INTERNATIONAL STANDARD

**ISO 9276-1**

Second edition
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# Representation of results of particle size analysis —

## Part 1:
Graphical representation

*Représentation de données obtenues par analyse granulométrique —*

*Partie 1: Représentation graphique*



Reference number
ISO 9276-1:1998(E)

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 9276-1 was prepared by Technical Committee ISO/TC 24, *Sieves, sieving and other sizing methods*, Subcommittee SC 4, *Sizing by methods other than sieving.*

This second edition cancels and replaces the first edition (ISO 9276-1:1990), of which it constitutes a technical revision.

ISO 9276 consists of the following parts, under the general title *Representation of results of particle size analysis*:

— *Part 1: Graphical representation*

— *Part 2: Calculation of average particles sizes/diameters and moments from particle size distributions*

Annex A of this part of ISO 9276 is for information only.





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# Representation of results of particle size analysis —

## Part 1:
Graphical representation



## 1 Scope

This part of ISO 9276 specifies rules for the graphical representation of particle size analysis data in histograms, density distributions and cumulative distributions. It also establishes a standard nomenclature to be followed to obtain the distributions mentioned above from the measured data.

This part of ISO 9276 applies to the graphical representation of distributions of solid particles, droplets or gas bubbles covering all size ranges.

## 2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this part of ISO 9276. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this part of ISO 9276 are encouraged to investigate the possibility of applying the most recent edition of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 565:1990, *Test sieves — Metal wire cloth, perforated metal plate and electroformed sheet — Nominal sizes of openings.*

## 3 Symbols

### 3.1 General

In this part of ISO 9276, the symbol $x$ is used to denote the particle size or the diameter of a sphere. However, it is recognized that the symbol $d$ is also widely used to designate these values. Therefore, in the context of this part of ISO 9276, the symbol $x$ may be replaced by $d$ where it appears.

Symbols for the particle size other than $x$ or $d$ should not be used.

### 3.2 Symbol explanation

$d$ particle size, diameter of a sphere (see 3.1)

$i$ (subscript) number of the size class with upper limit $x_i$: $\Delta x_i = x_i - x_{i-1}$

| | |
|---|---|
| $\nu$ | (integer, see subscript $i$) |
| $n$ | total number of size classes |
| $q_0(x)$ | density distribution by number |
| $q_1(x)$ | density distribution by length |
| $q_2(x)$ | density distribution by surface or projected area |
| $q_3(x)$ | density distribution by volume or mass |
| $q_r(x)$ | density distribution (general) |
| $q_r^*(\ln x)$ | density distribution in a representation with a logarithmic abscissa |
| $\bar{q}_{r,i}$ | average density distribution of the class $\Delta x_i$: $\bar{q}_{r,i} = \bar{q}_r(\Delta x_i) = \bar{q}_r(x_{i-1}, x_i)$ |
| $\bar{q}_r(x)$ | histogram (general) |
| $Q_0(x)$ | cumulative distribution by number |
| $Q_1(x)$ | cumulative distribution by length |
| $Q_2(x)$ | cumulative distribution by surface or projected area |
| $Q_3(x)$ | cumulative distribution by volume or mass |
| $Q_r(x)$ | cumulative distribution (general) |
| $Q_{r,i}$ | $= Q_r(x_i)$ |
| $\Delta Q_{r,i}$ | increment of cumulative distribution within the class $\Delta x_i$: $\Delta Q_{r,i} = \Delta Q_r(x_{i-1}, x_i) = Q_r(x_i) - Q_r(x_{i-1})$ |
| $x$ | particle size, diameter of a sphere (see 3.1) |
| $x_{min}$ | size below which there are no particles |
| $x_{max}$ | size above which there are no particles |
| $x_i$ | upper size of a particle size interval |
| $x_{i-1}$ | lower size of a particle size interval |
| $\Delta x_i$ | $= x_i - x_{i-1}$, width of the particle size interval |
| $\xi$ | $= \xi(x)$ transformed coordinate |

# 4 Particle size, measures and types

## 4.1 General

In a graphical representation of particle size analysis data, the independent variable, i.e. the physical property chosen to characterize the size of the particles, is plotted on the abscissa (see figure 1). The dependent variable, which characterizes measure and type of quantity, is plotted on the ordinate.

## 4.2 Particle size *x*

Regarding the denotation of particle size, see 3.1.

There is no single definition of particle size. Different methods of analysis are based on the measurement of different physical properties. Independently of the particle property actually measured, the particle size is reported as a linear dimension. In this part of ISO 9276, the particle size is defined as the diameter of a sphere having the same physical properties; this is known as the equivalent spherical diameter. The physical property to which the equivalent diameter refers shall be indicated using a suitable subscript, for example:

The different measures are

- $x_S$: equivalent surface area diameter;
- $x_V$: equivalent volume diameter.

Other definitions are possible, such as those based on the opening of a sieve or a statistical diameter, e.g. the Feret diameter, measured by image analysis.

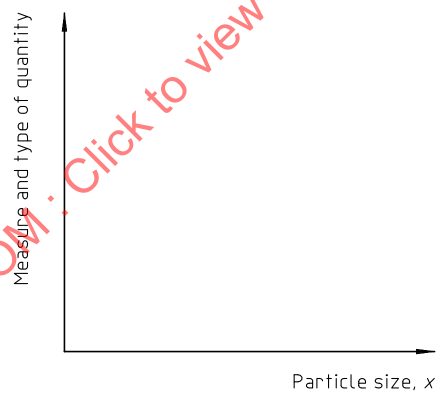


**Figure 1 — Coordinates for the representation of particle size analysis data**



## 4.3 Measures and types

The measures and types are distinguished with respect to the dependent variables by symbols as shown below.

The different measures are

- $Q$: cumulative measures, and
- $q$: density measures.

Each measure can be one of several types. The type is indicated by the general subscript, *r*, or by the appropriate value of *r* as follows:

number: $r = 0$

length: $r = 1$

area: $r = 2$

volume or mass: $r = 3$

The summary of the symbols used to designate density and cumulative distributions is shown in table 1.

**Table 1 — Symbols for distributions**

| **Type** | **Mathematical symbol for** | |
|---|---|---|
| | **density distribution** | **cumulative distribution** |
| Distribution by | | |
| number | $q_0(x)$ | $Q_0(x)$ |
| length | $q_1(x)$ | $Q_1(x)$ |
| area | $q_2(x)$ | $Q_2(x)$ |
| volume or mass | $q_3(x)$ | $Q_3(x)$ |
| General symbol | $q_r(x)$ | $Q_r(x)$ |

## 5 Graphical representation

Examples of the graphical representation of particle size analysis data are shown in figures 2 to 4.

### 5.1 Histogram $\bar{q}_r(x)$

Figure 2 shows the normalized histogram, $\bar{q}_r(x)$, of a density distribution $q_r(x)$. It comprises a successive series of series of rectangular columns, the area of each of which represents the relative quantity $\Delta Q_{r,i}(x)$, where

$$\Delta Q_{r,i} = \Delta Q_r(x_{i-1}, x_i) = \bar{q}_r(x_{i-1}, x_i)\,\Delta x_i \qquad (1)$$

or

$$\bar{q}_{r,i} = \bar{q}_r(x_{i-1}, x_i) = \frac{\Delta Q_r(x_{i-1}, x_i)}{\Delta x_i} = \frac{\Delta Q_{r,i}}{\Delta x_i} \qquad (2)$$

The sum of all the relative quantities, $\Delta Q_{r,i}$, forms the area beneath the histogram $\bar{q}_r(x)$, normalized to 100 % or 1 (condition of normalization). Therefore, the following equation holds:

$$\sum_{i=1}^{n} \Delta Q_{r,i} = \sum_{i=1}^{n} \bar{q}_{r,i}\Delta x_i = 1 = 100\ \% \qquad (3)$$

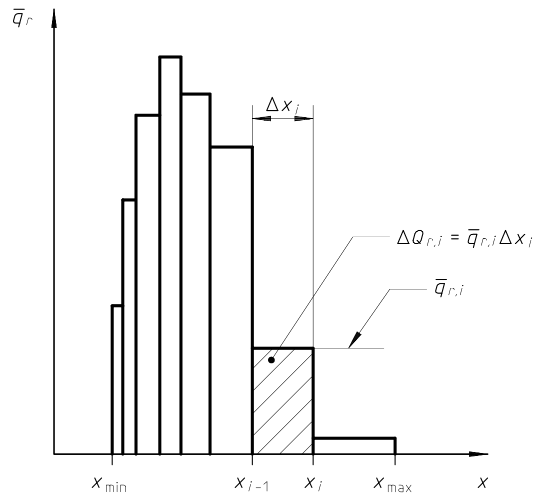


**Figure 2 — Histogram of a density distribution function $\bar{q}_r(x)$**



## 5.2 Cumulative distribution $Q_r(x)$

Figure 3 shows a typical normalized cumulative distributions, $Q_r(x)$. If the cumulative distribution is calculated from the histogram data, only individual points $Q_{r,i} = Q_r(x_i)$ are obtained, as indicated in figure 3.

Each individual point of the distribution, $Q_r(x_i)$, defines the relative amount of particles smaller than or equal to $x_i$. The continuous curve is calculated by suitable interpolation algorithms. A first approximation is obtained by connecting successive points by straight lines.

The normalized cumulative distribution extends between 0 and 1, i.e. 0 and 100 %.

$$Q_{r,i} = \sum_{\nu=1}^{i} \Delta Q_{r,\nu} = \sum_{\nu=1}^{i} \bar{q}_{r,\nu} \Delta x_\nu \qquad (4)$$

with $1 \le \nu \le i \le n$.

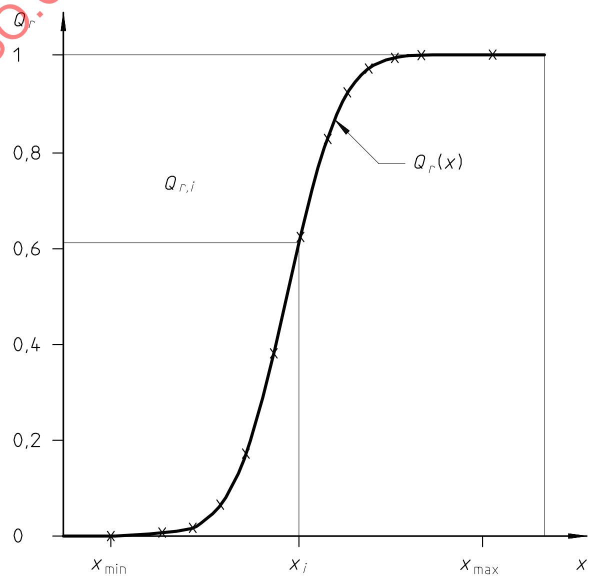


**Figure 3 — Cumulative distribution $Q_r(x)$**

### 5.3 Cumulative distribution $q_r(x)$

Under the presupposition that the cumulative distribution, $Q_r(x)$, is differentiable, the continuous density distribution, $q_r(x)$, is obtained from

$$q_r(x) = \frac{dQ_r(x)}{dx} \tag{5}$$

$q_r(x)$ is plotted in figure 4.

Conversely, the cumulative distribution, $Q_r(x)$, is obtained from the density distribution, $q_r(x)$, by integration:

$$Q_r(x_i) = \int_{x_{min}}^{x_i} q_r(x)dx \tag{6}$$

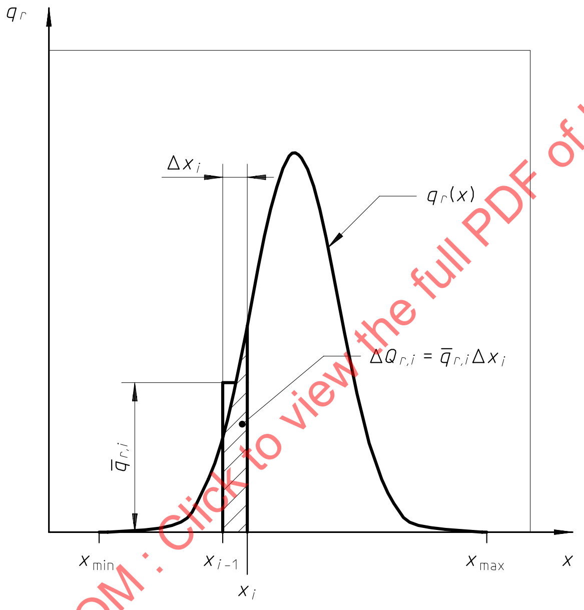


**Figure 4 — Density distribution $q_r(x)$**



## 6 Graphical representation of cumulative and density distributions on a logarithmic abscissa

Owing to the fact that a size distribution can cover several decades between its smallest particle size, $x_{min}$, and its largest particle size, $x_{max}$, plotting the data on a linear abscissa may not be suitable. In such a case, therefore, the results shall be plotted on graph paper with a logarithmic abscissa.

### 6.1 Cumulative distribution on a logarithmic abscissa

When plotted on graph paper with a logarithmic abscissa the cumulative values, $Q_{r,i}$, i.e. the ordinates of a cumulative distribution, do not change. Meanwhile, the course of the cumulative distribution curve changes but the relative amounts smaller than a certain particle size remain the same. Therefore, the following equation holds:

$$Q_r(x) = Q_r(\ln x) \tag{7}$$

### 6.2 Density distribution on a logarithmic abscissa

The density values of a histogram, $\bar{q}^*_{r,i} = \bar{q}^*_r\left(x_{i-1}, x_i\right)$, shall be recalculated using equation (8) which indicates that corresponding areas underneath the density distribution curve remain constant. In particular, the total area is equal to 1 or 100 %, independent of any transformation of the abscissa.

$$\bar{q}^*_r\left(\xi_{i-1},\xi_i\right)\Delta\xi_i = \bar{q}_r\left(x_{i-1},x_i\right)\Delta x_i \qquad (8)$$

where $\xi$ is any function of *x*.

Thus the following transformation shall be carried out to obtain the density distribution with a logarithmic abscissa:

$$\bar{q}^*_r\left(\ln x_{i-1}, \ln x_i\right) = \frac{\bar{q}_r\left(x_{i-1},x_i\right)\Delta x_i}{\ln x_i - \ln x_{i-1}} = \frac{\bar{q}_{r,i}\Delta x_i}{\ln\left(x_i / x_{i-1}\right)} = \frac{\Delta Q_{r,i}}{\ln\left(x_i / x_{i-1}\right)} \qquad (9)$$

Equation (9) also holds if the natural logarithm is replaced by the logarithm to the base 10.