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Hydrometry — Stage-fall-discharge relationships

Hydrométrie — Relations hauteur-dénivelé-débit

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 113, *Hydrometry*, Subcommittee SC 1, *Velocity area methods*.

This second edition cancels and replaces the first edition (ISO 9123:2001), which has been technically revised. The main changes were to improve the text relating to the stage-fall-discharge method and to revise the previous clause on uncertainty in accordance with HUG/GUM and similar related standards on the estimation of uncertainty in flow measurements.

Hydrometry — Stage-fall-discharge relationships

1 Scope

This document specifies methods for determining stage-fall-discharge relationships for a stream reach where variable backwater occurs either intermittently or continuously. Two gauging stations, a base reference gauge and an auxiliary gauge are required for gauge height measurements. A number of discharge measurements are required in order to calibrate the rating to the accuracy required by this document.

The preparation of rating curves is not described in detail in this document.

NOTE For a more detailed description of preparing rating curves, see the methods described in ISO 1100-2.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 772, *Hydrometry — Vocabulary and symbols*

ISO 1100-2, *Hydrometry — Measurement of liquid flow in open channels — Part 2: Determination of the stage-discharge relationship*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 772 apply.

Note, however that the application of the definition of backwater given in ISO 772 to the determination of discharge under intermittent or continuous backwater conditions should take into account that a higher gauge height would prevail for a given discharge than would be the case if the variable backwater was not present.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

4 Symbols and abbreviated terms

4.1 Symbols

Symbol	Meaning	Units
H	measured water level or stage at gauging station	m
H_{1e}	total effective upstream head	m
H_{2e}	total effective downstream head	m
H_{1max}	maximum upstream total head above crest elevation	m
h	measured fall (difference between stage at main gauging station and upstream or downstream secondary gauge)	m
h_c	reference fall or unit fall for constant fall methods	m

Symbol	Meaning	Units
h^*_r	reference fall or rating fall in limiting fall method	m
h_p	separation pocket head	m
H_o	stage/water level measurement at gauge zero/cease to flow level	m
$H_{u/s}$	upstream stage/water level	m
$H_{d/s}$	downstream stage/water level	m
$H_{u/s}- H_{d/s}$	effective fall	m
N	number of gaugings	non-dimensional
P	number of rating-curve parameters estimated from the N gaugings	non-dimensional
Q	measured discharge	m^3s^{-1}
Q_c	rating-derived discharge for unit and constant fall method	m^3s^{-1}
Q^*_r	rating-derived discharge for limiting fall method	m^3s^{-1}
S^2	estimated variance	
u_H	uncertainty in the stage/water level measurement	m or non-dimensional
$u(h)$	uncertainty in the measured fall	m or non-dimensional
$u(H)$	standard uncertainty in the recorded value of the stage	m
$u(H_0)$	standard uncertainty in the gauge zero	m
$u(H_{u/s})$	uncertainty in the recorded value of the upstream stage	m
$u(H_{d/s})$	uncertainty in the recorded value of the downstream stage	m
$u(H_{u/s}$-$H_{d/s})$	uncertainty in the effective fall	m or non-dimensional
$U(Q_e)$	uncertainty in the estimated (computed) discharge	
$u_{HC}(Q)$	uncertainty caused by neglecting all other physical parmeters that affect discharge	
$u_{RC}(Q)$	uncertainty in the stage-fall-discharge relation, mainly related to improper knowledge of hydraulic processes, shape of the assumed function and errors in parameter estimates	
$u(\theta)$	percentage uncertainty due to neglecting all other physical parameters	Non-dimensional
$U\ln(Q_{pr})$	standard uncertainty of prediction	Non-dimensional
α	scale factor that is numerically equal to the discharge when the effective depth of flow/stage $(H - H_0)$ is equal to 1	
β	slope of the rating curve when plotted on logarithm scales	
p	power parameter	
Subscripts		
u/s	denotes the upstream value	
d/s	denotes the downstream value	

4.2 Abbreviations

HUG Hydrometric uncertainty guidance

GUM Guide to the expression of uncertainty in measurement

SFD Stage-fall-discharge

5 General considerations

5.1 Importance of backwater

Most programmes for collecting records of discharge of streams are based on the fact that a relatively simple relationship exists between gauge height and discharge so that, by simply recording gauge height and developing the stage-discharge relationship, a continuous record of discharge can be computed. Several factors, however, can cause scatter of discharge measurements about the stage-discharge relationship at some stations. Backwater is one of these factors and is defined as a condition whereby the flow is retarded so that a higher gauge height is necessary to maintain a given discharge than would be necessary if the backwater were not present. Backwater is caused by constriction such as narrow reaches of a stream channel or downstream structures such as dams/bridges, downstream tributaries or tidal reach of a stream. All these factors can increase or decrease the energy gradient for a given discharge and cause variable backwater conditions. For example, in tidal streams the energy gradient during flood tides is less than the energy gradient during ebb tides.

5.2 Backwater conditions

Constant backwater, as caused by section controls for instance, will not adversely affect the stage-discharge relationship. The presence of variable backwater, on the other hand, does not allow the use of simple stage-discharge relationships for accurate determination of discharge. Regulated streams may have variable backwater virtually all of the time, while other streams will have only occasional backwater from downstream tributaries, vegetal growth, from the return of overbank flow or backwater from the sea.

Actually, the method is valid for steady, gradually varied flows. Large errors occur when the flow is unsteady, and/or it is rapidly varied. Then, the computed fall between the two gauges can be hydraulically meaningless. In such situations, other techniques including water volume balances might be used, not the stage-fall-discharge method. Further, this methodology appears to assume a constant linear relationship in terms of hydraulic gradient between the auxiliary and reference sites. It is possible for this relationship to be compromised by, for example, variable weed growth between the sites. Hence, the flow gaugings at these sites should be taken simultaneously or at least during conditions when the flow is the same at both sites.

5.3 Gauging requirements

Many of the backwater affected sites can be operated as stage-fall-discharge stations by using a base gauge at which gauge height is measured continuously and current-meter measurements of discharge are made occasionally. An auxiliary gauge some distance away from the base gauge, preferably downstream, is operated to measure gauge height continuously.

The auxiliary gauge should be located downstream of the base gauge because

a) it is preferable to set up the main station as far away as possible from the variable backwater cause and

b) when the upstream gauge becomes free from variable backwater (i.e. free-flowing conditions), the measured fall is not representative of the slope of the flow around each gauge and the downstream gauge is still impounded: then the upstream gauge can be rated using a stage-discharge curve, not the downstream one. There is generally no advantage in choosing the downstream gauge as the base gauge.

When the two gauges are set to the same datum, the difference between the two gauge height records is the water-surface fall and provides a measure of water-surface slope. Inflow between these gauges should be minimal. The locations of the base and auxiliary gauge are based on the characteristics of the slope reach. The length of the reach should be such that ordinary errors that occur in the determinations of the gauge heights at gauge stations will cause no more than minor error in computing the fall in the reach. Reliable discharge records can usually be computed when fall exceeds about 0,15 m. Precise time synchronization between the base and auxiliary gauges is very important when gauge height changes rapidly, or when fall is small. Timing and gauge-height errors that are trivial at high discharges become significant at very low flow[16].

It is also essential that the two gauges are levelled accurately to the same datum to minimize the errors not only in the individual stage readings but also the corresponding estimated fall. Therefore, the gauges shall be set to the same zero based on accurate survey techniques.

Channel slope in the reach should be as uniform as possible. The shorter the slope reach, the closer the relationship between measured fall and water-surface slope. On the other hand, the longer the slope reach, the smaller the percentage of error in the recorded fall. The reach should be as far upstream from the source of backwater as is practicable, and inflow between the two gauges should be negligible. If possible, reaches with frequent or appreciable overbank flow should be avoided, as should reaches with sharp bends or unstable channel conditions.

Rarely a slope reach will be found that has all of the above attributes, but these attributes should be considered in making a selection from the reaches that are available for slope measurement.

5.4 Types of stage-fall-discharge relationships

5.4.1 Under conditions of variable backwater, the fall as measured between the base gauge and the auxiliary gauge is used as a third parameter, and the rating becomes a stage-fall-discharge relationship. Stage-fall-discharge methods fall into the following two broad categories:

a) constant-fall method, of which the unit-fall method is a special case;

b) variable-fall method.

The applicable method for a stream reach depends to a large degree on whether the backwater is intermittent or always present.

5.4.2 The constant-fall method works best when backwater is always present at all gauge heights, but can sometimes be adapted to intermittent backwater conditions.

5.4.3 The unit-fall method is the simplest and requires the least amount of data for calibration. The unit-fall method should be used as a starting point before attempting more complex methods.

5.4.4 Variable-fall methods are the most complex and require the most data for calibration. The variable-fall method works best for the intermittent backwater condition.

NOTE The unit-fall method, the constant-fall method and the variable-fall method are also referred to in this document as the unit-fall rating, the constant-fall rating and the variable-fall rating.

6 Unit-fall method

6.1 General

The unit-fall method is a special case of the constant-fall method, where the constant fall is unity (1 m). The unit-fall method is used with the assumption that the relationship between the discharge ratio (Q/Q_c) and the fall ratio (h/h_c) is exactly a square root relationship, as given by the following formulae:

$$Q/Q_c = \left(h/h_c\right)^{0,5} = \left(h/1\right)^{0,5} = h^{0,5} \quad (1)$$

$$Q = Q_c h^{0,5} \text{ or } Q_c = Q/h^{0,5} \quad (2)$$

where

Q is the measured discharge, expressed in cubic metres per second;

h is the measured fall, expressed in metres;

Q_c is the discharge, expressed in cubic metres per second, from the rating curve corresponding to the constant fall and the base gauge height;

h_c is the constant fall, expressed in metres (1 m for the unit-fall method).

Note that the value 0,5 of the (h/h_c) exponent is justified by a channel control as modelled by the Chézy equation or the Manning-Strickler equation.

6.2 Method of analysis

The unit-fall rating shall be developed by plotting each measured discharge divided by the square root of the measured fall against the base gauge height for the discharge measurement. The rating curve shall then be fitted to these plotted points.

6.3 Computation of discharge

The rating shall be used to compute discharge by determining the value of Q_c from the rating for a given base gauge height, and multiplying this discharge by the square root of the measured fall. This type of rating will usually be satisfactory when backwater is always present, fall is greater than about 0,15 m, and the datum of the two gauges are within about 0,01 m.

If backwater is intermittent, it is also necessary to develop a free-fall rating or rating where backwater is not present. Gaugings not affected by backwater will normally be those that tend to plot to the right of the stage-discharge plot. If the stage-discharge points are plotted, an outer envelope stage-discharge curve can be derived which should hopefully reflect the gaugings which are backwater-free. Figure 4 may help illustrate this point.

The free-fall rating shall be used at all times except during periods when backwater is suspected, during which times discharge should be computed from both the free-fall and unit-fall ratings. The lower of the two discharges shall be considered to be the true value.

6.4 Example of unit-fall method

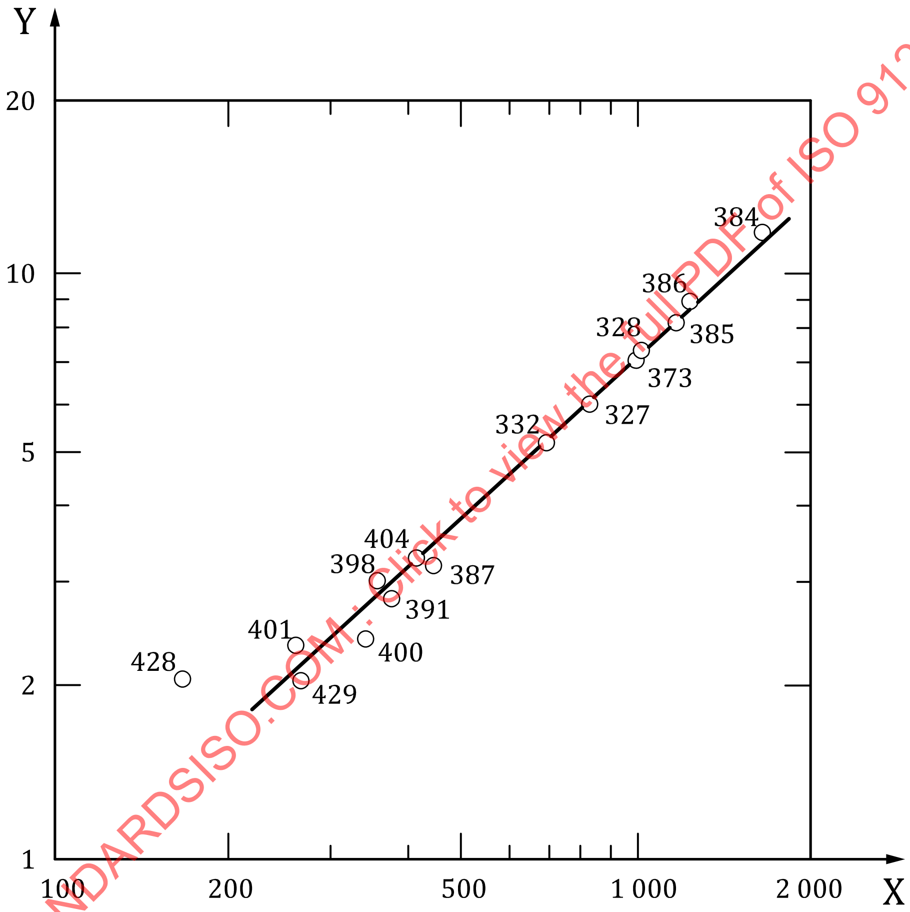
Figure 1 and Table 1 illustrate the unit-fall rating for a site with high backwater from a power dam. The backwater exists at all gauge heights and at all times.

Table 1 — Unit-fall calibration measurements

Measurement no.	Gauge height m	h m	Q m^3/s	$Q\sqrt{h}$	Q_c m^3/s	Difference %
327	5,907	1,917	1 160	838	840	−0,2
328	7,105	2,182	1 520	1 030	1 030	0
332	5,026	1,597	889	703	700	0,4
373	7,013	2,225	1 490	1 000	1 000	0
384	11,558	2,880	2 830	1 670	1 700	−1,8
385	8,108	1,920	1 640	1 180	1 190	−0,8
386	8,638	2,652	1 990	1 220	1 260	−3,3
387	3,139	0,808	399	444	410	7,7

Table 1 *(continued)*

Measurement no.	**Gauge height** m	h m	Q m³/s	$Q\sqrt{h}$	Q_c m³/s	**Difference** %
391	2,755	0,701	317	379	360	5,0
398	2,963	0,616	289	368	388	−5,4
400	2,359	0,204	156	345	300	13,0
401	2,286	0,290	145	269	290	−7,8
404	3,206	0,927	411	427	426	0,2
428	2,036	0,058	39,9	166	255	−53,6
429	2,012	0,061	66,0	267	250	6,4



Key

X discharge, Q_c, in m³/s

Y gauge height, in m

NOTE 1 Fall, h_c = 1 m.

NOTE 2 The numbers on the plot refer to the measurement number (see Table 1).

Figure 1 — Unit-fall rating

7 Constant-fall method

7.1 General

The constant-fall method is more complex than the unit-fall method in that it uses two relationship curves. In addition, it does not require that the constant fall be equal to unity, but can be any selected value. The constant fall is usually selected to be equal to the average fall in the gauging reach. The constant-fall method requires the use of the following two curves:

a) the relationship between gauge height and discharge for a constant fall of some specified value;

b) the relationship between measured fall, h, and the discharge ratio, Q/Q_c.

A unique feature of the constant-fall method is that the base gauge and auxiliary gauge need not be at the same datum.

7.2 Method of analysis

One method of developing a constant-fall rating is to compute first a unit-fall rating, as described in 6.2. This relationship between gauge height to discharge can then be used to compute discharge ratios, Q/Q_c, for each discharge measurement. These ratios shall be plotted against the measured fall, or gauge differences, to define the relationship between the fall and the discharge ratio. This curve shall then be used to refine the stage-discharge relationship. Alternate refinements of the two curves shall be continued until little or no improvement occurs. This usually takes only two or three trials. The resultant stage-fall-discharge relationship is similar to a unit-fall rating but without the assumption that the ratio curve varies as a square root function.

A second method of developing a constant-fall rating is to develop a stage-discharge relationship corresponding to the average fall in the slope reach. This will result in a stage-discharge rating corresponding more closely to average conditions. The average fall is computed by arithmetically averaging the measured falls occurring under various conditions of backwater. This number may be rounded to a convenient value and is designated as the constant fall, h_c.

To define the rating, first each measured discharge is divided by the square root of h/h_c, then this value is plotted against the corresponding gauge height at the base gauge. The square root shall only be used initially and shall be later adjusted. A curve shall be fitted to the plotted points and the curve value of discharge Q_c, shall be determined for each discharge measurement. The ratio of Q/Q_c shall be plotted against the measured fall, h, for each discharge measurement and a curve shall be fitted to these points.

The two curves, gauge height versus discharge, Q_c and measured fall, h, versus discharge ratio, Q/Q_c, shall be refined by alternately adjusting one while holding the other fixed. Two or three trials will usually be adequate. For clarity, variables denoted with a star (*) are those determined directly from a relationship curve.

7.3 Computation of discharge

Discharge is computed from constant-fall ratings by the following procedure.

a) Enter the constant-fall rating with the gauge height and determine the rating discharge Q^*_c.

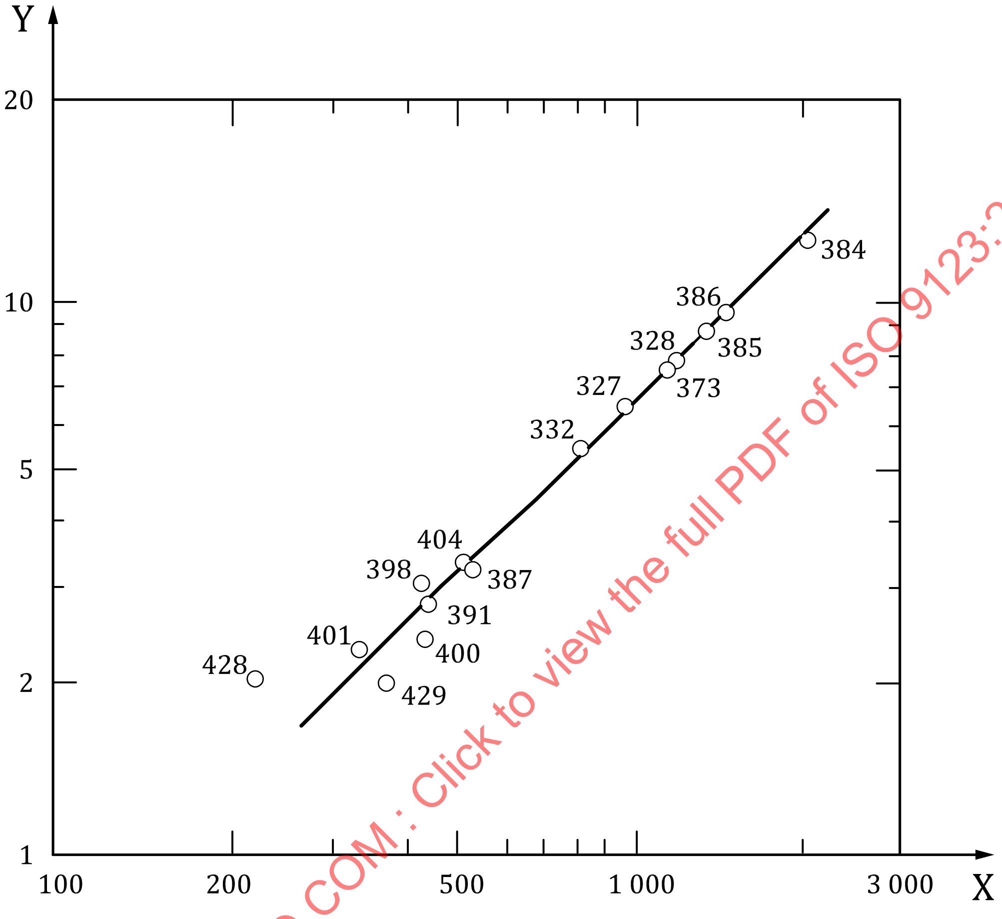
b) Enter the constant-fall ratio curve with the measured fall, h, and determine the ratio $(Q/Q_c)^*$.

c) Multiply the rating discharge, Q^*_c by the ratio $(Q/Q_c)^*$ to obtain the true discharge, Q.

7.4 Example of constant-fall method

Figures 2 and 3 and Table 2 illustrate the constant-fall method developed from the same data used in Table 1, and corresponding to a constant fall of 1,3 m. This rating was developed using the second procedure described in 7.2. The curves in Figures 2 and 3 are the final results of several trials and refinements.

The unit-fall example described in 6.4, and the constant-fall ratings described in this clause give essentially the same results[1)] and indicate the unit-fall method is as good as the constant-fall method in this instance. Both ratings indicate large percentage errors in the low-discharge range, as would be expected, because of the larger relative error in stage and discharge measurement.



Key

X discharge, Q_c, in m^3/s

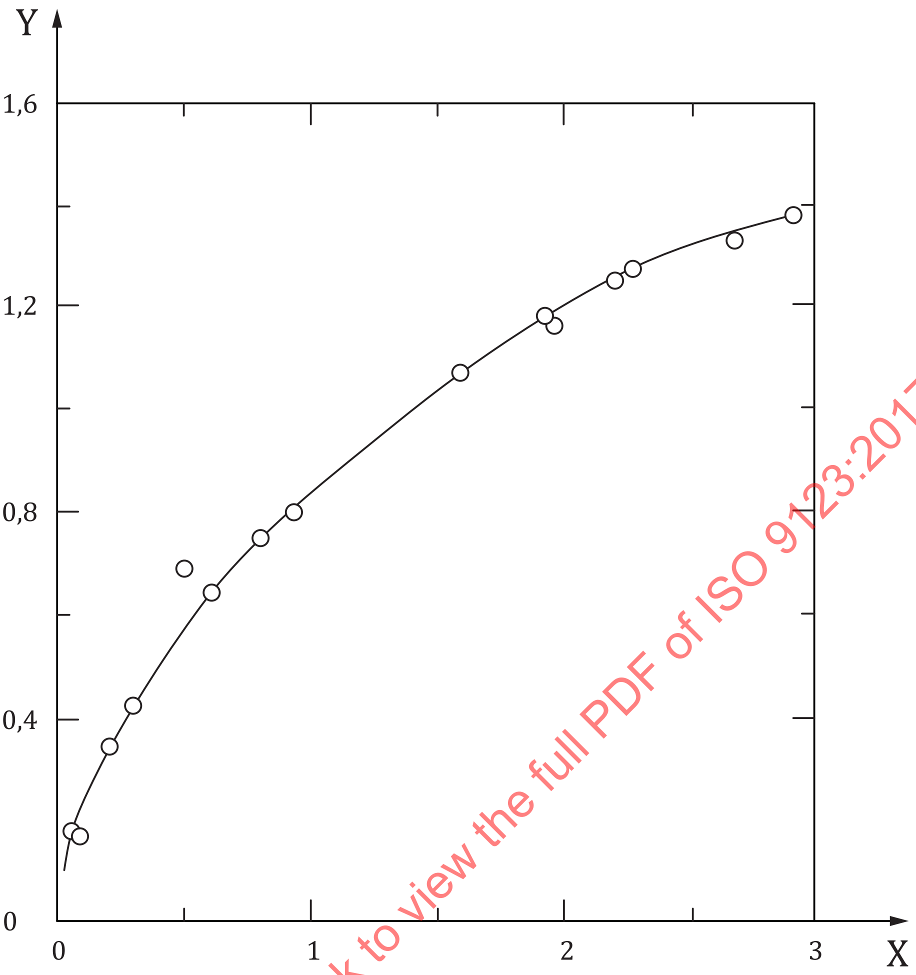
Y gauge height, in m

NOTE 1 Fall, h_c = 1 m.

NOTE 2 The numbers on the plot refer to the measurement number (see Table 1).

Figure 2 — Constant-fall stage discharge-rating curve

1) The unit-fall method yields good results in this example because the constant fall (1,3 m) is close to 1 m. This is not the general situation.



Key

X measured fall, h, in m

Y discharge ratio, Q/Q_c

Figure 3 — Constant-fall stage discharge-rating curve

Table 2 — Constant-fall calibration measurements (h_c = 1,3 m)

Measurement no.	Gauge height m	h m	Q m³/s	h/h_c	Q_c^* [a] m³/s	Q/Q_c^*	Q/Q_c^* [b]	Q_c [c] m³/s	Q [d] m³/s	Difference in measured value of Q %
327	5,907	1,917	1 160	1,475	980	1,184	1,185	979	1 160	0
328	7,105	2,182	1 520	1,678	1 200	1,267	1,260	1 210	1 510	0,7
332	5,026	1,597	889	1,228	825	1,078	1,075	827	887	0,2
373	7,013	2,225	1 490	1,712	1 190	1,252	1,270	1 190	1 510	−1,3
384	11,558	2,880	2 830	2,215	2 030	1,394	1,396	2 030	2 830	0
385	8,108	1,920	1 640	1,477	1 380	1,188	1,185	1 380	1 640	0
386	8,638	2,652	1 990	2,040	1 480	1,345	1,350	1 470	2 000	−1,5
387	3,139	0,808	399	0,622	500	0,798	0,755	530	378	5,3
391	2,755	0,701	317	0,539	440	0,720	0,700	440	308	2,8
398	2,963	0,616	289	0,474	465	0,622	0,660	438	307	−6,2
400	2,359	0,204	156	0,157	375	0,416	0,350	446	131	16,0
401	2,286	0,290	145	0,223	355	0,408	0,430	337	153	−5,5
404	3,206	0,927	411	0,713	510	0,806	0,805	511	411	0
428	2,036	0,058	39,9	0,045	305	0,131	0,180	222	54,9	−37,6
429	2,012	0,061	66,0	0,047	303	0,218	0,175	377	53	19,7

NOTE For clarity, variables denoted with a star (*) are those determined directly from a relationship curve.

a Value taken from the curve in Figure 2.

b Value taken from the curve in Figure 3.

c $Q_c = \frac{Q}{(Q/Q_c)*}$.

d $Q = Q_c^* \times (Q/Q_c)*$.

8 Variable-fall method

8.1 General

Variable-fall methods are the most complex of all stage-fall-discharge relationships[2)], and can be grouped into the following two types which differ according to how the stage-fall and stage-discharge ratings are defined:

a) the *normal-fall method*, in which the field data often fail to indicate a limiting position for the stage-discharge rating and the relation between gauge height and fall is defined by drawing a curve through the average fall experienced at each gauge height;

b) the *limiting-fall, or free-fall method* in which the relation between gauge height and discharge represents both non-backwater conditions and the maximum value of discharge for each gauge height and the fall is defined by drawing a curve through the minimum fall in the reach under those non-backwater conditions.

However, a transition towards a backwater-unaffected control for high enough discharge, and/or low enough downstream boundary condition can occur, especially upstream of a dam that is gradually opened when flood discharge increases. Then, segmentation of the curve has to be considered. The

2) A more complete description may be found in Rantz et al. (1982) Volume 2[16] and in the WMO *Manual on Stream Gauging* (2010)[18].

transition stages (breakpoints) between the SFD curves and the unique backwater-unaffected curve can be computed using continuity constraints.

8.2 Normal-fall method

The method for developing a normal-fall rating is similar to that for developing the limiting-fall rating and will not be described here in detail. In the normal-fall method, the stage-discharge relationship will correspond to average-fall conditions and can be used only when backwater is present. A separate rating is needed to compute discharge if there are times when backwater is not present. For this situation, discharge shall be computed by both ratings, and the lesser of the two values shall be considered to be correct.

8.3 Limiting-fall method

8.3.1 General

The limiting-fall method is best for gauging stations where there are times when backwater is not present. In this method, the rating curve of gauge height versus discharge defines a condition where backwater is not present. This same rating curve can be used to compute discharge at other times when backwater is present. This feature makes the limiting-fall method the most versatile of all stage-fall-discharge methods for streams where backwater is intermittent.

8.3.2 Method of analysis

The first step in the limiting-fall method is to plot all measured discharges against the base gauge height and label each point with the measured fall, h. A stage- discharge curve shall be drawn so as to pass through those measurements which are not affected by backwater.

Second, the measured fall, h, shall be plotted on a separate plot against the base gauge height. A curve shall be drawn through those points representing the minimum fall, but which are free of backwater. For most sites, there will be points both to the right and left of this curve. Points to the right represent falls exceeding the limiting, or minimum, fall which is affected by backwater. Thus, the name limiting-fall rating.

Thirdly, values of Q^*_r and h^*_r shall be determined from the discharge rating and from the fall rating, respectively, for each discharge measurement and the ratios Q/Q^*_r and h/h^*_r shall be computed. These ratios shall be plotted against each other and an average ratio curve shall be drawn.

Finally, each of the three curves, i.e. the stage-fall, stage-discharge and limiting fall ratio curves, shall be each in turn refined by holding two of them constant while recomputing and replotting the third one. Two or three trials will usually be adequate.

8.3.3 Computation of discharge

The three curves can be used to compute discharge by the following procedure.

a) Determine the gauge height and corresponding measured fall, h, for which discharge is to be computed.

b) Enter the measured discharge rating, Q, with the gauge height, and determine the rating discharge, Q^*_r, from the limiting-fall stage-discharge rating curve.

c) Enter the fall rating h with the gauge height, and determine the rating fall, h^*_r from the limiting-fall stage-fall relationship curve.

d) Compute the fall ratio by dividing the measured fall, h, by the rating fall, h^*_r.

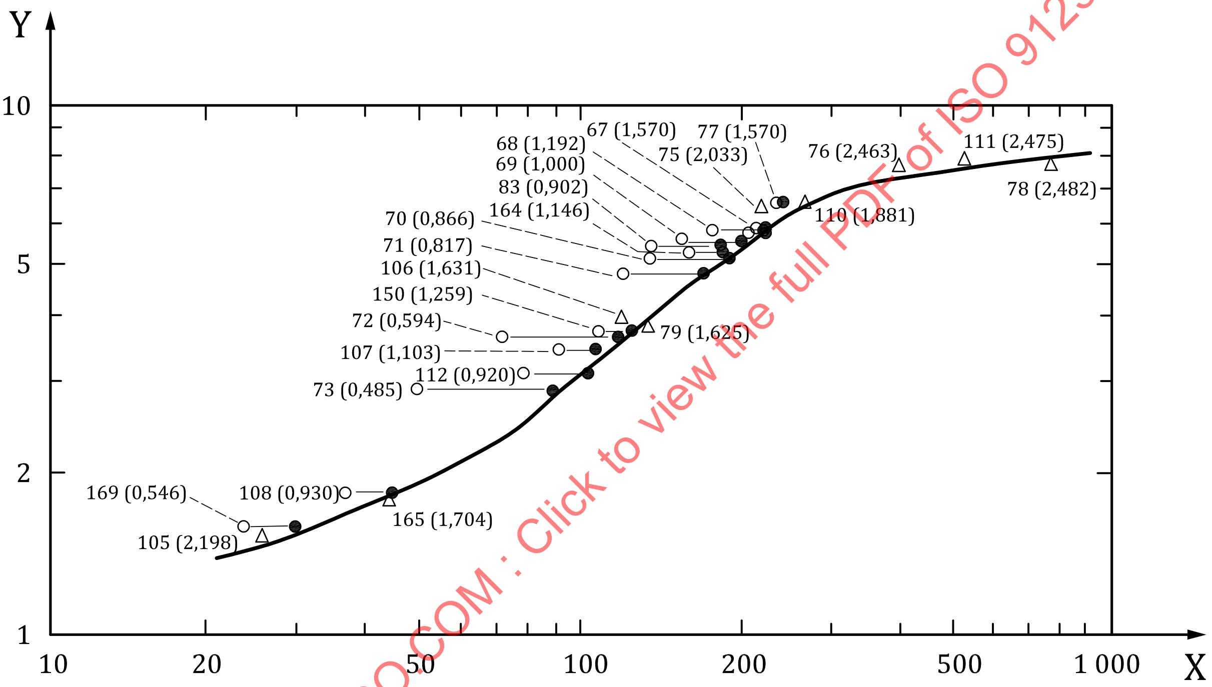
e) Enter the discharge ratio rating with the fall ratio, h/h^*_r, and determine the discharge ratio, (Q/Q^*_r) from the limiting-fall ratio curve.

f) Compute the true discharge, Q^*, by multiplying the rating discharge, Q^*_r, times the discharge ratio, $(Q/Q_r)^*$.

8.3.4 Example of limiting-fall method

Figures 4, 5 and 6 and Table 3 illustrate a limiting-fall rating for a site with intermittent backwater. These curves represent the final results after making several trials and refinements. In Figure 5, the plotted points show the fall after adjustment by the fall ratio. The measured fall has been omitted from this plot, except for those measurements where backwater is not present.

In the limiting-fall method, the stage-discharge rating is essentially a non-backwater rating and can be used to compute discharge either when backwater is present or when it is not present. This is an advantage of the limiting fall method, because a separate non-backwater rating is not required as in the normal-fall method. The limiting-fall method is the most complex of all the various fall ratings, but provides for the best use of available data.



Key

△ Measurements where no backwater is present.

○ Measurements affected by backwater.

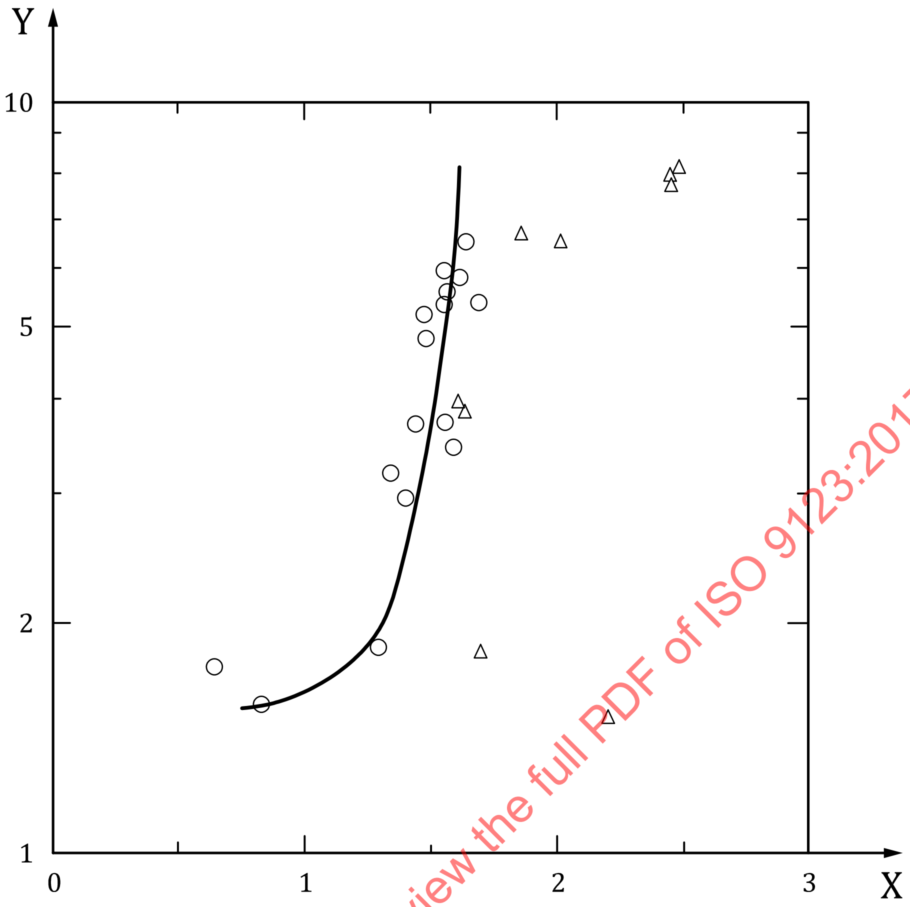
● Measurements adjusted for a condition when backwater is present.

X discharge, Q_r, in m^3/s

Y gauge height, in m

NOTE The numbers on the plot refer to the measurement number with the measured fall, h, given in parentheses (see Table 3).

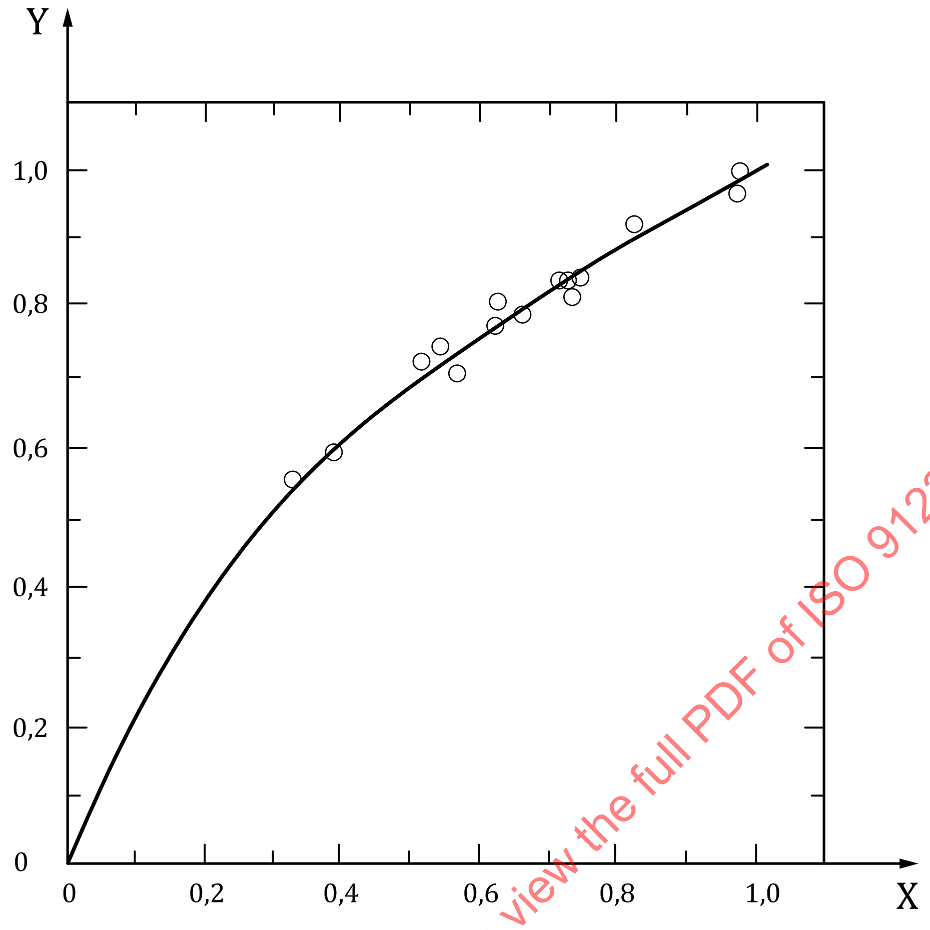
Figure 4 — Limiting-fall stage-discharge-rating curve



Key

△	Measurements where no backwater is present.
○	Measurements affected by backwater.
X	adjusted fall, h_r, in m
Y	gauge height, in m

Figure 5 — Limiting-fall stage-fall relationship curve



Key

X fall ratio, h/h_r

Y discharge ratio, Q/Q_r

Figure 6 — Limiting-fall ratio curve

Table 3 — Limiting-fall calibration measurements

Measurement no.	**Gauge height** m	h m	Q m^3/s	Q_r^* [a] m^3/s	Q/Q_r^*	h_c^* [b] m	h/h_r^*	$(Q/Q_r)^*$ [c]	Q_r [d] m^3/s	**Difference in value of Q_r** %	$(h/h_r)^*$ [c]	h_r [e] m^3/s
67	5,956	1,570	214	213	1,005	1,609	0,975	0,99	216	1,4	1,00	1,570
68	5,907	1,192	178	211	0,844	1,609	0,741	0,85	209	−0,9	0,73	1,630
69	5,614	1,000	154	198	0,778	1,600	0,625	0,77	200	1,0	0,63	1,587
70	5,246	0,866	134	181	0,741	1,588	0,545	0,72	186	2,9	0,58	1,492
71	4,865	0,817	119	165	0,721	1,573	0,519	0,70	170	3,0	0,55	1,485
72	3,725	0,594	70,5	119	0,591	1,527	0,389	0,60	118	−1,4	0,38	1,564
73	2,916	0,485	48,7	87,2	0,558	1,454	0,333	0,55	88,6	1,5	0,34	1,425
75f	6,559	2,033	217	242	—	1,628	—	—	217	−10,3	—	—
76f	7,705	2,463	391	379	—	1,646	—	—	391	3,2	—	—
77	6,514	1,570	233	240	0,973	1,625	0,966	0,98	238	−0,7	0,95	1,652
78f	8,077	2,482	767	736	—	1,646	—	—	767	4,2	—	—
79f	3,868	1,625	134	125	—	1,536	—	—	134	7,2	—	—
83	5,416	0,902	133	189	0,706	1,591	0,567	0,73	183	−3,3	0,53	1,702
105f	1,512	2,198	25,7	26,6	—	0,698	—	—	25,7	−3,4	—	—
106f	3,895	1,631	120	126	—	1,536	—	—	120	−4,8	—	—
107	3,487	1,103	89,8	110	0,817	1,509	0,731	0,84	107	−2,7	0,69	1,599
108	1,859	0,930	36,8	43,9	0,839	1,280	0,726	0,84	43,8	−0,2	0,72	1,291
110f	6,690	1,881	259	248	—	1,631	—	—	259	4,4	—	—
111f	8,001	2,475	524	549	—	1,646	—	—	524	−4,6	—	—
112	3,158	0,920	78,2	96,9	0,807	1,478	0,623	0,77	102	4,8	0,68	1,354
150	3,697	1,259	110	118	0,928	1,527	0,824	0,90	122	3,1	0,87	1,447
164	5,334	1,146	156	185	0,841	1,591	0,720	0,84	185	0,1	0,73	1,570
165f	1,817	1,704	43,0	41,9	—	1,237	—	—	43,0	2,6	—	—
169	1,585	0,546	23,8	30,0	0,792	0,832	0,656	0,80	29,7	−1,1	0,65	0,839

NOTE For clarity, variables denoted with a star (*) are those determined directly from a relationship curve.

[a] Value taken from the curve in Figure 4.

[b] Value taken from the curve in Figure 5.

[c] Value taken from the curve in Figure 6.

[d] $Q_r = \dfrac{Q}{(Q/Q_r)^*}$.

[e] $h_r = \dfrac{h}{(h/h_r)^*}$.

[f] Measurement where no backwater is present.

9 Rating curves and tables

All rating curves for stage-fall-discharge methods shall be prepared in accordance with methods described in ISO 1100-2 (under revision as ISO 18320)[3]. Each of the rating curves can be adapted to rating tables for easy application. Rating tables should show the discharges and falls corresponding to gauge heights in ascending order. The discharge ratios corresponding to either the measured fall (constant-fall method) or the fall ratio (limiting-fall method) should also be arranged in ascending order.

10 Method of computation

Computation of occasional discharge values can easily be performed by hand calculations directly from the curves and tables. However, if extended periods of hourly or daily discharges are needed, such as for a water year, it is best to perform these repetitive calculations by programming the method for computer calculations.

11 Periodic checking of stage-fall-discharge ratings

Stage-fall-discharge ratings should be checked periodically to ascertain that there have been no significant changes in the ratings. This can be done by making discharge measurements at regular intervals such as once every two or three months and plotting the fall-adjusted discharge on the stage-discharge rating. Deviations in excess of ± 10 % should alert the user to a possible shift, provided the possible error in the discharge measurement is not in excess of about ± 10 %. Several measurements over a period of time that all plot to one side of the rating, regardless of size of error, would indicate a shift of the rating.

12 Extrapolations

The stage-verse-$Q/\sqrt{h}$ curve may be extrapolated with caution, above or below the range of the observed data, by extending the curve at its end in accordance with the general trend of the curvature. Generally, the stage-fall-discharge rating can be extrapolated with more confidence when the data are such that they fit Formula (1) best.

Any abrupt change in hydraulic elements of the channel section will produce a corresponding change in the curvature of the curve. In this case, a correction for the change, if known, should be made in extrapolating the curve and such flow estimate should be identified as being approximate[7][16][4].

The extension of stage-fall-discharge relationship curves beyond the limits of a significant change in channel geometry, e.g. out of bank flow is not generally recommended. The extension of curves that are unaffected by backwater can be undertaken provided the physical properties of the channel are accounted for, as described in ISO 1100-2.

13 Evaluation of uncertainty in the stage-fall-discharge relation

13.1 General

This clause explains concisely the procedure for evaluating uncertainty in the stage-fall-discharge relation. These relationships are established in Clauses 6, 7 and 8 for a stream reach by computational conversion of the measurements to a normal fall (or a reference fall) using simultaneous measurement

3) Additional references regarding methods for building stage-fall-discharge curves and estimating their uncertainties may include Petersen and Reitan (2009)[15]; Coz et al. (2016)[8].

4) However, if the use to which the data are to be put justifies the increased workload and cost, the extension of rating curve can also be achieved by computational hydraulic modelling in up to three dimensions.

of discharge, stage, and fall[5]. This is graphically incorporated into a three dimensional stage-fall-discharge relationship manually using several trials.

The *Guide to the expression of uncertainty in measurement* (GUM) (ISO/IEC Guide 98-3:2008; JCGM 100:2008) is considered as a reference document for explaining and implementing the procedure for estimating the uncertainties in hydrometric measurements and flow determinations. The GUM was drawn upon to develop ISO/TS 25377:2007, *Hydrometric uncertainty guidance (HUG)*. The development of the uncertainties sections of hydrometric standards are usually based on the HUG which is aimed specifically at hydrometric practitioners. The HUG and the GUM have been used to develop the methods described in 13.2 to estimate the uncertainties in discharge estimates under backwater conditions.

The assessment of uncertainty in discharge estimates requires examination of hydraulic characteristics of the river channel and statistical interpretation of hydrometric data. A method is presented here for estimating uncertainty in a single discharge estimate from the use of an appropriate stage-fall-discharge relationship. A mathematical representation of the stage-fall-discharge relationship is made and both confidence level and prediction intervals are prepared and interpreted.

Subclause 13.2 explains the concept and methodology relevant to uncertainty analysis of stage-fall-discharge relation based on the GUM. Finally, an example of estimating the uncertainty of the stage-fall-discharge relation is demonstrated in 13.4. Annex A describes mathematical expressions and their matrix representation for explaining uncertainty of the stage-fall-discharge relationship.

13.2 Implementing the GUM procedure for evaluating uncertainty in the stage-fall-discharge relation and derived estimates

13.2.1 General

This subclause implements the above-described GUM procedure, to evaluate the uncertainty in the stage-fall-discharge relationship and discharge estimates. The description of various components of uncertainty, their propagation and evaluation are explained in different subclauses as given below:

13.2.2 Propagation of uncertainty for stage-fall-discharge estimates

The error in any river discharge estimate originates from several sources[6]. In operational practice, the river discharge estimates (under back-water conditions) are usually obtained by means of a functional relationship (rating curves), which is preliminarily estimated by the simultaneous measurements of stage, fall, and discharge. Thus in addition to the measurement errors, an additional error is induced by the imperfect estimation of rating curves. It is also important to note that the errors for each individual discharge measurement are often not evaluated but rather accepted as normalized values. These normalized errors are based on several test measurements for different gauging conditions and measurement techniques, and are usually given in hydrometric manuals[7]. In addition there are errors which arise due to difference between reference hydraulic conditions and actual hydraulic conditions during measurement. The combined uncertainty in discharge estimates, Q_e, may be expressed as follows:

$$U^2(Q_e) = u_{RC}^2(Q) + \left(\frac{\partial Q}{\partial H}\right)^2 u^2(H) + \left(\frac{\partial Q}{\partial h}\right)^2 u^2(h) + u_{HC}^2(Q) \quad (3)$$

where

5) This relationship can also be established by using Manning's formula by introducing slope (or fall) as a third parameter.

6) Dickinson (1967)[9] discussed several possible sources of systemic and random errors related to an individual measurement.

7) The HUG (ISO/TS 25377:2007) is specific to hydrometry and employs the methods from the GUM that are most applicable to hydrometry.

$U(Q_e)$ is the uncertainty in the estimated (computed) discharge;

$u_{RC}(Q)$ is the uncertainty in the stage-fall-discharge relation (rating curve) mainly related to improper knowledge of hydraulic processes, shape of the assumed function and errors in parameter estimates;

$u(H)$ is the uncertainty in the water level measurement; it propagates proportionally to the sensitivity coefficient i.e. $\frac{\partial Q}{\partial H}$;

$u(h)$ is the uncertainty in the fall measurement; this also propagates proportionally to the sensitivity coefficient $\frac{\partial Q}{\partial h}$;

$u_{HC}(Q)$ is the uncertainty caused by neglecting all other physical parameters that affect discharge.

The evaluation of Formula (3) expressing different uncertainty components of discharge estimation, is described as follows.

13.2.3 Uncertainty in rating curve

13.2.3.1 General

This subclause defines an appropriate relationship between stage, fall, and discharge measurements based on hydraulic characteristic and statistical interpretation to estimate uncertainty in river discharge estimates under back-water effects. However, it is important to make several assumptions before proceeding. It is assumed that

a) river control is stable during the period of discharge measurement,

b) there exists a stage-fall-discharge relationship, or at least a mean relationship about which the measured values vary randomly, and

c) the auxiliary gauge is preferred to be installed downstream.

Thus, it is desirable to choose a regression equation that adequately describes the relationship with minimum number of parameters.

13.2.3.2 Mathematical representation of stage-fall-discharge relationship

As a first step, it is required to define a functional relationship between stage-fall-discharge measurements [12][17]. The manual procedures explained in Clause 6 (unit fall), Clause 7 (constant fall) and Clause 8 (variable or normal fall) for operational purposes need to be simplified by mathematical fitting and optimization for uncertainty estimation. Considering the procedure explained in Clause 7, first a reference fall is selected from amongst the most frequently observed falls.

[The procedure explained in Clause 7 is for the constant fall method. However, the unit fall method is a special case of the constant fall method in which the reference fall is assumed to be equal to unity (i.e. $h_c = 1$). The variable fall method is used for streams where backwater is intermittent, i.e. there are times when backwater is not present. The hydraulic conditions (and controls) are different for backwater affected measurements and which are not affected by backwater. Therefore, the evaluation of uncertainty in such complex cases should be carried out separately for back water affected measurements and measurements which are not influenced by back water conditions.]

A rating curve, between stage H and the reference discharge Q_c, is then fitted directly by estimating:

$$Q = Q_c \left[\frac{h}{h_c} \right]^p \tag{4}$$

where p is a power parameter which typically varies between 0,4 and 0,6.

The relationship between the reference discharge, Q_c, and the gauge height can be approximated by a power function, which can be represented by a straight line as shown in Figure 2 (in the logarithm scale). This power function can be expressed as

$$Q_c = \alpha \left(H - H_0 \right)^{\beta} \tag{5}$$

where

$(H - H_0)$	is the effective depth of the water on the control;
H	is the gauge height;
H_0	is the effective gauge height of zero flow;
β	is the slope of the rating curve when plotted on logarithm paper;
α	is a scale factor that is numerically equal to the discharge when the effective depth of flow $(H - H_0)$ is equal to 1.

Combining Formulae (4) and (5) and solving for Q:

$$Q = \alpha \left(H - H_0 \right)^{\beta} \left[\frac{h}{h_c} \right]^p \tag{6}$$

To empirically fit a stage-fall-discharge relation, a least squares regression is employed as explained below.

Fitting the stage-fall-discharge relationship:

The method of least squares is chosen for estimating the parameters of the Formula (6). The method is employed on the logarithms[8)] of discharge, treating the equation in a linear form as expressed in Formula (7).

$$\ln(Q) = \ln\alpha + \beta \ln\left(H - H_0 \right) + p \left(\ln h - \ln h_c \right) + \varepsilon_i \tag{7}$$

For convenience, the Formula (7) may be represented as (by substituting $Y_i = \ln(Q); X_{1i} = \ln\left(H - H_0 \right)$; $X_{2i} = (\ln h - \ln h_c)$; and $C = \ln\alpha$):

$$Y_i = C + \beta X_{1i} + p X_{2i} + \varepsilon_i \tag{8}$$

where ε_i is the random variable. Considering the least squares techniques as applied to logarithms, the objective is to minimize the sum of squared deviations between the logarithms of the estimated and observed values. Draper and Smith (1966)[10] provided a details description of least squares estimation procedure for the multiple linear regressions. Annex A summarizes various results of multiple least squares regression in matrix formation. Important results of least squares estimation relevant to uncertainty evaluation of Formula (7) are discussed below.

8) The logarithmic transformation tends to stabilize the variance when it is proportional to the square of the discharge, and the least square approach is applicable on the logarithms.

13.2.3.3 Standard error of estimate

The statistical analysis of the scatter of the measurements around the stage-fall-discharge rating provides an estimation of uncertainty in the relation. This consists of comparing measured discharge with discharge estimated from the stage-fall-discharge relationship at the corresponding gauge height and fall. The standard error of estimate (or standard deviation of residuals), S, characterizes the uncertainty of the stage-fall-discharge relation as a whole, which is estimated from the dispersion of stage-fall-discharge data around the rating using Formula (9) as shown below:

$$S = \left[\frac{\sum \left[\left(\ln(Q_\mathrm{e}) - \ln(Q) \right) \right]^2}{(N-P)} \right]^{0,5} \tag{9}$$

where

$\ln(Q)$ is the log of observed discharge value;

$\ln(Q_\mathrm{e})$ is the log of corresponding discharge estimated from the stage-fall-discharge rating;

N is the number of gaugings in the rating curve;

P is the number of rating-curve parameters estimated from the N gaugings.

The value of P depends on how many parameters are adjusted to fit the rating curve to the gauging data.

13.2.3.4 Confidence intervals for the mean response[9)]

To place confidence limits on $\ln(Q_e)$ where $Q_e = \underline{\boldsymbol{X}\beta}$, it is necessary to have an estimate for the variance of $\ln(Q_\mathrm{e})$. $\ln(Q_\mathrm{e})$ is an estimate of $\ln(Q)$ at the point $\underline{\boldsymbol{X}}_{H,h}$ (a 1×R vector) in R dimensional space. $\boldsymbol{\beta}$^ is a R×1 vector consisting of the estimate of $\boldsymbol{\beta}$. The variance of $\ln(Q_\mathrm{e})$ given by Draper and Smith 1966[10] is

$$u^2\left[\ln Q_\mathrm{e}(H,h)\right] = \sigma^2 \underline{\boldsymbol{X}_{H,h}} \left(\underline{\boldsymbol{X}}\, \underline{\boldsymbol{X}}' \right)^{-1} \underline{\boldsymbol{X}_{H,h}'} \tag{10}$$

where $\mathbf{X}_{H,h}$ corresponds to the data points of interest. The variance of the mean response changes with respect to $\underline{\boldsymbol{X}_{H,h}'}$, increases as $\underline{\boldsymbol{X}_{H,h}'}$ moves away from the multi-dimensional centre of the data. In fact term $\underline{\boldsymbol{X}_{H,h}} \left(\underline{\boldsymbol{X}}\, \underline{\boldsymbol{X}}' \right)^{-1} \underline{\boldsymbol{X}_{H,h}'}$ is the leverage statistic, expressing the distance of $\underline{\boldsymbol{X}_{H,h}'}$ from the centre of the data. The variance in Formula (10) can be estimated by replacing σ^2 with S^2. The uncertainty interval around the estimated value $\ln Q_\mathrm{e}(H,h)$, which is expected to contain specified fraction of values that could reasonably be attributed to the discharge and can be expressed as

$$\ln Q_\mathrm{e}(H,h) \pm U\left[\ln Q_\mathrm{e}(H,h)\right] \tag{11}$$

The uncertainties and their intervals are expressed in the natural logarithm scale. The corresponding uncertainty interval for estimated discharge is found by taking anti-logarithm as shown in Formula (12) below:

$$Q_\mathrm{e}(H,h) e^{\pm U\left[\ln Q_\mathrm{e}(H,h)\right]} \tag{12}$$

9) Various matrix notations in BOLD are explained in Annex A.

13.2.4 Uncertainty in the measured stage

The percentage (or relative) standard uncertainty in stage (or head) may be estimated from the following equation:

$$u_r\left(H-H_0\right)=\frac{\sqrt{u^2\left(H\right)+u^2\left(H_0\right)}}{\left(H-H_0\right)} \tag{13}$$

where

H–H_0 is the effective stage, in meters;

$u(H)$ is the standard uncertainty in the recorded value of stage in meters (the recommended values for the shaft encoders = ± 0,003 m and for chart recorders = ± 0,005 m);

$u(H_0)$ is the standard uncertainty in the gauge zero, in meters (the recommended value = ± 0,003 m).

13.2.5 Uncertainty in the measured fall

The percentage (or relative) standard uncertainty in fall may be estimated from Formula (14):

$$u_\mathrm{r}\left(h_\mathrm{r}\right)=u_\mathrm{r}\left[hu_\mathrm{r}\left(H_{\mathrm{u/s}}-H_{\mathrm{d/s}}\right)\right]=\frac{\sqrt{u^2\left(H_{\mathrm{u/s}}\right)+u^2\left(H_{\mathrm{d/s}}\right)}}{\left(H_{\mathrm{u/s}}-H_{\mathrm{d/s}}\right)} \tag{14}$$

where

$H_{\mathrm{u/s}}$–$H_{\mathrm{d/s}}$ is the effective fall, in meters;

$\underline{u}(H_{\mathrm{u/s}})$ is the standard uncertainty in the recorded value of the upstream stage in meters (the recommended values for the shaft encoders = ±0,003 m and chart recorders = ±0,005 m);

$uH_{\mathrm{d/s}})$ is the standard uncertainty in the downstream gauge, in meters (the recommended values for the shaft encoders = ±0,003 m and chart recorders = ±0,005 m).

13.2.6 Prediction intervals of estimated discharge

The estimated discharge is generally considered to be a prediction of the discharge that would be observed corresponding to observed gauge height and fall. The magnitude of difference between the predicted discharge and the discharge that would be observed provides a measure of uncertainty of prediction denoted by $u\left[\ln(Q_{\mathrm{pr}})\right]$. The standard uncertainty of prediction is computed by combining uncertainty in the estimated discharge with standard error of estimate as shown in Formula (15):

$$u^2\left[\ln(Q_{\mathrm{pr}})\right]=\sigma^2\left[\mathbf{1}+\underline{\boldsymbol{X}_{H,h}}\left(\underline{\boldsymbol{X}}\,\underline{\boldsymbol{X}}'\right)^{-1}\underline{\boldsymbol{X}_{H,h}{}'}\right] \tag{15}$$

13.2.7 Uncertainty caused by neglecting all other physical parameters

The evaluation of this component of uncertainty requires consideration of what causes the spread of gaugings about the estimated rating function (stage-fall-discharge relation). This spread is mainly caused by error in discharge observations, errors in stage and fall observations and errors due to neglecting all other physical parameters. Assuming that these sources of errors are independent, their combined effect may be measured by S^2 (estimated variance)[10]. This can be represented as shown below:

$$S^2 = u^2(Q) + \left(\frac{\partial Q}{\partial H}\right)^2 u^2(H) + \left(\frac{\partial Q}{\partial h}\right)^2 u^2(h) + u^2(\theta) \qquad (16)$$

where

$u^2(Q)$ is the uncertainty in the discharge measurements. The percentage uncertainty (at a 95 % confidence level) for the velocity area method using a current meter is 5 %;

NOTE The attainable uncertainties in a single discharge measurement for different measurement techniques are given by Herschy (1995, p. 486)[13].)

$u^2(H)$ is the uncertainty in the stage measurement;

$u^2(h)$ is the uncertainty in the fall measurement;

$u^2(\theta)$ is the uncertainty due to neglecting all other physical parameters;

$\left(\frac{\partial Q}{\partial H}\right)$ slope or sensitivity coefficient of the rating function at H;

$\left(\frac{\partial Q}{\partial h}\right)$ slope or sensitivity coefficient of the rating function at h.

The Formula (16) can be solved for $u^2(\theta)$ as shown in Formula (17).

$$u^2(\theta) = S^2 - u^2(Q) - \left(\frac{\partial Q}{\partial H}\right)^2 u^2(H) - \left(\frac{\partial Q}{\partial h}\right)^2 u^2(h) \qquad (17)$$

Formula (17) shows that $u^2(\theta)$ in unavoidably very sensitive to gauging error. Instances can be envisaged where a small positive error in estimating gauging error can lead to a negative value of $u^2(\theta)$. In such cases, $u^2(\theta)$ should be set to zero.

Based on the procedure explained above, an example is worked out in 13.3.

13.3 Example

13.3.1 General

The various steps in the illustrated example follow the same computational sequences as described in Clause 13 and its subsclauses. The stage-fall-discharge observations given in Table 2 are considered to fit a relationship as shown in Formulae (7) and (8), using the least squares method (natural logarithm scale). The example in Table 4 shows the step-by-step calculation of the overall uncertainty in the estimated discharge.

10) Procedure explained by Dymond and Christian (1982)[11] for a rating curve.

