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ISO

INTERNATIONAL ORGANIZATION FOR STANDARDIZATION

ISO RECOMMENDATION

R 1153

SYMBOLS FOR FLIGHT DYNAMICS

PART III

DERIVATIVES OF FORCES, MOMENTS AND THEIR COEFFICIENTS

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BRIEF HISTORY

The ISO Recommendation R 1153, *Symbols for flight dynamics – Part III : Derivatives of forces, moments and their coefficients*, was drawn up by Technical Committee ISO/TC 20, *Aircraft and space vehicles*, the Secretariat of which is held by the British Standards Institution (BSI).

Work on this question led to the adoption of a Draft ISO Recommendation.

In November 1967, this Draft ISO Recommendation (No. 1486) was circulated to all the ISO Member Bodies for enquiry. It was approved, subject to a few modifications of an editorial nature, by the following Member Bodies :

Belgium	Italy	Switzerland
Czechoslovakia	Netherlands	Turkey
France	New Zealand	U.A.R.
Germany	Poland	United Kingdom
India	Spain	U.S.S.R.
Israel	Sweden	Yugoslavia

One Member Body opposed the approval of the Draft :

U.S.A.

The Draft ISO Recommendation was then submitted by correspondence to the ISO Council, which decided, in November 1969, to accept it as an ISO RECOMMENDATION.

FOREWORD

ISO Recommendation R 1153, *Symbols for flight dynamics – Part III : Derivatives of forces, moments and their coefficients*, is the third in a series of ISO Recommendations, the purpose of which is to define the principal terms used in flight dynamics and to specify symbols for these terms. *

In these ISO Recommendations, the term "aircraft" denotes an aerodyne having a fore-and-aft plane of symmetry. This plane is determined by the geometrical characteristics of the aircraft. When there are more than one fore-and-aft planes of symmetry, the reference plane of symmetry is arbitrary and it is necessary to indicate the choice made.

Angles of rotation, angular velocities and moments about any axis are positive clockwise when viewed in the positive direction of the axis.

All the axis systems used are three-dimensional, orthogonal and right-handed, which implies that a clockwise (positive) rotation through $\frac{\pi}{2}$ about the x-axis brings the y-axis into the position previously occupied by the z-axis.

Numbering of sections and clauses

Each of the ISO Recommendations represents a Part of the whole study on symbols for flight dynamics.

To permit easier reference to a section or a clause from one Part to another, a decimal numbering has been adopted which begins in each Recommendation with the number of the Part it represents.

* See in Appendix X the list of ISO Recommendations already published and the studies under way about symbols for flight dynamics.

CONTENTS *

* A section 3.5, "Derivatives of the non-dimensional coefficients of the components of the resultant force and moment with respect to motivator deflections", is at present at the stage of draft proposal.

ISO Recommendation | R 1153 | November 1969

SYMBOLS FOR FLIGHT DYNAMICS

PART III

DERIVATIVES OF FORCES, MOMENTS AND THEIR COEFFICIENTS

INTRODUCTION

This ISO Recommendation deals with the derivatives of forces, moments and their coefficients.

In this ISO Recommendation, the effects of the Earth's curvature are not considered; for the purpose of the definition of Earth axes, the Earth's surface is treated as a plane, that is, the Earth's radius is taken as infinite.

Aeroelastic effects would introduce further quantities, which are not considered in this ISO Recommendation.

Two groups of derivatives are usually involved in flight dynamic studies.

The first group is composed of partial derivatives of the non-dimensional coefficients of force and moment components with respect to normalized variables.

The second group is composed of normalized forms of the partial derivatives of force and moment components with respect to the basic variables of the motion.

Derivatives of the first group

Aerodynamic data are usually quoted in terms of these derivatives. They are meaningful when they relate to those forces and moments which obey the laws of aerodynamic similarity but their use may be extended to include forces not of this type, for example propulsion forces.

In this ISO Recommendation, it is assumed that the force and moment coefficients depend only on the following variables * :

- the angle of attack (1.2.2);
- the sideslip angle (1.2.1);
- the normalized angular velocities (1.3.7);
- the Mach number (1.3.3);
- the normalized forms of the derivatives, with respect to time, of the angle of attack, the angle of sideslip, and the airspeed (3.1).

Derivatives of the second group

The derivatives of the second group relate to components of the resultant force and moment arising from all forces and moments acting on the aircraft, excluding the gravitational, inertial and reaction forces due to contact with the Earth (1.5.1).

It is usual to restrict the number of variables used in forming these derivatives to the following ** (1.3.4 and 1.3.6) :

$u, v, w, p, q, r, \dot{u}, \dot{v}, \dot{w},$

where the dot refers to differentiation with respect to time.

These partial derivatives are then normalized by dividing by an appropriate constant quantity based on a datum flight condition (normally an equilibrium condition and denoted by a suffix e).

Remark on the written form of the derivative

The partial derivative, $\partial A/\partial\lambda$, of a quantity A, with respect to a variable λ, may be written A_λ or A^λ according to the custom of the country. The first form, A_λ, is used in this ISO Recommendation.

* In exceptional circumstances it may prove necessary to increase the number of variables, for example to include Reynolds number.

** In exceptional circumstances it may prove necessary to increase the number of variables, for example to include altitude.

3.1 NORMALIZED FORMS OF THE DERIVATIVES, WITH RESPECT TO TIME, OF THE ANGLES OF ATTACK AND SIDESLIP, AND AIRSPEED

No.	Term	Definition	Symbol
3.1.1	Normalized rate of change of the angle of attack	The derivative of the angle of attack (1.2.2) with respect to time multiplied by the factor l/V (1.3.1, 1.4.6) $\dot{\alpha}* = \frac{\dot{\alpha}\, l}{V}$	$\dot{\alpha}*$
3.1.2	Normalized rate of change of sideslip angle	The derivative of the angle of sideslip (1.2.1) with respect to time multiplied by the factor l/V (1.3.1, 1.4.6) $\dot{\beta}* = \frac{\dot{\beta}\, l}{V}$	$\dot{\beta}*$
3.1.3	Normalized tangential acceleration	The derivative of the airspeed (1.3.1) with respect to time multiplied by the factor l/V^2 (1.3.1, 1.4.6) $\dot{V}* = \frac{\dot{V}\, l}{V^2}$	$\dot{V}*$

3.2 DERIVATIVES OF THE NON-DIMENSIONAL COEFFICIENTS OF THE COMPONENTS OF THE RESULTANT FORCE AND MOMENT WITH RESPECT TO NORMALIZED QUANTITIES (FIRST GROUP)

No.	Term	Definition	Symbol
3.2.1	Derivatives with respect to angle of attack	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the angle of attack (1.2.2)	
		$\partial C_X/\partial\alpha$	$C_{X\alpha}$
		$\partial C_Y/\partial\alpha$	$C_{Y\alpha}$
		$\partial C_Z/\partial\alpha$	$C_{Z\alpha}$
		$\partial C_l/\partial\alpha$	$C_{l\alpha}$
		$\partial C_m/\partial\alpha$	$C_{m\alpha}$
		$\partial C_n/\partial\alpha$	$C_{n\alpha}$

No.	Term	Definition	Symbol
3.2.2	Derivatives with respect to the angle of sideslip	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the sideslip angle (1.2.1)	
		$\partial C_X / \partial \beta$	$C_{X\beta}$
		$\partial C_Y / \partial \beta$	$C_{Y\beta}$
		$\partial C_Z / \partial \beta$	$C_{Z\beta}$
		$\partial C_l / \partial \beta$	$C_{l\beta}$
		$\partial C_m / \partial \beta$	$C_{m\beta}$
		$\partial C_n / \partial \beta$	$C_{n\beta}$
3.2.3	Derivatives with respect to the normalized rate of roll	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of roll (1.3.7)	
		$\partial C_X / \partial p^*$	C_{Xp}
		$\partial C_Y / \partial p^*$	C_{Yp}
		$\partial C_Z / \partial p^*$	C_{Zp}
		$\partial C_l / \partial p^*$	C_{lp}
		$\partial C_m / \partial p^*$	C_{mp}
		$\partial C_n / \partial p^*$	C_{np}
3.2.4	Derivatives with respect to the normalized rate of pitch	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of pitch (1.3.7)	
		$\partial C_X / \partial q^*$	C_{Xq}
		$\partial C_Y / \partial q^*$	C_{Yq}
		$\partial C_Z / \partial q^*$	C_{Zq}
		$\partial C_l / \partial q^*$	C_{lq}
		$\partial C_m / \partial q^*$	C_{mq}
		$\partial C_n / \partial q^*$	C_{nq}

No.	Term	Definition	Symbol
3.2.5	Derivatives with respect to the normalized rate of yaw	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of yaw (1.3.7)	
		$\partial C_X / \partial r^*$	C_{Xr}
		$\partial C_Y / \partial r^*$	C_{Yr}
		$\partial C_Z / \partial r^*$	C_{Zr}
		$\partial C_l / \partial r^*$	C_{lr}
		$\partial C_m / \partial r^*$	C_{mr}
		$\partial C_n / \partial r^*$	C_{nr}
3.2.6	Derivatives with respect to Mach number	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to Mach number (1.3.3)	
		$\partial C_X / \partial M$	C_{XM}
		$\partial C_Y / \partial M$	C_{YM}
		$\partial C_Z / \partial M$	C_{ZM}
		$\partial C_l / \partial M$	C_{lM}
		$\partial C_m / \partial M$	C_{mM}
		$\partial C_n / \partial M$	C_{nM}
		NOTE. – M may be replaced by Ma or $\mathcal{M}$.	
3.2.7	Derivatives with respect to the normalized rate of change of the angle of attack	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of change of the angle of attack (3.1.1)	
		$\partial C_X / \partial \dot{\alpha}^*$	$C_{X\dot{\alpha}}$
		$\partial C_Y / \partial \dot{\alpha}^*$	$C_{Y\dot{\alpha}}$
		$\partial C_Z / \partial \dot{\alpha}^*$	$C_{Z\dot{\alpha}}$
		$\partial C_l / \partial \dot{\alpha}^*$	$C_{l\dot{\alpha}}$
		$\partial C_m / \partial \dot{\alpha}^*$	$C_{m\dot{\alpha}}$
		$\partial C_n / \partial \dot{\alpha}^*$	$C_{n\dot{\alpha}}$

No.	Term	Definition	Symbol
3.2.8	Derivatives with respect to the normalized rate of change of sideslip angle	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized rate of change of the sideslip angle (3.1.2)	
		$\partial C_X/\partial\dot{\beta}^*$	$C_{X\dot{\beta}}$
		$\partial C_Y/\partial\dot{\beta}^*$	$C_{Y\dot{\beta}}$
		$\partial C_Z/\partial\dot{\beta}^*$	$C_{Z\dot{\beta}}$
		$\partial C_l/\partial\dot{\beta}^*$	$C_{l\dot{\beta}}$
		$\partial C_m/\partial\dot{\beta}^*$	$C_{m\dot{\beta}}$
		$\partial C_n/\partial\dot{\beta}^*$	$C_{n\dot{\beta}}$
3.2.9	Derivatives with respect to the normalized tangential acceleration	Partial derivatives of the force and moment coefficients (1.5.3, 1.5.6) with respect to the normalized tangential acceleration (3.1.3)	
		$\partial C_X/\partial\dot{V}^*$	$C_{X\dot{V}}$
		$\partial C_Y/\partial\dot{V}^*$	$C_{Y\dot{V}}$
		$\partial C_Z/\partial\dot{V}^*$	$C_{Z\dot{V}}$
		$\partial C_l/\partial\dot{V}^*$	$C_{l\dot{V}}$
		$\partial C_m/\partial\dot{V}^*$	$C_{m\dot{V}}$
		$\partial C_n/\partial\dot{V}^*$	$C_{n\dot{V}}$

3.3 REFERENCE QUANTITIES USED TO FORM DERIVATIVES OF THE SECOND GROUP

No.	Term	Definition	Symbol
3.3.1	Datum speed	An arbitrary, constant value of the speed used in forming derivatives of the second group, usually the equilibrium airspeed	V_e
3.3.2	Datum (air) density	An arbitrary, constant value of the air density used in forming derivatives of the second group, usually that corresponding to equilibrium flight conditions	ρ_e

3.4 NORMALIZED DERIVATIVES OF THE COMPONENTS OF THE RESULTANT FORCE AND MOMENT WITH RESPECT TO THE MOTION VARIABLES (SECOND GROUP)

NOTE. In applications in which confusion may arise between the symbols for the non-dimensional derivatives and their ordinary dimensional counterparts, a suitable dressing may be added to the symbol to distinguish between these quantities.

No.	Term	Definition	Symbol
3.4.1	Aero-normalized derivatives with respect to the velocity component along the longitudinal axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the velocity component u (1.3.4) in the body axis system, expressed in non-dimensional form	
		$(\partial X/\partial u)/\frac{1}{2}\rho_e V_e S$	X_u
		$(\partial Y/\partial u)/\frac{1}{2}\rho_e V_e S$	Y_u
		$(\partial Z/\partial u)/\frac{1}{2}\rho_e V_e S$	Z_u
		$(\partial L/\partial u)/\frac{1}{2}\rho_e V_e Sl$	L_u
		$(\partial M/\partial u)/\frac{1}{2}\rho_e V_e Sl$	M_u
		$(\partial N/\partial u)/\frac{1}{2}\rho_e V_e Sl$	N_u
3.4.2	Aero-normalized derivatives with respect to the velocity component along the transverse axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the velocity component v (1.3.4) in the body axis system, expressed in non-dimensional form	
		$(\partial X/\partial v)/\frac{1}{2}\rho_e V_e S$	X_v
		$(\partial Y/\partial v)/\frac{1}{2}\rho_e V_e S$	Y_v
		$(\partial Z/\partial v)/\frac{1}{2}\rho_e V_e S$	Z_v
		$(\partial L/\partial v)/\frac{1}{2}\rho_e V_e Sl$	L_v
		$(\partial M/\partial v)/\frac{1}{2}\rho_e V_e Sl$	M_v
		$(\partial N/\partial v)/\frac{1}{2}\rho_e V_e Sl$	N_v

No.	Term	Definition	Symbol
3.4.3	Aero-normalized derivatives with respect to the velocity component along the normal axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the velocity component w (1.3.4) in the body axis system, expressed in non-dimensional form	
		$(\partial X/\partial w)/\frac{1}{2}\rho_e V_e S$	X_w
		$(\partial Y/\partial w)/\frac{1}{2}\rho_e V_e S$	Y_w
		$(\partial Z/\partial w)/\frac{1}{2}\rho_e V_e S$	Z_w
		$(\partial L/\partial w)/\frac{1}{2}\rho_e V_e Sl$	L_w
		$(\partial M/\partial w)/\frac{1}{2}\rho_e V_e Sl$	M_w
		$(\partial N/\partial w)/\frac{1}{2}\rho_e V_e Sl$	N_w
3.4.4	Aero-normalized derivatives with respect to the rate of roll (1.3.6)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the rate of roll (1.3.6), expressed in non-dimensional form	
		$(\partial X/\partial p)/\frac{1}{2}\rho_e V_e Sl$	X_p
		$(\partial Y/\partial p)/\frac{1}{2}\rho_e V_e Sl$	Y_p
		$(\partial Z/\partial p)/\frac{1}{2}\rho_e V_e Sl$	Z_p
		$(\partial L/\partial p)/\frac{1}{2}\rho_e V_e Sl^2$	L_p
		$(\partial M/\partial p)/\frac{1}{2}\rho_e V_e Sl^2$	M_p
		$(\partial N/\partial p)/\frac{1}{2}\rho_e V_e Sl^2$	N_p
3.4.5	Aero-normalized derivatives with respect to the rate of pitch (1.3.6)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the rate of pitch (1.3.6), expressed in non-dimensional form	
		$(\partial X/\partial q)/\frac{1}{2}\rho_e V_e Sl$	X_q
		$(\partial Y/\partial q)/\frac{1}{2}\rho_e V_e Sl$	Y_q
		$(\partial Z/\partial q)/\frac{1}{2}\rho_e V_e Sl$	Z_q
		$(\partial L/\partial q)/\frac{1}{2}\rho_e V_e Sl^2$	L_q
		$(\partial M/\partial q)/\frac{1}{2}\rho_e V_e Sl^2$	M_q
		$(\partial N/\partial q)/\frac{1}{2}\rho_e V_e Sl^2$	N_q

No.	Term	Definition	Symbol
3.4.6	Aero-normalized derivatives with respect to the rate of yaw (1.3.6)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to the rate of yaw (1.3.6), expressed in non-dimensional form	
		$(\partial X/\partial r)/\frac{1}{2}\rho_e V_e S l$	X_r
		$(\partial Y/\partial r)/\frac{1}{2}\rho_e V_e S l$	Y_r
		$(\partial Z/\partial r)/\frac{1}{2}\rho_e V_e S l$	Z_r
		$(\partial L/\partial r)/\frac{1}{2}\rho_e V_e S l^2$	L_r
		$(\partial M/\partial r)/\frac{1}{2}\rho_e V_e S l^2$	M_r
		$(\partial N/\partial r)/\frac{1}{2}\rho_e V_e S l^2$	N_r
3.4.7	Aero-normalized derivatives with respect to the rate of change of the velocity component along the longitudinal axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to $\dot{u}$ (see Introduction), expressed in non-dimensional form	
		$(\partial X/\partial \dot{u})/\frac{1}{2}\rho_e S l$	$X_{\dot{u}}$
		$(\partial Y/\partial \dot{u})/\frac{1}{2}\rho_e S l$	$Y_{\dot{u}}$
		$(\partial Z/\partial \dot{u})/\frac{1}{2}\rho_e S l$	$Z_{\dot{u}}$
		$(\partial L/\partial \dot{u})/\frac{1}{2}\rho_e S l^2$	$L_{\dot{u}}$
		$(\partial M/\partial \dot{u})/\frac{1}{2}\rho_e S l^2$	$M_{\dot{u}}$
		$(\partial N/\partial \dot{u})/\frac{1}{2}\rho_e S l^2$	$N_{\dot{u}}$
3.4.8	Aero-normalized derivatives with respect to the rate of change of the velocity component along the transverse axis (1.3.4)	Partial derivatives of the resultant force and moment components (1.5.2, 1.5.5) with respect to $\dot{v}$ (see Introduction), expressed in non-dimensional form	
		$(\partial X/\partial \dot{v})/\frac{1}{2}\rho_e S l$	$X_{\dot{v}}$
		$(\partial Y/\partial \dot{v})/\frac{1}{2}\rho_e S l$	$Y_{\dot{v}}$
		$(\partial Z/\partial \dot{v})/\frac{1}{2}\rho_e S l$	$Z_{\dot{v}}$
		$(\partial L/\partial \dot{v})/\frac{1}{2}\rho_e S l^2$	$L_{\dot{v}}$
		$(\partial M/\partial \dot{v})/\frac{1}{2}\rho_e S l^2$	$M_{\dot{v}}$
		$(\partial N/\partial \dot{v})/\frac{1}{2}\rho_e S l^2$	$N_{\dot{v}}$