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Geometrical product specifications (GPS) — Filtration —

Part 32: Robust profile filters: Spline filters

Spécification géométrique des produits (GPS) — Filtrage —

Partie 32: Filtres de profil robustes: Filtres splines

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Foreword

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*.

This document cancels and replaces ISO/TS 16610-32:2009, which has been technically revised.

The main changes are as follows:

— conversion to a Technical Report;

— inclusion of spline filtration for non-uniform sampling points;

— addition of a generalized filter equation with a revision of the equation of the robust spline filter harmonizing the statistical estimator with that of ISO 16610-31;

— inclusion of a termination criterion of the iterations for the robust, therefore nonlinear, filter;

— addition of specifications of the tension parameter.

A list of all parts in the ISO 16610 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document develops the terminology and concepts for spline filters. Spline filters have the advantage of being implementable for non-uniform sampling positions and for closed profiles. An example of application of spline filters is given in Annex A.

Robust filters are tolerant against outliers. Spline filters offer one method for form removal.

For more detailed information of the relation of this document to the filtration matrix and the ISO GPS standards, see Annex B and Annex C.

Geometrical product specifications (GPS) — Filtration —

Part 32:
Robust profile filters: Spline filters

1 Scope

This document provides information on a generalized version of the linear spline filter for uniform and non-uniform sampling and the robust spline filters for surface profiles. It supplements ISO 16610-22, ISO 16610-30 and ISO 16610-31.

This document provides information on how to apply the robust estimation to the spline filter as specified in ISO 16610-22, as well as its generalized form for non-uniform sampling. The weight function chosen for the M-estimator is the Tukey biweight influence function as specified in ISO 16610-31.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
robust filter
filter that is insensitive against specific phenomena in the input data

Note 1 to entry: A robust filter is a filter that delivers output data with robustness.

Note 2 to entry: Robust filters are nonlinear filters.

[SOURCE: ISO 16610-31:2016, 3.1, modified — Definition revised and notes to entry added.]

3.2
spline
linear combination of piecewise polynomials, with a smooth fit between the pieces

[SOURCE: ISO 16610-22:2015, 3.1, modified — Note 1 to entry removed.]

3.3
spline filter
linear filter based on *splines* (3.2)

Note 1 to entry: An example of spline filter application is given in Annex A.

3.4
robust spline filter
robust filter (3.1) based on *splines* (3.2)

3.5
uniform sampling
sampling of data points at equidistant positions, i.e. with the width of spacing intervals between neighbouring probing points being constant

3.6
non-uniform sampling
sampling of data points with non-equidistant spacing points

3.7
robust statistical estimator
rule that indicates how to calculate an estimate based on sample data from a population that is insensitive against specific phenomena in the input data

Note 1 to entry: An example of specific phenomena is significant deviation of the distribution of the input data (amplitude distribution in the case of surface profiles) from a Gaussian distribution mostly in the form of long tails.

3.8
M-estimator
robust statistical estimator (3.7) which uses an influence function, i.e. a function which is asymmetric and scale invariant, to weight points according to their signed distance from the reference line

[SOURCE: ISO 16610-30:2015, 3.5, modified — Definition revised.]

3.9
Tukey's biweight influence function
influence function which supresses specific phenomena in the input data x and is defined by:

$$\psi(x) = \begin{cases} x\left(1-\left(\frac{x}{c}\right)^2\right)^2 & \text{for } |x| \le c \\ 0 & \text{for } |x| > c \end{cases}$$

where c is a scale parameter

4 Spline filter for uniform and non-uniform sampling

4.1 General

The following low-pass filter equation for spline profile filters is based on cubic splines with a regularization parameter depending on the nesting index, which complies with the cut-off wavelength in the case of linear filters, for the smoothness of the resultant waviness profile (low-passed signal) and a tension parameter influencing the slope of the transfer function.

4.2 Filter equation for cubic spline filter

4.2.1 General

The filter equation is given in Formula (1):

$$\boldsymbol{w} = \left(\boldsymbol{V} + \beta\alpha^2\boldsymbol{P} + (1-\beta)\alpha^4\boldsymbol{Q}\right)^{-1}\boldsymbol{V}\boldsymbol{z} \tag{1}$$

where

$\boldsymbol{z}$	is the n-dimensional column vector of input data, e.g. the primary profile of n sampling points;
$\boldsymbol{w}$	is the column vector of output data, e.g. the waviness profile or smoothed profile;
$\boldsymbol{V}$	is the unity matrix in the case of the linear filter and the weighting matrix in the case of the robust filter;
$\boldsymbol{P}$ and $\boldsymbol{Q}$	are the matrices for the discretized differentiation;
β	is the tension parameter (see also 4.2.3);
α	is the parameter (see 4.2.2) depending on the smoothness, the nesting index (cut-off wavelength in the case of linear filters) of the spline.

Formula (1) is obtained by minimization of the objective (cost) function J as indicated in Formula (2):

$$\min_{\boldsymbol{w}} J \tag{2}$$

with the objective function defined in Formula (3):

$$J=(\boldsymbol{z}-\boldsymbol{w})^T\,\boldsymbol{V}\,(\boldsymbol{z}-\boldsymbol{w})+\beta\,\alpha^2\,\boldsymbol{w}^{\mathrm{T}}\,\boldsymbol{P}\,\boldsymbol{w}+(1-\beta)\,\alpha^4\,\boldsymbol{w}^{\mathrm{T}}\boldsymbol{Q}\,\boldsymbol{w} \tag{3}$$

where $\boldsymbol{Q}=\boldsymbol{P}^{\mathrm{T}}\boldsymbol{P}$.

A sufficient condition of a minimum is $\nabla_{\boldsymbol{w}}\,J=0$ leading to the filter equation in Formula (1).

NOTE 1 After extending the matrices of Formula (1) to tensors, the filter is also applicable to areal data[11].

NOTE 2 Usually the objective function of smoothing splines is defined with a regularization parameter μ also fitted during the optimization process with an additional condition for the smoothness measured according to the deviations $z_i-s(x_i)$. Objective functions of the more common type of smoothing splines do not include non-zero tension $J=\sum_{i=1}^{n}(z_i-s(x_i))^2+\mu\int_{x_1}^{x_n}\left(\frac{\partial}{\partial x}s(x)\right)^2\mathrm{d}x$ with $s(x)$ being the spline polynomials and the regularization parameter μ determining the degree of smoothing and hence following the data points vs approximating them.

4.2.2 Regularization parameter

The parameter μ specifies the regularization, i.e. the degree of smoothing. In the case of minimum tension, it holds $\mu=\alpha^4$ and is therefore related to the nesting index n_{i}, which is in the case of linear filtration equal to the cut-off wavelength λ_{c} as given in Formula (4):

$$\alpha=\frac{1}{2\sin\left(\frac{\pi\,\Delta}{n_{\mathrm{i}}}\right)} \tag{4}$$

where Δ is the sampling interval for uniformly sampled data and the average sampling interval as given in Formula (5):

$$\Delta=\frac{1}{n-1}\sum_{i=1}^{n-1}(x_{i+1}-x_i) \tag{5}$$

for data sampled non-uniformly at positions x_i with i = 1, ..., n–1.

NOTE 1 Formula (4) is derived in Reference [12].

NOTE 2 For sampling intervals $\Delta \ll n_{\mathrm{i}}$ the regularization parameter tends to infinity $\alpha^4 \to \infty$.

NOTE 3 For non-minimal tension the factor μ of the second order derivative term is also dependent on the tension parameter β : $\mu = (1-\beta)\alpha^4$.

4.2.3 Tension parameter

The product $\beta\alpha^2$ is the tension factor with parameter β lying between 0 and 1. The parameter β controls the degree of subsequent topography curvatures, where curvature means a local property of a curve or a surface, which is defined at every point quantifying second-order deviations of a curve from a straight line or a surface from a plane.

Following curvatures closely means optimal shape retainment of the low-pass result, the output data $\boldsymbol{w}$.

For $\beta = 0$ the characteristics of the transfer function conform to Formula (1) in ISO 16610-22, a minimum tension which is equivalent to the steepest slope of the transfer function and therefore a better shape retainment than for $\beta > 0$.

For β = 0,625 242 the characteristics of the transfer function is similar to that of the Gaussian filter[14] as specified in ISO 16610-21 and ISO 16610-61.

NOTE The shape retainment by the spline filter for $\beta = 0$ is global, while the shape retainment by the Gaussian regression with a parabolic regression ($p = 2$) is local.

4.2.4 Matrix *V* for linear cubic spline filter

Matrix $\boldsymbol{V}$ for linear filters is the $n \times n$-dimensional unity matrix as given in Formula (6):

$$\boldsymbol{V} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} \tag{6}$$

4.2.5 Matrix *V* for robust cubic spline filter

Matrix $\boldsymbol{V}$ contains the weights suppressing specific phenomena in the input data. They are derived from Tukey's biweight influence function as given in Formula (7):

$$\boldsymbol{V}^{(m)} = \begin{pmatrix} \delta_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_n \end{pmatrix} \text{with} \quad \delta_i = \begin{cases} \left(1 - \left(\dfrac{z_i - w_i^{(m)}}{c^{(m)}}\right)^2\right)^2 & \text{for} \left|z_i - w_i^{(m)}\right| \le c^{(m)} \\ 0 & \text{for} \left|z_i - w_i^{(m)}\right| > c^{(m)} \end{cases} \tag{7}$$

where

$i = 1, \ldots, n$;

$\boldsymbol{z} = (z_1, \ldots, z_n)^{\mathrm{T}}$;

$\boldsymbol{w}_{\boldsymbol{i}}^{(m)} = \left(w_1^{(m)}, \ldots, w_n^{(m)}\right)^{\mathrm{T}}$;

superscript m denotes the iteration put into brackets, which is not to be confused with an exponent;

superscript T denotes transposed.

Furthermore, the parameter c is specified as given in ISO 16610-31:2016, Formula (8), and as shown in Formula (8):

$$c = a \,\text{median}|z - \boldsymbol{w}| \quad \text{with} \quad a \cong 4{,}447\,8 \tag{8}$$

NOTE The exact value of a is obtained by the inverse error function erf^{-1}:

$$a = \frac{3}{\sqrt{2} \quad \text{erf}^{-1}(0{,}5)}$$

4.2.6 Termination of the iteration of robust estimation

The matrix $\boldsymbol{V}$ containing weights δ_i being dependent on output data w_i starts with w obtained by linear non-robust filtration, i.e. $\boldsymbol{V}^{(0)}$ is the unity matrix. The iteration process terminates if the condition given in Formula (9) is reached:

$$\frac{\left|c^{(m+1)} - c^{(m)}\right|}{c^{(m)}} \le 10^{-5} \quad \text{or} \quad m \ge 12 \tag{9}$$

4.2.7 Matrices of differentiation *P* and *Q*

4.2.7.1 General

A profile can be sampled at lateral positions x_i that are not necessarily equidistant. The lateral positions are strictly monotonically increasing, i.e. $x_i < x_{i+1}$.

The samplings intervals are denoted $\Delta_{i,j} = x_i - x_j$ and the quotient of the average sampling interval Δ and the distance between sampling positions x_i and x_j is denoted by $D_{i,j} = \dfrac{\Delta}{\Delta_{i,j}}$.

4.2.7.2 Differentiation matrix for first discretized derivative

For open profiles, matrix $\boldsymbol{P}$ is tri-diagonal as shown in Formula (10):

$$\boldsymbol{P} = \begin{pmatrix} P_{1,1} & P_{1,2} & 0 & \cdots & & \\ P_{2,1} & P_{2,2} & P_{2,3} & 0 & \cdots & \\ 0 & P_{3,2} & P_{3,3} & P_{3,4} & 0 & \cdots \\ & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \tag{10}$$

For rows $i = 2,\ldots,n-1$ the main diagonal has the elements: $P_{i,i} = D_{i,i-1}^2 + D_{i+1,i}^2$.

The two off-diagonals have the elements $P_{i,i-1} = -D_{i,i-1}^2$ and $P_{i,i+1} = -D_{i+1,i}^2$.

The first two elements of the first row and the last two elements of the last row are as follows:

$$P_{1,1} = D_{1,2}^2 \text{ and } P_{1,2} = -D_{1,2}^2 \text{ and } P_{n,n-1} = -D_{n,n-1}^2 \text{ and } P_{n,n} = D_{n,n-1}^2$$

For closed profiles the first row and the last row of the matrix differ, having additional non-zero entries at $P_{1\,n}$ and at $P_{n\,1}$ for the wrap around, i.e. the right neighbour of x_n is x_1 and the left neighbour of x_1 is x_n.

Then the first row has the following non-zeros entries:

$$P_{1,n} = -D_{1,n}^2 \text{ and } P_{1,1} = D_{1,n}^2 + D_{2,1}^2 \text{ and } P_{1,2} = -D_{2,1}^2$$

and the last row has the following entries:

$$P_{n,n-1} = -D_{n,n-1}^2 \text{ and } P_{n,n} = D_{n,n-1}^2 + D_{1,n}^2 \text{ and } P_{n,1} = -D_{1,n+1}^2$$

4.2.7.3 Differentiation matrix for second discretized derivative

For open profiles matrix $\boldsymbol{Q}$ is penta-diagonal as shown in Formula (11):

$$\boldsymbol{Q} = \begin{pmatrix} Q_{1,1} & Q_{1,2} & Q_{1,3} & 0 & \cdots & \\ Q_{2,1} & Q_{2,2} & Q_{2,3} & Q_{2,4} & \ddots & \\ Q_{3,1} & Q_{3,2} & Q_{3,3} & Q_{3,4} & Q_{3,5} & \\ 0 & Q_{4,2} & Q_{4,3} & Q_{4,4} & Q_{4,5} & \ddots \\ & \ddots & \ddots & \ddots & \ddots & \ddots \end{pmatrix} \quad (11)$$

For rows $i = 3, \ldots, n-2$ the elements of the main diagonal are as follows:

$$Q_{i,i} = 4\left(D_{i,i-1}^2 \left(D_{i,i-2}^2 + D_{i+1,i-1} D_{i+1,i} \right) + D_{i+1,i}^2 \left(D_{i+1,i-1} D_{i,i-1} + D_{i+2,i}^2 \right) \right)$$

The elements of the off-diagonal next to the main are as follows in the case of the upper:

$$Q_{i,i+1} = -4\, D_{i+1,i} \left(D_{i+1,i} D_{i+1,i-1} D_{i,i-1} + D_{i+2,i}^2 \left(D_{i+2,i+1} + D_{i+1,i} \right) \right)$$

and in the case of the lower as follows:

$$Q_{i,i-1} = -4\, D_{i,i-1} \left(D_{i+1,i} D_{i+1,i-1} D_{i,i-1} + D_{i,i-2}^2 \left(D_{i-1,i-2} + D_{i,i-1} \right) \right)$$

The elements of the second off-diagonals are in the case of the upper as follows:

$$Q_{i,i+2} = 4\, D_{i+1,i} D_{i+2,i+1} D_{i+2,i}^2$$

and in the case of the lower as follows:

$$Q_{i,i-2} = 4\, D_{i,i-2}^2 D_{i-1,i-2} D_{i,i-1}$$

For the first row and in the case of open profiles, the elements are as follows:

$$Q_{1,1} = 4\, D_{2,1}^2 D_{3,1}^2 \text{ and } Q_{1,2} = -4\, D_{2,1}^2 D_{3,2} D_{3,1} \text{ and } Q_{1,3} = 4\, D_{3,2} D_{3,1}^2 D_{2,1}$$

For the second row and in the case of open profiles they are as follows:

$$Q_{2,1} = Q_{1,2} \text{ and } Q_{2,2} = 4\, D_{3,2}^2 \left(D_{2,1}^2 + D_{4,2}^2 \right) \text{ and } Q_{2,3} = -4\, D_{3,2}^2 \left(D_{2,1} D_{3,1} + D_{4,2} D_{4,3} \right)$$

$$Q_{2,3} = -4\, D_{3,2} \left(D_{3,2} D_{3,1} D_{2,1} + D_{4,2}^2 \left(D_{4,3} + D_{3,2} \right) \right)$$

For closed profiles the first row has the following non-zero entries:

$$Q_{1,1} = 4\left(D_{1,n}^2 \left(D_{1,n-1}^2 + D_{2,n}\, D_{2,1} \right) + D_{2,1}^2 \left(D_{2,n}\, D_{1,n} + D_{3,1}^2 \right) \right)$$

$$Q_{1,2} = -4\, D_{2,1} \left(D_{2,1} D_{2,n} D_{1,n} + D_{3,1}^2 \left(D_{3,2} + D_{2,1} \right) \right)$$

$$Q_{1,n} = -4\, D_{1,n} \left(D_{2,1} D_{2,n} D_{1,n} + D_{1,n-1}^2 \left(D_{n,n-1} + D_{1,n} \right) \right)$$

$$Q_{1,3} = 4\, D_{2,1} D_{3,2} D_{3,1}^2 \text{ and } Q_{1,\,n-1} = 4\, D_{1,n-1}^2 D_{n,n-1} D_{1,n}$$

The last row, in the case of closed profiles, has the following non-zero entries:

$$Q_{n,n} = 4\left(D_{n,n-1}^2 \left(D_{n,n-2}^2 + D_{1,n-1}\, D_{1,n-1} \right) + D_{1,n}^2 \left(D_{1,n-1}\, D_{n,n-1} + D_{1,n}^2 \right) \right)$$

$$Q_{n\,1} = -4\, D_{1,n} \left(D_{1,n} D_{1,n-1} D_{n,n-1} + D_{2,n}^2 \left(D_{2,1} + D_{1,n} \right) \right)$$

$$Q_{n,\,n-1} = -4\, D_{n,n-1} \left(D_{1,n} D_{1,n-1} D_{n,n-1} + D_{n,n-2}^2 \left(D_{n-1,n-2} + D_{n,n-1} \right) \right)$$

$$Q_{n,\,2} = 4\, D_{n+1,n} D_{2,1} D_{2,n}^2 \text{ and } Q_{n,\,n-2} = 4\, D_{n,n-2}^2 D_{n-1,n-2} D_{n,n-1}$$

For closed profiles, the second row has the following non-zero entries:

$$Q_{2,2} = 4\left(D_{2,1}^2 \left(D_{2,n}^2 + D_{3,1}\, D_{3,2} \right) + D_{3,2}^2 \left(D_{3,1}\, D_{2,1} + D_{4,2}^2 \right) \right)$$

$$Q_{2,3} = -4\, D_{3,2} \left(D_{3,2} D_{3,1} D_{2,1} + D_{4,2}^2 \left(D_{4,3} + D_{3,2} \right) \right)$$

$$Q_{2,1} = -4\, D_{2,1} \left(D_{3,2} D_{3,1} D_{2,1} + D_{2,n}^2 \left(D_{1,n} + D_{2,1} \right) \right)$$

$$Q_{2,4} = 4\, D_{3,2} D_{4,3} D_{4,2}^2 \text{ and } Q_{2,\,n} = 4\, D_{2,n}^2 D_{1,n} D_{2,1}$$

and the second-last row is as follows:

$$Q_{n-1,\,n-1} = 4\left(D_{n-1,n-2}^2 \left(D_{n-1,n-3}^2 + D_{n,n-2}\, D_{n,n-2} \right) + D_{n,n-1}^2 \left(D_{n,n-2}\, D_{n-1,n-2} + D_{n,n-1}^2 \right) \right)$$

$$Q_{n-1,\,n} = -4\, D_{n,n-1} \left(D_{n,n-1} D_{n,n-2} D_{n-1,n-2} + D_{1,n-1}^2 \left(D_{1,n} + D_{n,n-1} \right) \right)$$

$$Q_{n-1,\,n-2} = -4\, D_{n-1,n-2} \left(D_{n,n-1} D_{n,n-2} D_{n-1,n-2} + D_{n-1,n-3}^2 \left(D_{n-2,n-3} + D_{n-1,n-2} \right) \right)$$

$$Q_{n-1,\,1} = 4\, D_{n,n-1} D_{1,n} D_{1,n-1}^2 \text{ and } Q_{n-1,\,n-3} = 4\, D_{n-1,n-3}^2 D_{n-2,n-3} D_{n-1,n-2}$$

4.3 Transmission characteristics

The transmission characteristics of the linear case, i.e. matrix ***V***, is a unity matrix as given in ISO 16610-22:2015, 4.3. The transmission characteristics of a robust filter do not exist, because of its nonlinearity.

4.4 Alternative robust spline filter

4.4.1 General

The weighting by the Tukey biweight function represents an M-estimator that causes a distribution of the residuals $z_i - w_i^{(m)}$ to be reshaped. The Tukey biweight influence function suppresses significantly large tails of a distribution or significant asymmetries. Hence the residuals contributing to filtration are almost Gaussian distributed. ISO 16610-30 allows alternative robust estimators that can be applied to spline filters in a generalized linear and nonlinear way. [13]

To retain the shape of profiles obtained on significantly curved surfaces within the waviness and to exclude this trend from roughness in ISO 16610-31, the default for the regression polynomial of the Gaussian regression is the parabola (i.e. order $p=2$). Retaining shape, i.e. passing curvature to the waviness profile, in the case of spline filters is obtained by minimizing the tension energy, which is equivalent to setting $\beta=0$.

When employing the least absolute deviation method, i.e. applying the L1-norm on the residuals, for robust estimation special care needs to be taken when defining the filter equation.

4.4.2 Objective function with L2-norm without tension energy for the linear filter equation

The filter equation, Formula (1), already represents the partial derivatives of the objective (cost) function J of the optimization problem[8, 9] as given by Formula (12):

$$J=(\boldsymbol{z}-\boldsymbol{w})^T(\boldsymbol{z}-\boldsymbol{w})+\alpha^4\boldsymbol{w}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{w}=\|\boldsymbol{z}-\boldsymbol{w}\|^2+\alpha^4\|\boldsymbol{P}\boldsymbol{w}\|^2 \tag{12}$$

For the case of minimum tension energy, i.e. $\beta=0$, and in the case of matrix $\boldsymbol{V}$ being the unity matrix $\boldsymbol{I}$, which applies to the linear filter derived from the least mean square (LMS) method, use the L2-norm denoted with $\|\cdot\|^2$. The filter equation without tension energy reduces as given by Formula (13):

$$\boldsymbol{w}=\left(\boldsymbol{I}+\alpha^4\boldsymbol{Q}\right)^{-1}z \tag{13}$$

where $\boldsymbol{I}$ is the unit matrix.

4.4.3 Objective function with L1-norm without tension energy for robust filtration

For distributions of the residuals $r_i=z_i-w_i$ with extremely long tails and a very narrow kernel, i.e. for a kurtosis much greater than 3 (the kurtosis value 3 complies with a Gaussian distribution), assuming a Laplacian distribution rather than a Gaussian one is more appropriate. The Laplacian distribution has a kurtosis of 6. Assuming the residuals z_i-w_i to follow a Laplacian distribution with zero mean, the target function is given in Formula (14):

$$J=\left(\frac{\pi}{2}\frac{1}{n}\|\boldsymbol{z}-\boldsymbol{w}^{(m-1)}\|\right)\|\boldsymbol{z}-\boldsymbol{w}\|+\alpha^4\|\boldsymbol{P}\boldsymbol{w}\|^2 \tag{14}$$

where $\|\cdot\|$ denotes the L1-norm.

The factor $\frac{\pi}{2}\frac{1}{n}\|\boldsymbol{z}-\boldsymbol{w}^{(m-1)}\|$ provides for an appropriate scaling carrying the dimension of a length,[14] with n being the number of sampling points and m the iteration step. The sum of absolute values of the residuals of the previous iteration is used to estimate this factor.

Dividing the objective function by c for further processing, the optimization problem is written in a form suitable for the split-Bregman algorithm. The optimization problem is expressed by Formula (15):

$$\min_{\boldsymbol{w},\boldsymbol{r}}\left\{\|\boldsymbol{r}\|+\mu\|\boldsymbol{P}\,\boldsymbol{w}\|^2\right\} \tag{15}$$

where $\boldsymbol{r} = \boldsymbol{z} - \boldsymbol{w}$ and the regularization parameter μ is given by Formula (16):

$$\mu=\frac{\alpha^4}{\frac{\pi}{2}\frac{1}{n}\|\boldsymbol{z}-\boldsymbol{w}^{(m-1)}\|} \tag{16}$$

which is equivalent to the optimization problem as given by Formula (17):

$$\min_{w,r}\left\{\|\boldsymbol{r}\|+\mu\|\boldsymbol{P}\,\boldsymbol{w}\|^2+\lambda\|\boldsymbol{z}-\boldsymbol{w}\|^2\right\} \tag{17}$$

where λ is Lagrange multipliers carrying the dimension of an inverse length.

The Lagrange multipliers are proportional to $1\Big/\frac{\pi}{2}\frac{1}{n}\|\boldsymbol{z}-\boldsymbol{w}^{(m-1)}\|$ (see Reference [15]).

5 Filter designation

The linear spline filter, i.e. the filter with matrix $\boldsymbol{V}$ being the unity matrix, is designated:

FPLS

and the robust spline filter, the filter with matrix $\boldsymbol{V}$ being a weighting matrix, as given in Formula (7), is designated:

FPRS

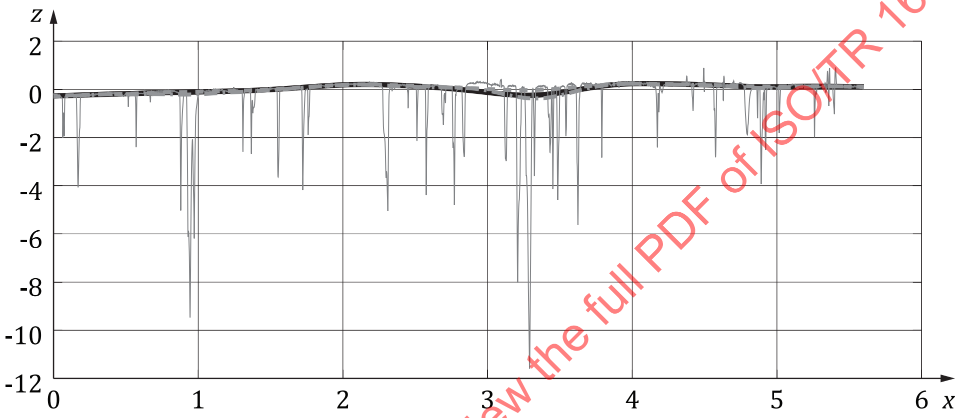
See also ISO 16610-1.

Annex A
(informative)

Example of spline filter applied to plateau structured profile

Typical use cases for robust filters include determining material ratio parameters of measurement data on plateau structured surfaces, i.e. profiles with extremely skewed amplitude distributions.

Figure A.1 displays a profile of a sintered surface filtered with a nesting index of $n_i = 0,8$ mm. The split-Bregman algorithm for L1-norm optimization complies with one or two iterations when employing M-estimation with the Tukey biweight function as weighting function depending on the residuals to suppress long tails.



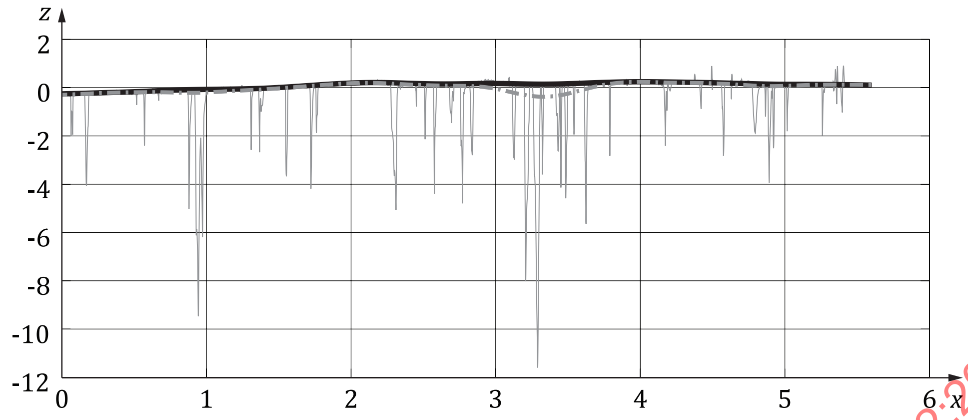
Key

z	height values of the profile, in µm
x	positions along the lateral scan axis, in mm
-·-·-·-	Tukey 1 iteration
- - - - -	Tukey 2 iteration
———	L1-norm

NOTE It is obtained by applying the M-estimator approach with the Tukey biweight function with only 1 and 2 iterative steps in comparison with the result of the L1-norm obtained by the Bregmen split procedure, with a nesting index of n_i = 0,8 mm and a tension parameter of $\beta = 0$.

Figure A.1 — Profile of sintered surface and its spline filtered waviness profiles

Figure A.2 illustrates the effect of iteratively applying the M-estimator approach until it converges to a stable waviness curve no longer following the deep grooves. It shows the difference between the waviness profile obtained by applying the M-estimator approach with the Tukey biweight function with only 1 iteration and with as many iterative steps as to guarantee one stable result, with a nesting index of n_i = 0,8 mm and a tension parameter of β = 0.

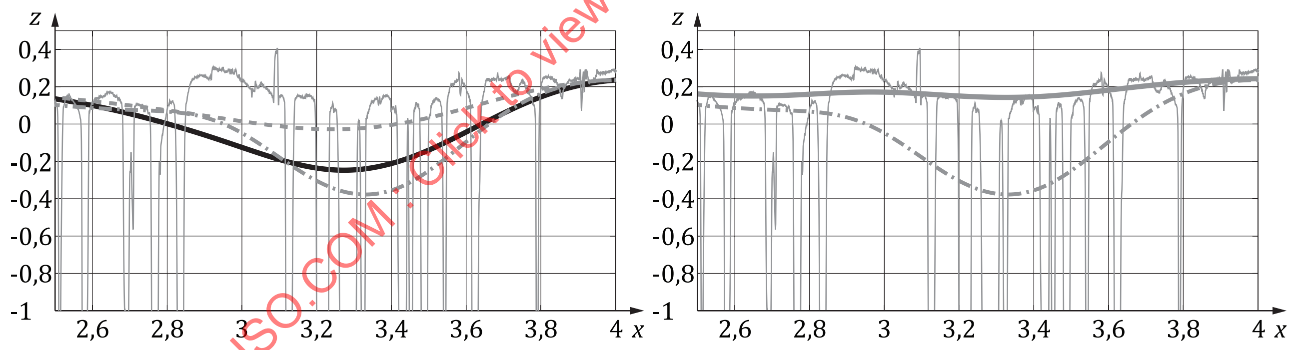


Key

z	height values of the profile, in um
x	positions along the lateral scan axis, in mm
(solid black line)	Tukey 21 iterations
(dash-dot grey line)	Tukey 1 iteration

Figure A.2 — Profile of Figure A.1

The feature of the profile that lies between 3 mm and 4 mm clearly reveals the idea of the robust filtration to obtain a waviness that represents the surface as a plateau. The waviness is a straight bridge across all the grooves. Figure A.3 therefore depicts this detail as an example. It shows the idea of a waviness profile to represent the plateau of a surface with grooves, which can be interpreted as bridges crossing the grooves.



Key

z	height values of the profile, in um
x	positions along the lateral scan axis, in mm
(dash-dot grey line)	Tukey 1 iteration
(dotted grey line)	Tukey 2 iterations
(solid black line)	Tukey 21 iterations
(solid grey line)	Tukey 1 iteration

Figure A.3 — Detail of a small interval of the profile of Figures A.1 and A.2