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**Information technology — Security  
techniques — Hash-functions —**

**Part 4:**  
Hash-functions using modular arithmetic

*Technologies de l'information — Techniques de sécurité — Fonctions  
de brouillage —*

*Partie 4: Fonctions de hachage utilisant l'arithmétique modulaire*

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## Foreword

ISO (the International Organization for Standardization) and IEC (the International Electrotechnical Commission) form the specialized system for worldwide standardization. National bodies that are members of ISO or IEC participate in the development of International Standards through technical committees established by the respective organization to deal with particular fields of technical activity. ISO and IEC technical committees collaborate in fields of mutual interest. Other international organizations, governmental and non-governmental, in liaison with ISO and IEC, also take part in the work.

In the field of information technology, ISO and IEC have established a joint technical committee, ISO/IEC JTC 1. Draft International Standards adopted by the joint technical committee are circulated to national bodies for voting. Publication as an International Standard requires approval by at least 75 % of the national bodies casting a vote.

International Standard ISO/IEC 10118-4 was prepared by Joint Technical Committee ISO/IEC JTC 1, *Information technology*, Subcommittee SC27, *IT Security techniques*.

ISO/IEC 10118 consists of the following parts, under the general title *Information technology – Security techniques – Hash-functions*:

- Part 1:        *General*
- Part 2:        *Hash-functions using an n-bit block cipher algorithm*
- Part 3:        *Dedicated hash-functions*
- Part 4:        *Hash-functions using modular arithmetic*

Annexes A, B and C of this part of ISO/IEC 10118 are for information only.

# Information technology — Security techniques — Hash functions —

## Part 4: Hash-functions using modular arithmetic

### 1 Scope

This part of ISO/IEC 10118 specifies two hash-functions which make use of modular arithmetic. These hash-functions, which are believed to be collision-resistant, compress messages of arbitrary but limited length to a hash-code whose length is determined by the length of the prime number used in the reduction-function defined in 7.3. Thus, the hash-code is easily scaled to the input length of any mechanism (e.g., signature algorithm, identification scheme).

The hash-functions specified in this part of ISO/IEC 10118, known as MASH-1 and MASH-2 (Modular Arithmetic Secure Hash) are particularly suitable for environments in which implementations of modular arithmetic of sufficient length are already available. The two hash-functions differ only in the exponent used in the round-function.

### 2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this part of ISO/IEC 10118. At the time of publication, the edition indicated was valid. All standards are subject to revision and parties to agreements based on this part of ISO/IEC 10118 are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO/IEC 10118-1: 1994, *Information technology – Security techniques – Hash-functions – Part 1: General*.

### 3 Terms and definitions

For the purposes of this part of ISO/IEC 10118, the following definitions apply.

#### 3.1 From ISO/IEC 10118-1

- collision-resistant hash-function
- data string (data)
- hash-code
- hash-function
- initializing value
- padding.

#### 3.2 Unique to this part of ISO/IEC 10118

##### 3.2.1 block

a string of bits of length  $L_\phi$ , which shall be an integer multiple of 16 (see also clause 6.1)

EXAMPLE The length of the output  $H_j$  of the round-function.

**3.2.2****half-block**

a string of bits of length  $L_\phi/2$

EXAMPLE Half the length of the block  $H_j$ .

**3.2.3****hash-function identifier**

a byte identifying a specific hash-function

**3.2.4****modulus**

a parameter which is a positive integer and a product of two distinct prime numbers

**3.2.5****reduction-function**

a function *RED* that is applied to the block  $H_q$  of length  $L_\phi$  to generate the hash-code  $H$  of length  $L_p$

**3.2.6****round-function**

a function  $\phi(\cdot, \cdot)$  that transforms two binary strings of length  $L_\phi$  to a binary string of length  $L_\phi$

NOTE It is used iteratively as part of a hash-function, where it combines an 'expanded' data block of length  $L_\phi$  with the previous output of length  $L_\phi$ .

**3.3 Conventions****3.3.1 Bit ordering**

Bit ordering in this part of ISO/IEC 10118 is as described in clause 3 of ISO/IEC 10118-1.

**3.3.2 Converting a number to a string**

During computation of the round-function, integers need to be converted to strings of  $L$  bits. Where this is required, the string of bits shall be made equal to the binary representation of the integer, with the left-most bit of the string corresponding to the most significant bit of the binary representation. If the resulting string of bits has less than  $L$  bits, then the string shall be left-padded with the appropriate number of zeros to make it of length  $L$ .

**3.3.3 Converting a string to a number**

During computation of the round-function, strings of bits need to be converted into integers. Where this is required, the integer shall be made equal to the number having binary representation equal to the binary string, where the left-most bit of the string is considered as the most significant bit of the binary representation.

**3.4 Hash-function identifier**

Identifiers are defined for each of the two MASH hash-functions specified in this standard. The hash-function identifiers for the hash-functions specified in clause 8.1 and 8.2 are equal to 41 and 42 (hexadecimal) respectively. The range of values from 43 to 4f (hexadecimal) are reserved for future use as hash-function identifiers by this part of ISO/IEC 10118.

**4 Symbols and abbreviated terms**

Throughout this part of ISO/IEC 10118, the following symbols and abbreviations apply.

**4.1 From ISO/IEC 10118-1**

<i>D</i>	Data
<i>H</i>	Hash-code
<i>IV</i>	Initializing value
$X \oplus Y$	Exclusive-or of strings of bits $X$ and $Y$

## 4.2 Unique to this part of ISO/IEC 10118

- $B_j$  The  $j$ th block derived from the data string  $D$  after the padding, splitting, and expansion process.
- $D_j$  The  $j$ th half-block derived from the data string  $D$  after the padding and the splitting process.  $D_{q+1}$  through  $D_{q+8}$  are additional data blocks computed in the reduction-function.
- $e$  The exponent used in the round-function.
- $E$  A constant block equal to four ones (in the left-most position) followed by  $L_\phi - 4$  zeros.
- $H_j$  The output of the round-function in the  $j$ th round.  $H_j$  has length  $L_\phi$ .
- $L_D$  The length of the input string  $D$  in bits.
- $L_\phi$  The length of the output  $H_j$  of the round-function  $\phi$ . It shall be an integer multiple of 16.
- $L_N$  The length of the modulus  $N$  used in the round-function.
- $L_p$  The length of the prime number  $p$  used in the reduction-function.
- mod If  $Z_1$  is an integer and  $Z_2$  is a positive integer, then  $Z_1 \bmod Z_2$  denotes the unique integer  $Z_3$  which satisfies
- $0 \leq Z_3 < Z_2$ , and
  - $Z_1 - Z_3$  is an integer multiple of  $Z_2$ .
- $N$  A composite integer, used as the modulus in the round-function.
- NOTE For the determination of the value of  $N$ , see clause 5.
- $p$  A prime number used in the reduction-function.
- NOTE For the determination of the value of  $p$ , see clause 5.
- $q$  The number of half-blocks in the data string  $D$  after the padding and splitting processes, also the number of blocks after the padding, splitting, and expansion process.
- RED* The reduction-function, that is applied as the last operation of the hashing procedure to reduce the block  $H_q$  of length  $L_\phi$  to the hash-code  $H$  of length  $L_p$ .
- $Y_j$  The  $j$ th sub-string of length  $L_\phi/4$  bits used in the reduction-function.
- $\phi$  A round-function. If  $X$  and  $Y$  denote strings of  $L_\phi$  bits, then  $\phi(X, Y)$  denotes a string of  $L_\phi$  bits obtained by applying  $\phi$  to  $X$  and  $Y$ .
- $\vee$  The bit-wise inclusive OR operation on strings of bits, i.e., if  $X$  and  $Y$  are strings of the same length, then  $X \vee Y$  denotes the string obtained as the bit-wise inclusive OR of  $X$  and  $Y$ .
- $\sim$  A symbol denoting the truncate operation. If  $X$  is a bitstring then  $X \sim j$  denotes the bitstring obtained by taking the right-most  $j$  bits of  $X$ .
- $:=$  A symbol denoting the 'set equal to' operation. It is used in the procedural specification of the round-function and of the reduction-function, where it indicates that the block on the left side of the symbol shall be changed to equal the value of the expression on the right side of the symbol.
- $X || Y$  Concatenation of bit-strings  $X$  and  $Y$  in the indicated order.

## 5 Requirements

**5.1** To employ either of the hash-functions specified in this part of ISO/IEC 10118, two integers shall be selected: the modulus  $N$  used in the round-function and the prime  $p$  used in the reduction-function.

Both integers,  $N$  and  $p$ , are determined by the security requirements of the application for which these hash-functions are used.

**5.1.1** The modulus  $N$  shall be chosen so that factoring it is computationally infeasible.

**5.1.2** The modulus  $N$  shall be generated in a way that the factors remain secret. This can be accomplished by a trusted third party or by a secure multiparty computation.

NOTE 1 Generating a modulus  $N$  with the property that its factors remain secret can be accomplished by using a trusted third party, trusted hardware, and/or a secure multiparty computation. Examples can be found in Boneh [1], Cocks [2], and Frankel [3].

NOTE 2 If the factors of the modulus are kept secret, and if the size of the prime  $p$  is sufficiently large, then the best known algorithm to find a collision takes approximately  $2^{L_p/2}$  evaluations of the round-function, and the best known algorithm to find a (2nd) pre-image requires approximately  $2^{L_p}$  evaluations of the round-function. Thus in these circumstances MASH-1 and MASH-2 are believed to be collision-free hash-functions.

**5.1.3** The reduction-function prime  $p$  shall not be a factor of the modulus  $N$  of the round-function.

**5.1.4** The length  $L_p$  of the prime  $p$  shall be at most half of the length of the modulus  $N$ ,  $L_p \leq L_\phi/2$ .

**5.1.5** The three high order bits of prime  $p$  shall consist of ones.

**5.2** To employ one of the hash-functions, MASH-1 or MASH-2, the user has to select one of the two exponents  $e$  used in the round-function  $\phi$ .

**5.3** MASH-1 and MASH-2 can be applied to all data strings  $D$  containing at most  $2^{L_\phi/2}-1$  bits.

## 6 Variables and values needed for the hash operation

### 6.1 The length of the hash-code and of the modulus

The length of the modulus  $N$  and the length of the blocks  $H_j$  are related in the following manner:

$$L_\phi + 1 \leq L_N \leq L_\phi + 16$$

The length  $L_\phi$  of the block  $H_q$  shall be an integer multiple of 16.

NOTE 1 If the length  $L_\phi$  is chosen, then the length  $L_N$  is constrained by the inequalities above. If the length  $L_N$  is chosen, then the length  $L_\phi$  will be the largest multiple of 16 less than  $L_N$ .

NOTE 2 Knowledge of  $N$  is sufficient to determine  $L_N$ , and consequently  $L_\phi$ .

### 6.2 The modulus of the round-function

The modulus  $N$  used in the round-function is a composite integer generated as a product of two prime numbers of about the same length such that it is computationally infeasible to factorize  $N$ .

NOTE 1 In addition to the infeasibility of the factorization of the modulus, the security of the MASH hash-functions is based in part on the difficulty of extracting modular roots.

NOTE 2 The choice of a specific modulus  $N$  of appropriate length is outside the scope of this part of ISO/IEC 10118.

### 6.3 Initializing value

The initializing value  $IV$  is defined to be the string of  $L_\phi$  binary zeros.

### 6.4 Exponent

For MASH-1 the value of the exponent  $e$  in the round-function equals 2. For MASH-2 the value of the exponent  $e$  in the round-function equals 257.

### 6.5 Reduction-function prime number

The reduction-function specified in 7.3 requires a prime  $p$ . The length  $L_p$  of prime  $p$  is determined by the security requirements, and by the input length of any mechanism using the hash-code. The length  $L_p$  shall be at most half of the length of the modulus  $N$ ,  $L_p \leq L_\phi/2$ .

NOTE 1 The choice of a specific prime  $p$  of appropriate length is outside the scope of this part of ISO/IEC 10118.

NOTE 2 To avoid unbalanced results by the reduction modulo  $p$ , the prime number  $p$  shall be selected with the three high order bits equal to ones.

## 7 Hashing procedure

The hash-code  $H$  of the data string  $D$  shall be calculated using the following steps (see Figure 1):

### 7.1 Preparation of the data string

The data string  $D$  is transformed into a sequence of blocks for input to the round-function  $\phi$ . The preparation consists of padding, splitting, and expanding as detailed in the following sub-clauses.

#### 7.1.1 Padding the data string

If the length  $L_D$  of the data string  $D$  is not an integer multiple of  $L_\phi/2$ ,  $D$  is right-padded with binary zeros according to padding method 1 described in ISO/IEC 10118-1, Appendix B.

#### 7.1.2 Appending the length

An additional half-block is right-appended to the data string. It contains the binary representation of the length  $L_D$  of the original (unpadded) data string  $D$ , left-padded with binary zeros (see 3.3.2).

NOTE – If the data block  $D$  is empty, only the length block is input to the hashing procedure.

#### 7.1.3 Splitting the data string

The resulting string is divided into a sequence of  $q$  half-blocks  $D_1, D_2, \dots, D_{q-1}, D_q$ .

#### 7.1.4 Expansion

Every half-block  $D_j$ ,  $j = 1, 2, \dots, q$ , is now doubled in length from  $L_\phi/2$  bits to  $L_\phi$  bits. This is achieved by dividing  $D_j$  into half-bytes and preceding each half-byte of  $D_j$  with a half-byte consisting of four ones (1111), for  $j=1, 2, \dots, q$ . The result of this process when applied to half-block  $D_j$  is denoted as  $B_j$ ,  $j = 1, 2, \dots, q$ .

### 7.2 Application of the round-function

The round-function  $\phi$  on which the MASH hash-functions are based takes as input two blocks,  $H_{j-1}$  and  $B_j$ , both of

length  $L_\phi$ . It returns a block  $H_j$  of length  $L_\phi$ . It is defined as follows:

$$\phi(B_j, H_{j-1}) = (((H_{j-1} \oplus B_j) \vee E)^e \bmod N) \sim L_\phi \oplus H_{j-1}$$

This round-function is applied sequentially to the data blocks  $B_j$  as follows:

$$H_0 := IV$$

$$H_j := \phi(B_j, H_{j-1}) \quad j = 1, 2, \dots, q$$

### 7.3 The Reduction-function

The reduction-function *RED* consists of eight applications of the round-function with a data input derived from  $H_q$ . The hash-code  $H$  shall be calculated using the following four steps: splitting the block  $H_q$ , extending the data string, processing the additional data blocks, and reducing the block  $H_{q+8}$ .

#### 7.3.1 Splitting the block $H_q$

The block  $H_q$  is divided into four strings of length  $L_\phi/4$  bits, denoted with  $H_{q1}, H_{q2}, H_{q3}, H_{q4}$ , i.e.,

$$H_q := H_{q1} \parallel H_{q2} \parallel H_{q3} \parallel H_{q4}$$

#### 7.3.2 Extending the data string

Define  $Y_0 := H_{q3}$ ,  $Y_1 := H_{q1}$ ,  $Y_2 := H_{q4}$ , and  $Y_3 := H_{q2}$ . Then for  $i = 4$  to 15 let

$$Y_i := Y_{i-1} \oplus Y_{i-4}$$

Define then eight additional data half-blocks  $D_{q+1}$  through  $D_{q+8}$  as follows: for  $i = 1$  to 8 let

$$D_{q+i} := Y_{2i-2} \parallel Y_{2i-1}$$

#### 7.3.3 Processing the half-blocks

The eight half-blocks  $D_{q+1}$  through  $D_{q+8}$  are processed as in 7.1.4 and 7.2, but with *IV* equal to  $H_q$ , yielding the result  $H_{q+8}$ .

#### 7.3.4 Reduction

The hash-code  $H$  of length  $L_p$  is computed as follows:

$$H := H_{q+8} \bmod p$$

## 8 Hash-functions

The two hash-functions specified in this part of ISO/IEC 10118 differ in the value of the exponent  $e$  used in the round-function  $\phi$ .

### 8.1 MASH-1

For MASH-1, the round-function  $\phi$  as specified in clause 7 becomes:

$$\phi(B_j, H_{j-1}) = (((H_{j-1} \oplus B_j) \vee E)^2 \bmod N) \sim L_\phi \oplus H_{j-1}$$

The ISO/IEC hash-function identifier for the hash-function MASH-1 is equal to 41 (hexadecimal).

## 8.2 MASH-2

For MASH-2, the round-function  $\phi$  as specified in clause 7 becomes:

$$\phi(B_j, H_{j-1}) = (((H_{j-1} \oplus B_j) \vee E)^{257 \bmod N} \sim L\phi) \oplus H_{j-1}$$

The ISO/IEC hash-function identifier for the hash-function MASH-2 is equal to 42 (hexadecimal).

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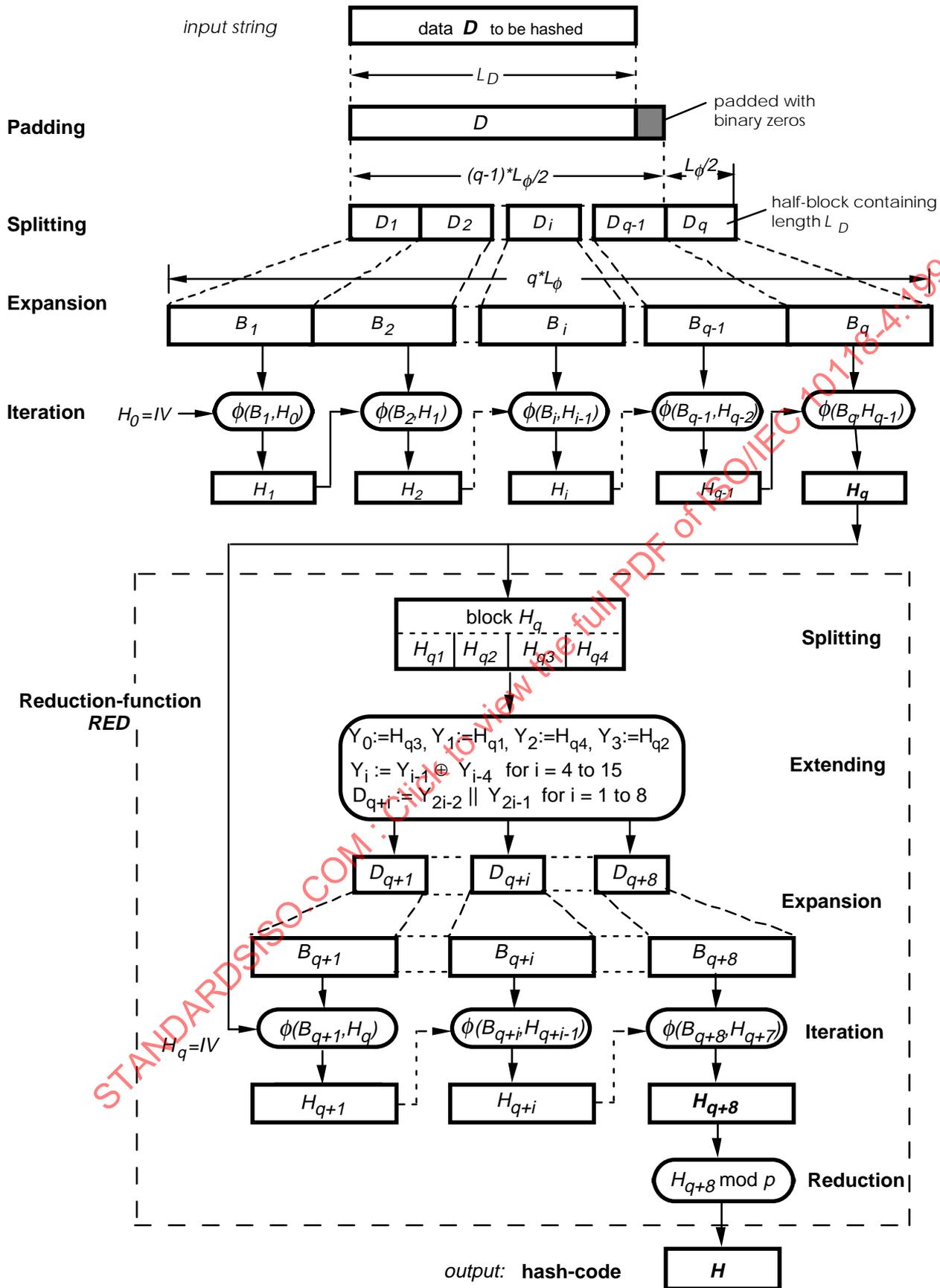


Figure 1 — The MASH hash-function

## Annex A (informative)

### Examples

#### A.1 The hashing procedure

In practice, the data string may be supplied as a stream. Its overall length  $L_D$  may be unknown before the end of the stream is reached. As the stream passes through, the hash calculation is performed. For convenience of the description, a constant  $k = L_\phi/2$  is introduced. Only the following four registers are required for the variables:  $i$ ,  $A$ ,  $B$ ,  $C$ .

- $i$  bit counter for the length  $L_D$ .
- $A$  buffer to hold the hash-code.
- $B$  accumulator to hold the intermediate results.
- $C$  holding one half-block of data.

The hash-code  $H$  of the data  $D$  is calculated in the following steps:

##### A.1.1 Step 1 (Initialisation)

- The buffer  $A$  is set to zero:  $A := 0$  (initializing value  $IV = H_0 = 0$ )
- The accumulator  $B$  is set to zero:  $B := 0$
- The bit counter is set to zero:  $i := 0$  (counts the number of effective data bits hashed)

##### A.1.2 Step 2a (Reading a data block)

If at least  $k$  bits of the data are remaining, read  $k$  bits of the data  $D$  into  $C$ . If  $k'$  (with  $0 < k' < k$ ) bits of the data are remaining, read  $k'$  bits of the data  $D$  into  $C$  and fill up  $C$  with  $k-k'$  binary zeros on the right.  $k$  (respectively  $k'$ ) is added to  $i$ . If no more data bits are available ( $k' = 0$ ), go to Step 3a.

##### Step 2b (Expansion)

Each byte of  $C$  is split into halves and each half is preceded with four binary ones. The result is put into the accumulator  $B$ .

##### Step 2c (Combination with previous hash-value)

Calculate:  $B := B \oplus A$  (combination)

##### Step 2d

$B = B \vee E$  (the four highest valued bits are set to 1)

##### Step 2e (Exponentiation)

Calculate:  $B := B^e \bmod N$  where  $e=2$  (squaring) with MASH-1, and  $e=257$  with MASH-2.

##### Step 2f (Truncation)

(the leftmost bits of  $C$  which exceed the length  $L_\phi$  are erased)

##### Step 2g (Feed-forward with previous hash-value)

Calculate:  $A := B \oplus A$

##### Step 2h

Return to Step 2a.

**A.1.3 Step 3a (Reading the length counter)**

$C := i$  (the content of the length counter is put into  $C$ ).  
The integer is converted into a string as specified in 3.3.2.

**Step 3b (Hash with length counter)**

Perform steps 2b, 2c, 2d, 2e, 2f and 2g, then go to Step 4.

**A.1.4 Step 4 (Output the result)**

The block  $H_q$  is contained in the buffer  $A$  as the rightmost  $L_\phi$  bits.

**A.1.5 The reduction procedure**

The following registers are required for the variables:  $A, B, C, C_0, C_1, C_2, C_3, i$ .

$A$  holding the block  $H_q$  to be reduced.  
 $B$  buffer used in the round-function.  
 $C$  accumulator of length  $L_\phi/2$ , to hold a half-block.  
 $C_0, C_1, C_2, C_3$  four buffers of length  $L_\phi/4$ .  
 $i$  a counter.

The hash-code  $H$  is calculated in the following steps.

**A.1.5.1 Step 4a (Initialisation)**

The counter is set to 8:  $i := 8$  (the number of half-blocks to be processed)

**A.1.5.2 Step 4b (Splitting of the block  $H_q$ )**

The block  $H_q$  contained in buffer  $A$  is divided into four parts  $H_{q1}, H_{q2}, H_{q3}, H_{q4}$  each of length  $L_\phi/4$  (see Figure 1) and saved:

$C_0 := H_{q3}$   
 $C_1 := H_{q1}$   
 $C_2 := H_{q4}$   
 $C_3 := H_{q2}$

**A.1.5.3 Step 4c (Extension and iteration)**

Calculate:  $C := C_0 \parallel C_1$  (concatenate)  
 and perform Steps 2b through 2g (apply the round-function  $\phi$ ) (Round number  $q+1$ )  
 Calculate:  $i := i-1$ . (reduce counter)

**Step 4d (Extension and iteration)**

Calculate:  $C := C_2 \parallel C_3$  (concatenate)  
 and perform Steps 2b through 2g (apply the round-function  $\phi$ ) (Round number  $q+2$ )  
 Calculate:  $i := i-1$  (reduce counter).

**A.1.5.4 Step 4e (Combination, extension and iteration)**

Calculate:  $C_0 := C_0 \oplus C_3$  (combination)  
 $C_1 := C_1 \oplus C_0$





Round no. q+2

(2b) B = f9fafaf8fbf8f1f6f2f8f2fbf2fcf6f4f0fefbf9f7f6f2fff7fef1f2fdf9fbf8h  
 (2c) B = 647d845abc037298fe29fe6da0482a57784f6e3355d332374169ede148f1448dh  
 (2d) B = f47d845abc037298fe29fe6da0482a57784f6e3355d332374169ede148f1448dh  
 (2e) B = 00000741c0abe2c27ac0bf2384d377ab7b07f3374c402f87ec26baafabd6f2e360f7000bh  
 (2f) A = c0abe2c27ac0bf2384d377ab7b07f3374c402f87ec26baafabd6f2e360f7000bh  
 (2g) A = 5d2c9c603d3b3c4d88027b3d29b32f94c4f1ba4d4e037a671d41eef0d5ffbf7eh

Round no. q+3

(4e) C<sub>0</sub> = C<sub>0</sub> ⊕ C<sub>3</sub> = 17e87113d745092ah  
 (4e) C<sub>1</sub> = C<sub>1</sub> ⊕ C<sub>0</sub> = 50607789cda41fdeh  
 (4e) C = C<sub>0</sub> || C<sub>1</sub> = 17e87113d745092a50607789cda41fdeh  
 (2b) B = f1f7fef8f7f1f1f3fdf7f4f5f0f9f2faf5f0f6f0f7f7f8f9fcfdaf4f1fffdfeh  
 (2c) B = acdb6298cacacdbe75f58fc8d94add6e31014cbdb9f4829ee1bc140424004280h  
 (2d) B = fcd6298cacacdbe75f58fc8d94add6e31014cbdb9f4829ee1bc140424004280h  
 (2e) B = 0000085f0e394019c43439b100f3865b24c54196d0a6612bc8298ee325583501f5fe139bh  
 (2f) A = 0e394019c43439b100f3865b24c54196d0a6612bc8298ee325583501f5fe139bh  
 (2g) A = 5315dc79f90f05fc88f1fd660d766e021457db66862af4843819dbf1200lace5h

Round no. q+4

(4f) C<sub>2</sub> = C<sub>2</sub> ⊕ C<sub>1</sub> = cac8cf9fe58f33bah  
 (4f) C<sub>3</sub> = C<sub>3</sub> ⊕ C<sub>2</sub> = c471b9b09b9dea02h  
 (4f) C = C<sub>2</sub> || C<sub>3</sub> = cac8cf9fe58f33bac471b9b09b9dea02h  
 (2b) B = fcfafcf8fcfff9fffef5f8fff3f3fbfafcf4f7f1fbf9fbf0f9fbf9fdfefaf0f2h  
 (2c) B = afef208105f0fc0376040599fe8595f8e8a32c977dd30f74c1e2220cdefb5c17h  
 (2d) B = ffef208105f0fc0376040599fe8595f8e8a32c977dd30f74c1e2220cdefb5c17h  
 (2e) B = 000009a0a067af1cb50e126496355f4f8033d62330dd0c151703d398534f4a1e89e28d4ah  
 (2f) A = a067af1cb50e126496355f4f8033d62330dd0c151703d398534f4a1e89e28d4ah  
 (2g) A = f37273654c0117981ec4a2298d45b821248ad7739129271c6b5691efa9e321afh

Round no. q+5

(4e) C<sub>0</sub> = C<sub>0</sub> ⊕ C<sub>3</sub> = d399c8a34cd8e328h  
 (4e) C<sub>1</sub> = C<sub>1</sub> ⊕ C<sub>0</sub> = 83f9bf2a817cfcf6h  
 (4e) C = C<sub>0</sub> || C<sub>1</sub> = d399c8a34cd8e32883f9bf2a817cfcf6h  
 (2b) B = fdf3f9f9fcf8faf3f4fcdf8fef3f2f8f8f3fff9fbfff2faf8f1f7fcfffccfff6h  
 (2c) B = 0e818a9cb0f9ed6bea385fd173b64ad9dc79288a6ad6d5e693a76613561fde59h  
 (2d) B = fe818a9cb0f9ed6bea385fd173b64ad9dc79288a6ad6d5e693a76613561fde59h  
 (2e) B = 0000013eb89f8f9208f8dd6af54caf0c5783bcb901c9ed5dc70ef255eb64da4fbafe19a2h  
 (2f) A = b89f8f9208f8dd6af54caf0c5783bcb901c9ed5dc70ef255eb64da4fbafe19a2h  
 (2g) A = 4bedfcf744f9caf2eb880d25dac6049825433a2e5627d54980324ba0131d380dh

Round no. q+6

(4f) C<sub>2</sub> = C<sub>2</sub> ⊕ C<sub>1</sub> = 493170b564f3cf4ch  
 (4f) C<sub>3</sub> = C<sub>3</sub> ⊕ C<sub>2</sub> = 8d40c905ff6e254eh  
 (4f) C = C<sub>2</sub> || C<sub>3</sub> = 493170b564f3cf4c8d40c905ff6e254eh  
 (2b) B = f4f9f3f1f7f0fbf5f6f4fff3fcfff4fcf8fdf4f0fcf9f0f5fffff6fef2f5f4feh  
 (2c) B = bf140f06b30931071d7cf2d62639f064ddbecedeaade25bc7fcd6d5ee1e8ccf3h  
 (2d) B = ff140f06b30931071d7cf2d62639f064ddbecedeaade25bc7fcd6d5ee1e8ccf3h  
 (2e) B = 000002cd631f7f2804f90933a27240f80ea77e9668d319c6157c23f7b11684f2d7080aceh  
 (2f) A = 631f7f2804f90933a27240f80ea77e9668d319c6157c23f7b11684f2d7080aceh  
 (2g) A = 28f283df4000c3c149fa4ddd4617a0e4d9023e8435bf6be3124cf52c41532c3h

Round no. q+7

(4e)  $C_0 = C_0 \oplus C_3 = 5ed901a6b3b6c666h$   
 (4e)  $C_1 = C_1 \oplus C_0 = dd20be8c32ca3a90h$   
 (4e)  $C = C_0 \parallel C_1 = 5ed901a6b3b6c666dd20be8c32ca3a90h$   
 (2b)  $B = f5fefdf9f0f1faf6fbf3fbf6fcf6f6f6fdfd2f0fbfef8fcf3f2fcfaf3faf9f0h$   
 (2c)  $B = dd0c7e26b0f13937b209b62b28978cf8b06dd118b8a50e42c2d633a837efcb33h$   
 (2d)  $B = fd0c7e26b0f13937b209b62b28978cf8b06dd118b8a50e42c2d633a837efcb33h$   
 (2e)  $B = 00000b5c780401af81fcfafb61d0af544bb9533b074260c5b9536bc8cc8d89c021c8cc326h$   
 (2f)  $A = 780401af81fcfafb61d0af544bb9533b074260c5b9536bc8cc8d89c021c8cc326h$   
 (2g)  $A = 50f68270c1fc6c7754f0b8996ff449be39b62fb3d66d4a32f9fc5350d899f1e5h$

Round no. q+8

(4f)  $C_2 = C_2 \oplus C_1 = 9411ce395639f5dc h$   
 (4f)  $C_3 = C_3 \oplus C_2 = 1951073ca957d092h$   
 (4f)  $C = C_2 \parallel C_3 = 9411ce395639f5dc1951073ca957d092h$   
 (2b)  $B = f9f4f1f1fcfef3f9f5f6f3f9fff5fdcf1f9f5f1f0f7f3fcfaf9f5f7fd0f9f2h$   
 (2c)  $B = a90273813d029f8ea1064b609001b442c84fda42269ab9ce0305a6a725690817h$   
 (2d)  $B = f90273813d029f8ea1064b609001b442c84fda42269ab9ce0305a6a725690817h$   
 (2e)  $B = 00000d67969376fb27ef9563bd5bd2b08156dd01904bc7ac090552607e69252ba9265edh$   
 (2f)  $A = 7969376fb27ef9563bd5bd2b08156dd01904bc7ac090552607e69252ba9265edh$   
 (2g)  $A = 299fb51f738295216f2505b267e1246e20b293c916fd1f14fe1ac102620b9408h$   
**H<sub>q+8</sub> = 299fb51f738295216f2505b267e1246e20b293c916fd1f14fe1ac102620b9408h**

Reduction mod p

(5)  $A = 497a26f552e76e98ea6eaae0e54b6d7d h$   
 (6)  $H = 497a26f552e76e98ea6eaae0e54b6d7d h$

Hash-code calculated by MASH-1

A.2.2 Example hash calculation using MASH-2

Initialisation step:  $A := 0$   
 $B := 0$   
 $i := 0$

First round:

(2a)  $C = 4e6f77206973207468652074696d6520h$  (the first 16<sub>d</sub> bytes of the data string D)  
 $i = 80h$   
 (2b)  $B = f4fef6fff7f7f2f0f6f9f7f3f2f0f7f4f6f8f6f5f2f0f7f4f6f9f6fdf6f5f2f0h$   
 (2c)  $B = f4fef6fff7f7f2f0f6f9f7f3f2f0f7f4f6f8f6f5f2f0f7f4f6f9f6fdf6f5f2f0h$   
 (2d)  $B = f4fef6fff7f7f2f0f6f9f7f3f2f0f7f4f6f8f6f5f2f0f7f4f6f9f6fdf6f5f2f0h$   
 (2e)  $B = 0000083871d3c375922382b7996401171c44350ffc63f85a11aebbf07e182108ba9d1f3fh$   
 (2f)  $A = 71d3c375922382b7996401171c44350ffc63f85a11aebbf07e182108ba9d1f3fh$   
 (2g)  $A = 71d3c375922382b7996401171c44350ffc63f85a11aebbf07e182108ba9d1f3fh$

Second round:

(2a)  $C = 666f7220616c6c20h$  (the last 8<sub>d</sub> bytes of the data string D)  
 (2a)  $C = 666f7220616c6c200000000000000000h$  (... padded with binary zeros)  
 $i = c0h$   
 (2b)  $B = f6f6f6fff7f7f2f0f6f1f6fcf6fcf2f0f0f0f0f0f0f0f0f0f0f0f0f0f0f0h$   
 (2c)  $B = 8725358a65d170476f95f7ebeb8c7ff0c9308aae15e4b008ee8d1f84a6defcfc h$   
 (2d)  $B = f725358a65d170476f95f7ebeb8c7ff0c9308aae15e4b008ee8d1f84a6defcfc h$   
 (2e)  $B = 0000064adfe7bc5d78f2c00b3c11cc020a4c7e798de85ff6bc98c78061e803f40e5b2b7ah$



(2f) A = 5aaa22afa6266de454fa9395ae40ff1dc667735f1e6e5f603e5c1ea79c94d8e9h  
 (2g) A = 22ef0072e132c7fc755afb60f1f1067158297ebf98d02472f63ad4aabd256bb6h

Round no. q+4

(4f) C<sub>2</sub> = C<sub>2</sub> ⊕ C<sub>1</sub> = 674878af6b6a5c88h  
 (4f) C<sub>3</sub> = C<sub>3</sub> ⊕ C<sub>2</sub> = c5357a9df9e44fd1h  
 (4f) C = C<sub>2</sub> || C<sub>3</sub> = 674878af6b6a5c88c5357a9df9e44fd1h  
 (2b) B = f6f7f4f8f7f8fafff6fbf6faf5fcf8f8fcf5f3f5f7faf9fdfff9fef4f4ffdfdf1h  
 (2c) B = d418f48a16ca3d0383a10d9a040dfe89a4dc8d4a6f2add8f09c32a5e49da9647h  
 (2d) B = f418f48a16ca3d0383a10d9a040dfe89a4dc8d4a6f2add8f09c32a5e49da9647h  
 (2e) B = 000006444434f5a752c3635d88ea144e3d28e4613eb15035e1b9c47c8ec14e50b95623ebh  
 (2f) A = 4434f5a752c3635d88ea144e3d28e4613eb15035e1b9c47c8ec14e50b95623ebh  
 (2g) A = 66dbf5d5b3f1a4a1fdb0ef2eccd9e21066982e8a7969e00e78fb9afa0473485dh

Round no. q+5

(4e) C<sub>0</sub> = C<sub>0</sub> ⊕ C<sub>3</sub> = ca8a24444a15464ah  
 (4e) C<sub>1</sub> = C<sub>1</sub> ⊕ C<sub>0</sub> = b9365305495ed5eeh  
 (4e) C = C<sub>0</sub> || C<sub>1</sub> = ca8a24444a15464ab9365305495ed5eeh  
 (2b) B = fcfaf8faf2f4f4f4f4faf1f5f4f6f4fafbf9f3f6f5f3f0f5f4f9f5fefdf5fefeh  
 (2c) B = 9a210d2f41055055094aledb382f16ea9d61dd7c8c9a10fb8c026f04f986b6a3h  
 (2d) B = fa210d2f41055055094aledb382f16ea9d61dd7c8c9a10fb8c026f04f986b6a3h  
 (2e) B = 00000b472e2404e982562be4723c0f7818292619525274b24a8b64f89ea41d1d89a764a0h  
 (2f) A = 2e2404e982562be4723c0f7818292619525274b24a8b64f89ea41d1d89a764a0h  
 (2g) A = 48fff13c31a78f458f8ce056d4f0c40934ca5a3833e284f6e65f87e78dd42cfdh

Round no. q+6

(4f) C<sub>2</sub> = C<sub>2</sub> ⊕ C<sub>1</sub> = de7e2baa22348966h  
 (4f) C<sub>3</sub> = C<sub>3</sub> ⊕ C<sub>2</sub> = 1b4b5137dbd0c6b7h  
 (4f) C = C<sub>2</sub> || C<sub>3</sub> = de7e2baa223489661b4b5137dbd0c6b7h  
 (2b) B = fdfef7fef2fbfafaf2f2f3f4f8f9f6f6f1fbf4fbf5f1f3f7fdfbfd0fcf6fbf7h  
 (2c) B = b50106c2c35c75bf7d7e13a22c0932ffc531aec3c61377011ba47a177122d70ah  
 (2d) B = f50106c2c35c75bf7d7e13a22c0932ffc531aec3c61377011ba47a177122d70ah  
 (2e) B = 000002cac6f3b0a9d6b41c106blef6465385af56258886f78d80f016914d005f66f51db2h  
 (2f) A = c6f3b0a9d6b41c106blef6465385af56258886f78d80f016914d005f66f51db2h  
 (2g) A = 8e0c4195e7139355e492161087756b5f1142dccf6e6274e0771287b8eb21314fh

Round no. q+7

(4e) C<sub>0</sub> = C<sub>0</sub> ⊕ C<sub>3</sub> = d1c1757391c580fdh  
 (4e) C<sub>1</sub> = C<sub>1</sub> ⊕ C<sub>0</sub> = 68f72676d89b5513h  
 (4e) C = C<sub>0</sub> || C<sub>1</sub> = d1c1757391c580fd68f72676d89b5513h  
 (2b) B = fdf1fcf1f7f5f7f3f9f1fcf5f8f0ffdf6f8fff7f2f6f7f6fd8f9fbf5f5f1f3h  
 (2c) B = 73fdbd6410e664a61d63eae57f8594a2e7ba23384c9483168aea7e431ed4c0bch  
 (2d) B = f3fdbd6410e664a61d63eae57f8594a2e7ba23384c9483168aea7e431ed4c0bch  
 (2e) B = 0000013ef76551c6e3b17dfb5ecb837566eec9c7fb39f16c39351b9fd4bdacb352eaf01dh  
 (2f) A = f76551c6e3b17dfb5ecb837566eec9c7fb39f16c39351b9fd4bdacb352eaf01dh  
 (2g) A = 7969105304a2eeae5a599565e19ba298ea7b2da387576f7fa3af2b0bb9cbc152h

Round no. q+8

(4f) C<sub>2</sub> = C<sub>2</sub> ⊕ C<sub>1</sub> = b6890ddcfaafdc75h  
 (4f) C<sub>3</sub> = C<sub>3</sub> ⊕ C<sub>2</sub> = adc25ceb217f1ac2h  
 (4f) C = C<sub>2</sub> || C<sub>3</sub> = b6890ddcfaafdc75adc25ceb217f1ac2h

(2b) B = fbf6f8f9f0fdfdcfffafafffdcf7f5fafdfcf2f5fcfefbf2f1f7fff1fafcf2h  
 (2c) B = 829fe8aaf45f135245a36f9a1c67556d1086d15172ab9184515edcf448313da0h  
 (2d) B = f29fe8aaf45f135245a36f9a1c67556d1086d15172ab9184515edcf448313da0h  
 (2e) B = 00000a8bf084975e7ac8e096b464172ed9a5cbffeb32c5512faec0b3702f427608497abd<sub>h</sub>  
 (2f) A = f084975e7ac8e096b464172ed9a5cbffeb32c5512faec0b3702f427608497abd<sub>h</sub>  
 (2g) A = 89ed870d7e6a0e380e3d824b383e69670149e8f2a8f9afccd380697db182bbef<sub>h</sub>  
 H<sub>q+8</sub> = 89ed870d7e6a0e380e3d824b383e69670149e8f2a8f9afccd380697db182bbef<sub>h</sub>

**Reduction mod p**

(5) A = 8ad87c2de674c2e82de5769806e1bb28<sub>h</sub>  
 (6) H = 8ad87c2de674c2e82de5769806e1bb28<sub>h</sub>

**Hash-code calculated by MASH-2**

**A.3 Sample test messages and their hash-codes**

In the calculation of the hash-code for the set of test messages (A.3.1 to A.3.9) the following integers are used:

1) the composite modulus

N = 8b251fa16f8b7a3c8a1ec50da421de6bfdcf4db6cf4452d0df98ad327b9f6feca66422e1434938ffe3576d7b7a76b8c94a90dc9d0cc576bd6f9a128f4af1d907e3b4dbb67f52683e7992a4cf031f885533d21d07c7e14811adb600fb78d62a789f<sub>h</sub>

2) the prime number

p = cde7e6f6e432331d896a7b02d031a09d7b2c77a1<sub>h</sub>

The length of the modulus N used in the round-function is: L<sub>N</sub> = 780<sub>d</sub> = 30C<sub>h</sub>

The length of the output of the round-function is: L<sub>φ</sub> = 768<sub>d</sub> = 300<sub>h</sub>

The length of the prime p used in the reduction-function is: L<sub>p</sub> = 160<sub>d</sub> = A0<sub>h</sub>

The length of the hash-code is: L<sub>p</sub> = 160<sub>d</sub> = A0<sub>h</sub>

**The contents of the fields:**

*length of text* contains the number of bits of the message text. It is given in decimal first (with subscript d) followed by the equivalent hexadecimal representation (with subscript h).

*text* contains the message to be hashed delimited by single quotation marks.

*text in ASCII* contains the message in ASCII code. The blanks are inserted for readability.

H<sub>q</sub> contains the output of the round-function in hexadecimal representation after q applications.

H<sub>q+8</sub> contains the output of the round-function in hexadecimal representation after q+8 applications.

H contains the hash-code in hexadecimal representation after the modulo p reduction.

**A.3.1 Example 1**

*length of text* 0<sub>d</sub> = 0<sub>h</sub>

*text* "

*text in ASCII* (empty string)

MASH-1 H<sub>q</sub> a69e5191337162387c3a2701bc8a49ad385d648194fa1e188ca2d4f5f149fdf89cea0e9c1324fd96270a1fee9a77e00925a014432896dd6a54232947f222b28b786259c45de5496b166e4955cd0c84f2539c49fb61f6fc66763915dd006e8b8c

H<sub>q+8</sub> c42af288cfd830875401c6e622a417058076b4869183cf5d79721a33c5d297937f90506891cbd0e0d307d8515ec3dd5d64bd6257e90ba06acf53c14c47cb007e54ae429dfa0329fff22cd1fdb2702598258d081d4f0d25a612084f906c2f5627

H b7231fa49ce0249adef0ce0a1429796e8f19ad8b

MASH-2  $H_q$  499bcae76a9b15f8061fc0fc2da49d7d8f1ca7436a70c36d564ac35875cdcf310f3a8e8791e358774669341bd07f7917cd35928f9ee60a88f7c53cecb2d5e82f9778049b24243b896a3e66448801ab6f0b975018a7732a87baa3c1ac07e95eaf

$H_{q+8}$  416000041591700f319a8e2bbc2bab56e88cd39dd24194da16e81b5b6db15d9ca22849f7ad7ae6d83987d92c1cdfdc65775c610740b935d1258f568af3038a79d8a7b97000aec763b960c27a63245d91d3c58f21bd386fb9b8600f4765d94efe

$H$  bcce0b6f5646e7eb0cfb1ed6dbe787016ce4b18e

**A.3.2 Example 2**

length of text  $8_d = 8_h$   
 text 'a'  
 text in ASCII 61

MASH-1  $H_q$  843d8fbd095cb55514b0017843541ea3003c03b6b9753501ad28a28f8e3d5a125461f8a6db9fc6bdb3475916915bb50dd71a833647710db0b71c4696570b37de678d185c99e285e496be7c5415130c2ee51108e4fe38f0e80ec4cfed7bb3c9f2

$H_{q+8}$  b7cd378165eb4ed4ca34f1372643d57d395810e696959eb44f3514be6413c9131817ac827a8f501961ac3cb0a51e2ecb32c990443f2eac32890b81ecf5874eabccc97ef292870ab945c00eb0555a4ede7ab9e23cca450bfdaff46921e2a40d25

$H$  c7c5e8262a0067a3f0479fdc5c814e90e636610b

MASH-2  $H_q$  4851adcd1809c0b66018a4f62a0008ea08e25fa5a6a8f970d0f09e3be81276301d68e5da78314ecae818a32589132a22caacdbe5f4e06afe7273632ef218255f4bd3c58d3fc3fd4357411c21047e77017c0358401a013f251638f3f4764ef5a4

$H_{q+8}$  597292939e469da0fa542227c5e8c6a4eade4497433cc3ca8db32e473c23287af7dfcfd eb7e1fe8372369772acc19ef5206d681bdab2b6fa3253fec58980284dda2fc077fbd0492e125c204af1f4625f439968115619e517ab05b74ee60eb0c

$H$  c8ae72da006544d524a6ca09c9d7702628bfb683

**A.3.3 Example 3**

length of text  $24_d = 18_h$   
 text 'abc'  
 text in ASCII 61 62 63

MASH-1  $H_q$  329f3abba8d9ffde27478023bcfba35bflabaa31fdf7e32a9a314fb43769a3809bb654a94838382b544702263e6ff3bd8a21fc38f5ff5f06859d11aab1d49af96912fed50ed7755ed3d1da0776ede4985aelfd56dbe8cff52e37eb7b724ec694

$H_{q+8}$  5910c9e2d3fa19bda8c4143aeeeeac65f7acdf8d067b43891ee2d4626c0fc570188083ee86d86d5fe89fded34e2b3ee75dfc5229bcce85b250b312339355b74c62e9429021b5a965e028ea3f5477687ae5d4858b46a480976b7e741dbba0c8d0

$H$  9e76dd088a4e88ea72bb9ce3727f34bd27c75912

MASH-2  $H_q$  3bbd7e23260830c085a6282c5ef2f2d35d7e78a74d0f3de4964bbea9a74780b43662d9a666daba357abef2c3186129e3202f80a015de4ddc6811db472cf36420e0ccb17a9a937e1a89647eb7f84105c023617b9144ba2faf8113070acb36edf6

$H_{q+8}$  344d4e0f7627fbb5affdaa62610d096ce6951f9a17f191e56a25ba5d3dcc4b08a361ffe917b8eef53d0e0a39193fd43274b34f931b96e469518d5e7d60b1d794fe78c1740b0c42c7273e2b7b1fa7d8092d7a0c590957eb76caf7868339e6aa6b

$H$  18086f7c5c96e1bdc659c1c7b5c957f3dfbae7c8

**A.3.4 Example 4**

length of text  $112_d = 70_h$   
 text 'message digest'  
 text in ASCII 6D 65 73 73 61 67 65 20 64 69 67 65 73 74