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TECHNICAL REPORT

ISO TR 3313

Second edition
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Measurement of pulsating fluid flow in a pipe by means of orifice plates, nozzles or Venturi tubes

Mesurage du débit d'un écoulement pulsatoire de fluide dans une conduite au moyen de diaphragmes, tuyères ou tubes de Venturi

Reference number
ISO/TR 3313:1992(E)

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 3313, which is a Technical Report of type 2, was prepared by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits*, Sub-Committee SC 2, *Pressure differential devices*.

This second edition cancels and replaces the first edition (ISO/TR 3313:1974), of which it constitutes a technical revision.

Annexes A, B and C of this Technical Report are for information only.

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Introduction

Methods of measuring fluid flow in a pipe by means of orifice plates, nozzles or Venturi tubes are described in ISO 5167-1. However, it is stipulated that the rate of flow shall be constant or, in practice, vary only slightly and slowly with time (see ISO 5167-1:1991, 6.3.1). ISO 5167-1 does not provide for the measurement of pulsating flow.

The reasons for the publication of this document in the form of a Technical Report are the following:

- the restricted field of application of the document;
- the lack of available data on the relationship between measuring installation parameters and errors in measurement;
- if no appropriate action is taken, pulsation can cause very large errors.

Measurement of pulsating fluid flow in a pipe by means of orifice plates, nozzles or Venturi tubes

1 Scope

This Technical Report defines pulsating flow, compares it with steady flow, and indicates the requirements that allow accurate measurement of the mean rate of flow by means of orifice plates, nozzles and Venturi tubes in pipes where pulsations are present.

This Technical Report applies to flow in which the pulsations are generated at a single source which is situated either upstream or downstream of the flowmeter's primary element. Its applicability is restricted to conditions where the flow direction does not reverse in the measuring section. It includes recommendations which apply to liquid flows and gas flows in which the density changes in the measuring section are small (expansibility factor $\varepsilon \geqslant 0{,}99$ and density fluctuation ratio $\rho_{rms}/\overline{\rho} \leqslant 1/40$). Critical flow Venturi nozzles are treated as special cases.

There is no restriction on the wave-form of the flow pulsation, but the pulsation wavelength must be long compared with the dimensions of the flowmeter.

This Technical Report gives practical criteria for ensuring that flow pulsations are damped to such an extent that systematic errors in the indicated time-mean flow rate do not exceed a specified value. Recommendations concerning the appropriate design of the differential pressure secondary device are also given.

Finally, this Technical Report describes how uncertainty in the measurement of mean flow rate under pulsating flow conditions can be established.

Annex A of this Technical Report defines the theoretical and experimental basis for the determination of the total measuring error.

Annex B gives the criteria for adequate damping.

2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this Technical Report. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this Technical Report are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 5167-1:1991, *Measurement of fluid flow by means of pressure differential devices — Part 1: Orifice plates, nozzles and Venturi tubes inserted in circular cross-section conduits running full.*

3 Definitions

For the purposes of this Technical Report, the following definitions apply.

3.1 steady flow: Flow in which parameters such as velocity, pressure, density and temperature do not vary significantly enough with time to affect the required uncertainty of measurement.

3.2 pulsating flow: Flow in which the flow rate in a measuring section is a function of time but has a constant mean value when averaged over a sufficiently long period of time.

Pulsating flow can be divided into:

— periodic pulsating flow, and

— randomly fluctuating flow.

Unless otherwise stated in this Technical Report, the term "pulsating flow" is always used to describe periodic pulsating flow.

4 Symbols and indices

4.1 Symbols

A	Area
A_*	Area of the throat of a Venturi nozzle
a_r, c_r, b_r	Amplitude of the r^{th} harmonic component in the damped or undamped pulsations
$\overline{B}$	Time-mean value of quantity B (see B.3)
B'	Fluctuation of quantity B such that $B = \overline{B} + B'$
b	The amplitude of pulsations of density
C_c	Contraction coefficient
c	Speed of sound
D	Internal diameter of the tube
d	Internal diameter of orifice or throat bore
E_T	Total error in the indicated time mean flow-rate
f	Pulsation frequency (Hz)
H	Harmonic distorsion factor
L, L_e, l_e	Effective axial lengths
$m = \beta^2$	Orifice area/pipe area ratio
p	Pressure (absolute)
q_m	Mass flow-rate
q_V	Volume flow-rate
t	Time
U	Axial velocity
V	Volume
X	Temporal inertia term for short pulsation wavelengths
β	Orifice diameter/pipe diameter ratio
γ	Ratio of specific thermal capacities
Δp	Differential pressure
$\Delta \varpi$	Pressure loss
θ	Phase angle
κ	Isentropic index
ρ	Fluid density
$\tau = p_2/p_1$	Ratio of pressure
ϕ	Maximum permissible fluctuation in density
ψ, ψ_1	Maximum allowable percentage error
$\omega = 2\pi f$	Angular frequency

4.2 Indices

0	Index for the pulsating source
1, 2	Index for measuring sections
rms	Root mean square

5 General

5.1 The nature of pipe flows

In practice, the flow observed in pipes is mostly a statistically steady flow. When the pipe Reynolds Number is sufficiently high, the flow is always turbulent and is, therefore, a fluctuating flow as there are irregular and random variations in quantities such as flow rate, pressure, density and temperature. If the conditions are similar to those which are typical of fully developed turbulent pipe flow and there is no periodic pulsation, the provisions of ISO 5167-1 apply.

The presence of pulsations is also very common in industrial pipe flows. Pulsating flows are generated by both rotary and reciprocating positive displacement engines, compressors, blowers and pumps. Hydrodynamic oscillations such as vortex shedding can also be a source of pulsation.

It is possible that the damping effect of the flow system between the pulsation source and the flow meter is so great that flow pulsation in the metering sections cannot be detected. In such a situation, the flow is regarded as steady and the methods of ISO 5167-1 are applicable. When flow pulsations in the metering section have an amplitude above the threshold value, however, ISO 5167-1 does not apply and the procedure outlined in this Technical Report should be followed.

As a guideline, the threshold between steady and pulsating flow can be defined in terms of the velocity pulsation amplitude such that if

$$U'_{rms}/\overline{U} < 0{,}05$$

and if the recommendations given in 6.4 are correctly followed,

$$\Delta p'_{rms}/\overline{\Delta p_p} \leqslant 0{,}10$$

U is the axial velocity component, such that

$$U = \overline{U} + U'$$

where U' is the velocity fluctuation,

Δp_p is the differential pressure in the primary element, such that

$$\Delta p_p = \overline{\Delta p}_p + \Delta p'$$

where $\Delta p'$ is the differential pressure fluctuation.

The barred quantities, $\overline{U}$ and $\overline{\Delta p}_p$ are time mean values.

5.2 The detection of pulsations

There is often no obvious indication of the presence of pulsations at the flowmeter. The secondary device used in industrial flowmeter installations is usually a slow response heavily damped instrument which may not show any oscillation. The best method of detecting flow pulsations is to place a hot wire anemometer or similar device on the axis of the pipe upstream of the measuring section. If this is not possible, a fast response differential pressure transducer may be connected across the primary element, provided that the recommendations given in 6.4 are strictly followed.

6 Determination of the mean flow rate of a pulsating flow

6.1 Pulsation effects on the primary device

The most important effect is due to the square root relationship between flow and differential pressure. In pulsating flow, the time-mean flow rate should be computed from the mean of the instantaneous values of the square root of differential pressure.

The practice of using slow, large displacement secondary devices for pulsating flow, and averaging by damping before taking the square root, results in an indicated flow rate greater than the true value. The amount greater depends on the ratio of the root-mean-square amplitude of the flow rate fluctuation to the mean value ($q'_{V,\text{rms}}/\overline{q}_V$).

Modern, fast, small displacement differential pressure transmitters with square root circuits and subsequent averaging exist. These can theoretically provice a more accurate output, but there is insufficient data to establish the magnitude of the improvement.

The recommended procedure for measuring the mean flow rate of a pulsating flow involves the installation of sufficient damping into the flow system such that the pulsation amplitude of the flow rate in the measuring section will be reduced to such a level that the square root error is less than a given allowable value.

The flow rate is calculated in the same way as for steady flow, using the discharge coefficients given in ISO 5167-1. When the amplitude of the residual damped pulsation can be measured, the discharge coefficient may possibly be reduced to compensate for the calculated square root error which is always a positive systematic error. Methods for error calculation are given in clause 8.

6.2 Determination of pulsation amplitude

When damping of the flow stream is required, it is necessary to determine the r.m.s. amplitude of the undamped flow pulsation, $q'_{V0,\text{rms}}$, at the pulsation source, in order to calculate the required damping or throttling volume using the criteria described in 6.3.

There are a number of methods which can be used to obtain the amplitude. These are, in order of preference.

a) Direct measurement using a linearized hot wire anemometer or similar device and r.m.s. voltmeter. The r.m.s. value of the fluctuating component of the voltage output must be measured on a true r.m.s. meter. Mean sensing r.m.s. meters must not be used as these will only read correctly for sinusoidal waveforms.

b) Use of a linearized hot wire anemometer or similar device and computing $q'_{V0,\text{rms}}$ from a recorded time trace.

c) Estimation of $q'_{V0,\text{rms}}$ from the action of the pulsation generator (this method is possible when a positive displacement compressor or motor is situated near the meter location).

d) If the methods described in a), b) or c) cannot be used, the maximum probable value of $q'_{V0,\text{rms}}$ can be approximately inferred from a measurement of $\Delta p'_{p0,\text{rms}}$ using one of the following two inequalities:

$$\frac{q'_{V0,\text{rms}}}{\overline{q}_V} \leqslant \frac{1}{2} \frac{\Delta p'_{p0,\text{rms}}}{\Delta p_{ss}}$$

$$\frac{q'_{V0,\text{rms}}}{\overline{q}_V} \leqslant$$

$$\leqslant \left\{ \frac{2}{1 + \left[1 - \left(\Delta p'_{p0,\text{rms}}/\overline{\Delta p}_{p0}\right)^2\right]^{1/2}} - 1 \right\}^{1/2}$$

where

Δp_{ss} is the differential pressure that would be measured across the primary element under steady flow conditions with the same time-mean flow rate;

$\overline{\Delta p}_{p0}$ is the time-mean differential pressure that would be measured across the primary element under undamped pulsating flow conditions;

$\Delta p'_{\text{p0,rms}}$ is the r.m.s. value of the fluctuating component of the differential pressure across the primary element measured using a fast response pressure transducer;

Δp_{p0} is the instantaneous differential pressure across the primary element under undamped pulsating flow conditions where $\Delta p_{\text{p0}} = \overline{\Delta p_{\text{p0}}} + \Delta p'_{\text{p0}}$.

Note that reliable measurements of $\overline{\Delta p_{\text{p0}}}$ and $\Delta p'_{\text{p0,rms}}$ can only be obtained if the recommendations given in 6.4 are strictly adhered to.

Note also that, if it is possible to determine Δp_{ss}, equation (1) is to be preferred. Equation (2) will only give reliable results if $(\Delta p'_{\text{p0,rms}}/\overline{\Delta p_{\text{p0}}}) < 0,5$.

6.3 Installation requirements

6.3.1 General

Pulsation in gases or vapours can be damped by a combination of volumetric capacity and throttling placed between the pulsation source and the flowmeter (see figure 1). The volumetric capacity can include the volume of any receivers and the pipeline itself, provided the axial lengths involved are short compared with the pulsation wavelength. The throttling can be provided by frictional pressure losses in the pipeline can also contribute towards the throttling effect. The straight pipe length between the meter and any additional throttling device must be in accordance with the installation requirements of ISO 5167-1. Care should be taken that the selected throttling device does not create hydrodynamic oscillations.

Figure 1 shows the installation arrangement of a pulsation source.

Pulsations in liquid flow can be similarly damped by sufficient capacity and throttling, the capacity being provided by an air vessel. Alternatively, a surge chamber can be used instead of the air vessel.

The equations in 6.3.2 and 6.3.3 are used for calculating whether damping is adequate to keep metering errors due to pulsation below a given level. These formulae are insufficiently exact to be used to predict actual values of pulsation errors in a given system.

6.3.2 Gas flow

A parameter which can be used as a criterion of adequate damping in gas flow is the Hodgson number, Ho, defined by

$$Ho = \frac{V}{\bar{q}_V/f} \times \frac{\overline{\Delta \varpi}}{\bar{p}}$$

where

V is the volume of the receiver and the pipework between pulsation source and flowmeter;

$\bar{q}_V/f$ is the time-mean volume flow per pulsation cycle;

$\overline{\Delta \varpi}$ is the overall time-mean pressure loss between the receiver and the source of supply (or discharge) at constant pressure;

$\bar{p}$ is the mean absolute static pressure in the receiver.

Damping will be adequate provided that the following condition is fulfilled:

$$\frac{Ho}{\kappa} \geqslant 0{,}056\,3 \frac{1}{\sqrt{\psi}} \times \frac{q'_{m0,\text{rms}}}{\bar{q}_m}$$

where

κ is the isentropic index of the gas or vapour ($\kappa = \gamma$ is the ratio of the specific thermal capacities for an ideal gas);

$q'_{m0,\text{rms}}$ is the root-mean-square value of the fluctuating component of the flow rate measured at the pulsation source;

$\bar{q}_m$ is the time-mean value of the flow rate;

ψ is the maximum allowable percentage error in the indicated flow rate due to pulsation at the flowmeter.

The pulsation amplitude ratio can be expressed in terms of mass or volume flow rate or bulk mean velocity.

Thus

$$\frac{q'_{m0,\text{rms}}}{\bar{q}_m} = \frac{q'_{V0,\text{rms}}}{\bar{q}_V} = \frac{U'_{0,\text{rms}}}{\bar{U}}$$

The derivation of the criteria for adequate damping implies that the dimensions of the damping chamber and the lengths of pipe between the damping chamber and the flowmeter are short compared with the pulsation wavelength.

The following guidelines may be used.

a) The length of the damping tank, L_1, should not be greater than 1/10 of the pulsation wavelength. Thus the frequency limit is given by

$$f < \frac{c}{10\,L_1}$$

where c is the speed of sound.

b) The length of the pipe, L_2, between damping tank and flowmeter should not be greater than 1/5 of the pulsation wavelength. Thus the limiting frequency must also fulfil the condition

$$f < \frac{c}{5\,L_2}$$

Another very important rule is that the undamped velocity pulsation amplitude $(U'_{0,\mathrm{rms}}/\bar{U}_0)$ must be determined very close to the inlet to the damping tank.

6.3.3 Liquid flow

Pulsating liquid flow can be damped by placing either a surge chamber or an air vessel between the pulsating flow source and the primary element (see figures 2 and 3).

In the diagrams, the flow is shown with the pulsation source upstream. It is equally possible, however, to have a flow with a constant head source upstream and pulsation source downstream. In this case, the surge chamber or air vessel should be placed downstream of the primary element.

6.3.3.1 Use of a surge chamber

The criterion for adequate damping when a surge chamber is used is that

$$\frac{\bar{Z}A}{\bar{q}_V/f} \geqslant 0{,}056\,3\,\frac{1}{\sqrt{\psi}} \times \frac{q'_{V0,\mathrm{rms}}}{\bar{q}_V}$$

where

$\bar{Z}$ is the time-mean value of the difference in liquid level between the surge chamber and the constant head vessel;

A is the cross-sectional area of the surge chamber.

6.3.3.2 Use of an air vessel

The criterion for adequate damping when an air vessel is used is that

$$\frac{1}{\kappa} \times \frac{V_0}{\bar{q}_V/f} \times \frac{\overline{\Delta\varpi}}{p_0} \times \frac{1}{(1 + V_0\rho g/p_0\kappa A)} \geqslant$$

$$\geqslant 0{,}056\,3\,\frac{1}{\sqrt{\psi}} \times \frac{q'_{V0,\mathrm{rms}}}{\bar{q}_V}$$

where

V_0 is the volume of the air in the air vessel;

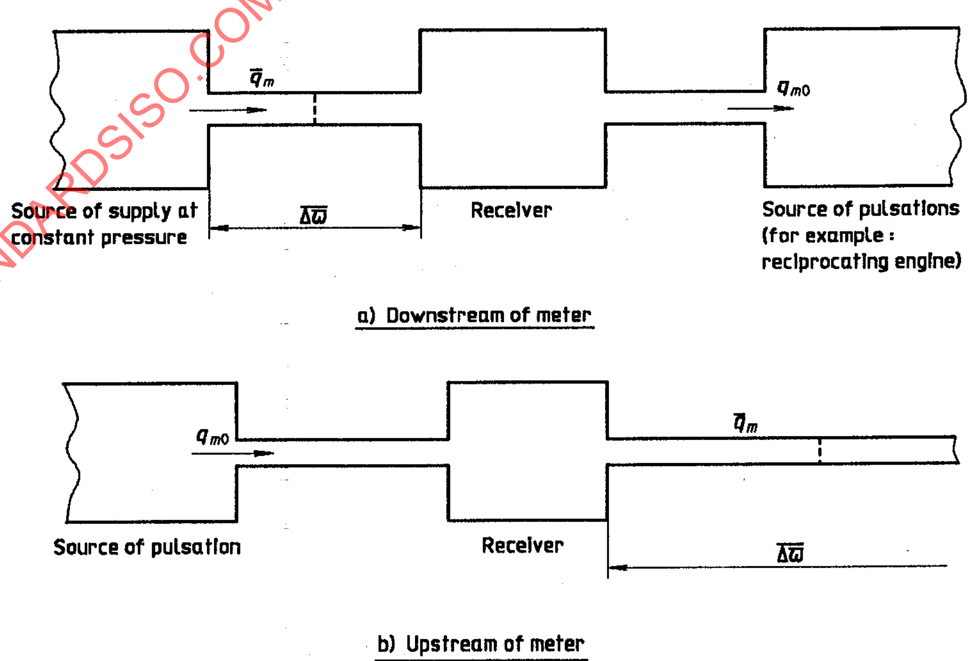


Figure 1 — Installation arrangement of a pulsation source

p_0 is the pressure of the air in the air vessel;

κ is the isentropic index for air;

ρ is the density of the liquid;

g is the acceleration due to gravity;

A is the free surface area of the liquid in the air vessel;

$\overline{\Delta\varpi}$ is the mean value of the difference in pressure between the air vessel and the constant head vessel.

6.4 Pulsation effects on the differential pressure secondary device

Pulsations at the pressure tappings can cause serious errors in the indicated time-mean differential pressure. These errors are due to wave action in the connecting leads and to non-linear damping both in the leads and in the differential pressure sensor itself. The magnitude of the errors depends not only on the pulsation characteristics, but also on the geometry of the secondary device. At present, it is not possible to define a threshold level of negligible pulsation applicable to all designs of secondary devices. However, it is possible to recommend a number of design rules.

For slow response secondary devices used to indicate the time-mean differential pressure in pulsating flow conditions, the rules are as follows.

a) The bore of the pressure tapping must be uniform and not too small. Piezometer rings must not be used.

b) The tube connecting the pressure tapping to the manometer must be as short as possible and of the same bore as the tappings. Lengths of head near the pulsation quarter-wave length should be avoided.

c) Volumes of gas must not be included in the connecting tubes or sensing element.

d) Damping resistances in the connecting tubes and sensing element must be linear. Throttle cocks must not be used.

e) The natural frequency of the sensing element must be much lower than the pulsation frequency.

f) When the above rules cannot be observed, the secondary device may be effectively isolated from pulsations by the insertion of identical linear-resistance dumping plugs into both connecting tubes, as close as possible to the pressure tappings.

It should be understood that observance of the rules listed in items a) to f) for a slow response device cannot eliminate square root error but merely reduces the error in the measurement of the time-mean differential pressure.

For a fast response electronic differential-pressure transducer, rules a), b) and c) and the following apply.

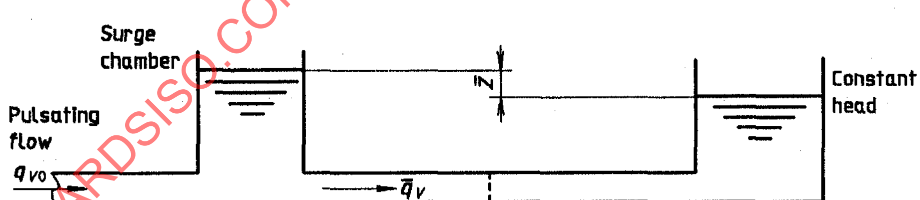


Figure 2 — Surge chamber

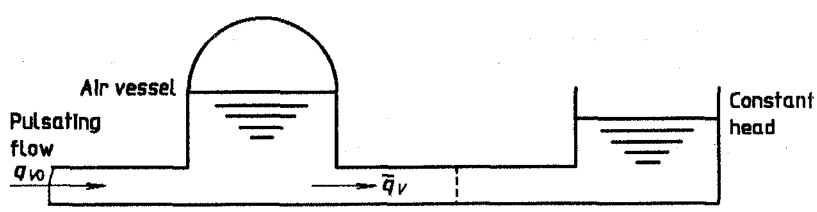


Figure 3 — Air vessel

g) The mechanical and electronic frequency limits of the transducer should be at least 10 times greater than the flow pulsation frequency.

h) The distance along the pressure passage from tapping to sensing element must be small compared with the pulsation quarter-wave length.

i) The device must be geometrically similar on both upstream and downstream sides.

7 Flow measurement

When the conditions specified in this Technical Report concerning the determination of the mean rate of flow and the installation have been satisfied, the methods of measurement given in ISO 5167-1 can then be used. It is, of course, necessary to respect all the other requirements specified in this Technical Report, apart from the need for steady flow through the primary element.

8 Errors

The formulae given in 6.3 allow adequate damping to be calculated for a given maximum allowable relative error, ψ, in the indicated flow-rate due to the residual damped pulsation. It is also possible to estimate the actual error, E_T, directly after measuring the pulsating amplitude at the orififce. The actual error, E_T, should be less than ψ.

In theory, E_T is always a positive systematic error, but in practice there will also be an additional random uncertainty mostly due to pulsation effects in the secondary device. Calculations of the errors and additional uncertainty are feasible provided that the pulsation amplitudes are not too large.

Limiting pulsation amplitudes for error calculations are

$$\frac{q'_{V,\mathrm{rms}}}{\bar{q}_V} = \frac{U'_{\mathrm{rms}}}{\bar{U}} \leqslant 0{,}32$$

or

$$\frac{\Delta p'_{\mathrm{p,rms}}}{\Delta p_{\mathrm{ss}}} \leqslant 0{,}64$$

or

$$\frac{\Delta p'_{\mathrm{p,rms}}}{\overline{\Delta p_{\mathrm{p}}}} \leqslant 0{,}58$$

The following equations can be used to estimate the actual error, E_T, for the low amplitude pulsations.

$$E_T = \left[1 + \left(\frac{U'_{\mathrm{rms}}}{\bar{U}}\right)^2\right]^{1/2} - 1$$

or

$$E_T = \left[1 + \frac{1}{4}\left(\frac{\Delta p'_{\mathrm{p,rms}}}{\Delta p_{\mathrm{ss}}}\right)^2\right]^{1/2} - 1$$

or

$$E_T = \frac{1}{2}\left\{1 + \left[1 - \left(\frac{\Delta p'_{\mathrm{p,rms}}}{\overline{\Delta p_{\mathrm{p}}}}\right)^2\right]^{1/2}\right\}^{-1/2} - 1$$

The systematic error can be compensated for by reducing the discharge coefficient by a percentage equal to $(1 - E_T)$.

There will be an additional uncertainty in the value of the discharge coefficient due to the pulsation, even after allowing for the systematic effect.

This percentage additional uncertainty is equal to 100 E_T and should be added to the uncertainty calculated for steady flow.

If the frequency response of the entire secondary system, including the connecting tubes, can be proved to be flat from 0 to $10f$ (where f is the fundamental pulsation frequency), the additional percentage uncertainty may be reduced to 100 $E_T/2$.

Annex A
(informative)

Theoretical considerations and supporting experimental evidence

A.1 General

The recommendations in this Technical Report are based on the fact that, unless the amplitude of the pulsations in a pulsating flow is reduced to an acceptable level, an accurate value of time-mean flow rate will not be obtained by substituting the time-mean differential pressure in the ordinary steady flow formula. At sufficiently small pulsation amplitudes, it is possible to relate the error in the indicated flow rate to pulsation amplitude and this has been done in the derivation of the formulae for adequate damping in this Technical Report.

A.2 Derivation of equations relating error in indicated flow rate to pulsation amplitude

These equations are derived from the assumption that the fluid can be regarded as incompressible. Experimental results obtained by placing orifice meters in pulsating gas flows indicate that this assumption is only valid if

- the expansibility factor is very nearly unity, i.e. if $\varepsilon \geqslant 0{,}99$, and
- the amplitude of the fluctuations in upstream density are very small compared with the mean value, i.e. if $\rho'_{rms}/\bar{\rho} \leqslant 1/40$.

It is also assumed that the throat of the meter is small compared with the pulsation quarter-wave length.

The special case of critical flow nozzles is covered in B.2.

A.3 Total error in the indicated flow rate

The definition of the total error, E_T, is based on the fact that the indicated flow rate during pulsating flow is calculated by using the measured time-mean differential pressure in the steady flow equation. It is assumed that there are no secondary-device errors.

Hence

$$E_T = \sqrt{\frac{\overline{\Delta p_p}}{\Delta p_{ss}}} - 1 \qquad \ldots (A.1)$$

where

$\overline{\Delta p_p}$ is the time-mean differential pressure measured in pulsating conditions;

Δp_{ss} is the differential pressure that would have been measured under steady conditions at a flow rate equal to the time-mean flow rate of the pulsating flow.

A.4 Quasi-steady temporal inertia theory

The one-dimensional flow momentum equation is

$$\frac{\partial u}{\partial t} + \frac{u \partial u}{\partial x} + \frac{1}{\rho} \times \frac{\partial p}{\partial x} = 0 \qquad \ldots (A.2)$$

and the continuity equation is

$$A \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u A)}{\partial x} = 0 \qquad \ldots (A.3)$$

If it is assumed that the flow is incompressible, these two equations can be combined and integrated with respect to x between the pressure tappings to give an equation for the *instantaneous* differential pressure, Δp_p:

$$\Delta p_p = \frac{q_m^2 (1 - C_c^2 \beta^4)}{2\rho (C_c \pi d^2/4)^2} + \rho \int_1^2 \frac{\partial u}{\partial t} \, \partial x \qquad \ldots (A.4)$$

The first term on the right-hand side of the equation is the differential presure associated with the convective acceleration of the fluid through the restriction. It is identical to the expression for the differential pressure generated by a steady flow with a mass flow rate equal to instantaneous mass flow rate, q_m, thus

$$\Delta p_p = \Delta p_s + \rho \int_1^2 \frac{\partial u}{\partial t} \, dx$$

The second term is the differential pressure associated with the temporal acceleration of the fluid. Its magnitude increases with pulsation frequency and is zero for steady flow. The integral cannot be evaluated exactly but can be replaced by an equivalent term:

$$\frac{L_e}{(C_c \pi d^2/4)} \times \frac{dq_m}{dt}$$

where L_e is the effective axial length of the restriction.

The instantaneous mass flow rate, q_m, can be expressed by

$$q_m = \bar{q}_m [1 + \phi(t)] \quad \text{. . . (A.5)}$$

where

$$\phi(t) = \sum_{r=1}^{\infty} a_r \sin(r\omega t + \theta_r) \quad \text{. . . (A.6)}$$

and

$$\frac{dq_m}{dt} = \bar{q}_m \sum_{r=1}^{\infty} r\omega\, a_r \cos(r\omega t + \theta_r) \quad \text{. . . (A.7)}$$

For a steady flow:

$$\Delta p_{ss} = \frac{\bar{q}_m^2 (1 - C_c^2 \beta^4)}{2\rho (C_c \pi d^2/4)^2} \quad \text{. . . (A.8)}$$

and hence for pulsating flow:

$$\Delta p_p = \Delta p_{ss} \left\{ [1 + \phi(t)]^2 + \frac{2J\phi'(t)}{\omega} \right\} \quad \text{. . . (A.9)}$$

where

$$J = \frac{2 \pi C_c}{(1 - C_c^2 \beta^4)} \times \frac{L_e}{d} \times \frac{fd}{\bar{U}_d} \quad \text{. . . (A.10)}$$

where C_c is the contraction coefficient for an orifice ($C_c = 1$ for a nozzle or Venturi tube).

By integrating equation (A.9) with respect to time we obtain:

$$\overline{\Delta p_p} = \Delta p_{ss} (1 + \overline{\phi^2})$$

and thus

$$E_T = \sqrt{\frac{\overline{\Delta p_p}}{\Delta p_{ss}}} - 1 = [1 + (U'_{rms}/\bar{U})^2]^{1/2} - 1 \quad \text{. . . (A.11)}$$

where U'_{rms} is the root-mean-square amplitude of the fluctuating component (i.e. the a.c. component) of velocity U.

i.e.

$$U'_{rms} = \bar{U} \left(\sum_{r=1}^{\infty} \frac{a_r^2}{2} \right)^{1/2} \quad \text{. . . (A.12)}$$

The ratio $(U'_{rms}/\bar{U})$ is equal to $(q_{m,rms}/\bar{q}_m)$ for incompressible flow and is the velocity pulsation amplitude ratio.

Amplitudes of differential pressure pulsation may be easier to measure than those of velocity pulsations, however, and it is, therefore, useful to define:

$$\Delta p'_{p,rms} = \sqrt{\overline{(\Delta p_p - \overline{\Delta p_p})^2}} \quad \text{. . . (A.13)}$$

From equation (A.9) and neglecting terms of secondary importance we obtain:

$$\frac{\Delta p'_{p,rms}}{\Delta p_{ss}} = \sqrt{4\overline{\phi^2} + \frac{4J^2}{\omega^2} \overline{\phi'^2}}$$

and introducing the Harmonic Distorsion Factor H:

$$H = \left[\frac{\sum_{r=1}^{\infty} (r^2 a_r^2)}{\sum_{r=1}^{\infty} a_r^2} \right]^{1/2} \quad \text{. . . (A.14)}$$

where a_r is the amplitude of the r^{th} harmonic in the Fourier series defined in equation (A.6).

For a sine wave, H has a value of 1 and is greater than 1 for all other waveforms.

Thus

$$\frac{\Delta p'_{p,rms}}{\Delta p_{ss}} = 2 \sqrt{\overline{\phi^2}(1 + H^2 J^2)} \quad \text{. . . (A.15)}$$

Hence, from equation (A.11) the following alternative formulae for total error, E_T, may be obtained:

$$E_T = \left[1 + \frac{1}{4} (\Delta p'_{p,rms}/\Delta p_{ss})^2 / (1 + H^2 J^2) \right]^{1/2} - 1 \quad \text{. . . (A.16)}$$

and also, from

$$\frac{\overline{\Delta p_p}}{\Delta p_{ss}} = 1 + \overline{\phi^2}$$

we obtain

$$E_T = \frac{1}{2}$$

$$\{1 + [1 - (\Delta p'_{p,rms}/\overline{\Delta p_p})^2 / (1 + H^2 J^2)]^{1/2}\}^{-1/2} - 1 \quad \text{. . . (A.17)}$$

Both equations (A.16) and (A.17) involve inertia terms H^2J^2, where J is defined by equation (A.10) and is proportional to the Strouhal Number

$$\frac{fL_e}{U_d}$$

The effective length, L_e, cannot be measured directly but it is likely that $L_e \approx d$.

Temporal inertial effects are negligible when

$$H^2J^2 \ll 1$$

When this is so, the total error, E_T, is theoretically entirely due to the square root effect and is only dependent on the pulsation amplitude.

In the derivation of all equations from equation (A.4), it was assumed that the coefficient of discharge and the expansibility factor behave as constants with their steady flow values. It was also assumed that the upstream density remained constant and that U was the axial velocity in the upstream section.

A.5 Experimental works

Despite the considerable number of publications on the subject of pulsation errors in orifice meters, during the last 60 years, there is a scarcity of reliable data. For data to be reliable it is essential that

- the investigators appreciated the importance of the effects of the secondary element and that instantaneous differential pressures were measured using reliable fast response pressure transducers, and
- that the r.m.s. value of pulsating amplitudes was measured in order to account for the effect of waveform variations.

Two investigations (by Yazici[1] and by Downing and Mottram[2]) in which these conditions were fulfilled are described in A.5.1 and A.5.2.

A.5.1 Yazici's experiment

In Yazici's experiment, pulsations in the range 10 Hz to 150 Hz were generated by an eccentric disc rotating at the outlet of a sonic air nozzle. The flow from the sonic nozzle passed through a cylindrical chamber 200 mm long and 200 mm in diameter and then into a 100 mm bore pipe. The orifice meter under test was located down this pipe 1 m from the cylindrical chamber. Yazici found that the measured errors in indicated flow rate agreed very well with the errors predicted using equation (A.11) provided the ratio $\left(U'_{rms}/\overline{U}\right)$ was small.

A.5.2 Downing and Mottram's experiments

A.5.2.1 The flow rig and instrumentation

Downing and Mottram carried out experiments in which pulsations were generated in an air flow rig of 80 mm bore in a frequency range 5 Hz to 50 Hz. A variable stroke piston pulsator in which the pulsator cylinder was co-axial with the meter run was located about 45 pipe diameters upstream of the square edged orifice meter. This design was adopted in order to avoid distortion of the normal pipe flow velocity profile. The layout of the flow rig is shown in figure A.1.

The dynamic differential pressure was measured using a variable reluctance transducer of symmetrical design having a central diaphragm with identical pressure passages and connecting leads to the upstream and downstream corner tappings on the orifice meter installation.

A hot-wire anemometer probe was used to monitor the centre line velocity, three pipe diameters upstream of the meter.

A true r.m.s. voltmeter and digital d.c. voltmeter were used for measuring signal amplitudes and time-mean values respectively.

A.5.2.2 Results

A series of tests was carried out in which the differential pressure pulsation amplitude ratio, $\Delta p'_{p,rms}/\Delta p_{ss}$, was kept constant. The metering error

$$E_T = \sqrt{\frac{\overline{\Delta p_p}}{\Delta p_{ss}}} - 1$$

was measured at different frequencies.

The tests were repeated using different orifice sizes, and different Reynolds numbers for both sinusoidal and non-sinusoidal pulsations. The results of these tests are shown in figure A.2 in which metering errors, E_T, are plotted against effective Strouhal numbers $\left(Hfd/\overline{U}_d\right)$. It should be noted from equation (A.10) that the Strouhal number $\left(fd/\overline{U}_d\right)$ is the variable factor in the temporal inertia term J. The remaining part of the term

$$\frac{2\,\pi\,C_c}{\left(1 - C_c^2\,\beta^4\right)} \times \frac{L_e}{d}$$

is a geometrical constant the value of which is in the order of 5.

The results broadly verify the effects predicted by equation (A.16) which is also plotted on the graph for the three values of $\Delta p'_{p,rms}/\Delta p_{ss}$. It can be observed that the scatter in the results diminishes with the pulsation amplitude indicating that agreement with the theory is better at small amplitudes.

A further series of tests was carried out at small pulsation amplitudes giving pulsation errors in the range of 0 to 6 %. The results of these tests are shown on figure A.3 with the metering error, E_T, plotted against the pulsation amplitude ratio, $\Delta p'_{p,rms}/\overline{\Delta p_p}$. The continuous curve shown on figure A.3 is the theoretical square root error corresponding to equation (A.17) with the H^2J^2 term assumed to be negligible. Inspection of the results shows that the metering errors measured for values of effective Strouhal number ⩽ 0,05 agree very well with the theoretical curve. Measured errors for tests where the Strouhal number exceeded 0,05 were less than the theoretical square root error.

Dimensions in meters

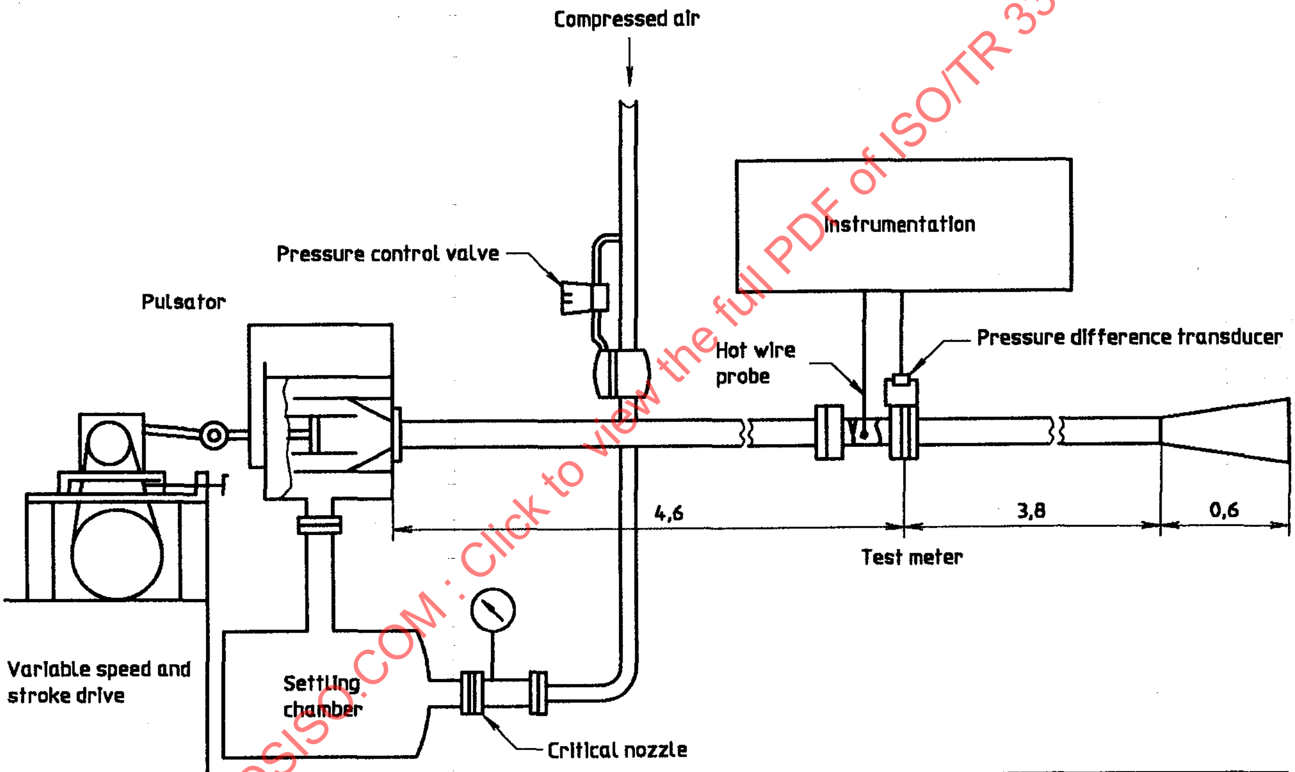


Figure A.1 — Diagram of an air flow rig in Downing and Mottram's experiments

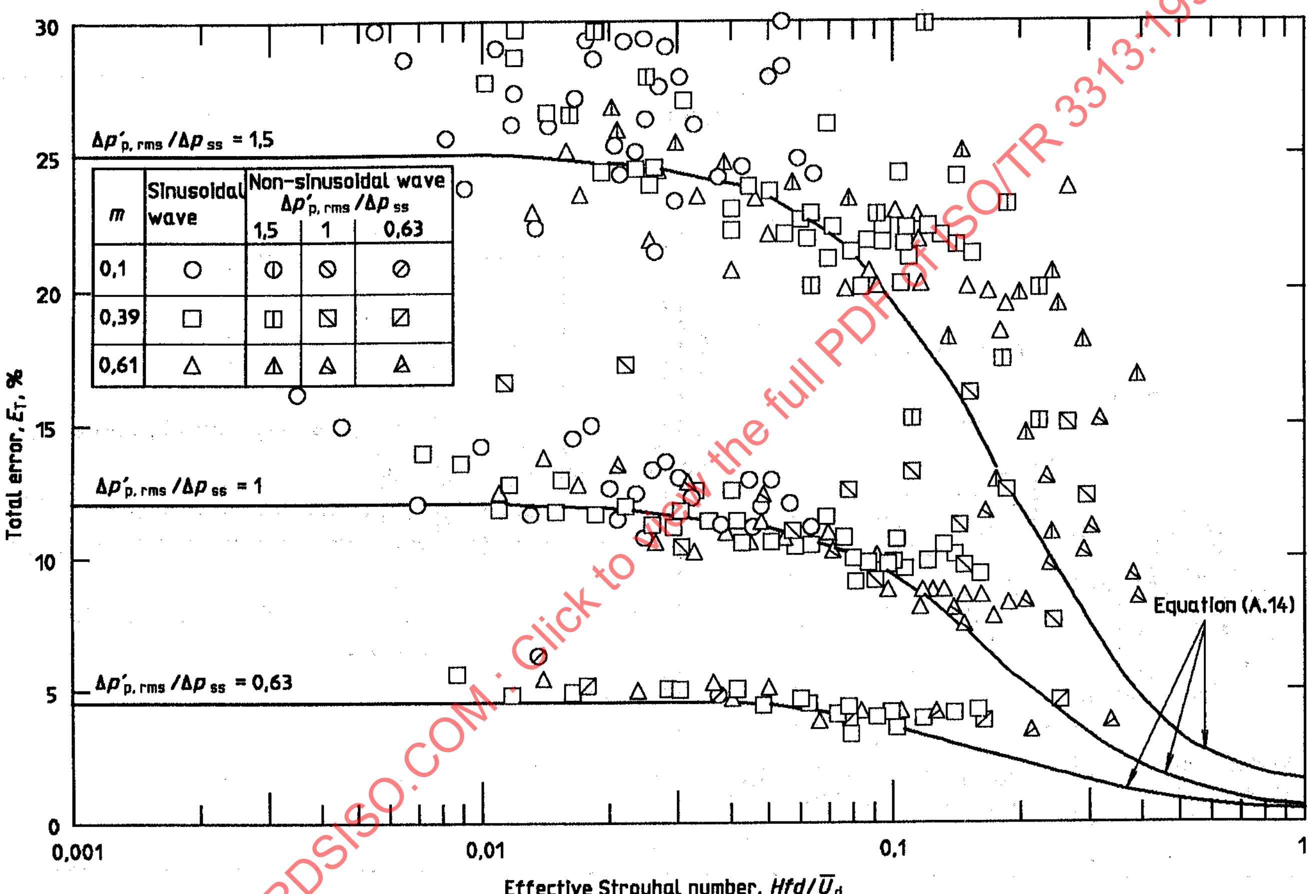
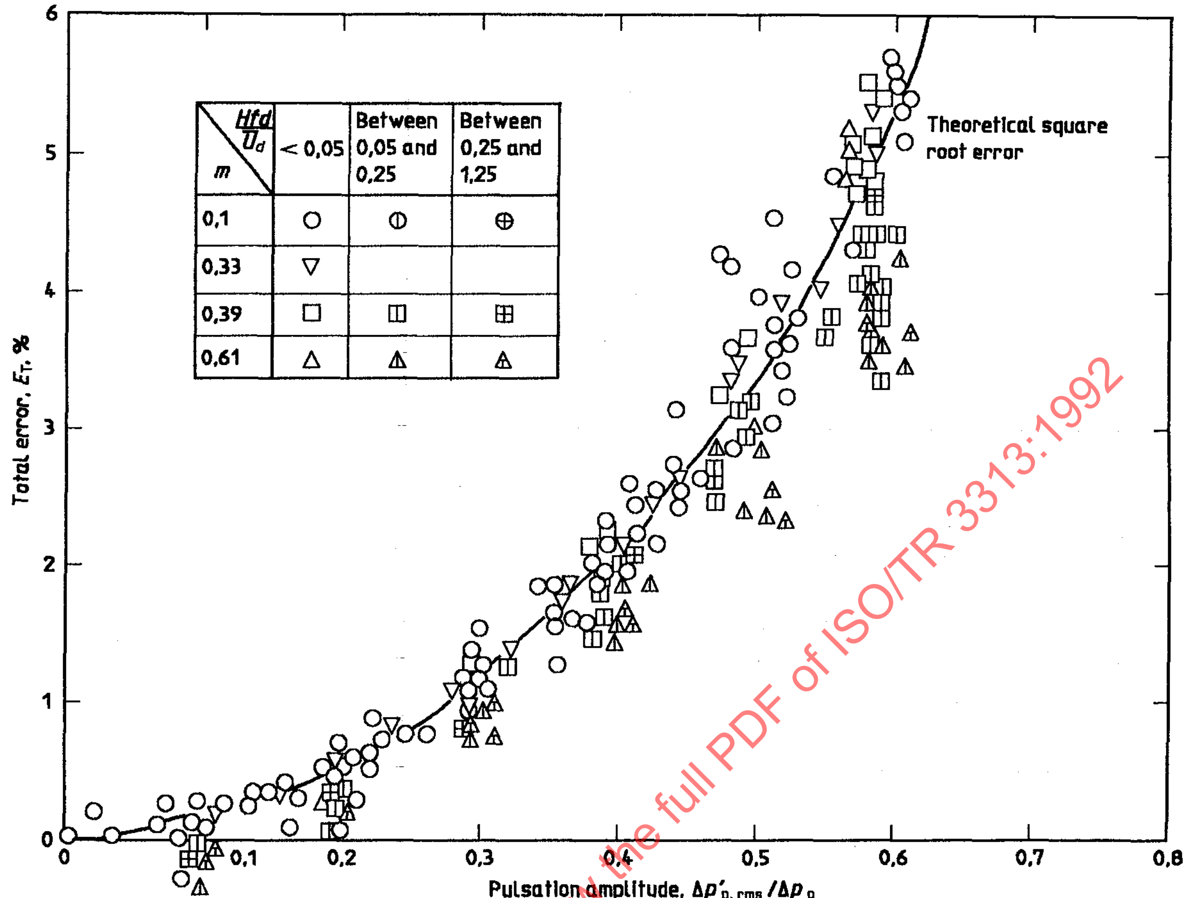


Figure A.2 — Curve of the metering error, E_T, as a function of the effective Strouhal number, $Hfd/\overline{U}_d$



NOTE — E_T is measured using an electronic secondary device.

Figure A.3 — Curve of the metering error, E_T, as a function of the pulsation amplitude, at low pulsation amplitudes (for $f = 5$ Hz to 50 Hz)

When the pulsation frequency is high, the meter length can no longer be short compared with the pulsation wavelength. When this is the case, the required theoretical treatment is slightly different from the quasi-steady/temporal inertia theory outlined above, and Mohammed and Mottram[11] derive expressions for total error, E_T, which are identical to equations (A.14) and (A.15) except that the temporal inertial term H^2J^2 is replaced by a term X, where X is a strong function of L_e/λ, L_e is the effective axial length of the meter and λ is the pulsation wavelength.

In fact

$$\frac{L_e}{\lambda} = \frac{fL_e}{\bar{U}_d} \times \frac{\bar{U}_d}{c}$$

where c is the speed of sound.

Mohammed and Mottram obtained experimental results with air at frequencies in the range of 50 Hz to 500 Hz which show a good correlation between total error, E_T, and the modified pulsation amplitude

$$\frac{\Delta p'_{p,rms}}{\Delta p_{ss}} (1 + X)^{-1/2}$$

However, at low pulsation amplitudes experimental results indicate that X is not a significant parameter and the square root error effect is predominant even at higher frequencies. The low amplitude results obtained for frequencies between 50 Hz and 500 Hz are shown in figure A.4 and can be seen to be very similar to those obtained for frequencies between 5 Hz and 50 Hz shown in figure A.3.

Equation (A.11) shows that the total error in the indicated flow rate, E_T, is theoretically only dependent on the velocity pulsation amplitude, $U'_{rms}/\bar{U}$. Figure A.5 is a plot of results obtained using Downing's thesis[13] showing a good correlation between E_T and $(U'_{rms}/\bar{U})$. It should be noted that the velocity amplitude was measured by a hot wire probe on the pipe centreline, $3D$ upstream from the orifice.

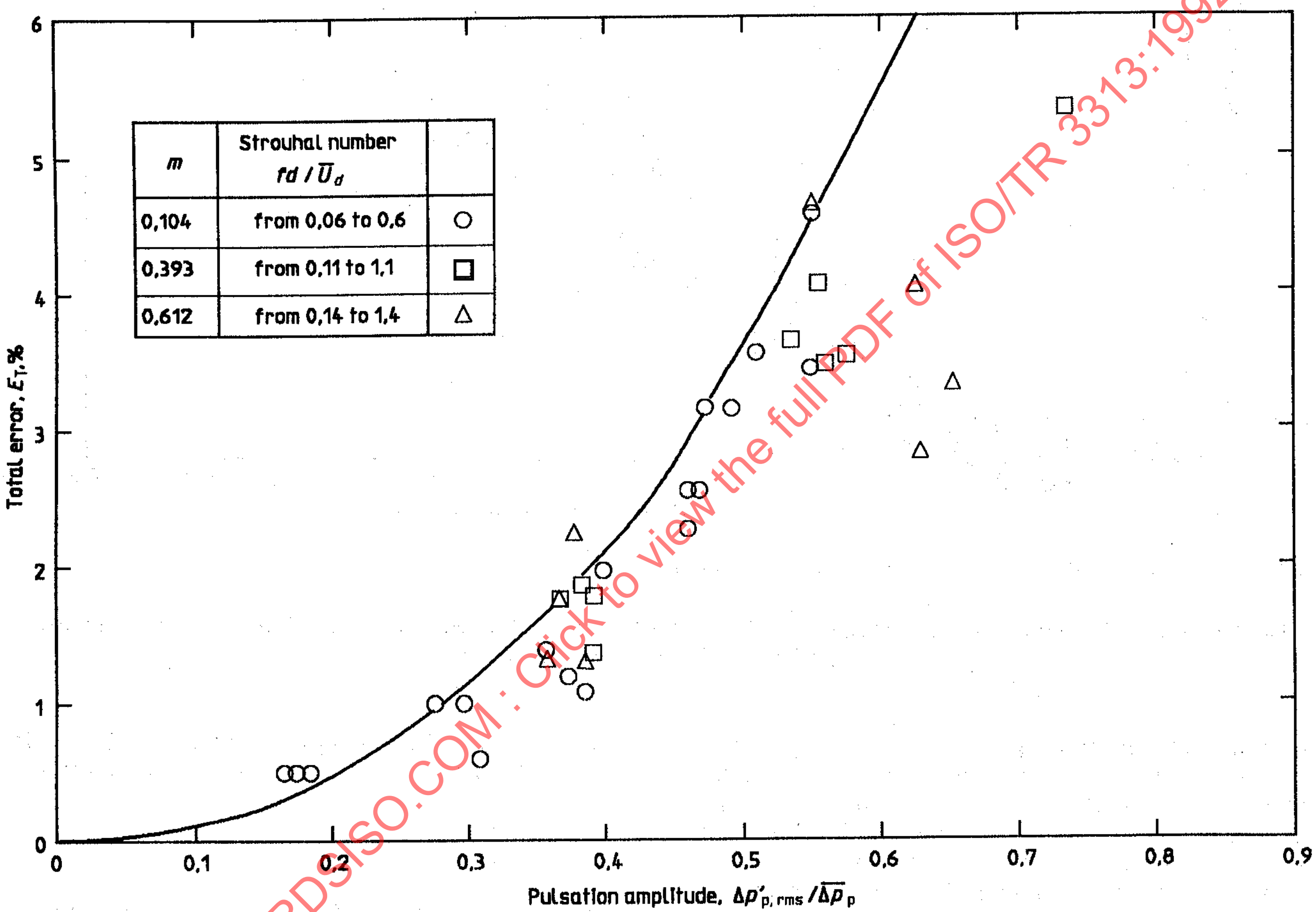
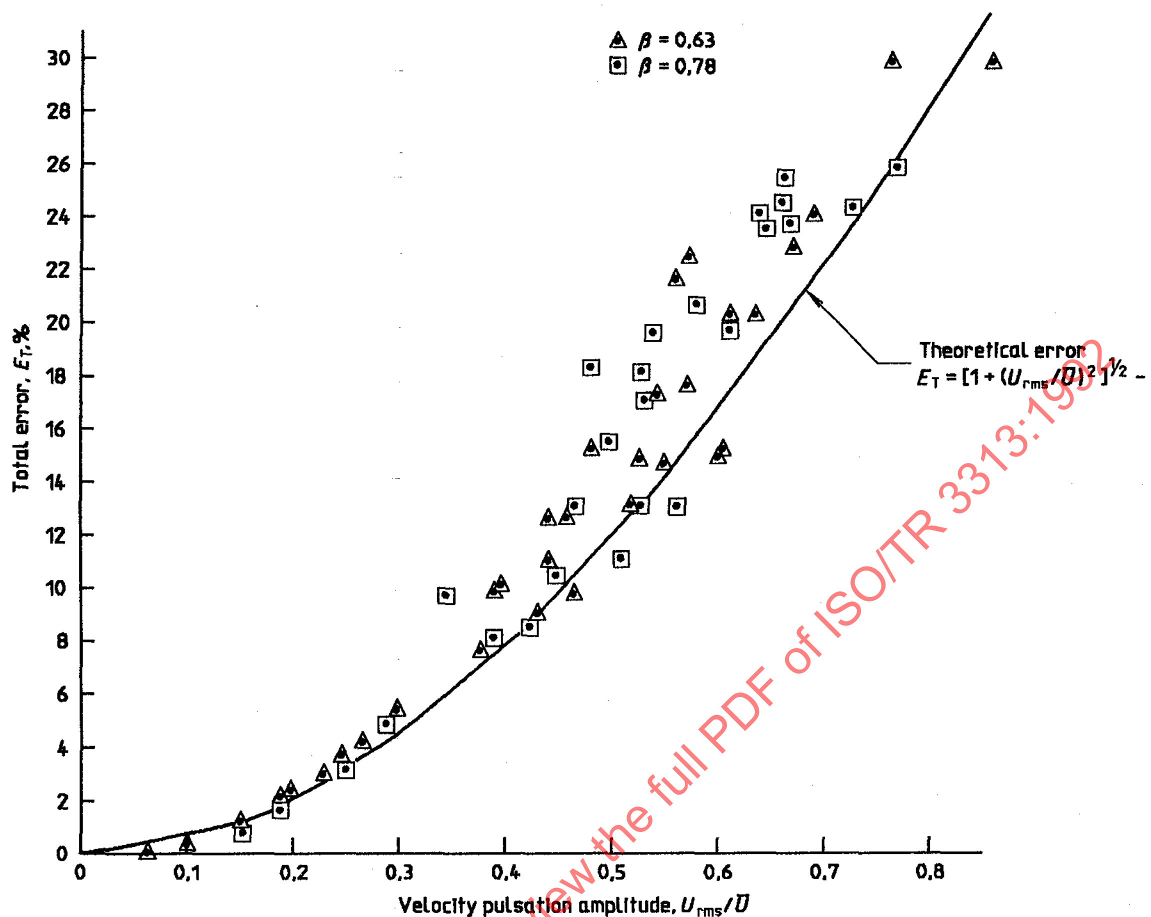


Figure A.4 — Curve of the metering error, E_T, as a function of the pulsation amplitude, at low pulsation amplitudes (for f = 50 Hz to 500 Hz)



NOTE — The velocity pulsation amplitude is measured on the pipe centerline, $3D$ upstream of the orifice.

Figure A.5 — Curve of the measuring error, E_T, as a function of the velocity pulsation amplitude

A.6 Conclusions on experimental work

The experimental results obtained by Yazici show that the pulsation error, E_T, correlates well with the velocity pulsation amplitude, $(U'_{rms}/\bar{U})$ at small values of this parameter.

The results obtained by Downing and Mottram, and Mohammed and Mottram show that E_T correlates well with the differential-pressure amplitude $(\Delta p'_{p,rms}/\Delta p_{ss})$ and a parameter based on the Strouhal number $fL_e/\bar{U}_d$.

The results obtained by Downing also show a good correlation between E_T and the velocity pulsation amplitude $(U'_{p,rms}/\bar{U})$.

However, at low pulsation amplitudes the experimental results show that the Strouhal number effects are not significant, and that the pulsation error can be adequately predicted by the simple square root theory with the amplitude $(\Delta p'_{p,rms}/\overline{\Delta p_p})$ as the only parameter.

The above conclusions are only valid in the absence of significant compressibility effects and when true measurement of $\Delta p'_{p,rms}$ and $\overline{\Delta p_p}$ can be obtained.

Annex B
(informative)

Criterion for adequate damping

B.1 Introduction

In the 1920's, J.L. Hodgson published work[3 and 4] demonstrating that a dimensionless number could be used as a criterion for adequate damping of pulsating gas flow. Later, this criterion was slightly modified by Ruppel[5] and the number which has become known universally as the Hodgson number is defined as follows:

$$Ho = \frac{V}{\bar{q}_V/f} \times \frac{\overline{\Delta\varpi}}{\bar{p}}$$

where

V is the volume of the receiver and the pipework between source and flowmeter;

$\bar{q}_V$ is the time-mean volumetric flow rate at the mean density in the receiver;

f is the fundamental pulsation frequency;

$\overline{\Delta\varpi}$ is the overall time-mean pressure loss between the receiver and the source of supply (or discharge) at constant pressure;

$\bar{p}$ is the mean absolute static pressure in the receiver.

Hodgson himself related his criterion to certain specific pulsation amplitudes and waveforms typical of slow speed reciprocating steam engines.

Later Lutz[6], Herning and Schmidt[7], and Kastner[8] presented work concerning the measurement of airflow inducted by internal combustion engines. This work broadly confirmed the validity of Hodgson's damping criterion. More recently, Fortier[9] presented a theoretical analysis in which values of the Hodgson number required for adequate damping could be predicted for both sinusoidal and rectangular waveforms.

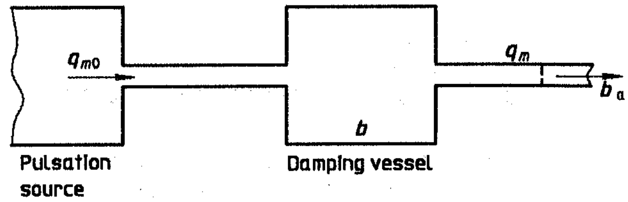
Mottram[10] presented an analysis similar to Fortier's but made it applicable to pulsations of any waveform. A summary of these analyses, slightly modified, is presented below.

B.2 Theoretical analysis for adequate damping — Low Mach number flow in the throttling device

The following analysis is based on the "lumped element" theory, i.e. the pipe lengths are assumed to be short compared with pulsation wavelengths. Damping vessel design based on a more sophisticated "distributed element" analysis requires knowledge of the acoustic characteristics of each particular flow system.

Consider a system in which a pulsating gas flow passes through a receiver to a throttling device including an orifice meter located in a length of pipe terminating in a constant pressure reservoir, e.g. the atmosphere (see figure B.1).

The dimensions of the vessel and the lengths of pipe including the throttling device are short compared with the pulsation wavelength and it is assumed that everywhere in the throttling device the Mach number is less than 1.



Key:

q_{m0} Instantaneous mass flow rate at the pulsation source,
q_m Instantaneous mass flow rate on the throttling device side of the damping vessel.

Figure B.1 — Diagram of a system with a damping vessel

The undamped and damped mass flow rates can be represented by a Fourier series and hence

$$q_{m0} = \bar{q}_m \left[1 + \sum_{r=1}^{\infty} b_r \sin(r\omega t + \theta_{0r}) \right] \qquad \ldots \text{(B.1)}$$

and

$$q_m = \bar{q}_m\left[1 + \sum_{r=1}^{\infty} a_r \sin(r\omega t + \theta_r)\right] = \bar{q}_m[1 + \phi(t)] \qquad \ldots \text{(B.2)}$$

where

b_r and a_r are the amplitudes of the r^{th} harmonic components in the series representing the undamped and damped flows respectively;

θ_{0r} and θ_r are phase angles which may be different for high harmonic components.

The instantaneous differential pressure Δp_p across the throttling device is always given by an equation which has the general form

$$\Delta p_p = \Delta p_{ss}\,\{[1 + \phi(t)]^n + B\phi'(t)\} \qquad \ldots \text{(B.3)}$$

where

Δp_{ss} is the differential pressure for a steady flow of mass flow rate $\bar{q}_m$ through the throttling device;

n is an exponent which depends on the structure of the throttling device but is generally equal to 2;

$$B = \frac{1}{l_e} \times \frac{\bar{q}_m}{\Delta p_s}$$

where l_e is an effective length of the throttling device.

Since the dimensions of the damping vessel are assumed to be small compared with the pulsation wavelength, continuity of mass flow is preserved if

$$V\frac{d\rho}{dt} + q_m = q_{m0} \qquad \ldots \text{(B.4)}$$

If the processes in the damping chamber can be assumed to be isentropic, then

$$\frac{d\rho}{dt} = \frac{1}{c^2} \times \frac{dp}{dt} \qquad \ldots \text{(B.5)}$$

where c is the speed of sound ($= \sqrt{\kappa p/\rho}$ for an ideal gas).

The term dp/dt can be evaluated from equation (B.3) if we further assume that

$$\Delta p_p = p - p_a$$

where

p is the instantaneous pressure in the damping vessel;

p_a is a constant pressure.

If we take $n = 2$, then we obtain

$$\frac{dp}{dt} = \Delta p_{ss}\,[2\phi'(1 + \phi) + B\phi''] \qquad \ldots \text{(B.6)}$$

where

$$\phi'' = -\sum_{r=1}^{\infty} a_r r^2\omega^2 \sin(r\omega t + \theta_r) \qquad \ldots \text{(B.7)}$$

If we assume that $\phi \ll 1$ (a reasonable hypothesis since ϕ represents the fluctuating components in the damped pulsating flow), equation (B.4) can now be written as

$$\frac{V\,\Delta p_{ss}}{c^2\,\bar{q}_m}(2\phi' + B\phi'') + \phi = \sum_{r=1}^{\infty} b_r \sin(r\omega t + \theta_{0r})$$

Both the left and right-hand sides of this equation are Fourier series and thus the mean square values of the amplitudes must be egal. Hence it is possible to obtain

$$\frac{\sum_{r=1}^{\infty} a_r^2}{\sum_{r=1}^{\infty} b_r^2} = \frac{1}{\dfrac{\sum_{r=1}^{\infty} a_r^2\left(1 - \dfrac{r^2\omega^2}{\omega_0^2}\right)^2}{\sum_{r=1}^{\infty} a_r^2} + \left(\dfrac{2V}{c^2} \times \dfrac{\Delta p_{ss}\omega}{\bar{q}_m}\right)^2 H^2} \qquad \ldots \text{(B.8)}$$

where

$$\frac{1}{\omega_0^2} = \frac{BV}{c^2} \times \frac{\Delta p_{ss}}{\bar{q}_m} = \frac{V}{c^2} \times \frac{1}{l_e}$$

Equation (B.8) shows that the ratio of the r.m.s. amplitudes of the damped and undamped pulsations is a minimum when the denominator of the right-hand side is a minimum. This happens when the angular pulsation frequency, ω, reaches the resonance frequency, ω_0, of the damping chamber. At resonance, the waveform becomes predominantly sinusoidal and only the first harmonic ($r = 1$) is significant. Hence it can be seen that resonance in the damping chamber represents the worst case.

Hence

$$\frac{\sum_{r=1}^{\infty} a_r^2}{\sum_{r=1}^{\infty} b_r^2} \leqslant \frac{1}{\left(\frac{2V\,\Delta p_{ss}\omega}{c^2\,q_m}\right)^2} \qquad \text{... (B.9)}$$

or

$$\frac{2V\,\Delta p_{ss}\omega}{c^2\,\bar{q}_m} = \frac{4\pi}{\kappa}\,Ho$$

where

$$Ho = \frac{V}{q_V/f} \times \frac{\Delta p_{ss}}{\bar{p}}$$

and

$$\bar{q}_V = \bar{q}_m/\bar{\rho}$$

q_V is the volume flow rate at density $\bar{\rho}$ in the damping chamber.

According to equations (A.11) and (A.12) the error in the indicated flow rate due to pulsation, is given by

$$E_T = \left[1 + \left(\frac{U'_{rms}}{\bar{U}}\right)^2\right]^{1/2} - 1$$

$$= \frac{1}{2}\left(\frac{U'_{rms}}{\bar{U}}\right)^2 = \frac{1}{4}\sum_{r=1}^{\infty} a_r^2$$

Thus we have

$$E_T \leqslant \frac{\frac{1}{4}\sum_{r=1}^{\infty} b_r^2}{\left(\frac{4\pi}{\kappa}\,Ho\right)^2}$$

and since

$$\frac{1}{2}\sum_{r=1}^{\infty} b_r^2 = \left(\frac{q'_{m0,rms}}{\bar{q}_m}\right)^2$$

we have

$$\frac{Ho}{\kappa} \geqslant \frac{q'_{m0,rms}}{\bar{q}_m} \times \frac{1}{4\pi\sqrt{2}} \times \frac{1}{\sqrt{E_T}} \qquad \text{... (B.10)}$$

$$\frac{Ho}{\kappa} \geqslant \frac{q'_{m0,rms}}{\bar{q}_m} \times \frac{1}{4\pi\sqrt{2}} \times \frac{1}{\sqrt{\psi}} \qquad \text{... (B.11)}$$

$$\frac{Ho}{\kappa} \geqslant \frac{q'_{m0,rms}}{\bar{q}_m} \times \frac{0{,}056\,3}{\sqrt{\psi}} \qquad \text{... (B.12)}$$

where ψ is the maximum allowable relative error due to pulsation.

Under incompressible flow conditions (low Mach number, $M \ll 1$)

$$\frac{q'_{m0,rms}}{\bar{q}_m} = \frac{U'_{rms}}{\bar{U}}$$

Mottram and Mohammed[12] describe experimental work to verify the criteria for adequate damping in pulsating gas flows, i.e. that

$$E_T \leqslant \left(\frac{U'_{rms}}{\bar{U}} \times \frac{\kappa}{4\pi\sqrt{2}} \times \frac{1}{Ho}\right)^2$$

Figure B.2 shows some of their results for a particular value of the velocity pulsation amplitude. It can be seen that for a given Hodgson number, the experimentally measured errors are always less than the theoretical maximum allowable error represented by the curve. It should be remembered that the curve represents the worst possible condition, i.e. when resonance in the damping chamber is assumed and $H = 1$.

The same experimental work confirmed that the damping criterion [see inequalities (B.10) to (B.12)] is only valid when the dimensions of the receiver and the lengths of pipe including the throttling device are short compared with the pulsation wavelength (see 6.3.2).

B.3 Case of critical flow Venturi nozzles

Equations (A.1) and (A.2) in annex A are the one-dimensional momentum and continuity equations which can be integrated with respect to x between a section (indice 1) which contains the upstream pressure tapping and the throat (indice 2) of a nozzle or Venturi.

If the fluid is assumed to be a perfect gas and if the expansion is isentropic, we obtain the following equations:

$$\frac{\kappa}{\kappa-1} \times \frac{p_2}{\rho_2} - \frac{\kappa}{\kappa-1} \times \frac{p_1}{\rho_1} + \frac{U_2^2}{2} - \frac{U_1^2}{2} + \int_1^2 \frac{\partial U}{\partial t}\,dx = 0 \qquad \text{... (B.13)}$$

$$q_{m_2} - q_{m_1} = \int_1^2 A\,\frac{\partial \rho}{\partial t}\,dx \qquad \text{... (B.14)}$$

When the pulsating gas flow passes through a receiver to a critical flow Venturi nozzle located in a pipe terminating in a constant pressure reservoir, e.g. the atmosphere, equations (B.13) and (B.14) are applicable from a section 1 in the receiver to section 2 which is the throat of the Venturi nozzle.