TECHNICAL REPORT

ISO/TR 13487

First edition
1997-07-15

Braking of road vehicles — Considerations on the definition of mean fully developed deceleration

Freinage des véhicules routiers — Considérations sur la définition de la décélération moyenne en régime

Reference number
ISO/TR 13487:1997(E)

Contents

International Organization for Standardization
Case postale 56 • CH-1211 Genève 20 • Switzerland
Internet central@iso.ch
X.400 c=ch; a=400net; p=iso; o=isocs; s=central

Printed in Switzerland

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 13487, which is a Technical Report of type 3, was prepared by Technical Committee ISO/TC 22, *Road vehicles*, Subcommittee SC 2, *Brake systems and equipment*.

Introduction

ECE Regulation No. 13 "Uniform Provisions Concerning the Approval of Vehicles of Categories M, N and O with Regard to Braking" determines the minimum legal braking performance for new road vehicles at the time of type-approval.

This braking performance is specified in terms of "stopping distance" and in terms of "mean fully developed deceleration" (mfdd).

The 08 series of amendments to ECE-R13 requires that both the above parameters must be fulfilled; furthermore, the 08 series of amendments prescribes the method of calculating the mean fully developed deceleration.

The chosen method of calculating the mfdd is based on the work done in ISO/TC 22/SC 2 Working Groups 6 and 10.

For this reason, it is useful to summarize the background information on this subject in this ISO Technical Report, by describing the physical fundamentals and the connection between stopping distance and mfdd; this will enable the persons responsible for determining the braking performance to analyse the results of testing, which are never exactly reproducible.

Because the legislative text does not stipulate the measuring equipment nor specific measuring procedures, this Technical Report may indicate alternative solutions to the Technical Services and to the manufacturers of measuring equipment; it will also address prospective computer-supported possibilities. In addition, the transition between different systems of units will be facilitated by this contribution.

The prospective legislative text will concede alternative methods of measuring the mfdd; these are explicitly explained in this Technical Report.

The report clearly indicates that in addition to the exact solutions for the mfdd (see equations 16 and 29), certain approximations (see equations 30, 31 and 33) are also permissible within the required accuracy, as documented by theoretical considerations and corresponding practical measurements with a vehicle.

For this reason, equation 31 was developed for computer-aided and equation 33 for graphical evaluations.

Symbols

Symbol	Unit	Description
a (s)	m/s^2	distance-dependent deceleration
a (t)	m/s^2	time-dependent deceleration
$\bar{a}$	m/s^2	representative constant deceleration
$\lvert\bar{a}\rvert$	m/s^2	absolute value of $\bar{a}$
a_B, a_E	m/s^2	decelerations at the beginning and end of evaluation range on the linear approximate solution for a (t)
a_F (t)	m/s^2	analytically given deceleration path with temporal drop
a_i, a_j	m/s^2	individual deceleration values
a_L (t)	m/s^2	linear approximate solution for a (t)
a_{max}	m/s^2	maximum value in a time-dependent deceleration path
a_{ms}	m/s^2	distance-related mean deceleration
a_{ms1}, a_{ms2}, a_{ms3}	m/s^2	examples of different distance-related mean deceleration
$\tilde{a}_{ms}$	m/s^2	approximate value for the distance-related mean deceleration in accordance with Equation (33)
a_{msN}	m/s^2	numerical approximate value for the distance-related mean deceleration in accordance with Equation (31)
a_{mt}	m/s^2	time-related mean deceleration
$\tilde{a}_{mt}$	m/s^2	approximate value for the time-related mean deceleration in accordance with Equation (34)
a_R (t)	m/s^2	analytically given deceleration path with temporal rise
a_1 (s), a_2 (s), a_3 (s)	m/s^2	different distance-dependent deceleration paths
a_1 (t), a_2 (t), a_3 (t)	m/s^2	different time-dependent deceleration paths
d_m	m/s^2	mfdd according to ECE Regulation No. 13
ds	m	distance differential
dt	s	time differential
dv	m/s	speed differential
mfdd	m/s^2	mean fully developed deceleration
s	m	distance
s_B, s_E	m	distances at the start and end of evaluation range
s_D	m	braking distance during the period of mfdd
s_F (t)	m	distance path during the analytically given deceleration a_F (t)
s_i	m	individual distance values
s_R	m	braking distance during response time and pressure build-up time

s_R (t)	m	distance path during the analytically given deceleration a_R (t)
s_1, s_2, s_3	m	braking distances with different deceleration paths
s_1 (t), s_2 (t), s_3 (t)	m	different distance paths
T	s	total braking time
t, t^i	s	time
t_B, t_E	s	points in time for the start and end of the evaluation range
t_R	s	sum of response time and pressure build-up time
t_S	s	time at the end of a stop
t_1	s	point in time at which the deceleration takes on the value 1/2 a_{max} at first time
t_2	s	point in time at which the deceleration takes on the value 1/2 a_{max} at last time
v	km/h	test speed
v (t)	m/s, km/h	variable speed
v_B, v_E	m/s	speeds at the start and end of evaluation range
v_F (t)	m/s	speed path during the analytically given deceleration a_F (t)
v_i	m/s	individual speed values
v_R (t)	m/s	speed path during the analytically given deceleration a_R (t)
v_0	m/s, km/h	initial speed
v_1 (t), v_2 (t), v_3 (t)	m/s	different speed paths
Δa_{msN}, $\Delta \tilde{a}_{ms}$ $\Delta \tilde{a}_{mt}$, $\Delta \tilde{a}_{msN}$	%	related differences of mean decelerations (Tables 5 and 6)
Δt	s	time increment
τ	s	time

Braking of road vehicles — Considerations on the definition of mean fully developed deceleration

1 Technical considerations

The ECE Regulation No. 13 "Uniform Provisions Concerning the Approval of Vehicles with Regard to Braking" deals in Annex 4 "Braking Tests and Performance of Braking Systems" with the observance of certain stopping distances and certain "mean fully developed decelerations" under defined test conditions.

The formulae applied for the judgement of the stopping distance ordinarily have the structure

$$s \leq s_R + s_D = t_R \times v + \frac{v^2}{2 \times d_m} \qquad (1)$$

where :

s is the measured stopping distance, s_R the distance correlated to the response and pressure build-up time t_R, s_D the distance correlated to the mean fully developed deceleration phase, v the test speed and d_m the so-called "mean fully developed deceleration". e.g. for passenger cars (vehicles of category M_1 according to ECE-R13) the following values are valid:

$t_R = 0{,}36$ s, $v = 22{,}22$ m/s and $d_m = 5{,}8$ m/s^2

If the stopping distance shall be measured in the dimension m and in addition the dimension km/h shall be used for the speed, we get from (1) in the case of M_1-vehicles the formula as it is known from the Regulation No. 13:

$$s \leq 0{,}1 \times v + \frac{v^2}{150} \qquad (2)$$

The problem is that there is until today no rule for determining the "mean fully developed deceleration" (mfdd) in such a way that it is commensurate to the existing legal requirements for stopping distances.

A procedure which establishes mfdd in such a way that it is in accordance with the stopping distance should additionally fulfil the following demands:

- Mfdd shall not be design-restrictive concerning the measuring devices i.e. even pure deceleration measurements shall be evaluable.

- The evaluation of the mfdd shall allow the use of modern computers as well as conventional methods.

- A representative part of the deceleration process must be chosen for the evaluation.

Until now, in national regulations in Europe, mean values have generally been based on time. There is no indication on any of the analytical processes that mean value formation based on time can lead to considerable errors, if the stopping distance or speed path is calculated with this time-related mean value.

Using various deceleration paths with the same time-related mean value, the following example shows that both the distance-related mean decelerations and the appertaining stopping distances assume different values.

Starting from the speed v_0 = 30 m/s, in a time interval of 0 to 6 s, the following deceleration paths are taken (Figure 1):

$$a_1(t) = -5\,m/s^2$$
$$a_2(t) = -(8\,m/s^2 - 1\,m/s^3 \, x\, t) \qquad (3)$$
$$a_3(t) = -(2\,m/s^2 + 1\,m/s^3 \, x\, t)$$

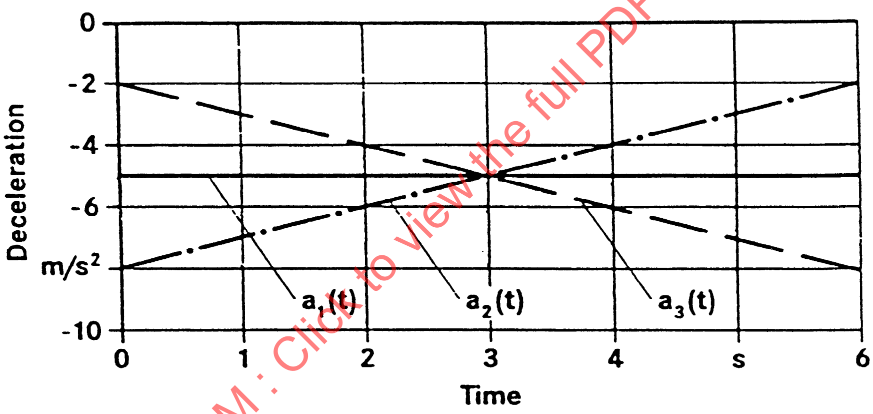

Figure 1 - Different deceleration paths with the same time-related mean value

Integration over time gives the speeds (Figure 2):

$$v_1(t) = 30\,m/s - 5\,m/s^2 \times t$$
$$v_2(t) = 30\,m/s - (8\,m/s^2 - 0{,}5\,m/s^3 \, x\, t) \times t \qquad (4)$$
$$v_3(t) = 30\,m/s - (2\,m/s^2 + 0{,}5\,m/s^3 \, x\, t) \times t$$

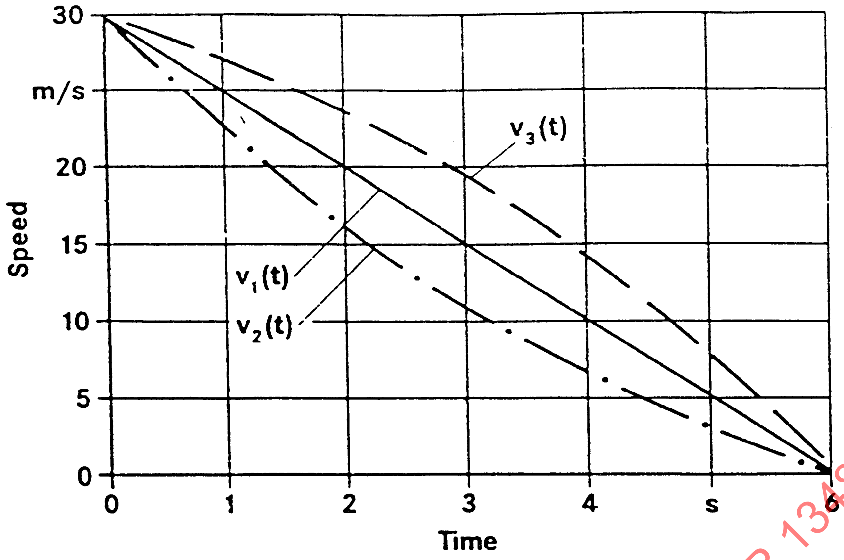

Figure 2 - Speed paths correlated to the decelerations in figure 1

Integration of the speeds over time gives the distance paths (figure 3):

$$s_1\,(t) = 30\,m/s \times t - 2{,}5/s^2 \times t^2$$

$$s_2\,(t) = 30\,m/s \times t - (8\,m/s^2 - 0{,}\overline{3}\,m/s^3 \times t) \times \frac{t^2}{2} \tag{5}$$

$$s_3\,(t) = 30\,m/s \times t - (2\,m/s^2 + 0{,}\overline{3}\,m/s^3\ x\, t) \times \frac{t^2}{2}$$

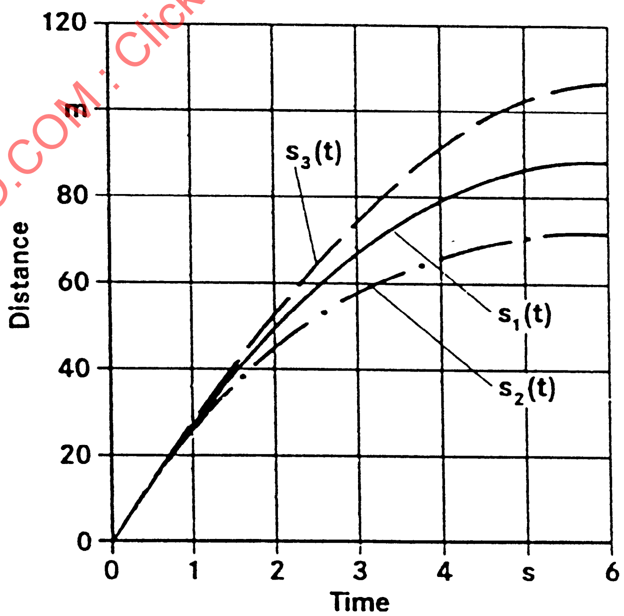

Figure 3 - Distance paths correlated to the decelerations in Figure 1

In the case of different decelerations a (t) with the same time-related mean value, major deviations in the braking distance are apparent (Figure 3):

$s_1 = 90$ m $\qquad s_2 = 72$ m $\qquad s_3 = 108$ m

If the deceleration paths correlated to Figure 1 are plotted against distance, the result is shown in Figure 4.

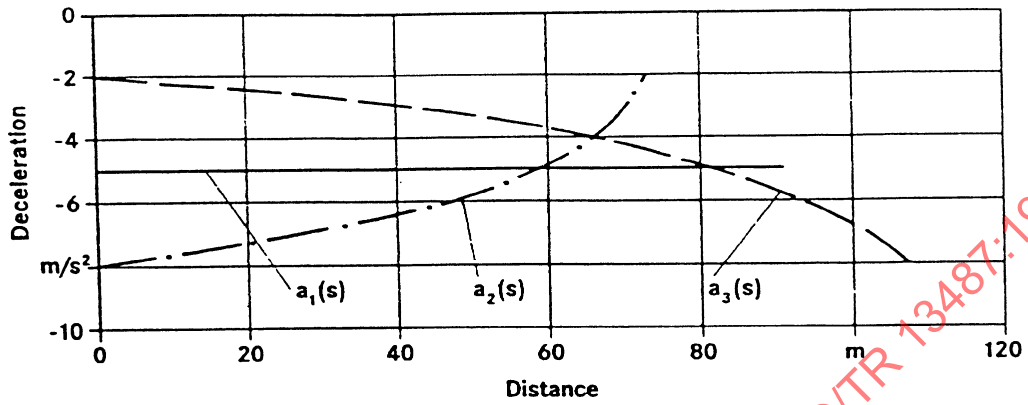

Figure 4 - Deceleration paths over distance based on the distances in Figure 3

With the help of the equation:

$$a_{msj} = -\frac{v_0^2}{2 \times s_j}, j = 1, 2, 3 \qquad (6)$$

which represents a special case of the equation (11) developed in chapter 2, the results is as follows:

a_{ms1} = - 5,0 m/s² a_{ms2} = - 6,25 m/s² a_{ms3} = - 4,17 m/s²

It can be seen that the distance-related mean decelerations deviate from one another. Only in the case of a_{ms1} = - 5.0 m/s^2 the value is equal to the time-related value.

The previous discussions indicate that it is necessary to carefully interpret the mean fully developed deceleration, designated as d_m in equation (1). Particularly so because the legal requirements based on the current edition of ECE Regulation No. 13 allow the possibility of using the mfdd d_m in addition to evaluation of the stopping distance for approval tests.

2 Theoretical considerations

2.1 Basic equations

The physical derivation is required to help understand the second term s_D from equation (1) containing mfdd. Starting with the basic equations:

$$v(t) = \frac{ds}{dt} \tag{7}$$

and

$$a(t) = \frac{dv}{dt} \tag{8}$$

the following relationship can be obtained by eliminating the time differential dt in (8) by substitution using (7):

$$ds = \frac{1}{a(t)} \times v(t) \times dv \tag{9}$$

The question of a suitable definition of mfdd is equivalent to the question which representative constant deceleration rate $\bar{a}$ can describe a given deceleration process. Considering the difference between distance s_B at the start and distance s_E at the end of the evaluation period it can be obtained by integrating (9) within the associated speed limits v_B and v_E

$$\int_{s_B}^{s_E} ds = s_E - s_B = \frac{1}{\bar{a}} \times \int_{v_B}^{v_E} v(t) \times dv = \frac{v_E^2 - v_B^2}{2 \times \bar{a}} \tag{10}$$

or:

$$\bar{a} = \frac{v_E^2 - v_B^2}{2 \times (s_E - s_B)} \tag{11}$$

When the vehicle is braked to a full stop V_E goes to zero. Equating:

$$s_E - s_B = s_D \tag{12}$$

$$|\bar{a}| = d_m \tag{13}$$

and:

$$v_B = v \tag{14}$$

it follows from (10) or (11) the second term of (1). This clearly describes its physical background.

Corresponding to the literal sense of mfdd the same result can be obtained by calculating a mean value a_{ms} for the distance-dependent deceleration a(s) using the usual mathematical definition of a mean:

$$a_{ms} = \frac{1}{s_E - s_B} \times \int_{s_B}^{s_E} a(s) \times ds = \frac{1}{s_E - s_B} \times \int_{v_B}^{v_E} \frac{dv}{dt} \times v(t) \times dt \qquad (15)$$

This results in:

$$a_{ms} = \frac{v_E^2 - v_B^2}{2 \times (s_E - s_B)} \qquad (16)^{1)}$$

The representative deceleration $\overline{a}$ according to (11) and the mean value a_{ms} according to (16) are therefore identical. So it is proved that only a distance-related mean deceleration is in harmony with the stopping distance.

A time-related mean value a_{mt} between the times t_B at the beginning and t_E at the end of the evaluation period can also be calculated analogous to (15) according to the equation:

$$a_{mt} = \frac{1}{s_E - t_B} \times \int_{t_B}^{t_E} a(t) \times dt = \frac{1}{s_E - t_B} \times \int_{v_B}^{v_E} \frac{dv}{dt} \times dt \qquad (17)$$

This results in:

$$a_{mt} = \frac{v_E - v_B}{t_E - t_B} \qquad (18)$$

When the deceleration process is not constant, a_{mt} deviates from a_{ms} as it was already shown in the examples in the introduction. The following discussion describes the effects of the difference between a_{ms} and a_{mt} in greater detail.

2.2 Determination of the distance-related mean deceleration from a(t)

Before continuing with the derivation of the physical laws required for a comparative discussion of a_{ms} and a_{mt}, a few deficiencies in the previous discussion need to be cleared up. In equation (13) it was only possible to achieve the transition to d_m by using the absolute value of $\overline{a}$. This was necessary, because the legal regulations only allow the use of positive deceleration values, while deceleration rates have negative values in exact physical terms, as are obtained in equations (11), (16) and (18). Moreover substitution of v for v_B in equation (14) is critical, because legal regulations define v to be the testing velocity, whereby v_B as used below means the velocity at the start of the evaluation period.

1) The anticipated legislative text in ECE-R13 is based on this equation.

This paper further uses in its theoretical part and while discussing analytical functions (see chap. 4.1) the physically precise denotation. But all deceleration rates and curves based on braking tests have furtheron positive values as it is usual in every-day measurement practice. Index "E" is used to designate all values at the end of the evaluation period and not at the end of a braking manoeuvre to a full stop, so that the descriptions below are generally valid without limitations.

With (16) and (18) the basic equations for discussion of the two methods of determining a mean deceleration are provided. This assumes that at any time suitable values are available in the case of (16) for the velocities v_E and v_B and for the distances s_E and s_B, and in the case of (18) for the velocities v_E and v_B and for the times t_E and t_B, which is not always the case in practice. Often only a recording of the time-dependent deceleration sequence a(t) is available. In such cases it is necessary to find suitable solutions based on (16) and/or (18). Moreover a solution has to be pointed out for evaluation of mean decelerations using approximation.

For derivation of the required solutions equation (8) is integrated within the variable upper limits v(t) and t, and the fixed lower limits v_B and t_B. This results in:

$$\int_{v_B}^{v(t)} dv = \int_{t_B}^{t} a(t) \times dt \qquad (19)$$

or:

$$v(t) = v_B + \int_{t_B}^{t} a(t) \times dt \qquad (20)$$

Integration of (8) within the fixed limits V_E, V_B, T_E and T_B yields:

$$\int_{v_B}^{v_E} dv = v_E - v_B = \int_{t_B}^{t_E} a(t) \times dt \qquad (21)$$

Squaring (21) we obtain:

$$v_B^2 = v_E^2 - 2 \times v_E \int_{t_B}^{t_E} a(t) \times dt + \left(\int_{t_B}^{t_E} a(t) \times dt \right)^2 \qquad (22)$$

Integration of (7) within the fixed limits s_E, S_B, t_E and t_B and utilization of (20) results in:

$$\int_{s_B}^{s_E} ds = \int_{t_B}^{t_E} v(t) \times dt = \int_{t_B}^{t_E} \left(v_B + \int_{t_B}^{t} a(\tau) \times d\tau \right) \times dt \qquad (23)$$

or:

$$s_E - s_B = (t_E - t_B) \times v_B + \int_{t_B}^{t_E} \int_{t_B}^{t} a(\tau) \times d\tau \times dt \qquad (24)$$

Combining (21) and (24) we obtain:

$$s_E - s_B = (t_E - t_B) \times v_E - (t_E - t_B) \times \int_{t_B}^{t_E} a(t) \times dt + \int_{t_B}^{t_E} \int_{t_B}^{t} a(\tau) \times d\tau \times dt \quad (25)$$

With (22) and (25) we obtain from (16) the following equation:

$$a_{ms} = \frac{2 \times v_E \times \int_{t_B}^{t_E} a(t) \times dt - \left(\int_{t_B}^{t_E} a(t) \times dt \right)^2}{2 \times \left[(t_E - t_B) \times v_E - (t_E - t_B) \times \int_{t_B}^{t_E} a(t) \times dt + \int_{t_B}^{t_E} \int_{t_B}^{t} a(\tau) \times d\tau \times dt \right]} \quad (26)$$

To express v_E by a(t) in (26), we derive the following from (8) analogous to (21):

$$\int_{v_E}^{o} dv = 0 - v_E = \int_{t_E}^{t_S} a\,(t) \times dt \quad (27)$$

or:

$$v_E = - \int_{t_E}^{t_S} a(t) \times dt \quad (28)$$

whereby t_S is the time at the end of a stop.

With (28) we obtain the following from (26):

$$a_{ms} = \frac{2 \times \int_{t_E}^{t_S} a(t) \times dt \times \int_{t_B}^{t_E} a(t) \times dt + \left(\int_{t_B}^{t_E} a(t) \times dt \right)^2}{2 \times \left[(t_E - t_B) \times \int_{t_B}^{t_s} a(t) \times dt - \int_{t_B}^{t_E} \int_{t_B}^{t} a(\tau) \times d\tau \times dt \right]} \quad (29)$$

If t_E for a full stop braking manoeuvre is very close to t_S and since $v_E \approx 0$ in these cases (29) as well as (26) can be simplified to:

$$a_{ms} = \frac{\left(\int_{t_B}^{t_E} a(t) \times dt \right)^2}{2 \times \left[(t_E - t_B) \times \int_{t_B}^{t_E} a(t) \times dt - \int_{t_B}^{t_E} \int_{t_B}^{t} a(\tau) \times d\tau \times dt \right]} \quad (30)$$

Equations (29) and (30) provide the solutions sought, allowing calculation of a distance-related mean for time-dependent deceleration sequences a(t).

2.3 Evaluation procedures

For the numerical determination of a_{ms} by help of computers a value a_{msN} can be derived from individual deceleration data a_i, registered at the time increments Δt, using the formula in (30):

$$a_{ms} = \frac{\frac{1}{2} \times \left(\sum_{i=B+1}^{E} \frac{a_{i-1} + a_i}{2} \times \Delta t \right)^2}{(t_{E} - t_B) \times \sum_{i=B+1}^{E} \frac{a_{i-1} + a_i}{2} \times \Delta t - \sum_{i=B+1}^{E} \sum_{j=B+1}^{i} \frac{a_{j-1} - 1 + a_j}{2} \times \Delta t^2 + \sum_{i=B+1}^{E} \frac{a_{i-1} + a_i}{4} \times \Delta t^2} \quad (31)$$

In practical situations deceleration diagrams often must be evaluated graphically. Here the equation for a straight line is taken for approximation:

$$a_L(t) = \frac{a_E - a_B}{t_E - t_B} \times t + \frac{a_B \times t_E - a_E \times t_B}{t_E - t_B} \quad (32)$$

whereby a_E and a_B are the deceleration values belonging to the times t_E and t_B compatible with the equation for a straight line and therefore positioned on that line. They should not be confused with these values associated with these times on the deceleration curve a(t).

By this procedure the common method of approximation will be simulated drawing the most accurately adapted line through a given time-dependent deceleration curve by means of visual estimation. The distance-related approximation can be obtained from (30) with (32):

$$\tilde{a}_{mt} = \frac{3}{4} \times \frac{(a_E + a_B)^2}{2 \times a_E + a_B} \quad (33)$$

With (32) the time-related approximation can be obtained from (17):

$$\tilde{a}_{mt} = \frac{a_E + a_B}{2} \quad (34)$$

Summarizing, it can be stated that for comparable considerations of time- and/or distance-related evaluation methods, the equations (16), (18), (29), (30), (31), (33) and (34) can be applied.

Equations (16) and (18) are generally valid. (29) is the exact solution for the distance-related mean deceleration based exclusively on time-dependent deceleration measurements. Based on equation (29), equation (30) represents an approximation solution for the distance-related mean deceleration. Equation (31) permits the numerical evaluation of the distance-related mean deceleration of measured deceleration curves on the basis of equation (30). Equations (33) and (34) are graphical approximation solutions, whereby (33) constitutes the distance-related and (34) the time-related mean value.

3 Evaluation Limits for the Determination of the Mean Fully Developed Deceleration

Since the transition from the initial braking phase to that of fully developed deceleration, and the transition from the fully developed deceleration to the stop, occur continuously, the selection of evaluation limits, that means the selection of t_B, s_B, v_B, a_B and t_E, s_E, v_E, a_E respectively, cannot be strictly objective or unequivocal.

Due to this background, the most diversified evaluation limits have been discussed in the different technical panel meetings. Essentially, the evaluation limits illustrated in the following Figures 5 and 6 have resulted from such discussions. To facilitate further discussions, chapter identifications have been attributed to the limits.

The possibilities for the evaluation limits are:

3.1 Evaluation limits in connection with the velocity signal

3.1.1 $t_B = t(0{,}8 \times v_0)$ for start of the evaluation range and
$t_E = t(0{,}1 \times v_0)$ for its completion [2)]

3.1.2 $t_B = t(0{,}9 \times v_0)$ for start of the evaluation range and
$t_E = t(0{,}05 \times v_0)$ for its completion.

3.2 Evaluation limits in connection with the deceleration signal (see figure 6)

3.2.1 $t_B = t_1\,(1/2\ a_{max}) + 0{,}3$ s for start of the evaluation range and
$t_E = t_2\,(1/2\ a_{max}) - 0{,}1$ s for its completion.

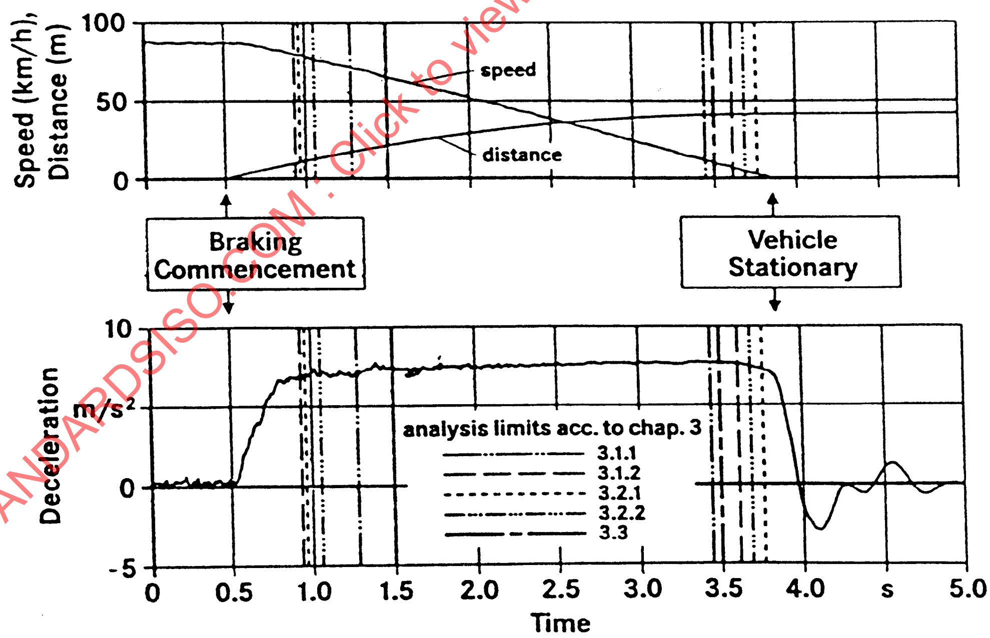

Figure 5 - Analysis limits drawn in time signals of a braking test

2) These evaluation limits are dicated in the anticipated legislative text in ECE R13

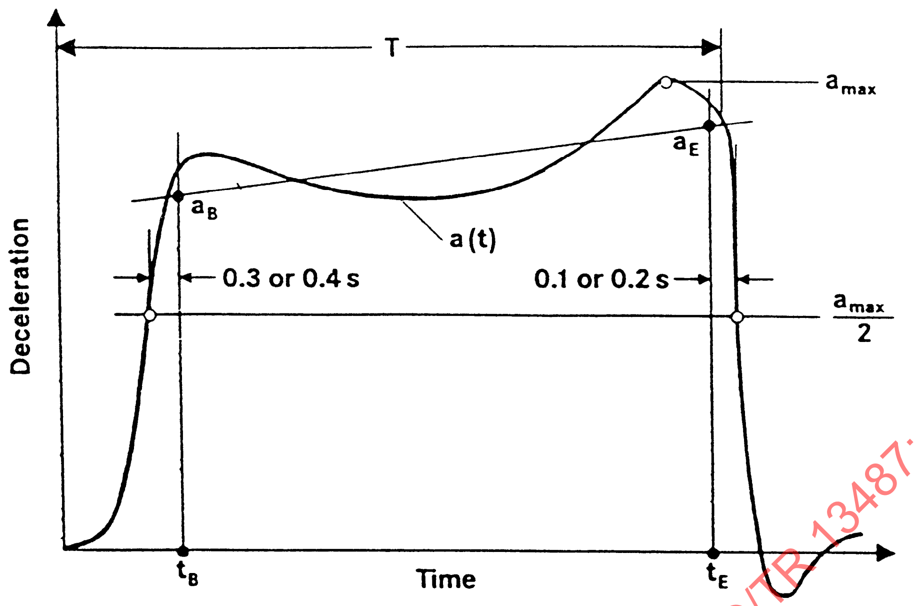

Figure 6 - Determination of the evaluation limits in connection with a time-dependent deceleration path

3.2.2 $t_B = t_1\,(1/2\ a_{max}) + 0{,}4$ s for start of the evaluation range and
$t_E = t_2\,(1/2\ a_{max}) - 0{,}2$ s for its completion.

3.3 Evaluation limits in connection with the total braking time

$t_B = 0{,}3$ T for start of the evaluation range and
$t_E = 0{,}9$ T for its completion.

3.4 Evaluation of limits by engineering judgement

Limits are evaluated based on an engineering assessment. The start of the evaluation period lies at the point at which a clear transition from deceleration increase to fully developed deceleration occurs. The end of the evaluation period lies at the point at which the fully developed deceleration is succeeded by a sharp drop.

The choice of the evaluation limits should be in conjunction with the choice of the equation used for the determination of mfdd; in other words, with the time signals employed to determine mfdd.

If the evaluation of mfdd is to be carried out in respect to equation (16), it is advantageous to choose the evaluation limits according to 3.1.1 or 3.1.2, since information regarding the time-dependent velocity curve must be available for the evaluation.

If however, equation (31) or (33) is chosen, the information regarding the time-dependent deceleration curve will be fully sufficient, so that, logically speaking, the evaluation limits mentioned in 3.2.1 or 3.2.2 are to be used.

The evaluation limits according to 3.3 can be used for all evaluation procedures. These limitations defined over the entire braking time have a certain disadvantage, since additional information will be necessary regarding the points in time of braking commencement and complete standstill respectively, which in turn means additional measuring efforts. As Figure 5 illustrates, these evaluation limits define in fact the smallest representative evaluation interval too.

The evaluation limits mentioned in 3.2.1 describe the largest evaluation interval, whereby however, the termination of the evaluation is placed shortly before the point in time of vehicle standstill. During excessively damped pitch motions or accelerometer characteristics the slope of the accelerations curve is planar as shown in Figure 5 which brings about the fact, that due to the definition of t_E, the limit will be set shortly after vehicle standstill. Therefore, the decelerations attained during the test run, with conditions mentioned in 3.2.2, are to be considered as the more suitable alternative.

The limits relating to the vehicle speed according to 3.1, cause difficulties to be able to accurately determine the defined final value 0,05 x v_0 during practical testing and evaluation procedures, when the data transducers do not exhibit corresponding measurement and resolution accuracies for low vehicle speeds.

What influence the different evaluation limits have on the mean deceleration values will be discussed in chapter 4 where the different evaluation procedures aided by analytical functions and by measurements recorded during driving tests are judged.

4 Application

4.1 Analytical functions

The essential equations for a comparative discussion of the methods for determining the distance-related and time-related mean values were described in the preceding chapters. In this section a comparison is to be performed by means of analytical functions, which allow recognition of the influence of the various evaluation methods in precise mathematical form. If only real test results were used for the judgement of the methods, the inevitably unavoidable measurement errors would also affect the judgement of the methods. This will be dealt with in chapter 4.2.

Deceleration curves (Figure 7) which can be described by the following polynomials of the tenth order, are used for comparison of equations (16), (18), (29), (30), (31), and (33):

$$a_F(t) = \sum_{i=2}^{10} a_{Fi} \times t^i \qquad (35)$$

$$a_R(t) = \sum_{i=2}^{10} a_{Ri} \times t^i \qquad (36)$$

with:

Table 1 - Coefficient of the above mentioned polynomials

i	a_{Fi}	a_{Ri}
2	- 74,910577	- 42,423426
3	153,60554	76,923847
4	- 144,35635	- 61,932803
5	77,628303	27,761645
6	- 25,629472	- 7,3937696
7	5,2783113	1,1581775
8	- 6,6020059 x 10^{-1}	- 9,6665526 x 10^{-2}
9	4,5796351 x 10^{-2}	2,9506892 x 10^{-3}
10	- 1,3477421 x 10^{-3}	4,4239856 x 10^{-5}

The range of variation for the deceleration described by these functions is certainly sufficient to cover even worst-case situations in practice. The high order of the polynomials was necessary to obtain various maximums and minimums in the curves. In particular it was necessary to ensure that the curves start at zero with a horizontal tangent and end again very abruptly at zero for simulation of actual situations. The high order of the polynomials required extended accuracy in the computer evaluation.

Figure 7 also illustrates the time-dependent velocity and braking distance curves obtained by integration. Curve $a_F(t)$, described by equation (35), was selected in such a manner that the final velocity of zero is reached after t_S = 6 s from an initial velocity of v_0 = 36,47 m/s, with a braking distance of s_F = 102,05 m. This curve indicates a decreasing trend over its entirety. Correspondingly for curve $a_R(t)$, described by equation (36), are t_S = 6 s, v_0 = 35,86 m/s and s_R = 121,25 m. This curve shows a primarily increasing trend.

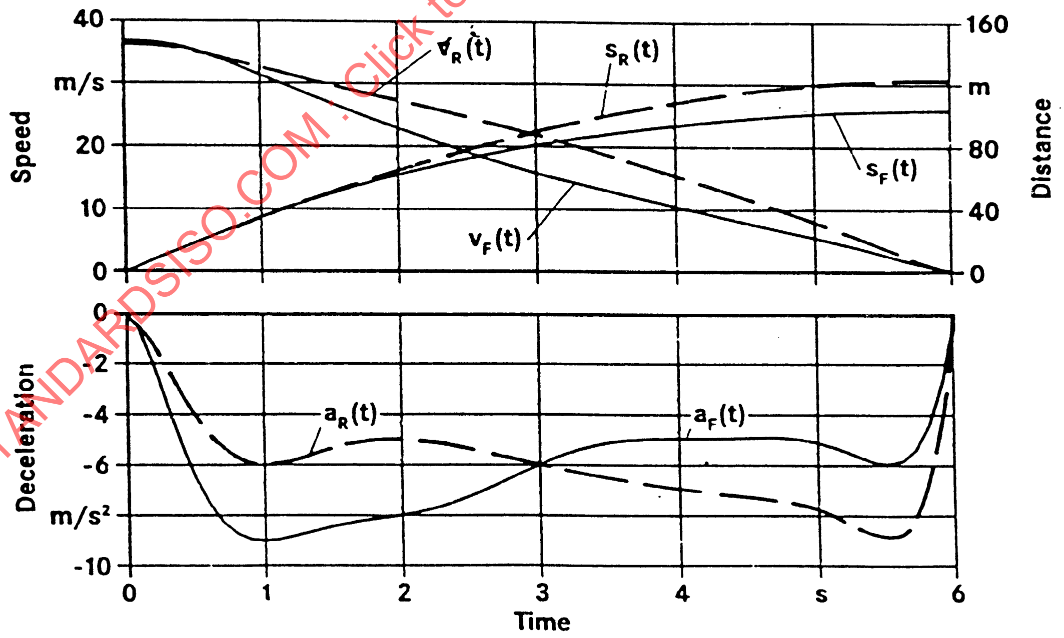

Figure 7 - Analytical functions simulating two different braking processes

To provide the marginal conditions required for evaluation the mean values according to the equations (16), (18), (29), (30), (31), and (33) the conditions 3.1.1, 3.1.2 and 3.2.1 were selected from chapter 3.

Using these marginal conditions and the separate time-dependent values a_i derived from equations (35) and (36) for the deceleration curves, v_i for the velocity curves and s_i for the braking distance curves, the marginal values were calculated for v_B, v_E, t_B, t_E, s_B and s_E listed in Table 2.

Table 2 - Marginal values

Deceleration curve		a_F (t)			a_R (t)		
Evaluation limit		3.1.1	3.1.2	3.2.1	3.1.1	3.1.2	3.2.1
v_B	m/s	29,18	32,82	33,85	28,69	32,27	33,16
v_E	m/s	3,65	1,82	1,01	3,59	1,79	0,87
t_B	s	1,19	0,78	0,66	1,64	1,00	0,85
t_E	s	5,25	5,56	5,70	5,46	5,67	5,79
s_B	m	40,32	27,53	23,52	54,11	34.,65	29,74
s_E	m	100,88	101,73	101,94	120,44	121,01	121,18
a_B	m/s^2	- 8,41	- 8,68	- 8,69	- 4,65	- 4,71	- 4,80
a_E	m/s^2	- 4,19	- 4,30	- 4,32	- 8,50	- 8,38	- 8,28

Here a_i, v_i and s_i were determined for time increments of $\Delta t = 0,1$ s. The marginal values listed in the first six lines were obtained from a_i, v_i and s_i by interpolation, if they could not be derived directly from the marginal conditions. To determine a_B and a_E first the linear approximation curves were calculated for the individual deceleration values a_i within the time interval t_B to t_E according to the Gaussian method of smallest error distribution. However, to get the final results for a_B and a_E no interpolation method was used. Corresponding to the selected time increment Δ t values rounded off to 0,1 s were used for t_B and t_E and these values were inserted in the calculated linear equation.

The results of the different calculation methods are compared in Table 3.

Table 3 - Comparison of different calculation methods

line	deceleration curve		a_F (t)			a_R (t)		
	evaluation limit		3.1.1	3.1.2	3.2.1	3.1.1	3.1.2	3.2.1
1	$\frac{v_E^2 - v_B^2}{2 \times (s_E - s_B)}$	m/s²	- 6,92	- 7,24	- 7,30	- 6,11	- 6,01	- 6,01
2	integration, see eq. (30)	m/s²	- 7,12	- 7,34	- 7,35	- 5,99	- 5,96	- 5,98
3	$\frac{L2 - L1}{L1}$	%	+ 2,9	+ 1,4	+ 0,7	- 2,0	- 0,8	- 0,5
4	sum, see eq. (31)	m/s²	- 7,09	- 7,32	- 7,34	- 5,97	- 5,96	- 5,99
5	$\frac{L4 - L1}{L1}$	%	+ 2,5	+ 1,1	+ 0,5	- 2,3	- 0,8	- 0,3
6	$\frac{3\,(a_E + a_B)^2}{4\,(2 \times a_E + a_s)}$	m/s²	- 7,10	- 7,32	- 7,33	- 5,99	- 5,99	- 6,01
7	$\frac{L6 - L1}{L1}$	%	+ 2,6	+ 1,1	+ 0,4	- 2,0	- 0,3	0,0
8	$\frac{3\,(a_E + a_B)^2}{4\,(2 \times a_E + a_B)}$	m/s²	- 6,99	- 7,16	- 7,12	- 5,98	- 6,04	- 6,05
9	$\frac{L8 - L6}{L6}$	%	- 1,5	- 2,2	- 2,9	- 0,2	+ 0,8	+ 0,7
10	δ - deviation of 11 persons	%	+ 2,1	+ 2,9	+ 2,0	+ 0,8	+ 1,8	+ 2,0
11	$\frac{v_E - v_B}{t_E - t_B}$	m/s²	- 6,29	- 6,49	- 6,52	- 6,57	- 6,53	- 6,54
12	$\frac{L11 - L1}{L1}$	%	- 9,1	- 10,4	- 10,7	+ 7,5	+ 8,7	+ 8,8

Line 1 lists the exact mfdd values according to equation (16). A check with equation (29) using equations (35) and (36) provides the same results as expected. This applies only approximately for the values in line 2, which reflect analogous evaluation according to equation (30). The differences between lines 1 and 2 are understandable, since equation (30) represents an approximation in relation to equation (29), because t_S is set equal to t_E and it is assumed that $v_E = 0$.

Line 3, demonstrating the deviation between line 1 and line 2 was calculated in percent, clearly shows that the deviation becomes significantly smaller as t_E approaches t_S in the sequence of marginal conditions 3.1.1, 3.1.2 and 3.2.1. This applies accordingly for lines 4 and 6, which deviate from line 2 only slightly.

Line 4 lists the results according to equation (31) offering a possibility for computer evaluation of deceleration curves actually measured. Even line 6, which lists approximated values according to equation (33), clearly shows that in principle this method is also permissible for worst-case situations.

Line 8 indicates whether this can also be expected in practical situations. The results in this line were obtained by having eleven test persons graphically evaluate independently of one another the deceleration curves given in Figure 7 with their differing marginal conditions 3.1.1, 3.1.2 and 3.2.1 by means of linear approximation.

Line 9, which lists the deviation in percent of the values actually expected in line 6, and line 10, which indicates the range of variation from one test person to another in percent, shows an irrefutable increase in the error in comparison to the previous results.

Line 11 summarizes the results for the time-related mean deceleration according to (18). Line 12 with values deviating from the exact values in line 1 by more than 10 %, underscores the fallacy of a time-related definition of mfdd.

Remembering that in line 3 the reduction in the error was already significant in the sequence 3.1.1, 3.1.2, 3.2.1, it appears logical to give marginal conditions 3.1.2 and 3.2.1 preference. This also has the advantage that highly compatible results can be obtained with highly differing evaluation methods. Even the fact that the conditions selected under 3.1.2 for the velocities v_B and v_E do not indicate any logically compulsory and mathematically justifiable relationship to the conditions selected under 3.2.1 for the times t_B and t_E, does not lead to grave deviations in lines 1, 2, 4, 6 and 8.

Summarized it can be said in terms of the method used that evaluation of mfdd according to equations (16), (29), (30), (31) and (33), preferably under marginal conditions 3.1.2 and 3.2.1, is permissible. The following chapters show that this conclusion also applies for practical situations.

4.2 Measurements

The last section showed the methodically conditioned influences of the different evaluation procedures with the support of synthetically generated analytical functions. In contrast to this, the following chapter will exemplify attainable results on the basis of real measurements acquired with the different evaluation methods, so that, besides the methodically conditioned influence, the measurement and evaluation errors can be additionally considered as they occur in practice.

The procedure consisted of braking tests performed with a 2,5 t van. The investigations were exclusively carried out below the wheel-locking threshold. It is advisable to take the evaluation results arising from equation (16) as basis of comparison for the other evaluation methods, since equation (16) is based on the directly measurable quantities "speed" and "distance" and it thereby offers physically exact results. But only then accurate results are rendered, when extremely precise speed and distance measurements have been recorded. Therefore, a statement regarding the precision, with which the braking distance can be determined, has to be placed before the discussion of the evaluation methods.

To be able to formulate statements regarding the evaluation methods in a general valid manner, one had to generate within the tests deceleration curves, deviating from each other. In this way it is possible to demonstrate the dependence of the mean values results on the different evaluation procedures. So that all variations of averaging based on different information acquired during field testing, can be directly compared to each other, the main objective of these tests was to simultaneously acquire, with a high level of precision, all physical values necessary for the different evaluation methods during one test run.

4.2.1 Measurement equipment of the test vehicle

The van was equipped with the following transducers:

– fifth wheel with 500-pole ring gear ;
– correlating optical speed transducer ;
– and a gyroscope measuring platform with an acceleration pick-up element.

With the aid of the fifth-wheel the stopping distance and vehicle speed were determined. The speed signal of the correlating optical transducer rendered through subsequent integration the stopping distance as well. The acceleration pick-up measured horizontal vehicle deceleration values.

In addition, the stop light switch of the vehicle was used as information transducer for the braking commencement and to trigger a cartridge filled with paint. The cartridge, after being fired, marked the braking commencement on the road surface. So it was possible to accurately measure the distance travelled by the vehicle up to its standstill with a tape-measure.

The data were correspondingly amplified and stored on magnetic tape. Such a test has already been shown in Figure 5. At this point it must be added, that the oscillations appearing in the deceleration signal after vehicle standstill, are caused by the pitching motion of the vehicle executed when it returns to its original static state of equilibrium.

4.2.2 Reproducibility of stopping distance measurements

The measurement of the stopping distance plays a major role in legislation, because in ECE Regulation No. 13 it will not only be used for the judgement of vehicle braking performance, but will be employed in future times for the determination of mfdd according to equation (16). In this way the quality of stopping distance determination appears as an immediate connection to the topic discussed in this paper. A comparison of certain stopping distances determined by different methods during a series of seven tests is illustrated in Table 4. The vehicle was decelerated in a range of 6,5 to 7,0 m/s² from a test speed of 80 km/h on a dry asphalt road surface.

Table 4 - Stopping distances measured by different methods

Test no	Stopping distances			Relative error	
	Tape-measure (m)	Optical speed transducer (m)	Fifth wheel (m)	Optical speed transducer (%)	Fifth wheel (%)
A14	39,46	39,60	39,40	+ 0,35	- 0,15
A15	38,41	38,34	38,34	- 0,18	- 0,18
A16	38,54	38,54	38,43	0,00	- 0,29
A17	37,91	37,94	37,85	+ 0,08	- 0,16
A18	40,93	41,09	40,89	+ 0,39	- 0,.10
A19	40,06	40,10	39,94	+ 0,10	- 0,30
A20	41,07	41,13	40,96	+ 0,15	- 0,27

The column "tape-measure" in Table 4 lists the manually measured stopping distances of the vehicle. To the stopping distance values, the travelling distance was added, which the vehicle transgressed during the so-called lag time period of the paint cartridge. This lag time period starts once the stop light switch has been triggered at braking commencement and ends at that point in time when the paint has met the road surface. The corresponding distance was determined in a test with the aid of a road-fixed trigger mechanism for the paint cartridge charge. The distance amounted to 0,25 m for a test speed of 80 km/h.

The column "optical speed transducer" contains the stopping values determined by integration of the correlation optical speed signal, the column "fifth-wheel" demonstrates the stopping distances determined with the aid of the fifth-wheel. The impulses triggered by the ring gear were counted, starting at braking commencement up to vehicle standstill and corresponding to a previously recorded calibration value, mathematically converted into a distance travelled by the vehicle. Since the fifth-wheel was attached to the rear end of the vehicle, from the above-mentioned distance was subtracted that portion which the fifth-wheel has travelled additionally by pitching motions of the vehicle, due to its geometric lay-out. If the manually measured stopping distance has been predetermined to be the absolutely accurate stopping distance value, then Table 4 demonstrates that the mean stopping distance is 0,1 % longer when determined with the optical speed transducer and in turn 0,2 % shorter when determined with the fifth-wheel. For both systems the maximum occurring error for the test series lies clearly below 0,5 %.

4.2.3 Evaluation of the mean fully developed deceleration

Out of the entire amount of field tests five characteristic deceleration curves were chosen (Figure 8). Aside from the vehicle specific deceleration curves (tests A16, A20 in Figure 8) artificial deceleration curves were generated with sharply dropping slopes. This was performed by the driver, in that the brake pedal was depressed as fast as possible and, while the vehicle was decelerating, the brake pedal was again released very slowly. The results being the deceleration curves of tests A22, A26 and A30. The different evaluation methods were applied to these braking procedures characterized by the deceleration curves, shown in Figure 8.