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Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes —

Part 2:
Rating methods

Transmissions hydrauliques — Essais de fatigue des enveloppes métalliques sous pression —

Partie 2: Méthodes de classement

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

In exceptional circumstances, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example), it may decide by a simple majority vote of its participating members to publish a Technical Report. A Technical Report is entirely informative in nature and does not have to be reviewed until the data it provides are considered to be no longer valid or useful.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO/TR 10771-2 was prepared by Technical Committee ISO/TC 131, *Fluid power systems*, Subcommittee SC 8, *Product testing*.

ISO/TR 10771 consists of the following parts, under the general title *Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes*:

— *Part 1: Test method*

— *Part 2: Rating methods*

Introduction

In hydraulic fluid power systems, power is transmitted and controlled under pressure within a closed circuit. It is important for the manufacturer and user of hydraulic components to have information on their global reliability because of the importance of the fatigue failure mode and the relationship with their functional safety and service life. This part of ISO 10771 provides a method for fatigue-testing in order to verify the rating of a pressure-containing envelope.

During operation, components in a system can be subjected to loads that arise from:

— internal pressure;

— external forces;

— inertia and gravitational effects;

— impact or shock;

— temperature changes or gradients.

The nature of these loads can vary from a single static application to continuously varying amplitudes, repetitive loadings and even shocks. It is important to know how well a component can withstand these loads, but this part of ISO 10771 addresses only the loads due to internal pressure.

There are several International Standards already in existence for pressure rating of individual components (e.g. for determining maximum allowable rated pressure) and this part of ISO 10771 is not intended to replace them. Instead, a method of fatigue verification is provided.

This part of ISO 10771 describes a universal verification test to give credibility to the many in-house and other methods of determining the pressure rating of the components. Credibility is based upon the fundamental nature of metal fatigue with its statistical treatment and a mathematical theory of statistical verification. Nevertheless, it is necessary to have design knowledge of the component and its representative specimens to maximize accuracy of the verification method. The use of this test method can reduce the risk of fatigue failure for a hydraulic component regardless of sample size.

In order to rate components in accordance with this part of ISO 10771, it is necessary to propose a rating for the component, select test specimens and select a test pressure. A fatigue test is then conducted in accordance with ISO 10771-1. If the test is successful, the proposed rating is verified for the family of components represented by the sample.

This part of ISO 10771 is based on ANSI/(NFPA) T 2.6.1, a standard which was developed and has been used in the United States for over 25 years and has been adopted for use in Japan as JSME S006-1985. If sufficient experience is gained in other parts of the world, and additional data on materials are obtained, this part of ISO 10771 might be re-drafted as an International Standard in the future.

It should be noted that the test factors in Annex A are based on material data obtained from sources originating in the USA. One of the objectives in issuing this part of ISO 10771 is to obtain material data from other countries. The test factors are based only on the material properties and not on any tolerances of the elements in the pressure-containing envelope.

Annex C describes a possible method for accelerating testing. The example shows how material property data can be used to determine an acceleration factor and shows that they have to be carefully chosen. Another objective of this part of ISO 10771 is to seek additional data as described in Annex C. Contributors are asked to submit any available data to the secretary of ISO TC 131/SC 8.

Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes —

Part 2:
Rating methods

1   Scope

This part of ISO 10771 specifies a test method for fatigue rating of the pressure-containing envelopes of components used in hydraulic fluid power systems, as tested under steady internal cyclic pressure loads in accordance with ISO 10771-1.

This part of ISO 10771 is only applicable to components whose failure mode is the fatigue of any element in the pressure-containing envelope, and that:

⎯ are manufactured from metals;

⎯ are operated at temperatures that exclude creep and low-temperature embrittlement;

⎯ are only subjected to pressure-induced stresses;

⎯ are not subjected to loss of strength due to corrosion or other chemical action;

⎯ can include gaskets, seals and other non-metallic components; however, these are not considered part of the pressure-containing envelope being tested (see note 3 of 5.5 of ISO 10771-1:2002).

This part of ISO 10771 does not apply to piping as defined in ISO 4413 (i.e. connectors, hose, tubing, pipe).

NOTE   See ISO 19879, ISO 6803 and ISO 6605 for methods of fatigue testing of tube connectors, hoses and hose assemblies.

This part of ISO 10771 establishes a general rating method that can be applied to many hydraulic fluid power components. In addition, EN 14359 has been developed for accumulators.

2   Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 5598, *Fluid power systems and components — Vocabulary*

ISO 10771-1:2002, *Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes — Part 1: Test method*

3   Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 5598, ISO 10771-1 and the following apply.

3.1
rated fatigue pressure
P_{RF}
maximum pressure that a component pressure-containing envelope, selected at random, has been verified to sustain for the rated cycle life without failure, with a known probability

3.2
assurance level
probability that the fatigue strength of a randomly selected test specimen exceeds its rated fatigue pressure

3.3
verification level
probability that the fatigue strength of a randomly selected test specimen is not less than its cyclic test pressure

3.4
coefficient of variation
k_o
standard deviation of the fatigue strength distribution of a material at a given fatigue life, divided by its mean

NOTE   Adapted from ISO 3534-1:2006 [1].

3.5
variability factor
K_V
ratio of cyclic test pressure to rated fatigue pressure

3.6
element
part of a component; for example, tie rods on a cylinder, end caps on a valve, bolts on a pump housing

4   Selection of material factors

4.1   Select a coefficient of variation, k_o, for each type of material in the pressure-containing envelope. The k_o factor should be obtained from fatigue tests on coupons for the particular temper of material used in the pressure-containing envelope. The fatigue test method used to obtain this data should be in accordance with a recognized national or international Standard.

4.2   As an alternative to testing the specific material, coefficients described in Annex A can be used for the k_o factor.

5   Determination of cyclic test pressure

5.1   Select an assurance level for the fatigue pressure rating. A nominal value is 90 %.

5.2   Select a verification level for the fatigue pressure rating. A nominal value is 90 %.

NOTE   See Annex D for a tutorial that describes these terms.

5.3   Select a number of component specimens to be tested, then determine the number of element specimens that will be tested in the components.

NOTE   The verification is independent of sample size because the test pressure compensates for different quantities.

5.4   Determine the variability factor, K_V, for each element in the component using Table 1 and the procedure described in the example given in Annex B. Use the largest K_V factor so obtained, for the calculations described in the example.

5.5   Propose a rated fatigue pressure for the pressure-containing envelope of the component.

5.6   Calculate the cyclic test pressure, P_{CT}, using Equation (1):

$$P_{CT} = K_V \times P_{RF} \qquad (1)$$

where

K_V is the variability factor;

P_{RF} is the rated fatigue pressure of the component pressure-containing envelope.

Table 1 — Variability factor, K_V (at a verification level of 90 %)

Assurance level	No. of specimens n [a]	Material coefficient of variation, k_o [b]															
		0	0,02	0,04	0,06	0,08	0,10	0,12	0,14	0,16	0,18	0,20	0,22	0,24	0,26	0,28	0,30
99,9 %	1	1,00	1,09	1,20	1,32	1,46	1,63	1,83	2,08	2,38	2,77	3,29	—	—	—	—	—
	2	1,00	1,08	1,16	1,26	1,38	1,52	1,68	1,88	2,13	2,45	2,87	—	—	—	—	—
	3	1,00	1,07	1,15	1,23	1,34	1,46	1,61	1,78	2,01	2,29	2,66	3,18	—	—	—	—
	4	1,00	1,06	1,13	1,22	1,31	1,42	1,56	1,72	1,93	2,19	2,54	3,02	—	—	—	—
	5	1,00	1,06	1,13	1,20	1,29	1,40	1,53	1,68	1,87	2,12	2,44	2,89	—	—	—	—
99 %	1	1,00	1,08	1,16	1,25	1,35	1,47	1,60	1,75	1,92	2,12	2,35	2,63	2,96	—	—	—
	2	1,00	1,06	1,12	1,20	1,28	1,37	1,47	1,58	1,72	1,87	2,05	2,26	2,52	2,85	—	—
	3	1,00	1,05	1,11	1,17	1,24	1,32	1,40	1,50	1,62	1,75	1,90	2,09	2,31	2,59	2,94	—
	4	1,00	1,05	1,10	1,15	1,21	1,28	1,36	1,45	1,55	1,67	1,81	1,98	2,18	2,43	2,74	3,16
	5	1,00	1,04	1,09	1,14	1,20	1,26	1,33	1,41	1,51	1,62	1,75	1,90	2,08	2,31	2,60	2,98
90 %	1	1,00	1,05	1,11	1,17	1,23	1,29	1,36	1,44	1,52	1,60	1,69	1,79	1,89	2,00	2,12	2,25
	2	1,00	1,04	1,07	1,11	1,16	1,20	1,25	1,30	1,35	1,41	1,47	1,54	1,61	1,69	1,77	1,86
	3	1,00	1,03	1,06	1,09	1,12	1,16	1,19	1,23	1,28	1,32	1,37	1,42	1,48	1,54	1,60	1,67
	4	1,00	1,02	1,05	1,07	1,10	1,13	1,16	1,19	1,23	1,26	1,30	1,34	1,39	1,44	1,49	1,55
	5	1,00	1,02	1,04	1,06	1,08	1,11	1,13	1,16	1,19	1,22	1,25	1,29	1,33	1,37	1,41	1,46

[a]   Test twice the number of specimens if a 99 % verification level is chosen.

[b]   Use an interpolation of k_o values between those tabulated here, or calculate K_V from Equation (D.14) in Annex D.

6   Conduct of fatigue test

6.1   Determine the number of cycles, between 1×10^5 and 1×10^7, for which the component will be rated.

6.2   Subject the test specimens to a fatigue pressure test in accordance with ISO 10771-1 for the number of cycles determined in 6.1, using the P_{CT} calculated from Equation (1).

6.3 The fatigue pressure test is successful if all of the element specimens selected in 5.3 do not fail as described in ISO 10771-1:2002, Clause 8.

7 Rating by similarity

It is permitted to extend a verified P_{RF} to other components of similar shape if it can be shown that differences between those components and the components tested do not result in any reduction of their fatigue strength capabilities. Examples of this are components that have smaller ports or different axial lengths but are otherwise identical in geometry to the component tested.

8 Rating declaration

The P_{RF} proposed in 5.5 will be verified if the requirements of 6.3 are met. A code should be applied to the component to declare its rating as:

$P_{RF} = P_{RF}$ (in megapascals)/assurance level/verification level/K_V in the test/number of test cycles

EXAMPLE   The rated fatigue pressure (12,5 MPa) of a component's pressure-containing envelope that was tested at an assurance level of 99 %, a verification level of 90 %, a K_V of 1,36 for 2×10^6 cycles, would be declared as:

$P_{RF} = 12,5$ MPa/ 99 %/ 90 %/ 1,36/ 2×10^6 cycles

9 Identification statement (reference to this part of ISO 10771)

Use the following statement in test reports, catalogues and sales literature when complying with this part of ISO 10771:

"Method for fatigue pressure rating conforms to ISO TR 10771-2:2008, *Hydraulic fluid power — Fatigue pressure testing of metal pressure-containing envelopes — Part 2: Rating methods*".

Annex A
(informative)

Material factor database

A.1 Values of coefficient of variation, k_o, for commonly used metals

Table A.1 tabulates data calculated from the sources listed in the bibliography.

Table A.1 — Values of coefficient of variation, k_o, for commonly used metals

Metal		k_o
Steel	Alloy, low	0,14
	Carbon, plain	0,08
	Nickel	0,10
	Stainless	0,09
	Tool	0,10
Iron		0,14
Nonferrous	Aluminium (except unalloyed)	0,13
	Unalloyed aluminium	0,23
	Cobalt	0,13
	Copper	0,09
	Magnesium	0,17
	Monel[1]	0,27
	Titanium	0,12

A.2 Procedures used to establish values of coefficient of variation, k_o, for the metals listed in Table A.1

A.2.1 Values of k_o were calculated from fatigue test data on test coupons that were published in the references cited in the bibliography. The types of data taken from these references were one of the following:

a) Means, μ, and standard deviations, σ, of normal distributions;

b) parameters of Weibull distributions;

c) raw data points on S-N curves. From these data, individual coefficients of variation, k_o, were calculated at 10^6 cycles for:

1) normal distributions; k_o equals the standard deviation divided by the mean;

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1) This is an example of a suitable product available commercially. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of this product.

2) Weibull distributions; k_o were calculated from a formula given in Reference [12]. The formula includes a gamma function, the value of which was selected as a constant at 0,89 because its variations were generally less than $\pm$ 2 % in the range of interest (a few data points went to a difference of $\pm$ 4 %);

3) *S-N* curves; the references had either included limit bands (assumed to be two sigma from the mean) or actual standard deviation points. These were then used to calculate k_o in the same manner as a normal distribution.

A.2.2   The resulting k_o values (shown as individual values in Table A.2 to Table A.13) include a mix of notched and unnotched specimens, several different tempers, plus different methods of testing (e.g. axial, rotating beam). However, only those tested at room temperature were used. No attempt was made to segregate these data. It is reasoned that the components to be tested will have a variety of tempers and notches, so an application of these published data to components can only be justified if the data are treated statistically at a conservative value.

A.2.3   Therefore, the values given in Table A.1 were derived by assuming that all k_o data for a particular metal group are part of a normal distribution, and a value that is greater than 90 % of this distribution was selected. This ensures that the selection is substantially conservative. However, this part of ISO 10771 allows the use of a more accurate k_o value, which is representative of the specific alloy and temper of the elements being tested, if sufficient testing is performed to obtain those data, as described in 4.1. This approach will likely yield a value that would be more advantageous for a particular application, but less than the conservative values presented in Table A.1.

A.2.4   Table A.2 to Table A.13 describe all of the k_o calculations made from the data obtained from Reference [10], Reference [11], and Reference [13] to Reference [17]. Most of the data are based on the strength distribution at 10^6 cycles, but some data are at the endurance limit and these are identified in each table, if applicable.

Table A.2 — Summary of k_o calculations for iron

Type	Reference	Number of distributions	k_o values [a]
Armco[1)]	[11]	1	0,0220
Pearlitic (grey)	[15];[17]	8	0,0402; 0,042[a]; 0,044[a]; 0,0652; 0,126[a]; 0,146[a]; 0,137[a]
Ferritic (malleable)	[15];[17]	7	0,055[a]; 0,0596; 0,0649; 0,065[a]; 0,075[a]; 0,109[a]
Nodular	[15]	10	0,029[a]; 0,040[a]; 0,049[a]; 0,065[a]; 0,086[a]; 0,094[a]; 0,095[a]; 0,098[a]; 0,173[a]; 0,185[a]
Fe; 5,5 % Mo; 2,5 % Cr; 0,5 % C	[13]	2	0,0286; 0,0477
Summary of all data above	[13];[15];[11];[17]	28	(k_o) 90 % = 0,1335; (μ = 0,0771; σ = 0,0440) Conclusion: k_o value selected = 0,14
[a] Data from reference [15] are at endurance limit.			

Table A.3 — Summary of k_o calculations for aluminium

Alloy	Reference	Number of distributions	k_o values [a]
Duraluminum[1)	[13]	1	0,0720
356	[15]	2	0,038; 0,042
355	[14]	1	0,0766
1100	[14]	2	0,1742; 0,2377
2014	[14];[15]	13	0,017; 0,0400; 0,0527; 0,0534; 0,0541; 0,0556; 0,0702; 0,0732; 0,1152; 0,1164; 0,1215; 0,1386; 0,1400
2024	[14];[15]	14	0,026; 0,0498; 0,0542; 0,0561; 0,0613; 0,0708; 0,0717; 0,0765; 0,0825; 0,0974; 0,1039; 0,1190; 0,1404; 0,1840
2025	[14]	3	0,0347; 0,0549; 0,0947
2026	[14]	2	0,0507; 0,0834
2219	[14]	2	0,0701; 0,0705
5052	[14]	2	0,0845; 0,0914
5056	[14]	1	0,0947
5086	[14]	1	0,0640
5154	[14]	1	0,0662
5456	[14];[15]	2	0,012; 0,0708
6061	[14];[15]	4	0,018; 0,027; 0,0478; 0,087
7039	[14]	1	0,1405
7075	[14];[15]	8	0,040; 0,0505; 0,059; 0,0615; 0,0689; 0,0906; 0,1686; 0,2157
7076	[10]	2	0,0413[a]; 0,0593[a]
7079	[14]	3	0,0560; 0,0942; 0,1486
7178	[14]	2	0,0484; 0,0881
R303	[14]	1	0,0434
5 Mg Al	[14]	2	0,0934; 0,1302
7,5 Zn 2,5 Mg Al	[14]	1	0,0570
Summary of all data above	[13];[14];[15];[10]	71	(k_o) 90 % = 0,1390; $\mu = 0,0811$; $\sigma = 0,0452$
Remove the 1100 data		69	(k_o) 90 % = 0,1288; $\mu = 0,0775$; $\sigma = 0,0400$ Conclusion: select k_o = 0,13 for alloyed aluminium; select k_o = 0,23 for unalloyed aluminium
[a] Data from reference [10] are at endurance limit.			

**Table A.4 — Summary of k_o calculations for low alloy steels
(containing silicon at less than 1 % and 1 or more of the following: nickel - less than 4 %;
chrome - less than 2 %; molybdenum - less than 0,5 %)**

Alloy	Reference	Number of distributions	k_o values [a]
2340	[13]	6	0,0190; 0,0296; 0,0311; 0,0374; 0,0622 0,0696
3140	[13]	4	0,0145; 0,0283; 0,0435; 0,0919
4140	[13]	2	0,0650; 0,1102
4330	[10]	6	0,0313; 0,0372; 0,0498; 0,0644; 0,1063; 0,1129
4340	[13]	14	0,0525; 0,0819; 0,1023; 0,1037; 0,1219; 0,1285; 0,1301; 0,1335; 0,1428; 0,1438; 0,1476; 0,1484; 0,1600; 0,2035
4340	[11]	4	0,0253; 0,0301; 0,0321; 0,0627
4340	[10]	10	0,0497; 0,0509; 0,0607; 0,0608; 0,0640; 0,0795; 0,0822; 0,0833; 0,0880; 0,0966
4350	[10]	8	0,0759; 0,0837; 0,0878; 0,0895; 0,1025; 0,1055; 0,1209; 0,1213
AMS 5727	[13]	3	0,0341; 0,0385; 0,1002
Summary of all data above	[13];[11];[10]	57	(k_o) 90 % = 0,1348; μ = 0,0812; σ = 0,0418 Conclusion: k_o value selected = 0,14
[a] Data from reference [10] are at endurance limit.			

Table A.5 — Summary of k_o calculations for cobalt

Metal and alloy	Reference	Number of distributions	k_o values
Stellite 31[1)	[14]	3	0,0771; 0,1202; 0,1472
S-816 (AMS5765)	[13];[14]	4	0,0448; 0,0456; 0,0730; 0,0777
S-816 (AMS5534)	[13]	1	0,0646
Summary of all data above	[13];[14]	8	(k_o) 90 % = 0,1269; μ = 0,0813; σ = 0,0355 Conclusion: k_o value selected = 0,13

Table A.6 — Summary of k_o calculations for copper

Metal and alloy	Reference	Number of distributions	k_o values [a]
100 % Cu	[14]	2	0,0522; 0,0804
70/30 Brass	[14]	2	0,0153; 0,0735
Cu-7,3 Al BRNZ	[14]	2	0,0496; 0,1037
Al Ni BRNZ	[10]	1	0,0938[a]
Beryllium	[10]	1	0,0740[a]
Copper casting alloys	[16]	8	0,0153; 0,0173; 0,0181; 0,0185; 0,0200; 0,0211; 0,0234; 0,0514
Summary of all data above	[14];[16];[10]	16	(k_o) 90 % = 0,0853; μ = 0,0455; σ = 0,0311 Conclusion: k_o value selected = 0,09

[a] Data from reference [10] are at endurance limit.

Table A.7 — Summary of k_o calculations for magnesium

Metal and alloy	Reference	Number of distributions	k_o values
AZ31A	[14]	1	0,0331
AZ31B	[14]	1	0,0714
AZ61A	[14]	3	0,0854; 0,1705; 0,1735
AZ80A-F	[14]	1	0,0457
AZ81	[14]	2	0,0458; 0,0761
ZK60A	[14]	1	0,1053
2,5 Al Mg	[14]	3	0,1152; 0,1444; 0,1702
Summary of all data above	[14]	12	(k_o) 90 % = 0,1694; μ = 0,1031; σ = 0,0517 Conclusion: k_o value selected = 0,17

Table A.8 — Summary of k_o calculations for plain carbon steel

Group and alloy	Reference	Number of distributions	k_o values
1045	[13]	3	0,0273; 0,0581; 0,0682
1050	[11]	1	0,0171
Summary of all data above	[13];[11]	4	(k_o) 90 % = 0,0739; μ = 0,0427; σ = 0,0244 Conclusion: k_o value selected = 0,08

Table A.9 — Summary of k_o calculations for stainless steel

Metal and alloy	Reference	Number of distributions	k_o values
321	[13]	3	0,0439; 0,0606; 0,0755
A-286	[13]	2	0,0958; 0,1303
347	[13]	4	0,0302; 0,0491; 0,0802; 0,1162
Multimet N-155[1]	[13]	14	0,0163; 0,0180; 0,0230; 0,0313; 0,0313; 0,0315; 0,0325; 0,0367; 0,0381; 0,0407; 0,0427; 0,0544; 0,0547; 0,0574
PH 15-7	[13]	2	0,0676; 0,0936
17-7 PH	[13]	4	0,0135; 0,0145; 0,0168; 0,0505;
403	[13]	2	0,0160; 0,0381
Summary of all data above	[13]	31	(k_o) 90 % = 0,0868; μ = 0,0484; σ = 0,0300 Conclusion: k_o value selected = 0,09

Table A.10 — Summary of k_o calculations for tool steel

Metal and alloy	Reference	Number of distributions	k_o values[a]
Tricent[1]	[10]	3	0,0350[a];0,0483[a]; 0,0513[a]
Ferrovac[1]	[10]	1	0,0909[a]
H-23	[10]	1	0,0648[a]
M-10	[13];[10]	6	0,0672; 0,0704[a]; 0,0707; 0,0714[a]; 0,1032; 0,1087
D (6AC)	[13]	4	0,0400; 0,0546; 0,0572; 0,1178; 0,0258; 0,0266
H-11	[13]	19	0,0307; 0,0325; 0,0399; 0,0444; 0,0544; 0,0555; 0,0633; 0,0635; 0,0690; 0,0726; 0,0760; 0,0766; 0,0798; 0,0843; 0,0923; 0,1195; 0,1219
Thermold J[1]	[13]	2	0,0251; 0,0724
Timken 16-25-6[1]	[13]	1	0,0231
Summary of all data above	[13];[10]	37	(k_o) 90 % = 0,0999; μ = 0,0649; σ = 0,0273 Conclusion: k_o value selected = 0,10
[a] Data from reference [10] are at endurance limit.			

Table A.11 — Summary of k_o calculations for titanium

Alloy	Reference	Number of distributions	k_o values
Ti-140 (AMS 493)	[13]	1	0,0956
Ti-6Al-4V	[13];[14]	13	0,0385; 0,0480; 0,0514; 0,0514; 0,0556; 0,0557; 0,0616; 0,0741; 0,0792; 0,0841; 0,0882; 0,1086; 0,1226
Ti-A55	[14]	8	0,0272; 0,0507; 0,0637; 0,0794; 0,0837; 0,1003; 0,1033; 0,1515
Ti-75A	[14]	6	0,0298; 0,0380; 0,0413; 0,0508; 0,0621; 0,0626
Ti-150A	[14]	2	0,0707; 0,0882
Ti-0,2 (0_2)	[14]	3	0,0422; 0,0461; 0,0906
Ti-0,2C	[14]	1	0,0691
Ti-4Al-3Mo-1V	[14]	5	0,0961; 0,0979; 0,1338; 0,1450; 0,2041
Ti-5Al-2,5Sn-0,07(N_2)	[14]	2	0,0581; 0,0697
Ti-5Al-2,5Sn-0,2(0_2)	[14]	1	0.07760
Ti-6Al	[14]	4	0,0442; 0,0472; 0,0480; 0,0821
Summary of all data above	[13];[14]	46	(k_o) 90 % = 0,1203 ; μ = 0,0754; σ = 0,0350 Conclusion: k_o value selected = 0,12

Table A.12 — Summary of k_o calculations for nickel steel (nickel content at least 40 %)

Alloy	Reference	Number of distributions	k_o values
GMR-235	[13]	1	0,0301
Udimet 500[1]	[13]	1	0,0642
Hastelloy C[1]	[14]	1	0,0753
Hasteloy R235	[14]	2	0,0592; 0,1166
Incoloy 901[1] (AMS 5560A)	[14]	1	0,0570
Inconel 718[1]	[14]	3	0,0705; 0,0933; 0,0952
Waspaloy[1]	[14]	1	0,0714
Rene-41 (AMS 5713)[1]	[14]	3	0,0336; 0,0741; 0,0876
6 Mo Waspaloy[1]	[14]	2	0,0327; 0,0741
Summary of all data above	[13];[14]	15	(k_o) 90 % = 0,1003; μ = 0,0690; σ = 0,0244 Conclusion: k_o value selected = 0,10

Table A.13 — Summary of k_o calculations for Monel

| Alloy	Reference	Number of distributions	k_o value
Monel	[14]	1	0,2643 Conclusion: k_o value selected = 0,27

Annex B
(normative)

Calculation of variability factor K_V

B.1 General

Annex B provides an example of how to calculate variability factor K_V.

B.2 Method

Consider a simple pressure-containing envelope consisting of a cylindrical tube with a square head on each end, held together by tie rods and nuts in each of the square's four corners (see Figure B.1).
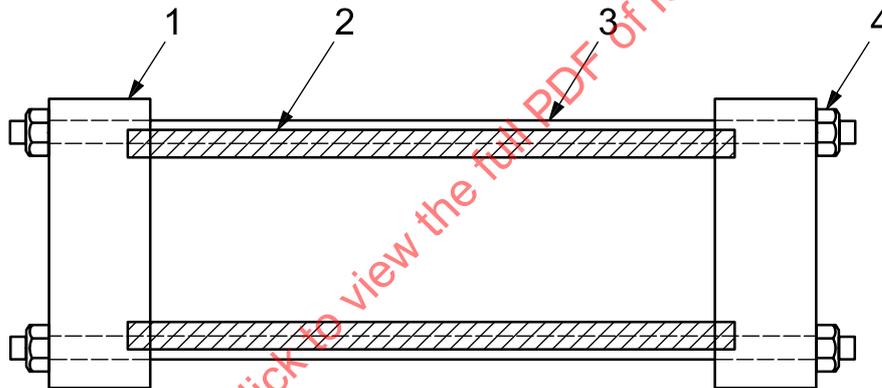
Key
1   square head
2   cylinder tube
3   tie rod
4   tie rod nuts

Figure B.1 — Pressure-containing envelope

The test operator has chosen an assurance level of 90 % and a verification level of 99 %. Information needed to obtain K_V values from Table 1 is shown in Table B.1.

Table B.1 — Other information needed to calculate K_V

Element	Material	k_o	Number of actual test specimens	Number of specimens chosen from Table 1	K_v
Cylinder tube	Aluminium	0,13	2	1	1,40
End head	Magnesium	0,17	4	2	1,38
Tie rod	Steel	0,08	8	4	1,10
Tie rod nut	Steel	0,08	16	See example	1,05

The value of K_V for the last row in Table B.1 is calculated as described in the example that follows. Equation (D.6) in Annex D is used, and is shown here as Equation (B.1).

$$K_V = \frac{1 + k_o Z_4}{1 - k_o Z_2}$$

(B.1)

EXAMPLE   The value of K_V for the tie rod nut was calculated as:

For an assurance level of 90 %, tail area $A_2 = 0,10$; then, $Z_2 = 1,282$.

For a verification level of 99 %, tail area $A_1 = 0,01$, $n = 16$.

$(A_1 + A_4) = A_1^{1/n} = (0,01)^{1/16} = 0,7499$, and $Z_4 = -0,674$

$$K_V = \frac{1 + (0,08)(-0,674)}{1 - (0,08)(1,282)} = 1,054 \text{ which is rounded to } 1,05$$

NOTE 1 $Z_4 = 0$ at $(A_1 + A_4) = 0,5$ and is negative above 0,5.

In the case described above, the highest value of K_V for any of the elements is 1,40, therefore, this is the value of K_V that is used to calculate the P_{CT}.

NOTE 2   The highest K_V value does not always correspond to the highest k_o value.

NOTE 3   When a verification level of 99 % is chosen, the number of test specimens is always an even number.

Annex C
(informative)

Proposal for an acceleration factor

C.1 General

It is proposed that a product can be rated for 10^7 cycles, but tested for only 10^6 cycles if it is tested at a higher pressure.

If material data for a fatigue strength distribution at both the 10^6 and 10^7 lives are available, then an acceleration factor can be determined. This acceleration factor would be a simple ratio of the fatigue strength at 10^6 cycles, to that at 10^7 cycles. The acceleration factor would be applied to the test pressure in order to raise the stress level in the test samples.

Data at the characteristic life are often available for Weibull distributions, and ratios of the characteristic strengths between those two levels can be used to calculate the acceleration factor. Likewise, data at the median life are often available for Normal distributions, and ratios of the median strengths between those two levels can be used for the acceleration factor.

For example, an AISI 4140 steel with a Weibull distribution at 10^6 cycles has a characteristic fatigue strength of 530 MPa; and the distribution at 10^7 cycles has a characteristic fatigue strength of 486 MPa. The ratio of these fatigue strengths is 1.09. Therefore, the test pressure would be raised by this factor if a 10^7 cycle rating was desired with a 10^6 cycle test. This factor would be in addition to any other factors imposed on the test pressure.

Considerable judgment would be required in proposing a pressure rating when using acceleration factors, because the probabilities of failure during the test are greater.

C.2 Extrapolating data to 10^7 cycles

The number of fatigue strength distributions available at 10^7 cycles is not very abundant. Therefore, a method to extrapolate data to 10^7 cycles is proposed if data are available at three lower levels, but spaced well apart, such as 10^4, 10^5 and 10^6 cycles.

Consider an *S-N* curve as shown in Figure C.1:
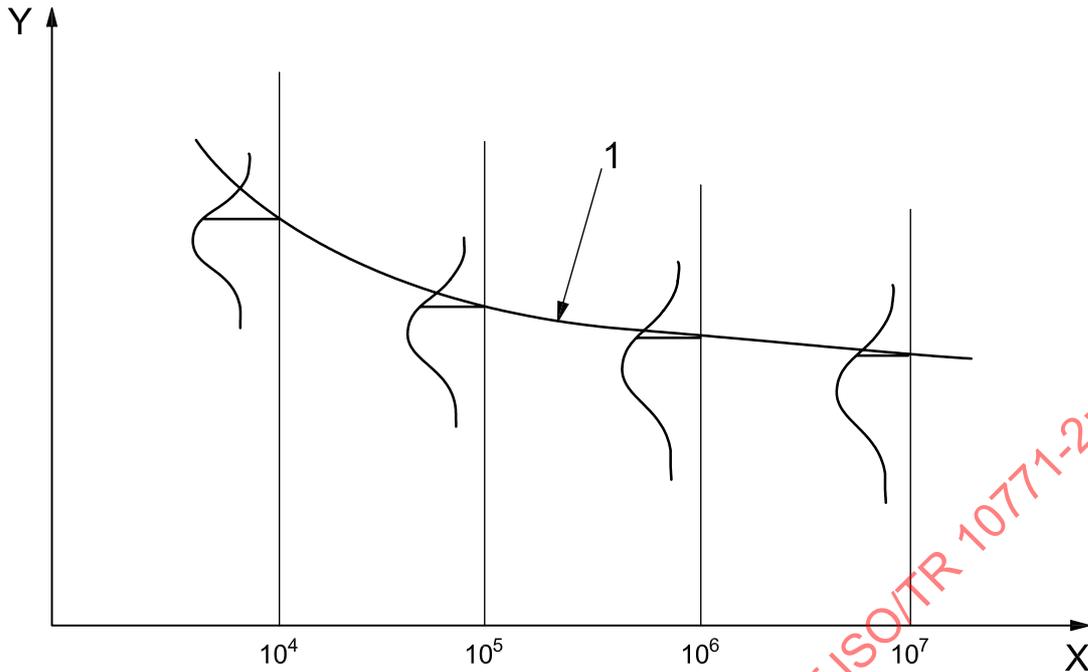
Key

X   life, expressed in number of cycles

Y   applied stress from testing; and material strength from failures

1   $S = CN^{-D} + E$, where S is strength, N is life in cycles; C, D and E are coefficients

Figure C.1 — *S-N* curve

At each level of life, there is a fatigue strength distribution. A constant probability curve joins each point of the fatigue strength from each of the distributions, and is suggested to have an equation of the form shown in Equation (C.1).

$$S = CN^{-D} + E \tag{C.1}$$

Since Equation (C.1) has three unknown coefficients, it is theoretically possible to determine them from three sets of data points. Therefore let these data points be the pairs:

S_1, N_1 @10^4 S_2, N_2 @10^5 S_3, N_3 @10^6

Then, knowing the coefficients, the corresponding value of S can be projected to $N = 10^7$ cycles. This value can then be used to determine the acceleration factor discussed in the previous section.

To begin, re-arrange Equation (C.1) as:

$$(S - E) = CN^{-D} \text{ and } \ln(S - E) = \ln C - D \ln N \tag{C.2}$$

Inserting data points into Equation (C.2) gives:

$$\ln(S_1 - E) = \ln C - D \ln N_1$$

$$\ln(S_2 - E) = \ln C - D \ln N_2$$

$$\ln(S_3 - E) = \ln C - D \ln N_3$$

Subtracting:

$$\ln(S_1 - E) - \ln(S_2 - E) = D \ln N_2 - D \ln N_1 = D(\ln N_2 - \ln N_1) \tag{C.3}$$

Similarly:

$$\ln(S_1 - E) - \ln(S_3 - E) = D \ln N_3 - D \ln N_1 = D(\ln N_3 - \ln N_1) \tag{C.4}$$

Dividing Equation (C.3) by Equation (C.4):

$$\frac{\ln(S_1 - E) - \ln(S_2 - E)}{\ln(S_1 - E) - \ln(S_3 - E)} = \frac{D(\ln N_2 - \ln N_1)}{D(\ln N_3 - \ln N_1)} = \frac{\ln N_2 - \ln N_1}{\ln N_3 - \ln N_1} = C_1 \tag{C.5}$$

It is observed that the right-hand side of Equation (C.5) is a constant.

Continuing from Equation (C.5):

$$\ln(S_1 - E) - \ln(S_2 - E) = C_1 \ln(S_1 - E) - C_1 \ln(S_3 - E)$$

$$\ln\left(\frac{S_1 - E}{S_2 - E}\right) = C_1 \ln\left(\frac{S_1 - E}{S_3 - E}\right) = \ln\left(\frac{S_1 - E}{S_3 - E}\right)^{C_1} \tag{C.6}$$

Equating the logs of both sides, and expanding using the binomial theorem to 3 terms gives Equation (C.7).

$$\frac{S_1 - E}{S_2 - E} = \left(\frac{S_1 - E}{S_3 - E}\right)^{C_1} = \frac{S_1^{C_1} - C_1 S_1^{(C_1 - 1)} E + \frac{1}{2} C_1 (C_1 - 1) S_1^{(C_1 - 2)} E^2 + \cdots}{S_3^{C_1} - C_1 S_3^{(C_1 - 1)} E + \frac{1}{2} C_1 (C_1 - 1) S_1^{(C_1 - 2)} E^2 + \cdots} \tag{C.7}$$

It is observed that the coefficients of the binomial can be replaced by constants:

$$C_2 = C_1 S_1^{(C_1 - 1)} \qquad C_3 = \frac{1}{2} C_1 (C_1 - 1) S_1^{(C_1 - 2)} \qquad C_4 = C_1 S_3^{(C_1 - 1)} \qquad C_5 = \frac{1}{2} C_1 (C_1 - 1) S_1^{(C_1 - 2)}$$

Resulting in Equation (C.8):

$$\frac{S_1 - E}{S_2 - E} = \frac{S_1^{C_1} - C_2 E + C_3 E^2}{S_3^{C_1} - C_4 E + C_5 E^2} \tag{C.8}$$

Cross multiplying and expanding Equation (C.8):

$$(S_1 - E)\left(S_3^{C_1} - C_4 E + C_5 E^2\right) = (S_2 - E)\left(S_1^{C_1} - C_2 E + C_3 E^2\right)$$

$$S_1 S_3^{C_1} - S_1 C_4 E + S_1 C_5 E^2 - S_3^{C_1} E + C_4 E^2 - C_5 E^3 = S_2 S_1^{C_1} - S_2 C_2 E + S_2 C_3 E^2 - S_1^{C_1} E + C_2 E^2 - C_3 E^3$$

Combining terms results in a cubic equation:

$$(C_3 - C_5) E^3 + \left[(C_4 - C_2) + (S_1 C_5 - S_2 C_3)\right] E^2 + \left(S_2 C_2 - S_1 C_4 + S_1^{C_1} - S_3^{C_1}\right) E + \left(S_1 S_3^{C_1} - S_2 S_1^{C_1}\right) = 0$$

Dividing by the first coefficient gives Equation (C.9):

$$E^3 + \frac{\left(C_4 - C_2 + S_1 C_5 - S_2 C_3\right)}{\left(C_3 - C_5\right)} E^2 + \frac{\left(S_2 C_2 - S_1 C_4 + S_1^{C_1} - S_3^{C_1}\right)}{\left(C_3 - C_5\right)} E + \frac{\left(S_1 S_3^{C_1} - S_2 S_1^{C_1}\right)}{\left(C_3 - C_5\right)} = 0 \qquad (C.9)$$

Substituting for the coefficients:

$$p = \frac{\left(C_4 - C_2 + S_1 C_5 - S_2 C_3\right)}{\left(C_3 - C_5\right)} \qquad q = \frac{\left(S_2 C_2 - S_1 C_4 + S_1^{C_1} - S_3^{C_1}\right)}{\left(C_3 - C_5\right)} \qquad r = \frac{\left(S_1 S_3^{C_1} - S_2 S_1^{C_1}\right)}{\left(C_3 - C_5\right)}$$

This results in the classic textbook cubic Equation (C.10):

$$E^3 + pE^2 + qE + r = 0 \qquad (C.10)$$

which can be solved with software (also available on the Internet [18]). There are three roots to Equation (C.10), and one of them should be selected for continued use. This choice is made by examining the values of the three roots and selecting the one that is a real number, and less than the value of S_3 (it might even be negative). It can also be necessary to try more than one root and examine the results.

Returning to Equation (C.1), and substituting the chosen value of E:

$$S = CN^{-D} + E \text{ and } (S - E) = CN^{-D}$$

Inserting data points and dividing gives:

$$\left(\frac{S_1 - E}{S_2 - E}\right) = \frac{CN_1^{-D}}{CN_2^{-D}} = \left(\frac{N_1}{N_2}\right)^{-D} \text{ and } \ln\left(\frac{S_1 - E}{S_2 - E}\right) = -D \ln\left(\frac{N_1}{N_2}\right)$$

which can be solved for D:

$$-\frac{\ln\left(\dfrac{S_1 - E}{S_2 - E}\right)}{\ln\left(\dfrac{N_1}{N_2}\right)} = D \qquad (C.11)$$

Finally, substituting known values into Equation (C.1) again gives:

$$S_1 - E = CN_1^{-D} \text{ and}$$

$$\frac{S_1 - E}{N_1^{-D}} = C \qquad (C.12)$$

Thus, all three coefficients for Equation (C.1) are now known, and one of the fatigue strength values (characteristic or mean) at 10^7 cycles can be determined using Equation (C.13):

$$S_4 = CN_4^{-D} + E \qquad (C.13)$$

C.3 Examples

C.3.1 General

Weibull fatigue strength distributions for five materials (both smooth and notched specimens) were found for cycle lives at 10^4, 10^5, 10^6 and 10^7 cycles. The equations in Clause C.2 were used to calculate the fatigue strengths at 10^7 cycles, and then compared to the published value at 10^7 cycles. From this, the acceleration factors were calculated. Results are summarized in Table C.1 and Table C.2.

Table C.1 — Smooth specimen examples

Material		4140 steel	H11 steel	4340 steel	Alloy steel	2024 Aluminium
Characteristic life	Published at 10^7	485,5 MPa	673,1 MPa	620,7 MPa	1079,3 MPa	186,2 MPa
	Calculated at 10^7	487,2 MPa	684,0 MPa	609,3 MPa	1082,9 MPa	186,8 MPa
	Error	+0,36 %	+1,61 %	−1,83 %	+0,33 %	−0,04 %
	Acceleration factor published	1,09	1,12	1,13	1,08	1,22
	Acceleration factor calculated	1,09	1,10	1,15	1,08	1,21
	Error	−0,36 %	−1,59 %	+1,86 %	−0,33 %	−0,28 %

Table C.2 — Notched specimen examples

Material		4140 steel	H11 steel	4340 steel	Alloy steel	2024 Aluminium
Characteristic life	Published at 10^7	219,3 MPa	675,9 MPa	87,6 MPa	497,2 MPa	92,3 MPa
	Calculated at 10^7	219,7 MPa	677,4 MPa	80,6 MPa	503,1 MPa	92,2 MPa
	Error	+0,17 %	+0,22 %	−1,83 %	+1,17 %	−0,19 %
	Acceleration factor published	1,18	1,16	1,81	1,05	1,41
	Acceleration factor calculated	1,18	1,16	1,97	1,04	1,41
	Error	−0,17 %	−0,22 %	+8,70 %	−1,15 %	+0,19 %

The materials used in this comparison were all with three-parameter Weibull distributions and the data sources were Reference [13] and Reference [14].

C.3.2 Observations

The following observations were made.

a) The accuracy of projecting the characteristic and mean life to 10^7 cycles was very good for all the example materials, except the 4340 notched steel. However, data for this material are questionable because the notched values for minimum life were very low, resulting in only one real root to its cubic equation and that was larger than the minimum life at 10^6 cycles.

b) The acceleration factor from projected life had good accuracy compared to the acceleration factor calculated directly from published data.

c) There is a significant difference in the acceleration factor between smooth and notched specimens, for some of the materials, for example:

1) the pressure increase would double for notched specimens over smooth ones in the 4140 steel. (For example, if the test pressure for a 10^6 cycle test were 10 MPa [2], the smooth specimen data would result in a 0,9 MPa increase in test pressure, to 10,9 MPa, for rating at 10^6 MPa. If notched data were used, the increase would be 1,8 MPa, resulting in a test pressure of 11,8 MPa);

2) the pressure increase is about 33 % more for notched specimens than smooth ones in the H11 steel;

3) the pressure increase is actually less for notched specimens than smooth samples in the alloy steel;

4) the acceleration factor is unreasonably high for notched specimens in aluminium and the 4340 steel.

C.3.3 Proposal

The examples justify the accuracy of the analysis technique, but selection of material data for use in a standard should be explored further. The sources cited for the data had some questionable values (as noted for the 4340 steel), but they had values from a Weibull distribution and not just endurance values. Therefore, new data should be sought with Normal, Weibull, or other fatigue distribution data.

With enough data, a proposal could be made for a table of acceleration factors to be used in an International Standard.

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[2] 1 bar = 0,1 MPa = 0,1 N/mm^2 = 105 N/m^2.

Annex D
(informative)

Basis of fatigue pressure rating[3] [9]

D.1 Basis of pressure rating

D.1.1 Abstract

A theoretical development of the NFPA verification method for fatigue pressure rating fluid power components is presented in Annex D. In addition, a method is shown for generating a fatigue strength distribution at a desired life point, using raw data from a fatigue test. Finally, examples are given for using the methods developed and discussion offered for this technique.

D.1.2 Introduction

Fluid power products are designed to operate in a pressurized condition, which necessitates some claim by its manufacturer about pressure-containing ability. There are many standards which describe, in various degrees of detail, just how a fluid power product might be designed for a given pressure rating. It has generally been found, however, that the ingenuity of designers tends to outpace the rate at which design rules can be standardized. The philosophy of this part of ISO 10771 on pressure rating has recognized this fact and suggests that manufacturers begin with their own methods for assigning a fatigue pressure rating. Then apply this part of ISO 10771, with its methods of determining material strength variability and usage of statistical tools to substantiate the proposed fatigue pressure rating. That this method employs fundamental concepts, lends itself to a universal application for all fluid power products, if made and used within the limits of its scope (i.e. metal only, environment, temperature, etc.).

The purpose of this part of ISO 10771 then is to define a method by which the claimed pressure rating can be verified on a fatigue basis.

Annex D is an analytical development of the basis theory for verification. It is offered as an academic tutorial, as well as a guide for those wishing to calculate their own test factors.

This paper is divided into three subject matters: Statistical Analysis Theory (Clause D.2), Fatigue Distribution Data (Clause D.3) and a Data Calculation Example (Clause D.4). Clause D.2 assumes that fatigue strength distribution data for the material are available, and develops a verification theory. Clause D.3 describes how raw data from a fatigue test are transformed into the statistical distribution for use in the verification theory.

Clause D.4 uses data extracted from several industry interlaboratory tests in a single example of the theoretical results.

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3) Annex D is reproduced from the Proceedings of the 43rd National Conference on Fluid Power, Chicago, Illinois, USA, October, 1988.

D.2 Statistical analysis theory

D.2.1 Objective

Fatigue data necessary for this analysis are a statistical distribution of the material strength at some rated life, as shown in Figure D.1.

Key
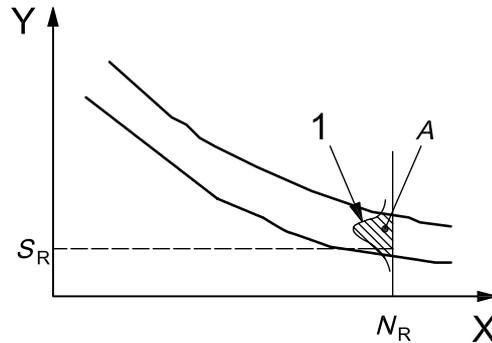
X   life, expressed in number of cycles

Y   fatigue strength

1   strength distribution at rated fatigue life

S_R rated strength

N_R rated life

A area under curve above S_R

Figure D.1 — Fatigue strength at rated life

From the parameters of this distribution, it is possible to define some level near the lower end for rating purposes. The objective of verification, then, is to demonstrate by test that a population of products belongs to this distribution.

One way to make this demonstration is to test several products to destruction. From the data, a sample distribution curve can be made and statistically compared to the population distribution. If the sample size is sufficiently large, the results could even be used as a new population and the rating obtained in a direct manner. These are both accurate methods for verification, but costly.

This part of ISO 10771 proposes a method of non-failure testing (verification testing), using a few sacrificial specimens. By incorporating conservative principles, the verification process can be greatly simplified with an assurance of product integrity.

D.2.2 Planned conservatism

The area below the population strength distribution curve, above the rated strength (A in Figure D.1) represents the probability of surviving N_R cycles of operation *at the rated strength*.

However, if a few specimens were tested at the rated strength and survived, they could not yield statistical confidence about that rated strength. It would only suggest that the sample strength level was above the level of testing conducted. A conservative test, however would be one conducted in the upper tail as depicted in Figure D.2.
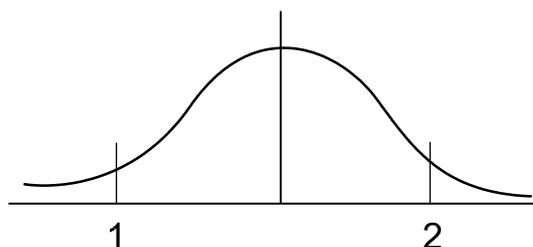
Key

1 rated load
2 test level

Figure D.2 — Conservative test level

Now the survivors of such a test level would offer more confidence for some rated load located in the left-hand tail.

Surviving a fatigue test in the upper tail, however, is very unlikely. But if an artificial shift of the distribution curve is made to a lower strength region, as shown in Figure D.3, then the test level becomes relocated into a left-hand tail where passing is more probable. The new curve is defined as a rated strength distribution and the rating level is correspondingly lowered.

Conservatism in such a rating scheme is now apparent:

a) the selected test level gives high probability $(1 - A_1)$ that survivors are not representative of a strength distribution lower than the new, shifted rated one;

b) the rating level gives high probability $(1 - A_2)$ that product strength will exceed this rating level;

c) there is high probability $(1 - A_3)$ that the product is strong enough to pass the test. Using the tail areas so described, the following terms are defined:

— $(1 - A_1)$ is the verification level;

— $(1 - A_2)$ is the assurance level;

— $(1 - A_3)$ is the test success level.

Key
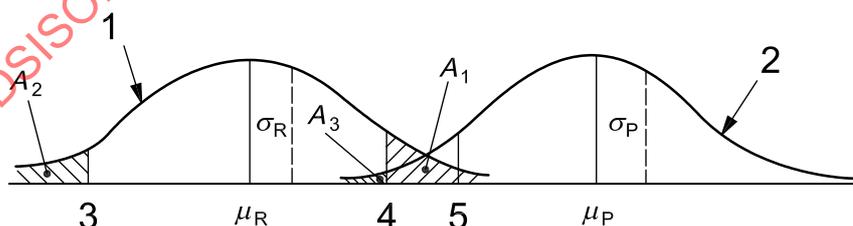
1 rated strength distribution
2 population strength distribution
3 rating level
4 test level
5 rated strength (S_R in Figure D.1.)

Figure D.3 — Shifted curve to a rated position

An examination of Figure D.3 reveals that some relationship between the two distributions should be established otherwise the shape of the rated strength distribution might be arbitrary. Therefore select Equation (D.1):

$$\frac{\sigma_R}{\mu_R} = \frac{\sigma_P}{\mu_P} = k_o \tag{D.1}$$

where

k_o is the coefficient of variation;

μ_R is the median of the rated strength distribution;

μ_P is the median of the population strength distribution;

σ_R is the rated standard deviation;

σ_P is the population standard deviation.

The amount of shift between the two distributions will be examined later.

D.2.3 Number of test specimens

When random specimens are subjected to a verification test, they will either survive or fatigue before N_R cycles. Since there are only these two possibilities, a binomial distribution can be used to describe the probability of success (achieving fatigue) in a set of specimens:

where

$$P(y) = C_y^n p^y q^{(n-y)} \quad \text{where} \quad C_y^n = \frac{n!}{y!(n-y)!} \tag{D.2}$$

and:

$P(y)$ is the probability that from a group of n specimens, y units will fatigue at or before N_R cycles;

p is the probability of an individual specimen becoming fatigued at or before N_R cycles;

q is the probability of an individual specimen surviving.

Since the basis of this verification test is that all specimens survive: $y = 0$

Then:

$$C_n^y = 1$$

$$p^y = 1$$

$$q = 1 - p$$

and

$$P(0) = (1-p)^n \tag{D.3}$$

This means that the probability of no failures in a test group is related to the fatigue probability of a typical unit and the number of units in the test group. Assuming the probability of fatigue is related to strength, and Equation (D.3) is applied to the rated strength distribution of Figure D.3, then the area under this curve to the left of the test level represents the proportion of units weaker than the imposed load and is the probability of failure in a typical unit.

$p_1 = 1 - A_1$ (the verification level)

For a random specimen, the probability of passing the verification test would be:

$$P(0) = (1 - p_1)^n = (1 - 1 + A_1)^1 = A_1 \qquad (D.4)$$

which is the right-hand tail area of the rated strength distribution.

If the number of specimens is increased but the probability of passing the verification test is held constant:

$$P(0) = (1 - p_n)^n = A_1 \qquad (D.5)$$

$$p_n = 1 - (A_1)^{1/n} \qquad (D.6)$$

From this, it is observed that the probability of failure in an individual unit of multiple specimens, p_n, is smaller than p_1. This means that the tail area, A_1, has to be larger for multiple specimens than for a single specimen, if the verification level value is maintained. Such a case can be depicted by returning the rated strength distribution curve back slightly towards the population curve, but maintaining area, A_3, constant, as shown in Figure D.4.

Key
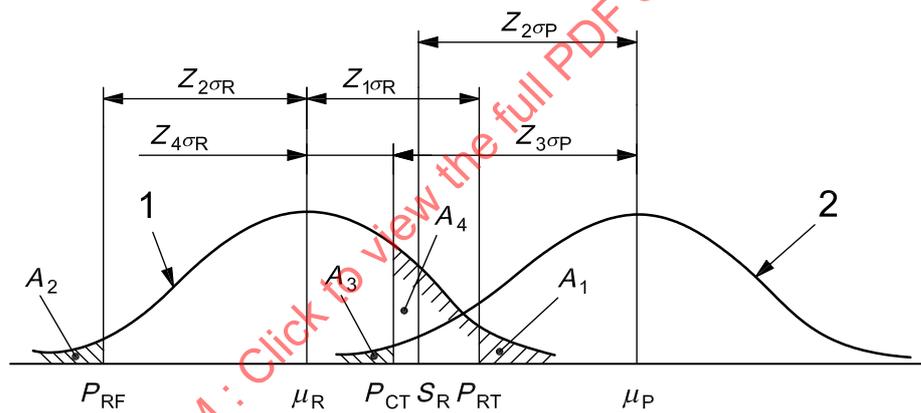
1   rated strength distribution
2   population strength distribution

Figure D.4 — Shifted curve for multiple specimens

where

P_{CT} is the cyclic test pressure for a multi-specimen test;

P_{RT} is the rated test pressure for a single specimen;

P_{RF} is the rated fatigue pressure;

A_4 is the expanded tail area, A_1;

Z_i is the number of standard deviations for a normal probability distribution.

Other factors are the same as previously defined.

If the overall probability for passing the verification test is held constant again, Equation (D.3) yields:

$$P(0) = (1-p)^n$$

$$A_1 = \{1 - [1 - (A_1 + A_4)]\}^n$$

$$A_1^{1/n} = A_1 + A_4 \tag{D.7}$$

Using the value of $(A_1 + A_4)$ in a normal probability distribution table, a corresponding value of Z_4 can be determined which will be used in further calculations.

Note that for $N = 1$, $A_4 = 0$, $Z_4 = Z_1$ ($Z_4 \neq 0$) and the area under the curve becomes $(1 - A_1)$, the verification level. Thus, the verification level becomes a fixed parameter in the rating system.

D.2.4 Rating selection level

The degree to which the rated strength distribution is displaced from the population strength distribution can be now judged from a ratio:

$$F_R = \frac{S_R}{P_{RF}} = \frac{\mu_p - Z_2\sigma_p}{\mu_R - Z_2\sigma_R} = \frac{\mu_p(1 - Z_2 k_o)}{\mu_R(1 - Z_2 k_o)} = \frac{\mu_p}{\mu_R} \tag{D.8}$$

where

F_R is the rating ratio.

The degree of conservation thus introduced is similar to the "safety factor" concept commonly used in static stress analysis. The rating ratio is dependent upon a probability selected to pass the verification test and can be evaluated from an examination of Figure D.4.

$$F_R = \frac{\mu_p}{\mu_R} = \frac{\mu_R + Z_4\sigma_R + Z_3\sigma_p}{\mu_R} \tag{D.9}$$

$$= 1 + Z_4 k_o + Z_3 \frac{\sigma_p}{\mu_R} \tag{D.10}$$

From Equation (D.1):

$$k_o = \frac{\sigma_p}{\mu_p} = \frac{\sigma_R}{\mu_R}$$

and from Equation (D.9):

$$F_R = \frac{\mu_p}{\mu_R}$$

$$\sigma_p = \frac{\mu_p}{\mu_R}\sigma_R = F_R\sigma_R$$

Substituting into Equation (D.10):

$$F_R = 1 + Z_4 k_o + Z_3 \frac{F_R \sigma_R}{\mu_R} = 1 + Z_4 k_o + Z_3 k_o F_R$$

$$F_R (1 - Z_3 k_o) = 1 + Z_4 k_o$$

$$F_R = \frac{1 + Z_4 k_o}{1 - Z_3 k_o} \tag{D.11}$$

Factor Z_3 is obtained from a normal probability distribution table, based upon the one-sided tail area, A_3. From Figure D.4, it is seen that the complementary area represents the probability of passing the verification test. Thus the degree of conservation introduced into this rating process can be measured from the parameters chosen.

D.2.5 Calculation of K_V factor

From all of the factors now determined, a relationship can be calculated between P_{CT} and P_{RF}. From Figure D.4:

$$P_{RF} + Z_2 \sigma_R + Z_4 \sigma_R = P_{CT} \tag{D.12}$$

$$\sigma_R = \frac{P_{CT} - P_{RF}}{Z_2 + Z_4}$$

Also from Figure D.4:

$$P_{RF} + Z_2 \sigma_R = \mu_R = \frac{\sigma_R}{k_o} \tag{D.13}$$

$$P_{RF} = \sigma_R \left(\frac{1}{k_o} - Z_2 \right)$$

$$\frac{P_{RF}}{\frac{1}{k_o} - Z_2} = \sigma_R$$

Equating the two σ_R expressions:

$$\frac{P_{CT} - P_{RF}}{Z_2 + Z_4} = \frac{P_{RF}}{\frac{1}{k_o} - Z_2}$$

$$\frac{P_{CT}}{P_{RF}} - 1 = \frac{Z_2 + Z_4}{\frac{1}{k_o} - Z_2}$$

$$\frac{P_{CT}}{P_{RF}} = \frac{1 - k_o Z_2}{1 - k_o Z_2} + \frac{k_o Z_2 + k_o Z_4}{1 - k_o Z_2}$$

$$\frac{P_{CT}}{P_{RF}} = \frac{1 + k_o Z_4}{1 - k_o Z_2} = Z_V \tag{D.14}$$

Constant K_V is thus determined as the ratio between P_{CT} and P_{RF}. The K_V constant is a function of material property k_o plus a combination of sample size, verification level and assurance level.

D.3 Fatigue distribution data

D.3.1 Objective

From Clause D.2, it is seen that the verification procedure requires a statistical distribution of fatigue strength at some desired life. Clause D.3 describes how a distribution, as shown in Figure D.1 can be generated from the raw data of a fatigue test.

It is important to point out that collecting the raw data is not a simple matter. It is important to follow careful controls, as described by the rules of statistical experiments, in order to obtain samplings from a population.

D.3.2 Generating a best fit curve

Consider the raw data plotted on an S-N curve, as shown in Figure D.5

NOTE   This is a linear plot, not logarithmic on either axis.
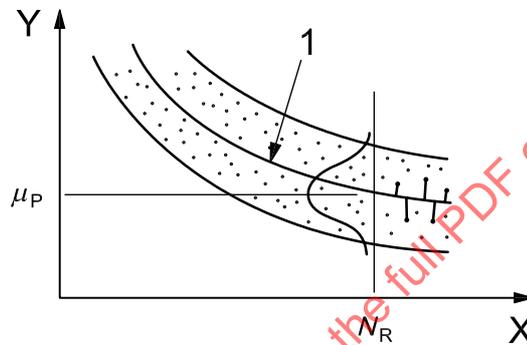
Key

X   life, expressed in number of cycles

Y   applied stress from testing; and material strength from failures, S

1   best fit curve to the data

Figure D.5 — S-N curve with data points and bounds

Consider that a best fit line passes through the data, in the form:

$$S = C_o N^{-D} + E_o \quad \text{where } N > N_{min}$$

for values of N greater than some minimum N.

This qualification is necessary to overcome the difficulty of large values of S satisfying the model equation at low values of N but not representative of the actual fatigue strength. Thus, the range of applicability is limited.

Now a linear proportionality exists between pressure, P, and stress, S, such that:

$$P = CN^{-D} + E \tag{D.15}$$

let

$$(P - E)^2 = C^2 N^{-2D}$$

and

$$\ln (P - E)^2 = 2 \ln C - 2 D \ln N \tag{D.16}$$

During the conduct of a fatigue test, the independent variable is actually the vertical axis Y (S or P). However, if a linear regression analysis is applied with P as the independent variable, the variance of each data point from the best fit line is parallel to the N axis. Those data points below the asymptotic portion of the best fit line will then not make an intersection, resulting in an undefined variance. More likely, however, the best fit line will become distorted from the results, pushing the asymptotic portion below the data points so that the horizontally directed variance makes an intersection. This results in an untrue representation of the best fit line.

Therefore, an arbitrary choice is made to allow the N axis to become the independent variable, at least for the purpose of obtaining a best fit line in the data field and also for values above the N minimum. Now the variances are taken in a vertical direction, parallel to the Y (S or P) axis and will intersect the line in the region of interest. But yet another difficulty has to be overcome in performing the regression analysis. If Equation (D.15) were not squared, the left-hand expression of Equation (D.16) would be ln (P–E). Since E is the asymptotic limit of the curve, those data points below the curve would have values of P less than E. When used in a variance calculation, (P–E) would be zero or negative and the logarithm is undefined. Therefore, the squared function is chosen to overcome this difficulty.

Then, from Equation (D.16) let:

$$y = \ln(P - E)^2 \; ; \; x = \ln N \; ; \; A = 2\ln C \; ; \; B = -2D$$

then

$$y = A + B_x \tag{D.17}$$

Performing a least squares technique:

$$\overline{x} = \frac{\sum\limits_{1}^{k} x_i}{k} \quad \overline{y} = \frac{\sum\limits_{1}^{k} y_i}{k} \tag{D.18}$$

$$S_{Sx} = \sum\limits_{1}^{k} x_i^2 - \frac{\left(\sum\limits_{1}^{k} x_i\right)^2}{k} \tag{D.19}$$

$$S_{Sy} = \sum\limits_{1}^{k} y_i^2 - \frac{\left(\sum\limits_{1}^{k} y_i\right)^2}{k} \tag{D.20}$$

$$S_{Sxy} = \sum\limits_{1}^{k} x_i y_i - \frac{\left(\sum\limits_{1}^{k} x_i\right)\left(\sum\limits_{1}^{k} y_i\right)}{k} \tag{D.21}$$

where

k is the number of data points.

From these, the following parameters may be determined:

$$B = \frac{S_{Sxy}}{S_{Sx}} \quad D = \frac{-B}{2} = \frac{-S_{Sxy}}{2}$$

$$A = \overline{y} - B\overline{x} \quad C = e^{A/2}$$

A measure of how well the data fit the resulting equation is judged from the coefficient of determination:

$$r^2 = \frac{(S_{Sxy})^2}{(S_{Sx})(S_{Sy})} \tag{D.22}$$

Now the preceding analysis includes a parameter, E, which has not been calculated. Unless arbitrary values are initially assumed, calculation of Y_i values cannot proceed. If arbitrary values are indeed assumed, the effect will be observed in the value of r^2. Therefore, the best assumption to make for E would be one that results in the maximum value of r^2 or:

$$\frac{\mathrm{d}(r^2)}{\mathrm{d}E} = 0 \tag{D.23}$$

from

$$r^2 = \frac{(S_{Sxy})^2}{(S_{Sx})(S_{Sy})}$$

$$\frac{\mathrm{d}(r^2)}{\mathrm{d}E} = 0 = \frac{(S_{Sx})(S_{Sy})2(S_{Sxy})\dfrac{\mathrm{d}}{\mathrm{d}E}(S_{Sxy}) - (S_{Sxy})^2(S_{Sx})\dfrac{\mathrm{d}}{\mathrm{d}E}(S_{Sy}) - (S_{Sxy})^2(S_{Sy})\dfrac{\mathrm{d}}{\mathrm{d}E}(S \nearrow^0 S_x)}{(S_{Sx})^2(S_{Sy})^2} \tag{D.24}$$

Since S_{Sx} is not a function of E, it is constant relative to E

and $\dfrac{\mathrm{d}S_{Sx}}{\mathrm{d}E} = 0$

rearranging

$$2(\cancel{S_{Sx}})(S_{Sy})(\cancel{S_{Sxy}})\frac{\mathrm{d}}{\mathrm{d}E}(S_{Sxy}) = (S_{Sxy})^{\cancel{2}}(\cancel{S_{Sx}})\frac{\mathrm{d}}{\mathrm{d}E}(S_{Sy})$$

$$2(S_{Sy})\frac{\mathrm{d}}{\mathrm{d}E}(S_{Sxy}) = (S_{Sxy})\frac{\mathrm{d}}{\mathrm{d}E}(S_{Sy}) \tag{D.25}$$

Evaluating the differentials individually (again noting that only y_i is a function of E):

$$\frac{\mathrm{d}}{\mathrm{d}E}(S_{Sxy}) = \frac{\mathrm{d}}{\mathrm{d}E}\left[\sum x_i y_i - \frac{\sum x_i \sum y_i}{k}\right] = \left[\sum\left(x_i \frac{\mathrm{d}y_i}{\mathrm{d}E}\right) - \frac{1}{k}\left(\sum x_i\right)\left(\sum \frac{\mathrm{d}y_i}{\mathrm{d}E}\right)\right] \tag{D.26}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}E}(S_{Sy}) = \frac{\mathrm{d}}{\mathrm{d}E}\left[\sum y_i^2 - \frac{(\sum y_i)^2}{k}\right] = \left[2\left(\sum y_i \frac{\mathrm{d}y_i}{\mathrm{d}E}\right) - \frac{2}{k}\left(\sum y_i\right)\left(\sum \frac{\mathrm{d}y_i}{\mathrm{d}E}\right)\right] \tag{D.27}$$

Consider the equation for y_i in two forms:

$y_i = \ln(P_i - E)^2 \qquad y_i = \ln(P_i - E)^2$

$y_i = 2\ln(P_i - E) \qquad y_i = 2\ln(P_i - E)$

$$\mathrm{e}^{\frac{1}{2}y_i} = P_i - E \quad \frac{\mathrm{d}y_i}{\mathrm{d}E} = \frac{-2}{P_i - E}$$

Substituting the left-hand equation into the right-hand equation yields Equation (D.28):

$$\frac{\mathrm{d}y_i}{\mathrm{d}E} = -2\mathrm{e}^{-\frac{1}{2}y_i} \tag{D.28}$$

Substituting this last expression into the two differential Equations (D.26) and (D.27) yields:

$$\frac{\mathrm{d}}{\mathrm{d}E}(S_{Sxy}) = \left[-2\left(\sum x_i \mathrm{e}^{-\frac{1}{2}y_i}\right) + \frac{2}{k}\left(\sum x_i\right)\left(\sum \mathrm{e}^{-\frac{1}{2}y_i}\right) \right]$$

$$\frac{\mathrm{d}}{\mathrm{d}E}(S_{Sy}) = \left[-4\left(\sum y_i \mathrm{e}^{-\frac{1}{2}y_i}\right) + \frac{4}{k}\left(\sum y_i\right)\left(\sum \mathrm{e}^{-\frac{1}{2}y_i}\right) \right]$$

Now these differentials are substituted back into Equation (D.25), yielding Equation (D.29):

$$2\left[S_{Sy}\right]\left[-2\left(\sum x_i \mathrm{e}^{-\frac{1}{2}y_i}\right) + \frac{2}{k}\left(\sum x_i\right)\left(\sum \mathrm{e}^{-\frac{1}{2}y_i}\right)\right] =$$

$$\left[S_{Sxy}\right]\left[-4\left(\sum y_i \mathrm{e}^{-\frac{1}{2}y_i}\right) + \frac{4}{k}\left(\sum y_i\right)\left(\sum \mathrm{e}^{-\frac{1}{2}y_i}\right)\right] \tag{D.29}$$

Substituting back other terms into Equation (D.29) yields Equation (D.30):

$$Z = \left[S_{Sy}\right]\left[\overline{x}\left(\sum \mathrm{e}^{-\frac{1}{2}y_i}\right) - \sum x_i \mathrm{e}^{-\frac{1}{2}y_i}\right]$$

$$- \left[S_{Sxy}\right]\left[\overline{y}\left(\sum \mathrm{e}^{-\frac{1}{2}y_i}\right) - \sum y_i \mathrm{e}^{-\frac{1}{2}y_i}\right] = 0 \tag{D.30}$$

Equation (D.30) now has to be evaluated with trials on E to find a value that results In $Z = 0$. That value of E is then used in the preceding least squares technique [Equation (D.17) to Equation (D.21)] to determine parameters C and D. Those results then yield the highest possible value of r^2 to be obtained from the set of data.

It should be noted that division by S_{Sx} and S_{Sxy} occurred in developing Equation (D.25), necessitating that their values not be zero. This is impossible for S_{Sx} if there is more than one data point, and if they are not equal to each other. If S_{Sxy} were zero, then B = 0 and the best fit curve S-N would be constant, an obvious incongruity to the facts. Thus, division by these factors is considered valid.

It is possible that more than one solution might occur for Equation (D.30), so those values of E have to be tried for best fits and the results judged for acceptance.

After a value for E has been selected, constants B and A can be determined from the least squares technique; then constants C and D are derived, which define the best fit curve (Equation D.15). Values of μ_P and σ_P at N_R are then determined by translation as explained D.3.3.

D.3.3 Translation of individual data points

A return to Figure D.5 shows the individual data points clustered about the best fit curve. Each of these data points lies on an individual "probability" curve, proportional to the best fit describing a family of curves for all of the data. Thus, the best fit curve represents a 50 % probability of the data and the other curves, likewise, describe other probabilities.

This is shown in more detail in Figure D.6. A typical data point at P_i, N_i is on the individual probability curve that intersects the N_R rated life at the "translated point" of the P_{Ri} value.

Key
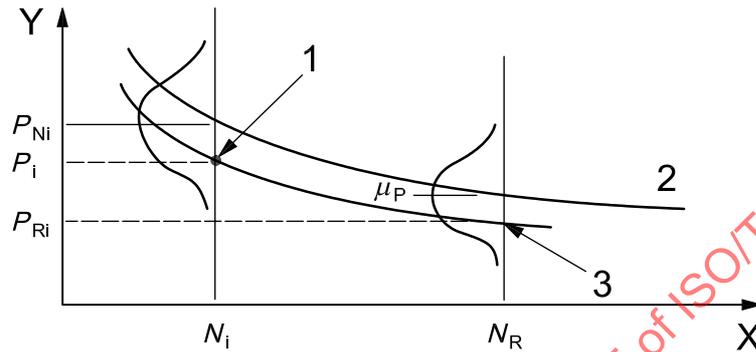
X   life, expressed in number of cycles

Y   fatigue pressure

1   arbitrary data point

2   best fit curve $P = CN^{-D} + E$

3   translated point

Figure D.6 — Translation of data points to rated life

The collection of such translated points at N_R provides a statistical distribution of strength as shown. In fact, a strength distribution exists at each value of life, including the one shown in Figure D.6 passing through the data point. A relationship exists between these distributions, at least in the mid-to-long life regimes, which is that their coefficient of variation is constant. Thus a proportionality can be written between the means, data points and translated points as:

$$\frac{\mu_\rho}{P_{Ri}} = \frac{P_{Ni}}{P_i}$$

$$P_{Ri} = P_i \left(\frac{\mu\rho}{P_{Ni}} \right)$$

$$P_{Ri} = P_i \left[\frac{CN_R^{-D} + E}{CN_i^{-D} + E} \right] \tag{D.31}$$