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Measurement of liquid flow in open channels — Mixing length of a tracer

Mesure de débit des liquides dans les canaux découverts — Longueur de bon mélange d'un traceur

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Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of technical committees is to prepare International Standards, but in exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 11656, which is a Technical Report of type 3, was prepared by Technical Committee ISO/TC 113, *Measurement of liquid flow in open channels*, Sub-Committee SC 4, *Dilution methods*.

Annexes A, B and C of this Technical Report are for information only.

Introduction

A variety of formulae have been developed for estimating mixing length in open channels. Some of these formulae have been developed for special flow conditions.

Most mixing-length formulae have been developed for injection of a tracer at the centre of flow. Mixing theory will also allow these formulae to be used for the injection of a tracer at one edge of the flow. However, there are times when, for a variety of reasons, a tracer is injected at a point other than the centre or edge of flow. Also, the tracer may be injected from a line source or from a multiple-point source. Thus, a mixing-length formula is needed that can estimate the mixing length for different injection situations.

Mixing-length formulae are generally developed for a condition which assumes complete mixing. However, an examination of the mixing process indicates that an infinite distance is required for theoretically complete mixing (100 %). If this theory is correct, then the existing mixing-length formulae only approximate complete mixing. Experience shows that satisfactory flow measurements can be made with less than complete mixing. Special methods can be used to minimize errors resulting from measurements at considerably less than complete mixing.

For these reasons, it is important to provide an objective means of defining the degree of mixing, and to estimate the mixing distance associated with various specified degrees of mixing.

These results may lead to the elaboration of a future International Standard.

Measurement of liquid flow in open channels — Mixing length of a tracer

1 Scope

This Technical Report investigates cross-channel mixing characteristics of solutes injected in streams. Specifically, it relates to the use of tracers for the measurement of discharge. A tracer must be well mixed, or compensating measures taken, in order to obtain a satisfactory dilution-type discharge measurement.

The purposes of this Technical Report are as follows:

a) to compare methods of defining the degree of mixing of a solute in a stream and to recommend a method;

b) to compare methods of estimating the mixing length (the downstream distance required for a solute to thoroughly mix across a stream) and to recommend a particular method;

c) to investigate the errors in dilution measurements associated with incomplete mixing;

d) to discuss methods of reducing errors in dilution-discharge measurements when mixing is incomplete.

2 Reference

The following standard contains provisions which, through reference in this text, constitute provisions of this Technical Report. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this Technical Report are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 772:1988, *Liquid flow measurement in open channels — Vocabulary and symbols.*

3 Definitions

For the purposes of this Technical Report, the definitions given in ISO 772, except where noted, and the following definitions apply.

3.1 complete mixing: Mixing which occurs at a channel section, when the constant-injection method is used, if the steady-state tracer concentrations are equal at all points in the cross-section. Similarly, for the sudden injection method, mixing which occurs if the areas under the time/concentration curves are equal at all points sampled in a section.

3.2 degree of mixing: Measure of the extent to which mixing has been achieved in a cross-section downstream from the injection of a tracer.

The degree of mixing may vary from nearly 0 % in a cross-section immediately downstream from the injection to 100 % at a cross-section in which the tracer has been completely mixed across the entire cross-section.

3.3 mixing length: Distance, measured along the general path of flow between the injection cross-section and the downstream cross-section, at which the specified degree of mixing is obtained.

For given conditions, the mixing length is not a fixed value. It varies according to the specified degree of mixing. The higher the specified degree of mixing, the longer the mixing length.

4 Units of measurement

The units of measurements used in this Technical Report are those of the International System (SI).

5 Determination of the degree of mixing

5.1 Criteria and concepts

Determination of the degree of mixing should be readily conceptualized. It should also provide a unique value for the degree of mixing which can be rationally related to the mixing observed in the channel.

The values describing the degree of mixing range from 0 to 100 %. Where mixing has just begun, near the injection source, the degree of mixing approximates 0, and where mixing is complete, it reaches 100 %. If a tracer has been injected such that it is completely mixed in half of the flow and is not mixed at all in the other half, the degree of mixing is 50 %. The concept should hold for conditions where the tracer is fully mixed in other specified parts of the flow and is not mixed at all in the remaining flow.

For a selected downstream cross-section, the degree of mixing is determined by using the areas under N curves of concentration as a function of time for the sudden injection method, or by using N concentration values on the plateau for the constant-injection method. The areas of the concentrations must be related to some cross-sectional flow characteristic. Because of the mass balance of tracer, the appropriate characteristic is cumulative discharge, or relative cumulative discharge, measured from one edge of flow. The preferred index is the relative cumulative discharge, ranging in value from 0 to 1. Width or other cross-section characteristics vary from one cross-section to another, and are not usually adequate for accounting for the mass balance of tracer.

5.2 Formulae defining the degree of mixing

Various formulae have been proposed for defining the degree of mixing. Five such formulae are presented below.

Coefficient of variation (see [1], page 6 and [7], page 1073)

$$M_{\mathrm{CV}} = \frac{s_C}{\overline{C}} \times 100$$

$$= \frac{\left[N \sum_{i=1}^{N} C_i^2 - \left(\sum_{i=1}^{N} C_i \right)^2 \right]^{1/2}}{N\overline{C}} \times 100 \qquad \ldots (1)$$

Rimmar equation (see [1], page 6)

$$M_{\mathrm{R}} = \left[\frac{\hat{C} - \overline{C}}{\overline{C}} \right] \times 100 \qquad \ldots (2)$$

Schuster equation (see [1], page 6 and [8], page 134)

$$M_{\mathrm{S}} = \left[1 - \frac{\sum_{i=1}^{N} | C_i - \overline{C} |}{N\overline{C}} \right] \times 100 \qquad \ldots (3)$$

Cobb-Bailey equation (see [9], page C5 and [5], page 48)

$$M_{\mathrm{CB}} = \left[1 - \frac{1}{2} \int_{q/Q=0}^{1} \left| \frac{C - \overline{C}}{\overline{C}} \right| d(q/Q) \right] \times 100 \qquad \ldots (4)$$

or, in discrete form

$$M_{\mathrm{CB}} = \left\{ 1 - \frac{1}{2} \sum_{i=1}^{N} \left[\left| \frac{C_i - C}{\overline{C}} \right| \Delta(q/Q) \right] \right\} \times 100 \qquad \ldots (5)$$

Graphic (Cobb and Bailey, communication)

$$M_{\mathrm{G}} = \left(\frac{A}{A + B} \right) \times 100 \qquad \ldots (6)$$

Definition of symbols

In the above equations:

M is the degree of mixing;

s_C is the standard deviation of the concentrations observed in a section;

C is the observed tracer concentration; this is the steady-state concentration observed at a selected cross-section for the constant-injection method, or the area under the time/concentration curve for the sudden injection method;

C_i is the discrete value of C at the ith observation point across the channel;

$\overline{C}$ is the average concentration in the channel cross-section;

$\hat{C}$ is the concentration observed in the cross-section having the greatest departure from the average concentration $\overline{C}$;

N is the number of observation points across a section; the observations are taken at the centre of equal increments of flow;

Q is the total stream discharge;

q is the cumulative discharge at any point in a channel cross-section; the value of q is 0 at one bank and Q at the opposite bank;

A and B are the areas associated with the cross-sectional distribution of tracer (see figure 1).

The characteristics of the various definitions of mixing may best be seen by looking at a number of examples. The examples A, B, C and D which follow assume that the concentration was observed at 10 points across the section. The assumed concentration distributions are shown in figure 2. The degrees of mixing computed by the various formulae are shown in table 1.

The concentration distributions shown in figure 2 are idealized distributions which can be approximated by a line injection across a part of a section. The distributions shown in figure 2 are used to demonstrate various characteristics of the formulae.

Table 1 — Degree of mixing computed by application of the various equations to the concentration distributions shown in figure 2

Values in percentage

Equation	Degree of mixing, M, for example A	B	C	D
(1)	200	100	50	0
(2)	400	100	− 100	0
(3)	− 60	0	60	100
(5)	20	50	80	100
(6)	20	50	80	100

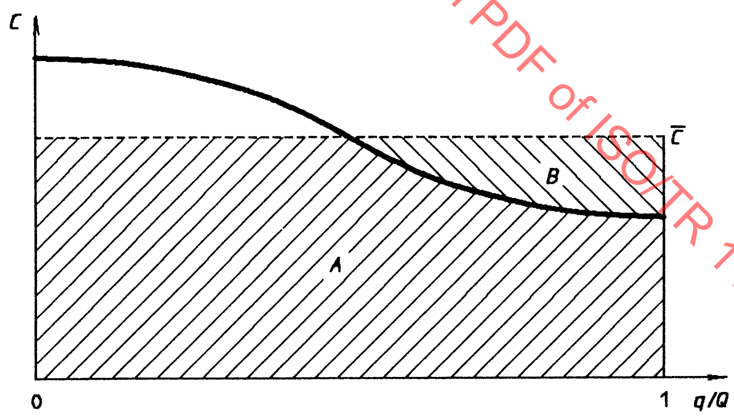
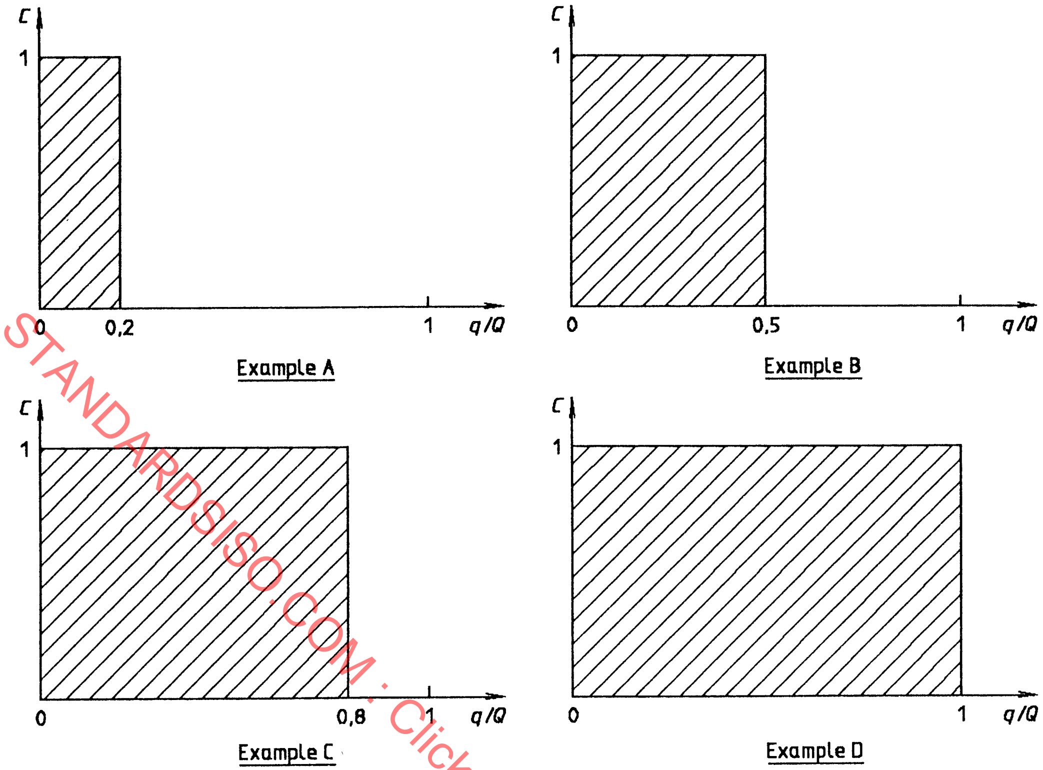


Figure 1 — Graphic description of the degree of mixing



Example	q/Q	Concentration observed, C, for an average concentration, $\bar{C}$, across a section of									
		0,05	0,15	0,25	0,35	0,45	0,55	0,65	0,75	0,85	0,95
A	0,2	1,0	1,0	0	0	0	0	0	0	0	0
B	0,5	1,0	1,0	1,0	1,0	1,0	0	0	0	0	0
C	0,8	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	0	0
D	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0

Figure 2 — Hypothetical concentration distributions across a section (constant-injection method)

A number of observations can be made from this exercise. Equations (1) and (2) can give values which exceed 100 %. Equation (3) gives negative values for low degrees of mixing. Equation (2) can have negative values for some degrees of mixing.

The definition of the coefficient of variation [equation (1)] appears to be uniquely valued but shows an inverse relation with the degree of mixing. That is, the lower the computed value, the higher the degree of mixing.

The Cobb-Bailey formula [equations (4) and (5)] and the graphic formula [equation (6)] give identical numerical values. In fact, the Cobb-Bailey formula was developed to be consistent with the graphic definition. Examination of the concentration distribution curve across the injection point cross-section will reveal by the graphic method that M_G approaches zero. This can also be shown mathematically by the Cobb-Bailey formula. It is seen that only the Cobb-Bailey and the graphic formulae, [equations (4), (5) and (6)] fully meet the recommended criteria.

5.3 Recommended formula

Because of the above characteristics, as revealed in table 1, the definitions of mixing given by Rimmar and Schuster may be discarded. These equations do not define a unique degree of mixing at every condition of mixing.

The coefficient of variation [equation (1)] exhibits an inverse relation with the degree of mixing and may also give values greater than 100 %.

The graphic definition [equation (6)] provides a clear and easy-to-follow conceptual definition. Both the Cobb-Bailey and the graphic formulae [equations (4), (5) and (6)] fit the criteria established earlier and provide identical results. These two definitions thus are recommended for determining the degree of mixing.

In most cases, the discrete form, equation (5), will be necessary. The computed degree of mixing may vary slightly with the number of cross-channel observations.

In a practical situation, the problem arises of how to determine the relative flow, q/Q, across a channel without increasing the number of sub-area and velocity measurements across the channel. At times, information is available for defining the variation of the relative flow across a section. Otherwise, an approximate calculation can be made on the basis of width by substituting relative width in place of relative discharge, q/Q, in equations (4) and (5). Whether the values of q/Q are approximated or not, it is conceptually essential to have a definition that requires each concentration observation to be weighted by flow, because this is the only means of accounting for the mass balance of the tracer.

6 Mixing length

6.1 Concepts

In an open channel, mixing takes place in three directions: vertical, lateral (cross-flow) and longitudinal (parallel to the flow).

Vertical mixing takes place in most open channels relatively quickly if the tracer solution density is near that of water. Lateral mixing approaches the completely mixed condition within a finite distance depending on channel and flow characteristics. Longitudinal mixing continues to take place throughout the length of the channel.

The validity of flow measurement is dependent on adequate vertical and lateral mixing. The distance required for lateral mixing is of primary concern when making dilution-type flow measurements, since vertical mixing usually occurs rapidly.

Complete mixing as defined in 3.1 theoretically never occurs but is approached asymptotically. In a practical sense, complete mixing needs to be defined for a finite distance. For the purposes of this Technical Report, an adequate degree of mixing for most discharge measurements is considered to be 98 % as defined by the Cobb-Bailey equation.

6.2 Methods of estimating mixing length

There are three general approaches to estimating mixing lengths: direct observation, the empirical method and the theoretical method.

6.2.1 Direct observation

Direct observation involves injecting a tracer into a channel and, from measurements, determining the distance required for mixing. The results are generally valid only for that channel and for those flow conditions for which the observations were made.

6.2.2 Empirical method

A number of researchers have used the empirical approach. This involves the fitting of observed data from a limited number of streams to a set of channel or flow characteristics, either by regression analysis or by some other technique. Usually, the number of independent variables is very limited and involves measures of stream width, depth and discharge. The resulting equation is usually valid only for a limited area or type of flow condition.

The empirical formula generally takes the form:

$$X = aB^{b_1}D^{b_2}Q^{b_3}$$

where

X	is the mixing length;
B	is the channel width;
D	is the channel depth;
Q	is the flow rate;
a	is a constant;
b_1, b_2, b_3	are exponents, any one of which can be positive or negative, or in some cases equal to zero.

6.2.3 Theoretical method

The theoretical approach is usually more tedious and an attempt is made to account, in some manner, for all of the significant channel and flow characteristics which may affect the mixing process. The resulting equation for estimating mixing distance should be quasi-universal in application. The assumptions made and the lack of understanding of how some variables affect the mixing process may limit this universality, but this property qualifies the theoretical method as a satisfactory approach for an international standard.

Most theoretical equations have been developed for the injection of tracer at the centre or side of a channel. They provide an approximate value of the distance required for adequate mixing, but seldom specify the actual degree of mixing for which a formula has been developed, which can cause confusion when comparing equations.

6.3 Formulae for estimating mixing length

A review of the literature reveals that a number of relations have been developed to estimate mixing length. The following relations were considered and computational results compared with observed data for a degree of mixing, M, of 98 %.

6.3.1 Empirical relations

André formula

$$X = a_1 B Q^{1/3} \qquad \ldots (7)$$

Day formula

$$X = 25B \qquad \ldots (8)$$

Hull formula

$$X = a_2 Q^{0,33} \qquad \ldots (9)$$

6.3.2 Theoretical relations

Elder formula

$$X = 10 \frac{UD}{U_*} \qquad \ldots (10)$$

Fischer formula

$$X = K_1 \frac{UB^2}{E} \qquad \ldots (11)$$

Rimmar formula

$$X = 0{,}13B^2 \times \frac{C(0{,}7C + 2g^{1/2})}{gD} \qquad \ldots (12)$$

USSR formula

$$X = \frac{B_{mC}^2}{16\alpha_y D g^{1/2} \ln(1 - \alpha_c)} \qquad \ldots (13)$$

Ward formula

$$X = K_2 \frac{B^2}{0{,}02D} \qquad \ldots (14)$$

Yotsukura formula

$$X = \frac{1}{2\alpha^2\beta} \times \frac{UD^2}{\overline{uy}^2} \times \frac{U}{U_*} \times \frac{B^2}{D} \qquad \ldots (15)$$

6.3.3 Definition of symbols

In the formulae in 6.3.1 and 6.3.2:

X is the mixing distance, in metres;

a_1 is a constant varying from 8 to 28; use 10 for most conditions but a higher value for very steep turbulent streams;

a_2 is a constant, equal to 150 for centre injection and 600 for side injection;

B is the average stream surface width between injection and sampling sites, in metres;

B_m is the distance between the point of injection and the more distant bank, in metres;

C is the Chezy coefficient;

D is the average stream depth between injection and sampling sites, in metres;

E is the transverse mixing coefficient, in square metres per second;

g is the acceleration of gravity ($g = 9{,}807$ m/s^2);

K_1 is a constant, equal to 0,1 for centre injection and 0,4 for side injection;

K_2 is a variable related to the degree of mixing, M, and the injection site (see figure 3);

Q is the discharge, in cubic metres per second;

S is the water-surface slope;

u is the velocity in a stream segment, in metres per second;

U is the mean stream velocity between the injection and sampling sites, in metres per second;

U_* is the shear velocity [$U_* = (gDS)^{1/2}$], in metres per second;

$\frac{(UD^2)}{(\overline{uy}^2)}$ is a ratio that is usually determined as a whole and ranges in value from 0,3 to 0,9, with a value of 0,6 used for most streams;

y is the water depth in a stream segment, in metres;

α is a distance parameter which is a function of the degree of mixing, M, and the point of injection (see figures 4 and 5);

α_c is a coefficient for the degree of tracer concentration variability at the site of sufficiently complete mixing, and varies from 0,15 to 0,20;

α_y is a coefficient varying from 0,23 to 0,25;

β is a coefficient ranging from 0,2 to 0,3 for straight channels up to 0,6 for channels with minor bends; a value of 0,2 is generally recommended for conservative estimates of the mixing length.

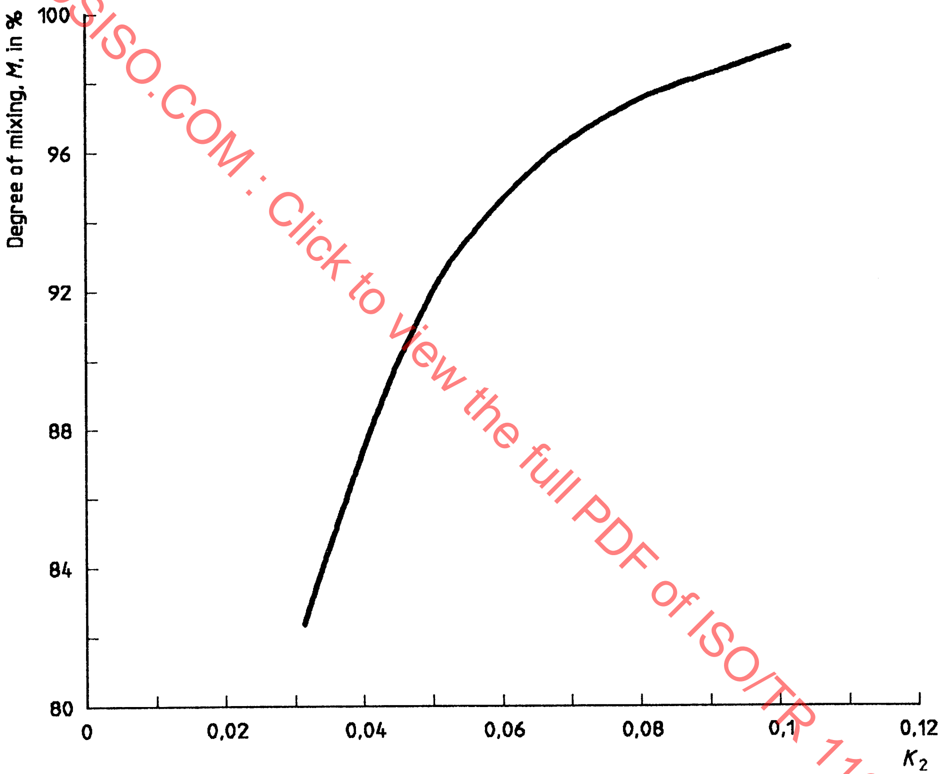


Figure 3 — Relationship between K_2 in the Ward mixing-length formula and the degree of mixing for centre injection of tracer

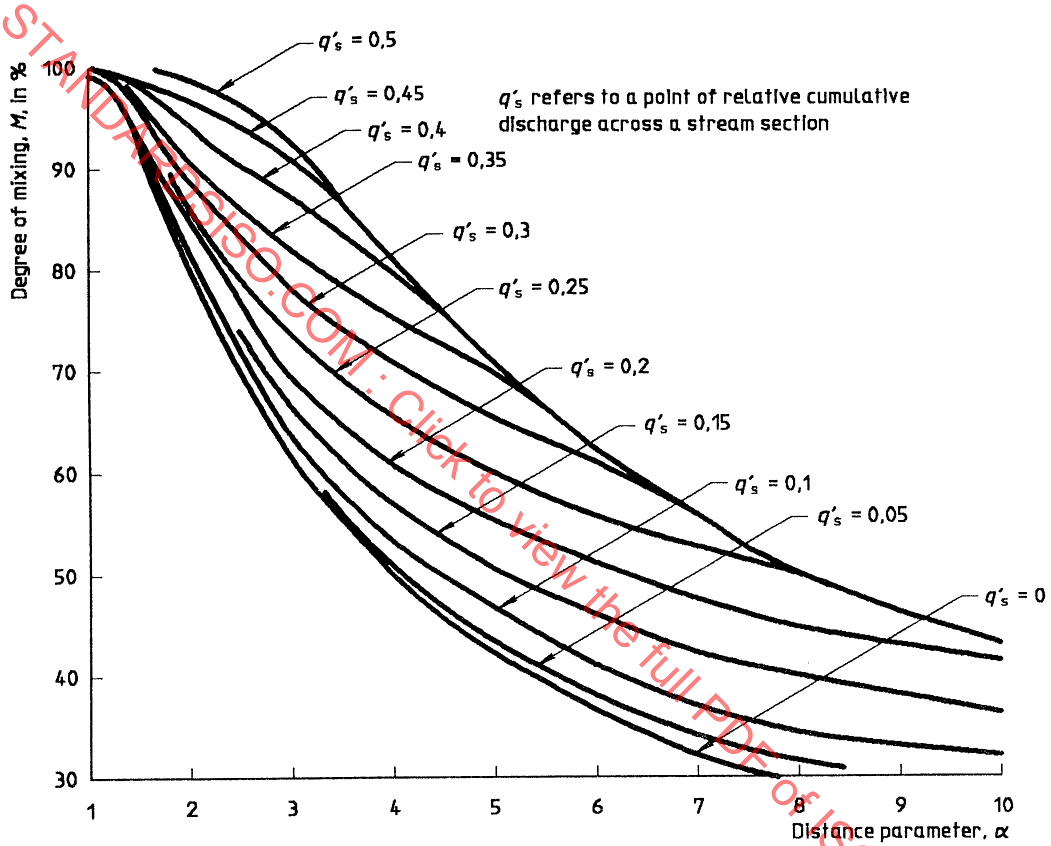


Figure 4 — Relation of the degree of mixing, M, to the distance parameter, α, and the injection site in terms of relative discharge, q'_s, for use in the Yotsukura mixing-length formula for point-source injections

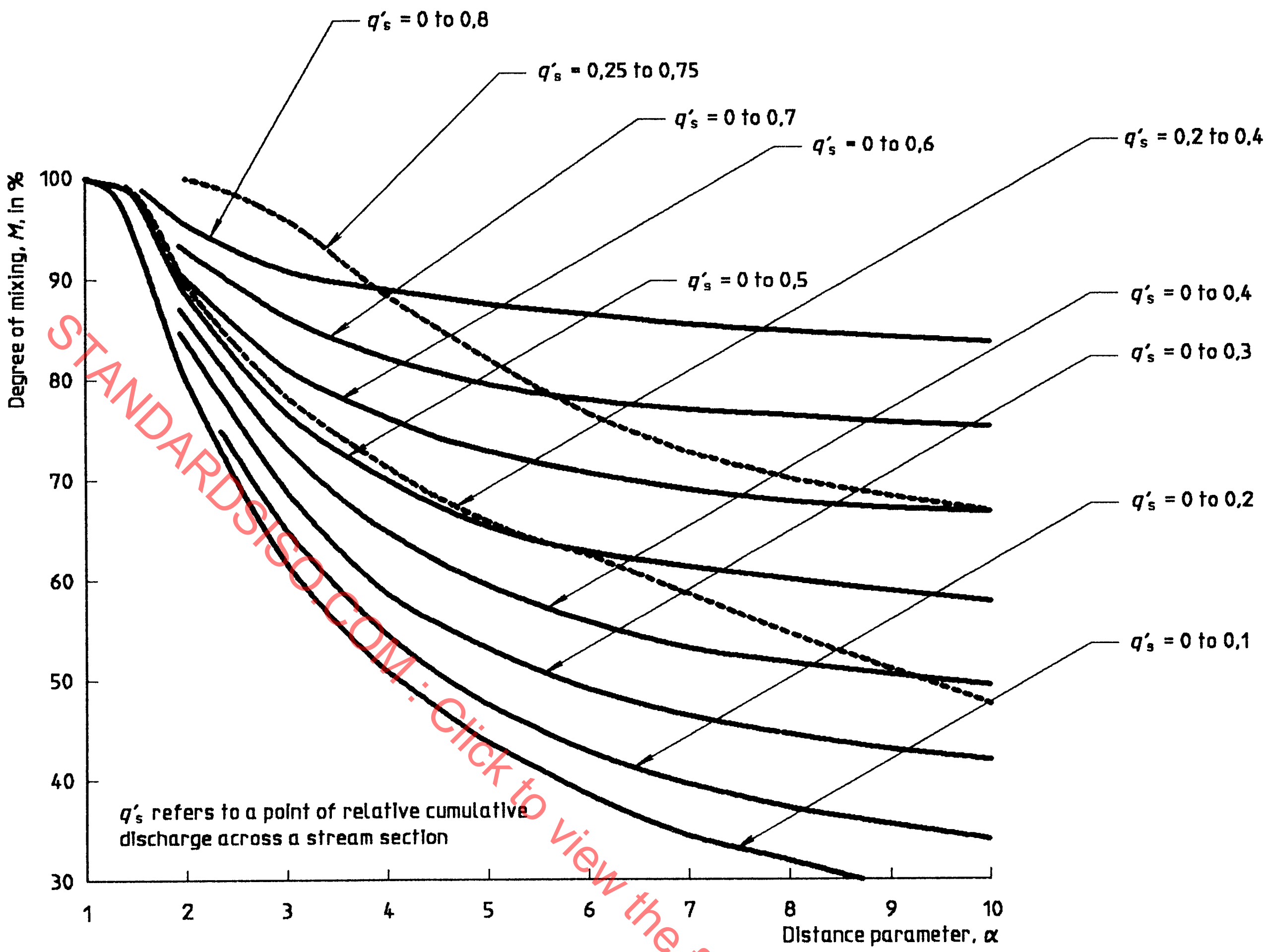


Figure 5 — Relation of the degree of mixing, M, to the distance parameter, α, and the injection site in terms of relative discharge, q'_s, for use in the Yotsukura mixing-length formula for line-source injections

6.4 Comparison of mixing-length estimates

6.4.1 Data set

Data were obtained from 22 fairly complete studies and could be used as observed data to compare with mixing-length estimates from equations (7) to (15). The data are shown in annex A. The values in parentheses are computed values and were not provided with the original data set.

When the dispersion coefficient, E, was provided but not the shear velocity, U_*, the shear velocity was determined from the following relation:

$$U_* = \frac{E}{0{,}2D} \qquad \ldots (16)$$

or, conversely, E was determined from U_* using the same relation. In all cases, values for slope, S, the Manning coefficient, n, and the Chezy coefficient, C, had to be computed. Slope was determined from the following relation:

$$S = \frac{U_*^2}{gD} \qquad \ldots (17)$$

The coefficient n is determined from the equation:

$$n = \frac{AR^{2/3}S^{1/2}}{Q} \qquad \ldots (18)$$

where

A is the average cross-sectional area between the injection and sampling sites, in square metres ($A = DB$);

R is the average hydraulic radius in the stream reach, in metres ($R = A/WP$), where WP is the wetted perimeter estimated as $B + 2D$.

The Chezy coefficient C is determined from the equation:

$$C = \frac{1}{n} R^y \qquad \ldots (19)$$

where

n is the Manning roughness coefficient;

R is the hydraulic radius;

y is an exponent estimated from the following formulae:

$$y = 1{,}5\sqrt{n} \text{ for } R < 1{,}0 \text{ m}$$

$$y = 1{,}3\sqrt{n} \text{ for } R > 1{,}0 \text{ m}$$

Table 2 provides a summary of computed and observed mixing distances. Observed mixing distances have been plotted as a function of corresponding degrees of mixing (see annex A). The observed values shown in table 2 for various degrees of mixing were then taken from the curve drawn through the plotted points. Values given in parentheses were extrapolated or interpolated from observed data and thus possess greater uncertainty than other values shown.

Where provision was made in the mixing equation for the degree of mixing, the mixing distance was computed for 98 % mixing. Unless provision was made in the mixing-distance equation for the point of injection, centre injection of the tracer was assumed. If the tracer is injected near the bank, for the purposes of computation, the stream is considered to be twice as wide as shown, or the mixing distance is increased to four times the distance computed by the equation. If the equation takes into consideration the point of injection, this adjustment is made automatically. In one set of data, the tracer was injected at points 0,25 and 0,75 across the section. In this case, for computational purposes, the stream width was considered to be one-half of the width shown, or the mixing distance reduced to one-fourth of the computed distance. These adjustments were made on the basis that most theoretical relations indicate a mixing distance proportional to the square of the width.

Table 2 — Summary of computed and observed mixing lengths

Values in metres

Test	Mixing length calculated according to equation										Observed mixing length		
	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15a)*	(15b)*	X_{90}	X_{95}	X_{98}
1	216	458	177	53	1 300	2 050	4 220	1 300	4 370	1 220	—	640	(2 800)
2	211	458	173	20	418	2 500	1 220	1 200	1 150	394	(960)	(1 800)	(2 600)
3	103	229	134	10	105	555	282	300	388	98	240	410	580
4	360	458	293	72	540	5 760	1 610	717	1 360	509	1 100	1 500	1 800
5	355	458	289	72	540	5 120	1 500	717	933	509	800	1 220	1 950
6	357	915	1 160	144	2 160	23 100	6 420	2 860	2 040	2 040	2 600	3 100	3 400
7	525	502	388	141	1 160	26 600	3 950	827	2 010	1 100	1 570	1 980	2 280
8	18 100	4 580	1 450	3 400	29 100	86 700	152 000	45 300	73 100	73 100	14 000	17 000	19 500
9	2 660	2 860	1 380	120	25 200	255 000	73 000	33 500	—	23 800	(22 000)	(30 000)	(36 000)
10	2 280	2 640	1 290	139	29 800	345 000	73 900	31 200	—	28 200	(14 000)	(19 000)	(23 000)
11	45	162	105	3	18	28	29	252	—	17	30	41	56
12	25	102	92	6	33	105	73	142	—	31	55	90	130
13	21	98	80	3	20	38	35	145	—	19	30	44	65
14	22	102	80	4	35	93	71	172	—	33	43	68	90
15	101	282	135	5	145	298	258	913	—	136	(240)	(300)	(350)
16	74	155	178	34	107	1 130	330	158	—	101	110	170	210
17	444	305	539	26	96	41 000	3 210	333	—	91	77	140	210
18	11	55	77	4	3	7	6	28		3	12	25	50
19	18 100	4 580	1 450	723	27 000	640 000	85 700	41 700	67 900	25 400	13 000	15 000	17 000
20	52 000	11 200	6 830	1 680	104 000	235 000	346 000	70 900	98 100	98 100	(12 000)	(14 000)	(15 000)
21	9 880	3 800	3 830	993	8 570	149 000	30 100	9 030	8 100	8 100	6 000	7 200	7 700
22	56 800	14 200	5 880	1 270	105 000	192 000	369 000	104 000	99 000	99 000	(120 000)	(165 000)	(190 000)

* Equation (15a) uses the applicable value of α determined using relative discharge across the section and equation (15b) uses the value of α determined using relative width across the section.

The Yotsukura formula [equation (15)] introduces a coefficient that is partly dependent on the site of injection in the section. It is intended that this site be determined on the basis of cumulative discharge from one bank of the stream. For practical reasons, the injection site must often be estimated based on the relative distance from the stream bank to the point of injection. The first set of data obtained by equation (15) [indicated in table 2 as equation (15a)] is for an injection position defined by cumulative discharge for 12 sets of data points. The second set of data obtained by equation (15) [indicated in table 2 as equation (15b)] is for an injection position defined by relative distance across the channel.

6.4.2 Comparison of computed and observed mixing distances

Comparison of computed distances with observed distances (table 2) for 98 % mixing is made using regression analysis. A logarithmic transform is applied to all mixing distance data prior to the regression analysis of the data. The resulting regression equation is of the form:

$$X_c = aX_o^b$$

where

X_c is the computed mixing distance;

X_o is the observed mixing distance;

a is a regression constant;

b is an exponent.

a and b are both defined by the regression analysis.

Table 3 shows the resulting regression equation constant a, the standard error of estimate S_E in log units, and the coefficient of correlation R^2 in percent.

Table 3 — Summary of comparative data for mixing-length computations

Equation	Regression constant a	Exponent b	S_E 1)	R^2 %	Corrected S_E 2)
(7)	0,32	0,995	0,483	81,9	0,485
(8)	5,50	0,646	0,239	88,7	0,370
(9)	8,65	0,528	0,319	74,3	0,604
(10)	0,136	0,820	0,460	77,1	0,562
(11)	0,077	1,25	0,335	93,8	0,268
(12)	0,327	1,31	0,691	79,2	0,529
(13)	0,170	1,30	0,455	89,7	0,350
(14)	1,43	0,960	0,358	88,5	0,372
(15a)3)	0,185	1,21	0,393	80,5	0,325
(15b)4)	0,066	1,27	0,358	93,1	0,281

1) In log units.
2) In log units, about the first bisector for mixing distances computed using equations modified to reduce bias.
3) For tracer-injection sites based on cumulative discharge.
4) For tracer-injection sites based on relative distance across the section.

Table 3 indicates that the relations generally show some bias. An unbiased relation will have an equation constant and an exponent of 1,0.

The empirical formula (8) developed by Day had the lowest standard error of estimate but also had considerable bias. Based on the regression equations, the supporting statistical information and on plots of the data, the least biased and best-fitting formula was that of Ward [equation (14)]. Annex B shows plots of the data and the corresponding regression curves.

The next step in the comparison of the mixing-length equations was to reduce or eliminate the bias in the equations based on the available observed mixing-length data. This was done by adjusting the mixing-length equations for the information obtained in the regression analysis.

Biases in the mixing-length equations were adjusted for by applying a multiplier of $1/a$ to the mixing length equation and by applying an exponent of $1/b$ to the resulting equation, where a and b are derived from the appropriate regression equation in table 3. The result provides a scatter of a plot of the observed data versus the computed data about the first bisector. The standard error of estimation about this bisector for the data adjusted for bias are listed in the last column of table 3. The corrected values are plotted in the figures in annex B.

6.4.3 Discussion and recommendations

Comparison of the standard errors for the adjusted equations shows that the formula of Fischer [equation (11)] most closely approximates the observed mixing distances for 98 % mixing. For practical purposes, the formula of Yotsukura [equation (15)] gives similar results. Several of the adjusted equations agree fairly closely with observed mixing distances. The adjusted formulae of Fischer and Yotsukura seem to best agree with the observed values.

The Fischer formula is the easier to use when tracer injection is at the centre or side of the streamflow. For tracer injections made at an arbitrary but defined location in the cross-section, the Yotsukura equation allows adjustment for the mixing distance based on an arbitrary site of the tracer injection.

The adjusted mixing formulae of Fischer and Yotsukura are as follows:

Adjusted Fischer formula

$$X = \left[\frac{K_1 U B^2}{0{,}077 E} \right]^{0{,}80} \qquad \ldots (20)$$

Adjusted Yotsukura formula

$$X = \left[\frac{1}{0{,}132 \alpha^2 \beta} \times \frac{U D^2}{\overline{uy}^2} \times \frac{U}{U_*} \times \frac{B^2}{D} \right]^{0{,}79} \qquad \ldots (21)$$

They are used where the injection site is defined by relative distance across the section.

The adjustments made to the various mixing equations are based on only a small amount of data and may include some data from mountain-type streams. Additional data may indicate other adjustments and may show that some of the original formulae closely agree with observed data without adjustment.

Because of the small amount of available data and the possibility of mixed types of data, the adjusted formulae are not generally recommended for use. The adjusted formulae are provided to show what is required to make the formula best fit the observed data.

For streams having sharp contractions and expansions in the measuring reach, as is common in many mountain streams, the empirical equations may more accurately represent mixing length than the theoretical equations. The empirical equation should, however, either be developed from data collected from representative streams in the area or checked using such data.

7 Errors in dilution measurements associated with incomplete mixing

This clause provides information on the error in the degree of mixing when, for weighting purposes, it is computed using incremental width instead of incremental flow. The error in the computation of discharge using dilution methods, when mixing is incomplete, is also examined.

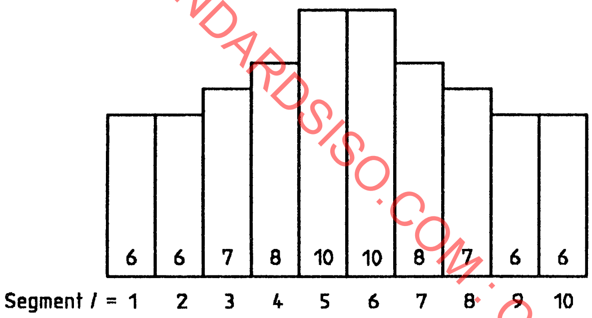
Earlier in this Technical Report, it was recommended that a formula for the degree of mixing be used which is based on an incremental flow weighting procedure. Often it is impractical to determine incremental flows; therefore, it is recommended in this case that incremental width be substituted in place of incremental flow. This clause explores the error which results when such a substitution is made.

A degree of mixing of 98 % has been recommended as adequate for a good flow measurement. This clause also examines the error associated with the calculation of discharge when mixing is less than 100 %.

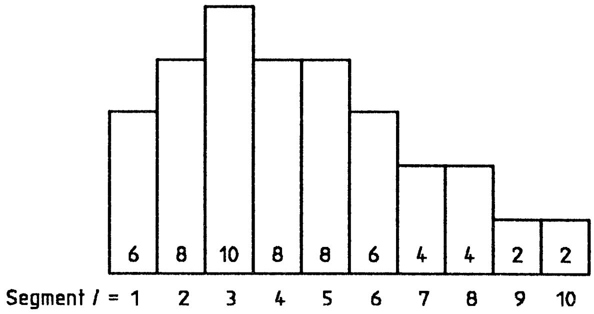
7.1 Error in the degree of mixing when weighted by width

7.1.1 Method of analysis

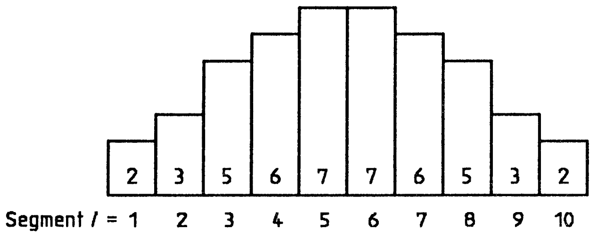
Let us assume a number of distributions of concentration and flow, as shown in figures 6 and 7. In these distributions, each segment i, numbered from 1 to 10 is assumed to have equal widths of one-tenth of the channel width. Mean concentrations and degrees of mixing, using both width and flow weighting procedures, were computed for each concentration/flow combination and are listed in table 4.



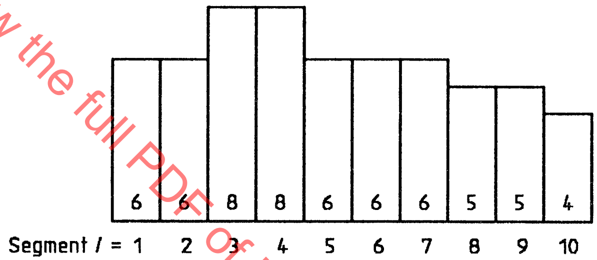
6 a) Distribution 1



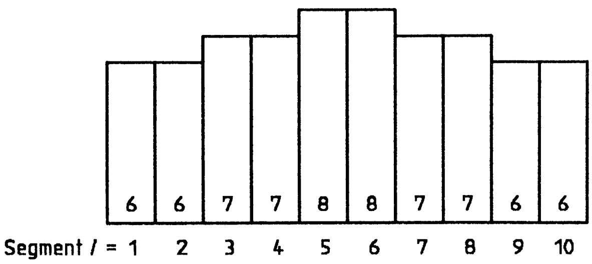
6 d) Distribution 4



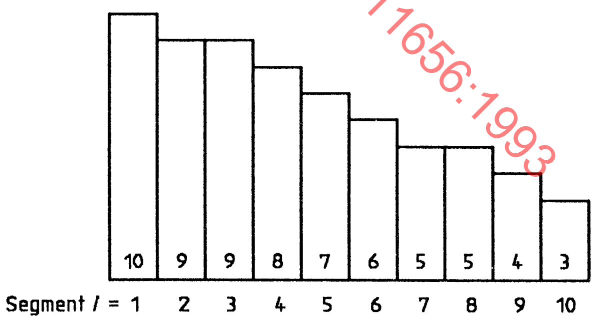
6 b) Distribution 2



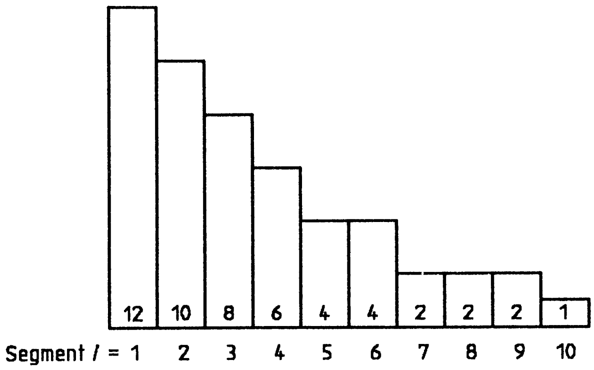
6 e) Distribution 5



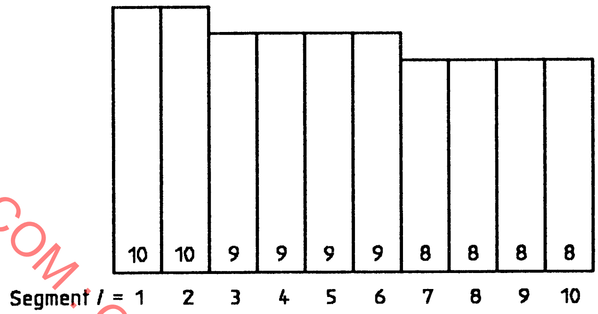
6 c) Distribution 3



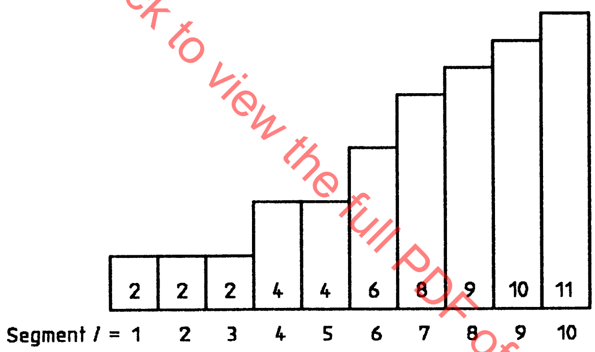
6 f) Distribution 6



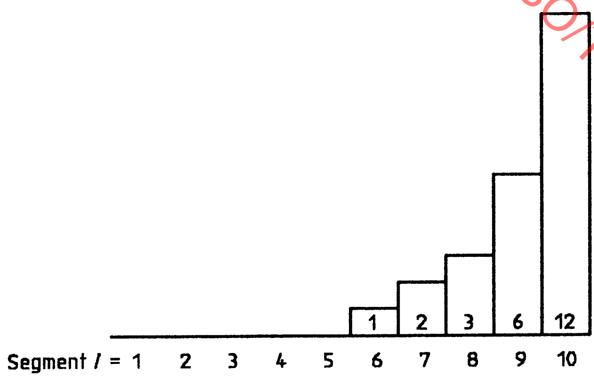
6 g) Distribution 7



6 h) Distribution 8



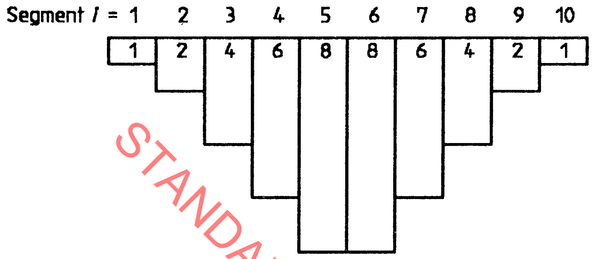
6 i) Distribution 9



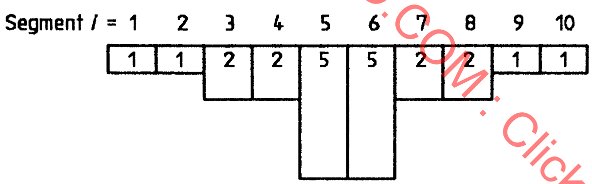
6 j) Distribution 10

NOTE — The number in each segment refers to the number of concentration units in that segment.

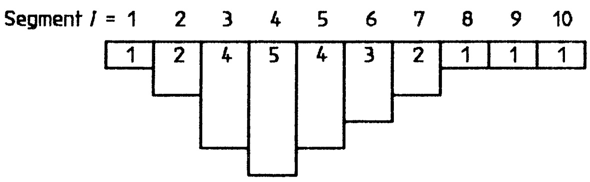
Figure 6 — Assumed concentration distribution



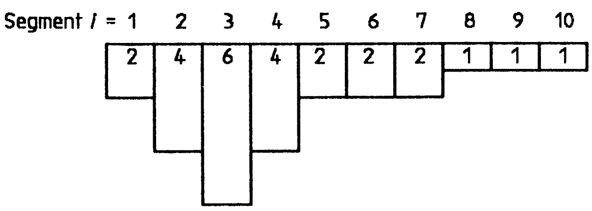
7 a) Distribution A



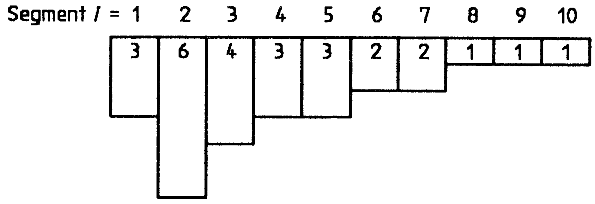
7 b) Distribution B



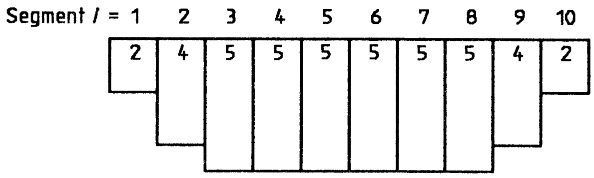
7 c) Distribution C



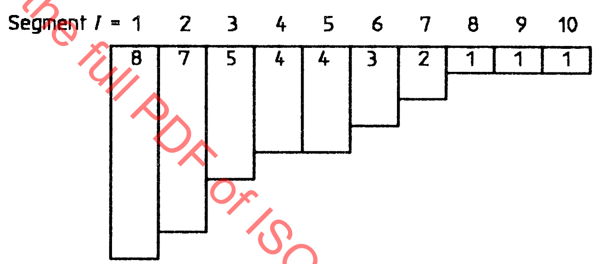
7 d) Distribution D



7 e) Distribution E



7 f) Distribution F



7 g) Distribution G

NOTE — The number in each segment refers to the number of flow units in that segment.

Figure 7 — Assumed flow distribution

Table 4 — Mean concentrations and degrees of mixing for various concentration and flow distributions

Concentration/flow distribution[1)]	Mean concentration			Degree of mixing		
	Width-weighted[2)]	Flow-weighted[3)]	Error[4)] %	Width-weighted[5)]	Flow-weighted[6)]	Error[4)] %
1A	7,40	8,29	− 11	91	92	− 1
1B	7,40	8,36	− 11	91	91	0
1C	7,40	7,96	− 7	91	92	− 1
1D	7,40	7,40	0	91	92	− 1
1E	7,40	7,35	1	91	91	0
1F	7,40	7,67	− 4	91	92	− 1
1G	7,40	7,28	2	91	91	0
2A	4,60	5,71	− 19	82	90	− 9
2B	4,60	5,64	− 18	82	88	− 7
2C	4,60	5,38	− 14	82	88	− 7
2D	4,60	4,80	− 4	82	86	− 5
2E	4,60	4,58	0	82	82	0
2F	4,60	5,05	− 9	82	86	− 5
2G	4,60	4,36	6	82	80	3
3A	6,80	7,24	− 6	95	96	− 1
3B	6,80	7,27	− 6	95	95	0
3C	6,80	7,08	− 4	95	96	− 1
3D	6,80	6,84	− 1	95	96	− 1
3E	6,80	6,77	0	95	95	0
3F	6,80	6,95	− 2	95	96	− 1
3G	6,80	6,72	1	95	95	0
4A	5,80	6,38	− 9	81	85	− 5
4B	5,80	6,36	− 9	81	85	− 5
4C	5,80	7,00	− 17	81	86	− 6
4D	5,80	7,20	− 19	81	86	− 6
4E	5,80	7,00	− 17	81	87	− 7
4F	5,80	6,10	− 5	81	82	− 1
4G	5,80	7,00	− 17	81	88	− 8

Concentration/flow distribution[1]	Mean concentration			Degree of mixing		
	Width-weighted[2]	Flow-weighted[3]	Error[4] %	Width-weighted[5]	Flow-weighted[6]	Error[4] %
5A	6,00	6,29	– 5	93	93	0
5B	6,00	6,14	– 2	93	94	– 1
5C	6,00	6,58	– 9	93	92	1
5D	6,00	6,64	– 10	93	92	1
5E	6,00	6,38	– 6	93	93	0
5F	6,00	6,17	– 3	93	93	0
5G	6,00	6,39	– 6	93	94	– 1
6A	6,60	6,60	0	85	89	– 4
6B	6,60	6,59	0	85	89	– 4
6C	6,60	7,17	– 8	85	90	– 6
6D	6,60	7,60	– 13	85	89	– 4
6E	6,60	7,65	– 14	85	89	– 4
6F	6,60	6,62	0	85	87	– 2
6G	6,60	8,00	– 17	85	90	– 6
7A	5,10	4,50	13	69	78	– 12
7B	5,10	4,59	11	69	78	– 12
7C	5,10	5,46	– 7	69	78	– 12
7D	5,10	5,96	– 14	69	77	– 10
7E	5,10	6,73	– 24	69	77	– 10
7F	5,10	4,86	5	69	72	– 4
7G	5,10	7,42	– 31	69	78	– 12
8A	8,80	8,76	0	96	97	– 1
8B	8,80	8,82	0	96	97	– 1
8C	8,80	8,92	– 1	96	98	– 2
8D	8,80	9,04	– 3	96	97	– 1
8E	8,80	9,15	– 4	96	97	– 1
8F	8,80	8,76	0	96	97	– 1
8G	8,80	9,28	– 5	96	97	– 1

Concentration/flow distribution[1)]	Mean concentration			Degree of mixing		
	Width-weighted[2)]	Flow-weighted[3)]	Error[4)] %	Width-weighted[5)]	Flow-weighted[6)]	Error[4)] %
9A	5,80	5,55	5	74	80	− 7
9B	5,80	5,50	5	74	80	− 7
9C	5,80	4,75	22	74	77	− 4
9D	5,80	4,24	37	74	73	1
9E	5,80	4,15	40	74	73	1
9F	5,80	5,69	2	74	76	− 3
9G	5,80	3,78	53	74	74	0
10A	2,40	1,33	80	43	45	− 4
10B	2,40	1,50	60	43	42	2
10C	2,40	1,17	105	43	32	34
10D	2,40	1,08	122	43	27	59
10E	2,40	1,04	131	43	27	59
10F	2,40	1,86	29	43	45	− 4
10G	2,40	0,78	208	43	22	95

1) The number refers to the concentration distribution and the letter to the flow distribution (see figures 6 and 7).

2) Width-weighted mean concentration:

$$\bar{C}_N = \frac{1}{N} \sum C_i$$

3) Flow-weighted mean concentration:

$$\bar{C}_Q = \frac{1}{Q} \sum C_i q_i$$

4) Percent errors are relative to the flow-weighted values.

5) Width-weighted degree of mixing:

$$\bar{M}_N = \left[1 - \frac{1}{2N} \sum_{i=1}^{N} \left(\left| \frac{C_i - \bar{C}_N}{\bar{C}_N} \right| \right) \right] \times 100$$

6) Flow-weighted degree of mixing:

$$\bar{M}_Q = \left[1 - \frac{1}{2} \sum_{i=1}^{N} \left(\left| \frac{C_i - \bar{C}_Q}{\bar{C}_Q} \right| \times \frac{q_i}{Q} \right) \right] \times 100$$

where

C_i is the concentration in the ith channel segment;

N is the total number of flow segments;

Q is the total flow in the channel cross-section;

q_i is the flow in segment i.

The mean concentration, $\overline{C}_N$, and the degree of mixing, $\overline{M}_N$, using weighting by equal increments of width, are determined by the following relations:

$$\overline{C}_N = \frac{1}{N} \sum_{i=1}^{N} C_i \qquad \ldots (22)$$

$$\overline{M}_N = \left[1 - \frac{1}{2N} \sum_{i=1}^{N} \left(\left| \frac{C_i - \overline{C}_N}{\overline{C}_N} \right| \right) \right] \times 100 \qquad \ldots (23)$$

where

C_i is the tracer concentration in the ith flow segment;

N is the total number of flow segments in a cross-section.

The mean concentration, $\overline{C}_Q$, and the degree of mixing, $\overline{M}_Q$, using flow weighting, were determined by the following equations:

$$\overline{C}_Q = \frac{1}{Q} \sum_{i=1}^{N} C_i q_i \qquad \ldots (24)$$

$$\overline{M}_Q = \left\{ 1 - \frac{1}{2} \sum_{i=1}^{N} \left[\left| \frac{C_i - \overline{C}_Q}{\overline{C}_Q} \right| \left(\frac{q_i}{Q} \right) \right] \right\} \times 100 \qquad \ldots (25)$$

where

Q is the total flow in the channel cross-section;

q is the flow in segment i.

The error in the determination of the degree of mixing using width weighting was determined for each assumed concentration/flow combination. The error, E_m, was determined by the following equation:

$$E_m = \left(\frac{M_N - M_Q}{M_Q} \right) \times 100 \qquad \ldots (26)$$

7.1.2 Results

Figure 8 shows data points from table 4 and an envelope curve for the absolute values of the errors obtained from the concentration/flow combinations used in this study. A different set of concentration/flow distributions would have resulted in a different envelope curve. Extreme non-uniformity of flow and/or low degrees of mixing could move the curve to the right, giving increased errors. The envelope curve shown illustrates that, as the degree of mixing approaches 100 %, the error from using a width-weighting method approaches zero. At low degrees of mixing, errors can become quite large. The curve shows that, at 90 % mixing, errors can be 5 % or more.

Moderate errors in computing the degree of mixing seem to occur when both the concentration and flow distributions are moderately non-uniform. When the concentration distribution is fairly uniform, as in distributions 1, 3, 5 and 8 (see figure 6), errors will generally be quite small. When the concentration distribution is rather non-uniform, as in distributions 2, 4, 6 and especially in distributions 7, 9 and 10 (see figure 6), the errors can vary considerably depending on the flow distribution.

On the other hand, when flows are fairly uniform, as in flow distribution F (see figure 7), the resulting errors will generally be fairly low. This is to be expected, since the percentage of flow in each segment approaches the percentage of width in each segment, for conditions approaching uniform flow.

7.1.3 Significance

Most of the time, it is impractical to determine the flow distribution in order to calculate the degree of mixing by means of equation (25). Consequently, the use of equation (23) will often be necessary for calculating the degree of mixing. The user should, however, be aware that under non-uniform flow conditions, the computed degree of mixing using equation (23) can be in error and considerably so if the degree of mixing is fairly low.

For some purposes, a 5 % error in the calculation of the degree of mixing may not be very significant. If studies are being made to determine mixing distance, it is important that the flow distribution be determined so that the degree of mixing can be computed based on flow-weighted data.

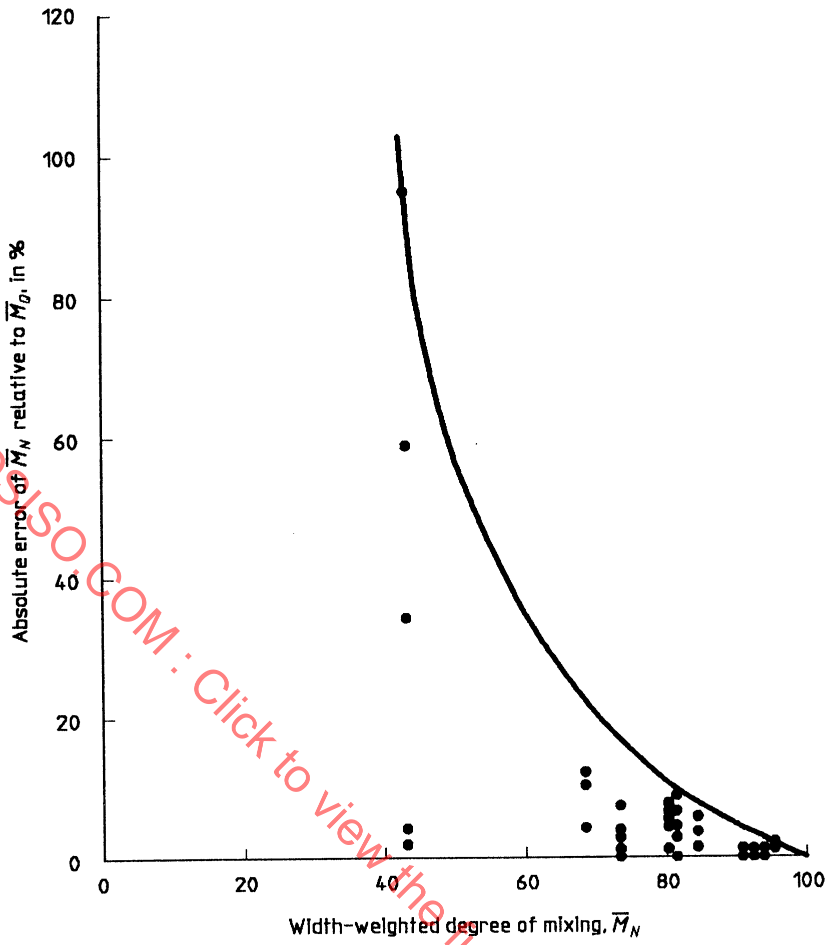


Figure 8 — Envelope curve of observed error in the width-weighted computation of the degree of mixing

An example of the significance of obtaining the best value of the degree of mixing when conducting mixing length studies can be seen by examining the relative theoretical lengths corresponding to mixing of 90 %, 95 % and 98 % based on equation (15), as follows:

$$X = \frac{1}{2\alpha^2\beta} \times \frac{UD^2}{\overline{uy}^2} \times \frac{U}{U_*} \times \frac{B^2}{D} \qquad \ldots (27)$$

For the purpose of this comparison, assume that

$$R = \frac{1}{2\beta} \times \frac{UD^2}{\overline{uy}^2} \times \frac{U}{U_*} \times \frac{B^2}{D}$$

and thus

$$X = \frac{R}{\alpha^2}$$

The values of α for a centre injection for 90 %, 95 % and 98 % mixing are 3,29, 2,78 and 2,32 respectively. This results in values of X for 90 %, 95 % and 98 % mixing of 0,092 4R, 0,129 4R and 0,185 8R respectively. From this, it can be seen that 95 % mixing requires 40 % more mixing length than required for 90 % mixing, and 98 % mixing requires 44 % more mixing length than required for 95 % mixing. From this exercise, it can be seen that it is very important to have the best value of the degree of mixing when conducting mixing-length studies.

7.2 Error in calculation of discharge when mixing is incomplete

7.2.1 Method of analysis

The mass balance principle of dilution gaging is that the mass inflow of the tracer at the injection site must be equal to the mass outflow at the measurement site, which is given by the product of discharge and the uniformly mixed tracer concentration. From this principle, discharge weighting of non-uniformly-mixed tracer concentration, as expressed by equation (24), is the correct way to evaluate the discharge.

When samples are obtained in the centre of equal increments of width, or the sample concentrations are otherwise weighted by width instead of by flow, errors can be introduced into the flow measurement unless the tracer concentration is completely uniform across the channel. The concentration/flow distributions shown in figures 6 and 7 indicate the magnitude of error which can be introduced.

Mean concentrations weighted by width were computed using equation (22) and those weighted by discharge were computed using equation (24). The results are listed in table 4. The error in the determination of the mean concentration using width weighting was determined for each assumed concentration/flow combination.

The error, E_C, was determined by the following equation:

$$E_C = \left(\frac{\overline{C}_N - \overline{C}_Q}{\overline{C}_Q} \right) \times 100 \qquad \ldots (28)$$

7.2.2 Results

Figure 9 shows an envelope curve for the absolute values of the errors obtained from the concentration/flow combinations used in this study. A different set of concentration/flow distributions would have resulted in a different envelope curve. Extreme non-uniformity of flow and/or low degrees of mixing could move the curve to the right, giving increased errors. The envelope curve shown illustrates that, as the degree of mixing approaches 100 %, the error from using a width-weighting method approaches zero. The curve shows that for low degrees of mixing, the error can be quite large. For 95 % mixing, errors can be 10 % or more, while for 90 % mixing, they can be 20 % or more.

Even with moderately good mixing, as with concentration distribution 3 in figure 6, errors can be significant for certain flow distributions such as distributions A and B in figure 7. As the flow distribution approaches a uniform condition, the error will approach zero; however, even with a fairly uniform distribution, as shown by distribution F, there can be rather large errors if the degree of mixing is low, as illustrated by concentration distribution 10.

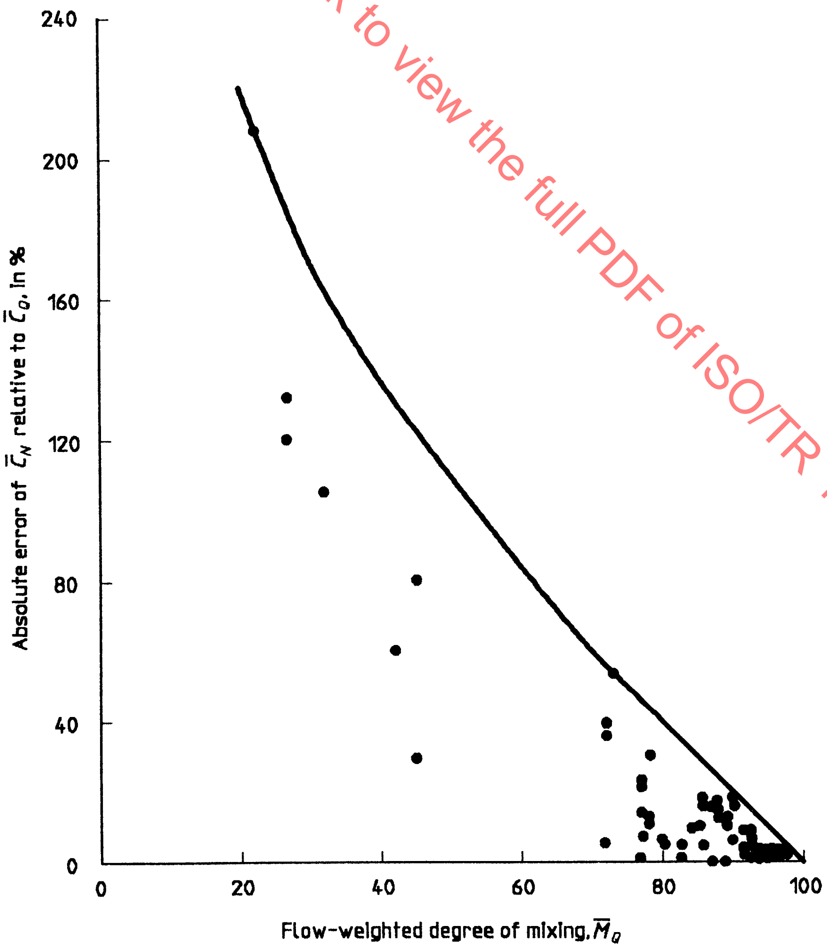


Figure 9 — Envelope curve of observed error in the width-weighted computation of mean concentration

The error in the mean concentration was determined to be primarily a function of two variables: the degree of mixing, M, and a skewness function, S_k, where

$$S_k = (5,5 - S_C)(5,5 - S_q) \qquad \ldots (29)$$

where

S_C is the centroid of the concentration in the segments (see figure 6);

S_q is the centroid of the discharge in the segments (see figure 7).

The value of S_k is large in the positive direction if the skews of both the flow and the concentration are large and toward the same side of the channel. It is large in a negative direction if there is a large skew of flow to one side of the channel and a large skew of concentration toward the opposite side of the channel.

The error in the mean concentration, $E_{\bar{C}}$, is determined by multiple regression analysis to be

$$E_{\bar{C}} = 101 - 1,14M - 14,1S_k \qquad \ldots (30)$$

A standard error, E_s, of 10,3 % and a coefficient of variation, R_2, of 93,1 % are obtained.

7.2.3 Significance

This exercise has shown that unless the flow is extremely uniform, or the degree of mixing is quite high, significant errors can be introduced into the measurement of flow.

Earlier, the recommendation was made that the degree of mixing should be at least 98 % for an acceptable discharge measurement. The envelope curve of figure 9 shows that even with 98 % mixing, errors of 4 % or greater can be experienced if the flow distribution is not taken into account.

This exercise shows that it is important to obtain a correct measure of the degree of mixing in order to define the possible error that can be associated with a dilution-type discharge measurement. There are other sources of error in a dilution measurement, but unless the degree of mixing is quite high, the error resulting from inadequately considering the flow distribution may be overwhelming.

7.3 Discussion

Using assumed distributions of concentration and flow, the errors associated with computing the degree of mixing and discharge using weighting methods based on width rather than discharge have been examined. Errors were computed for each combination of concentration and flow, and the errors plotted against the degree of mixing. An envelope curve was drawn showing the upper limits of the observed errors.

As the degree of mixing approaches 100 %, the error approaches zero. At 90 % mixing, the error in the computation of the degree of mixing can be as much as 5 % or more. For the same degree of mixing, the error in the computation of discharge can be as much as 20 % or greater. In most situations, the error will be less. For mixing of 95 %, the error in the computation of discharge can be as much as 10 % or more and for 98 % mixing, the error can be 4 % or greater.

It is thus recommended that unless flow-weighting is used, the degree of mixing should be at least 98 % for an acceptable discharge measurement for most purposes.

8 Dilution discharge measurements when mixing is incomplete

When the degree of mixing is less than 98 %, the concentration obtained at the various points across the section should be flow-weighted. This can be accomplished in a number of ways, but one of the easiest is to sample the tracer using an equal-velocity depth-integrating sampler with sediment-sampling techniques.

The equal transit rate (ETR) method is recommended for flow-weighting samples for dilution measurements. Samples are obtained at equally spaced verticals in the section using a depth-integrating sampler. This requires that the constant-rate injection method be used. Ten to twenty verticals are recommended, depending on the uniformity of flow and tracer concentration across the section. The higher number should be used if the flow or tracer concentration is quite skewed in the section. ETR methods should probably not be used for most discharge measurements when mixing is less than 50 % to 60 %, as errors in sampling may introduce fairly large errors into the discharge measurement.

9 Discussion and recommendations

A measure of the degree of mixing is needed such that the value will vary from zero, when the tracer is first introduced into a stream by point injection, to 100 % when the tracer is completely mixed across the section. The Cobb-Bailey formulae (4) and (5) fit these criteria and are recommended for use in defining the degree of mixing of a solute across a stream section.

Three general approaches are used to estimate mixing lengths: direct observation, the empirical method and the theoretical method. The direct observation approach has little transfer value. The empirical approach may be best for specific areas with streams having similar mixing characteristics. The empirical relationship should be defined or verified with data collected in the area of use. The theoretical approach provides the most universal relationships and is best suited for estimating mixing distances in alluvial type streams where there are not many sharp contractions and expansions in the cross-section. The Fischer formula (11) is recommended for most uses where the tracer is injected in the centre or at the side of the flow. Where the tracer is injected in another part of the flow or as a line source, the Yotsukura formula (15) is recommended.

The errors that can result from incomplete lateral mixing of a tracer in the flow have been investigated. Errors in dilution-type discharge measurements, for mixing of 90 %, can be 20 % or more; for mixing of 95 %, they can be as much as 10 % or more and for 98 % mixing, 4 % or more. It is therefore recommended that mixing be about 98 % for dilution discharge measurements. When mixing is incomplete, an equal transit rate method can be used with 10 to 20 verticals. Samples are collected using an equal-velocity depth-integrating sampler. This method should probably not be used in most cases if mixing is less than 50 % to 60 %.

Annex A
(informative)

Statistical data obtained from different streams for mixing-length comparisons

1. South River, Virginia, USA
June 8, 1966
Reference: [9]

$z = 0,5$
$q = 0,75$
$Q = 1,64$ m^3/s
$\overline{U} = 0,23$ m/s
$\overline{D} = 0,37$ m
$B = 18,3$ m
$S = (0,000\ 074$ m/m)
$E_z = 0,012\ 9$ m^2/s
$U_* = (0,016$ m/s)
$n = (0,018)$
$c = (45)$

X, m	M
244	0,93
366	0,94
488	0,94
610	0,95

2. South River, Virginia, USA
June 9, 1966
Reference: [9]

$z = 0,5$
$q = 0,35$
$Q = 1,53$ m^3/s
$\overline{U} = 0,20$ m/s
$\overline{D} = 0,40$ m
$B = 18,3$ m
$S = (0,000\ 40$ m/m)
$E_z = 0,005\ 39$ m^2/s
$U_* = 0,040$ m/s
$n = (0,050)$
$c = (14)$

X, m	M
122	0,78
183	0,74
305	0,77
393	0,83

3. South River, Virginia, USA
June 9, 1966
Reference: [9]

$z = 0,25$ and $0,75$
$q = 0,10$ and $0,85$
$Q = 1,42$ m^3/s
$\overline{U} = 0,20$ m/s
$\overline{D} = 0,40$ m
$B = 18,3$ m
$S = (0,000\ 40$ m/m)
$E_z = 0,004\ 27$ m^2/s
$U_* = 0,040$ m/s
$n = (0,054)$
$c = (13)$

X, m	M
122	0,82
183	0,87
305	0,92
393	0,94

4. Atrisco Feeder Canal, New Mexico, USA
June 21, 1966
Reference: [9]

$z = 0,5$
$q = 0,4$
$Q = 7,62$ m^3/s
$\overline{U} = 0,67$ m/s
$\overline{D} = 0,67$ m
$B = 18,3$ m
$S = (0,000\ 59$ m/m)
$E_z = 0,011\ 6$ m^2/s
$U_* = 0,062$ m/s
$n = (0,029)$
$c = (31)$

X, m	M
122	0,49
183	0,58
244	0,63
305	0,66
732	0,88
975	0,88
1 220	0,92

5. Atrisco Feeder Canal, New Mexico, USA
June 22, 1966
Reference: [9]

$z = 0,5$
$q = 0,45$
$Q = 7,31$ m^3/s
$\overline{U} = 0,67$ m/s
$\overline{D} = 0,67$ m
$B = 18,3$ m
$S = (0,000\ 59$ m/m)
$E_z = 0,012\ 4$ m^2/s
$U_* = 0,062$ m/s
$n = (0,030)$
$c = (29)$

X, m	M
488	0,78
610	0,85
732	0,87
853	0,91
975	0,93
1 100	0,96
1 220	0,96
1 980	0,98

6. Atrisco Feeder Canal, New Mexico, USA
June 23, 1966
Reference: [9]

$z = 0,0$
$q = 0,0$
$Q = 7,45$ m^3/s
$\overline{U} = 0,67$ m/s
$\overline{D} = 0,67$ m
$B = 18,3$ m
$S = (0,000\ 59$ m/m)
$E_z = 0,009\ 48$ m^2/s
$U_* = 0,062$ m/s
$n = (0,029)$
$c = (31)$

X, m	M
122	0,30
183	0,33
488	0,48
975	0,63
1 220	0,69
1 980	0,82

7. Bernardo Conveyance Channel, New Mexico, USA
February 14, 1967
Reference: [9]

$z = 0,5$
$q = 0,45$
$Q = 17,8\ m^3/s$
$\overline{U} = 1,25\ m/s$
$\overline{D} = 0,70\ m$
$B = 20,1\ m$
$S = (0,000\ 56\ m/m)$
$E_z = 0,017\ 3\ m^2/s$
$U_* = 0,062\ m/s$
$n = (0,014)$
$c = (66)$

X, m	M
305	0,51
762	0,79
1 070	0,81
1 370	0,87
1 680	0,92
1 980	0,95

8. Missouri River, Nebraska, USA
November 17, 1967
Reference: [9]

$z = 0,4$
$q = 0,4$
$Q = 966\ m^3/s$
$\overline{U} = 1,74\ m/s$
$\overline{D} = 2,74\ m$
$B = 183\ m$
$S = (0,000\ 20\ m/m)$
$E_z = 0,131\ m^2/s$
$U_* = 0,073\ m/s$
$n = (0,014)$
$c = (83)$

X, m	M
2 660	0,42
3 610	0,56
4 720	0,59
5 710	0,67
6 830	0,70
8 440	0,78
10 050	0,82

9. Grand River, Ontario, Canada
August 12, 1975
Reference: [4]

$z = 0,06$
$q = —$
$Q = 12,5\ m^3/s$
$\overline{U} = 0,42\ m/s$
$\overline{D} = 0,56\ m$
$B = 57,3\ m$
$S = (0,000\ 28\ m/m)$
$E_z = 0,004\ 39\ m^2/s$
$U_* = (0,039\ m/s)$
$n = (0,029)$
$c = (30)$

X, m	M
122	0,25
427	0,35
1 082	0,55
2 240	0,51
3 398	0,64

10. Grand River, Ontario, Canada
August 12, 1975
Reference: [4]

$z = 0,06$
$q = —$
$Q = 10,1\ m^3/s$
$\overline{U} = 0,41\ m/s$
$\overline{D} = 0,51\ m$
$B = 52,7\ m$
$S = (0,000\ 19\ m/m)$
$E_z = 0,003\ 11\ m^2/s$
$U_* = (0,030\ m/s)$
$n = (0,023)$
$c = (37)$

X, m	M
183	0,24
533	0,39
1 082	0,44
2 240	0,51
3 429	0,71

11. Brenig, Test 1
Reference: [10]

$z = 0,5$
$q = —$
$Q = 0,34$ m^3/s
$\underline{U} = 0,21$ m/s
$\overline{D} = 0,24$ m
$B = 6,5$ m
$S = (0,017\ 0$ m/m)
$E_z = 0,009\ 60$ m^2/s
$U_* = 0,2$ m/s
$n = (0,220)$
$c = (1,6)$

X, m	M
30,5	0,92
61,0	0,99

12. Alwen, Test 2
Reference: [10]

$z = 0,5$
$q = —$
$Q = 0,23$ m^3/s
$\underline{U} = 0,33$ m/s
$\overline{D} = 0,17$ m
$B = 4,1$ m
$S = (0,006\ 0$ m/m)
$E_z = (0,003\ 40$ m^2/s)
$U_* = 0,10$ m/s
$n = (0,068)$
$c = (7,1)$

X, m	M
15,2	0,33
30,5	0,83
61,0	0,91
82,3	0,94

13. Alwen, Test 3
Reference: [10]

$z = 0,5$
$q = —$
$Q = 0,15$ m^3/s
$\underline{U} = 0,26$ m/s
$\overline{D} = 0,15$ m
$B = 3,9$ m
$S = (0,011\ 5$ m/m)
$E_z = (0,003\ 90$ m^2/s)
$U_* = 0,13$ m/s
$n = (0,112)$
$c = (3,3)$

X, m	M
18,3	0,77
48,8	0,96
94,5	0,996

14. Alwen, Test 4
Reference: [10]

$z = 0,5$
$q = —$
$Q = 0,15$ m^3/s
$\underline{U} = 0,26$ m/s
$\overline{D} = 0,14$ m
$B = 4,1$ m
$S = (0,005\ 9$ m/m)
$E_z = (0,002\ 52$ m^2/s)
$U_* = 0,09$ m/s
$n = (0,076)$
$c = (5,7)$

X, m	M
15,2	0,78
30,5	0,86
61,0	0,94
82,3	0,97

15. Alwen, Test 5
Reference: [10]

$z = 0{,}5$
$q = —$
$Q = 0{,}72\ m^3/s$
$\overline{U} = 0{,}34\ m/s$
$\overline{D} = 0{,}20\ m$
$B = 11{,}3\ m$
$S = (0{,}011\ 5\ m/m)$
$E_z = (0{,}006\ 00\ m^2/s)$
$U_* = 0{,}15\ m/s$
$n = (0{,}112)$
$c = (3{,}9)$

X, m	M
61,0 122,0	0,60 0,75

16. Wye, Test 6A
Reference: [10]

$z = 0{,}5$
$q = —$
$Q = 1{,}67\ m^3/s$
$\overline{U} = 0{,}78\ m/s$
$\overline{D} = 0{,}35\ m$
$B = 6{,}2\ m$
$S = (0{,}001\ 86\ m/m)$
$E_z = (0{,}005\ 60\ m^2/s)$
$U_* = 0{,}08\ m/s$
$n = (0{,}026)$
$c = (29)$

X, m	M
30,5 70,1 235,0	0,77 0,84 0,99

17. Devon, Test 7A
Reference: [10]

$z = 0{,}5$
$q = —$
$Q = 48{,}1\ m^3/s$
$\overline{U} = 0{,}62\ m/s$
$\overline{D} = 0{,}64\ m$
$B = 12{,}2\ m$
$S = (0{,}003\ 6\ m/m)$
$E_z = (0{,}019\ 2\ m^2/s)$
$U_* = 0{,}15\ m/s$
$n = (0{,}007)$
$c = (133)$

X, m	M
30,5 61,0 152,0 244,0	0,82 0,88 0,97 0,991

18. Hambledon Brook, Test 9
Reference: [10]

$z = 0{,}5$
$q = —$
$Q = 0{,}13\ m^3/s$
$\overline{U} = 0{,}23\ m/s$
$\overline{D} = 0{,}25\ m$
$B = 2{,}2\ m$
$S = (0{,}008\ 0\ m/m)$
$E_z = (0{,}007\ 00\ m^2/s)$
$U_* = 0{,}14\ m/s$
$n = (0{,}131)$
$c = (3{,}2)$

X, m	M
21,3 39,6 76,2	0,94 0,97 0,995

19. Missouri River near river mile 648, Iowa, USA
Reference: [10]

$z = —$
$q = 0{,}40$
$Q = 966\ \mathrm{m^3/s}$
$\bar{U} = 1{,}76\ \mathrm{m/s}$
$\bar{D} = 3{,}0\ \mathrm{m}$
$B = 183\ \mathrm{m}$
$S = (0{,}000\ 181\ \mathrm{m/m})$
$E_z = (0{,}043\ 8\ \mathrm{m^2/s})$
$U_* = 0{,}073\ \mathrm{m/s}$
$n = (0{,}016)$
$c = (74)$

X, m	M
2 660	0,42
3 610	0,56
4 720	0,59
5 710	0,66
6 830	0,69
8 440	0,78
10 050	0,82

20. Missouri River near river mile 532, Iowa, USA
Reference: [10]

$z = —$
$q = 0{,}975$
$Q = 1{,}590\ \mathrm{m^3/s}$
$\bar{U} = 1{,}78\ \mathrm{m/s}$
$\bar{D} = 4{,}0\ \mathrm{m}$
$B = 223\ \mathrm{m}$
$S = (0{,}000\ 184\ \mathrm{m/m})$
$E_z = (0{,}068\ 0\ \mathrm{m^2/s})$
$U_* = 0{,}085\ \mathrm{m/s}$
$n = (0{,}019)$
$c = (67)$

X, m	M
1 180	0,40
2 000	0,39
2 780	0,45
3 590	0,60
5 200	0,54
6 810	0,71
8 420	0,71
9 850	0,92

21. Ijssel near Rk 883
Reference: [10]

$z =$ (a) 1,00 (b) 0,0
$q =$ (a) 1,00 (b) 0,0
$Q = 275\ \mathrm{m^3/s}$
$\bar{U} = 0{,}99\ \mathrm{m/s}$
$\bar{D} = 3{,}66\ \mathrm{m}$
$B = 76\ \mathrm{m}$
$S = (0{,}000\ 149\ \mathrm{m/m})$
$E_z = (0{,}053\ 4\ \mathrm{m^2/s})$
$U_* = 0{,}073\ \mathrm{m/s}$
$n = (0{,}028)$
$c = (46)$

X, m	M
(a) 500	0,49
1 350	0,66
2 400	0,65
3 500	0,69
4 400	0,81
(b) 500	0,26
1 350	0,40
2 400	0,57

22. Waal near Rk 930
Reference: [10]

$z =$ (a) 1,00 (b) 0,0
$q =$ (a) 1,00 (b) 0,0
$Q = 1\ 010\ \mathrm{m^3/s}$
$\bar{U} = 0{,}82\ \mathrm{m/s}$
$\bar{D} = 4{,}41\ \mathrm{m}$
$B = 283\ \mathrm{m}$
$S = (0{,}000\ 075\ \mathrm{m/m})$
$E_z = (0{,}050\ 3\ \mathrm{m^2/s})$
$U_* = 0{,}057\ \mathrm{m/s}$
$n = (0{,}028)$
$c = (49)$

X, m	M
(a) 1 880	0,26
4 180	0,31
5 890	0,39
9 830	0,42
(b) 1 540	0,13
3 840	0,28
5 560	0,32
9 490	0,48

Annex B
(informative)

Comparison of computed and observed mixing distances

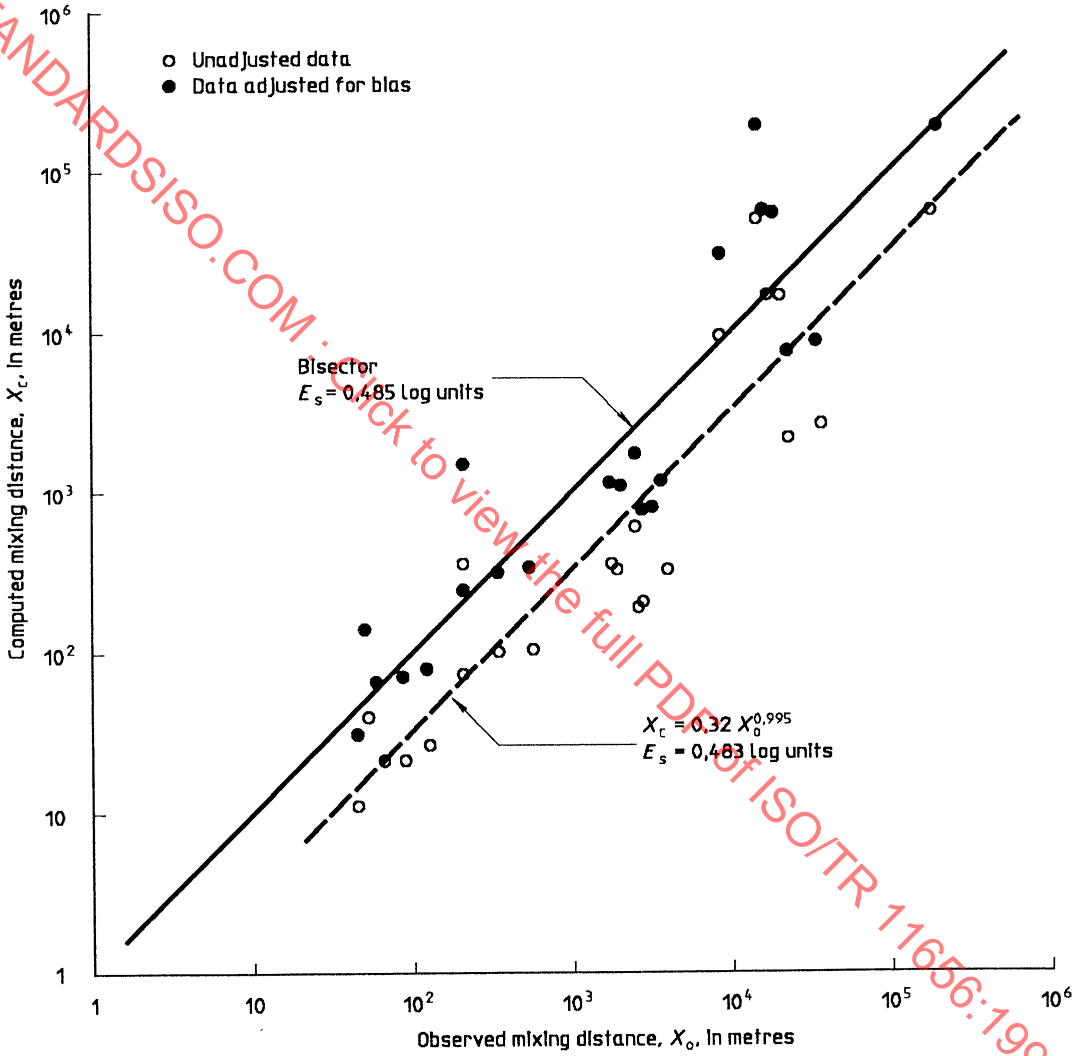


Figure B.1 — Observed mixing distances for 98 % **mixing versus mixing distances computed by the André equation** (see 6.3.1)

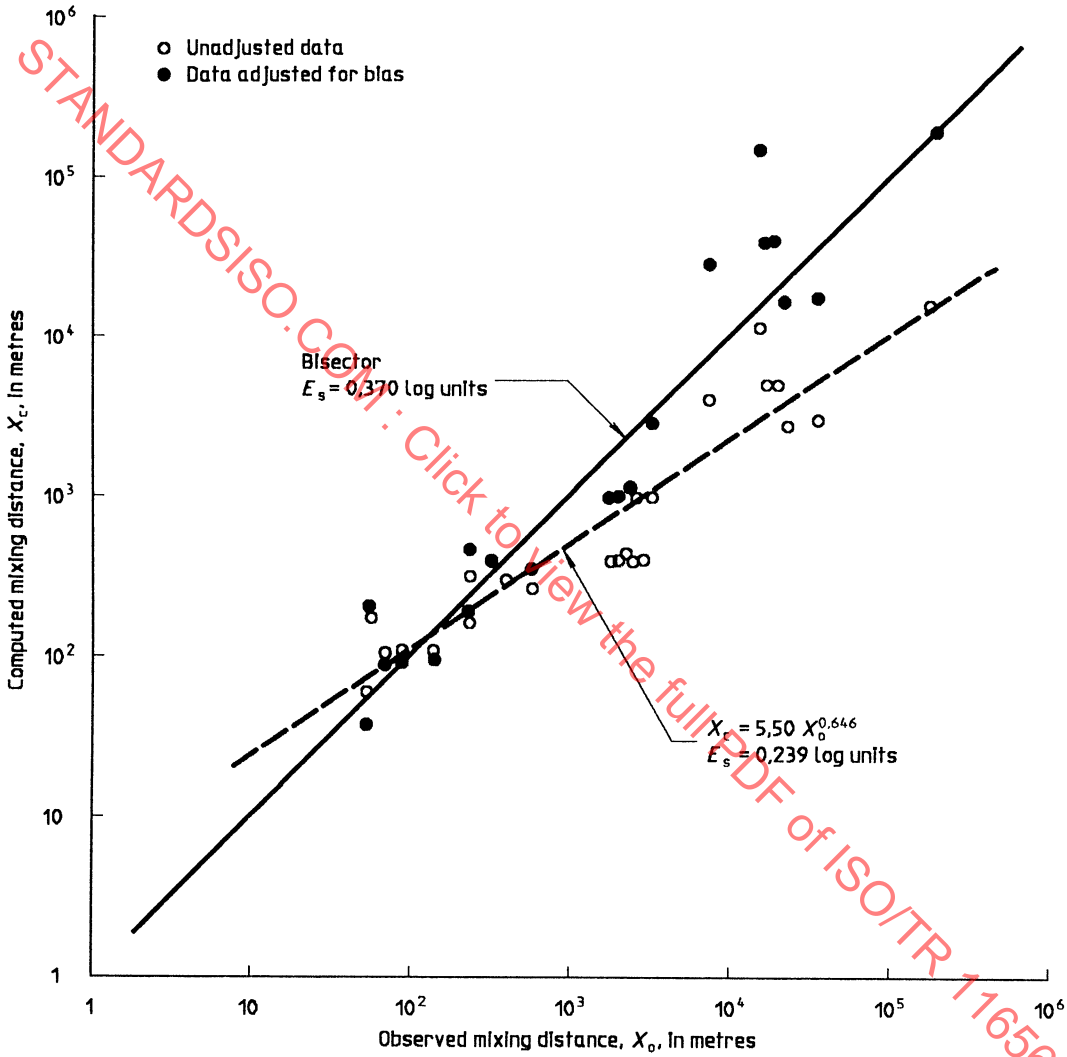


Figure B.2 — Observed mixing distances for 98 % **mixing versus mixing distances computed by the Day formula** (see 6.3.1)

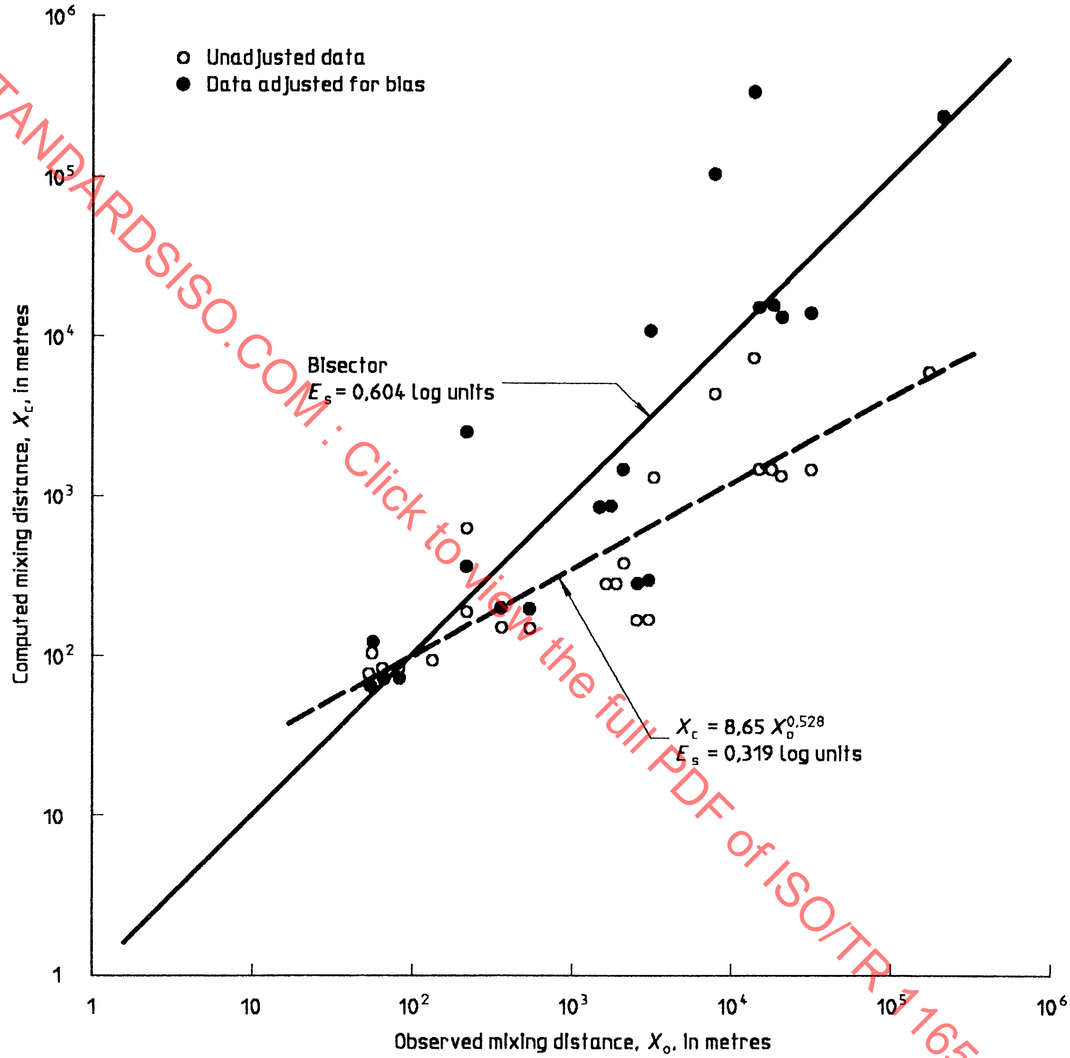


Figure B.3 — Observed mixing distances for 98 % **mixing versus mixing distances computed by the Hull formula** (see 6.3.1)

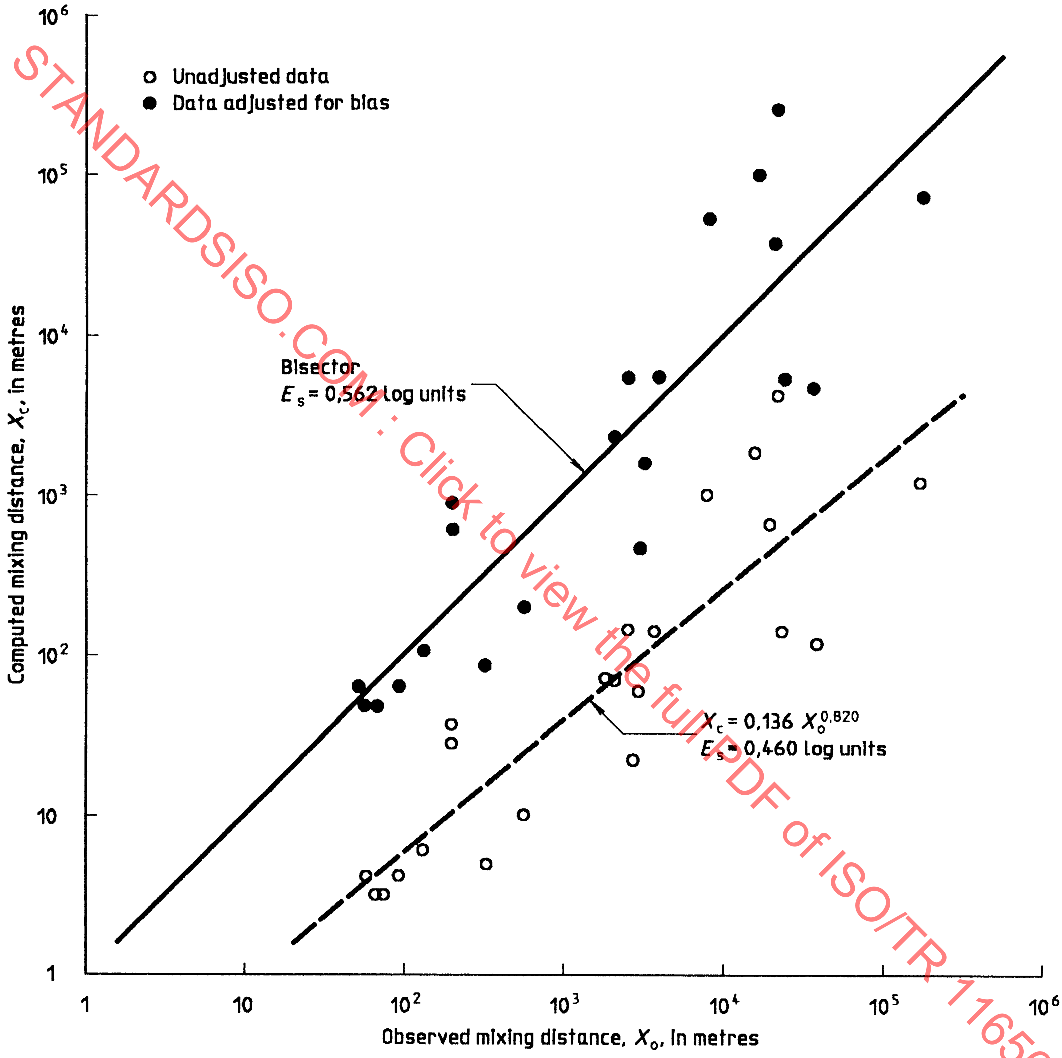


Figure B.4 — Observed mixing distances for 98 % **mixing versus mixing distances computed by the Elder formula** (see 6.3.2)