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Explanatory notes on ISO 76

Notes explicatives sur l'ISO 76

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Contents

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Annexes

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The main task of ISO technical committees is to prepare International Standards. In exceptional circumstances a technical committee may propose the publication of a Technical Report of one of the following types:

— type 1, when the required support cannot be obtained for the publication of an International Standard, despite repeated efforts;

— type 2, when the subject is still under technical development or where for any other reason there is the future but not immediate possibility of an agreement on an International Standard;

— type 3, when a technical committee has collected data of a different kind from that which is normally published as an International Standard ("state of the art", for example).

Technical Reports of types 1 and 2 are subject to review within three years of publication, to decide whether they can be transformed into International Standards. Technical Reports of type 3 do not necessarily have to be reviewed until the data they provide are considered to be no longer valid or useful.

ISO/TR 10657, which is a Technical Report of type 3, was prepared by Technical Committee ISO/TC 4, *Rolling bearings*, Sub-Committee SC 8, *Load ratings and life*.

ISO/TR 10657 has been prepared for the guidance of users of ISO 76 : 1987, *Rolling bearings — Static load ratings*. It is a purely scientific document intended for use by specialists in this field, and it is not envisaged that it will become an International Standard.

Annexes A, B and C form an integral part of this Technical Report.

Explanatory notes on ISO 76

1 Scope

This Technical Report gives supplementary background information regarding the derivation of formulae and factors given in ISO 76, Rolling bearings - Static load ratings.

2 Brief History

2.1 ISO/R 76 - 1958

The ISO Recommendation R 76, Ball and Roller Bearings - Methods of Evaluating Static Load Ratings, was drawn up by Technical Committee ISO/TC 4, Ball and Roller Bearings, the Secretariat of which is held by the Sveriges Standardiseringskommission (SIS).

This Recommendation is based on the studies [1]*, [2] by A. Palmgren and so on. It is defined in the Recommendation that the basic static load ratings correspond to a total permanent deformation of rolling element and raceway at the most heavily stressed rolling element/raceway contact of 0,0001 of the rolling element diameter. And then the Standard values confined to the basic static load ratings for special inner design rolling bearings are laid down.

* Figures in brackets indicate literature references in annex C.

Technical Committee ISO/TC 4 discussed the questions dealt with by the ISO Recommendation at the following meetings:

the third meeting, held in Göteborg, in September 1953,

the fourth meeting, held in Madrid, in May 1955,

the fifth meeting, held in Vienna, in September 1956.

At the third meeting of the Technical Committee, Working Group No. 3 was appointed to assist the ISO/TC 4 Secretariat in preparatory work and in drawing up proposals. The Working Group composed of Germany, Sweden and USA held the following meetings:

the first meeting, held in Madrid, in May 1955,

the second meeting, held in Vienna, in September 1956.

On 4th June 1957, the Draft ISO Recommendation was sent out to all the ISO Member Bodies and was approved by 28 (out of a total of 38) Member Bodies.

The Draft ISO Recommendation was then submitted by correspondence to the ISO Council, which decided, in December 1958, to accept it as an ISO Recommendation.

2.2 ISO 76 - 1978

The Working Group No. 3 was transformed into SC 8 at the 13th meeting held in Paris in May 1972. The SC 8 Secretariat proposed to include the revision of ISO/R 76 in the future work at the first meeting held in London in November 1973, and SC 8 requested its Secretariat to prepare a Draft Proposal for an International Standard replacing ISO/R 76 and it was decided that this proposal should be submitted to the SC 8 member Bodies for consideration prior to 1 October 1974 (SC 8 RESOLUTION 21, 21 London 1973).

The SC 8 Secretariat distributed a DRAFT PROPOSAL (Revision of ISO/R 76) in July 1974.

The TC 4 decided to include the revision of ISO/R 76 in its programme of work (TC 4 RESOLUTION 514, item No. 132) and SC 8 Secretariat was requested to prepare a Draft Proposal (SC 8 RESOLUTION 38, 13 Miami Beach 1974). As a result, the Secretariat submitted a DP [3] in January 1976.

The Draft Proposal DP 76 was accepted by correspondence by 6 of the 8 Members of SC 8. Of the remaining two, Japan would prefer further study and USA its counter proposal, document 4/8 N 64 [4]. The DIS 76 was then submitted to the ISO Central Secretariat. After the DIS had been approved by the ISO Member Bodies, the ISO Council decided in June 1978 to accept it as an International Standard.

This Standard adopted the SI unit newton and was revised in total, but without essential changes of substance. However, values of X_0 and Y_0 for the nominal contact angles 15° and 45° for angular contact groove ball bearings were added to the table which shows the values of X_0 and Y_0 in the formulae to calculate the static equivalent radial loads of radial ball bearings.

2.3 ISO 76 - 1987

During the revision of ISO/R 76 - 1958, USA had in 1975 submitted a counter proposal [4] for the basic static load ratings based on a calculated contact stress.

The Secretariat requested a vote on the revision of the static load ratings based on a contact stress level in January 1978 and afterward circulated the voted results in June 1978, and the item No. of revision work had became No. 157 of the programme of work of TC 4.

This Draft Proposal DP 76 was dealt with at the following TC 4 meetings:

15th meeting, held in Moscow, in April 1977,

16th meeting, held in London, in November 1979,

17th meeting, held in Budapest, in May 1983,

and then, dealt with at the following SC 8 meetings:

the third meeting, held in London, in November 1979,

the fourth meeting, held in Budapest, in May 1983,

the fifth meeting, held in Arlington, in November 1984.

The following resolutions for the contents of the Standard were adopted during the third meeting to the fifth meeting:

SC 8, considering the proposals made in the documents 4/8 N 75 [5] and 4 N 865 [6], as well as the comments made by TC 4 Members and that several SC 8 Members expressed a need for updating ISO 76, agreed to continue its study taking into account the possibility of using either permanent deformation or stress level as a basis for static load ratings (SC 8 RESOLUTION 45, 5 London 1979), and SC 8 requested its Secretariat to prepare a new draft. The new draft should be prepared with the principles and formulae of the document 4/8 N 75, and to include levels of contact stress for various rolling element contact stated to be generally corresponding to a permanent deformation of $10^{-4} D_w$ at the centre of the most heavily stressed rolling element/raceway contact. For roller bearings a stress level of 4000 MPa was agreed (SC 8 RESOLUTION 51, 4 Budapest 1983) and then SC 8 agreed, by a majority vote, that static load ratings should correspond to calculated contact stresses of

4000 MPa for roller bearings,

4600 MPa for self-aligning ball bearings and

4200 MPa for all other ball bearings to which the

Standard applies (SC 8 RESOLUTION 56, 3 Arlington 1984). Moreover, SC 8 recommended the document 4/8 N 121 [7] , amended in accordance with SC 8 Resolution 56, as a revised ISO 76 (SC 8 RESOLUTION 57, 4 Arlington 1984).

For these calculated contact stresses, a total permanent deformation occures at the centre of the most heavily stressed rolling element /raceway contact, and its deformation is approximately 0,0001 of the rolling element diameter.

The DIS 76 was submitted to the ISO Central Secretariat 1985, and after it had been approved by the ISO Members, the ISO Council decided in February 1987 to accept it as an International Standard.

Furthermore, SC 8 decided at its fifth meeting in April 1986 that supplementary background information, regarding the derivation of formulae and factors given in ISO 76, should be published as a Technical Report (SC 8 RESOLUTION 71, 11 Hangzhou 1986).

3 Basic Static Load Ratings

(1) Basic equation for point contact The relationship between a calculated contact stress and a rolling element load within an elliptical contact area is given as follows [8],

$$\sigma = \frac{3Q}{2\pi ab}\left[1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2\right]^{1/2}, \qquad (3\text{-}1)$$

where

σ = calculated contact stress, MPa,

a = major semi axis of the contact ellipse, mm,

b = minor semi axis of the contact ellipse, mm,

Q = normal force between rolling element and raceway, N,

x = distance in a-direction, mm,

y = distance in b-direction, mm.

It is concluded that the maximum calculated contact stress (σ_{max}) occurs at the point of x = 0 and y = 0,

$$\sigma_{max} = \frac{3Q}{2\pi ab}, \quad \text{or} \quad Q = \frac{2\pi ab}{3}\sigma_{max}. \qquad (3\text{-}2)$$

According to the Hertz's theory,

$$a = \left(\frac{2\kappa^2 E(\kappa)}{\pi}\right)^{1/3}\left[\frac{3Q}{2\Sigma\rho}\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)\right]^{1/3}, \qquad (3\text{-}3)$$

$$b = \left(\frac{2E(\kappa)}{\pi\kappa}\right)^{1/3}\left[\frac{3Q}{2\Sigma\rho}\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)\right]^{1/3}, \qquad (3\text{-}4)$$

where

$\varkappa = a/b$

$E(\varkappa)$ = complete elliptic integral of the second kind

$$= \int_0^{\pi/2} \left[1 - \left(1 - \frac{1}{\varkappa^2}\right)\sin^2\phi\right]^{1/2} d\phi$$

E_1, E_2 = modulus of elasticity (Young's modulus), MPa,

ν_1, ν_2 = Poisson's ratio,

$\sum\rho = \rho_{11} + \rho_{12} + \rho_{21} + \rho_{22}$,

$\rho_{11}, \rho_{21} = \frac{2}{D_w}$ = principal curvature of body 1 (ball),

ρ_{12}, ρ_{22} = principal curvature of body 2 (ring) at the point contact.

Substituting equations (3-3) and (3-4) into equation (3-2) for the case of $E_1 = E_2 = E$ and $\nu_1 = \nu_2 = \nu$,

$$Q = \sigma_{max}^3 \frac{32\pi}{3E_o^2} \varkappa \left(\frac{E(\varkappa)}{\sum\rho}\right)^2, \qquad (3\text{-}5)$$

and

$$1 - \frac{2}{\varkappa^2 - 1}\left(\frac{K(\varkappa)}{E(\varkappa)} - 1\right) - F(\rho) = 0 , \qquad (3\text{-}6)$$

where

$E_o = \frac{E}{1 - \nu^2}$,

$E = 2{,}07 \cdot 10^5$ MPa ,

$\nu = 0{,}3$

$K(\varkappa)$ = complete elliptic integral of the first kind

$$= \int_0^{\pi/2} \left[1 - \left(1 - \frac{1}{\varkappa^2}\right)\sin^2\phi\right]^{-1/2} d\phi ,$$

$$F(\rho) = \frac{\rho_{11} - \rho_{12} + \rho_{21} - \rho_{22}}{\rho_{11} + \rho_{12} + \rho_{21} + \rho_{22}} .$$

Consequently,

$$Q = 6,4762065 \times 10^{-10} \kappa \left(\frac{E(\kappa)}{\Sigma \rho}\right)^2 \sigma_{max}^3 \quad . \qquad (3\text{-}7)$$

(2) Basic equation for line contact The relationship between a calculated contact stress and a rolling element load for a line contact is given as follows [9] ,

$$\sigma = \frac{2Q}{\pi L_{we} b}\left[1 - \left(\frac{y}{b}\right)^2\right]^{1/2} , \qquad (3\text{-}8)$$

where

σ = calculated contact stress, MPa ,

b = semiwidth of the contact surface, mm ,

L_{we} = length of roller applicable to calculate load ratings, mm ,

Q = normal force between rolling element and raceway, N ,

y = distance in b-direction, mm .

It is concluded that the maximum calculated contact stress (σ_{max}) occurs at the line of y = 0,

$$\sigma_{max} = \frac{2Q}{\pi L_{we} b} , \quad \text{or} \quad Q = \frac{\pi L_{we} b}{2} \sigma_{max} \quad . \qquad (3\text{-}9)$$

And also b is given by the following equation,

$$b = \left[\frac{4Q}{\pi L_{we} \Sigma \rho}\left(\frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}\right)\right]^{1/2} , \qquad (3\text{-}10)$$

where

$$\Sigma\rho = \rho_{11} + \rho_{12} + \rho_{21} + \rho_{22} ,$$

$$\rho_{11} = \frac{2}{D_{we}} , \quad \rho_{12} = \pm \frac{2}{D_{we}} \frac{\gamma}{1 \mp \gamma} , \rho_{21} = 0 , \quad \rho_{22} = 0 ,$$

the upper sign applies to inner ring contact and
the lower to outer ring contact,

D_{we} = roller diameter applicable to calculate load ratings, mm ,

$$\gamma = \frac{D_{we} \cos\alpha}{D_{pw}} ,$$

D_{pw} = pitch diameter of roller set, mm .

Substituting equation (3-10) into equation (3-9) for the case of

$E_1 = E_2 = E$ and $\nu_1 = \nu_2 = \nu$,

$$Q = 2\pi\sigma_{max}^2 \frac{L_{we}}{E_o \Sigma\rho} ,$$

where

$$E_o = \frac{E}{1 - \nu^2} ,$$

$E = 2,07 \times 10^5$ MPa ,

$\nu = 0,3$.

Consequently,

$$Q = 2,7621732 \times 10^{-5} \frac{L_{we}}{\Sigma\rho} \sigma_{max}^2 . \qquad (3\text{-}11)$$

3.1 Basic static radial load rating C_{or} for radial ball bearings

3.1.1 Radial and angular contact groove ball bearings

The curvature sum $\Sigma\rho$ and curvature difference $F(\rho)$ of radial and angular contact groove ball bearings is given by the following equation,

$$\Sigma\rho = \frac{2}{D_w}\left(2 \pm \frac{\gamma}{1 \mp \gamma} - \frac{1}{2f_{i(e)}}\right) , \qquad (3\text{-}12)$$

$$F(\rho) = \frac{\pm \dfrac{\gamma}{1 \mp \gamma} + \dfrac{1}{2f_{i(e)}}}{2 \pm \dfrac{\gamma}{1 \mp \gamma} - \dfrac{1}{2f_{i(e)}}} . \qquad (3\text{-}13)$$

where

the upper sign applies to inner ring contact and the lower to outer ring contact,

D_w = ball diameter, mm ,

$\gamma = \dfrac{D_w \cos\alpha}{D_{pw}}$,

D_{pw} = pitch diameter of ball set, mm ,

$f_i = \dfrac{r_i}{D_w}$,

$f_e = \dfrac{r_e}{D_w}$,

r_i = inner ring groove radius, mm ,

r_e = outer ring groove radius, mm .

Substituting equation (3-12) into equation (3-7),

$$Q = 6{,}4762065 \times 10^{-10} \varkappa \left(\frac{D_w}{2} \frac{E(\varkappa)}{2 \pm \dfrac{\gamma}{1 \mp \gamma} - \dfrac{1}{2f_{i(e)}}}\right)^2 \sigma_{max}^3 . \qquad (3\text{-}14)$$

Substituting equations (3-12) and (3-14) into the following equation [10] , furthermore exchanging Q for Q_{max},

$$C_{or} = \frac{1}{S} Z Q_{max} \cos\alpha , \qquad (3\text{-}15)$$

where

C_{or} = basic static radial load rating, N ,

Z = number of balls per row ,

Q_{max} = maximum normal force between rolling element and raceway, N ,

S is a function of the loaded zone parameter ε.
If one half of the balls are loaded then S = 4,37 applies.
A common value used in general bearing calculations is S = 5, which leads to a rather conservative estimate of the maximum ball load,

α = nominal contact angle, °,

$$C_{or} = 0{,}2ZQ_{max}\cos\alpha \quad . \tag{3-16}$$

Consequently,

$$C_{or} = 0{,}2 \times 6{,}4762065 \times 10^{-10} \times (4000)^3 \left(\frac{\sigma_{max}}{4000}\right)^3 \kappa \times \frac{1}{4}\left(\frac{E(\kappa)}{2 \pm \dfrac{\gamma}{1 \mp \gamma} - \dfrac{1}{2f_{i(e)}}}\right)^2 ZD_w^2\cos\alpha \; ,$$

where the upper sign refers to the inner ring and the lower sign refers to the outer ring. Therefore, introducing the number of rows i of balls,

$$C_{or} = f_o iZD_w^2\cos\alpha \; , \tag{3-17}$$

where

f_o = factor which depends on the geometry of the bearing components and on applicable stress level

$$= 2{,}072\left(\frac{\sigma_{max}}{4000}\right)^3 \kappa \left(\frac{E(\kappa)}{2 \pm \dfrac{\gamma}{1 \mp \gamma} - \dfrac{1}{2f_{i(e)}}}\right)^2 \; . \tag{3-18}$$

The formula applies to bearings with a cross-sectional raceway groove radius not larger than 0,52 D_w in radial and angular contact groove ball bearing inner rings, and 0,53 D_w in radial and angular contact groove ball bearing outer rings and self-aligning ball bearing inner rings.

The load-carrying ability of a bearing is not necessarily increased by the use of a smaller groove radius, but is reduced by the use of a larger groove radius. In the latter case, a correspondingly reduced value of f_o shall be used.

For an inner ring with f_i = 0,52,

$$f_o = 2{,}072\left(\frac{\sigma_{max}}{4000}\right)^3 \kappa\left(\frac{E(\kappa)}{2 + \frac{\gamma}{1-\gamma} - \frac{1}{1{,}04}}\right)^2 , \qquad (3\text{-}19)$$

and for an outer ring with f_e = 0,53,

$$f_o = 2{,}072\left(\frac{\sigma_{max}}{4000}\right)^3 \kappa\left(\frac{E(\kappa)}{2 - \frac{\gamma}{1+\gamma} - \frac{1}{1{,}06}}\right)^2 . \qquad (3\text{-}20)$$

The smaller value between the f_o values calculated from equations (3-19) and (3-20) shall be adopted.

The values of factor f_o on table 1 in ISO 76 - 1987 are calculated from substituting the values for κ, $E(\kappa)$ and $\gamma = D_w \cos\alpha / D_{pw}$ shown in table A.1, and σ_{max} = 4200 MPa into the above equation.

3.1.2 Self-aligning ball bearings

The curvature sum $\sum\rho$ of self-aligning ball bearings is given by the following equation for an outer ring,

$$\sum\rho = \frac{4}{D_w}\left(\frac{1}{1+\gamma}\right) . \qquad (3\text{-}21)$$

Substituting equation (3-21) into equation (3-7),

$$Q = 6{,}4762065 \times 10^{-10} \kappa \left[\frac{D_w}{4}(1 + \gamma)E(\kappa)\right]^2 \sigma_{max}^3 \quad . \qquad (3\text{-}22)$$

In general, $\kappa = a/b = 1$ for the case of contact between an outer ring raceway and balls of self-aligning ball bearings. Consequently,

$$E(\kappa) = \int_0^{\pi/2} \left[1 - (1 - \frac{1}{\kappa^2})\sin^2\phi\right]^{1/2} d\phi = \int_0^{\pi/2} d\phi = \frac{\pi}{2} \quad .$$

Therefore, equation (3-22) yields

$$Q = 6{,}4762065 \times 10^{-10} \kappa \left[\frac{D_w}{8}(1 + \gamma)\pi\right]^2 \sigma_{max}^3 \quad . \qquad (3\text{-}23)$$

Substituting equation (3-23) into equation (3-16) and moreover exchanging Q for Q_{max},

$$C_{or} = 2{,}072\left(\frac{\sigma_{max}}{4000}\right)^3 \left[\frac{\pi}{4}(1 + \gamma)\right]^2 Z D_w^2 \cos\alpha \quad .$$

Introducing the number of rows i of balls,

$$C_{or} = f_o i Z D_w^2 \cos\alpha \quad , \qquad (3\text{-}24)$$

where

$$f_o = 2{,}072\left(\frac{\sigma_{max}}{4000}\right)^3 \left[\frac{\pi}{4}(1 + \gamma)\right]^2 \quad . \qquad (3\text{-}25)$$

The values of factor f_o on table 1 in ISO 76 are calculated from substituting σ_{max} = 4600 MPa and values of $\gamma = D_w \cos\alpha / D_{pw}$ shown in the table of ISO 76 into equation (3-25).

3.2 Basic static axial load rating C_{oa} for thrust ball bearings

The curvature sum $\Sigma\rho$ and curvature difference $F(\rho)$ of thrust ball bearings is given by the following equations,

$$\Sigma\rho = \frac{2}{D_w}\left(2 \pm \frac{\gamma}{1 \mp \gamma} - \frac{1}{2f}\right) , \qquad (3\text{-}26)$$

$$F(\rho) = \frac{\pm \frac{\gamma}{1 \mp \gamma} + \frac{1}{2f}}{2 \pm \frac{\gamma}{1 \mp \gamma} - \frac{1}{2f}} , \qquad (3\text{-}27)$$

where

$f = r/D_w$,

r = groove radius of housing washer, mm .

Substituting equation (3-26) into equation (3-7),

$$Q = 6{,}4762065 \times 10^{-10} \varkappa \left(\frac{D_w}{2} \frac{E(\varkappa)}{2 \pm \frac{\gamma}{1 \mp \gamma} - \frac{1}{2f}}\right)^2 \sigma_{max}^3 . \qquad (3\text{-}28)$$

Substituting equation (3-28) into the following equation (3-29),

$$C_{oa} = ZQ_{max} \sin\alpha , \qquad (3\text{-}29)$$

where

C_{oa} = basic static axial load rating, N ,

Z = number of balls carrying load in one direction ,

Q_{max} = maximum normal force between rolling element and raceway, N .

Therefore,

$$C_{oa} = 10{,}362\left(\frac{\sigma_{max}}{4000}\right)^3 \varkappa \left(\frac{E(\varkappa)}{2 \pm \frac{\gamma}{1 \mp \gamma} - \frac{1}{2f}}\right)^2 Z D_w^2 \sin\alpha . \qquad (3\text{-}30)$$

The formula applies to bearings with a cross-sectional raceway groove radius not larger than 0,54 D_w.

The load-carrying ability of a bearing is not necessarily increased by the use of a smaller groove radius, but is reduced by the use of a larger groove radius. In the latter case, a correspondingly reduced value of f_o shall be used.

The smaller value C_{oa} calculated from equation (3-30) shall be adopted. For washers with f = 0,54, using the upper sign,

$$C_{oa} = f_o Z D_w^2 \sin\alpha \quad , \qquad (3\text{-}31)$$

where

$$f_o = 10{,}362\left(\frac{\sigma_{max}}{4000}\right)^3 \kappa\left(\frac{E(\kappa)}{2 + \frac{\gamma}{1-\gamma} - \frac{1}{1{,}08}}\right)^2 \quad . \qquad (3\text{-}32)$$

The values of factor f_o on table 1 in ISO 76 are calculated from substituting the values for κ, $E(\kappa)$ and $\gamma = D_w \cos\alpha / D_{pw}$ shown in table A.2 , and σ_{max} = 4200 MPa into equation (3-32).

3.3 Basic static radial load rating C_{or} for radial roller bearings

The curvature sum $\sum\rho$ for radial roller bearings is given by the following equation,

$$\sum\rho = \frac{2}{D_{we}} \frac{1}{1 \mp \gamma} \quad . \qquad (3\text{-}33)$$

Substituting equation (3-33) into equation (3-11) and adopting the smaller Q,

$$Q = 1{,}3810867 \times 10^{-5} (1 - \gamma) L_{we} D_{we} \sigma_{max}^2 \quad . \qquad (3\text{-}34)$$

Substituting equation (3-34) into the following equation,

$$C_{or} = \frac{1}{S} ZQ_{max} \cos \alpha \, , \qquad (3\text{-}15)$$

where

C_{or} = basic static radial load rating, N ,

Z = number of rollers per row ,

Q_{max} = maximum normal force between rolling element and raceway, N ,

S is a function of the loaded zone parameter ε.
If one half of the rollers are loaded then S = 4,08 applies.
A common value used in general bearing calculations is
S = 5, which leads to a rather conservative estimate of
the maximum roller load.

α = nominal contact angle, °,

$$C_{or} = 44{,}194774\left(\frac{\sigma_{max}}{4000}\right)^2 (1 - \gamma) ZL_{we} D_{we} \cos \alpha \, .$$

Consequently, adopting σ_{max} = 4000 MPa and introducing the number of rows i of rollers,

$$C_{or} = 44\left(1 - \frac{D_{we} \cos \alpha}{D_{pw}}\right) iZL_{we} D_{we} \cos \alpha \, . \qquad (3\text{-}35)$$

3.4 Basic static axial load rating C_{oa} for thrust roller bearings

The curvature sum $\Sigma\rho$ of thrust roller bearings is given by the equation (3-33) and Q is given by the equation (3-34).

Substituting equations (3-33) and (3-34) into equation (3-29),

$$C_{oa} = 220{,}97387\left(\frac{\sigma_{max}}{4000}\right)^2 (1 - \gamma) ZL_{we} D_{we} \sin \alpha \, .$$

Consequently, adopting σ_{max} = 4000 MPa,

$$C_{oa} = 220\left(1 - \frac{D_{we} \cos \alpha}{D_{pw}}\right) ZL_{we} D_{we} \sin \alpha \, . \qquad (3\text{-}36)$$

4 Static Equivalent Load

4.1 Theoretical static equivalent radial load P_{or} for radial bearings

4.1.1 Single row radial bearings and radial contact groove ball bearings (nominal contact angle $\alpha = 0$)

Assuming both the bearing rings will yield a parallel displacement when a radial and axial loads act simultaneously on a single row radial bearings, the maximum rolling element load Q_{max} (N) is given by the following equation [11]

$$Q_{max} = \frac{F_r}{Z \cos\alpha \, J_r} = \frac{F_a}{Z \sin\alpha \, J_a} , \qquad (4-1)$$

where

F_r = radial load, N ,

F_a = axial load, N ,

J_r = radial load integral ,

J_a = axial load integral ,

Z = number of rolling elements per row ,

α = nominal contact angle, ° .

The radial and axial load integrals are given by the following equations

$$\left.\begin{aligned} J_r = J_r(\varepsilon) &= \frac{1}{2\pi}\int_{-\psi_0}^{+\psi_0}\left[1 - \frac{1}{2\varepsilon}(1 - \cos\psi)\right]^t \cos\psi \, d\psi \ , \\ J_a = J_a(\varepsilon) &= \frac{1}{2\pi}\int_{-\psi_0}^{+\psi_0}\left[1 - \frac{1}{2\varepsilon}(1 - \cos\psi)\right]^t d\psi \ , \end{aligned}\right\} \quad (4\text{-}2)$$

where

t = 3/2 for point contact

= 1.1 for line contact ,

ψ_0 = one half of the loaded arc ,

ε = a parameter indicating the width of the loaded zone in the bearing .

Assuming the bearing has no radial internal clearance under mounting, the static equivalent radial load $P_{or} = F_r$ when the rings displace in the radial direction ($\varepsilon = 0{,}5$). Consequently, since we can obtain the following equation from equation (4-1)

$$Q_{max} = \frac{P_{or}}{Z\cos\alpha J_r(0{,}5)} \ ,$$

the following relationship yields

$$\frac{F_r}{P_{or}} = \frac{J_r}{J_r(0{,}5)} \ , \quad (4\text{-}3)$$

$$\frac{F_a \cot\alpha}{P_{or}} = \frac{J_a}{J_r(0{,}5)} \ . \quad (4\text{-}4)$$

The values calculated from equations (4-3) and (4-4) for a constant contact angle α are given in table 4-1. In accordance with the functional relationship given by this table, the static equivalent radial load P_{or} for the given values of F_r , F_a and α may be obtained. The relationship between F_r/P_{or} and $F_a \cot\alpha/P_{or}$ also is shown by figure 4-1.

Table 4-1 Values for F_r/P_{or} and $F_a \cot\alpha/P_{or}$ vs. $F_r \tan\alpha/F_a$ for single row radial bearings

ε	ball bearings			roller bearings		
	$F_r \tan\alpha/F_a$	F_r/P_{or}	$F_a \cot\alpha/P_{or}$	$F_r \tan\alpha/F_a$	F_r/P_{or}	$F_a \cot\alpha/P_{or}$
0,5	0,8225	1	1,2158	0,7940	1	1,2595
0,6	0,7835	1,0558	1,3475	0,7482	1,0469	1,3993
0,7	0,7427	1,0949	1,4743	0,7000	1,0746	1,5353
0,8	0,6995	1,1183	1,5988	0,6484	1,0834	1,6709
0,9	0,6529	1,1255	1,7239	0,5917	1,0711	1,8102
1	0,6000	1,1128	1,8547	0,5238	1,0286	1,9638
1,25	0,4538	1,0003	2,2043	0,3600	0,8474	2,3541
1,67	0,3080	0,8165	2,6512	0,2333	0,6464	2,7703
2,5	0,1850	0,5852	3,1637	0,1372	0,4382	3,1948
5	0,0831	0,3108	3,7400	0,0611	0,2218	3,6317
∞	0	0	4,3706	0	0	4,0766

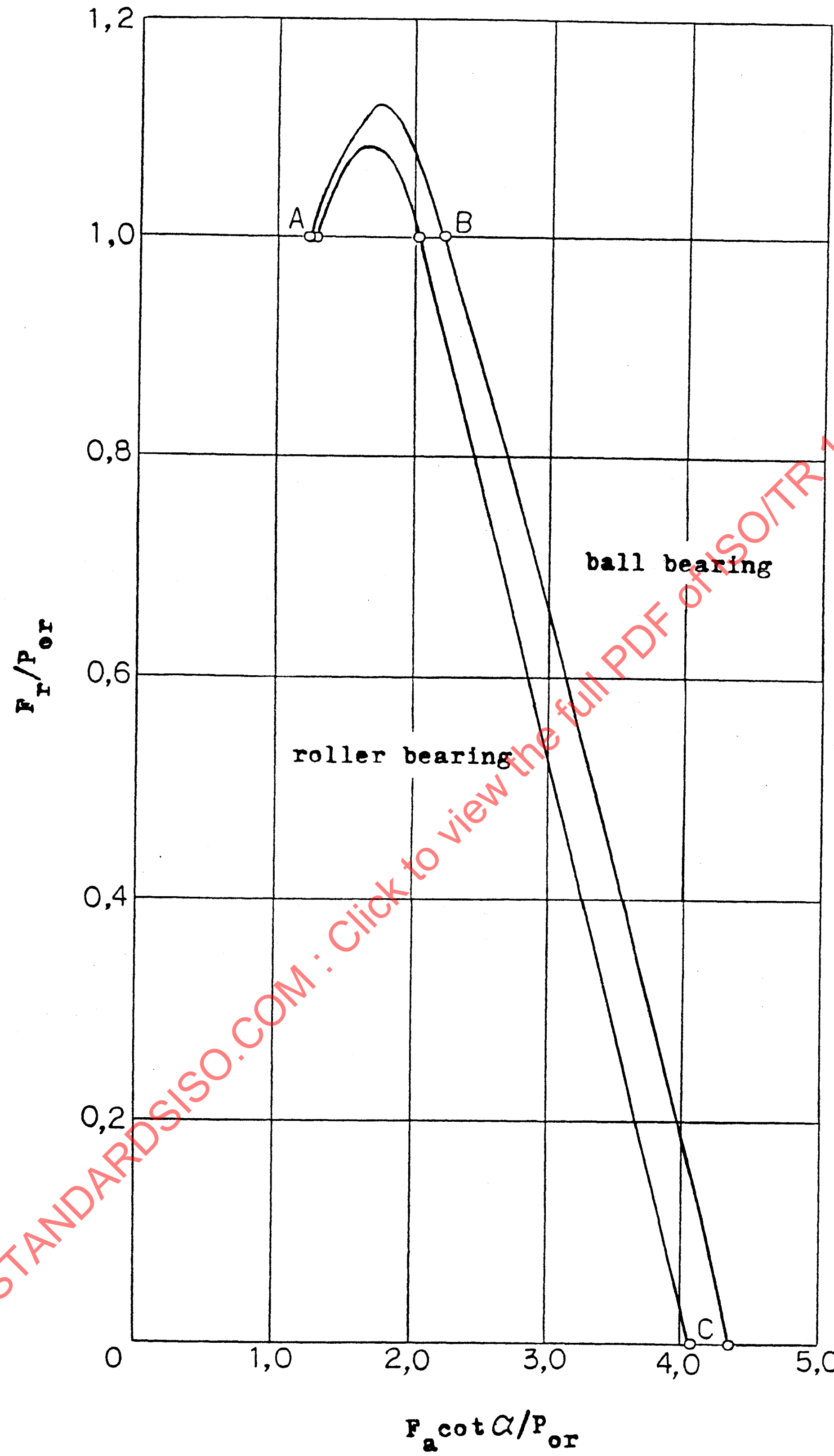

Figure 4-1 Theoretical relationship between radial and axial loads vs, static equivalent load for single row radial bearings

Table 4-1 and figure 4-1 are calculated and plotted based on the assumption of a constant contact angle. However, the above relationship is also approximately applicable to ball bearings (angular contact groove ball bearings etc.), a contact angle of which varies with the load, if $\cot\alpha'$ given by the following equation (4-5) is substituted for $\cot\alpha$ [12]

$$\frac{\cos\alpha}{\cos\alpha'} = 1 + \frac{c}{2r/D_w - 1}\left(\frac{F_a}{ZD_w^2\sin\alpha'}\right)^{2/3} \qquad (4\text{-}5)$$

c in the above equation is a compression constant depending on the elastic modulus and conformity $2r/D_w$, where r is a curvature radius of a raceway cross-section and D_w is a ball diameter (see table 4-2).

Table 4-2 Values of c and $\frac{c}{2r/D_w - 1}$

unit N and mm

$2r/D_w$		1,0325	1,035	1,0375	1,06
$c \times 10^4$	units in N, mm	4,3217	4,3871	4,4745	4,9547
$\frac{c}{2r/D_w - 1}$		0,01323	0,01253	0,01193	0,008258
$c \times 10^3$	units in kgf, mm	1,98	2,01	2,05	2,27
$\frac{c}{2r/D_w - 1}$		0,06062	0,05743	0,05440	0,03783

For an example of $2r/D_w = 1,035$, the values of $\cot\alpha'$ for each value of F_a/ZD_w^2 for angular contact groove ball bearings with $\alpha = 15° \sim 45°$ are given in table 4-3.

Furthermore, for single and double row radial contact groove ball bearings, table 4-4 may be obtained from equations

Table 4-3 Example of values of $\cot\alpha'$ for angular contact groove ball bearings

α	F_a/ZD_w^2 *				
	0,5	1	2	5	10
	$\cot\alpha'$				
15°	3,024	2,793	2,526	2,154	1,865
20°	2,450	2,322	2,164	1,905	1,691
25°	1,997	1,929	1,834	1,664	1,511
30°	1,651	1,613	1,552	1,444	1,337
35°	1,381	1,356	1,317	1,248	1,171
40°	1,163	1,146	1,122	1,072	1,018
45°	0,975	0,969	0,952	0,920	0,879

* units in N and mm. Since $C_{or} = f_o ZD_w^2 \cos\alpha$, $F_a/ZD_w^2 = (F_a/C_{or}) f_o \cos\alpha$.

(4-3), (4-4) and the following equation (4-6) [12]

$$\sin\alpha' \approx \tan\alpha' \approx \left(\frac{2c}{2r/D_w - 1}\right)^{3/8} \left(1 - \frac{1}{2\varepsilon}\right)^{3/8} \left(\frac{F_a}{J_a iZD_w^2}\right)^{1/4} , \qquad (4\text{-}6)$$

where

i = number of rows of balls ,

Z = number of balls per row .

For given values of F_r and F_a a provisional value of α' is found using equation (4-7). Next, table 4-4 is used to find ε and F_r/P_{or} or $F_a \cot\alpha'/P_{or}$ and then P_{or} can be determined.

Table 4-4 Values of F_r/P_{or} and $F_a \cot\alpha'/P_{or}$ vs. $F_r \tan\alpha'/F_a$ for radial contact groove ball bearings

ε	$F_r \tan\alpha'/F_a$	F_r/P_{or}	$F_a \cot\alpha'/P_{or}$
0,5	∞	1	0
0,6	1,1432	1,0558	0,9238
0,7	0,9055	1,0949	1,2096
0,8	0,7859	1,1183	1,4231
0,9	0,7013	1,1255	1,6051
1	0,6280	1,1128	1,7721
1,25	0,4632	1,0003	2,1600
1,67	0,3105	0,8165	2,6035
2,5	0,1855	0,5852	3,1548
5	0,0831	0,3108	3,7377
∞	0	0	4,3706

$$\tan\alpha' \approx \left(\frac{2c}{2r/D_w - 1}\right)^{3/8} \left(\frac{F_a}{iZD_w^2}\right)^{1/4} . \qquad (4\text{-}7)$$

For $2r/D_w = 1{,}035$, for example, the values of $\tan\alpha'$ for each value of F_a/iZD_w^2 are given in table 4-5.

Table 4-5 Example of values of contact angle for radial contact groove ball bearings

F_a/iZD_w^2 *	0,5	1	2	5	10
$\tan\alpha'$	0,2110	0,2510	0,2985	0,3753	0,4463

* units in N and mm. $F_a/iZD_w^2 = (F_a/C_{or})f_o$.

Moreover, the relationship between F_r/P_{or} and $F_a \cot\alpha'/P_{or}$ is given in figure 4-2.

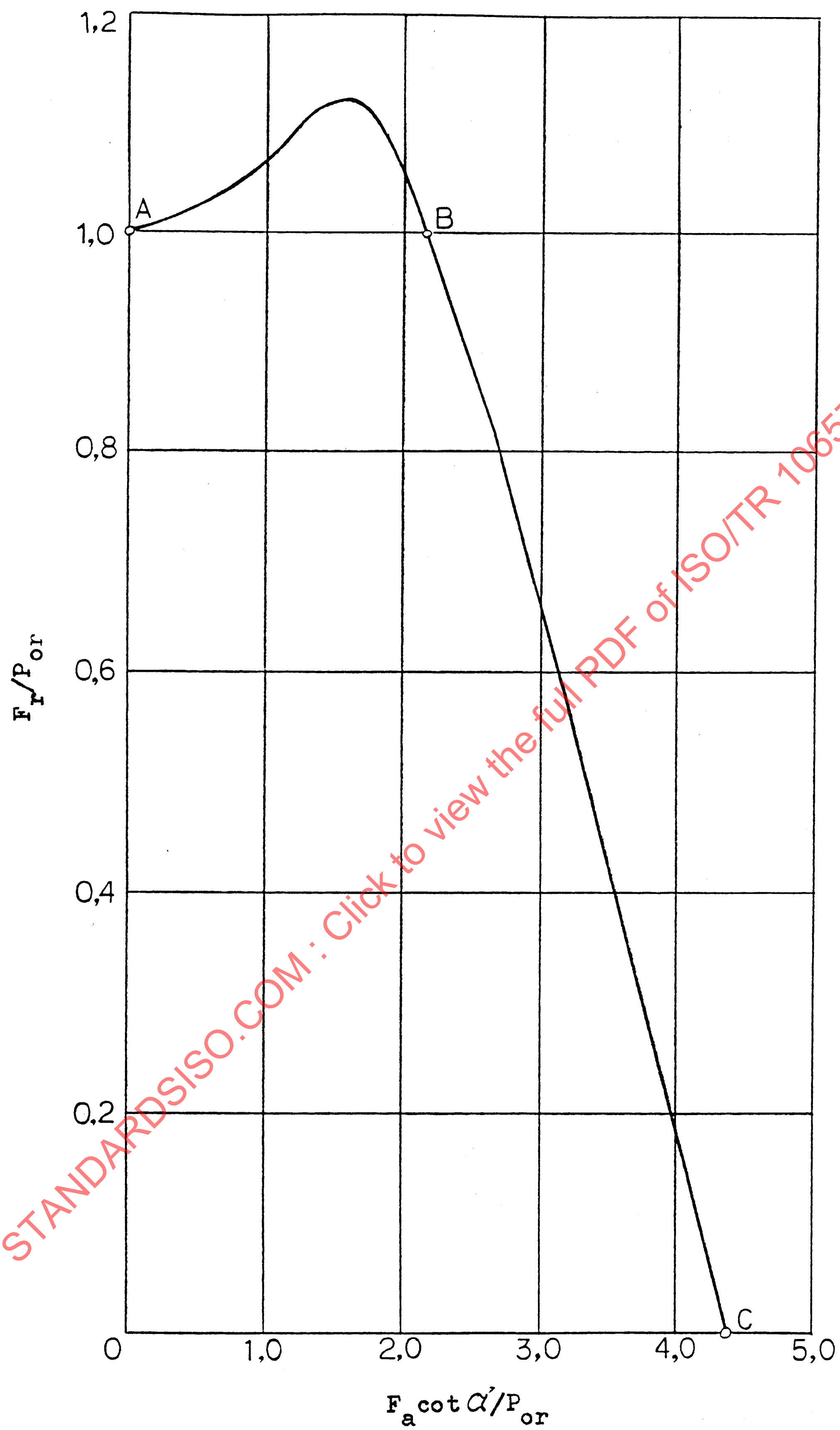

Figure 4-2 Theoretical relationship between radial and axial loads vs. static equivalent load for radial contact bearings

4.1.2 Double row radial bearings

Assuming both the bearing rings will yield a parallel displacement when a radial load and an axial load act simultaneously on a double row radial bearing and designating each row as I and II,

$$F_r = F_{rI} + F_{rII} \quad , \quad F_a = F_{aI} - F_{aII}$$

and the maximum rolling element load for each row (the number of rolling elements is Z, respectively.) is given by the following equations [11]

$$\left.\begin{aligned} Q_{max} &= \frac{F_r}{Z\cos\alpha J_r} = \frac{F_a}{Z\sin\alpha J_a} \quad , \\ Q_{max II} &= Q_{max I}\left(\frac{\varepsilon_{II}}{\varepsilon_I}\right)^t \quad , \end{aligned}\right\} \qquad (4-8)$$

where

$$\left.\begin{aligned} J_r &= J_r(\varepsilon_I) + \left(\frac{\varepsilon_{II}}{\varepsilon_I}\right)^t J_r(\varepsilon_{II}) \quad , \\ J_a &= J_a(\varepsilon_I) - \left(\frac{\varepsilon_{II}}{\varepsilon_I}\right)^t J_a(\varepsilon_{II}) \quad . \end{aligned}\right\} \qquad (4-9)$$

Assuming the bearing has no radial internal clearance, the static equivalent load $P_{or} = F_r$, when the bearing rings displace in the radial direction ($\varepsilon_I = \varepsilon_{II} = 0,5$), and

$$Q_{max} = \frac{P_{or}}{Z\cos\alpha J_r(0,5)} \quad ,$$

that is, in this case equations (4-3) and (4-4) are valid. The values calculated from equations (4-3) and (4-4) for a constant contact angle α are given in table 4-6. In accordance with the functional relationship given by this table, the static equivalent radial load P_{or} for the given values of F_r, F_a and α may be obtained. The relationship between F_r/P_{or} and $F_a \cot\alpha/P_{or}$ is shown by figure 4-3. Furthermore, for double row radial contact groove ball bearings, the contact angle of which varies with the load, P_{or} may be obtained approximately by using α' by equation (4-5) instead of α in table 4-6.

Table 4-6 Values of F_r/P_{or} and $F_a \cot\alpha/P_{or}$ vs. $F_r \tan\alpha/F_a$ for double row radial bearings

ε_I	ε_{II}	ball bearings			roller bearings		
		$F_r \tan\alpha/F_a$	F_r/P_{or}	$F_a \cot\alpha/P_{or}$	$F_r \tan\alpha/F_a$	F_r/P_{or}	$F_a \cot\alpha/P_{or}$
0,5	0,5	∞	1	0	∞	1	0
0,6	0,4	2,0465	0,7797	0,3810	2,3908	0,8217	0,3437
0,7	0,3	1,0916	0,6634	0,6078	1,2101	0,7022	0,5803
0,8	0,2	0,8005	0,6026	0,7528	0,8229	0,6187	0,7518
0,9	0,1	0,6713	0,5721	0,8523	0,6340	0,5586	0,8811
1	0	0,6000	0,5564	0,9274	0,5238	0,5143	0,9819
1,25	0	0,4538	0,5001	1,1021	0,3600	0,4237	1,1771
1,67	0	0,3080	0,4083	1,3256	0,2333	0,3232	1,3852
2,5	0	0,1850	0,2926	1,5819	0,1372	0,2191	1,5974
5	0	0,0831	0,1554	1,8699	0,0611	0,1109	1,8158
∞	0	0	0	2,1850	0	0	2,0383

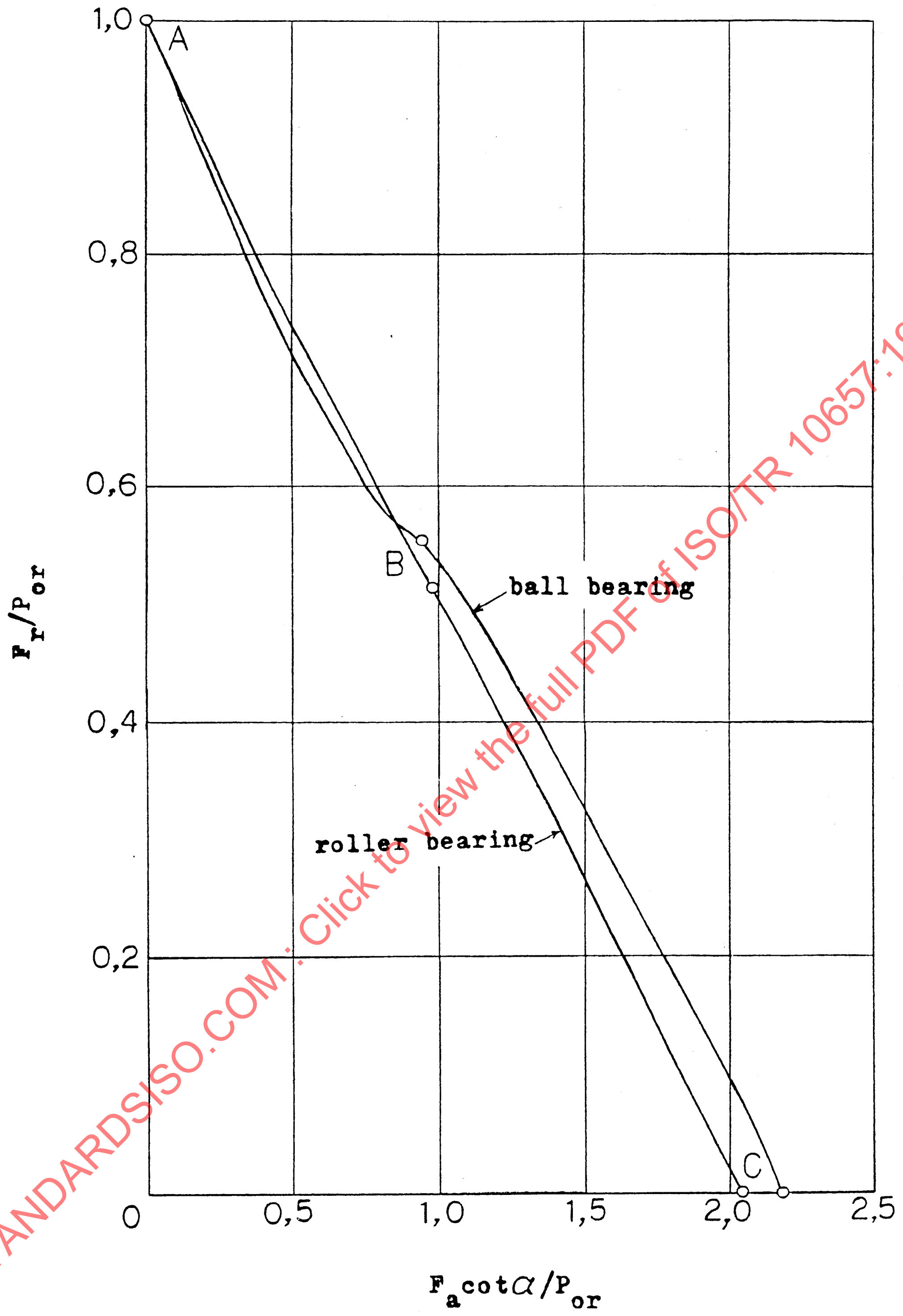

Figure 4-3 Theoretical relationship between radial and axial loads vs. static equivalent load for double row radial bearings

4.2 Theoretical static equivalent axial load P_{oa} for thrust bearings

4.2.1 Single direction thrust bearings

Single direction thrust bearings which can support radial loads may be considered as single row radial contact bearings with a large contact angle.

When bearing washers displace in the axial direction in the equation (4-4) which is valid for single row radial contact bearings with a constant contact angle, $\varepsilon = \infty$ and $J_a = 1$, and since the static equivalent axial load $P_{oa} = F_a$, substituting this relationship, the following equation yields

$$P_{or} = P_{oa} \cot\alpha J_r(0,5) \quad .$$

Substituting this equation into equations (4-3) and (4-4), the following equations yield

$$\frac{F_r \tan\alpha}{P_{oa}} = J_r \quad , \qquad (4-10)$$

$$\frac{F_a}{P_{oa}} = J_a \quad . \qquad (4-11)$$

The table 4-7 may be obtained from equations (4-10) and (4-11). In accordance with the functional relationship given by this table, the static equivalent axial load P_{oa} for the given values of F_r, F_a and α may be obtained. The relationship between F_a/P_{oa} and $F_r \tan\alpha/P_{oa}$ is given in figure 4-4.

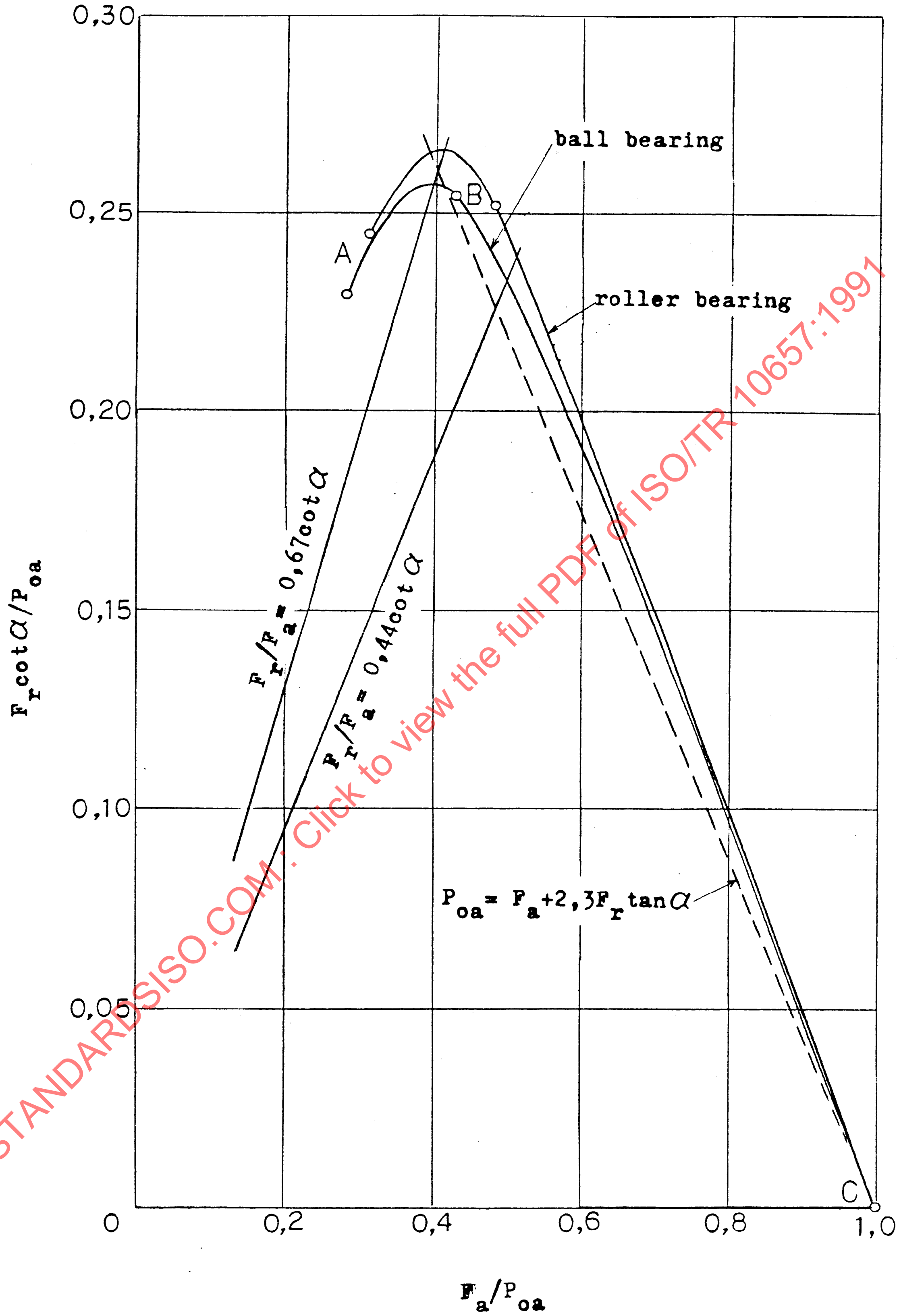

Figure 4-4 Theoretical relationship between axial and radial loads vs. static equivalent load for single direction thrust bearings

Table 4-7 Values of F_a/P_{oa} and $F_r \tan\alpha/P_{oa}$ vs. $F_r \tan\alpha/F_a$ for single direction thrust bearings

ε	ball bearings			roller bearings		
	$F_r \tan\alpha/F_a$	F_a/P_{oa}	$F_r \tan\alpha/P_{oa}$	$F_r \tan\alpha/F_a$	F_a/P_{oa}	$F_r \tan\alpha/P_{oa}$
0,5	0,8225	0,2782	0,2288	0,7940	0,3090	0,2453
0,6	0,7835	0,3084	0,2416	0,7482	0,3433	0,2568
0,7	0,7427	0,3374	0,2505	0,7000	0,3766	0,2636
0,8	0,6995	0,3658	0,2559	0,6484	0,4099	0,2658
0,9	0,6529	0,3945	0,2576	0,5917	0,4441	0,2628
1	0,6000	0,4244	0,2546	0,5238	0,4817	0,2523
1,25	0,4538	0,5044	0,2289	0,3600	0,5775	0,2079
1,67	0,3080	0,6067	0,1868	0,2333	0,6796	0,1586
2,5	0,1850	0,7240	0,1339	0,1372	0,7837	0,1075
5	0,0831	0,8558	0,0711	0,0611	0,8909	0,0544
∞	0	1	0	0	1	0

4.2.2 Double direction thrust bearings

Double direction thrust bearings which can support radial loads may be considered as double row radial contact bearings with a large contact angle.

For this case the same equations (4-10) and (4-11) as for single direction thrust bearings are valid and table 4-8 may be obtained. In accordance with the functional relationship given by this table, the static equivalent axial load P_{oa} for the given values of F_r, F_a and α may be obtained. The relationship between F_a/P_{oa} and $F_r \tan\alpha/P_{oa}$ is given in figure 4-5.

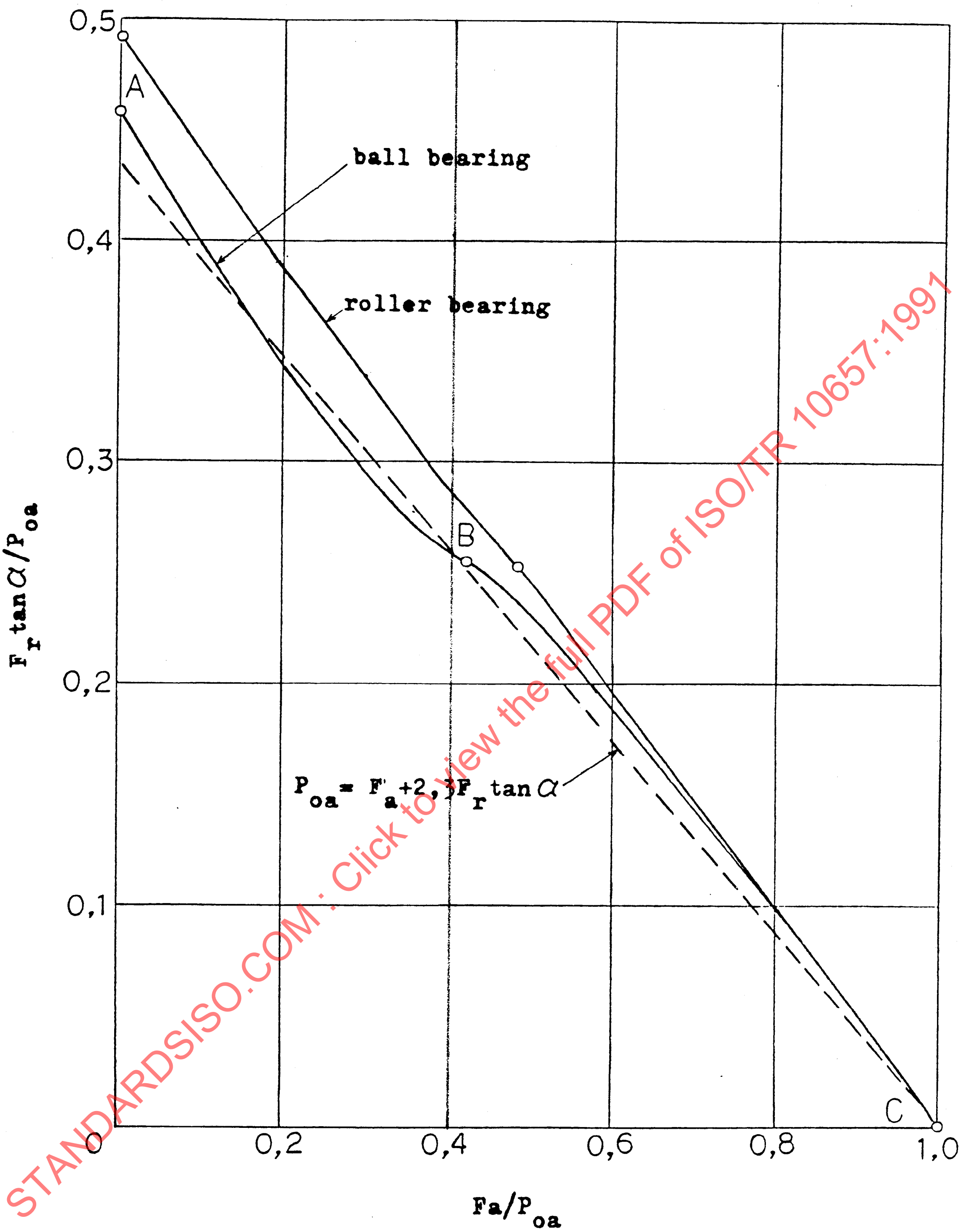

Figure 4-5 Theoretical relationship between axial and radial loads vs. static equivalent load for double direction thrust bearings

Table 4-8 Values of F_a/P_{oa} and $F_r \tan\alpha/P_{oa}$ vs. $F_r \tan\alpha/P_{oa}$ for double direction thrust bearings

ε_I	ε_{II}	ball bearings			roller bearings		
		$F_r \tan\alpha/F_a$	F_a/P_{oa}	$F_r \tan\alpha/P_{oa}$	$F_r \tan\alpha/F_a$	F_a/P_{oa}	$F_r \tan\alpha/P_{oa}$
0,5	0,5	∞	0	0,4577	∞	0	0,4906
0,6	0,4	2,0465	0,1744	0,3568	2,3908	0,1686	0,4031
0,7	0,3	1,0916	0,2782	0,3036	1,2101	0,2847	0,3445
0,8	0,2	0,8005	0,3445	0,2758	0,8229	0,3689	0,3035
0,9	0,1	0,6713	0,3900	0,2618	0,6340	0,4323	0,2741
1	0	0,6000	0,4244	0,2546	0,5238	0,4817	0,2523
1,25	0	0,4538	0,5044	0,2289	0,3600	0,5775	0,2079
1,67	0	0,3080	0,6067	0,1868	0,2333	0,6796	0,1586
2,5	0	0,1850	0,7240	0,1339	0,1372	0,7837	0,1075
5	0	0,0831	0,8558	0,0711	0,0611	0,8909	0,0544
∞	0	0	1	0	0	1	0

4.3 Approximate formulae for theoretical static equivalent load

4.3.1 Radial bearings

From a practical standpoint it is preferable to replace the theoretical curves in figures 4-1 and 4-3 for radial contact bearings with a constant contact angle by two straight line segments AB, BC and a straight line AC, respectively.

The static equivalent radial load P_{0r} given by the straight line segments and straight lines is shown in tables 4-9 and 4-10, respectively.

For radial ball bearings the contact angle of which varies with

Table 4-9 Approximate formulae for theoretical static equivalent radial loads for single row radial bearings (straight line segments AB and BC in figure 4-1)

bearing type	abscissa			approximate formulae of P_{or}	
	point A	point B	point C	segment AB	segment BC
single row ball bearings	1,22	2,21	4,37	$P_{or} = F_r$	$P_{or} = 0,494F_r + 0,229\cot\alpha F_a$
single row roller bearings	1,26	2,03	4,08	$P_{or} = F_r$	$P_{or} = 0,502F_r + 0,245\cot\alpha F_a$

Table 4-10 Approximate formulae for theoretical static equivalent radial loads for double row radial bearings (straight line AC in figure 4-3)

bearing type	abscissa of point C	approximate formulae of P_{or}
double row ball bearings	2,18	$P_{or} = F_r + 0,459\cot\alpha F_a$
double row roller bearings	2,04	$P_{or} = F_r + 0,490\cot\alpha F_a$

the load, the formulae given by tables 4-9 and 4-10 are approximately applicable if $\cot\alpha'$ by equation (4-5) is substituted for $\cot\alpha$ in the formulae.

For radial contact groove ball bearings replacing theoretical curve by straight line segments AB and BC in figure 4-2, the static equivalent radial load P_{or} may be given in table 4-11, and the value of $\cot\alpha'$ in the table is given by the equation (4-7).

Table 4-11 Approximate formula for theoretical static equivalent radial load for radial contact groove ball bearings (straight line segments AB and BC in figure 4-2)

bearing type	abscissa		approximate formula of P_{or}	
	point B	point C	segment AB	segment BC
radial contact groove ball bearings	2,16	4,37	$P_{or} = F_r$	$P_{or} = 0{,}506F_r + 0{,}229\cot\alpha' F_a$

4.3.2 Thrust bearings

Replacing theoretical curves in figures 4-4 and 4-5 by the straight lines BC and AC, respectively, the static equivalent axial load P_{oa} may be given by tables 4-12 and 4-13.

Table 4-12 Approximate formulae for theoretical static equivalent axial loads for single direction thrust bearings (straight line BC in figure 4-4)

bearing type	coordinates of point B	approximate formulae of P_{oa}
single direction ball bearings	(0,424 , 0,255)	$P_{oa} = 2{,}26\tan\alpha F_r + F_a$
single direction roller bearings	(0,482 , 0,252)	$P_{oa} = 2{,}06\tan\alpha F_r + F_a$

Table 4-13 Approximate formulae for theoretical static equivalent axial loads for double direction thrust bearings (straight line AC in figure 4-5)

bearing type	coordinates of point A	approximate formulae of P_{oa}
double direction ball bearings	(0 , 0,458)	$P_{oa} = 2{,}18\tan\alpha F_r + F_a$
double direction roller bearings	(0 , 0,491)	$P_{oa} = 2{,}04\tan\alpha F_r + F_a$

4.4 Practical formulae of static equivalent load

4.4.1 Radial bearings

Assuming the bearing has no radial internal clearance, maximum rolling element loads under radial load for single radial contact bearings with a contact angle α are given as follows since $J_r(0,5)$ = 0,2288 (ball bearings), $J_r(0,5)$ = 0,2453 (roller bearings) for ε = 0,5 in equation (4-1),

$$Q_{max} = \frac{F_r}{0,2288 Z \cos\alpha} = \frac{4,37 F_r}{Z \cos\alpha} \quad \text{(ball bearings)},$$

$$Q_{max} = \frac{F_r}{0,2453 Z \cos\alpha} = \frac{4,08 F_r}{Z \cos\alpha} \quad \text{(roller bearings)}.$$

However, taking into account internal clearances in practice, the following equation for either ball bearings or roller bearings has been adopted since R. Stribeck (1901)

$$Q_{max} = 5 \frac{F_r}{Z \cos\alpha} \quad . \qquad (4\text{-}12)$$

Moreover, the factor f_o involved in the formulae for basic radial load rating C_{or} of radial bearings also is based on the assumption by which load distributions depend on equation (4-12) (see Annotation 1) in table 1 of ISO 76).

For single row radial bearings with a constant contact angle under combined radial load F_r and axial load F_a, the following equation may be obtained owing to H. Stellrecht [13] where all of rolling elements are subjected to the load (see figure 4-6)

$$\frac{5P_{or}}{Z\cos\alpha} = \frac{F_a}{Z\sin\alpha} + \frac{2,5F_r}{Z\cos\alpha} . \qquad (4\text{-}13)$$

In the case of the first term $\geqq$ the second term in the right hand of the above equation, that is $F_r \leqq 0,4\cot\alpha F_a$ (corresponds to $\varepsilon > 1,25$ in table 4-1 and segment BC in figure 4-1), this equation is valid.

The following equation may be obtained from equation (4-13)

$$P_{or} = 0,5F_r + 0,2\cot\alpha F_a . \qquad (4\text{-}14)$$

When $F_r = 0,4\cot\alpha F_a$, the right hand terms in equation (4-14) will equal F_r, and for the segment AB in $F_r < 0,4\cot\alpha F_a$ (figure 4-7)

$$P_{or} = F_r .$$

When a double row radial bearing with a number of rolling elements per row Z is subjected to a load by its one row only and all of the rolling elements Z are subjected to the load (corresponds to $\varepsilon \geqq 1$ in table 4-6 and segment BC in figure 4-3), the following equation may be obtained from the same consideration as for the above single row bearings

$$\frac{5P_{or}}{2Z\cos\alpha} = \frac{F_a}{Z\sin\alpha} + \frac{2{,}5F_r}{Z\cos\alpha} \quad . \qquad (4\text{-}15)$$

Consequently,

$$P_{or} = F_r + 0{,}4\cot\alpha F_a \quad . \qquad (4\text{-}16)$$

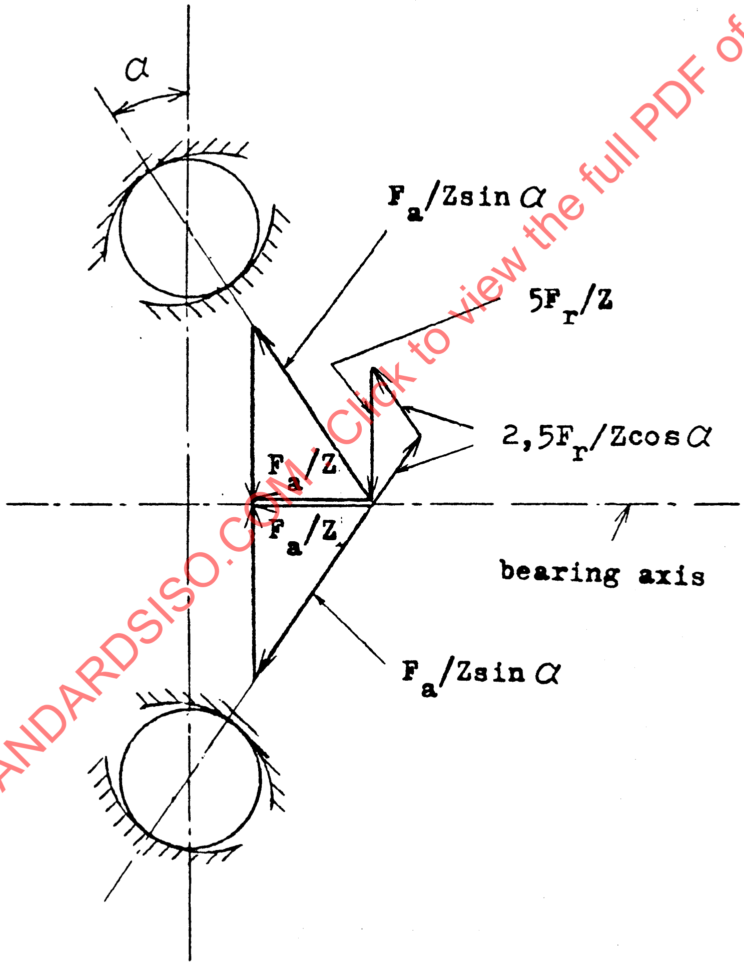

Figure 4-6 Rolling element load in a single row radial bearing

This equation is valid where $F_r \leqq 0{,}4\cot\alpha F_a$ (segment B_1C_1 in figure 4-7), and $P_{or} = 2F_r$ (point B_1) where $F_r = 0{,}4\cot\alpha F_a$. $P_{or} = F_r$ at point A_1 in figure 4-7, and it may be considered that equation (4-16) also is valid for segment A_1B_1.

For radial bearings contact angles of which vary with the axial loads, we may adopt α' instead of α in equations (4-14) and (4-16). That is,

single row bearings : $P_{or} = 0{,}5F_r + 0{,}2\cot\alpha' F_a$, (4-17)

double row bearings : $P_{or} = F_r + 0{,}4\cot\alpha' F_a$. (4-18)

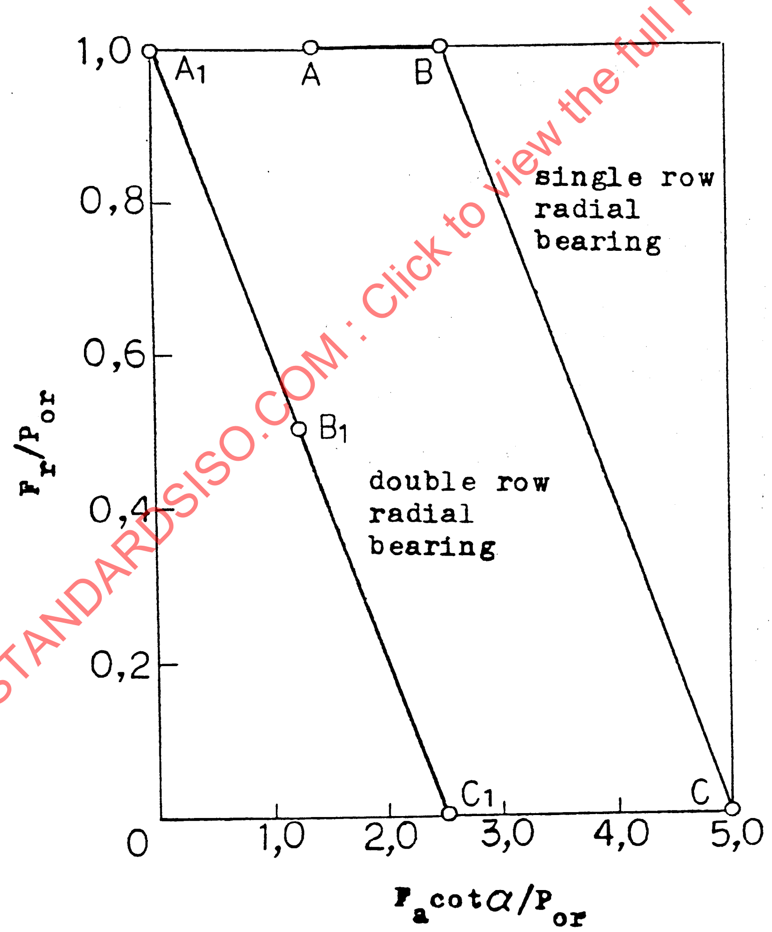

Figure 4-7 Relationship between radil and axial loads vs. static equivalent load for radial bearings

However, strictly speaking, α only of the right hand in equations (4-13) and (4-15) shall be replaced by α', and it is necessary to take into account that the static load capacity decreases because of manufacturing inaccuracy as an axial runout where a radial bearing is subjected to an axial load. Therefore, taking a reduction factor $(1 - k_o \sin\alpha)$, the formulae for P_{or} become as follows;

single row bearings : $$P_{or} = \frac{0,5\cos\alpha}{\cos\alpha'} F_r + \frac{0,2\cos\alpha}{\sin\alpha'(1 - k_o\sin\alpha)} F_a , \qquad (4\text{-}19)$$

double row bearings : $$P_{or} = \frac{\cos\alpha}{\cos\alpha'} F_r + \frac{0,4\cos\alpha}{\sin\alpha'(1 - k_o\sin\alpha)} F_a . \qquad (4\text{-}20)$$

There is the experiment by A. Palmgren[14] as for the magnitude of rolling element load under combined load for the single row radial contact groove ball bearing. Figure 4-8 plots his results on figure of F_r/P_{or} —— $F_a\cot\alpha/P_{or}$.

NOTE - This experiment had been done for bearings which have $D_w = 16,5$ mm, groove radius $r = 0,53D_w$, $Z = 12$, and the value of α' for the experiment values in figure 4-8 is based on the following equation (unit for F_a is kgf)

$$\tan\alpha' \approx \left(\frac{2c}{2r/D_w - 1}\right)^{3/8} \left(\frac{F_a}{ZD_w^2}\right)^{1/4} = 0,05024 F_a^{1/4} . \qquad (4\text{-}21)$$

Taking the straight line AC_1 inside (in safety side of) all of experiment values shown in figure 4-8, the following equation yields

$$P_{or} = F_r + 0,2\cot\alpha' F_a . \qquad (4\text{-}22)$$

And also, in figure 4-8, we may take the straight line segments AB and BC [coordinates of point B (1,7 , 1,0)] taking into account theoretical curve (see figure 4-2) and experiment results. In this case, the following equation yields

$$P_{or} = 0{,}575F_r + 0{,}25\cot\alpha' F_a \quad . \qquad (4\text{-}23)$$

4.4.2 Thrust bearings

Since single and double direction thrust bearings with a contact angle $\alpha \neq 90^\circ$ may be regarded as single and double row radial bearings with a large constant contact angle, respectively, the formula for the static equivalent axial load of thrust bearings based on equations (4-14) and (4-16). That is, since the static radial load capacity/ the static axial load capacity = $0{,}2\cot\alpha$ or $0{,}4\cot\alpha$, the following equation yields by means of dividing both hands in equations (4-14) and (4-16) by $0{,}2\cot\alpha$ and $0{,}4\cot\alpha$, respectively,

$$P_{oa} = 2{,}5F_r \tan\alpha + F_a \quad . \qquad (4\text{-}24)$$

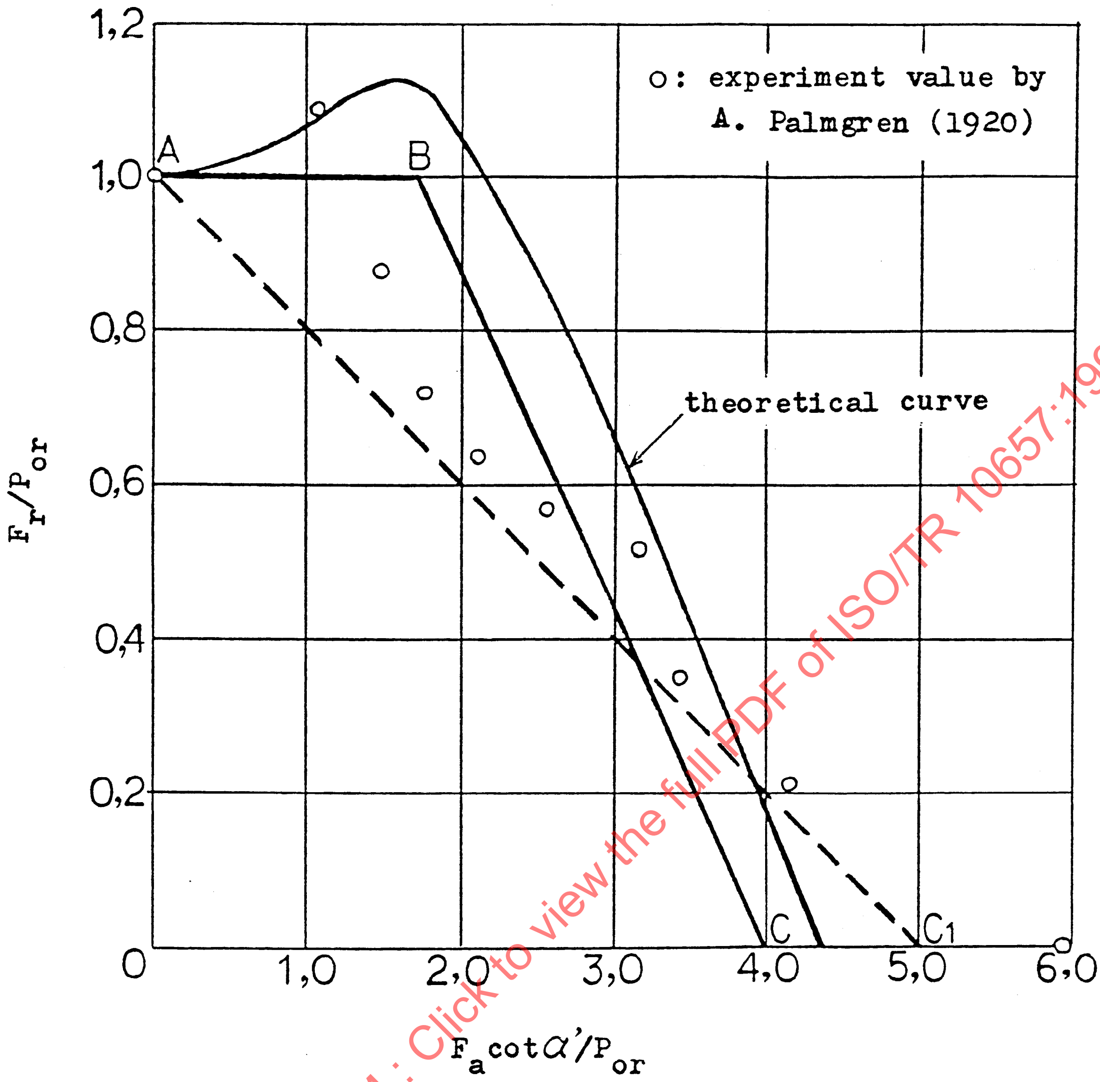

Figure 4-8 Relationship between radial and axial loads vs. static equivalent load for radial contact groove ball bearings

4.5 Static radial load factor X_o and static axial load factor Y_o

4.5.1 Radial bearings

4.5.1.1 Radial contact groove ball bearings

For radial contact groove ball bearings, from equation (4-22) for the static equivalent radial load,

$$X_o = 1 \quad , \quad Y_o = 0{,}2\cot\alpha' \ . \qquad (4\text{-}25)$$

However, using the contact angle $\alpha' \approx 15^o$ corresponding to (1) conformity $2r/D_w = 1{,}035$ and axial load $F_a \approx 0{,}1C_{or}$ (relatively small in safety side) or (2) $2r/D_w = 1{,}035$ and $F_a/iZD_w^2\sin\alpha' = 0{,}5$ kgf/mm^2 = 4,9033 N/mm^2, Y_o yields

$$Y_o = 0{,}2 \times 3{,}732 = 0{,}7464 \approx 0{,}75 \ .$$

Factors $X_o = 1$ and $Y_o = 0{,}75$ had been adopted at the initial period [15], [16] .

NOTE – For (1), from equation (4-7), $\tan\alpha' = 0{,}2636$ (see table 4-14). Consequently, $\alpha' = 14{,}8^o$. And for (2), from equation (4-5), $\cos\alpha' = 0{,}96508\cos\alpha$ yields. Consequently, $\alpha' = 15{,}2^o$.

Secondly, from equation 4-23,

$$X_o = 0{,}575 \approx 0{,}6 \quad , \quad Y_o = 0{,}25\cot\alpha \ . \qquad (4\text{-}26)$$

However, if $2r/D_w = 1,035$, from equation (4-7) (units : kgf and mm),

$$\tan\alpha' = 0,44418\left(\frac{F_a}{iZD_w^2}\right)^{1/4} = 0,46872\left(\frac{F_a}{C_{or}}\right)^{1/4}$$

yields $[(C_{or} = 1,24iZD_w^2$ (kgf)],values of Y_o versus each value for F_a/C_{or} become as shown in table 4-14.

Table 4-14 Values of factor Y_o for radial contact groove ball bearings

F_a/C_{or} *	0,05	0,1	0,2	0,5	1
$\tan\alpha'$	0.2216	0,2636	0,3135	0,3941	0,4687
$Y_o = 0,25\cot\alpha'$	1,128	0,948	0,797	0,634	0,533
practical values for Y_o	1,1	0,9	0,8	0,6	0,5

* units in kgf and mm

$X_o = 0,6$ and practical values for Y_o in the lowest line of the above table had been adopted after $X_o = 1$ and $Y_o = 0,75$ [17], [18] . However, ISO/R 76 and ISO 76 have the given $X_o = 0,6$ and $Y_o = 0,5$ (corresponds to $F_a/C_{or} = 1$). As for this, A. Palmgren had mentioned the following sentence in his book [19] ,"The Y_o factors given in Table 3.6 for radial contact groove ball bearings are calculated for thrust loads of the same magnitude as basic static load rating. With lighter thrust loads, smaller contact angles are developed, and then somewhat greater Y_o factors can be justified."

It is considered that the fact that ISO Standard specified the values of Y_o only corresponding to $F_a \approx C_{or}$ is based on that its influence is relatively small except F_a/C_{or} ($\alpha \neq 0°$) and F_r/C_{or} being particularly same (consequently, P_{or}/C_{or} being small) owing to there being the condition of $P_{or} \geq F_r$, and calculation of the static equivalent load necessary to design is done to ascertain $P_{or} \leq C_{or}$ usually.

Moreover, if $\alpha' = 5°$ is adopted taking into account radial internal clearances, the value of α' for $2r/D_w = 1{,}035$ and $F_a = C_{or} = 1{,}25iZD_w^2$ (kgf) $= 12{,}258iZD_w^2$ (N) is 26,6° from equation (4-5) and $Y_o = 0{,}25\tan\alpha'$ $= 0{,}4992 \approx 0{,}5$ yields from equation (4-26). And from equation (4-19),

$$X_o = 0{,}5\,\frac{\cos\alpha}{\cos\alpha'} = 0{,}5571 \approx 0{,}6 \quad ,$$

$$Y_o = \frac{0{,}2\cos\alpha}{\sin\alpha'(1 - 0{,}2\sin\alpha)} = 0{,}4529 \approx 0{,}5 \quad .$$

4,5.1.2 <u>Angular contact groove ball bearings</u>

For angular contact groove ball bearings, from equations (4-14) and (4-16),

$$\text{single row bearings}: X_o = 0{,}5 \quad , \quad Y_o = 0{,}2\cot\alpha \quad , \qquad (4\text{-}27)$$

$$\text{double row bearings}: X_o = 1 \quad , \quad Y_o = 0{,}4\cot\alpha \qquad (4\text{-}28)$$

had been adopted initially [15].

However, afterward Y_o becomes as follows from equations (4-14) and (4-18):

$$Y_o = 0{,}2\cot\alpha' \text{ (single row bearings)} \quad ,$$

$$Y_o = 0{,}4\cot\alpha' \text{ (double row bearings)} \quad ,$$

and substituting the values of α' obtained from the equation $\cos\alpha' = 0{,}972402\cos\alpha$ in the above equations, the values shown in table 4-15 have been adopted [17].

NOTE – Equation $\cos\alpha' = 0{,}972402\cos\alpha$ may be obtained by substituting $2r/D_w = r_i/D_w + r_e/D_w = 0{,}5175 + 0{,}53 = 1{,}0475$, $c = 0{,}00214$ (units in kgf,mm) or $c = 0{,}00046709$ (units in N,mm) and $F_a/ZD_w^2\sin\alpha' = 0{,}5$ kgf/mm^2 = 4,9033 N/mm^2 into equation (4-5).

Table 4-15 Values of Y_o for angular contact groove ball bearings

α	α'	single row bearings		double row bearings	
		$Y_o = 0{,}2\cot\alpha'$	Practical values of Y_o	$Y_o = 0{,}4\cot\alpha'$	practical values of Y_o
15°	20,1°	0,547	0,55	1,093	1,09
20°	24,0°	0,449	0,45	0,898	0,90
25°	28,2°	0,373	0,37	0,746	0,75
30°	32,6°	0,313	0,31	0,625	0,63
35°	37,2°	0,263	0,26	0,527	0,53
40°	41,8°	0,224	0,22	0,447	0,45

Values of factors X_o and Y_o in ISO 76 are based on equations (4-19) and (4-20) (however, $k_o = 0{,}2$). That is , for single row bearings

$$X_o = 0{,}5\,\frac{\cos\alpha}{\cos\alpha'}, \quad Y_o = \frac{0{,}2\cos\alpha}{\sin\alpha'(1 - 0{,}2\sin\alpha)}, \qquad (4\text{-}29)$$

and adopting the following approximate equation to the value of

$$\cot\alpha' = \frac{\cot\alpha}{\sqrt{1 + \cot^2\alpha/6}}, \quad \text{or } \tan\alpha' = 0{,}4082\sqrt{1 + 6\tan^2\alpha}, \qquad (4\text{-}30)$$

the values in table 4-16 may be obtained.

Table 4-16 Values of factors X_o and Y_o for single row angular contact groove ball bearings [Values for α' are based on an approximate equation (4-30).]

α	α'	$X_o = \frac{0,5\cos\alpha}{\cos\alpha'}$	Standard values of X_o	$Y_o = \frac{0,2\cos\alpha}{\sin\alpha'(1 - 0,2\sin\alpha)}$	Standard values of Y_o
15°	26,0°	0,5373	0,5	0,4647	0,46
20°	28,7°	0,5357	0,5	0,4201	0,42
25°	31,8°	0,5332	0,5	0,3757	0,38
30°	35,3°	0,5306	0,5	0,3330	0,33
35°	39,0°	0,5270	0,5	0,2941	0,29
40°	43,0°	0,5237	0,5	0,2578	0,26
45°	47,2°	0,5204	0,5	0,2245	0,22

NOTE – Values of factors X_o and Y_o for α = 15° and 45° have not been included in ISO/R 76, but have been laid down as supplement in ISO 76 - 1978.

Moreover, values for factors X_o and Y_o for double row angular contact groove ball bearings are 2 times the factors for single row angular contact groove ball bearings.

NOTE – Table 4-17 shows the comparison between values for α' obtained by substituting $2r/D_w$ = 1,0325, 1,035 and 1,0375 and $F_a = C_{or}/2$ $= 0,625ZD_w^2\cos\alpha$ (kgf) = $6,1292ZD_w^2\cos\alpha$ (N) into equation (4-5) and values of α' by the approximate equation (4-30).

Table 4-17 Comparison between values of α' by approximate equation (4-30) and equation (4-5)

α			5°	10°	15°	20°	25°	30°	35°	40°	45°
approximate equation (4-30)			22,7°	24,0°	26,0°	28,7°	31,8°	35,3°	39,0°	43,0°	47,2°
equation (4-5)	$\frac{2r}{D_w}$	1,0325	23,1°	24,2°	26,1°	28,5°	31,6°	35,1°	38,9°	43,0°	47,3°
		1,035	22,6°	23,8°	25,7°	28,2°	31,3°	34,9°	38,7°	42,9°	47,2°
		1,0375	22,2°	23,4°	25,3°	27,9°	31,1°	34,7°	38,6°	42,8°	47,1°

Furthermore, values in table 4-18 may be obtained by adopting the values of α' for $2r/D_w = 1,0325$ and $F_a = C_{or}/2$.

Table 4-18 Values of factors X_o and Y_o for single row angular contact groove ball bearings [Values for α' are based on equation (4-5) for $2r/D_w = 1,0325$ (see table 4-17).]

α	α'	$X_o = \frac{0,5\cos\alpha}{\cos\alpha'}$	Standard values of X_o	$Y_o = \frac{0,2\cos\alpha}{\sin\alpha'(1 - 0,2\sin\alpha)}$	Standard values of Y_o
15°	26,1°	0,5378	0,5	0,4631	0,46
20°	28,5°	0,5346	0,5	0,4228	0,42
25°	31,6°	0,5320	0,5	0,3779	0,38
30°	35,1°	0,5293	0,5	0,3347	0,33
35°	38,9°	0,5263	0,5	0,2947	0,29
40°	43,0°	0,5237	0,5	0,2578	0,26
45°	47,3°	0,5213	0,5	0,2241	0,22

4.5.1.3 Self-aligning ball bearings and radial roller bearings

The contact angles for self-aligning ball bearings and radial roller bearings with $\alpha \neq 0°$ (tapered roller bearings and self-aligning roller bearings) are constant and initially the following factors by equations (4-14) and (4-16) have been adopted [17]

single row bearings : $X_o = 0,5$, $Y_o = 0,2\cot\alpha$, (4-31)

double row bearings : $X_o = 1$, $Y_o = 0,4\cot\alpha$. (4-32)

The above factors were given in document ISO/TC 4/SC 1 N 43 but the values of Y_o was revised by the increase of 10 % as result of the discussion at WG 3 meeting, Vienna, 1956 and specified in ISO/R 76 (similarly in ISO 76).