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Representation of results of particle size analysis —

Part 4:
Characterization of a classification process

Représentation de données obtenues par analyse granulométrique —

Partie 4: Caractérisation d'un processus de triage

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Foreword

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International Standard ISO 9276-4 was prepared by Technical Committee ISO/TC 24, *Sieves, sieving and other sizing methods*, Subcommittee SC 4, *Sizing by methods other than sieving*.

ISO 9276 consists of the following parts, under the general title *Representation of results of particle size analysis*:

— *Part 1: Graphical representation*

— *Part 2: Calculation of average particle sizes/diameters and moments from particle size distributions*

— *Part 3: Fitting of an experimental cumulative curve to a reference model*

— *Part 4: Characterization of a classification process*

— *Part 5: Validation of calculations relating to particle size analyses using the logarithmic normal probability distribution*

Annex A of this part of ISO 9276 is for information only.

Introduction

In classification processes used in particle size analysis, such as occurring in impactors, sieves, etc., the mass of the supply or feed material, m_s, or its number, n_s, of particles, the particle size distribution of which is described by its density distribution, $q_{r,s}(x)$, is separated into at least one fine fraction of mass, m_f, or number, n_f, and of density distribution, $q_{r,f}(x)$ and a coarse fraction of mass, m_c, or number, n_c, and a density distribution, $q_{r,c}(x)$. The type of quantity chosen in the analysis is described by the subscript, r, the supply or feed material and the fine and coarse fractions by the additional subscripts: s; f and c respectively. See Figure 1.

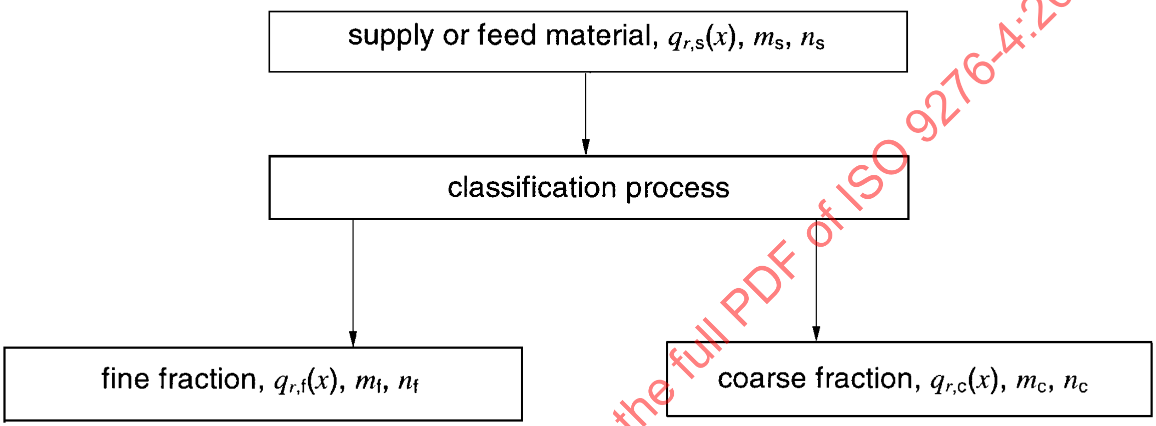

Figure 1 — Fractions and distributions produced in a one step classification process

For the characterization of processes with more than one coarse fraction, e.g. cascade impactors, s, f and c can be replaced by numbers 0, 1 and 2. In this case e.g. number 3 describes a second coarse fraction containing larger particles than fraction 2.

It is assumed that the size, x, of a particle is described by the diameter of a sphere. Depending on the problem, the particle size, x, may also represent an equivalent diameter of a particle of any other shape.

Representation of results of particle size analysis —

Part 4:
Characterization of a classification process

1 Scope

The main object of this part of ISO 9276 is to provide the mathematical background for the characterization of a classification process. This part of ISO 9276 is not limited to an application in particle size analysis, the same procedure may be used for the characterization of a technical classification process (e.g. air classification, centrifugal classification) or a separation process (e.g. gas or hydrocyclones).

In clause 3 the characterization of a classification process is described under the presupposition that the density distribution curves describing the feed material and the fractions, as well as the overall mass balance, are free from errors. In clause 4 the influence of systematic errors on the efficiency of a classification process is described. The effect of stochastic errors in the characterization of a classification process is described in annex A.

2 Symbols

2.1 Symbols for specific terms

See Table 1.

Table 1 — Symbols for specific terms

Symbol	Term
A	Parameters derived from cumulative distribution curves
E	Mass balance error, cumulative distributions
I	Imperfection
$K(x)$	Corrected cumulative distribution
m	Mass
n	Total number of size classes, number of particles
$q_r(x)$	Density distribution curve
$Q_r(x)$	Cumulative distribution curve
$\Delta Q_{r,i}$	Difference of two cumulative distribution values, relative amount in the ith particle size interval, Δx_i
s^2	Variance
t	Student's factor
T	Grade efficiency
T_{o}	Overall classification or separation efficiency
$T(x)$	Grade efficiency curve
x	Particle diameter, diameter of a sphere
x_{a}	Analytical cut size
x_{e}	Equiprobable cut size, median particle size of a grade efficiency curve
x_i	Upper particle size of the ith particle size interval
x_{i-1}	Lower particle size of the ith particle size interval
Δx_i	Width of the ith particle size interval
x_{max}	Particle size above which there are no particles in a given size distribution
x_{min}	Particle size below which there are no particles in a given size distribution
α	Angle of slope, weighted sum of variances
ε	Mass balance error, density distributions
$\eta_{r,i} = Q_{r,\text{s},i} - Q_{r,\text{c},i}$	Variable
κ	Sharpness of cut parameters formed with characteristic particle sizes
ν	Relative amount
$\xi_{r,i} = Q_{r,\text{f},i} - Q_{r,\text{c},i}$	Variable
τ	Amount of particles not participating in a classification process
ϕ	Variable

2.2 Subscripts

See Table 2.

Table 2 — Subscripts

Symbol	Significance
c	Coarse fraction (second subscript after r)
f	Fine fraction (second subscript after r)
i	Number of the size class with upper particle size: x_i
r	Type of quantity of a density distribution [a] (general description)
s	Supply or feed material (second subscript after r)
0	Replaces s in case of more than one coarse fraction
1	Replaces f in case of more than one coarse fraction
2	Replaces c in case of more than one coarse fraction

[a] For example, $r = 3$ if type of quantity = volume or mass.

3 Characterization of a classification process based on error-free distribution curves and mass balances

3.1 Density distribution curves representing a classification process

In a classification process a given supply or feed material (subscript s) is classified into at least two parts, which are called the fine (subscript f) and the coarse (subscript c) fractions. If an *ideal* classification were possible, the fine fraction would, as shown in Figure 2, contain particles below or equal to a certain size, x_e, the so-called cut size, and the coarse fraction would contain all particles above that size.

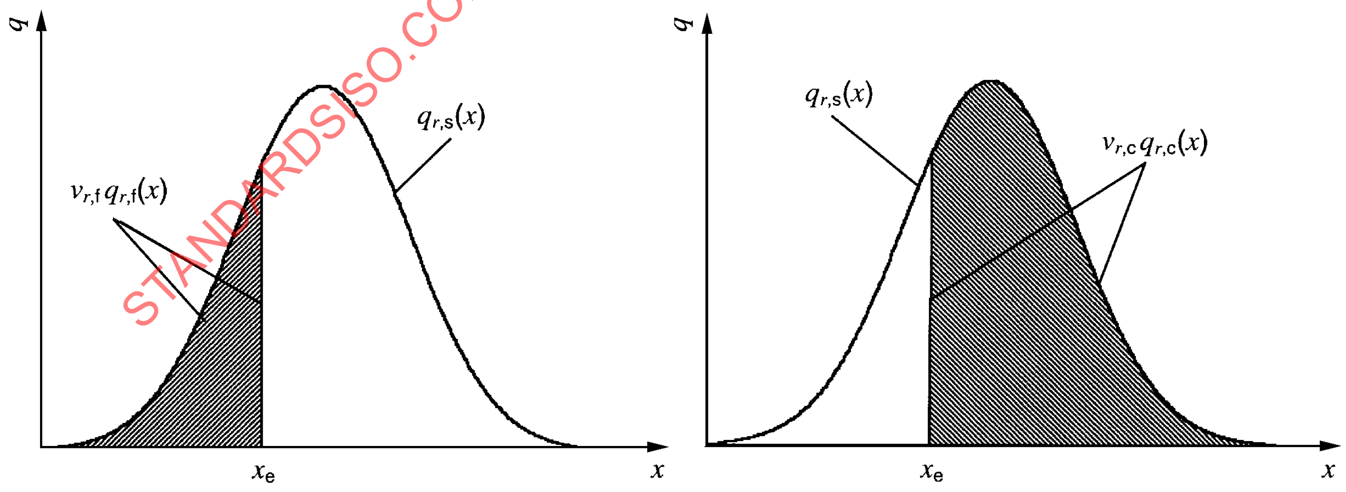

Figure 2 — Weighted density distributions of the feed material $q_{r,s}(x)$ and the fine and coarse fractions of an *ideal* classification process

The shaded areas beneath the weighted density distributions of the fine and the coarse product represent the relative mass, $v_{3,f}$, or number, $v_{0,f}$, of the fine, $v_{r,f}$, and the coarse fraction, $v_{r,c}$, the sum which equals 100 % or unity.

In *reality*, however, in a certain range of sizes $x_{min,c} < x < x_{max,f}$ particles of the same size, x, are present in both the fine and the coarse fractions. The density distribution curves of the fine and the coarse fractions overlap and intersect each other in this size range, The point of intersection as shown in Figure 3 corresponds to a cut size, which is called the equiprobable cut size, x_e(see 3.3.2).

The particles below the cut size, x_e, in the coarse or above x_e in the fine fraction have been incorrectly classified.

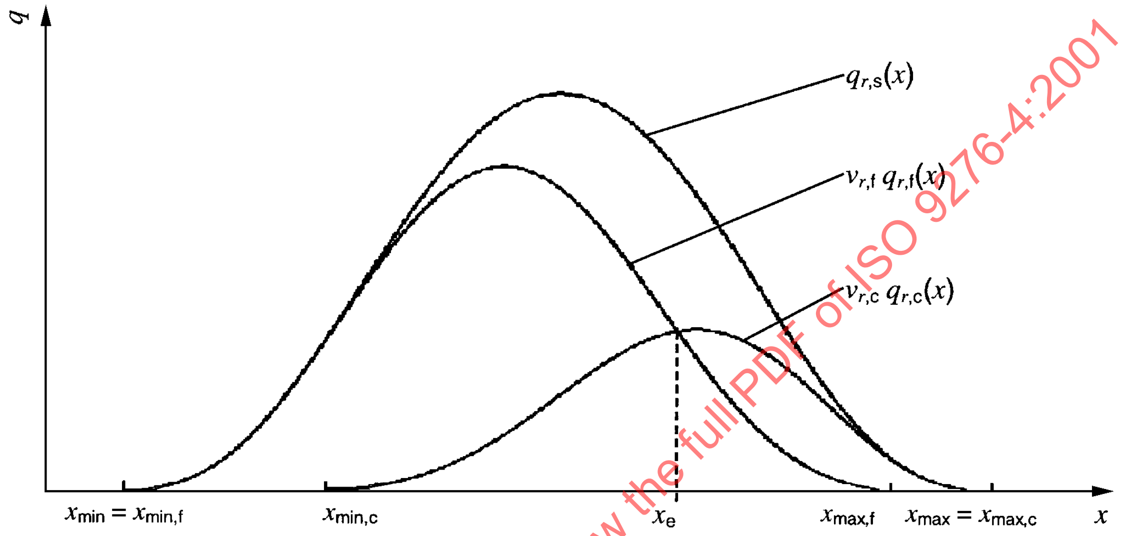

Figure 3 — Weighted density distributions of feed material, $q_{r,s}(x)$, and the fine, $v_{r,f}\, q_{r,f}(x)$, and the coarse fraction, $v_{r,c}\, q_{r,c}(x)$, of an *real* classification process

3.2 Mass and number balances

3.2.1 Mass and number balance in the size range from x_{min} to x_{max}

Due to the classification process, the mass, m_s, or number, n_s, of the feed material, is split into the mass, m_f, or number, n_f, of the fine material and the mass, m_c, or number, n_c, of the coarse material. One obtains:

$$m_s = m_f + m_c \quad \text{or} \quad n_s = n_f + n_c \tag{1}$$

and

$$1 = \frac{m_f}{m_s} + \frac{m_c}{m_s} \quad \text{or} \quad 1 = \frac{n_f}{n_s} + \frac{n_c}{n_s} \tag{2}$$

$$1 = v_{3,f} + v_{3,c} \quad \text{or} \quad 1 = v_{0,f} + v_{0,c} \tag{3}$$

$v_{r,f}$ represents the relative amount of the fine fraction, $v_{r,c}$ the relative amount of the coarse fraction.

In Figures 2 and 3, $v_{r,f}$ and $v_{r,c}$ are represented by the areas beneath the weighted density distribution curves of the fine, $v_{r,f}\, q_{r,f}(x)$, and the coarse, $v_{r,c}\, q_{r,c}(x)$, fractions. The area beneath the density distribution curve of the feed material, $q_{r,s}(x)$, equals unity.

3.2.2 Mass and number balance in the size range from x to $x + dx$

Particles of a certain size, x, present in the feed material, are either transferred in the classification process to the fine or to the coarse fractions. The amount of these particles in the feed material, $dQ_{r,s}(x)$, is therefore split into two fractions: $v_{r,f}\, dQ_{r,f}(x)$ and $v_{r,c}\, dQ_{r,c}(x)$.

$$dQ_{r,s}(x) = v_{r,f}\, dQ_{r,f}(x) + v_{r,c}\, dQ_{r,c}(x) \quad (4)$$

Replacing $dQ_r(x)$ by equation 5:

$$dQ_r(x) = q_r(x)\, dx \quad (5)$$

one obtains:

$$q_{r,s}(x) = v_{r,f}\, q_{r,f}(x) + v_{r,c}\, q_{r,c}(x) \quad (6)$$

Equation 6 must be used to construct the set of density distribution curves of Figure 3. It should be realized that in plotting Figure 3 only three of the variables of equation 6 can be chosen arbitrarily. If, two density distributions and the relative amount of the fine or the coarse material, e.g., $q_{r,s}(x)$, $q_{r,f}(x)$ and $v_{r,f}$ are given, $q_{r,c}(x)$, and $v_{r,c}$ are fixed.

3.2.3 Mass and number balance in the size range from x_{min} to x

Integrating equation 6 between x_{min} and x yields:

$$Q_{r,s}(x) = v_{r,f}\, Q_{r,f}(x) + v_{r,c}\, Q_{r,c}(x) \quad (7)$$

3.2.4 The indirect evaluation of $v_{r,f}$ and $v_{r,c}$

In many cases of practical application $v_{r,f}$ and $v_{r,c}$ cannot be calculated from the relevant masses or mass flow rates, due to the fact that these are not available or difficult to measure, etc. If however, representative samples of the feed material and the fine and the coarse fraction have been measured equations 3 and 6 or 7 may be used to calculate $v_{r,f}$ or $v_{r,c}$. Introducing equation 3 into equations 6 and 7 and solving with respect to $v_{r,f}$ yields:

$$v_{r,f} = 1 - v_{r,c} = \frac{Q_{r,s}(x) - Q_{r,c}(x)}{Q_{r,f}(x) - Q_{r,c}(x)} = \frac{q_{r,s}(x) - q_{r,c}(x)}{q_{r,f}(x) - q_{r,c}(x)} \quad (8)$$

If the cumulative distributions $Q_{r,s}(x)$, $Q_{r,f}(x)$ and $Q_{r,c}(x)$ are free from errors, i.e. the mass balance according to equations 6 or 7 leave no remainder, $v_{r,f}$ or $v_{r,c}$ will be constant and independent of size x.

3.3 Definitions of cut size, x_e

3.3.1 General

In principle, any value of x between $x_{min\ c}$ and $x_{max\ f}$, i.e. the size range in which the density distributions of the fine and the coarse fractions overlap, can be used as cut size.

Two definitions are commonly used as described in 3.3.2 and 3.3.3.

3.3.2 The equiprobable cut size, x_e, the median of the grade efficiency curve

In Figure 3 the weighted density distribution curves of the fine and the coarse fraction intersect at a certain size x_e. This particle size, which represents the median of the grade efficiency curve, $T(x)$, as defined in 3.4, is the equiprobable cut size, x_e:

$$x_e = x\,(T = 0{,}5) \quad (9)$$

Independently from other particle sizes, particles of this size have the equal probability of being classified into the fine and into the coarse fraction. Therefore the length of the dashed vertical line from the intersection of the weighted fine and coarse density distributions in Figure 3 is equal to the vertical distance of that point to the weighted feed density distribution.

In the result particles of the equiprobable size are equally present in the fine and the coarse fraction:

$$v_{r,\mathrm{f}}\ q_{r,\mathrm{f}}(x_\mathrm{e}) = v_{r,\mathrm{c}}\ q_{r,\mathrm{c}}(x_\mathrm{e}) \tag{10}$$

3.3.3 The analytical cut size, x_a

An analytical air classifier, e.g. a single stage of an impactor, represents itself to the user as a black box (see Figure 1). A known mass, m_s, for example is supplied to the classifier. At the end of the classification process, one quantitatively obtains, in most cases, the mass, m_c, of the coarse product only. The mass of the fine product m_f can be calculated from the difference from the supplied mass. Since the relative mass of the fine material, $v_{3,\mathrm{f}} = m_\mathrm{f} / m_\mathrm{s}$, as determined by the experiment, is taken to be equal to the relative mass of the undersize material in the feed, that is $Q_{3,\mathrm{s}}(x)$, a cut size x corresponding to this definition has to be found. This cut size is called the analytical cut size, x_a. The general definition is:

$$1 - v_{r,\mathrm{c}} = v_{r,\mathrm{f}} = Q_{r,\mathrm{s}}(x_\mathrm{a}) \tag{11}$$

For a given particle size distribution of the supply or feed material the known relative amount of the fine material yields the analytical cut size shown in Figure 4.

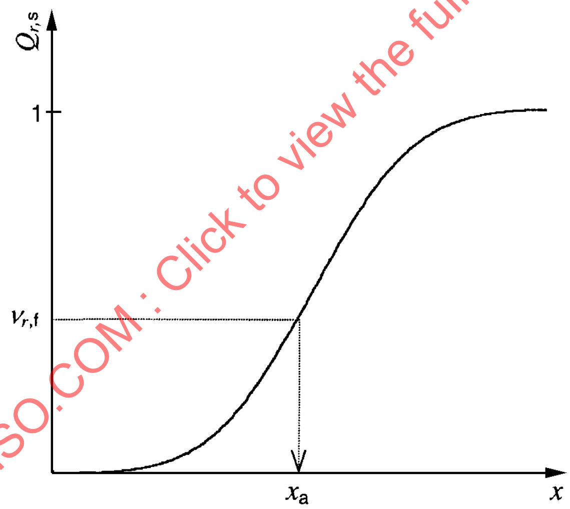

Figure 4 — The definition of the analytical cut size, x_a, taking the relative amount of the fine material, $v_{r,\mathrm{f}}$, to be equal to the relative amount of the undersize material in the feed, $Q_{r,\mathrm{s}}(x_\mathrm{a})$

Inserting equation 11 into the mass and number balance in equation 7 signifies that, with reference to this size, the coarse and the fine material contains equal quantities of misplaced material, i.e. the amount of coarse particles in the fine fraction $v_{r,\mathrm{f}}\ [1 - Q_{r,\mathrm{f}}(x_\mathrm{a})]$ is equal to the amount of fine in the coarse fraction $v_{r,\mathrm{c}}\ Q_{r,\mathrm{c}}(x_\mathrm{a})$. This special case $x = x_\mathrm{a}$ can be visualised in Figure 6, if the shaded areas A_3 and A_6 are equal. Then the shaded area A_1 will represent the $v_{r,\mathrm{f}}$ part of the complete area between x_{min} and x_{max}.

3.4 Grade efficiency, T, the grade efficiency curve, $T(x)$, (Tromp's curve)

In order to describe the efficiency of a classification process it is usual to deduce the so-called grade efficiency curve, $T(x)$, from the density distribution curves of Figure 3.

The grade efficiency, T, (or size selectivity) represents, for a certain particle size, the ratio of the amount of material present in the coarse material, $v_{r,c}\ q_{r,c}(x)\mathrm{d}x$, to the amount of the same size initially present in the feed material, $q_{r,s}(x)\mathrm{d}x$.

The grade efficiency curve, $T(x)$, can therefore be calculated from:

$$T(x) = \frac{v_{r,c}\ q_{r,c}(x)}{q_{r,s}(x)} = \frac{v_{r,c}\Delta Q_{r,c}(x_i, x_{i-1})}{\Delta Q_{r,s}(x_i, x_{i-1})} = \frac{v_{r,c}\left[Q_{r,c}(x_i) - Q_{r,c}(x_{i-1})\right]}{Q_{r,s}(x_i) - Q_{r,s}(x_{i-1})} \tag{12}$$

If one plots T against particle size x, the resulting curve is the grade efficiency curve, $T(x)$ shown in Figure 5. It should start at zero and remain there between x_{min} and $x_{min,c}$. It should reach the value of one at $x_{max,f}$ and above. Reasons why this may not happen are dealt with in clause 4.

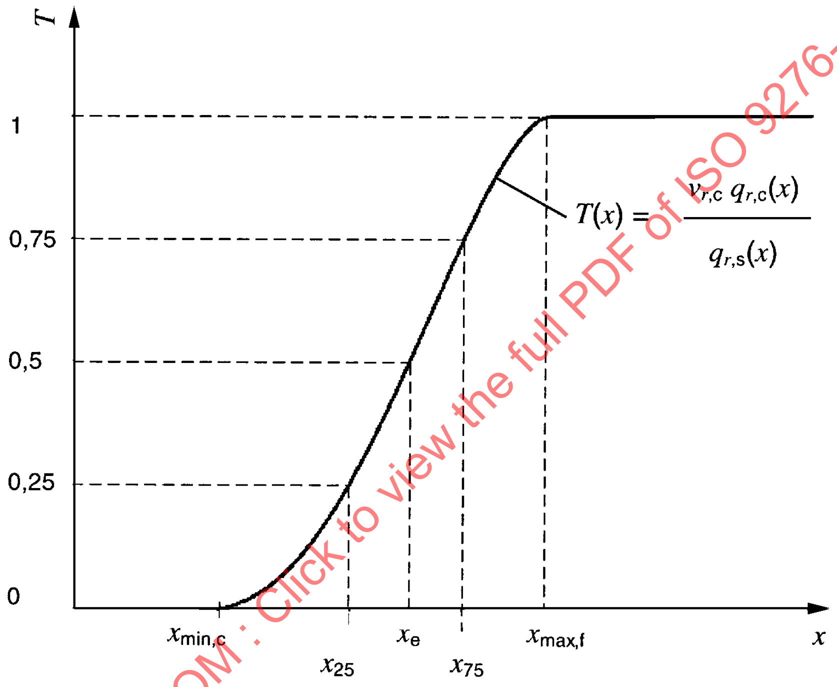

Figure 5 — The grade efficiency curve, $T(x)$

3.5 Measures of sharpness of cut

3.5.1 General

The smaller the overlapping size range between $x_{min,c}$ and $x_{max,f}$, or the lesser the amounts of misplaced material, the better the sharpness of cut or the quality of a classification process. In order to indicate quantitatively the sharpness, or the lack of sharpness, of a cut in a classification process, a great number of parameters has been proposed. These parameters can only be used meaningfully when selected with regard to the technical application for which the classification process has been used. Therefore one parameter alone will in many cases not be as adequate for a complete description of the classification process as a series or even a combination of different parameters. It should be kept in mind that most parameters given will only quantify parts or part of the information obtainable from the grade efficiency curve.

Three groups of parameter can be formed which suffice to include all hitherto suggested parameters as described in 3.5.2 and 3.5.3.

3.5.2 Parameters formed with characteristic particle sizes

These parameters indicate a difference or a ratio of characteristic particle sizes taken from the grade efficiency curve; x_y, is used in what follows to indicate the value of size x where the grade efficiency curve has a value of $T = y$ %.

For example, one distinguishes the imperfection:

$$I = \frac{x_{75} - x_{25}}{2 x_{50}} \quad (13)$$

or the sharpness

$$\kappa_{25/75} = \frac{x_{25}}{x_{75}} \quad (14)$$

Equations 13 and 14 are indications of the central slope of the grade efficiency curve.

3.5.3 Parameters derived from cumulative distribution curves

These parameters can be determined from cumulative distribution curves $Q_{r,0}(x)$, $Q_{r,f}(x)$ and $Q_{r,c}(x)$ and the relative amount of the fine material $v_{r,f}$ and $v_{r,c}$. The grade efficiency curve is not required for their determination.

One distinguishes in principle between six different areas, A_1 to A_6, beneath the three density distribution curves, as shown in Figure 6. From these areas the characteristic parameters given below can be derived.

The following parameters either indicate the amount of fine or coarse particles below or above a certain size in the feed material or the fine and the coarse fractions.

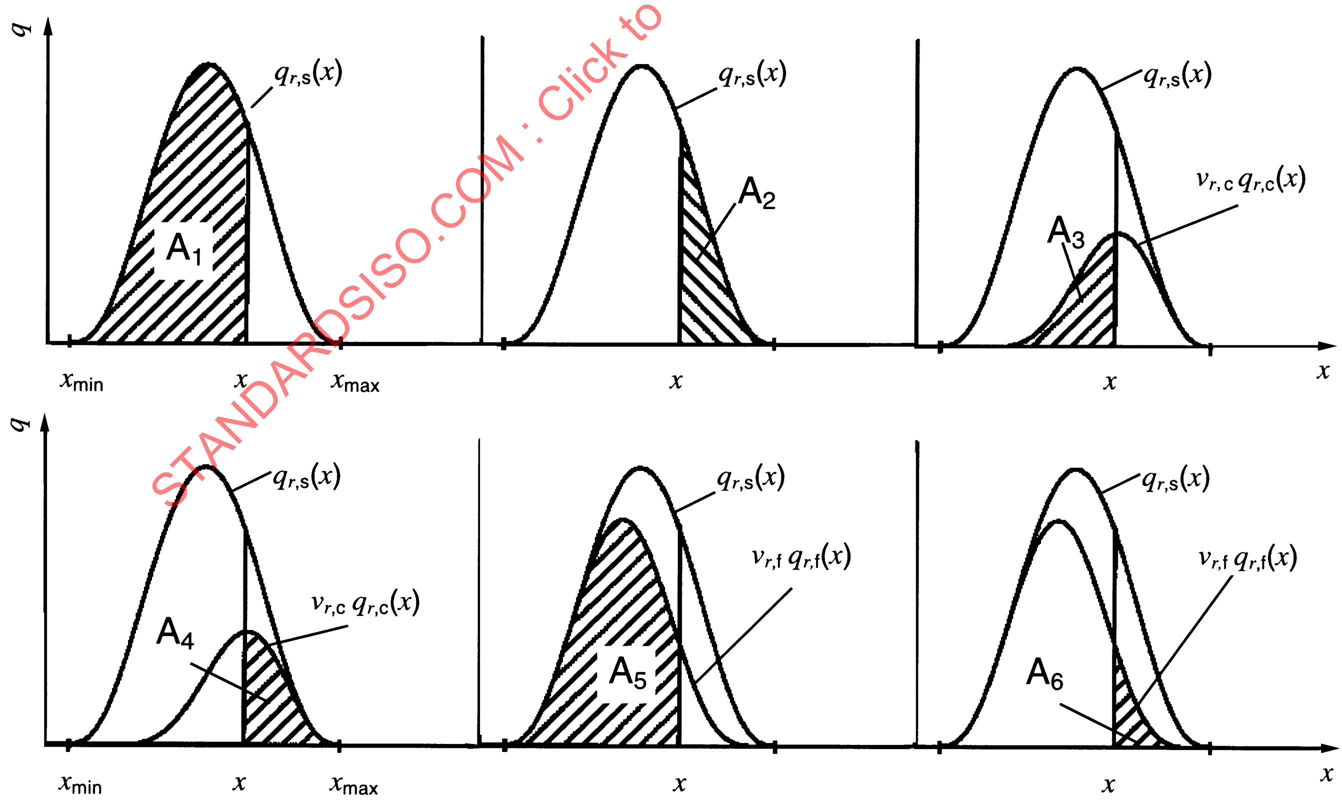

Figure 6 — Definition and representation of six areas beneath the three density distribution curves

Fine particles in the feed:

$$A_1 = Q_{r,s}(x) \tag{15}$$

Coarse particles in the feed:

$$A_2 = 1 - Q_{r,s}(x) \tag{16}$$

Fine particles in the coarse fraction:

$$A_3 = v_{r,c}\, Q_{r,c}(x) \tag{17}$$

Coarse particles in the coarse fraction:

$$A_4 = v_{r,c}\left[1 - Q_{r,c}(x)\right] \tag{18}$$

Fine particles in the fine fraction:

$$A_5 = v_{r,f} Q_{r,f}(x) \tag{19}$$

Coarse particles in the fine fraction:

$$A_6 = v_{r,f}\left[1 - Q_{r,f}(x)\right] \tag{20}$$

It should be noted that these areas A_1 to A_6, are dependent on particle size, x.

With these areas relative parameters may also be formed, e.g.:

The retrieval or recovery of fine particles related to those initially present in the feed material:

$$\frac{A_5}{A_1} = \frac{v_{r,f}\, Q_{r,f}(x)}{Q_{r,s}(x)} \tag{21}$$

The retrieval or recovery of coarse particles related to those initially present in the feed material:

$$\frac{A_4}{A_2} = \frac{v_{r,c}\left[1 - Q_{r,c}(x)\right]}{1 - Q_{r,s}(x)} \tag{22}$$

3.5.4 The total classification or separation efficiency, T_o

The total classification or separation efficiency T_o is generally used to describe the quality of dust removal systems, e.g. gas cyclones. It corresponds to the already defined relative amount of coarse material, $v_{r,c}$, and may be calculated from the grade efficiency curve, $T(x)$, and the density distribution curve of the feed material, $q_{r,s}(x)$, as follows:

$$T_o = v_{r,c} = \int_{x_{min}}^{x_{max}} T(x) q_{r,s}(x)\, dx = \int_0^1 T(x)\, dQ_{r,s}(x) = \sum_{i=1}^{n} T_i(\bar{x}_i) \cdot dQ_{r,s,i} \tag{23}$$

4 The influence of systematic errors on the determination of grade efficiency curve

4.1 General

Systematic deviations from the ideal course of a grade efficiency curve can be caused by:

a) systematic analytical errors of sampling and sample splitting;

b) superposition of a classification and a splitting process within the classifier;

c) undispersed agglomerated fine particles which are transferred to the coarse product;

d) comminution of the feed material in the classifier.

Assuming that the sampling from and the sample splitting of the samples of the feed material and the fine and coarse fractions have been performed with the greatest of care, the first systematic error listed above may disregarded.

4.2 Systematic error due to a splitting process in the classifier

If the grade efficiency curve, as shown in Figure 7, does not drop to zero at small particle sizes, but ends parallel to the abscissa at a certain grade efficiency $T = \tau$, it is most likely that the classification process is superimposed by a splitting process.

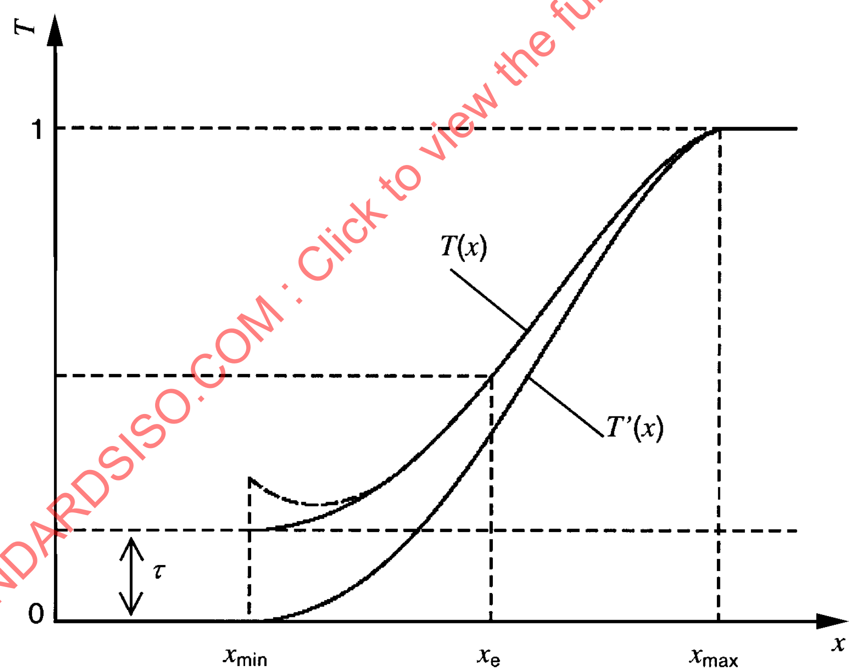

Figure 7 — Influence of a splitting process within the classifier on $T(x)$

In this case, part of the feed is removed into the coarse fraction without being subject to classification.

A modified grade efficiency curve, $T'(x)$, may be calculated from equation 24:

$$T'(x) = \frac{T(x) - \tau}{1 - \tau} \qquad (24)$$

The true grade efficiency curve is still represented by $T(x)$, but the use of $T'(x)$ and τ may simplify the description of the grade efficiency curve and may give additional technical information.

4.3 Incomplete dispersion of the feed material

If the feed material is not properly dispersed before entering the classification zone, coarse agglomerates, consisting of fine particles, are transferred to the coarse fraction. If in the particle size analysis the samples of the feed material and the fine and the coarse fraction are better dispersed than in the classifier, the coarse agglomerates i.e. the supposed coarse particles, will disappear. The grade efficiency curve then rises at small particle sizes, as shown in Figure 8.

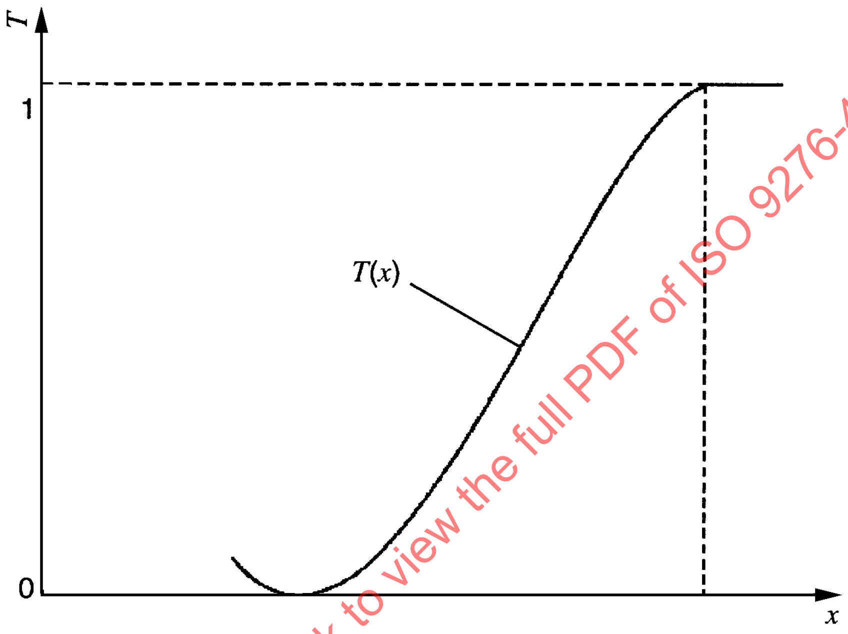

Figure 8 — Deviation from the ideal grade efficiency curve caused by incomplete dispersion of the feed material in the classifier

4.4 The influence of comminution of the feed in the classifier

If the feed material experiences a comminution process in the classification zone, i.e. the classifier partly acts as a grinding machine, the true "feed" material differs from the one obtained from a sample taken when feeding the material to the classifier. New fine material is produced and an equivalent amount of coarse particles disappears.

The errors, $\varepsilon(x)$, in the mass balance of the density distributions, as defined in annex A, can be used to form a new variable $\varphi(x)$, which indicates whether the deviations observed are either stochastic or systematic ones, and which course they follow.

$$\varphi(x) = \frac{v_{r,\mathrm{f}}\, q_{r,\mathrm{f}}(x) + v_{r,\mathrm{c}}\, q_{r,\mathrm{c}}(x)}{q_{r,\mathrm{s}}(x)} = 1 - \frac{\varepsilon(x)}{q_{r,\mathrm{s}}(x)} \qquad (25)$$

If the sum of the weighted density distributions of the fine and the coarse material equals the density distribution of the feed, then $\varepsilon(x)$ equals zero and $\varphi(x)$ equals one.

Stochastic deviations of $\varphi(x)$ from a value of one indicate stochastic errors $\varepsilon(x)$ in the mass balance. The suggested error correction and multiple analyses should then be used in the evaluation of the grade efficiency curve, as described in annex A.

If however $\varphi(x)$ deviates systematically from one, comminution of the feed material in the classifier might be responsible. In such a case the device is not recommended for use in particles size analysis, unless the operating conditions can be modified to avoid comminution.

Annex A
(informative)

The influence of stochastic errors on the evaluation of grade efficiency curves

A.1 General

The mass balances and the evaluation of the grade efficiency curve as described in clause 4, are only valid under the presupposition that equations 6 and 7 leave no remainder. In reality however, both equations will not hold when introducing $q_{r,\mathrm{s}}(x)$, $q_{r,\mathrm{f}}(x)$ and $q_{r,\mathrm{c}}(x)$ or $Q_{r,\mathrm{s}}(x)$, $Q_{r,\mathrm{f}}(x)$ and $Q_{r,\mathrm{c}}(x)$ as obtained in the particle size analysis for the feed material and the fine and coarse fractions. Further errors may arise when directly measuring $v_{r,\mathrm{f}}$ or $v_{r,\mathrm{c}}$.

Size dependent errors, $\varepsilon(x)$ and $E(x)$, are then observed and equations 6 and 7 must be rewritten as follows:

$$q_{r,\mathrm{s}}(x) - v_{r,\mathrm{f}}\, q_{r,\mathrm{f}}(x) - v_{r,\mathrm{c}}\, q_{r,\mathrm{c}}(x) = \varepsilon(x) \tag{A.1}$$

$$Q_{r,\mathrm{s}}(x) - v_{r,\mathrm{f}}\, Q_{r,\mathrm{f}}(x) - v_{r,\mathrm{c}}\, Q_{r,\mathrm{c}}(x) = E(x) \tag{A.2}$$

If one calculates, as usual, the average grade efficiency of a small size class, Δx_i, the average size is represented by:

$$\bar{x}_i = \frac{x_{i-1} + x_i}{2} \tag{A.3}$$

and the average grade efficiency equals:

$$\bar{T}(\bar{x}_i) = \frac{v_{r,\mathrm{c}}\left[Q_{r,\mathrm{c}}(x_i) - Q_{r,\mathrm{c}}(x_{i-1})\right]}{Q_{r,\mathrm{s}}(x_i) - Q_{r,\mathrm{s}}(x_{i-1})} \tag{A.4}$$

When using equations A.3 and A.4 with erroneous results of particle size analysis one obtains grade efficiency curves as shown in Figure A.1.

One realises a significant difference between theses curves and the grade efficiency curve as shown in Figure 5. A reliable average grade efficiency curve is difficult, if not impossible to draw.

A.2 The indirect evaluation of $v_{r,\mathrm{f}}$ and $v_{r,\mathrm{c}}$

In many practical cases of application the relative amounts of the fine, $v_{r,\mathrm{f}}$, and the coarse material, $v_{r,\mathrm{c}}$, cannot be obtained directly, as suggested in equation 1. If however, sampling is possible from the feed material and the fine and the coarse fraction, e.g., the cumulative size distributions, $Q_{r,\mathrm{s}}(x)$, $Q_{r,\mathrm{f}}(x)$ and $Q_{r,\mathrm{c}}(x)$ may be determined and used for a calculation of either $v_{r,\mathrm{f}}$ or $v_{r,\mathrm{c}}$.

$v_{r,\mathrm{f}}$ is calculated using equation 8. This equation may be interpreted as a linear equation, if one rewrites it as follows:

$$\eta_i = Q_{r,\mathrm{s}}(x_i) - Q_{r,\mathrm{c}}(x_i) = v_{r,\mathrm{f}}\left[Q_{r,\mathrm{f}}(x_i) - Q_{r,\mathrm{c}}(x_i)\right] = v_{r,\mathrm{f}}\, \xi_i \tag{A.5}$$

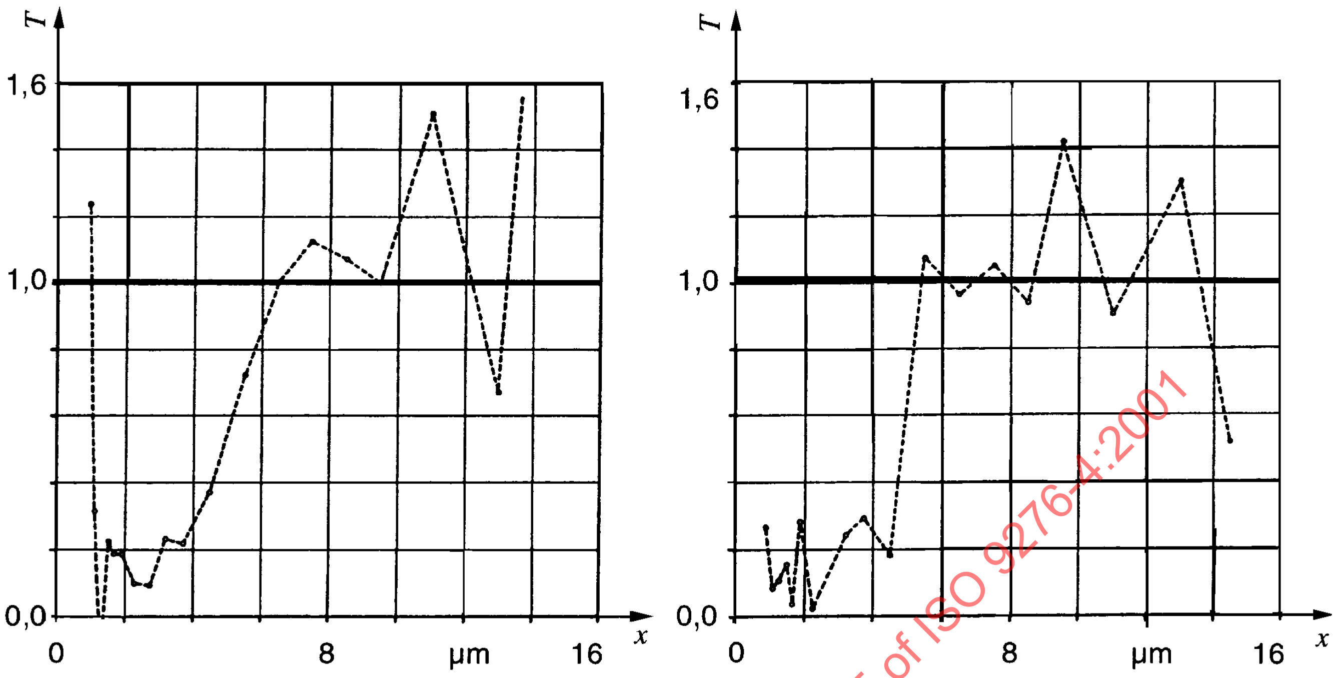

Figure A.1 — Uncorrected grade efficiency curves

If one plots $\eta_i = Q_{r,\text{s},i} - Q_{r,\text{c},i}$ on the ordinate against $\xi_{r,i} = Q_{r,\text{f},i} - Q_{r,\text{c},i}$ on the abscissa, one obtains a series of points in the neighbourhood of a straight line through the origin. Regression-analysis yields the average value of $v_{r,\text{f}}$ from:

$$\bar{v}_{r,\text{f}} = \frac{\sum_{i=1}^{n} \xi_i \eta_i}{\sum_{i=1}^{n} \xi_i^2} \tag{A.6}$$

Its variance, s^2, can be calculated from equation A.7:

$$s_{\bar{v}_{r,\text{f}}}^{\;2} = \frac{1}{n-1}\left(\sum \eta_i^2 - \frac{\left(\sum \xi_i \eta_i\right)^2}{\sum \xi_i^2}\right) \tag{A.7}$$

with the confidence interval being defined by:

$$\pm \frac{t\,s}{\sqrt{n}} \tag{A.8}$$

where t is Student's factor.

For example, t equals approximately 1,96 if a probability of 95 % is assumed and the number, n, of values used in the calculation is larger than 25.