TECHNICAL REPORT

ISO/TR 230-9

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Test code for machine tools —

Part 9: **Estimation of measurement uncertainty for machine tool tests according to series ISO 230, basic equations**

Code d'essai des machines-outils —

Partie 9: Estimation de l'incertitude de mesure pour les essais des machines-outils selon la série ISO 230, équations de base

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 230-9 was prepared by Technical Committee ISO/TC 39, *Machine tools*, Subcommittee SC 2, *Test conditions for metal cutting machine tools*.

ISO 230 consists of the following parts, under the general title *Test code for machine tools*:

— *Part 1: Geometric accuracy of machines operating under no-load or finishing conditions*

— *Part 2: Determination of accuracy and repeatability of positioning of numerically controlled axes*

— *Part 3: Determination of thermal effects*

— *Part 4: Circular tests for numerically controlled machine tools*

— *Part 5: Determination of the noise emissions*

— *Part 6: Determination of positioning accuracy on body and face diagonals (Diagonal displacement tests)*

— *Part 7: Geometric accuracy of axes of rotation*

— *Part 9: Estimation of measurement uncertainty for machine tool tests according to series ISO 230, basic equations* [Technical Report]

The following parts are under preparation:

— *Part 8: Determination of vibration levels* [Technical Report]

Introduction

In this part of ISO 230 equations for the estimation of the measurement uncertainty are presented.

Annex C is the special annex for the estimation of the measurement uncertainty for ISO 230-2.

Test code for machine tools —

Part 9: Estimation of measurement uncertainty for machine tool tests according to series ISO 230, basic equations

1 Scope

This part of ISO 230 provides information on a possible estimation of measurement uncertainties for measurements according to ISO 230.

The methods described here are aimed for practical use; therefore, standard uncertainties are mainly evaluated by type B evaluation (see Clause 4 and GUM).

Other methods complying with GUM may be used.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 230-2:—[1)], *Test code for machine tools — Part 2: Determination of accuracy and repeatability of positioning numerically controlled axes*

ISO/TR 16015:2003, *Geometrical product specifications (GPS) — Systematic errors and contributions to measurement uncertainty of length measurement due to thermal influences*

ISO/TS 14253-2, *Geometrical Product Specifications (GPS) — Inspection by measurement of workpieces and measuring equipment — Part 2: Guide to the estimation of uncertainty in GPS measurement, in calibration of measuring equipment and in product verification*

Guide to the expression of certainty in measurement, (GUM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1st edition, 1993, corrected and reprinted in 1995

3 Terms, definitions and symbols

For the purposes of this part of ISO 230, the terms, definitions and symbols given in ISO 230-2 and GUM apply.

1) To be published. (Revision of ISO 230-2:1997)

4 Estimation of measurement uncertainty U

The estimation of the measurement uncertainty, U, follows GUM, ISO/TS 14253-2 and ISO/TR 16015.

The individual contributors to the measurement uncertainty have to be identified (for examples, see Annex C) and expressed as standard uncertainties, u_i.

The combined standard uncertainty, u_c, is calculated according to Equation (1):

$$u_c = \sqrt{u_r^2 + \sum u_i^2} \quad (1)$$

where

u_c is the combined standard uncertainty, in micrometres (µm);

u_r is the sum of strongly positive correlated contributors, see Equation (2), in micrometres (µm);

u_i is the standard uncertainty of uncorrelated contributor, i, in micrometres (µm);

$$u_r = \sum u_j \quad (2)$$

where u_j is the standard uncertainty of strongly positive correlated contributor, j, in micrometres (µm).

The measurement uncertainty U is calculated according to Equation (3), where the coverage factor k is set to 2.

$$U = k \cdot u_c \quad (3)$$

where

U is the measurement uncertainty, in micrometres (µm);

k is the coverage factor,

$k = 2$

u_c is the combined standard uncertainty, in micrometres (µm);

A standard uncertainty u_i is obtained by statistical analysis of experimental data (type A evaluation) or by other means, such as knowledge, experience and scientific guess (type B evaluation).

If an estimation gives a possible range of $\pm a$ or $(a^+ - a^-)$ of a contributor, then the standard uncertainty u_i is given according to Equation (4), assuming a rectangular distribution.

$$u_i = \frac{a^+ - a^-}{2\sqrt{3}} \quad (4)$$

where

u_i is the standard uncertainty;

a^+ is the upper limit of rectangular distribution;

a^- is the lower limit of rectangular distribution.

5 Estimation of the uncertainty of parameters, basic equations

In Clause 4, the black box method of the uncertainty estimation is used. For the parameters that are calculated from individual measurement runs, from mean values, from multiples of the standard deviation, and/or sums of those, the uncertainty estimates are obtained using transparent box method. Positioning accuracy, repeatability and reversal value are such parameters. This can be written generally as

$$Y = f(X_i) \tag{5}$$

where

Y is the parameter (e.g. repeatability, reversal value, positioning accuracy);

X_i is the measured value i.

The combined standard uncertainty u_c is then calculated according Equation (6):

$$u_\mathrm{c} = \sqrt{u_\mathrm{r}^2 + \sum \left(\frac{\delta Y}{\delta X_i} \cdot u_{Xi} \right)^2} \tag{6}$$

where

u_c is the combined standard uncertainty;

u_r is the sum of strongly positive correlated components, see Equation (7);

u_{Xi} is the standard uncertainty of uncorrelated component i.

$$u_\mathrm{r} = \sum \frac{\delta Y}{\delta X_j} \cdot u_{Xj} \tag{7}$$

where u_{Xj} is the standard uncertainty of strongly positive correlated component j.

Annex A
(informative)

Measurement uncertainty of mean value

A.1 General

The mean value is defined by

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{A.1}$$

where

$\bar{x}$ is the mean value;

x_i is the measured value i;

n is the number of measurements.

If the mean value is calculated from measurements x_i, having a measurement uncertainty u_{xi}, then the mean value has also an uncertainty.

A.2 Calculation of the measurement uncertainty of the mean value, $u(\bar{x})$

A.2.1 General

The measurement uncertainty of the mean value $u(\bar{x})$ depends on the correlation between the uncertainties of the single measurements u_{xi}.

A.2.2 Uncertainty of the mean value $u(\bar{x})$ for strongly positive correlated uncertainties u_{xj}

If the uncertainties of the single measurements u_{xj} are strongly positive correlated, their influences on the uncertainty of the mean value $u(\bar{x})$ are simple summed, according to Equation (7).

NOTE A possible misalignment of a measuring instrument does not change in a series of measurements. Then this uncertainty contributor does not change between repeated measurements, and is regarded as strongly positive correlated.

If Equations (6) and (7) are applied to Equation (A.1) for strongly positive correlated contributors, the result is

$$u(\bar{x}) = \sum \frac{\delta \bar{x}}{\delta x_j} \cdot u_{xj} \tag{A.2}$$

where

$u(\bar{x})$ is uncertainty of the mean value for strongly positive correlated contributors;

$\bar{x}$ is the mean value;

x_j is the single measurement value;

u_{xj} is the strongly positive correlated measurement uncertainty contributor for measured value j.

The partial derivation of the mean value $\overline{x}$ to the single measurement value x_j is the following:

$$\frac{\delta \overline{x}}{\delta x_j} = \frac{1}{n} \tag{A.3}$$

It is assumed that the measurement uncertainty for the single measurement does not change, i.e.

$$u_{x1} = u_{x2} = \ldots = u_{xn} = u_x \tag{A.4}$$

Equations (A.3) and (A.4) are set into Equation (A.2), resulting in

$$\begin{aligned} u(\overline{x}) &= \frac{1}{n} \cdot u_{x1} + \frac{1}{n} \cdot u_{x2} + \ldots + \frac{1}{n} \cdot u_{xn} \\ u(\overline{x}) &= \frac{1}{n} \cdot u_x \cdot n \\ u(\overline{x}) &= u_x \end{aligned} \tag{A.5}$$

where

$u(\overline{x})$ is the uncertainty of the mean value for strongly positive correlated contributors;

u_x is the strongly positive correlated measurement uncertainty contributor for measured values.

Equation (A.5) tells us that the uncertainty of the mean value $u(\overline{x})$ is the uncertainty of the measured value u_x, if the uncertainty contributors are strongly positive correlated.

A.2.3 Uncertainty of mean value $u(\overline{x})$ for uncorrelated uncertainties u_{xi}

If the uncertainties of the individual measurements u_{xi} are not correlated, the square root of the squared sum is applied according to Equation (6), with $u_r = 0$.

NOTE The influence of an environmental thermal variation error, ETVE, in general will change from measurement value to measurement value. Therefore, this influence is regarded as uncorrelated.

If Equation (6) is applied to Equation (A.1) for uncorrelated contributors, the results is

$$u(\overline{x}) = \sqrt{\sum \left(\frac{\delta \overline{x}}{\delta x_i} \right)^2 \cdot u_{xi}^2} \tag{A.6}$$

où

$u(\overline{x})$ is the uncertainty of the mean value for non-correlated contributors;

$\overline{x}$ is the mean value;

x_i is the single measurement value;

u_{xi} is the non-correlated measurement uncertainty contributor for measured value i.

Equations (A.3) and (A.4) are set into Equation (A.6), resulting in

$$u(\bar{x}) = \sqrt{\left(\frac{u_{x1}}{n}\right)^2 + \left(\frac{u_{x2}}{n}\right)^2 + \ldots + \left(\frac{u_{xn}}{n}\right)^2}$$

$$u(\bar{x}) = u_x \cdot \sqrt{\left(\frac{1}{n^2}\right) \cdot n} \qquad \text{(A.7)}$$

$$u(\bar{x}) = \frac{1}{\sqrt{n}} \cdot u_x$$

The measurement uncertainty of the mean value is reduced by $\frac{1}{\sqrt{n}}$, if n is the number of repeated measurements, and if the uncertainties of the repeated measurements are uncorrelated.

Annex B
(informative)

Measurement uncertainty of estimator of standard deviation s

B.1 General

The estimator of the standard deviation is defined by

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \qquad \text{(B.1)}$$

where

s is the estimator of the standard deviation;

$\bar{x}$ is the mean value as defined in Equation (A.1);

x_i is the measured value i;

n is the number of measurements.

If the estimator of the standard deviation is calculated from measurements x_i having a measurement uncertainty u_{xi}, then the estimator also has an uncertainty. Therefore, any parameter defined as a function of s shows a measurement uncertainty.

B.1.1 Calculation of measurement uncertainty of estimator of standard deviation $u(s)$

The contributors to the measurement uncertainty are uncorrelated, otherwise there will be no standard deviation in repeated measurements.

It is assumed that the measurement uncertainty for the individual measurement does not change, i.e.

$$u_{x1} = u_{x2} = \ldots = u_{xn} = u_x \qquad \text{(B.2)}$$

Applying Equation (6) with u_r = 0 and Equation (B.2) to Equation (B.1) results in the following:

$$u(s) = \sqrt{\sum\left(\frac{\delta s}{\delta x_i}\right)^2 \cdot u_{xi}^2}$$
$$u(s) = u_x \cdot \sqrt{\left(\frac{\delta s}{\delta x_1}\right)^2 + \left(\frac{\delta s}{\delta x_2}\right)^2 + \ldots + \left(\frac{\delta s}{\delta x_n}\right)^2} \qquad \text{(B.3)}$$

The partial derivations of s are calculated as, assuming $s \neq 0$,

$$\frac{\delta s}{\delta x_1} = \frac{1}{2} \cdot \frac{1}{s} \cdot \frac{1}{n-1} \cdot \left[2 \cdot (x_1 - \bar{x}) \cdot \left(1 - \frac{1}{n}\right) + 2 \cdot (x_2 - \bar{x}) \cdot \left(-\frac{1}{n}\right) + \ldots + 2 \cdot (x_n - \bar{x}) \cdot \left(-\frac{1}{n}\right) \right]$$

$$\frac{\delta s}{\delta x_1} = \frac{1}{s} \cdot \frac{1}{n-1} \cdot \left[x_1 - \bar{x} - \frac{x_1}{n} + \frac{\bar{x}}{n} - \frac{x_2}{n} + \frac{\bar{x}}{n} - \ldots - \frac{x_n}{n} + \frac{\bar{x}}{n} \right]$$

$$\frac{\delta s}{\delta x_1} = \frac{1}{s} \cdot \frac{1}{n-1} \cdot \left[x_1 - \bar{x} + n \cdot \frac{\bar{x}}{n} - \frac{1}{n} \cdot (x_1 + x_2 + \ldots + x_n) \right]$$

$$\frac{\delta s}{\delta x_1} = \frac{1}{s} \cdot \frac{1}{n-1} \cdot \left[x_1 - \bar{x} + \bar{x} - \frac{1}{n} \sum x_i \right]$$

$$\frac{\delta s}{\delta x_1} = \frac{1}{s} \cdot \frac{1}{n-1} \cdot (x_1 - \bar{x})$$

$$\frac{\delta s}{\delta x_2} = \frac{1}{s} \cdot \frac{1}{n-1} \cdot (x_2 - \bar{x})$$

...

$$\frac{\delta s}{\delta x_n} = \frac{1}{s} \cdot \frac{1}{n-1} \cdot (x_n - \bar{x})$$

$$\left(\frac{\delta s}{\delta x_1}\right)^2 = \frac{1}{s^2} \cdot \frac{1}{(n-1)^2} \cdot (x_1 - \bar{x})^2$$

$$\left(\frac{\delta s}{\delta x_2}\right)^2 = \frac{1}{s^2} \cdot \frac{1}{(n-1)^2} \cdot (x_2 - \bar{x})^2 \qquad \text{(B.4)}$$

...

$$\left(\frac{\delta s}{\delta x_n}\right)^2 = \frac{1}{s^2} \cdot \frac{1}{(n-1)^2} \cdot (x_n - \bar{x})^2$$

Equations (B.4) and (B.1) are set into equation (B.2):

$$u(s) = u_x \cdot \sqrt{\frac{1}{s^2} \cdot \frac{1}{(n-1)^2} \cdot \left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2 \right]}$$

$$u(s) = u_x \cdot \sqrt{\frac{1}{s^2} \cdot \frac{1}{n-1} \cdot \frac{1}{n-1} \cdot \sum (x_i - \bar{x})^2} \qquad \text{(B.5)}$$

$$u(s) = u_x \cdot \sqrt{\frac{1}{s^2} \cdot \frac{1}{n-1} \cdot s^2}$$

$$u(s) = u_x \cdot \sqrt{\frac{1}{n-1}}$$

assuming $s \neq 0$.

The measurement uncertainty of the estimator of the standard deviation $u(s)$ is the uncertainty of the single measurement reduced by $\sqrt{\frac{1}{n-1}}$, if n is the number of repeated measurements and $s \neq 0$.

Annex C
(informative)

Measurement uncertainty estimation for linear positioning measurement according to ISO 230-2

C.1 Introduction

In this annex a possible estimation of the measurement uncertainty of the parameters evaluated according to ISO 230-2 is presented.

The equations of this annex are used for Annex A of ISO 230-2:—, which includes simplified equations and easy-to-read-and-use tables for the estimation of the measurement uncertainty for industrial applications and conditions.

C.2 Contributors to the measurement uncertainty

C.2.1 Overview

The main contributors to the measurement uncertainty for linear positioning measurements are

— the uncertainty of the calibration of the measurement device, i.e. the laser interferometer or the linear scale,

— the alignment of the measurement device to the machine axis under test,

— the compensation of the machine tool temperature when measuring at temperatures other than 20 °C,

— the environmental variation error (EVE or drift) during the time of measurement, e.g. the influence of temperature variation and air density variation to the measurement device and/or the machine tool under test, and

— the repeatability of the set-up of the measurement device.

The following assumptions are made:

— the measurement device is used correctly according to the guidelines of the equipment manufacturer/supplier,

— all necessary compensations (e.g. calibration values, compensation for temperature influences) for the measurement equipment and the machine tool are carried out,

— all additional sensors (e.g. for machine temperature) are mounted correctly,

— the measurement equipment is mounted statically and dynamically stiff and without any backlash, and

— the machine components holding the equipment behave as rigid bodies.

If these assumptions are not fulfilled, additional contributors to the measurement uncertainty have to be taken into account.

C.2.2 Uncertainty due to the measurement device, u_{DEVICE}

The measurement device should be calibrated. The uncertainty of the calibration $u_{\text{CALIBRATION}}$ should be given in the calibration certificate and is used to calculate u_{DEVICE} according to equation (C.1). For a laser-interferometer, the dead-path error is assumed to be zero.

$$u_{\text{DEVICE}} = \frac{U_{\text{CALIBRATION}}}{k} \tag{C.1}$$

where

u_{DEVICE} is the standard uncertainty due to the measurement device in micrometres (µm);

$U_{\text{CALIBRATION}}$ is the uncertainty of the calibration according to the calibration certificate in micrometres (µm);

k is the coverage factor for $U_{\text{CALIBRATION}}$ according to the calibration certificate.

Often, the uncertainty of the calibration is given in micrometres per metre (µm/m) or in parts per million (ppm). In these cases the uncertainty of the device is calculated according to Equation (C.2):

$$u_{\text{DEVICE}} = \frac{U_{\text{CALIBRATION}} \cdot L}{k} \tag{C.2}$$

where

u_{DEVICE} is the standard uncertainty due to the measurement device in micrometres (µm);

$U_{\text{CALIBRATION}}$ is the uncertainty of the calibration according to the calibration certificate in micrometres per metre (µm/m) or in parts per million (ppm);

L is the measuring length in metres;

k is the coverage factor for $U_{\text{CALIBRATION}}$ according to the calibration certificate.

If no calibration is available, one has to rely on the data given by the equipment manufacturer.

For a laser interferometer, the accuracy is given by the manufacturer, e.g. by an uncertainty value or by a ppm value, depending on the type of compensation of the air parameters and the temperature range of the environment, in which the instrument is used. Another contributor is the wavelength stability, e.g. given by another uncertainty value or ppm value. These ppm values have to be multiplied by the length measured to obtain the range of possible deviation. This range can be used to calculate the standard uncertainty $u_{\text{DEVICE,ESTIMATE}}$ according to Equations (1) and (4). The dead-path error is assumed to be zero.

The accuracy statement of the equipment manufacturer is based on assumptions related to the environment. For laser interferometers, fast changes of the air temperature, e.g. caused by air conditioning, is problematic, because the temperature sensors often do not follow fast changes, whereas the influence on the wavelength is without any delay. On the other hand, a temperature sensor may respond to changes while the laser beam may not, especially if the temperature sensor is far from the laser beam. If such conditions are suspected, an additional drift check (see C.2.5) for the equipment is needed to estimate that additional influence on the measurement uncertainty, which might reach a range of 2 µm on a length of 1 000 mm.

For a linear scale, the accuracy is given, e.g., by an uncertainty value or by a maximum deviation for the length of the scale. The maximum deviation is taken as a range and transferred to the standard uncertainty $u_{\text{DEVICE,ESTIMATE}}$ according to Equation (4). It is assumed that the output of the linear scale is compensated to measurement values at 20 °C. If this compensation is not part of the uncertainty statement, additional contributors have to be calculated for the temperature measurement and the expansion coefficient of the scale, as described in C.2.4 for the compensation of the machine temperature.

If no calibration is available, the uncertainty due to the resolution $u_{\text{DEVICE,RESOLUTION}}$ of the measurement device has to be estimated according to Equations (4) and (C.3). This uncertainty due to resolution has to be added to the uncertainty statement of the device manufacturer according to Equations (1) and (C.4).

$$u_{\text{DEVICE,RESOLUTION}} = \frac{r}{2\sqrt{3}} \tag{C.3}$$

where

$u_{\text{DEVICE,RESOLUTION}}$ is the standard uncertainty due to the resolution of the measurement device in micrometres (µm);

r is the resolution of the measurement device in micrometres (µm).

$$u_{\text{DEVICE}} = \sqrt{u^2_{\text{DEVICE,ESTIMATE}} + u^2_{\text{DEVICE,RESOLUTION}}} \tag{C.4}$$

where

u_{DEVICE} is the standard uncertainty due to the measurement device in micrometres (µm);

$u_{\text{DEVICE,ESTIMATE}}$ standard uncertainty due to the measurement device according to statement of device manufacturer, in micrometres (µm);

$u_{\text{DEVICE,RESOLUTION}}$ standard uncertainty due to the resolution of the measurement device in micrometres (µm);

C.2.3 Uncertainty due to misalignment of measurement device to machine axis under test, $u_{\text{MISALIGNMENT}}$

With a laser interferometer, the laser beam should be parallel to the machine axis under test. A misalignment is generally observed only in the change of the intensity of the return beam. Many systems allow a lateral change of up to ± 4 mm, which would cause a range of maximum misalignment of 4 mm. The misalignment of the beam can also be seen from the stability of the position of the reflected beam. By manual adjustment, this can be brought down to ± 1 mm, which means that the parallelism is within 1 mm.

Linear scales are often equipped with reference surfaces to align the scale. The accuracy of the reference surfaces and the accuracy of the alignment determine the maximum misalignment.

The misalignment is an influence of second order, according to Equation (C.5):

$$\Delta L_{\text{MISALIGNMENT}} = L \cdot (1 - \cos\gamma) \cdot 1\,000 \tag{C.5}$$

where

$\Delta L_{\text{MISALIGNMENT}}$ is the difference between measured and actual length due to misalignment, in micrometres (µm);

L measurement length in millimetres (mm);

γ angle of misalignment, $\sin\gamma = \text{misalignment (mm)}/L$

The misalignment can be on the order of millimetres, therefore this contribution might be significant on short distances. The difference between measured and actual length $\Delta L_{\text{ALIGNMENT}}$ is used to calculate the standard uncertainty according to Equation (4), $u_{\text{MISALIGNMENT}} = \dfrac{\Delta L_{\text{MISALIGNMENT}}}{2\sqrt{3}}$.

C.2.4 Uncertainty due to compensation of machine tool temperature, $u_{\text{TEMPERATURE}}$

If the measurements are taken at temperatures other than 20 °C, the temperature of the machine tool (or workpiece) has to be compensated (see 3.1 of ISO 230-2:—). With this compensation, uncertainties are introduced due to the uncertainty of the temperature measurement and due to the uncertainty of the expansion coefficient of the machine tool or the workpiece.

The most important influence on the uncertainty of the temperature measurement is the point where the temperature measurements are taken, i.e. the question of whether or not the measured temperatures are representative for the machine tool (or workpiece). The positions of the temperature sensors need some attention and shall be stated in the test report.

The temperature sensors should be calibrated. The calibration certificate should state the uncertainty of the calibration and the coverage factor.

If uncalibrated sensors are used, one has to rely on the statement of the equipment manufacturer. The uncertainty of the temperature measurement $u(\Theta)$ is given by the manufacturer of the equipment, e.g. by a standard uncertainty of the device, which is the preferred method, or as a maximum deviation. A maximum deviation is taken as a range and transferred to a standard uncertainty according to Equation (4). The uncertainty due to the temperature measurement u_{M} is calculated according to Equation (C.6).

If a mechanical device, e.g. a linear scale, is used for the length measurement and is set on the machine table, the measurement device adopts the temperature of the machine table. In this case, just the temperature difference between the measurement device and the workpiece holding part of the machine tool is relevant for the uncertainty due to the temperature measurement u_{M}.

$$u_{\text{M,MACHINE\_TOOL}} = \alpha \cdot L \cdot u(\Theta) \tag{C.6}$$

where

$u_{\text{M,MACHINE\_TOOL}}$ is the uncertainty due to temperature measurement of machine tool, in micrometres (µm);

α is the expansion coefficient of machine tool, or of axis under test, in micrometres per millimetre degrees Celsius (µm/mm °C);

L is the measuring length in millimetres (mm);

$u(\Theta)$ is the uncertainty of the temperature measurement device (standard uncertainty) and uncertainty due to the point of measurement, or uncertainty due to the temperature difference between the (mechanical) measurement device and the workpiece holding part of the machine tool, in degrees Celsius (°C).

If the uncertainty statement for the measurement device does not include the uncertainty of the temperature measurement of the device, or if the measurement device does not adopt the temperature of the workholding part of the machine tool, the uncertainty due to the temperature measurement, $u_{\text{M,DEVICE}}$, has to be calculated for the measurement device as well. $u_{\text{M, DEVICE}}$ is calculated using Equation (C.6), replacing the uncertainty of the temperature measurement and the expansion coefficient of the machine tool (or workpiece) by that of the measurement device instead. If the uncertainty statement for the measurement device includes the uncertainty of the temperature measurement of the device, or if the measurement device adopts the temperature of the workholding part of the machine tool, $u_{\text{M,DEVICE}}$ can be set to zero.

The uncertainty of the expansion coefficient $u(\alpha)$ of the machine tool or the workpiece can be estimated in most cases. A minimum range of 10 % of the nominal value of α, but not smaller than 0,002 µm/mm °C (= 2 µm/m °C) is suggested. This range is transferred according to Equation (4) to a standard deviation. With this assumption, $u(\alpha) = \frac{0{,}002}{2\sqrt{3}} = 0{,}000\,6$ µm/mm°C. The uncertainty due to the expansion coefficient u_E is calculated according to Equation (C.7).

$$u_{\text{E, MACHINE TOOL}} = \Delta T \cdot L \cdot u(\alpha) \quad \text{(C.7)}$$

where

$u_{\text{E,MACHINE TOOL}}$ is the uncertainty due to the thermal expansion coefficient of machine tool, in micrometres (µm)

ΔT is the difference to 20 °C in degrees Celsius (°C), $\Delta T = T - 20$ °C;

T is the temperature of the machine tool or workpiece in degrees Celsius (°C);

L is the measuring length in millimetres (mm);

$u(\alpha)$ is the uncertainty of expansion coefficient of machine tool or workpiece (standard uncertainty) in micrometres per millimetre degrees Celsius (µm/mm °C)

If the statement of the uncertainty of the measurement device does not include the uncertainty of the thermal expansion coefficient of the device, the uncertainty due to the expansion coefficient $u_{\text{E,DEVICE}}$ has to be calculated for the measurement device as well. $u_{\text{E,DEVICE}}$ is calculated using equation (C.7) replacing the temperature and the uncertainty of the expansion coefficient of the machine tool (or workpiece) by that of the measurement device instead. If the uncertainty statement for the device does include this uncertainty, $u_{\text{E,DEVICE}}$ can be set to zero.

The uncertainties of the temperature measurement and the expansion coefficient are assumed to be uncorrelated. According to Equation (1) this results in Equation (C.8):

$$u_{\text{TEMPERATURE}} = \sqrt{u^2_{\text{M,MACHINE\_TOOL}} + u^2_{\text{M,DEVICE}} + u^2_{\text{E,MACHINE\_TOOL}} + u^2_{\text{E,DEVICE}}} \quad \text{(C.8)}$$

where

$u_{\text{M,MACHINE\_TOOL}}$ is the uncertainty due to temperature measurement of machine tool or workpiece, or uncertainty due to temperature difference between measurement device and workholding part of machine tool, in micrometres (µm);

$u_{\text{M,DEVICE}}$ is the uncertainty due to temperature measurement of the device in micrometres (µm); can be set to zero, if the uncertainty statement for the device includes the uncertainty of the temperature measurement of the device (or if the uncertainty of the compensation of measurements at temperatures other than 20 °C is included); can be set to zero, if the (mechanical) device adopts the temperature of the workholding part of the machine tool;

$u_{\text{E,MACHINE TOOL}}$ is the uncertainty due to the thermal expansion coefficient of the machine tool or workpiece, in micrometres (µm);

$u_{\text{E,DEVICE}}$ is the uncertainty due to the thermal expansion coefficient of the measurement device, in micrometres (µm); can be set to zero, if the uncertainty statement for the device includes the uncertainty of the expansion coefficient of the device (or if the uncertainty of the compensation of measurements at temperatures other than 20 °C is included).

C.2.5 Uncertainty due to environmental variation error (E_{VE}, or drift), u_{EVE}

During the time of measurement, the environment, the instrument and/or the machine tool might drift or change, influencing the readout of the measuring system. This environmental variation error can be checked by a drift test: By setting up the measurement equipment on the machine tool under test and looking at the change of readout in the extreme position during the time necessary to do the positioning test, the magnitude of this influence, E_{VE}, can be obtained.

This drift value E_{VE} is preferably evaluated by calculating the standard deviation, u_{EVE}, from the data taken or is taken as a range and transferred to a standard uncertainty according to Equation (C.9) — in correspondence with Equation (4).

This drift value does not include the repeatability of the machine axis, as the axis should not be moved during the drift test.

$$u_{EVE} = \frac{E_{VE}}{2\sqrt{3}} \quad \text{(C.9)}$$

where

u_{EVE} is the uncertainty due to the environmental variation, not influenced by the repeatability of the moved axis;

E_{VE} is the drift value.

The drift value E_{TVE} does not contain pure drift, as drift shall be minimized for the measurements according to 3.3 of ISO 230-2:—, i.e. an ordered progression of deviations between successive approaches to any particular target position shall not be obtained. Therefore, this contributor is regarded as uncorrelated between the five consecutive measurement runs.

The environmental variation error E_{VE} always increases the standard deviation calculated for the unidirectional repeatability $R\uparrow$, $R\downarrow$ and for the bi-directional repeatability R. Therefore the repeatability values can be corrected for the environmental influences according to Equation (C.10), if the largest standard deviations from the repeated positioning measurements appear at longer measuring lengths. If the largest standard deviations appear at shorter measuring lengths, additional drift tests should be carried out at the relevant measuring lengths.

$$\begin{aligned}
s_{i,\text{corrected}}\uparrow &= \sqrt{s_i\uparrow^2 - u_{EVE}^2} \\
s_{i,\text{corrected}}\downarrow &= \sqrt{s_i\downarrow^2 - u_{EVE}^2} \\
R_{i,\text{corrected}}\uparrow &= 4 \cdot s_{i,\text{corrected}}\uparrow \\
R_{i,\text{corrected}}\downarrow &= 4 \cdot s_{i,\text{corrected}}\downarrow \\
R_{i,\text{corrected}} &= \max.\left[2 \cdot s_{i,\text{corrected}}\uparrow + 2 \cdot s_{i,\text{corrected}}\downarrow + \left|B_i\right|; R_{i,\text{corrected}}\uparrow; R_{i,\text{corrected}}\downarrow\right] \\
R_{\text{corrected}}\uparrow &= \max.\left[R_{i,\text{corrected}}\uparrow\right] \\
R_{\text{corrected}}\downarrow &= \max.\left[R_{i,\text{corrected}}\downarrow\right] \\
R_{\text{corrected}} &= \max.\left[R_{i,\text{corrected}}\right]
\end{aligned} \quad \text{(C.10)}$$

where

$s_{i,\text{corrected}}\uparrow,\downarrow$ is the corrected unidirectional standard uncertainty s_i, correction due to environmental influences;

s_i is the estimator of the unidirectional standard uncertainty of positioning, see 2.15 of ISO 230-2:—;

u_{EVE} is the uncertainty due to environmental variations;

$R_{i,corrected}\uparrow,\downarrow$ is the corrected unidirectional repeatability of positioning at a position i, correction due to environmental influences;

$R_{i,corrected}$ is the corrected bi-directional repeatability of positioning at a position i, correction due to environmental influences;

$R_{corrected}\uparrow,\downarrow$ is the corrected unidirectional repeatability of positioning, correction due to environmental influences;

$R_{corrected}$ is the corrected bi-directional repeatability of positioning, correction due to environmental influences.

C.2.6 Uncertainty due to the repeatability of the measurement set-up, u_{SETUP}

The linear positioning accuracy and repeatability varies between different lines of measurement, if the moving axis has pitch and/or yaw movements. Therefore, the line of measurement has to be stated exactly, as demanded in Clause 6 of ISO 230-2:—. If the data of accuracy of positioning according to this test is used for compensation purposes or for calculating a volumetric accuracy, the exact position of the line of measurement is essential. In practice it is not possible to state the position of the line of measurement exactly; therefore, this contributor to the measurement uncertainty has to be included.

This influence depends on the size of the angular deviations pitch and yaw of the axis under test and on the Abbe-offset between the measurement lines of two set-ups. The offset is measured square to the line of measurement. The resulting influence to the length measured is calculated according to Equation (C.11), assuming in the first estimate, that pitch and yaw have the same contribution to the change in length.

$$\Delta L_{SETUP} = \frac{\sqrt{2} \cdot O_{ABBE} \cdot D_{ANGLE}}{1000} \tag{C.11}$$

where

ΔL_{SETUP} is the change in measured length due to repeatability of measurement set-up in micrometres (µm);

O_{ABBE} is the Abbe offset between two possible lines of measurement in millimetres (mm);

D_{ANGLE} is the angular deviation (pitch/yaw) of axis under test in micrometres per metre (µm/m).

In general cases, the Abbe offset between two measurement set-ups will be about 50 mm; when customized fixtures are used, this can be reduced to 1 mm or less. The angular deviation of the axis under test can be either measured, or the ISO tolerances for pitch and yaw taken as a first estimate. The ISO tolerances for pitch and yaw for machine tools are of the order of 50 µm/m.

The influence due to the repeatability of the set-up on the length measured ΔL_{SETUP} is used to calculate the standard uncertainty according to Equation (4), $u_{SETUP} = \frac{\Delta L_{SETUP}}{2\sqrt{3}}$.

C.3 Uncertainty of measured point, u_{POINT}

The uncertainty of a single measured point is influenced by the device, the alignment, the temperature compensation, the environmental variation error, and by the repeatability of the measurement set-up. These contributors are uncorrelated; therefore u_r of Equation (1) can be set to zero, and u_{POINT} calculated — using Equation (1) — according to Equation (C.12).

$$u_{\text{POINT}} = \sqrt{u_{\text{DEVICE}}^2 + u_{\text{ALIGNMENT}}^2 + u_{\text{TEMPERATURE}}^2 + u_{E\text{VE}}^2 + u_{\text{SETUP}}^2} \quad \text{(C.12)}$$

where u_{POINT} is the measuring point uncertainty.

Most contributors depend on the length measured, so u_{POINT} is different at each measuring point along the machine axis. For a first uncertainty estimate, the largest length should be taken for all contributors.

C.4 Estimation of the uncertainty of parameters

C.4.1 General

In ISO 230-2, the parameters repeatability, reversal value, systematic deviation and accuracy of positioning are defined and shall be presented as results of the measurements. The uncertainties of these parameters, based on the measurement uncertainty, are estimated in C.4.2 to C.4.5.

C.4.2 Uncertainty estimation for unidirectional repeatability $U(R\uparrow, R\downarrow)$

This subclause is applicable for axes up to 2 000 mm only.

The unidirectional repeatability is the maximum of four times the standard deviation of the five measurement runs at each measuring point (see 2.17 of ISO 230-2:—). Only the differences at the same positions are contributing factors for this parameter. Thus, the uncertainty of the unidirectional repeatability $R\uparrow$, $R\downarrow$ does not depend on the calibration of the measurement device, alignment of the measurement device or compensation of the machine temperature. For the unidirectional repeatability, small changes in the set-up location are regarded as non-critical.

Therefore, the only remaining contributor is the environmental variation error $u_{E\text{VE}}$. The five measurement runs can be regarded as uncorrelated in respect to these contributors; therefore u_r of Equation (6) is zero. Using 2.10, 2.15, 2.16 of ISO 230-2:— and Equations (3), (6) and (B.5) of this part of ISO 230, the following result is obtained:

$$u(R\uparrow, R\downarrow) = 4 \cdot \sqrt{\frac{1}{n-1}} \cdot u_{E\text{VE}} \qquad U(R\uparrow, R\downarrow) = 2 \cdot u(R\uparrow, R\downarrow) \quad \text{(C.13)}$$

where

$U(R\uparrow,R\downarrow)$ is the uncertainty of unidirectional repeatability, $k = 2$;

n is the number of measurement runs, $n = 5$.

C.4.3 Uncertainty estimation for reversal value, $U(B)$

The reversal value is the maximum difference between two mean values (see 2.13 of ISO 230-2:—). Only differences between moving upwards and downwards at the same positions are contributing factors for this parameter. Thus, as for the unidirectional repeatability, the uncertainty for reversal value does not depend on the calibration of measurement device, the alignment of the measurement device, and the compensation of the machine tool temperature.

Therefore, the only remaining contributors are $u_{E\text{VE}}$ and u_{SETUP}. The contributor $u_{E\text{VE}}$ can be regarded as uncorrelated between the five measurement runs. The contributor u_{SETUP} does not change between the five measurements, therefore it is regarded as correlated. In a first estimate, the mean values from moving upwards and downwards are regarded as correlated, because values at the same measuring point are looked at. Using 2.10, 2.12 and 2.13 of ISO 230-2:—, and Equations (3), (6), (7), (A.5) and (A.7) of this part of ISO 230, the following result is obtained:

$$u(B) = 2 \cdot \sqrt{\frac{u_{\text{EVE}}^2}{n} + u_{\text{SETUP}}^2} \quad U(B) = 2 \cdot u(B) \tag{C.14}$$

where

$U(B)$ is the uncertainty of reversal value, $k = 2$;

n is the number of measurement runs,

$n = 5$ for axes up to 2 000 mm,

$n = 1$ for axes exceeding 2 000 mm.

C.4.4 Uncertainty of bi-directional repeatability, $U(R)$

This subclause is applicable for axes up to 2 000 mm only.

The bi-directional repeatability (see 2.17 of ISO 230-2:—) is in principle the sum of the unidirectional repeatability and the reversal value. As the repeatability is evaluated from the standard deviation of the five runs and as the reversal value is evaluated from the mean value, the two components are regarded as uncorrelated. Using Equations (6), (C.13) and (C.14), the following result is obtained:

$$u(R) = \sqrt{u(B)^2 + u(R\uparrow, R\downarrow)^2} \quad U(R) = 2 \cdot u(R) \tag{C.15}$$

where $U(R)$ is the uncertainty of bi-directional repeatability, $k = 2$.

C.4.5 Uncertainty of systematic deviations, $U(M, E, E\uparrow, E\downarrow)$

All systematic deviations of ISO 230-2 rely on mean values calculated from the five measurement runs (see 2.19 to 2.21 of ISO 230-2:—). For these values, the absolute accuracy of the device, the alignment, the compensation of the machine tool temperature and the repeatability of the set-up are of main importance. These four contributors remain the same for the five measurement runs; therefore, they have to be treated as correlated between the five runs. Only the environmental variation error can be regarded as uncorrelated between the five runs.

The systematic deviations are the difference between a maximum and a minimum value along the travel of the machine axis. As the single contributors are estimated for the total axis travel and minimum and maximum are in general located near the ends of the travel, the estimate with just the end point is in most cases sufficient. Using Equations (3), (6), (A.5) and (A.7) of this part of ISO 230 the result is

$$u(M, E, E\uparrow, E\downarrow) = \sqrt{u_{\text{DEVICE}}^2 + u_{\text{MISALIGNMENT}}^2 + u_{\text{TEMPERATURE}}^2 + u_{\text{SETUP}}^2 + \frac{u_{\text{EVE}}^2}{n}}$$

$$U(M, E, E\uparrow, E\downarrow) = 2 \cdot u\,(M, E, E\uparrow, E\downarrow) \tag{C.16}$$

where

$U(M, E, E\uparrow, E\downarrow)$ is the uncertainty of systematic deviations, $k = 2$;

n is the number of measurement runs,

for axes up to 2 000 mm:

$n = 5$ for E, $E\uparrow$, $E\downarrow$,

$n = 10$ for M,

for axes exceeding 2 000 mm:

n = 1 for E, $E\uparrow$, $E\downarrow$,

n = 2 for M.

C.4.6 Uncertainty of accuracy of positioning, $U(A, A\uparrow, A\downarrow)$

This subclause is applicable for axes up to 2 000 mm only.

The accuracy values are in principle a sum of systematic deviations and unidirectional repeatability values (see 2.22 and 2.23 of ISO 230-2:—), i.e. the sum of a mean value and a multiple of the standard deviation. These two components are regarded as not correlated, leading to the Equation (C.17).

$$u(A, A\uparrow, A\downarrow) = \sqrt{u(E)^2 + u(R\uparrow, R\downarrow)^2} \quad U(A, A\uparrow, A\downarrow) = 2 \cdot u(A, A\uparrow, A\downarrow) \tag{C.17}$$

where $U(A,A\uparrow,A\downarrow)$ is the uncertainty of accuracy of positioning, k = 2.

C.5 Example uncertainty estimation

C.5.1 Uncertainty estimation for laser interferometer under average industrial conditions

Average industrial conditions are defined as

— the instrument is not calibrated, the uncertainty values given by the equipment manufacturer are taken,

— the alignment is done in order to obtain a sufficient intensity of the return beam, instead of the recommended alignment via the return beam (see C.5.2),

— the temperature in the workshop is around 25 °C,

— the temperature sensors are not calibrated,

— the repeatability of the set-up is within 50 mm, the pitch and yaw movements are 50 µm/m as maximum, and

— the environmental variation error E_{VE}, taken from a drift test, is assumed to be 1,7 µm.

In this case, the uncertainty for the bi-directional accuracy of positioning $U(A)$ is 14 µm (k=2) for a measured length of 1 750 mm (see Table C.1).

C.5.2 Uncertainty estimation for laser interferometer under improved industrial conditions

Improved conditions are defined as

— a calibrated instrument is used,

— the instrument is aligned within 1 mm to the axis under test by aligning the return beam,

— the temperature in the workshop is stable and within 20°C ± 1 °C,

— the temperature sensors are improved,

— the measurement set-up is made with special fixtures in order to keep the repeatability at 1 mm maximum, the pitch and yaw movements are at 50 µm/m maximum, and

— the environmental variation error E_{VE}, taken from a drift test, is assumed to be 1,7 µm (although a better environment will cause — in general — a smaller E_{VE}).

Then, the uncertainty for the bi-directional accuracy of positioning $U(A)$ is 4 µm (k=2) for a measured length of 1 750 mm (see Table C.2).

C.5.3 Uncertainty estimation for linear scale under average industrial conditions

Average industrial conditions are defined as

— the instrument is not calibrated, the uncertainty values given by the equipment manufacturer are taken,

— the alignment is done on the side of the linear scale on the machine table within 0,5 mm,

— the temperature in the workshop is around 25 °C,

— the linear scale has adopted the temperature of the workpiece holding device, where it is clamped to, with a maximum difference of 0,1 °C,

— the repeatability of the set-up is within 50 mm, the pitch and yaw movements are 50 µm/m as maximum, and

— the environmental variation error E_{VE}, taken from a drift test, is assumed to be 1,7 µm.

In this case, the uncertainty for the bi-directional accuracy of positioning $U(A)$ is 15 µm (k=2) for a measured length of 1 750 mm (see Table C.3).

C.5.4 Uncertainty estimation for linear scale under improved industrial conditions

Improved conditions are defined as

— a calibrated instrument is used,

— the instrument is aligned (e.g. with a dial gauge) on its side within 0,5 mm to the axis under test,

— the temperature in the workshop is stable and within 20 °C ± 1 °C,

— the linear scale has adopted the temperature of the workpiece holding device, to which it is clamped, with a maximum difference of 0,05 °C,

— the measurement set-up is made with special fixtures in order to keep the repeatability at 1 mm maximum, the pitch and yaw movements are at 50 µm/m maximum, and

— the environmental variation error $E_{VE,}$ taken from a drift test, is assumed as 1,7 µm (although a better environment will cause — in general — a smaller E_{VE}).

Then the uncertainty for the bi-directional accuracy of positioning $U(A)$ is 4 µm (k=2) for a measured length of 1 750 mm (see Table C.4).