j=1}^{5} s_j^2} = \sqrt{s_1^2 + s_2^2 + s_3^2 + s_4^2 + s_5^2} \quad \text{. . . (22)}$$

7.3 Combining elemental systematic uncertainties

If there were only a few sources of elemental systematic uncertainties, it might be reasonable to add them together to obtain the overall systematic uncertainties. For example, if there were three sources, the probability that they would all be plus (or minus) would be one-half raised to the third power, or one-eighth. However, the probability that all three will have the same sign and be at the limit of the systematic uncertainties is extremely small. In actual practice, most measurements will have ten, twenty or more sources of systematic error. The probability that they would all be plus (or minus) and be at their limit is close to zero, and therefore it is more appropriate to combine them by root-sum-square.

If a measurement uncertainty analysis identifies four or less sources of systematic uncertainty, there should be some concern that some sources have been overlooked. The analysis should be random and expert help should be recruited to examine the calibration hierarchy, the data acquisition process and the data reduction procedure for additional sources.

Therefore, the systematic uncertainty will be used herein as the root-sum-square of the elemental systematic uncertainties from all sources.

$$B = \sqrt{\sum_{j} \sum_{i} B_{ij}^2} \quad \text{. . . (23)}$$

For example, the systematic uncertainty for the calibration hierarchy (table 1) is

$$B_1 = B_{\text{cal}} = \sqrt{B_{11}^2 + B_{21}^2 + B_{31}^2 + B_{41}^2} \quad \text{. . . (24)}$$

The systematic uncertainty for the basic measurement process is

$$B = \sqrt{B_1^2 + B_2^2 + B_3^2 + B_4^2 + B_5^2} \quad \text{. . . (25)}$$

If any of the elemental systematic uncertainties are nonsymmetrical, separate root-sum-squares are used to obtain B^+ and B^-. For example, assume B_{21}^+, B_{21}^-, B_{23}^+ and B_{23}^- are available. Then

$$B^+ = \sqrt{B_{11}^2 + \left(B_{21}^+\right)^2 + B_{31}^2 + B_{41}^2 + B_2^2 + B_{13}^2 + \left(B_{23}^+\right)^2} \quad \text{. . . (26)}$$

$$B^- = \sqrt{B_{11}^2 + \left(B_{21}^-\right)^2 + B_{31}^2 + B_{41}^2 + B_2^2 + B_{13}^2 + \left(B_{23}^-\right)^2} \quad \text{. . . (27)}$$

7.4 Propagation of measurement uncertainties

Fluid flow parameters are rarely measured directly; usually more basic quantities such as temperature and pressure are measured, and the fluid flow parameter is calculated as a function of the measurements. Uncertainty of the measurements is propagated to the parameter through the function. The effect of the propagation may be approximated with Taylor's series methods. It is convenient to introduce the concept of the sensitivity of a result to a measured quantity as the uncertainty propagated to the result due to unit uncertainty of the measurement. The "sensitivity coefficient" (also known as "influence coefficient") of each subsidiary quantity is most easily obtained in one of two ways.

a) Analytically

Where there is a known mathematical relationship between the result, R, and subsidiary quantities Y_i, Y_2, . . . Y_K, the dimensional sensitivity coefficient Θ_i of the result R to the quantity Y_i is obtained by partial differentiation. Thus, if $R = f(Y_1, Y_2, \ldots Y_K)$, then

$$\Theta_i = \frac{\partial R}{\partial Y_i} \quad \text{. . . (28)}$$

Analogously, the relative (nondimensional) sensitivity coefficient, Θ_i', is

$$\Theta_i' = \frac{\partial R / R}{\partial Y_i / Y_i} \quad \text{. . . (29)}$$

In this form, the sensitivity is expressed as "percent/percent". That is, Θ_i' is the percentage change in R brought about by a 1 % change in Y_i. This is the form used if the uncertainties to be combined are expressed as percentages of their associated variables rather than absolute values.

b) Numerically

Where no mathematical relationship is available or when differentiation is difficult, finite increments may be used to evaluate Θ_i. This is a convenient method with computer calculations.

Here Θ_i is given by

$$\Theta_i = \frac{\Delta R}{\Delta Y_i} \quad \text{. . . (30)}$$

The result is calculated using Y_i to obtain R, and then recalculated using $(Y_i + \Delta Y_i)$ to obtain $(R + \Delta R)$. The value of ΔY_i used should be as small as practicable.

If the experimental standard deviations of the measurement are small and the variables independent, the experimental standard deviation of the result R is given by:

$$s_R = \sqrt{\sum \left(\Theta_i s_{Y_i}\right)^2} \quad \text{. . . (31)}$$

and the systematic uncertainty by:

$$B_R = \sqrt{\sum \left(\Theta_i B_{Y_i} \right)^2} \qquad \ldots (32)$$

Care should be taken to ensure that the variables are independent. With complex parameters, the same measurement may be used more than once in the formula. This may increase or decrease the uncertainty depending on whether the sign of the measurement is the same or opposite. If the Taylor's series relates the most elementary measurements to the ultimate parameter or result, these "linked" relationships will be properly accounted for.

This effect can be covered by calculating a modified Θ by simultaneous perturbation of all the inputs likely to be affected, thus:

Θ_{link} = (change in output R due to a change in linked parameter which affects all inputs, Y_i, simultaneously)

An example of this is a change in barometric pressure, which affects all pressure inputs simultaneously in a "gauge-pressure" system. Another example is the use of a common working standard to calibrate all the pressure transducers.

The product of the sensitivity coefficient and the experimental standard deviation of the linked parameters can then be combined with independent ones, thus:

$$s_R = \sqrt{\left(\Theta_{\text{link}} s_{Y_{\text{link}}} \right)^2 + \sum \left(\Theta_i s_{Y_i} \right)^2} \qquad \ldots (33)$$

In equation (33), the product of one linked parameter is combined with the products of the remaining independent parameters. The products of other linked parameters may be combined in a similar way.

Examples of the propagation of measurement uncertainties to a parameter can be found in annex C.

8 Calculation of uncertainty

8.1 Summary of procedure

Select the additive or the root-sum-square model of combination and combine the systematic and random uncertainties of the test result to obtain the overall uncertainty. The test result plus and minus the uncertainty is the uncertainty interval that should contain the true value with high probability.

8.2 Uncertainty intervals

For simplicity of presentation, a single number (some combination of systematic and random uncertainties) is needed to express a reasonable limit for error. The single number should have a simple interpretation (e.g., the largest error reasonably expected) and be useful without complex explanation. It is usually impossible to define a single rigorous statistic because the systematic uncertainty is based on judgement which has unknown characteristics.[2)] This function is a hybrid combination of an estimated quantity based on judgement (systematic uncertainty) and a statistic (random uncertainty). If both numbers were statistics, a confidence interval would be recommended. 95 % or 99 % confidence levels would be available at the discretion of the analyst. Although rigorous statistical confidence levels are not available, two uncertainty intervals, with an associated coverage approximately analogous to 95 % and 99 % confidence levels, are recommended.

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2) If information exists to justify the assumption that the systematic uncertainties have a random distribution, a rigorous statistic can be defined.

8.3 Symmetrical intervals

Uncertainty (figure 10) for the symmetrical systematic uncertainty case is centred about the measurement, and the uncertainty interval is defined as:

$$R - U, R + U$$

where

$$U_{ADD} = (B + t_{95}s) \quad \text{. . . (34)}$$

$$U_{RSS} = \sqrt{B^2 + (t_{95}s)^2} \quad \text{. . . (35)}$$

If the experimental standard deviation is based on small samples, the methods in annex A may be used to determine a value of Student's t_{95}. For large samples (> 30), 2 may be substituted for t_{95} in equations (34) and (35).

If the test result is an average $(\bar{R})$ based on sample size N, instead of a single value (R), $s/\sqrt{N}$ should be substituted for s.

The uncertainty selected [equation (34) or (35)] should be provided in the presentation; the components (systematic uncertainty, random uncertainty, degree of freedom) should be available in an appendix or in supporting documentation. These three components may be required to substantiate and explain the uncertainty value, to provide a sound technical base for improved measurements, and to propagate the uncertainty from measured parameters to fluid flow parameters and from fluid flow parameters to other more complex performance parameters (e.g. engine, pump or fan performance and power station or chemical plant efficiency).

8.4 Nonsymmetrical interval

If there is a nonsymmetrical systematic uncertainty interval, the uncertainty (U) is no longer symmetrical about the measurement. The interval is defined by the positive systematic uncertainty (B^+) and the negative systematic uncertainty (B^-) (see 7.3).

Figure 11 shows the uncertainty (U) for nonsymmetrical systematic uncertainties. (See table 4.)

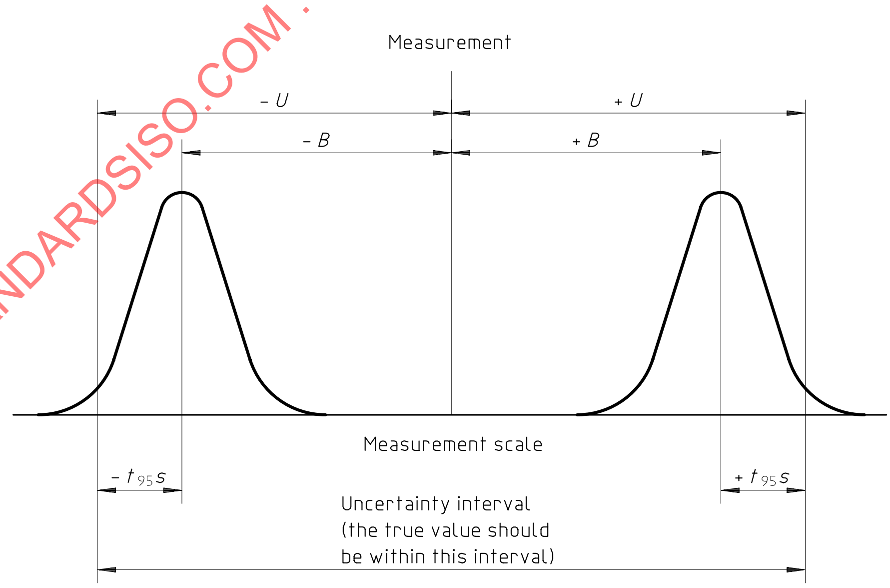


Figure 10 — Measurement uncertainty interval (U_{ADD}); symmetrical systematic uncertainty and additive combination

Table 4 — Uncertainty intervals defined by nonsymmetrical systematic uncertainties

B^-	B^+	$t_{95}s_{\bar{x}}$	U^-_{ADD}	U^+_{ADD}	U^-_{RSS}	U^+_{RSS}
0 K	+ 10 K	2 K	– 2 K	+ 12 K	– 2 K	+ 10,2 K
– 3 kg	+ 13 kg	4 kg	– 7 kg	+ 17 kg	– 5 kg	+ 13,6 kg
0 Pa	+ 7 Pa	2 Pa	– 2 Pa	+ 9 Pa	– 2 Pa	+ 7,3 Pa
– 8 K	0 K	2 K	– 10 K	+ 2 K	– 8,2 K	+ 2,0 K

$$U^+_{ADD} = B^+ + t_{95}s$$

$$U^+_{RSS} = \sqrt{{B^+}^2 + (t_{95}s)^2} \quad \text{. . . (36)}$$

$$U^-_{ADD} = B^- + t_{95}s$$

$$U^-_{RSS} = -\sqrt{{B^-}^2 + (t_{95}s)^2} \quad \text{. . . (37)}$$

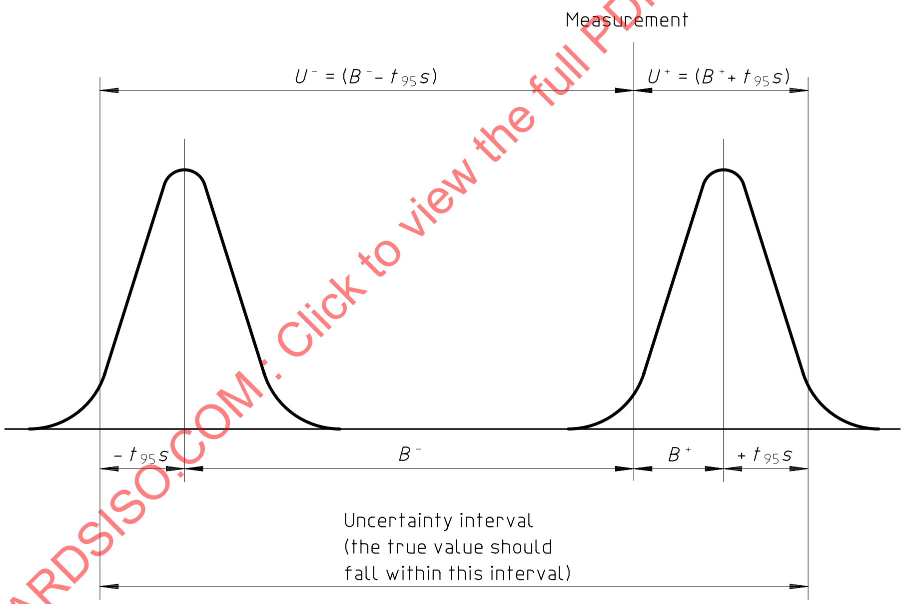


Figure 11 — Measurement uncertainty; nonsymmetrical systematic uncertainty and additive combination

9 Presentation of results

9.1 Summary of requirement

The summary report should contain the nominal level of the test result, the systematic uncertainty, the experimental standard deviation, the degrees of freedom and the overall uncertainty. The equation used to calculate uncertainty, U_{ADD} or U_{RSS} should be stated. The summary should reference a table of the elemental uncertainties considered and included in the uncertainty.

9.2 Before-measurement analysis and corrective action

Uncertainty is a function of the measurement process. It provides an estimate of the largest error that may reasonably be expected for that measurement process. Errors larger than the uncertainty should rarely occur. If the difference to be detected in an experiment is of the same size or smaller than the projected uncertainty, corrective action should be taken to reduce the uncertainty. Therefore, it is recommended that an uncertainty analysis always be done before the test or experiment. The recommended corrective action depends on whether the systematic or the random uncertainty is too large as shown in table 5.

Table 5 — Recommended corrective action if the predicted before-measurement uncertainty is unacceptable

Systematic uncertainty too large:	**Random uncertainty too large:**
• Improve calibration	• Larger test sample
• Independent calibrations for redundant meters	• More precise instrumentation
• Concomitant variable	• Redundant instrumentation
• *In-situ* calibration	• Data smoothing — Moving average — Filter — Regression
	• Improve design of experiment

9.3 After-measurement analysis and data validity

After-measurement analysis is required to confirm the before-measurement estimates or to identify data validity problems. Comparison of measurement test results with the before-measurement analysis is an excellent data validity check. The random uncertainty of the repeated points or redundant instruments should not be significantly larger than the before-measurement estimates. When redundant instrumentation or calculation methods are available, the individual uncertainty intervals should be compared for consistency with each other and with the before-measurement uncertainty analysis.

Three cases are illustrated in figure 12.

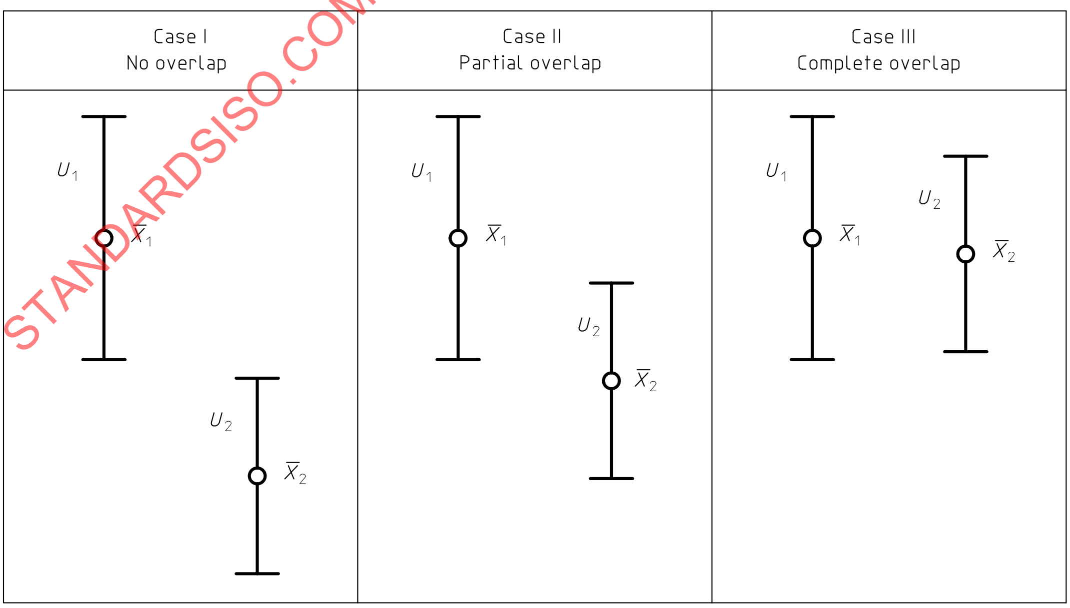


Figure 12 — Three after-measurement uncertainty interval comparisons

When there is no overlap between uncertainty intervals, as in Case I, a problem exists. The true value cannot be contained within both intervals. That is, there should be a very low probability that the true value lies outside any of the uncertainty intervals. Either the uncertainty analysis is wrong or a data validity problem exists. Investigation to identify bad readings, overlooked sources of systematic error, etc., is necessary to resolve this discrepancy. Redundant and dissimilar instrumentation should be compared. Partial overlap of the uncertainty intervals, as in Case II, also signals that a problem may exist. The magnitude of the problem depends on the amount of overlap. The only situation when one can be confident that the data is valid and the uncertainty analysis is correct is Case III, when the uncertainty intervals attain maximum overlap.

9.4 Summary for reporting error

The definition of the components, systematic uncertainty, experimental standard deviation and the overall uncertainty (U) suggests a summary format for reporting measurement uncertainty. The format will describe the components of uncertainty, which are necessary to estimate further propagation of the uncertainties, and a single value (U) which may be described as the largest error expected from the combined uncertainties. Additional information (degrees of freedom for the estimate of s) is required to use the experimental standard deviation if small samples were used to calculate s. These summary numbers provide the information necessary to accept or reject the measurement uncertainty. The reporting format should contain:

a) s, the experimental standard deviation, calculated from data;

b) for small samples, ν, the degrees of freedom associated with the experimental standard deviation (s). The degrees of freedom for small samples (less than 30) is obtained from the Welch-Satterthwaite procedure illustrated in annex A.

c) B, the systematic uncertainty of the measurement process or B^- and B^+ if the systematic uncertainty is nonsymmetrical;

d) the uncertainty formula $U_{\text{ADD}} = \left(B + t_{95}s\right)$ or $U_{\text{RSS}} = \left[B^2 + \left(t_{95}s\right)^2\right]^{1/2}$, the uncertainty interval within which the true value is expected to lie. If the systematic uncertainty is nonsymmetrical, $U_{\text{ADD}}^- = B^- - t_{95}s$ and $U_{\text{ADD}}^+ = B^+ + t_{95}s$ or $U_{\text{RSS}}^- = -\left[\left(B^-\right)^2 + \left(t_{95}s\right)^2\right]^{1/2}$ and $U_{\text{RSS}}^+ = \left[\left(B^+\right)^2 + \left(t_{95}s\right)^2\right]^{1/2}$. No more than two significant places should be reported. For small samples see annex A.

The model components, s, ν, B and U, are required to report the uncertainty of any measurement process. The first three components, s, ν, and B, are necessary : (1) to indicate corrective action if the uncertainty is unacceptably large before the test, (2) to propagate the uncertainty to more complex parameters, and (3) to substantiate the overall uncertainty.

9.5 Reporting uncertainty: Table of elemental sources

To support the measurement uncertainty summary, a table detailing the elemental error sources is needed for several purposes. If the projected uncertainty is deemed excessive for the purpose of the test, corrective action should be taken to reduce this uncertainty. Further, if the uncertainty quoted in the summary appears to be optimistically small, the list of sources considered should be reviewed to identify missing sources. For this reason, it is important to list all sources considered, even if negligible.

Table 6 gives the format for such a table; one line should be completed for each elemental error source. Note that all uncertainties in this table have been propagated from the basic measurement to the end result.

Table 6 — Elemental error sources

Subscript ij	**Source**	**Measurement value**	**Experimental standard deviation** s_{ij}	**Degrees of freedom** ν_{ij}	**Systematic uncertainty** B_{ij}	**Source of systematic uncertainty**
11 21 31 — — — 12 22 32 42 — — — 13 23 33 — — —						
Results		**Measurement value**	$s = \sqrt{\sum s_{ij}^2}$	ν	$B = \sqrt{\sum B_{ij}^2}$	t_{95}
			$U_{ADD} = B + t_{95}(s) =$ .... $U_{RSS} = \sqrt{B^2 + (t_{95}s)^2} =$			

Annex A
(normative)

Small sample methods

A.1 Student's t

When the experimental standard deviation is based on small samples $(N \leqslant 30)$, uncertainty is defined as:

$$U_{ADD} = B + \left(t_{95}\, s/\sqrt{N}\right) \qquad \text{. . . (A.1)}$$

$$U_{RSS} = \sqrt{B^2 + \left(t_{95}\, s/\sqrt{N}\right)^2} \qquad \text{. . . (A.2)}$$

For these small samples, the interval $\left[\bar{x} - \left(t_{95}\, s/\sqrt{N}\right), \bar{x} + \left(t_{95}\, s/\sqrt{N}\right)\right]$ will contain the true unknown average, μ, 95 % of the time. If the systematic uncertainty is negligible, this statistical confidence interval is the uncertainty interval. t_{95} is the 95th percentile point for the two-tailed Student's t-distribution. For small samples, t will be large, and for larger samples t will be smaller, approaching 1,96 as a lower limit. The t-value is a function of the number of degrees of freedom (ν) used in calculating s. Since 30 degrees of freedom (ν) yield a t of 2,05 and infinite degrees of freedom yield a t of 1,96, an arbitrary selection of $t = 2$ is used for simplicity for values of ν from 30 to infinity (see table A.1).

A.2 Number of degrees of freedom for small samples

In a sample, the number of degrees of freedom (ν) is equal to the sample size, N. When a statistic is calculated from the sample, the degrees of freedom associated with the statistic is reduced by 1 for every estimated parameter used in calculating the statistic. For example, from a sample of size N, $\bar{x}$ is calculated and has N degrees of freedom, and the experimental standard deviation, s, is calculated using equation (1) (see subclause 4.2), and has $N-1$ degrees of freedom because $\bar{x}$ is used to calculate s. In calculating other statistics, more than one degree of freedom may be lost. For example, in calculating the experimental standard deviation of a curve fit, the number of degrees of freedom is equal to $N-k$ where k is the number of estimated coefficients for the polynomial fitted to the data.

When all random uncertainties have large sample sizes (i.e. $\nu_{ij} > 30$) the calculation of number of degrees of freedom (ν) is unnecessary and 2 is substituted for t_{95}. However, for small samples, when combining experimental standard deviations by the root-sum-square method [see equation (21) in subclause 7.2 for example], the number of degrees of freedom (ν) associated with the combined experimental standard deviations is calculated using the Welch-Satterthwaite formula (A.3).

For example: the number of degrees of freedom for the calibration experimental standard deviation (s_1) given by equation (21), is:

$$\nu_1 = \frac{\left(\sum_{i=1}^{4} s_{i1}^2\right)^2}{\sum_{i=1}^{4} \frac{s_{i1}^4}{\nu_{i1}}} = \frac{\left(s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2\right)^2}{\frac{s_{11}^4}{\nu_{11}} + \frac{s_{21}^4}{\nu_{21}} + \frac{s_{31}^4}{\nu_{31}} + \frac{s_{41}^4}{\nu_{41}}} \qquad \text{. . . (A.3)}$$

where ν_{i1} is the number of degrees of freedom of each elemental experimental standard deviation in the calibration process.

The number of degrees of freedom for the measurement experimental standard deviation (s), as given by equation (22), is:

$$\nu = \frac{\left(\sum_{j=1}^{3} \sum_{i=1}^{K} s_{ij}^2\right)^2}{\sum_{j=1}^{3} \sum_{i=1}^{K} \frac{s_{ij}^4}{\nu_{ij}}} \qquad \text{. . . (A.4)}$$

If the test result is an average, $\bar{x}$, based on a sample of size N,

$$s_{\bar{x}} = \frac{s_x}{\sqrt{N}} \qquad \text{. . . (A.5)}$$

As $\sqrt{N}$ is a known constant, the number of degrees of freedom of $s_{\bar{x}}$ is the same as s, i.e.

$$\nu_{s_{\bar{x}}} = \nu \qquad \text{. . . (A.6)}$$

A.3 Propagating the degrees of freedom

The Student's t value from table A.1 to be used in calculating the uncertainty of the test result [equation (A.1) or (A.2)] is based on ν_r, the number of degrees of freedom of s_r. If the number of degrees of freedom of any measurement standard deviation is less than 30, the number of degrees of freedom of the result also may be less than 30. In such cases, the following small sample method may be used to determine ν_r. This is defined for the absolute experimental standard deviation according to the Welch-Satterthwaite formula by:

$$\nu_r = \frac{s_r^4}{\sum_{i=1}^{J} \frac{\left(\Theta_i s_{\bar{p}_i}\right)^4}{\nu_{p_i}}} \qquad \text{. . . (A.7)}$$

and for the relative experimental standard deviation by:

$$\nu_r = \frac{\left(s_r / r\right)^4}{\sum_{i=1}^{J} \frac{\left(\Theta_i s_{\bar{p}_i} / \bar{p}_i\right)^4}{\nu_{\bar{p}_i}}} \qquad \text{. . . (A.8)}$$

where

$$s_r = \sqrt{\sum_{i=1}^{J} \left(\Theta_i s_{\overline{p}_i}\right)^2}$$

and the number of degrees of freedom on the experimental standard deviation $\left(s_{p_i}\right)$ of the independent measurements is usually given by:

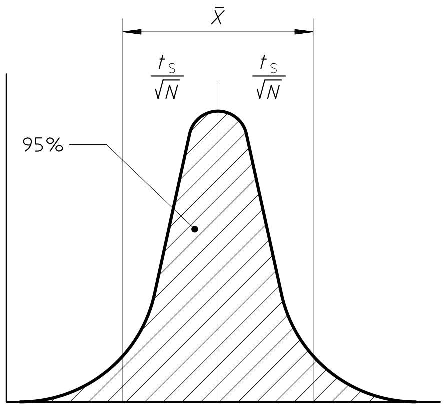
$$\nu_{\overline{p}_i} = \left(N_i - 1\right) \qquad \text{. . . (A.9)}$$

NOTE — The number of degrees of freedom for the relative and absolute experimental standard deviations are identical.

Welch-Satterthwaite degrees of freedom may contain fractional, decimal parts. The fractions should be dropped or truncated, as rounding down is conservative with Student's t, e.g. ν = 13,6 should be treated as ν = 13,0.

Table A.1 — Two-tailed Student's t table for small sample methods, < 30 degrees of freedom

Number of degrees of freedom	t_{95}	Number of degrees of freedom	t_{95}
1	12,706	16	2,120
2	4,303	17	2,110
3	3,182	18	2,101
4	2,776	19	2,093
5	2,571	20	2,086
6	2,447	21	2,080
7	2,365	22	2,074
8	2,306	23	2,069
9	2,262	24	2,064
10	2,228	25	2,060
11	2,201	26	2,056
12	2,179	27	2,052
13	2,160	28	2,048
14	2,145	29	2,045
15	2,131	∞	1,96



Annex B
(normative)

Outlier treatment

B.1 General

All measurement systems may produce spurious data points. These points may be caused by temporary or intermittent malfunctions of the measurement system or they may represent actual variations in the measurement. Errors of this type should not be included as part of the uncertainty of the measurement. Such points are meaningless as test data. They should be discarded. Figure B.1 shows a spurious datum point, called an outlier.

All data should be inspected for spurious data points as a continuing check on the measurement process. Points should be rejected based on engineering analysis of instrumentation, fluid mechanics, flow profiles and past history with similar data. To ease the burden of scanning large masses of data, computerized routines are available to scan steady-state data and flag suspected outliers. The flagged points should then be subjected to an engineering analysis.

The effect of these outliers is to increase the random uncertainty of the system. A test is needed to determine if a particular point from a sample is an outlier. The test should consider two types of error in detecting outliers:

1) rejecting a good datum point;

2) not rejecting a bad datum point.

The probability of rejecting a good point is usually set at 5 %. This means that the odds of rejecting a good point are 20 to 1 (or less). The odds will be increased by setting the probability of 1) lower. However, this practice decreases the probability of rejecting bad data points. The lower probability of rejecting a good point will require that the rejected points be further from the calculated mean, and fewer bad data points will thus be identified. For large sample sizes, e.g. containing several hundred measurements, almost all bad data points can be identified. For small samples (five or ten measurements), bad data points are hard to identify.

One test in common usage for determining whether spurious data are outliers is Grubbs' method.

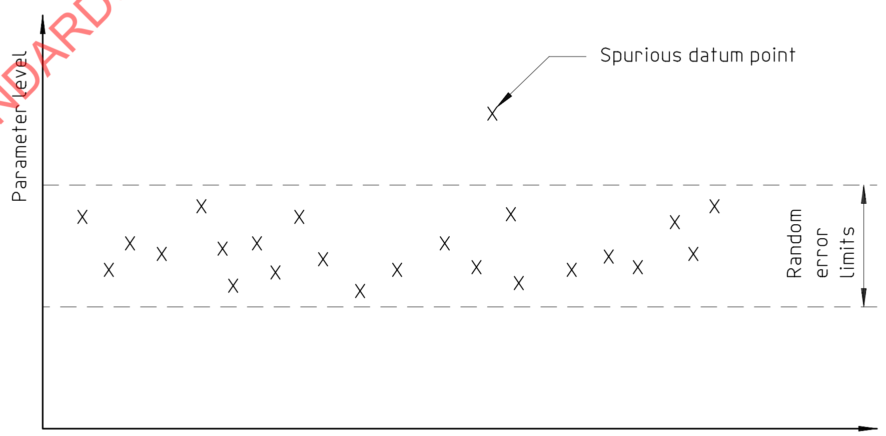


Figure B.1 — Outlier outside the range of acceptable data

B.2 Grubbs' method

Consider a sample (x_i) of N measurements. The mean $(\bar{x})$ and an experimental standard deviation (s) are calculated by equation (1). Suppose that (x_j), the jth observation, is the suspected outlier; the absolute statistic, T_n, calculated is then:

$$T_n = \left| \frac{x_j - \bar{x}}{s} \right| \quad \ldots \text{(B.1)}$$

Using table B.1, a value of T_n is obtained for the sample size (N) at the 5 % significance level (P). This limits the probability of rejecting a good point to 5 %. (The probability of not rejecting a bad datum point is not fixed. It will vary as a function of sample size.)

The test for the outlier is to compare the calculated T_n with the value for T_n given in table B.1.

If T_n calculated is larger than or equal to T_n as given in table B.1, we call x_j an outlier.

If T_n calculated is smaller than T_n as given in table B.1, we say x_j is not an outlier.

Table B.1 — Rejection values for Grubbs'method

Sample size N	**5 % significance level** (one-sided)	**Sample size** N	**5 % significance level** (one-sided)
3	1,150	20	2,56
4	1,46	21	2,58
5	1,67	22	2,60
6	1,82	23	2,62
7	1,94	24	2,64
8	2,03	25	2,66
9	2,11	30	2,75
10	2,18	35	2,82
11	2,23	40	2,87
12	2,29	45	2,92
13	2,33	50	2,96
14	2,37	60	3,03
15	2,41	70	3,09
16	2,44	80	3,14
17	2,47	90	3,18
18	2,50	100	3,21
19	2,53		

B.3 Example

In the following sample of 40 values which are deviations from an average,

26	79	58	24	1	− 103	− 121	− 220
− 11	− 137	120	124	129	− 38	25	− 60
− 148	− 52	− 216	− 12	− 56	89	8	− 29
− 107	20	9	− 40	40	2	10	166
126	− 72	179	41	127	− 35	**334**	**− 555**

the suspected outliers are 334 and − 555.

To illustrate the calculations for determining whether − 555 is an outlier from figure B.2:

Mean, $\bar{x}$ = 1,125

Experimental standard deviation, s = 140,813 6

Sample size, N = 40

$$T_{n,\text{calc}} = \left| \frac{-555 - 1{,}125}{140{,}813\,6} \right| = 3{,}95$$

From table B.1 using Grubbs' method for N = 40 at a 5 % level of significance (one-sided),

$$T_{n,\text{table}} = 2{,}87$$

Therefore, since 3,95 > 2,87, i.e.

$$T_{n,\text{calc}} > T_{n,\text{table}}$$

− 555 is an outlier according to the Grubbs's test.

Table B.2 gives the results of this and two further iterations. The two suspected outliers, − 555 and 334, are rejected by the Grubbs' test.

Figure B.2 is a normal probability plot of these data with the suspected outliers indicated. In this case, the engineering analysis indicated that the − 555 and 334 readings were outliers, agreeing with the Grubbs' test results.

Table B.2 — Results of Grubbs' test

Suspected outlier	Sample size N	Experimental standard deviation s	Mean $\bar{x}$	Calculated T_n	Table T_n for P = 5
− 555	40	140,8	1,125	3,95	2,87
334	39	109,6	15,385	2,91 (stop)	2,86
− 220	38	97,5	7,000	2,33	2,85

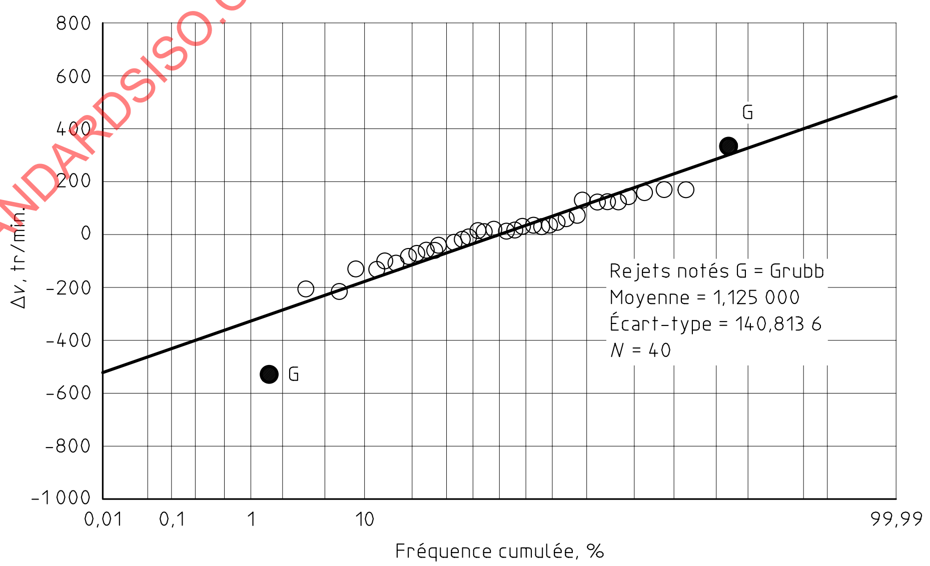


Figure B.2 — Results of outlier tests

Annex C
(informative)

Examples of estimation of uncertainty in airflow measurement

This annex contains two examples of fluid flow measurement uncertainty analysis. The first deals with airflow measurement for an entire facility (with several test stands) over a long period. It also applies to a single test with a single set of instruments. The second example demonstrates how comparative development tests can reduce the uncertainty of the first example.

C.1 General

Airflow measurements in gas turbine engine systems are generally made with one of three types of flowmeters: Venturis, nozzles and orifices. Selection of the specific type of flowmeter to use for a given application is contingent upon a trade-off between measurement accuracy requirements, allowable pressure drop and fabrication complexity and cost.

Flowmeters may be further classified into two categories: subsonic flow and critical flow. With a critical flowmeter, in which sonic velocity is maintained at the flowmeter throat, mass flowrate is a function only of the upstream gas properties. With a subsonic flowmeter, where the throat Mach number is less than sonic, mass flowrate is a function of both upstream and downstream gas properties.

Equations for the ideal mass flowrate through nozzles, Venturis and orifices are derived from the continuity equations:

$$q = \rho A V \qquad \text{. . . (C.1)}$$

In using the continuity equation as a basis for ideal flow equation derivations, it is normal practice to use the principles of conservation of mass and energy and to assume one-dimensional isentropic flow. Expressions for ideal flow will not yield actual flow, since actual conditions always deviate from ideal. An empirically determined correction factor, the discharge coefficient (C) is used to adjust ideal to actual flow:

$$C = q_{\text{actual}} / q_{\text{ideal}} \qquad \text{. . . (C.2)}$$

C.2 Example 1: Test facility

C.2.1 Definition of the measurement process

What is the airflow measurement capability of a given test facility? This question might relate to a guarantee in a product specification or a research contract. Note that this question implies that many test stands, sets of instrumentation and calibrations over a long period of time should be considered.

The same general uncertainty model is applied in the second example to a single stand process, the comparative test.

These examples will provide, step by step, the entire process of calculating the uncertainty of the airflow parameter. The first step is to understand the defined measurement process and then identify the source of every possible error. For each measurement, uncertainties associated with calibration will be discussed first, then data acquisition, data reduction, and finally, propagation of these uncertainties to the calculated parameter.

Figure C.1 depicts a sonic nozzle flowmeter installed in the inlet ducting upstream of a turbine engine under test for this example.

When a Venturi flowmeter is operated at critical pressure ratios, the flowrate through the Venturi is a function of the upstream conditions only and may be calculated from

$$q = \frac{\pi d^2}{4} C F_a \varphi * \frac{p_1}{\sqrt{T_1}} \qquad \text{. . . (C.3)}$$

C.2.2 Measurement error sources

Each of the variables in equation (C.3) must be carefully considered to determine how and to what extent uncertainties in the determination of the variable affect the calculated parameter. A relatively large uncertainty in some will affect the final answer very little, whereas small uncertainties in others have a large effect. Particular care should be taken to identify measurements that influence the fluid flow parameters in more than one way.

In equation (C.3), upstream pressure and temperature (p_1 and T_1) are of primary concern. Error sources for each of these measurements are: (1) calibration, (2) data acquisition and (3) data reduction.

C.2.2.1 Figure C.2 illustrates a typical calibration hierarchy. Associated with each comparison in the calibration hierarchy is a possible pair of elemental uncertainties, a systematic uncertainty and an experimental standard deviation. Table C.1 lists all the elemental uncertainties. Note that these elemental uncertainties of inlet pressure, p, are not cumulative, e.g. B_{21} is not a function of B_{11}. The systematic uncertainties should be based on interlaboratory tests if available, otherwise, the judgement of the best experts must be used. The experimental standard deviations are calculated from calibration history data banks.

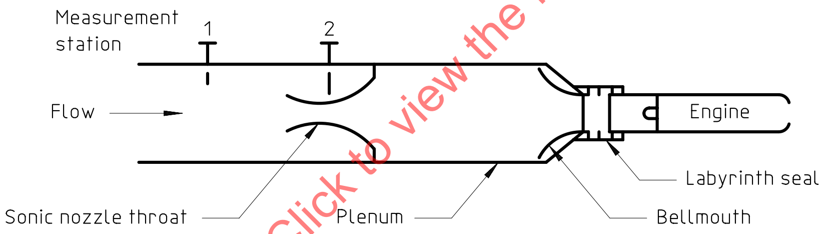


Figure C.1 — Schematic of sonic nozzle flowmeter installation upstream of a turbine engine

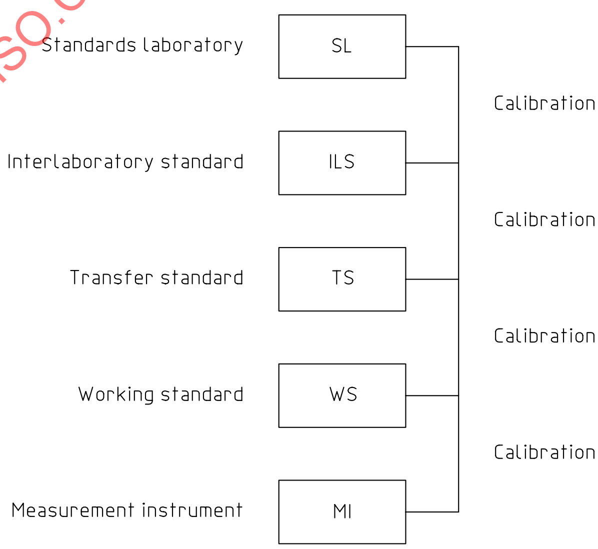


Figure C.2 — Typical calibration hierarchy

Table C.1 — Calibration hierarchy error sources

Calibration	Systematic uncertainty Pa	Experimental standard deviation Pa	Degrees of freedom
SL — ILS	B_{11} = 69	s_{11} = 13,8	ν_{11} = 10
ILS — TS	B_{21} = 69	s_{21} = 13,8	ν_{21} = 15
TS — WS	B_{31} = 69	s_{31} = 13,8	ν_{31} = 20
WS — MI	B_{41} = 124	s_{41} = 36,5	ν_{41} = 30

The experimental standard deviation for the calibration process is the root-sum-square of the elemental experimental standard deviations, i.e.

$$s_1 = \sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2}$$

$$= \sqrt{13{,}8^2 + 13{,}8^2 + 13{,}8^2 + 36{,}5^2} \qquad \text{. . . (C.4)}$$

$$= 43{,}6 \text{ Pa}$$

The number of degrees of freedom associated with s are calculated from the Welch-Satterthwaite formula as follows:

$$\nu_1 = \frac{\left(s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2\right)^2}{\left(\frac{s_{11}^4}{\nu_{11}} + \frac{s_{21}^4}{\nu_{21}} + \frac{s_{31}^4}{\nu_{31}} + \frac{s_{41}^4}{\nu_{41}}\right)}$$

$$= \frac{\left(13{,}8^2 + 13{,}8^2 + 13{,}8^2 + 36{,}5^2\right)^2}{\left(\frac{13{,}8^4}{10} + \frac{13{,}8^4}{15} + \frac{13{,}8^4}{20} + \frac{36{,}5^4}{30}\right)} \qquad \text{. . . (C.5)}$$

$$= 54$$

The systematic uncertainty for the calibration process is the root-sum-square of the elemental systematic uncertainties, i.e.

$$B_1 = \sqrt{B_{11}^2 + B_{21}^2\ B_{31}^2 + B_{41}^2}$$

$$= \sqrt{69^2 + 69^2 + 69^2 + 124^2} \qquad \text{. . . (C.6)}$$

$$= 172 \text{ Pa}$$

Data acquisition error sources for pressure measurement are listed in table C.2.

Table C.2 — Pressure transducer data acquisition error sources

Error source	Systematic uncertainty Pa	Experimental standard deviation Pa	Degrees of freedom
Excitation voltage	$B_{12} = 69$	$s_{12} = 34,5$	$\nu_{12} = 40$
Electrical simulation	$B_{22} = 69$	$s_{22} = 34,5$	$\nu_{22} = 90$
Signal conditioning	$B_{32} = 69$	$s_{32} = 34,5$	$\nu_{32} = 200$
Recording device	$B_{42} = 69$	$s_{42} = 34,5$	$\nu_{42} = 10$
Pressure transducer	$B_{52} = 69$	$s_{52} = 48,3$	$\nu_{52} = 100$
Environmental effects	$B_{62} = 69$	$s_{62} = 69$	$\nu_{62} = 10$
Probe errors	$B_{72} = 117$	$s_{72} = 48,3$	$\nu_{72} = 60$

The experimental standard deviation for the data acquisition process is

$$s_2 = \sqrt{s_{12}^2 + s_{22}^2 + s_{32}^2 + s_{42}^2 + s_{52}^2 + s_{62}^2 + s_{72}^2}$$

$$s_2 = \left(34,5^2 + 34,5^2 + 34,5^2 + 34,5^2 + 48,3^2 + 69^2 + 48,3^2\right)^{1/2} \qquad \text{. . . (C.7)}$$

$$= 119 \text{ Pa}$$

$$\nu_2 = \frac{\left(s_{12}^2 + s_{22}^2 + s_{32}^2 + s_{42}^2 + s_{52}^2 + s_{62}^2 + s_{72}^2\right)^2}{\left(\frac{s_{12}^4}{\nu_{12}} + \frac{s_{22}^4}{\nu_{22}} + \frac{s_{32}^4}{\nu_{32}} + \frac{s_{42}^4}{\nu_{42}} + \frac{s_{52}^4}{\nu_{52}} + \frac{s_{62}^4}{\nu_{62}} + \frac{s_{72}^4}{\nu_{72}}\right)}$$

$$\nu_2 = \frac{\left(34,5^2 + 34,5^2 + 34,5^2 + 34,5^2 + 48,3^2 + 69^2 + 48,3^2\right)}{\left(\frac{34,5^4}{40} + \frac{34,5^4}{90} + \frac{34,5^4}{200} + \frac{34,5^4}{10} + \frac{48,3^4}{100} + \frac{69^4}{10} + \frac{48,3^4}{60}\right)} \qquad \text{. . . (C.8)}$$

$$= 77$$

The systematic uncertainty for the data acquisition process is

$$B_2 = \left(69^2 + 69^2 + 69^2 + 69^2 + 69^2 + 69^2 + 117^2\right)^{1/2} \qquad \text{. . . (C.9)}$$

$$= 206 \text{ Pa}$$

A computer operates on raw pressure measurement data to perform the conversion to engineering units. Errors in this process are called data reduction errors and stem from curve fits and computer resolution.

Computer resolution is the source of a small elemental uncertainty. Some of the smallest computers used in experimental test applications have six digits' resolution. The resolution error is $\pm$ 1 in 10^6. Even though this error is probably negligible, consideration should be given to rounding-off and truncating errors. Rounding-off results in a random uncertainty. Truncating always results in a systematic uncertainty (assumed in this example).

Table C.3 lists data reduction error sources.

Table C.3 — Pressure measurement data reduction error sources

Error source	Systematic uncertainty Pa	Experimental standard deviation Pa	Degrees of freedom
Curve fit	$B_{13} = 69$	$s_{13} = 0$	v_{13}
Computer resolution	$B_{23} = 6{,}89$	$s_{23} = 0$	v_{23}

The experimental standard deviation for the data reduction process is

$$s_3 = \sqrt{s_{13}^2 + s_{23}^2} \quad \text{. . . (C.10)}$$

$$= 0{,}0$$

The systematic uncertainty for the data reduction process is

$$B_3 = \sqrt{B_{13}^2 + B_{23}^2}$$

$$= \sqrt{69^2 + 6{,}89^2} \quad \text{. . . (C.11)}$$

$$= 69{,}3 \text{ Pa}$$

The experimental sample standard deviation for pressure measurement then is

$$s_p = \left(s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2 + s_{12}^2 + s_{22}^2 + s_{32}^2 + s_{42}^2 + s_{52}^2 + s_{62}^2 + s_{72}^2 + s_{13}^2 + s_{23}^2\right)^{1/2} \quad \text{. . . (C.12)}$$

or

$$s_p = \sqrt{s_1^2 + s_2^2 + s_3^2}$$

$$= \sqrt{43{,}7^2 + 119^2 + 0{,}0^2} \quad \text{. . . (C.13)}$$

$$= 127 \text{ Pa}$$

Degrees of freedom associated with the experimental standard deviation are determined as follows:

$$v_p = \frac{\left(s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2 + s_{12}^2 + s_{22}^2 + s_{32}^2 + s_{42}^2 + s_{52}^2 + s_{62}^2 + s_{72}^2 + s_{13}^2 + s_{23}^2\right)^2}{\left(\frac{s_{11}^4}{v_{11}} + \frac{s_{21}^4}{v_{21}} + \frac{s_{31}^4}{v_{31}} + \frac{s_{41}^4}{v_{41}} + \frac{s_{12}^4}{v_{12}} + \frac{s_{22}^4}{v_{22}} + \frac{s_{32}^4}{v_{32}} + \frac{s_{42}^4}{v_{42}} + \frac{s_{52}^4}{v_{52}} + \frac{s_{62}^4}{v_{62}} + \frac{s_{72}^4}{v_{72}} + \frac{s_{13}^4}{v_{13}} + \frac{s_{23}^4}{v_{23}}\right)} \quad \text{. . . (C.14)}$$

or

$$\nu_p = \frac{\left(s_1^2 + s_2^2 + s_3^2\right)^2}{\left(\frac{s_1^4}{\nu_1} + \frac{s_2^4}{\nu_2} + \frac{s_3^4}{\nu_3}\right)}$$

$$= \frac{\left(43{,}6^2 + 119^2 + 0{,}0^2\right)^2}{\left(\frac{43{,}6^4}{54} + \frac{119^4}{77} + \frac{0{,}0^4}{0}\right)} \quad \text{. . . (C.15)}$$

$$= 97$$

therefore $t_{95} = 2$

The systematic uncertainty for the pressure measurement is

$$B_p = \left(B_{11}^2 + B_{21}^2 + B_{31}^2 + B_{41}^2 + B_{12}^2 + B_{22}^2 + B_{32}^2 + B_{42}^2 + B_{52}^2 + B_{62}^2 + B_{72}^2 + B_{13}^2 + B_{23}^2\right)^{1/2} \quad \text{. . . (C.16)}$$

or

$$B_p = \sqrt{B_1^2 + B_2^2 + B_3^2}$$

$$= \sqrt{172^2 + 206^2 + 69{,}3^2} \quad \text{. . . (C.17)}$$

$$= 277 \text{ Pa}$$

Uncertainty for the pressure measurement is

$$U_{ADD} = \left(B_p + t_{95}s_p\right)$$

$$= \left[277 + \left(2 \times 127\right)\right] = 531 \text{ Pa} \quad \text{. . . (C.18)}$$

$$U_{RSS} = \sqrt{B_p^2 + \left(t_{95}s_p\right)^2}$$

$$= \sqrt{(277)^2 + (2 \times 127)^2} = 376 \text{ Pa} \quad \text{. . . (C.19)}$$

C.2.2.2 The calibration hierarchy for temperature measurements is similar to that for pressure measurements. Figure C.3 depicts a typical temperature measurement hierarchy. As in the pressure calibration hierarchy, each comparison in the temperature calibration hierarchy may produce elemental systematic and random uncertainties. Table C.4 lists temperature calibration hierarchy elemental uncertainties.

Table C.4 — Temperature calibration hierarchy elemental errors

Calibration	**Systematic uncertainty** K	**Experimental standard deviation** K	**Degrees of freedom**
SL — ILS	B_{11} = 0,056	s_{11} = 0,002	ν_{11} = 2
ILS — TS	B_{21} = 0,278	s_{21} = 0,028	ν_{21} = 10
TS — WS	B_{31} = 0,333	s_{31} = 0,028	ν_{31} = 15
WS — MI	B_{41} = 0,378	s_{41} = 0,039	ν_{41} = 30

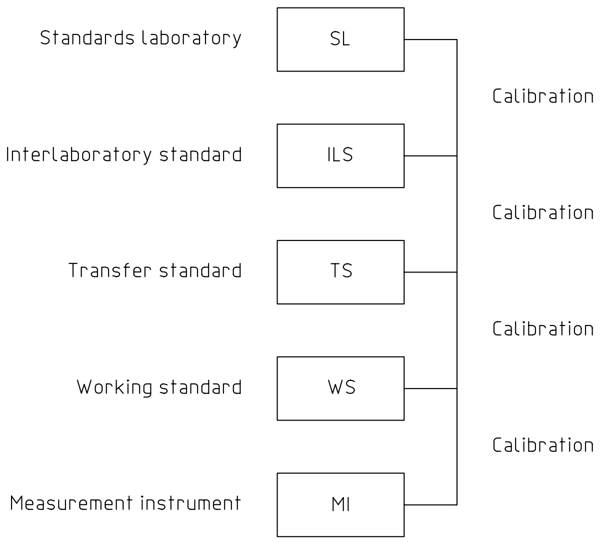


Figure C.3 — Temperature measurement calibration hierarchy

The calibration hierarchy experimental standard deviation is calculated as

$$s_1 = \sqrt{s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2}$$

$$= \sqrt{0{,}002^2 + 0{,}028^2 + 0{,}028^2 + 0{,}039^2} \qquad \text{. . . (C.20)}$$

$$= 0{,}056 \text{ K}$$

Degrees of freedom associated with s_1 are

$$\nu_1 = \frac{\left(s_{11}^2 + s_{21}^2 + s_{31}^2 + s_{41}^2\right)^2}{\left(\frac{s_{11}^4}{\nu_{11}} + \frac{s_{21}^4}{\nu_{21}} + \frac{s_{31}^4}{\nu_{31}} + \frac{s_{41}^4}{\nu_{41}}\right)}$$

$$= \frac{\left(0{,}002^2 + 0{,}028^2 + 0{,}028^2 + 0{,}039^2\right)^2}{\left(\frac{0{,}002^4}{2} + \frac{0{,}028^4}{10} + \frac{0{,}028^4}{15} + \frac{0{,}039^4}{30}\right)} \qquad \text{. . . (C.21)}$$

$$= 53 > 30$$

therefore $t_{95} = 2$

The calibration hierarchy systematic uncertainty is

$$B_1 = \sqrt{B_{11}^2 + B_{21}^2 + B_{31}^2 + B_{41}^2}$$

$$= \sqrt{0{,}056^2 + 0{,}278^2 + 0{,}333^2 + 0{,}378^2} \qquad \text{. . . (C.22)}$$

$$= 0{,}578 \text{ K}$$

A reference temperature-monitoring system will provide an excellent source of data for evaluating both data acquisition and reduction temperature random uncertainties.

Figure C.4 depicts a typical set-up for measuring temperature with chromel-alumel thermocouples.

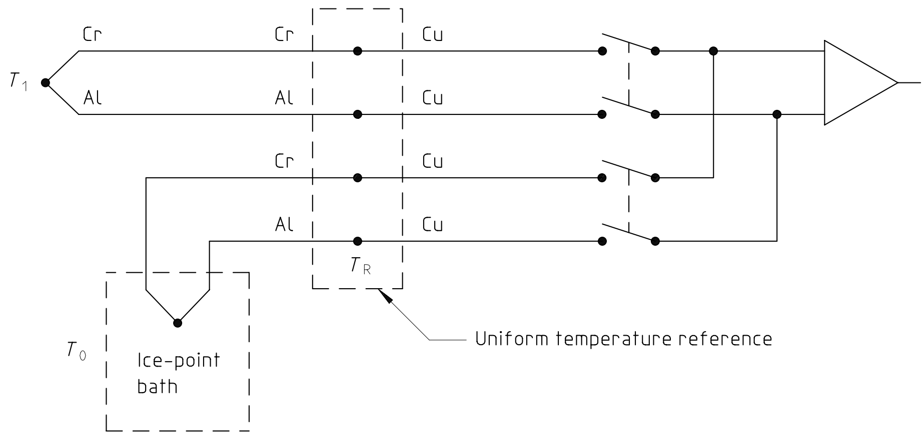


Figure C.4 — Typical thermocouple channel

If several calibrated thermocouples are utilized to monitor the temperature of an ice-point bath, statistically useful data can be recorded each time measurement data are recorded. Assuming that those thermocouple data are recorded and reduced to engineering units by processes identical to those employed for test temperature measurements, a stockpile of data will be gathered, from which data acquisition and reduction uncertainties may be estimated.

For the purposes of illustration, suppose N calibrated chromel-alumel thermocouples are employed to monitor the ice-bath temperature of a temperature-measuring system similar to that depicted by figure C.4. If each time measurement data are recorded, multiple-scan recordings are made for each of the thermocouples, and if a multiple-scan average $\left(\bar{x}_{ij}\right)$ is calculated for each thermocouple, then the average $\left(\bar{x}_j\right)$ for all recordings of the jth thermocouple is

$$\bar{x}_j = \frac{\sum_{i=1}^{K_j} \bar{x}_{ij}}{K_j} \qquad \text{. . . (C.23)}$$

where K_j is the number of multiple-scan recordings for the jth thermocouple.

The grand average $\left(\bar{\bar{x}}\right)$ is computed for all monitor thermocouples as

$$\bar{\bar{x}} = \frac{\sum_{j=1}^{N} \bar{x}_j}{N} \qquad \text{. . . (C.24)}$$

The experimental standard deviation $\left(s_{\bar{x}}\right)$ for the data acquisition and reduction processes is then

$$s_{\bar{x}} = \sqrt{\frac{\sum_{j=1}^{N} \bar{x}_j \sum_{i=1}^{K_j} \left(\bar{x}_{ij} - \bar{x}_j\right)^2}{\sum_{j=1}^{N} \left(K_j - 1\right)}} \qquad \text{. . . (C.25)}$$

$= 0{,}009\,4$ K (assumed for this example)

The degrees of freedom associated with $s_{\bar{x}}$ are

$$\nu_{\bar{x}} = \sum_{j=1}^{N} \left(K_j - 1\right) \qquad \text{. . . (C.26)}$$

$= 200$ (assumed for this example)

Data acquisition and reduction systematic uncertainties may be evaluated from the same ice-bath temperature data if the temperature of the ice bath is continuously measured with a working standard such as a calibrated mercury-in-glass thermometer. There the systematic uncertainty is the largest observed difference between $\bar{x}$ and the temperature indicated by the working standard acquisition and reduction process. In this example, it is assumed to be 0,56 K, i.e.:

$$B_{\bar{x}} = 0{,}56 \text{ K} \qquad \text{. . . (C.27)}$$

Error sources accounted for by this method are:

a) ice-point bath reference (random);

b) reference block temperature (random);

c) recording system resolution;

d) recording system electrical noise;

e) analog-to-digital conversion;

f) chromel-alumel thermocouple voltage versus temperature curve-fit;

g) computer resolution.

Several uncertainties which are not included in the monitoring system statistics are:

a) conduction uncertainty (B_C);

b) radiation uncertainty (B_R);

c) recovery uncertainty (B_Y);

d) calibration uncertainty (B_1).

These uncertainties are a function of probe design and environmental conditions. Detailed treatment of these uncertainties is beyond the scope of this work.

The experimental standard deviation for temperature measurements in this example is

$$\begin{aligned} s_T &= \sqrt{s_1^2 + s_{\bar{x}}^2} \\ &= \sqrt{0{,}056^2 + 0{,}094^2} \\ &= 0{,}11 \text{ K} \end{aligned} \qquad \text{. . . (C.28)}$$

where

s_1 = calibration hierarchy experimental standard deviation;

$s_{\bar{x}}$ = data acquisition and reduction experimental standard deviation.

The degrees of freedom associated with s_T are

$$\begin{aligned} \nu_T &= \frac{\left(s_1^2 + s_2^2\right)^2}{\left(\frac{s_1^4}{\nu_1} + \frac{s_2^4}{\nu_2}\right)} \\ &= \frac{\left(0{,}056^2 + 0{,}094^2\right)^2}{\left(\frac{0{,}056^4}{53} + \frac{0{,}094^4}{200}\right)} \\ &= 249, \text{ therefore } t_{95} = 2 \end{aligned} \qquad \text{(C.29)}$$

When v is less than 30, t_{95} is determined from a Student's t table at the value of v. Since v_T is greater than 30 here, use t_{95} = 2.

Systematic uncertainties for the measurements are

$$B_T = \sqrt{B_1^2 + B_{\bar{x}}^2 + B_C^2 + B_R^2 + B_Y^2}$$

$$= \sqrt{0{,}578^2 + 0{,}56^2} \qquad \text{. . . (C.30)}$$

$$= 0{,}805 \text{ K}$$

where

B_1 = calibration hierarchy systematic uncertainties;

$B_{\bar{x}}$ = data acquisition and reduction systematic uncertainties;

B_C = conduction error systematic uncertainties (negligible in this example);

B_R = radiation systematic uncertainties (negligible in this example);

B_Y = recovery factor systematic uncertainties (negligible in this example).

Uncertainty for the temperature measurement is

$$U_{ADD} = \left(B_T + t_{95} s_T\right) \qquad U_{RSS} = \sqrt{B_T^2 + \left(t_{95} s_T\right)^2}$$

$$= \left[0{,}805 + \left(2 \times 0{,}11\right)\right] \qquad = \sqrt{0{,}805^2 + \left(2 \times 0{,}11\right)^2} \qquad \text{. . . (C.31)}$$

$$= 1{,}03 \text{ K} \qquad = 0{,}83 \text{ K}$$

C.2.2.3 To minimize the uncertainty in the discharge coefficient, it should be calibrated using primary standards in a recognized laboratory. Such a calibration will determine a value of C and the associated systematic uncertainty and experimental standard deviation.

When an independent flowmeter is used to determine flowrates during a calibration for C, dimensional uncertainties are effectively calibrated out. However, when C is calculated or taken from a standard reference, uncertainties due to the measurement of pipe and throat diameters will be reflected as systematic uncertainties in the flow measurement.

Dimensional uncertainties in large Venturis, nozzles and orifices may be negligible. For example, an error of 0,001 mm in the throat diameter of a 5 mm critical flow nozzle will result in a 0,04 % systematic uncertainty in airflow. However, these uncertainties can be significant at smaller diameters.

C.2.2.4 Nonideal gas behaviour and changes in gas composition are accounted for by selection of the proper values for compressibility factor (Z), molecular weight (M) and ratio of specific heats (γ) for the specific gas flow being measured.

When values of γ and Z are evaluated at the proper pressure and temperature conditions, airflow uncertainties due to uncertainties of γ and Z will be negligible.

For the specific case of airflow measurement, the main factor contributing to variation of composition is the moisture content of the air. Though small, the effect of a change in air density due to water vapour on airflow measurement should be evaluated in every measurement process.

C.2.2.5 The thermal expansion correction factor (F_a) corrects for changes in throat area caused by changes in flowmeter temperature.

For steels, a 17 K flowmeter temperature difference, between the time of a test and the time of calibration, will introduce an airflow uncertainty of 0,06 % if no correction is made. If flowmeter skin temperature is determined to within 3 K and the correction factor applied, the resulting uncertainty in airflow will be negligible.

C.2.3 Propagation of uncertainty to airflow

For an example of propagation of uncertainties in airflow measurement using a critical-flow Venturi, consider a Venturi having a throat diameter of 0,554 m operating with dry air at an upstream total pressure of 88 126 Pa and an upstream total temperature of 265,9 K.

Equation (C.32) [identical to (C.3)] is the flow equation to be analysed:

$$q = \frac{\pi d^2}{4} C F_a \varphi^* \frac{p_1}{\sqrt{T_1}} \qquad \text{. . . (C.32)}$$

where

$$\varphi^* = \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left(\frac{\gamma M}{ZR}\right)}$$

Assume, for this example, that the theoretical discharge coefficient (C) has been determined to be 0,995. (To illustrate the uncertainty methodology, we will assume an experimental standard deviation of 0,000 5 in addition to a systematic uncertainty of 0,003.) Further assume that the thermal expansion correction factor (F_a) and the compressibility factor (Z) are equal to 1,0. Table C.5 lists measured values, systematic uncertainties, experimental standard deviations and degrees of freedom for each error source in the above equation.

Note that, in table C.5, airflow uncertainties resulting from uncertainties in F_a, Z, γ, M and R are considered negligible.

Table C.5 — Airflow measurement uncertainty sources

Error source	Units	Measured value	Systematic uncertainty	Experimental standard deviation	Degrees of freedom ν	Uncertainty U_{ADD}	Uncertainty U_{RSS}
p_1	Pa	88 126	277	127	96	531	376
T_1	K	266	0,8	0,11	250	1,02	0,83
d	m	0,554	$2,54 \times 10^{-5}$	$2,54 \times 10^{-5}$	100	$7,62 \times 10^{-5}$	$5,68 \times 10^{-5}$
C		0,995	0,003	0,000 5	—	0,003	0,003 2
F_a		1,0	—	—	—	—	—
Z		1,0	—	—	—	—	—
γ		1,40	—	—	—	—	—
M	kg/kmol	28,9	—	—	—	—	—
R	J/(K·kmol)	$8,314 \times 10^3$	—	—	—	—	—

From equation (C.32), airflow is calculated as

$$q = \frac{3{,}141\,59}{4}(0{,}554\,00)^2 \times 0{,}995 \times 1{,}0 \times \left[\left(\frac{2}{2{,}40}\right)^{\frac{2{,}40}{0{,}40}}\left(\frac{1{,}40 \times 28{,}9}{8\,314}\right)\right]^{1/2} \times \frac{88\,126}{\sqrt{266}}$$

$$= 52{,}3\ \mathrm{kg \cdot s^{-1}}$$

Taylor's series expansion of equation (C.32) with the assumptions indicated yields equations (C.33) and (C.34), from which the flow measurement experimental standard deviation and systematic uncertainties are calculated.

$$s_q = q\left[\left(\frac{s_{p_1}}{p_1}\right)^2 + \left(\frac{-s_{T_1}}{2T_1}\right)^2 + \left(\frac{s_C}{C}\right)^2 + \left(\frac{2s_d}{d}\right)^2\right]^{1/2}$$

$$= 52{,}3\left[\left(\frac{127}{88\,126}\right)^2 + \left(\frac{-0{,}11}{2 \times 266}\right)^2 + \left(\frac{0{,}000\,5}{0{,}995}\right)^2 + \left(\frac{2 \times 0{,}000\,025}{0{,}554}\right)^2\right]^{1/2} \quad \text{. . . (C.33)}$$

$$= 52{,}3\sqrt{(0{,}001\,4)^2 + (-0{,}000\,2)^2 + (0{,}000\,503)^2 + (0{,}000\,09)^2}$$

$$= 0{,}079\ \mathrm{kg \cdot s^{-1}}$$

$$B_q = q\left[\left(\frac{B_{p_1}}{p_1}\right)^2 + \left(\frac{-B_{T_1}}{2T_1}\right)^2 + \left(\frac{B_C}{C}\right)^2 + \left(\frac{2B_d}{d}\right)^2\right]^{1/2}$$

$$= 52{,}3\left[\left(\frac{277}{88\,126}\right)^2 + \left(\frac{-0{,}804}{532}\right)^2 + \left(\frac{0{,}003}{0{,}995}\right)^2 + \left(\frac{0{,}000\,05}{0{,}554}\right)^2\right]^{1/2} \quad \text{. . . (C.34)}$$

$$= 52{,}3\sqrt{(0{,}003\,1)^2 + (-0{,}001\,5)^2 + (0{,}003\,0)^2 + (0{,}000\,09)^2}$$

$$= 0{,}239\ \mathrm{kg \cdot s^{-1}}$$

By using the Welch-Satterthwaite formula, the degrees of freedom for the combined experimental standard deviation are determined from

$$\nu_q = \frac{\left[\left(\frac{\partial q}{\partial p_1}s_{p_1}\right)^2 + \left(\frac{\partial q}{\partial T_1}s_{T_1}\right)^2 + \left(\frac{\partial q}{\partial d}s_d\right)^2 + \left(\frac{\partial q}{\partial C}s_C\right)^2\right]^2}{\frac{\left(\frac{\partial q}{\partial p_1}s_{p_1}\right)^4}{\nu_{p_1}} + \frac{\left(\frac{\partial q}{\partial T_1}s_{T_1}\right)^4}{\nu_{T_1}} + \frac{\left(\frac{\partial q}{\partial d}s_d\right)^4}{\nu_d} + \frac{\left(\frac{\partial q}{\partial C}s_C\right)^4}{\nu_C}} \quad \text{. . . (C.35)}$$

$$= \frac{\left[\left(\frac{s_{p_1}}{p_1}\right)^2 + \left(\frac{-s_{T_1}}{2T_1}\right)^2 + \left(\frac{2s_d}{d}\right)^2 + \left(\frac{s_C}{C}\right)^2\right]^2}{\frac{\left(\frac{s_{p_1}}{p_1}\right)^4}{\nu_{p_1}} + \frac{\left(\frac{-s_{T_1}}{2T_1}\right)^4}{\nu_{T_1}} + \frac{\left(\frac{2s_d}{d}\right)^4}{\nu_d} + \frac{\left(\frac{s_C}{C}\right)^4}{\nu_C}}$$

which results in an overall degree of freedom > 30, and, therefore, a value of t_{95} of 2,0.

Total airflow uncertainty is then:

$$\begin{aligned} U_{\text{ADD}} &= \left(B_q + t_{95} s_q\right) \\ &= 0{,}239 + (2 \times 0{,}079) \\ &= 0{,}40 \text{ kg} \cdot \text{s}^{-1} \\ U'_{\text{ADD}} &= 0{,}8\ \% \end{aligned} \qquad \text{. . . (C.36)}$$

$$\begin{aligned} U_{\text{RSS}} &= \sqrt{B_q^2 + \left(t_{95} s_q\right)^2} \\ &= \sqrt{(0{,}239)^2 + (2 \times 0{,}079)^2} \\ &= 0{,}29 \text{ kg} \cdot \text{s}^{-1} \\ U'_{\text{RSS}} &= 0{,}55\ \% \end{aligned} \qquad \text{. . . (C.37)}$$

C.3 Example 2: Comparative test

C.3.1 Definition of the measurement process

The objective of a comparative test is to determine with the smallest measurement uncertainty the net effect of a design change, such as a new part. The first test is performed with the standard or baseline configuration. A second test, identical to the first except that the design change is substituted in the baseline configuration, is then carried out. The difference between the measurement results of the two tests is an indication of the effect of the design change.

As long as we only consider the difference or net effect between the two tests, all the fixed, constant, systematic errors will cancel out. The measurement uncertainty is composed of random uncertainties only.

For example, assume we are testing the effect on the gas flow of a centrifugal compressor from a change to the inlet inducer. At constant inlet and discharge conditions, and constant rotational speed, will the gas flow increase? If we test the compressor with the old and new inducers and take the difference in measured airflow as our defined measurement process, we obtain the smallest uncertainty. All the systematic errors cancel. Note that, although the comparative test provides an accurate net effect, the absolute value (gas flow with the new inducer) is not determined or, if calculated, as in Example 1, it will be inflated by the systematic uncertainties. Also, the small uncertainty of the comparative test can be significantly reduced by repeating it several times.

C.3.2 Measurement error sources

All errors result from random errors in data acquisition and data reduction. Systematic errors and hence systematic uncertainties are effectively zero. Random uncertainty values are identical to those in Example 1, except that calibration random uncertainties are classed as systematic uncertainties and, hence, become effectively zero.

C.3.2.1 Comparative tests shall use the same test facility and instrumentation for each test. All calibration errors are systematic and cancel out in taking the difference between the test results.

$$B_1 = 0$$

and

$$s_1 = 0,\ s_C = 0$$

C.3.2.2

$s_p = s_2$

$= 119$ Pa [see equation (C.7)]

$\nu_p = \nu_2 = 77$ [see equation (C.8)]

$s_T = s_{\bar{x}}$

$= 0{,}094$ K [see equation (C.25)]

$\nu_T = \nu_{\bar{x}}$

$= 200$ [see equation (C.26)]

C.3.2.3 The test result is the difference in flow between two tests.

$$\Delta q = q_1 - q_2$$

$$s_{\Delta q} = \sqrt{s_{q_1}^2 + (-1)^2 s_{q_2}^2} = s_q\sqrt{2}$$

$$U_{\Delta q_{\text{ADD}}} = \left(B_{\Delta q} + 2s_{\Delta q}\right) \qquad U_{\Delta q_{\text{RSS}}} = \sqrt{\left(B_{\Delta q}\right)^2 + \left(2s_{\Delta q}\right)^2}$$

$$= \left(0 + 2s_{\Delta q}\right) \qquad = \sqrt{0^2 + \left(2s_{\Delta q}\right)^2}$$

$$= 2s_{\Delta q} \qquad = 2s_{\Delta q}$$

$$U_{\Delta q_{\text{ADD}}} = 2s_q\sqrt{2} \qquad U_{\Delta q_{\text{RSS}}} = 2s_q\sqrt{2}$$

$$s_q = \pm 52{,}3\left[\left(\frac{119}{88\,126}\right)^2 + \left(\frac{-0{,}094}{2\times 266}\right)^2 + \left(\frac{0{,}000\,5}{0{,}995}\right)^2 + \left(\frac{0{,}000\,05}{0{,}554}\right)^2\right]^{1/2}$$

$$s_{\Delta q} = 0{,}107\,6\ \text{kg}\cdot\text{s}^{-1} \qquad s_{\Delta q} = 0{,}107\,6\ \text{kg}\cdot\text{s}^{-1}$$

$$U_{\Delta q_{\text{ADD}}} = 0{,}22\ \text{kg}\cdot\text{s}^{-1} \qquad U_{\Delta q_{\text{RSS}}} = 0{,}22\ \text{kg}\cdot\text{s}^{-1}$$

[see equations (C.36) and (C.37)]

C.3.2.4 Note that the differences shown in table C.6 are entirely due to differences in the measurement process definitions. The same fluid flow measurement system might be used in both examples. The comparative test has the smallest measurement uncertainty, but this uncertainty value does not apply to the measurement of absolute level of fluid flow, only to the difference.

Table C.6 — Uncertainty comparisons of Examples 1 and 2

	Example 1 facility	Example 2 facility
1) Experimental standard deviation, $\text{kg}\cdot\text{s}^{-1}$ (s)	0,079	0,076
2) Degrees of freedom (ν)	> 30	> 30
3) Systematic uncertainty, $\text{kg}\cdot\text{s}^{-1}$ (B)	0,246	0
4) Uncertainty, $\text{kg}\cdot\text{s}^{-1}$	0,40	0,22

C.4 Airflow example

In this example, airflow is determined by the use of a sonic nozzle and measurements of upstream stagnation temperature and stagnation pressure (figure C.5).

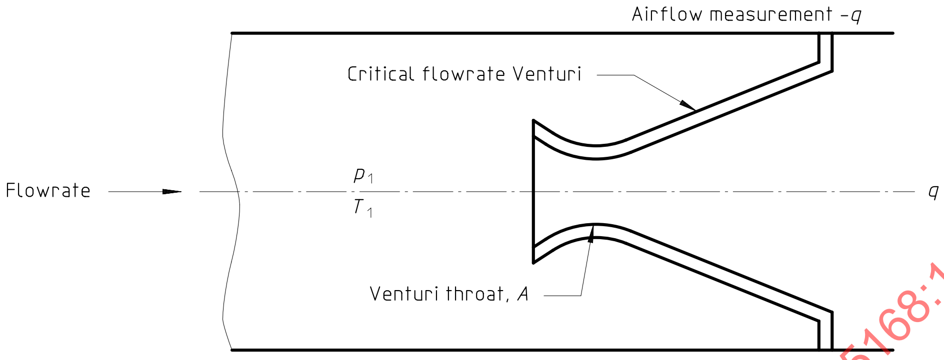


Figure C.5 — Flowrate through a sonic nozzle

The flowrate is calculated from

$$q = CAF_a\varphi^* \frac{p_{1t}}{\sqrt{T_{1t}}} \qquad \text{. . . (C.38)}$$

where

- q is the mass flowrate of air;
- F_a is the factor to account for thermal expansion of the Venturi;
- A is the Venturi throat area;
- p_{1t} is the upstream stagnation pressure;
- T_{1t} is the upstream stagnation temperature;
- φ^* is the factor to account for the properties of the air (critical flow constant);
- C is the discharge coefficient.

The experimental standard deviation for the flowrate (s_q) is calculated using the Taylor's series expansion.

Assuming C equals 1 and has negligible uncertainty

$$s_q = \left[\left(\Theta_{F_a} s_{F_a}\right)^2 + \left(\Theta_{\varphi^*} S_{\varphi^*}\right)^2 + \left(\Theta_A s_A\right)^2 + \left(\Theta_{p_{1t}} s_{p_{1t}}\right)^2 + \left(\Theta_{T_{1t}} s_{T_{1t}}\right)^2\right]^{1/2} \qquad \text{. . . (C.39)}$$

where

$$\Theta_{F_a} = \frac{\partial q}{\partial F_a}$$

denotes the partial derivative of q with respect to F_a.

$$s_q = C\left[\left(\frac{\varphi^* A p_{1t}}{\sqrt{T_{1t}}} s_{F_a}\right)^2 + \left(\frac{F_a A p_{1t}}{\sqrt{T_{1t}}} s_{\varphi^*}\right)^2 + \left(\frac{F_a \varphi^* A}{\sqrt{T_{1t}}} s_{p_{1t}}\right)^2 + \left(\frac{F_a \varphi^* p_{1t}}{\sqrt{T_{1t}}} s_A\right)^2 + \left(\frac{F_a \varphi^* A p_{1t}}{-2\sqrt{T_{1t}^3}} s_{T_{1t}}\right)^2\right]^{1/2} \qquad \text{. . . (C.40)}$$

By inserting the measured values and standard deviations from table C.7 into equation (C.40), the standard deviation of 0,16 kg·s^{-1} for airflow is obtained.

The systematic uncertainty in the flowrate calculation is propagated from the systematic uncertainties of the measured variables. Using the Taylor's series formula gives

$$B_q = \left[\left(\Theta_{x_1} B_{x_1}\right)^2 + \left(\Theta_{x_2} B_{x_2}\right)^2 + \left(\Theta_{x_3} B_{x_3}\right)^2 + \ldots + \left(\Theta_{x_m} B_{x_m}\right)^2 \right]^{1/2} \quad \ldots \text{(C.41)}$$

For this example,

$$B_q = \left[\left(\Theta_{F_a} B_{F_a}\right)^2 + \left(\Theta_{\varphi^*} B_{\varphi^*}\right)^2 + \left(\Theta_A B_A\right)^2 + \left(\Theta_{p_{1t}} B_{p_{1t}}\right)^2 + \left(\Theta_{T_{1t}} B_{T_{1t}}\right)^2 \right]^{1/2} \quad \ldots \text{(C.42)}$$

Taking the necessary partial derivatives gives

$$B_q = C \left[\left(\frac{\varphi^* A p_{1t}}{\sqrt{T_{1t}}} B_{F_a} \right)^2 + \left(\frac{F_a A p_{1t}}{\sqrt{T_{1t}}} B_{\varphi^*} \right)^2 + \left(\frac{F_a \varphi^* p_{1t}}{\sqrt{T_{1t}}} B_A \right)^2 + \left(\frac{F_a \varphi^* A}{\sqrt{T_{1t}}} B_{p_{1t}} \right)^2 + \left(\frac{F_a \varphi^* A p_{1t}}{-2\sqrt{T_{1t}^3}} B_{T_{1t}} \right)^2 \right]^{1/2} \quad \ldots \text{(C.43)}$$

By inserting the measured values and systematic uncertainties of the measured parameters from table C.7 into equation (C.43), a systematic uncertainty of 0,46 kg·s^{-1} is obtained for a nominal airflow of 113 kg·s^{-1}.

Table C.7 contains a summary of the measurement uncertainty analysis for this flow measurement. It should be noted the uncertainties listed only apply to the nominal values.

Table C.7 — Flowrate data

Parameter	Unit	Measured value	Experimental standard deviation	Systematic uncertainty
F_a	—	1,00	0,0	0,001
C	—	1,0	0,0	0,0
φ^*	kg·K$^{1/2}$·N^{-1}·s^{-1}	0,040 4	0,0	$4,04 \times 10^{-5}$
A	m^2	0,191	0,0	$6,85 \times 10^{-4}$
p_{1t}	Pa	$2,54 \times 10^5$	345,0	345,0
T_{1t}	K	303,0	0,17	0,17
Δq	kg·s^{-1}	113	0,16	0,46

Annex D
(informative)

Examples of estimating uncertainty in open channel flow measurement

D.1 General

Evaluation of the overall uncertainty of a flow in an open channel will be demonstrated by considering (1) the velocity-area method and (2) the weir method.

The method of measuring the flow is such that it is impractical to eliminate interdependent variables from the equation before estimating flow uncertainty. Therefore, it involves evaluation of the interdependent uncertainties specified in 7.4. In addition, measurement conditions often make it impossible to obtain the replicate measurements needed for evaluation of experimental standard deviations. Under these conditions, it is appropriate to assume that all the random uncertainties are equivalent to two experimental standard deviations. Under this assumption, the random uncertainties can be propagated with each other by means of the same root-sum-square formulas as the systematic uncertainties [see equations (20) and (23)].

D.2 Example 1: Velocity-area method

D.2.1 Equation for discharge in an open channel (velocity-area)

The channel cross-section under consideration is divided into segments by m verticals. The breadth, depth and mean velocity associated with any vertical i are denoted by b_i, d_i and $\bar{v}_i$ respectively (see figure D.1). The product $q_i = b_i d_i \bar{v}_i$ represents an approximation to the discharge (volumetric flowrate) in the ith segment. The sum over all segments,

$$Q_o = \sum_{i=1}^{m} q_i = \sum_{i=1}^{m} b_i d_i \bar{v}_i \qquad \text{. . . (D.1)}$$

represents an estimated or observed value of the total discharge.

If x and y are respectively horizontal and vertical coordinates of all the points in the cross section, and A is its total area, then the precise mathematical expression for Q, the true volumetric flowrate (discharge) across the cross-sectional area, can be written as

$$Q = \iint_A v(x,y)\, \mathrm{d}x\, \mathrm{d}y \qquad \text{. . . (D.2)}$$

The true discharge and the observed discharge are related by a proportionality factor representing the approximation of the integral equation (D.2) by the finite sum equation (D.1), thus:

$$Q = F_m Q_o = F_m \sum_{i=1}^{m} b_i d_i \bar{v}_i \qquad \text{. . . (D.3)}$$

where

$$F_m = \frac{\iint_A v(x,y)\,\mathrm{d}x\,\mathrm{d}y}{\sum_{i=1}^{m} b_i d_i \bar{v}_i}$$

In practice, F_m can be evaluated from analysis of measurements in which m is sufficiently large for the effects on Q_o of omitting verticals, in stages, to be determined. F_m is subject to a random uncertainty.

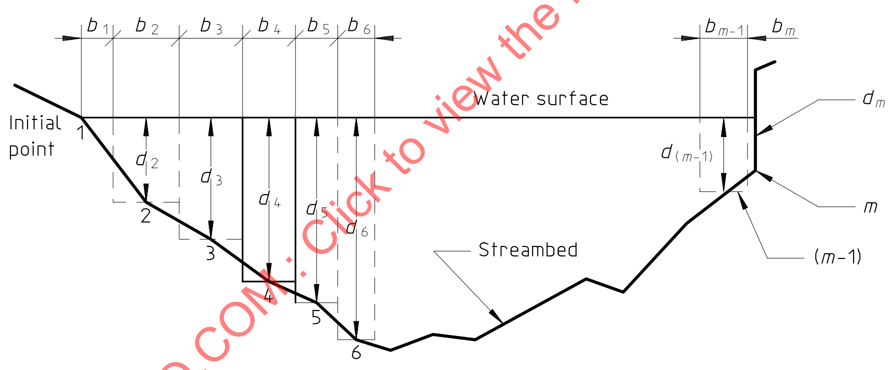
It may be convenient in practice to take an F_m variation with m that is a mean value of values for sections of several different rivers, taken together. Then the actual variations of F_m from river to river, as compared with the meaned variation, will involve both systematic and random uncertainties.

F_m is dependent on the number of verticals m, and tends to unity as m increases without limit. Thus, equation (D.3) can be written approximately as

$$Q = \sum_{i=1}^{m} \left(b_i d_i \bar{v}_i\right) \qquad \text{. . . (D.4)}$$

with increasing accuracy as m increases.

This last form is the one that is given in ISO 748.



Key

1, 2, 3, . . . , m Observation points

b_1, b_2, b_3, . . . , b_m Breadth (metres) of segment associated with the observation point

d_1, d_2, d_3, . . . , d_m Depth of water (metres) at the observation point

Dashed lines Boundary of segments: one heavily outlined

Figure D.1 — Schematic of velocity-area method of discharge measurement (midsection method)

D.2.2 Overall uncertainty of the flow determination

It is plausible to assume that, at a given m, F_m and Q_o can be treated as independent variables.

However, the q_i in principle are not independent of one another, since the value corresponding to any one vertical will be related to the values of adjacent verticals. Furthermore, there is an interdependence between the d_i and $\bar{v}_i$ corresponding to any particular vertical. Thus, by applying the principles for combining experimental standard deviations (see clause 7), the following expression for s_Q, the experimental standard deviation of Q, can be derived from equation (D.3).

$$\left(\frac{s_Q}{Q}\right)^2 = \left(\frac{s_{F_m}}{F_m}\right)^2 + \sum_{i=1}^{m}\left(\frac{q_i}{Q_o}\right)^2\left[\left(\frac{s_{b_i}}{b_i}\right)^2 + \left(\frac{s_{d_i}}{d_i}\right)^2 + \left(\frac{s_{\bar{v}_i}}{\bar{v}_i}\right)^2\right] + \frac{2}{Q_o^2}\left\{\sum_{i=1}^{m-1}\sum_{j=i+1}^{m} s_{ij}^2 + \sum_{i=1}^{m}\left[\left(\frac{q_i^2}{d_i\bar{v}_i}\right)s_{d\bar{v}i}^2\right]\right\} \qquad \text{. . . (D.5)}$$

where s_{ij} arise from the interdependence between q_i and q_j and $s_{d\bar{v}_i}$ from the interdependence between d_i and $\bar{v}_i$.

It is convenient to introduce the notation s' for relative experimental standard deviation. Thus s_{b_i}/b_i is written s'_{b_i}, s_{F_m}/F_m is written s'_{F_m}, and, neglecting s_{ij} and $s_{d\bar{v}_i}$, equation (D.5) becomes

$$s_Q'^2 = s_{F_m}'^2 + \sum_{i=1}^{m}\left(\frac{q_i}{Q_o}\right)^2\left(s_{b_i}'^2 + s_{d_i}'^2 + s_{\bar{v}_i}'^2\right) \qquad \text{. . . (D.6)}$$

If the relative experimental standard deviations s'_{b_i} are all nearly enough equal, of value s'_b, and similarly for the s'_{d_i} and $s'_{\bar{v}_i}$, then

$$s_Q'^2 = s_{F_m}'^2 + \left(s_b'^2 + s_d'^2 + s_{\bar{v}}'^2\right)\sum_{i=1}^{m}\left(q_i/Q_o\right)^2 \qquad \text{. . . (D.7)}$$

If the verticals are so located that $q_i \approx Q_o/m$, then

$$s_Q'^2 \approx s_{F_m}'^2 + \frac{1}{m}\left(s_b'^2 + s_d'^2 + s_{\bar{v}}'^2\right) \qquad \text{. . . (D.8)}$$

In multipoint velocity-area methods, velocity is measured at several points on a vertical, and the mean value is obtained by graphical integration or as a weighted average. The latter treatment can be expressed mathematically as

$$\bar{v} = \sum_{p=1}^{k} w_p v_p \qquad \text{. . . (D.9)}$$

where the w_p are constant weighting factors. The subscript i that identifies the particular vertical is omitted to simplify the symbolism. The weighting factors usually are chosen so that $\sum w_p = 1$. This equation can also represent the single-point method, by taking $k = 1$.

In all cases, the estimates $\bar{v}$ so computed are subject to errors. These errors are due to improper placement of the meter at depth, and to deviations of the actual velocity profile from the presumed profile. The effect of these errors can be expressed by means of a multiplicative coefficient P analogous to the coefficient F_m used for similar purposes in equation (D.3). The same analysis that led to equation (D.5) then yields the following expression for relative experimental standard deviation of the average velocity $\bar{v}$:

$$s_{\bar{v}}'^2 = s_p'^2 + s_v'^2 \frac{\sum\left(w_p v_p\right)^2}{\left(\sum w_p v_p\right)^2} \qquad \text{. . . (D.10)}$$

in which s' denotes relative experimental standard deviation in the subscript variable, v is measured point velocity, and the ratio of wv-sum expresses the variability of weighted velocity over the depth of the vertical. For a uniform k-point velocity profile, this ratio would equal $1/k$. For an extremely nonuniform profile, in which a single term dominated all the others, the ratio would equal 1. The latter value is adopted, at least from small k values, for the sake of conservatism, with the result

$$s'^2_{\bar{v}} = s'^2_p + s'^2_v \qquad \text{. . . (D.11)}$$

This choice also helps to represent the effect of any unaccounted-for correlations among point-velocity errors in the same vertical.

In practice, the random uncertainty in the velocity measurement at a point is assumed to be due to a meter-calibration relative experimental standard deviation, s'_c, together with a stream-pulsation relative experimental standard deviation, s'_e. Then the relative experimental standard deviation for point velocities is

$$s'^2_v = s'^2_c + s'^2_e$$

The corresponding relative experimental standard deviation for average velocity in the vertical is

$$s'^2_{\bar{v}} = s'^2_p + s'^2_c + s'^2_e \qquad \text{. . . (D.12)}$$

D.2.3 Example of calculation of uncertainty

It is required to calculate the uncertainty in a current-meter gauging from the following particulars:

Number of verticals used	20
Exposure time of current meter at each point in the vertical	3 min
Number of points taken on the vertical (single point, two points, etc.)	2
Type of current-meter rating (individual or group)	Individual
Average velocity in measuring section	above 0,3 m·s^{-1}

Details of the procedure are described in ISO 748.

The random and systematic uncertainties are combined by the root-sum-square and linear additive methods as stated in 8.3, i.e. if $2\,S'_Q$ and B'_Q are the percentage overall random and systematic relative uncertainties respectively, then U'_Q, the percentage uncertainty in the current-meter gauging, is

$$U'_{Q_{\text{RSS}}} = \sqrt{\left(2s'_Q\right)^2 + B'^2_Q} \text{ and}$$

$$U'_{Q_{\text{ADD}}} = B'_Q + 2s'_Q$$

D.2.3.1 The equation used for evaluating the overall relative experimental standard deviation is [see equation (D.8)]

$$s'_Q = \sqrt{s'^2_{F_m} + \frac{1}{m}\left(s'^2_b + s'^2_d + s'^2_{\bar{v}}\right)}$$

where

s'_Q is the overall percentage experimental standard deviation;

s'_{F_m} is the percentage experimental standard deviation due to the limited number of verticals used;

s'_b is the percentage experimental standard deviation in measuring the width of segments;

s'_d is the percentage experimental standard deviation in measuring the depth of segments;

$s'_{\bar{v}}$ is the percentage experimental standard deviation in estimating the average velocity in each vertical

and $s'_{\bar{v}} = \sqrt{s'^2_p + s'^2_c + s'^2_e}$ [see equation (D.12)]

where

s'_p is the percentage experimental standard deviation due to limited number of points taken in the vertical (in the present example the two-point method was used, i.e. at 0,2 m and 0,8 m from the surface respectively);

s'_c is the percentage experimental standard deviation of the current-meter rating (in the present example an individual rating was used at velocities of the order of 0,30 m/s);

s'_e is the percentage experimental standard deviation due to pulsations (uncertainty due to the random fluctuation of velocity with time; the time of exposure in the present example was three one-minute readings of velocity).

The percentage values of the above partial uncertainties at the 95 % confidence level are tabulated in D.2.3.2.

The equation for calculating the overall systematic uncertainty is

$$B'_Q = \sqrt{B'^2_b + B'^2_d + B'^2_c}$$

where

B'_Q is the overall percentage systematic uncertainty in discharge;

B'_b is the percentage systematic uncertainty in the instrument measuring width;

B'_d is the percentage systematic uncertainty in the instrument measuring depth; and

B'_c is the percentage systematic uncertainty in the current-meter rating tank.

The systematic uncertainties in the current-meter gauging are confined to the instruments measuring width, depth and velocity, and should be restricted to 1 % as shown in D.2.3.2.

D.2.3.2 The values of the elemental uncertainties affecting uncertainty in discharge are listed in table D.1 as percentage uncertainties at the 95 % confidence level. The numerical values are taken from ISO 748. It is recommended, however, that each user determine independently the values of the uncertainties for any particular measurement.

Table D.1 — Elemental uncertainties affecting uncertainty in discharge

Error source	Units	Relative random uncertainty (2s: 95 %) 2s'	Relative systematic uncertainty B'
F_m, number of verticals	—	5,0 %	—
b, segment width	m	0,5 %	1,0 %
d, segment depth	m	0,5 %	1,0 %
v_p, number of points in the vertical	$m \cdot s^{-1}$	7,0 %	—
v_c, current-meter calibration	$m \cdot s^{-1}$	2,0 %	1,0 %
v_e, current-meter exposure time	$m \cdot s^{-1}$	5,0 %	—

Then, the overall random uncertainty in discharge is given by

$$2s'_Q = 2\sqrt{s'^2_{Fm} + \frac{1}{m}\left(s'^2_b + s'^2_d + s'^2_p + s'^2_c + s'^2_e\right)}$$

$$= \sqrt{25 + \frac{1}{20}(0{,}25 + 0{,}25 + 49 + 4 + 25)}$$

$$= 5{,}4\ \%$$

The overall systematic uncertainty is

$$B'_Q = \sqrt{1^2 + 1^2 + 1^2}$$

$$= 1{,}7\ \%.$$

The combination of both random and systematic uncertainties then gives the overall percentage uncertainty in discharge, U'_Q.

$$U'_{Q_{RSS}} = \sqrt{(2s'_Q)^2 + B'^2_Q}$$

$$= \sqrt{5{,}4^2 + 1{,}7^2}$$

$$= 5{,}7\ \%$$

$$U'_{Q_{ADD}} = B'_Q + 2s'_Q$$

$$= 1{,}7 + 5{,}4$$

$$= 7{,}1\ \%$$

D.2.3.3 The discharge measurement may be expressed in the following form:

Discharge, Q	$m^3 \cdot s^{-1}$
(Combined) uncertainty, $U'_{Q_{RSS}}$	5,7 %
(Combined) uncertainty, $U'_{Q_{ADD}}$	7,1 %
Random uncertainty, $2s'_Q$	5,4 %
Systematic uncertainty, B'_Q	1,7 %

Uncertainties calculated in accordance with this Technical Report.