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Calculation of load capacity of spur and helical gears — Application to high speed gears and gears of similar requirements

Calcul de la capacité de charge des engrenages cylindriques à dentures droite et hélicoïdale — Application aux engrenages grande vitesse et aux engrenages d'exigences similaires

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ISO copyright office
Case postale 56 • CH-1211 Geneva 20
Tel. + 41 22 749 01 11
Fax + 41 22 749 09 47
E-mail copyright@iso.ch
Web www.iso.ch

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 3.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this International Standard may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

International Standard ISO 9084 was prepared by Technical Committee ISO/TC 60, *Gears*, Subcommittee SC 2, *Gear capacity calculation*.

Annexes A and B form a normative part of this International Standard. Annex C is for information only.

Introduction

Procedures for the calculation of the load capacity of general spur and helical gears with respect to pitting and bending strength appear in ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5. This International Standard is derived from ISO 6336-1, ISO 6336-2 and ISO 6336-3 by the use of specific methods and assumptions which are considered to be applicable to industrial gears. Its application requires the use of allowable stresses and material requirements which are to be found in ISO 6336-5.

Calculation of load capacity of spur and helical gears — Application to high speed gears and gears of similar requirements

1 Scope

The formulae specified in this International Standard are intended to establish a uniformly acceptable method for calculating the pitting resistance and bending strength capacity of high speed gears and gears of similar requirements with straight or helical teeth.

The rating formulae in this International Standard are not applicable to other types of gear tooth deterioration, such as plastic yielding, micropitting, scuffing, case crushing, welding and wear, and are not applicable under vibratory conditions where there may be an unpredictable profile breakdown. The bending strength formulae are applicable to fractures at the tooth fillet, but are not applicable to fractures on the tooth working profile surfaces, failure of the gear rim, or failures of the gear blank through web and hub. This International Standard does not apply to teeth finished by forging or sintering. It is not applicable to gears which have a poor contact pattern.

This International Standard provides a method by which different gear designs can be compared. It is not intended to assure the performance of assembled drive gear systems. It is not intended for use by the general engineering public. Instead, it is intended for use by the experienced gear designer who is capable of selecting reasonable values for the factors in these formulae based on knowledge of similar designs and awareness of the effects of the items discussed.

CAUTION — The user is cautioned that the calculated results of this International Standard should be confirmed by experience.

2 Normative references

The following normative documents contain provisions which, through reference in this text, constitute provisions of this International Standard. For dated references, subsequent amendments to, or revisions of, any of these publications do not apply. However, parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the normative documents indicated below. For undated references, the latest edition of the normative document referred to applies. Members of ISO and IEC maintain registers of currently valid International Standards.

ISO 1122-1:1998, *Vocabulary of gear terms — Part 1: Definitions related to geometry*.

ISO 1328-1:1995, *Cylindrical gears — ISO system of accuracy — Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth*[1)].

ISO 6336-1:1996, *Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors*.

ISO 6336-2:1996, *Calculation of load capacity of spur and helical gears — Part 2: Calculation of surface durability (pitting)*.

ISO 6336-3:1996, *Calculation of load capacity of spur and helical gears — Part 3: Calculation of tooth bending strength*.

ISO 6336-5:1996, *Calculation of load capacity of spur and helical gears — Part 5: Strength and quality of materials*.

1) This was corrected and reprinted in 1997.

3 Terms and definitions

For the purposes of this International Standard, the terms and definitions given in ISO 1122-1 apply. For the symbols, see Table 1.

Table 1 — Symbols and abbreviations used in this International Standard

Symbol	Description or term	Unit
a	centre distance[a]	mm
b	facewidth	mm
b_B	facewidth of an individual helix of a double helical gear	mm
B	total facewidth of a double helical gear including gap width	mm
c_γ	mean value of mesh stiffness per unit facewidth	N/(mm · µm)
c'	maximum tooth stiffness of one pair of teeth per unit facewidth (single stiffness)	N/(mm · µm)
$d_{a1,2}$	tip diameter of pinion (or wheel)	mm
$d_{b1,2}$	base diameter of pinion (or wheel)	mm
$d_{f1,2}$	root diameter of pinion (or wheel)	mm
d_i	internal diameter of pinion shaft	mm
$d_{w1,2}$	pitch diameter of pinion (or wheel)	mm
$d_{1,2}$	reference diameter of pinion (or wheel)	mm
$f_{f\alpha}$	profile form deviation (the value for the total profile deviation F_α may be used alternatively for this, if tolerances complying with ISO 1328-1 are used)	µm
f_{ma}	mesh misalignment due to manufacturing deviations	µm
f_{pb}	transverse base pitch deviation (the values of f_{pt} may be used for the calculations in accordance with ISO 6336-1, using tolerances complying with ISO 1328-1)	µm
f_{sh}	helix deviation due to elastic deflections	µm
$f_{H\beta}$	tooth alignment deviation	µm
g_α	path length of contact	mm
h	tooth depth	mm
h_{aP}	addendum of basic rack of cylindrical gear	mm
h_{fP}	dedendum of basic rack of cylindrical gear	mm
h_{Fe}	bending moment arm for load application at the outer point of single pair tooth contact	mm
l	bearing span	mm
m^*	relative individual gear mass per unit facewidth referenced to line of action	kg/mm
m_n	normal module	mm
m_{red}	reduced gear pair mass per unit facewidth referenced to line of action	kg/mm
m_t	transverse module	mm
$n_{1,2}$	rotation speed of pinion (or wheel)	min^{-1}
n_{E1}	resonance speed of pinion	min^{-1}
pr	protuberance of the tool	mm
p_{bn}	normal base pitch	mm
p_{bt}	transverse base pitch	mm
q	finishing stock allowance	mm
q_s	notch parameter	—

Table 1 — Symbols and abbreviations used in this International Standard (*continued*)

Symbol	Description or term	Unit
s_{pr}	residual fillet undercut	mm
s_{Fn}	tooth-root chord at the critical section	mm
s_R	rim thickness	mm
u	gear ratio[a] $\|u\| = \|z_2/z_1\| \geqslant 1$	—
v	circumferential velocity (without subscript: at reference circle ≈ circumferential velocity at working pitch circle)	m/s
$x_{1,2}$	rack shift coefficient of pinion (or wheel)	—
y_β	running-in allowance (equivalent misalignment)	µm
z_n	virtual number of teeth of a helical gear	—
$z_{1,2}$	number of teeth of pinion (or wheel)[a]	—
C_a	tip relief	µm
C_B	basic rack factor	—
C_R	gear blank factor	—
E	modulus of elasticity, Young's modulus	N/mm²
F_m	mean transverse load at the reference cylinder $(= F_t\, K_A\, K_v)$	N
F_t	(nominal) transverse tangential load at reference cylinder	N
$F_{t\,eq}$	equivalent tangential load at reference cylinder	N
F_β	total helix deviation	µm
$F_{\beta x}$	initial equivalent misalignment (before running-in)	µm
J^*	moment of inertia per unit facewidth	kg·mm²/mm
K_v	dynamic factor	—
K_A	application factor	—
$K_{F\alpha}$	transverse load factor (tooth-root stress)	—
$K_{F\beta}$	face load factor (tooth-root stress)	—
$K_{H\alpha}$	transverse load factor (contact stress)	—
$K_{H\beta}$	face load factor (contact stress)	—
K_γ	mesh load factor (takes into account the uneven distribution of the load between meshes for multiple transmission paths)	—
$M_{1,2}$	auxiliary values for the determination of $Z_{B,D}$	—
N	resonance ratio	—
N_L	number of cycles	—
P	transmitted power	kW
Ra	arithmetic mean roughness value (as specified in ISO 4287)	µm
Rz	mean peak-to-valley roughness (as specified in ISO 4287)	µm
S_F	safety factor (tooth breakage)	—
$S_{F\,min}$	minimum safety factor (tooth breakage)	—
S_H	safety factor (pitting)	—
$S_{H\,min}$	minimum safety factor (pitting)	—
$T_{1,2}$	nominal torque at the pinion (or wheel)	Nm
Y_F	form factor, for the influence on nominal tooth-root stress with load applied at the outer point of single pair tooth contact	—

Table 1 — Symbols and abbreviations used in this International Standard (*continued*)

Symbol	Description or term	Unit
$Y_{\mathrm{R\,rel\,T}}$	relative surface factor	—
Y_{S}	stress correction factor	—
Y_{X}	size factor (tooth-root)	—
Y_{β}	helix angle factor (tooth-root)	—
$Y_{\delta\,\mathrm{rel\,T}}$	relative notch sensitivity factor	—
Z_{v}	speed factor	—
$Z_{\mathrm{B,D}}$	single pair tooth contact factors for the pinion (or wheel)	—
Z_{E}	elasticity factor	$\sqrt{\mathrm{N/mm^2}}$
Z_{H}	zone factor	—
Z_{L}	lubricant factor	—
Z_{R}	roughness factor (pitting)	—
Z_{W}	work-hardening factor	—
Z_{X}	size factor (pitting)	—
Z_{β}	helix angle factor (pitting)	—
Z_{ϵ}	contact ratio factor (pitting)	—
α_{n}	normal pressure angle	°
α_{t}	transverse pressure angle	°
α_{wt}	transverse pressure angle at the pitch cylinder	°
α_{P}	pressure angle of the basic rack for cylindrical gears	°
β	helix angle (without subscript — at the reference cylinder)	°
β_{b}	base helix angle	°
ϵ_{α}	transverse contact ratio	—
$\epsilon_{\alpha\mathrm{n}}$	transverse contact ratio of virtual spur gear pairs	—
ϵ_{β}	axial overlap ratio	—
ϵ_{γ}	total contact ratio $(\epsilon_{\gamma} = \epsilon_{\alpha} + \epsilon_{\beta})$	—
κ_{β}	running-in factor (equivalent misalignment)	—
ρ_{fP}	root fillet radius of the basic rack for cylindrical gears	mm
ρ_{F}	tooth-root fillet radius at the critical section	mm
σ_{F}	tooth-root stress	$\mathrm{N/mm^2}$
$\sigma_{\mathrm{F\,lim}}$	nominal stress number (bending)	$\mathrm{N/mm^2}$
σ_{FE}	allowable stress number (bending)	$\mathrm{N/mm^2}$
σ_{FG}	tooth-root stress limit	$\mathrm{N/mm^2}$
σ_{FP}	permissible bending stress	$\mathrm{N/mm^2}$
σ_{F0}	nominal contact stress	$\mathrm{N/mm^2}$
σ_{H}	calculated contact stress	$\mathrm{N/mm^2}$
$\sigma_{\mathrm{H\,lim}}$	allowable stress number (contact)	$\mathrm{N/mm^2}$
σ_{HG}	modified allowable stress number $(= \sigma_{\mathrm{HP}}\ S_{\mathrm{H\,min}})$	$\mathrm{N/mm^2}$
σ_{HP}	permissible contact stress	$\mathrm{N/mm^2}$
$\omega_{1,2}$	angular velocity of pinion (or wheel)	rad/s

[a] For external gear pairs a, u, z_1 and z_2 are positive; for internal gear pairs a, u and z_2 are negative with z_1 positive.

4 Application

4.1 Design, specific applications

4.1.1 General

Gear designers shall recognize that requirements for different applications vary considerably. Use of the procedures of this International Standard for specific applications demands a careful appraisal of all applicable considerations, in particular:

— the allowable stress of the material and the number of load repetitions;

— the consequences of any percentage of failure (failure rate);

— the appropriate safety factor.

Design considerations to prevent fractures emanating from stress raisers in the tooth flank, tip chipping and failures of the gear blank through the web or hub should be analysed by general machine design methods.

Any variances according to the following shall be reported in the calculation statement.

a) If a more refined method of calculation is desired or if compliance with the restrictions given in 4.1 is for any reason impractical, relevant factors may be evaluated according to the basic standard or another application standard.

b) Factors derived from reliable experience or test data may be used instead of individual factors according to this International Standard. Concerning this, the criteria for Method A in ISO 6336-1:1996, 4.1.8.1, are applicable.

In other respects, rating calculations shall be strictly in accordance with this International Standard if stresses, safety factors, etc. are to be classified as being in accordance with this International Standard.

This International Standard recognizes all high speed gears and gears of similar requirements besides high speed, and special purpose gear units used in petroleum, chemical and gas industries. For these ISO 13691 may apply.

This International Standard is applicable when the wheel blank, shaft/hub connections, shafts, bearings, housings, threaded connections, foundations and couplings conform to the requirements regarding accuracy, load capacity and stiffness which form the basis for the calculation of the load capacity of gears.

Although the method described in this International Standard is mainly intended for recalculation purposes, by means of iteration it can also be used to determine the load capacities of gears. The iteration is accomplished by selecting a load and calculating the corresponding safety factor against pitting, S_{H1}, for the pinion. If S_{H1} is greater than $S_{\mathrm{H\,min}}$ the load is increased, if it is smaller than $S_{\mathrm{H\,min}}$ the load is reduced. This is done until the load chosen corresponds to $S_{\mathrm{H1}} = S_{\mathrm{H\,min}}$. The same method is used for the wheel ($S_{\mathrm{H2}} = S_{\mathrm{H\,min}}$) and also for the safety factors against tooth breakage, $S_{\mathrm{F1}} = S_{\mathrm{F2}} = S_{\mathrm{F\,min}}$.

4.1.2 Gear data

This International Standard is applicable within the following constraints.

a) Types of gear

— external and internal, involute spur, helical and double helical gears;

— for double helical gears, it is assumed that the total tangential load is evenly distributed between the two helices; if this is not the case (e.g. due to externally applied axial forces), this shall be taken into account; the two helices are treated as two single helical gears in parallel;

— planetary and other gear trains with multiple transmission paths.

b) Range of speeds

n_1 more than or equal to 3 600 min^{-1} (synchronous speed of two-pole motor at 60 Hz current frequency); it is also applicable for gears of high accuracy needed for special requirements at lower speeds.

c) Gear accuracy

accuracy grade 6 or better according to ISO 1328-1 (affects K_v, $K_{\text{H}\alpha}$, $K_{\text{H}\beta}$ and $K_{\text{F}\beta}$).

d) Range of the transverse contact ratios of virtual spur gear pairs

1,2 $<\epsilon_\alpha<$ 2,5 (affects c', c_γ, K_v, $K_{\text{H}\beta}$, $K_{\text{F}\alpha}$, $K_{\text{H}\alpha}$ and $K_{\text{F}\beta}$).

e) Range of helix angles

β less than or equal to 30° (affects c', c_γ, K_v, $K_{\text{H}\beta}$ and $K_{\text{F}\beta}$).

f) Basic racks

no restriction[2)], but see d).

4.1.3 Pinion and pinion shaft

This International Standard is applicable to pinions integral with shafts or bored pinions mounted symmetrically between their bearings. It is assumed that the bored pinions will be mounted on solid shafts or on hollow shafts with $d_\text{i}/d_\text{shi}<$ 0,5 (this affects $K_{\text{H}\beta}$ and $K_{\text{F}\beta}$).

4.1.4 Wheel blank, wheel rim

This International Standard is applicable when s_R, the thickness of the wheel rim under the tooth roots of internal and external gears, is $>$ 3,5 m_n.

4.1.5 Materials

These include steel materials (affects Z_E, $\sigma_{\text{H lim}}$, σ_FE, K_v, $K_{\text{H}\beta}$, $K_{\text{F}\beta}$, $K_{\text{H}\alpha}$ and $K_{\text{F}\alpha}$). For materials and their abbreviations used in this International Standard, see Table 2. For information on other materials, see ISO 6336-1, ISO 6336-2, ISO 6336-3 and ISO 6336-5.

Table 2 — Materials

Material	Abbreviation
Through-hardening steel, alloy or carbon, through hardened ($\sigma_\text{B}\geqslant$ 800 N/mm^2)	V
Case-hardened steel, case hardened	Eh
Steel, flame- or induction-hardened	IF
Nitriding steel, nitrided	NT (nitr.)
Through-hardening and case-hardening steel, nitrided	NV (nitr.)
Through-hardening and case-hardening steel, nitrocarburized	NV (nitrocar.)

4.1.6 Lubrication

The calculation procedures are valid subject to the condition that the gears are spray lubricated at all times of operation with a lubricant approved by the manufacturer/designer of the gears and the lubricant is sprayed at a tem-

2) For all practical purposes, it may be assumed that the proportions of the basic rack of the tool are equal to those of the basic rack of the gear.

perature and rate which ensures that temperatures assumed for purposes of calculations are not exceeded (affects lubricant film formation, i.e. factors Z_L, Z_v and Z_R).

4.2 Safety factors

It is necessary to distinguish between the safety factor relative to pitting, S_H, and the safety factor relative to tooth breakage, S_F.

For a given application, adequate gear load capacity is demonstrated by the computed values of S_H and S_F being equal to or greater than the values $S_{H\,min}$ and $S_{F\,min}$, respectively.

Choice of the value of a safety factor should be based on the degree of confidence in the reliability of the available data and the consequences of possible failures.

Important factors to be considered are the following:

— the allowable stress numbers used in the calculation are valid for a given probability of failure (the material values in ISO 6336-5 are valid for 1 % probability of damage);

— the specified quality and the effectiveness of quality control at all stages of manufacture;

— the accuracy of specification of the service duty and external conditions;

— tooth breakage is often considered to be a greater hazard than pitting.

Therefore, the chosen value for $S_{F\,min}$ should to be greater than the value chosen for $S_{H\,min}$. It is recommended that the minimum values of the safety factors should be agreed upon between the purchaser and the manufacturer.

For calculation of the actual safety factor, see 6.1.5 (S_H, pitting) and 7.1.4 (S_F, tooth breakage).

4.3 Input data

The following data shall be available for the calculations:

a) gear data:

a, z_1, z_2, m_n, d_1, d_{a1}, d_{a2}[3], b, x_1, x_2, α_n, β, ϵ_α, ϵ_β, basic rack profile;

b) design and manufacturing data:

C_{a1}, C_{a2}, Ra_1, Ra_2, Rz_1, Rz_2;

materials, material hardnesses and heat treatment details; material quality grades, gear accuracy grades, bearing span, gear dimensions, polar or mass moments of inertia of pinion and wheel and when applicable, profile and helix modification;

c) operating data:

P or T or F_t, n_1, v_1, working characteristics of driving and driven machines.

Requisite geometrical data can be calculated according to national standards.

Information to be exchanged between the manufacturer and purchaser should include data specifying material preferences, lubrication, safety factor and externally applied forces due to vibrations and overloads (application factor).

3) When tooth tips are chamfered or rounded, substitute $d_{N1,2}$ for $d_{a1,2}$.

4.4 Numerical equations

The units listed in Clause 3 shall be used in all calculations. Information which will facilitate the use of this International Standard is provided in annex C of ISO 6336-1:1996.

5 Influence factors

5.1 General

The influence factors K_{v}, $K_{\mathrm{H}\alpha}$, $K_{\mathrm{H}\beta}$, $K_{\mathrm{F}\alpha}$ and $K_{\mathrm{F}\beta}$ are all dependent on the tooth load. Initially this is the applied load (nominal tangential load multiplied by the application factor).

These factors are also interdependent and shall therefore be calculated successively as follows:

a) K_{v} with the applied tangential load $F_{\mathrm{t}}K_{\mathrm{A}}$ (equivalent load, multiple mesh trains with $F_{\mathrm{t}}K_{\mathrm{A}}K_{\gamma}$ [4]);

b) $K_{\mathrm{H}\beta}$ or $K_{\mathrm{F}\beta}$ with the recalculated load $F_{\mathrm{t}}K_{\mathrm{A}}K_{\mathrm{v}}$.

5.2 Nominal tangential load, F_{t}, nominal torque, T, nominal power, P

The nominal tangential load, F_{t}, is determined in the transverse plane at the reference cylinder. It is based on the input torque to the driven machine. This is the torque corresponding to the heaviest regular working condition. Alternatively, the nominal torque of the prime mover can be used as a basis if it corresponds to the torque requirement of the driven machine, or some other suitable basis can be chosen.

$$F_{\mathrm{t}} = \frac{2\,000\,T_{1,2}}{d_{1,2}} = \frac{19\,098 \times 1\,000 P}{d_{1,2} n_{1,2}} = \frac{1\,000 P}{v} \qquad (1)$$

$$T_{1,2} = \frac{F_{\mathrm{t}} d_{1,2}}{2\,000} = \frac{1\,000 P}{\omega_{1,2}} = \frac{9\,549 P}{n_{1,2}} \qquad (2)$$

$$P = \frac{F_{\mathrm{t}} v}{1\,000} = \frac{T_{1,2}\omega_{1,2}}{1\,000} = \frac{T_{1,2} n_{1,2}}{9\,549} \qquad (3)$$

$$v = \frac{d_{1,2}\omega_{1,2}}{2\,000} = \frac{d_{1,2} n_{1,2}}{19\,098} \qquad (4)$$

$$\omega_{1,2} = \frac{\pi n_{1,2}}{30} = \frac{2\,000 v}{d_{1,2}} = \frac{n_{1,2}}{9\,549} \qquad (5)$$

5.3 Non-uniform load, non-uniform torque, non-uniform power

When the transmitted load is not uniform, consideration should be given not only to the peak load and its anticipated number of cycles, but also to intermediate loads and their numbers of cycles. This type of load is classed as a *duty cycle* and may be represented by a load spectrum. In such cases, the cumulative fatigue effect of the duty cycle is considered in rating the gearset. A method of calculating the effect of the loads under this condition is given in ISO/TR 10495.

4) The total tangential load in the case of gear trains with multiple transmission paths, planetary gear systems, or split-path gear trains is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This is to be taken into consideration by inserting a distribution factor K_{γ} to follow K_{A}, as appropriate, to adjust the average tangential load per mesh as necessary.

5.4 Maximum tangential load, $F_{t\,max}$, maximum torque, T_{max}, maximum power, P_{max}

This is the maximum tangential load $F_{t\,max}$ (or corresponding torque T_{max}, corresponding power P_{max}) in the variable duty range. Its magnitude can be limited by a suitably responsive safety clutch. $F_{t\,max}$, T_{max} and P_{max} shall be known when safety from pitting damage and from sudden tooth breakage due to loading corresponding to the static stress limit is to be determined (see 5.5).

5.5 Application factor, K_A

5.5.1 General

The factor K_A adjusts the nominal load F_t, in order to compensate for incremental gear loads from external sources. These additional forces are largely dependent on the characteristics of the driving and driven machines, as well as the masses and stiffness of the system, including shafts and couplings used in service.

It is recommended that the purchaser and manufacturer/designer agree on the value of the application factor.

5.5.2 Method A — Factor $K_{A\text{-}A}$

K_A is determined in this method by means of careful measurements and a comprehensive analysis of the system, or on the basis of reliable operational experience in the field of application concerned (see 5.3).

5.5.3 Method B — Factor $K_{A\text{-}B}$

If no reliable data, obtained as described in 5.5.2, are available, or even as early as the first design phase, it is possible to use the guideline values for K_A as described in annex C with a minimum safety factor of 1,25.

5.6 Internal dynamic factor, K_v

5.6.1 General

The dynamic factor relates the total tooth load, including internal dynamic effects of a "multi-resonance" system, to the transmitted tangential tooth load.

Method B of ISO 6336-1:1996 is used in this International Standard.

In this procedure it is assumed that the gear pair consists of an elementary single mass and spring system comprising the combined masses of pinion and wheel, and the mesh stiffness of the contacting teeth. It is also assumed that each gear pair functions as a single stage pair, i.e. the influence of other stages in a multiple-stage gear system is ignored. This assumption is only tenable when the torsional stiffness (measured at the base radius of the gears), of the shaft common to a wheel and a pinion is less than the mesh stiffness. See 5.6.3 and clause B.1 for the procedure dealing with very stiff shafts.

Forces caused by torsional vibrations of the shafts and coupled masses are not covered by K_v. These forces should be included with other externally applied forces (e.g. with the application factor).

In multiple mesh gear trains there are several natural frequencies. These can be higher or lower than the natural frequency of a single gear pair which has only one mesh. When such gears run in the supercritical range, analysis by Method A is recommended. See ISO 6336-1:1996, 6.3.1.

The specific loading for the calculation of K_v is $(F_t K_A/b)$ or alternatively $F_{t\,eq}/b$.

If $(F_t K_A)/b > 100$ N/mm, then $F_m/b = (F_t K_A)/b$.

If $(F_t K_A)/b \leqslant 100$ N/mm, then $F_m/b = 100$ N/mm.

Similarly for $F_{\text{t eq}}/b$.

When the specific loading $(F_{\text{t}}K_{\text{A}})/b < 50$ N/mm, a particular risk of vibration exists (under some circumstances, with separation of working tooth flanks), above all for spur or helical gears of coarse quality grade running at high speed.

5.6.2 Calculation of the parameters required for evaluation of K_{v}

5.6.2.1 Calculation of the relative mass, m_{red}

a) Calculation of the relative mass m_{red} of a single-stage gear pair

$$m_{\text{red}} = \frac{m_1^* m_2^*}{m_1^* + m_2^*} = \frac{J_1^* J_2^*}{J_1^* r_{\text{b2}}^2 + J_2^* r_{\text{b1}}^2} \quad \text{with } m_{1,2}^* = \frac{J_{1,2}^*}{r_{\text{b1,2}}^2} \tag{6}$$

where

m_{red} is the relative mass of a gear pair, i.e. of the mass per unit facewidth of each gear, referred to its base radius or to the line of action;

$J_{1,2}^*$ are the polar moments of inertia per unit face width;

$r_{\text{b1,2}}$ are the base radii $(= 0{,}5\, d_{\text{b1,2}})$.

b) Calculation of relative mass, m_{red}, of a multistage gear pair

See clause B.1.

c) Calculations of relative mass, m_{red}, of gears of less common designs

For information on the following cases, see clause B.1:

— pinion shaft with diameter at mid-tooth depth, d_{m1}, about equal to the shaft diameter;

— two rigidly connected, coaxial gears;

— one large wheel driven by two pinions;

— planetary gears;

— idler gears.

5.6.2.2 Determination of the resonance running speed (main resonance) of a gear pair

a) Resonance running speed, n_{E1}, of the pinion, in reciprocal minutes:

$$n_{\text{E1}} = \frac{30 \times 10^3}{\pi z_1} \sqrt{\frac{c_\gamma}{m_{\text{red}}}} \quad \text{min}^{-1} \tag{7}$$

with c_γ from annex A.

b) Resonance ratio, N

The ratio of pinion speed to resonance speed, the resonance ratio, N, is determined as follows:

$$N = \frac{n_1}{n_{E1}} = \frac{n_1 \pi z_1}{30\,000} \sqrt{\frac{m_{red}}{c_\gamma}} \qquad (8)$$

The resonance running speed may be above or below the running speed calculated from equation (8) because of stiffnesses which have not been included, e.g. the stiffnesses of shafts, bearings, housings, etc. and as a result of damping. For reasons of safety, the resonance range is defined by the following:

$$N_S < N \leqslant 1{,}15 \qquad (9)$$

At loads such that $(F_t K_A)/b$ is less than 100 N/mm, the lower limit of resonance ratio N_S is determined as follows:

— if $(F_t K_A)/b < 100$ N/mm, then

$$N_S = 0{,}5 + 0{,}35 \sqrt{\frac{F_t K_A}{b \cdot 100}} \qquad (10)$$

— if $(F_t K_A)/b \geqslant 1\,000$ N/mm, then

$$N_S = 0{,}85 \qquad (11)$$

5.6.2.3 Gear accuracy and running-in parameters, B_p, B_f, B_k

B_p, B_f and B_k are non-dimensional parameters used to take into account the effect of tooth deviations and profile modifications on the dynamic load:[5)]

$$B_p = \frac{c' f_{pb\,eff}}{F_t K_A / b} \qquad (12)$$

$$B_f = \frac{c' f_{f\,eff}}{F_t K_A / b} \qquad (13)$$

$$B_k = \left| 1 - \frac{c' C_a}{F_t K_A / b} \right| \qquad (14)$$

with

c' from annex A;

C_a $C_a = C_{ay}$ from Table 3 for gears without a specified profile modification.

The effective base pitch and profile deviations are those which are present after running-in. The values of $f_{pb\,eff}$ and $f_{f\,eff}$ are determined by deducting the estimated running-in allowances y_p and y_f as follows:

$$f_{pb\,eff} = f_{pb\,1} - y_{p\,1} \text{ or } f_{pb\,eff} = f_{pb\,2} - y_{p\,2} \qquad (15)$$

5) The amount C_a of tip relief may only be allowed for gears of quality grades in the range 0 to 6 as specified in ISO 1328-1.

whichever is the greater;

$$f_{\text{f eff}} = f_{\text{f}\alpha 1} - y_{\text{f 1}} \text{ or } f_{\text{f eff}} = f_{\text{f}\alpha 2} - y_{\text{f 2}} \tag{16}$$

whichever is the greater.

5.6.2.4 Running-in allowances, y_{p}, y_{f}

a) For V[6)]

$$y_{\text{p}} = \frac{160}{\sigma_{\text{H lim}}} f_{\text{pb}} \tag{17}$$

$$y_{\text{f}} = \frac{160}{\sigma_{\text{H lim}}} f_{\text{f}\alpha} \tag{18}$$

b) For Eh, IF, NT (nitr.), NV (nitr.) and NV (nitrocar.)[6)]

$$y_{\text{p}} = 0{,}075\, f_{\text{pb}} \tag{19}$$

$$y_{\text{f}} = 0{,}075\, f_{\text{f}\alpha} \tag{20}$$

c) When the materials differ, $y_{\text{p1,f1}}$ should be determined for the pinion material and $y_{\text{p2,f2}}$ for the wheel material. The average value is used for the calculation:

$$y_{\text{p,f}} = 0{,}5\,(y_{\text{p1,f1}} - y_{\text{p2,f2}}) \tag{21}$$

5.6.3 Dynamic factor in the subcritical range ($N \leqslant N_{\text{S}}$)

In this sector resonances may exist if the tooth mesh frequency coincides with $N = 1/2$ and $N = 1/3$. The risk of this is slight in the case of precision helical or spur gears, if the latter have suitable profile modification (gears as specified in ISO 1328-1:1995, accuracy grade 6 or better).

When the contact ratio of straight spur gears is small or if the quality is of low grade, K_{v} can be just as great as in the main resonance-speed range. If this occurs, the design or operating parameters should be altered.

Resonances at $N = 1/4$, 1/5, ... are seldom troublesome because the associated vibration amplitudes are usually small.

For gear pairs where the stiffnesses of the driving and driven shafts are not equal, in the range $N \approx 0{,}2 \ldots 0{,}5$, the tooth contact frequency can excite natural frequencies when the torsional stiffness c of the stiffer shaft, referred to the line of action, is of the same order of magnitude as the tooth stiffness; i.e. if c/r_{b}^2 is of the order of magnitude of c_γ. When this is so, dynamic load increments can exceed values calculated using equation (22):

$$K_{\text{v}} = (NK) + 1 \tag{22}$$

$$K = (C_{\text{v1}}\, B_{\text{p}}) + (C_{\text{v2}}\, B_{\text{f}}) + (C_{\text{v3}}\, B_{\text{k}}) \tag{23}$$

where C_{v1} and C_{v2} allow for pitch and profile deviations, while C_{v3} allows for the cyclic variation of mesh stiffness. See Table 3.

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6) See Table 2 for an explanation of the abbreviations used.

5.6.4 Dynamic factor in the main resonance range ($N_S < N \leqslant 1{,}15$)

High quality helical gears with high total contact ratios can function satisfactorily in this sector. This also applies to spur gears of grade 6 or better as specified in ISO 1328-1:1995 which have suitably modified profiles.

Subject to the above, this factor is equal to:

$$K_v = (C_{v1}\, B_p) + (C_{v2}\, B_f) + (C_{v4}\, B_k) + 1 \qquad (24)$$

For C parameters, refer to Table 3.

5.6.5 Dynamic factor in the supercritical range ($N \geqslant 1{,}5$)

Resonance peaks can occur at $N = 2,\ 3,\ \ldots$ in this range. However, in the majority of cases, vibration amplitudes are small, since excitation forces with frequencies lower than meshing frequency are usually small.

For gears in the supercritical range it is also necessary to consider possible dynamic loads due to transverse vibrations of the gear and shaft assemblies. When the critical transverse vibration frequency is near to the rotation frequency, and if this condition cannot be avoided, such loads shall be taken into account:

$$K_v = (C_{v5}\, B_p) + (C_{v6}\, B_f) + C_{v7} \qquad (25)$$

For C parameters refer to Table 3, and for c' refer to annex A.

Table 3 — Equations for the calculation of factors C_{v1} to C_{v7} and C_{ay}

	$1 < \epsilon_\gamma \leqslant 2$	$\epsilon_\gamma > 2$	
C_{v1}	0,32	0,32	
C_{v2}	0,34	$\dfrac{0{,}57}{\epsilon_\gamma - 0{,}3}$	
C_{v3}	0,23	$\dfrac{0{,}096}{\epsilon_\gamma - 1{,}56}$	
C_{v4}	0,90	$\dfrac{0{,}57 - 0{,}05\,\epsilon_\gamma}{\epsilon_\gamma - 1{,}44}$	
C_{v5}	0,47	0,47	
C_{v6}	0,47	$\dfrac{0{,}12}{\epsilon_\gamma - 1{,}74}$	
	$1 < \epsilon_\gamma \leqslant 1{,}5$	$1{,}5 < \epsilon_\gamma \leqslant 2{,}5$	$\epsilon_\gamma > 2{,}5$
C_{v7}	0,75	$0{,}125 \sin\left[\pi\,(\epsilon_\gamma - 2)\right] + 0{,}875$	1,0

$$C_{ay} = \frac{1}{18}\left(\frac{\sigma_{H\,lim}}{97} - 18{,}45\right)^2 + 1{,}5$$

NOTE When the material of the pinion (1) is different from that of the wheel (2), C_{ay1} and C_{ay2} are calculated separately: then $C_{ay} = 0{,}5\,(C_{ay1} + C_{ay2})$. A value C_{ay} resulting from running-in is substituted for C_a in equation (14) in the case of gears without a specified profile modification.

5.6.6 Dynamic factor in the intermediate range ($1{,}15 < N < 1{,}5$)

In this range, the dynamic factor is determined by linear interpolation between K_v at $N = 1{,}15$ and at $N = 1{,}5$ as specified in 5.6.4 and 5.6.5.

$$K_v = K_{v(N=1{,}5)} + \frac{K_{v(N=1{,}15)} - K_{v(N=1{,}5)}}{0{,}35} (1{,}5 - N) \tag{26}$$

See annex A for single tooth stiffness c'.

5.7 Face load factor, $K_{H\beta}$

5.7.1 General

The face load factor adjusts gear tooth stresses, to allow for the effects of uneven load distribution over the facewidth.

Method C1 of ISO 6336-1:1996 is used in this International Standard.

The use of method C1 is appropriate for gears having the following characteristics:

a) pinion on solid or hollow shaft, $d_{shi}/d_{sh} < 0{,}5$ positioned symmetrically between bearings (an asymmetrically positioned pinion leads to an additional bending deformation which shall be evaluated and added to f_{ma}, or additionally compensated in the full helix modification);

b) pinion diameter about equal to shaft diameter;

c) stiff wheel and case, stiff wheel shaft, stiff bearings;

d) the contact pattern, under load, extends over the entire facewidth;

e) no additional external loads act on the pinion shaft (e.g. from shaft couplings);

f) running-in allowance $y_\beta \leqslant y_{\beta\,max}$ as specified in 5.7.2.2; a computed value for $F_{\beta x}$ may be verified using the equation:

$$F_{\beta x} = \frac{K_{H\beta} - 1}{\chi_\beta \left(\dfrac{c_\gamma/2}{F_m/b} \right)} \tag{27}$$

g) it is recommended that the values used for f_{ma} be verified by inspection checks, such as the contact pattern in the working progress.

Refer to clause B.2 for application to planetary gears.

5.7.2 Values required for the calculations

5.7.2.1 Assumed mesh misalignment based on manufacturing tolerances f_{ma}

f_{ma} is the maximum separation between the tooth flank of the meshing teeth of mating gears. For a pair of gears, the larger of the values $f_{H\beta}$ of a gear pair shall be substituted in equations (28) to (30).

a) Assembly of gears without any modification or adjustment:

$$f_{ma} = 1{,}0\, f_{H\beta} \tag{28}$$

b) Gear pairs with provision for adjustment (lapping or running-in under light load, adjustable bearings or appropriate helix modification) and gear pairs suitably crowned:

$$f_{\text{ma}} = 0{,}5\ f_{\text{H}\beta} \tag{29}$$

c) Gear pairs with well-designed end relief:

$$f_{\text{ma}} = 0{,}7\ f_{\text{H}\beta} \tag{30}$$

5.7.2.2 Running-in allowance y_β, running-in factor κ_β

y_β is the amount by which the initial equivalent misalignment is reduced by running-in after start of operation. κ_β is the factor characterizing the equivalent misalignment after running-in. The use of κ_β in calculations is valid only as long as y_β is proportional to $F_{\beta\text{x}}$.

a) For V:

$$y_\beta = \frac{320}{\sigma_{\text{H lim}}}\ F_{\beta\text{x}};\ \ \kappa_\beta = 1 - \frac{320}{\sigma_{\text{H lim}}} \tag{31}$$

when $v \leqslant 5$ m/s: no restriction for $F_{\beta\text{x}}$

when 5 m/s $< v \leqslant 10$ m/s: $F_{\beta\text{x}} \leqslant 80$ µm

when $v > 10$ m/s: $F_{\beta\text{x}} \leqslant 40$ µm

b) For Eh, IF, NT (nitr.), NV (nitr.) and NV (nitrocar.):

$$y_\beta = 0{,}15\ F_{\beta\text{x}};\ \ \kappa_\beta = 0{,}85 \tag{32}$$

$F_{\beta\text{x}}$ max. $= 40$ µm

When the material of the pinion differs from that of the wheel, $\kappa_{\beta 1}$ and $\kappa_{\beta 2}$ shall be determined separately for each material.

$$y_\beta = \left(y_{\beta 1} + y_{\beta 2}\right)/2;\ \ \kappa_\beta = \left(\kappa_{\beta 1} + \kappa_{\beta 2}\right)/2 \tag{33}$$

5.7.3 Determination of the face load factor $K_{\text{H}\beta}$

5.7.3.1 Gears with unmodified helices

In the following equations and those in 5.7.3.2, use

κ_β from 5.7.2.2;

c_γ from annex A;

f_{ma} from 5.7.2.1.

a) Spur and single helical gears[7)]

$$K_{\text{H}\beta} = 1 + \frac{4\,000}{3\pi}\,\kappa_\beta\,\frac{c_\gamma}{E}\left(\frac{b}{d_1}\right)^2\left[5{,}12 + \left(\frac{b}{d_1}\right)^2\left(\frac{l}{b} - \frac{7}{12}\right)\right] + \frac{\kappa_\beta c_\gamma f_{\text{ma}}}{2F_{\text{m}}/b} \tag{34}$$

7) It is assumed that the entire torque input is at one shaft end. If the torque input is at both shaft ends or in between helices of a double helical gear, a more accurate analysis is necessary.

b) Double helical gears[7), 8)]

$$K_{\mathrm{H}\beta} = 1 + \frac{4\,000}{3\pi}\,\kappa_\beta\,\frac{c_\gamma}{E}\left[3{,}2\left(\frac{2b_{\mathrm{B}}}{d_1}\right)^2 + \left(\frac{B}{d_1}\right)^4\left(\frac{l}{B} - \frac{7}{12}\right)\right] + \frac{\kappa_\beta c_\gamma f_{\mathrm{ma}}}{F_{\mathrm{m}}/b_{\mathrm{B}}} \qquad (35)$$

5.7.3.2 Gears with modified helices

a) Spur and single helical gears[7)]

— With partial helix modification[9)] (torsional deformation only compensated for);

$$K_{\mathrm{H}\beta} = 1 + \frac{4\,000}{3\pi}\,\kappa_\beta\,\frac{c_\gamma}{E}\left(\frac{b}{d_1}\right)^4\left(\frac{l}{b} - \frac{7}{12}\right) + \frac{\kappa_\beta c_\gamma f_{\mathrm{ma}}}{2F_{\mathrm{m}}/b} \qquad (36)$$

— With full helix modification (torsional and bending deflection compensated):

$$K_{\mathrm{H}\beta} = 1 + \frac{\kappa_\beta c_\gamma f_{\mathrm{ma}}}{2F_{\mathrm{m}}/b} \text{ and } K_{\mathrm{H}\beta} \geqslant 1{,}05 \qquad (37)$$

b) Double helical gears[7), 8)]

— With full[10)] helix modification (torsional and bending deflection compensated):

$$K_{\mathrm{H}\beta} = 1 + \frac{\kappa_\beta c_\gamma f_{\mathrm{ma}}}{F_{\mathrm{m}}/b_{\mathrm{B}}} \text{ and } K_{\mathrm{H}\beta} \geqslant 1{,}05 \qquad (38)$$

The validity of equations (34) to (38) depends upon compliance with 5.7.1, items a) to g).

5.8 Face load factor, $K_{\mathrm{F}\beta}$

$$K_{\mathrm{F}\beta} = K_{\mathrm{H}\beta}^{N_F} \qquad (39)$$

a) If $b/h \geqslant 3$, then

$$N_{\mathrm{F}} = \frac{(b/h)^2}{1 + b/h + (b/h)^2} = \frac{1}{1 + h/b + (h/b)^2} \qquad (40)$$

b) If $b/h < 3$, then

$$N_{\mathrm{F}} = 0{,}692\,3 \qquad (41)$$

8) The value of $K_{\mathrm{H}\beta}$ is for the more severely stressed helix, which is that nearer to the torqued end of the pinion; the tangential load is divided equally between the two helices: the gap width is small compared to the facewidth. Since for the calculation for $K_{\mathrm{H}\beta}$, half the tooth width (incorporating half the gap width) is used, the obtained values are large. Thus for double helical gears with a large gap width, method C2 of ISO 6336-1:1996 must be used for the calculation of $K_{\mathrm{H}\beta}$.

9) Torsional deflection can be almost completely compensated for by means of a linear tooth trace or helix angle modification. In addition, crowning is necessary when compensation for bending deflection is required.

10) Full modification of both helices is necessary. Partial modification of the helix angle merely to compensate for torsional deflection is not appropriate for double helical gears which are symmetrically positioned between bearings. Torsional and bending deflections can be almost completely compensated for by means of helix angle modification. However, it is often sufficient if only the helix nearest the torque input end is modified; torsional and bending deflections of the other helix tend to compensate each other. This should be verified.

where

b is the smaller of the facewidths of the pinion and wheel measured at the pitch circles; chamfers or rounding of tooth ends shall be ignored; for double helical gears, the width of one helix, b_B, shall be substituted;

h is the tooth height from tip to root: $h = (d_a - d_f)/2$.

5.9 Transverse load factors, $K_{H\alpha}$, $K_{F\alpha}$

The transverse load factors account for the effect of the non-uniform distribution of transverse load between several pairs of simultaneously contacting gear teeth as follows: $K_{H\alpha}$ for surface stress and $K_{F\alpha}$ for tooth-root stress.

For high speed gears

$$K_{H\alpha} = K_{F\alpha} = 1{,}0 \quad (42)$$

6 Calculation of surface durability (pitting)

6.1 Basic formulae

6.1.1 General

The calculation of surface durability is based on the contact stress, σ_H, at the pitch point or at the inner (lowest) point of single pair tooth contact. The higher of the two values obtained is used to determine capacity. σ_H and the permissible contact stress, σ_{HP}, shall be calculated separately for the wheel and pinion. σ_H shall be equal to, or less than, σ_{HP}.

6.1.2 Determination of contact stress, σ_H, for the pinion

The contact stress σ_H for the pinion is calculated as

$$\sigma_H = Z_B Z_H Z_E Z_\epsilon Z_\beta \sqrt{\frac{K_A K_v K_{H\beta} F_t}{d_1 b} \times \frac{u+1}{u}} \quad (43)$$

where

b is the facewidth (for a double helical gear $b = 2\,b_B$); the value b of mating gears is the smaller of the facewidths at the pitch circles of pinion and wheel ignoring any intentional transverse chamfers or tooth-end rounding; neither unhardened portions of surface-hardened gear tooth flanks nor the transition zones shall be included;

Z_B is the factor for the single tooth pair contact for the pinion (see 6.2).

6.1.3 Determination of contact stress, σ_H, for the wheel

The contact stress σ_H for the wheel is calculated as

$$\sigma_H = Z_D Z_H Z_E Z_\epsilon Z_\beta \sqrt{\frac{K_A K_v K_{H\beta} F_t}{d_1 b} \times \frac{u+1}{u}} \quad (44)$$

where Z_D is the factor for the single tooth pair contact for the wheel (see 6.2).

The total tangential load in the case of gear trains with multiple transmission paths, planetary gear systems, or split-path gear trains is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This is to be taken into consideration by substituting $K_{\gamma}K_{\mathrm{A}}$ for K_{A} in equation (43) and equation (44) to adjust the average tangential load per mesh as necessary; see clause 5.

6.1.4 Determination of permissible contact stress, σ_{HP}

In this International Standard, Method B of ISO 6336-2:1996 is used.

$$\sigma_{\mathrm{HP}} = \frac{\sigma_{\mathrm{H\,lim}}}{S_{\mathrm{H\,min}}} Z_{\mathrm{L}} Z_{\mathrm{v}} Z_{\mathrm{R}} Z_{\mathrm{W}} Z_{\mathrm{X}} = \frac{\sigma_{\mathrm{HG}}}{S_{\mathrm{H\,min}}} \qquad (45)$$

The permissible contact stress (endurance) shall be derived from equation (45), with the influence factors $S_{\mathrm{H\,lim}}$, Z_{L}, Z_{v}, Z_{R} and Z_{W} calculated according to this International Standard. However, according to ISO 6336-2 the values of $\sigma_{\mathrm{H\,lim}}$ are validated for $N_{\mathrm{L}} = 5 \times 10^{7}$ load cycles. This number is likely to be exceeded in the life of a high speed gear. Nevertheless values of σ_{HP} derived from equation (45) may be used, given optimum conditions, material, manufacturing and experience; otherwise the values shall be substituted as $\sigma_{\mathrm{HP\,ref}}$ in equation (46). Also see ISO 6336-2:1996, 4.2.

$$\sigma_{\mathrm{HP}} = 0{,}92\,\sigma_{\mathrm{HP\,ref}} \left(\frac{10^{10}}{N_{\mathrm{L}}}\right)^{0{,}019\,1} = \frac{\sigma_{\mathrm{HG}}}{S_{\mathrm{H\,min}}} \qquad (46)$$

NOTE This equation is not in ISO 6336-2 but can be deduced from Figure 8 of ISO 6336-2:1996.

6.1.5 Safety factor for surface durability, S_{H}

S_{H} shall be calculated separately for the pinion and wheel:

$$S_{\mathrm{H}} = \frac{\sigma_{\mathrm{HG}}}{\sigma_{\mathrm{H}}} > S_{\mathrm{H\,min}} \qquad (47)$$

with σ_{HG} for endurance according to equation (45), σ_{H} in accordance with equation (43) for the pinion and with equation (44) for the wheel (see 6.1).

NOTE This is the calculated safety factor with regard to contact stress (Hertzian pressure). The corresponding factor relative to torque capacity is approximately equal to the square of S_{H}.

Notes on minimum safety factor and probability of failure are given in 4.3 of ISO 6336-1:1996.

6.2 Single tooth pair contact factors, Z_{B} and Z_{D}

When $Z_{\mathrm{B}} > 1$ or $Z_{\mathrm{D}} > 1$, the factors Z_{B} and Z_{D} are used to transform the contact stress at the pitch point of spur gears to the contact stress at the inner (lowest) limit of single pair tooth contact of the pinion or the wheel. See 6.1.1.

a) Internal gears

Z_{D} is always to be taken as unity.

b) Spur gears

Determine M_1 (quotient of $\rho_{\mathrm{rel\,C}}$ at the pitch point by $\rho_{\mathrm{rel\,B}}$ at the inner limit (lowest point) of single tooth pair contact of the pinion) and M_2 (quotient of $\rho_{\mathrm{rel\,C}}$ by $\rho_{\mathrm{rel\,D}}$ - wheel) from:

$$M_1 = \frac{\tan\alpha_{\mathrm{wt}}}{\sqrt{\left(\sqrt{\frac{d_{\mathrm{a1}}^2}{d_{\mathrm{b1}}^2} - 1} - \frac{2\pi}{z_1}\right)\left[\sqrt{\frac{d_{\mathrm{a2}}^2}{d_{\mathrm{b2}}^2} - 1} - (\epsilon_{\alpha} - 1)\frac{2\pi}{z_2}\right]}} \qquad (48)$$

$$M_2 = \frac{\tan\alpha_{\mathrm{wt}}}{\sqrt{\left(\sqrt{\frac{d_{\mathrm{a2}}^2}{d_{\mathrm{b2}}^2}-1}-\frac{2\pi}{z_2}\right)\left[\sqrt{\frac{d_{\mathrm{a1}}^2}{d_{\mathrm{b1}}^2}-1}-(\epsilon_\alpha-1)\frac{2\pi}{z_1}\right]}} \tag{49}$$

(See 6.5.1 for calculation of the profile contact ratio ϵ_α)

if $M_1 > 1$ then $Z_{\mathrm{B}} = M_1$; if $M_1 \leqslant 1$ then $Z_{\mathrm{B}} = 1{,}0$

if $M_2 > 1$ then $Z_{\mathrm{D}} = M_2$; if $M_2 \leqslant 1$ then $Z_{\mathrm{D}} = 1{,}0$

c) Helical gears with $\epsilon_\beta \geqslant 1$

$$Z_{\mathrm{B}} = Z_{\mathrm{D}} = 1$$

d) Helical gears with $\epsilon_\beta < 1$

Z_{B} and Z_{D} are determined by linear interpolation between the values for spur and helical gearing with $\epsilon_\beta \geqslant 1$:

$$Z_{\mathrm{B}} = M_1 - \epsilon_\beta (M_1 - 1)\,;\ Z_{\mathrm{B}} \geqslant 1$$

$$Z_{\mathrm{D}} = M_2 - \epsilon_\beta (M_2 - 1)\,;\ Z_{\mathrm{D}} \geqslant 1 \tag{50}$$

If Z_{B} or Z_{D} is set to unity, the contact stresses calculated using equations (43) or (44) are the values for the contact stress at the pitch cylinder.

The methods in 6.2 apply to the calculation of contact stress when the pitch point lies in the path of contact. If the pitch point C is determinant and lies outside the path of contact, then Z_{B} and/or Z_{D} shall be determined for contact at the adjacent tip circle. For helical gears when ϵ_β is less than 1,0, Z_{B} and Z_{D} shall be determined by linear interpolation between the values (determined at the pitch point or at the adjacent tip circle, as appropriate) for spur gears and those helical gears with $\epsilon_\beta \geqslant 1$.

6.3 Zone factor, Z_{H}

The zone factor, Z_{H}, accounts for the influence on Hertzian pressure of tooth flank curvature at the pitch point and transforms the tangential force at the reference cylinder to normal force at the pitch cylinder.

$$Z_{\mathrm{H}} = \sqrt{\frac{2\cos\beta_{\mathrm{b}}\cos\alpha_{\mathrm{wt}}}{\cos^2\alpha_{\mathrm{t}}\sin\alpha_{\mathrm{wt}}}} \tag{51}$$

6.4 Elasticity factor, Z_{E}

The elasticity factor, Z_{E}, takes into account the influences of the material properties E (modulus of elasticity) and ν (Poisson's ratio) on the contact stress. For materials listed in Table 2:

$$Z_{\mathrm{E}} = 189{,}8 \tag{52}$$

6.5 Contact ratio factor, Z_ϵ

6.5.1 General

The contact ratio factor, Z_ϵ, accounts for the influence of the transverse contact and overlap ratios on the surface load capacity of cylindrical gears.

a) Spur gears:

$$Z_\epsilon = \sqrt{\frac{4-\epsilon_\alpha}{3}} \tag{53}$$

The conservative value of $Z_\epsilon = 1{,}0$ may be chosen for spur gears having a contact ratio less than 2,0.

b) Helical gears:

If $\epsilon_\beta < 1$ then

$$Z_\epsilon = \sqrt{\frac{4-\epsilon_\alpha}{3}(1-\epsilon_\beta) + \frac{\epsilon_\beta}{\epsilon_\alpha}} \tag{54}$$

If $\epsilon_\beta \geqslant 1$ then

$$Z_\epsilon = \sqrt{\frac{1}{\epsilon_\alpha}} \tag{55}$$

6.5.2 Transverse contact ratio, ϵ_α

$$\epsilon_\alpha = g_\alpha / p_{\text{bt}} \tag{56}$$

with a path length of contact

$$g_\alpha = \frac{1}{2}\left(\sqrt{d_{\text{a1}}^2 - d_{\text{b1}}^2} \pm \sqrt{d_{\text{a2}}^2 - d_{\text{b2}}^2}\right) - a \sin \alpha_{\text{wt}} \tag{57}$$

and transverse base pitch

$$p_{\text{bt}} = m_{\text{t}} \pi \cos \alpha_{\text{t}} \tag{58}$$

The positive sign is used for external gears, the negative for internal gears.

Equation (57) is only valid if the path of contact is effectively limited by the tip circle of the pinion and the wheel and not, for example, by undercut tooth profiles.

6.5.3 Overlap ratio, ϵ_β

$$\epsilon_\beta = \frac{b \sin \beta}{\pi m_{\text{n}}} \tag{59}$$

See 6.1.2 for definition of facewidth.

6.6 Helix angle factor, Z_β

The helix angle factor, Z_β, takes account of the influence on surface stress of the helix angle.

$$Z_\beta = \sqrt{\cos \beta} \tag{60}$$

6.7 Allowable stress numbers (contact), $\sigma_{\text{H lim}}$

ISO 6336-5 provides information on commonly used gear materials, methods of heat treatment, and the influence of gear quality on values for allowable stress numbers, $\sigma_{\text{H lim}}$, derived from test results of standard reference test gears.

Also see ISO 6336-5 for requirements concerning material and heat treatment for qualities ML, MQ, ME and MX. Material quality MQ is a minimum quality standard required for high speed gears, unless otherwise agreed.

6.8 Influences on lubrication film formation, Z_L, Z_v and Z_R

6.8.1 General

As described in ISO 6336-2, Z_L accounts for the influence of nominal viscosity of the lubricant, Z_v for the influence of tooth-flank velocities and Z_R for the influence of surface roughness on the formation of the lubricant film in the contact zone. Method B of ISO 6336-2:1996 is used in this International Standard.

Factors shall be determined for the softer material when the hardnesses of meshing gears are different.

6.8.2 Lubricant factor, Z_L

$$Z_L = C_{ZL} + \frac{4\,(1{,}0 - C_{ZL})}{\left(1{,}2 + \frac{80}{\nu_{50}}\right)^2} = C_{ZL} + \frac{4\,(1{,}0 - C_{ZL})}{\left(1{,}2 + \frac{134}{\nu_{40}}\right)^2} \tag{61}$$

a) If $\sigma_{\text{H lim}} < 850$ N/mm², then:

$$C_{ZL} = 0{,}83 \tag{62}$$

b) If 850 N/mm² $\leqslant \sigma_{\text{H lim}} \leqslant$ 1 200 N/mm², then:

$$C_{ZL} = \frac{\sigma_{\text{H lim}}}{4\,375} + 0{,}635\,7 \tag{63}$$

c) If $\sigma_{\text{H lim}} >$ 1 200 N/mm², then:

$$C_{ZL} = 0{,}91 \tag{64}$$

Alternatively, one can use:

$$Z_L = C_{ZL} + 4\,(1 - C_{ZL})\,\nu_f$$

where

$$\nu_f = 1/\left(1{,}2 + 80/\nu_{50}\right)^2$$

using viscosity parameters ν_f from Table 4.

Table 4 — Viscosity parameters

ISO viscosity class			VG 32[a]	VG 46[a]	VG 68[a]	VG 100	VG 150	VG 220	VG 320
Nominal viscosity	ν_{40}	mm^2/s	32	46	68	100	150	220	320
	ν_{50}	mm^2/s	21	30	43	61	89	125	180
Viscosity parameter	ν_f	—	0,040	0,067	0,107	0,158	0,227	0,295	0,370
[a] Only for high speed transmission.									

6.8.3 Velocity factor, Z_v[11)]

$$Z_v = C_{Zv} + \frac{2\,(1{,}0 - C_{Zv})}{\sqrt{0{,}8 + \dfrac{32}{v}}} \tag{65}$$

with

$$C_{Zv} = C_{ZL} + 0{,}02 \tag{66}$$

for C_{ZL}, see equations (62) to (64).

6.8.4 Roughness factor, Z_R

6.8.4.1 Calculation of Z_R

$$Z_R = \left(\frac{3}{Rz_{10}}\right)^{C_{ZR}} \tag{67}$$

or alternatively

$$Z_R = \left(\frac{1{,}293a^{1/3}}{Rz_1 + Rz_2}\right)^{C_{ZR}} \tag{68}$$

6.8.4.2 Roughness values

$$Rz = \frac{Rz_1 + Rz_2}{2} \tag{69}$$

$Rz_{1,2}$ is measured on several tooth flanks[12), 13)]

where

$$Rz_{10} = Rz\sqrt[3]{\frac{10}{\rho_{red}}} \tag{70}$$

---

11) Alternatively: $Z_v = C_{Zv} + 2\,(1 - C_{Zv})\;v_p$; where the velocity parameter, $v_p = 1/\left[0{,}8 + (32/v)\right]^{0,5}$.

12) The mean roughnesses Rz_1 (pinion flank) and Rz_2 (wheel flank) should be determined for their surface condition after manufacture, including any running-in treatment, planned as a manufacturing, commissioning or in-service process, when it is safe to assume that it will take place.

13) If the stated roughness is an Ra value (= CLA value) (= AA value), the following approximation may be used to convert:

Ra = CLA = AA = $Rz/6$

$$\rho_{\mathrm{red}} = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \tag{71}$$

where

$$\rho_{1,2} = 0{,}5\, d_{\mathrm{b1,2}} \tan \alpha_{\mathrm{wt}} \tag{72}$$

(also for internal gears, d_{b} has a negative sign)

6.8.4.3 Material dependent index, $C_{Z\mathrm{R}}$

a) If $\sigma_{\mathrm{H\,lim}} < 850$ N/mm^2 then:

$$C_{Z\mathrm{R}} = 0{,}15 \tag{73}$$

b) If 850 N/mm$^2 \leqslant \sigma_{\mathrm{H\,lim}} \leqslant$ 1 200 N/mm^2 then:

$$C_{Z\mathrm{R}} = 0{,}32 - 0{,}000\,2\, \sigma_{\mathrm{H\,lim}} \tag{74}$$

c) If $\sigma_{\mathrm{H\,lim}} > 1\,200$ N/mm^2 then:

$$C_{Z\mathrm{R}} = 0{,}08 \tag{75}$$

6.9 Work hardening factor, Z_{W}

As described in ISO 6336-2, the work hardening factor Z_{W} takes account of the increased surface durability due to meshing a steel wheel (structural steel, through-hardened steel) with a pinion which is significantly ($\approx$ 200 HV or more) harder than the wheel and with smooth tooth flanks ($Rz \leqslant 6$ µm, otherwise effects of wear are not covered by this International Standard). Method B of ISO 6336-2:1996 is applied, as follows:

if HB $<$ 130 then

$$Z_{\mathrm{W}} = 1{,}2 \tag{76}$$

if 130 $\leqslant$ HB $\leqslant$ 470 then

$$Z_{\mathrm{W}} = 1{,}2 - \frac{\mathrm{HB} - 130}{1\,700} \tag{77}$$

if HB $>$ 470 then

$$Z_{\mathrm{W}} = 1{,}0 \tag{78}$$

where HB is the Brinell hardness of the tooth flanks of the softer gear of the pair.

6.10 Size factor, Z_{X}

By means of Z_{X}, account is taken of statistical evidence indicating that the stress levels at which fatigue damage occurs decrease with an increase of component size (larger number of weak points in structure), as a consequence of the influence on subsurface defects of the smaller stress gradients which occur (theoretical stress analysis) and the influence of size on material quality (effect on forging process, variations in structure, etc.). Important influence parameters are:

a) material quality (furnace charge, cleanliness, forging);

b) heat treatment, depth of hardening, distribution of hardening;

c) radius of flank curvature;

d) module: in the case of surface hardening; depth of hardened layer relative to the size of teeth (core supporting-effect).

For through-hardened gears and for surface-hardened gears with adequate case depth relative to tooth size and radius of relative curvature, the size factor, Z_X, is taken to be 1,0.

7 Calculation of tooth bending strength

7.1 Basic formulae

7.1.1 General

As described in ISO 6336-3, the maximum tensile stress at the tooth-root, may not exceed the permissible bending stress for the material. This is the basis for rating the bending strength of gear teeth.

The actual tooth-root stress σ_F and the permissible bending stress σ_{FP} shall be calculated separately for the pinion and wheel; σ_F shall be less than σ_{FP}.

7.1.2 Determination of tooth root stress, σ_F

In this International Standard, Method B of ISO 6336-3:1996 is used.

Tooth root stress σ_F is calculated as follows:

$$\sigma_F = \sigma_{F0}\, K_A\, K_v\, K_{F\beta}\, K_{F\alpha} \leqslant \sigma_{FP} \tag{79}$$

with

$$\sigma_{F0} = \frac{F_t}{b\, m_n}\, Y_F\, Y_S\, Y_\beta \tag{80}$$

The total tangential load in the case of gear trains with multiple transmission paths, planetary gear systems, or split-path gear trains is not quite evenly distributed over the individual meshes (depending on design, tangential speed and manufacturing accuracy). This is to be taken into consideration by substituting $K_\gamma\, K_A$ for K_A in equation (79) to adjust the average tangential load per mesh as necessary; see clause 5.

When the facewidth b (for a double helical gear $b = 2\, b_B$) is larger than that of its mating gear, the bending strength of its teeth shall be based on the smaller facewidth plus a length, not exceeding one module of any extension at each end. However, if it is foreseen that because of crowning or end relief, contact does not extend to the end of face, then the smaller facewidth shall be used for both pinion and wheel. b is the facewidth at the root cylinder of the gear.

7.1.3 Determination of permissible tooth root stress, σ_{FP}

Equation (81) shall be used for the determination of the permissible tooth root stress:

$$\sigma_{FP} = \frac{\sigma_{FE}}{S_{F\,min}}\, Y_{\delta\,rel\,T}\, Y_{R\,rel\,T}\, Y_X = \frac{\sigma_{FG}}{S_{Fmin}} \tag{81}$$

According to ISO 6336-3, the values of $\sigma_{F\,lim}$ and σ_{FE} are validated for $N_L = 3 \times 10^6$ load cycles. This number is likely to be exceeded in the life of a high speed gear. However, values of σ_{FP} derived from equation (81) may be used, given optimum conditions, material, manufacturing and experience, otherwise the values shall be substituted as σ_{FPref} in equation (82). Also see ISO 6336-3:1996, 4.2.

$$\sigma_{\text{FP}} = 0{,}92\,\sigma_{\text{FP}} \left(\frac{10^{10}}{N_{\text{L}}}\right)^{0{,}02} = \frac{\sigma_{\text{FG}}}{S_{\text{Fmin}}} \tag{82}$$

NOTE This equation is not in ISO 6336-3 but can be deduced from Figure 36 of ISO 6336-3:1996.

7.1.4 Safety factor for bending strength, S_{F}

The factor S_{F} shall be calculated from the following equation:

$$S_{\text{F}} = \frac{\sigma_{\text{FG}}}{\sigma_{\text{F}}} \geqslant S_{\text{F min}} \tag{83}$$

S_{F} is calculated separately for the pinion and wheel, with σ_{FG} calculated in accordance with equation (81) or (82) as appropriate, and σ_{F} from equation (79).

More information on safety factors and probability of failure can be found in ISO 6336-1:1996, 4.3.

7.2 Form factor, Y_{F}

7.2.1 General

Y_{F} is the factor by means of which the influence of tooth form on nominal bending stress is taken into account. Y_{F} is relevant to application of load at the outer limit of single pair tooth contact (Method B of ISO 6336-3:1996).

Values of Y_{F} are determined for spur gears and the virtual spur gears of helical gears. Virtual spur gears have the virtual number of teeth z_{n}. See 7.2.4 for calculation of z_{n} and other virtual gear parameters.

Y_{F} shall be determined separately for the wheel and pinion from the following equation:

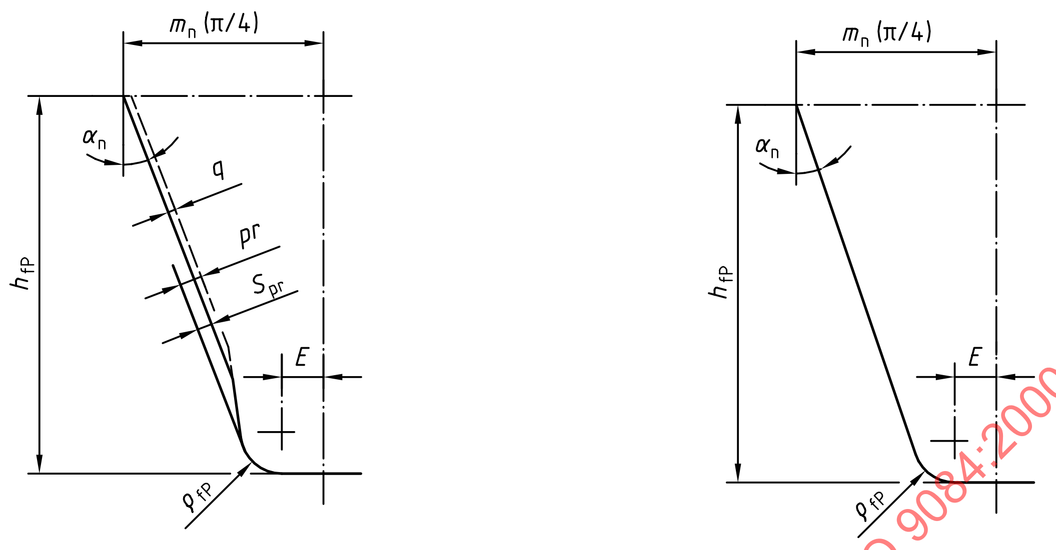
$$Y_{\text{F}} = \frac{\dfrac{6_{\text{h}_{\text{Fe}}}}{m_{\text{n}}} \cos \alpha_{\text{Fen}}}{\left(\dfrac{s_{\text{Fn}}}{m_{\text{n}}}\right)^2 \cos \alpha_{\text{n}}} \tag{84}$$

The relationships given below apply for all involute basic rack profiles, with or without undercut, however with the following restrictions:

a) the 30° tangents contact the tooth-root curve generated by the generating basic rack profile;

b) the generating basic rack has a root radius $\rho_{\text{fP}} > 0$;

c) the gear teeth are generated using a rack type tool.



a) With undercut b) Without undercut

Figure 1 — Dimensions and basic rack profile of the teeth (finished profile)

7.2.2 Parameters required for the determination of Y_F

Firstly determine the auxiliary values E, G and H for equation (84):

$$E = \frac{\pi}{4} m_n - h_{fP} \tan \alpha_n + \frac{s_{pr}}{\cos \alpha_n} - (1 - \sin \alpha_n) \frac{\rho_{fP}}{\cos \alpha_n} \qquad (85)$$

with

$s_{pr} = \quad pr - q$

$s_{pr} = 0$ when gears are not undercut.

$$G = \frac{\rho_{fP}}{m_n} - \frac{h_{fP}}{m_n} + x \qquad (86)$$

$$H = \frac{2}{z_n} \left(\frac{\pi}{2} - \frac{E}{m_n} \right) - \frac{\pi}{3} \qquad (87)$$

Next, use G and H together with $\theta = \pi/6$ as a seed value (on the right-hand side) in equation (88).

$$\theta = \frac{2\,G}{z_n} \tan \theta - H \qquad (88)$$

Use the newly calculated value for θ and apply equation (88) again. Continue using equation (88) until there is no significant change in successive values of θ. Generally the function converges after two or three iterations. Use this final value of θ in equations (89), (90) and (94).

Tooth root normal chord s_{Fn}:

$$\frac{s_{Fn}}{m_n} = z_n \sin \left(\frac{\pi}{3} - \theta \right) + \sqrt{3} \left(\frac{G}{\cos \theta} - \frac{\rho_{fP}}{m_n} \right) \qquad (89)$$

Radius of root fillet ρ_{F}:

$$\frac{\rho_{\mathrm{F}}}{m_{\mathrm{n}}} = \frac{\rho_{\mathrm{f\,P}}}{m_{\mathrm{n}}} + \frac{2G^2}{\cos\theta\,(z_{\mathrm{n}}\cos^2\theta - 2G)} \tag{90}$$

Bending moment arm h_{Fe}:

$$\alpha_{\mathrm{en}} = \arccos\left(\frac{d_{\mathrm{bn}}}{d_{\mathrm{en}}}\right) \tag{91}$$

$$\gamma_{\mathrm{e}} = \frac{0{,}5\pi + 2x\tan\alpha_{\mathrm{n}}}{z_{\mathrm{n}}} + \mathrm{inv}\,\alpha_{\mathrm{n}} - \mathrm{inv}\,\alpha_{\mathrm{en}} \tag{92}$$

$$\alpha_{\mathrm{Fen}} = \alpha_{\mathrm{en}} - \gamma_{\mathrm{e}} = \tan\alpha_{\mathrm{en}} - \mathrm{inv}\,\alpha_{\mathrm{n}} - \frac{0{,}5\pi + 2x\tan\alpha_{\mathrm{n}}}{z_{\mathrm{n}}} \tag{93}$$

$$\frac{h_{\mathrm{Fe}}}{m_{\mathrm{n}}} = 0{,}5\left[(\cos\gamma_{\mathrm{e}} - \sin\gamma_{\mathrm{e}}\tan\alpha_{\mathrm{Fen}})\frac{d_{\mathrm{en}}}{m_{\mathrm{n}}} - z_{\mathrm{n}}\cos\left(\frac{\pi}{3} - \theta\right) - \frac{G}{\cos\theta} + \frac{\rho_{\mathrm{f\,P}}}{m_{\mathrm{n}}}\right] \tag{94}$$

7.2.3 Internal gearing

It is assumed that the value of the form factor of a special rack may be substituted as an approximate value of the form factor of an internal gear. The profile of such a rack should be a version of the basic rack profile, so modified that it would generate the normal profile, including tip and root circles, of an exact counterpart gear of the internal gear. The tip load angle is α_{n}.

The values to be used in equation (84) are determined as follows.

Tooth root normal chord s_{Fn2}:

$$\frac{s_{\mathrm{Fn2}}}{m_{\mathrm{n}}} = 2\left[\frac{\pi}{4} + \frac{h_{\mathrm{fP2}} - \rho_{\mathrm{fP2}}}{m_{\mathrm{n}}}\tan\alpha_{\mathrm{n}} + \frac{\rho_{\mathrm{fP2}} - s_{\mathrm{pr}}}{m_{\mathrm{n}}\cos\alpha_{\mathrm{n}}} - \frac{\rho_{\mathrm{fP2}}}{m_{\mathrm{n}}}\cos\frac{\pi}{6}\right] \tag{95}$$

Bending moment arm h_{Fe2}:

$$\frac{h_{\mathrm{Fe2}}}{m_{\mathrm{n}}} = \frac{d_{\mathrm{en2}} - d_{\mathrm{fn2}}}{2m_{\mathrm{n}}} - \left[\frac{\pi}{4} + \left(\frac{h_{\mathrm{fP2}}}{m_{\mathrm{n}}} - \frac{d_{\mathrm{en2}} - d_{\mathrm{fn2}}}{2m_{\mathrm{n}}}\right)\tan\alpha_{\mathrm{n}}\right]\tan\alpha_{\mathrm{n}} - \frac{\rho_{\mathrm{fP2}}}{m_{\mathrm{n}}}\left(1 - \sin\frac{\pi}{6}\right) \tag{96}$$

with

d_{en2} to be derived from equation (108) with parameters having 2 added to subscripts

d_{fn2} to be derived in the same way as d_{an} [equation (107); note that $d_{\mathrm{fn2}} - d_{\mathrm{f2}} = d_{\mathrm{n2}} - d_2$].

Obtain h_{fP2} from equation (97); refer to equation (98) and related information for ρ_{fP2}.

$$h_{\mathrm{fP2}} = \frac{d_{\mathrm{n2}} - d_{\mathrm{fn2}}}{2} \tag{97}$$

$$\rho_{\mathrm{fP2}} = \frac{c_{\mathrm{p}}}{1 - \sin\alpha_{\mathrm{n}}} = \frac{h_{\mathrm{f2}} - h_{\mathrm{Nf2}}}{1 - \sin\alpha_{\mathrm{n}}} = \frac{d_{\mathrm{Nf2}} - d_{\mathrm{f2}}}{2\,(1 - \sin\alpha_{\mathrm{n}})} \tag{98}$$

where d_{Nf2} represents the diameter of a circle near the tooth root, containing limits of the usable flanks of an internal gear.

Root fillet radius ρ_{F2}:

When the root fillet radius is known, it is to be used. When it is not known, the following approximation may be used:

$$\rho_{F2} = 0{,}15\, m_n \quad (99)$$

Ensure that the correct sign is used; (see footnote a in Table 1).

7.2.4 Parameters for virtual gears

$$\beta_b = \arccos \sqrt{1 - (\sin\beta \cos\alpha_n)^2} = \arcsin\ (\sin\beta \cos\alpha_n) \quad (100)$$

$$z_n = \frac{z}{\cos^2\beta_b \cos\beta} \quad (101)$$

Approximation:

$$z_n \approx \frac{z}{\cos^3\beta} \quad (102)$$

$$\epsilon_{\alpha n} = \frac{\epsilon_\alpha}{\cos^2\beta_b} \quad (103)$$

$$d_n = \frac{d}{\cos^2\beta_b} = m_n\, z_n \quad (104)$$

$$p_{bn} = \pi\, m_n \cos\alpha_n \quad (105)$$

$$d_{bn} = d_n \cos\alpha_n \quad (106)$$

$$d_{an} = d_n + d_a - d \quad (107)$$

$$d_{en} = 2\,\frac{z}{|z|}\sqrt{\left[\sqrt{\left(\frac{d_{an}}{2}\right)^2 - \left(\frac{d_{bn}}{2}\right)^2} - \frac{\pi\, d \cos\beta \cos\alpha_n}{|z|}\,(\epsilon_{\alpha n} - 1)\right]^2 + \left(\frac{d_{bn}}{2}\right)^2} \quad (108)$$

The number z is positive for external gears and negative for internal gears (see footnote a in Table 1).

7.3 Stress correction factor, Y_S

The stress correction factors Y_S is used to convert the nominal bending stress to local tooth root stress. Y_S shall be determined separately for the pinion and wheel.

$$Y_S = (1{,}2 + 0{,}13\, L)\, q_s^{\,[1/(1{,}21 + 2{,}3/L)]} \quad (109)$$

where

$$L = \frac{s_{Fn}}{h_{Fe}} \quad (110)$$

$$q_s = \frac{s_{Fn}}{2\,\rho_F} \quad (111)$$

with

s_{Fn} is taken from equation (89) for external gears;

s_{Fn} is taken from equation (95) for internal gears;

h_{Fe} is taken from equation (94) for external gears;

h_{Fe} is taken from equation (96) for internal gears;

ρ_F is taken from equation (90) for external gears;

ρ_F is taken from equations (98) and (99) for internal gears.

7.4 Helix angle factor, Y_β

The tooth-root stress of a virtual spur gear, calculated as a preliminary value, is converted by means of the helix factor Y_β to that of the corresponding helical gear. By this means, the oblique orientation of the lines of mesh contact is taken into account (lower tooth-root stress).

If $\epsilon_\beta > 1$ and $\beta \leqslant 30^\circ$ then

$$Y_\beta = 1 - \frac{\beta}{120^\circ} \tag{112}$$

If $\epsilon_\beta > 1$ and $\beta > 30^\circ$ then

$$Y_\beta = 0{,}75 \tag{113}$$

If $\epsilon_\beta \leqslant 1$ and $\beta \leqslant 30^\circ$ then

$$Y_\beta = 1 - \epsilon_\beta \frac{\beta}{120^\circ} \tag{114}$$

If $\epsilon_\beta \leqslant 1$ and $\beta > 30^\circ$ then

$$Y_\beta = 1 - 0{,}25\,\epsilon_\beta \tag{115}$$

7.5 Tooth-root reference strength, σ_{FE}

ISO 6336-5 provides information on values of $\sigma_{F\,lim}$ and σ_{FE} for the more popular gear materials. The requirements for heat treatment processes and material quality for quality grades ML, MQ and ME are also included.

The quality MQ is a minimum quality standard required for high speed gears unless otherwise agreed. Method B from ISO 6336-3:1996 is used in this International Standard.

7.6 Relative notch sensitivity factor, $Y_{\delta\,rel\,T}$

$Y_{\delta rel\,T}$ indicates, approximately, the overstress tolerance of the material in the root fillet region. In this International Standard, Method C of ISO 6336-3:1996 is used.

— If $q_s \geqslant 1{,}5$ then $Y_{\delta\,rel\,T} = 1{,}0$

— If $q_s < 1{,}5$ then $Y_{\delta\,rel\,T} = 0{,}95$

The notch parameter, q_s, may be obtained from equation (111).

7.7 Relative surface factor, $Y_{\text{R rel T}}$

7.7.1 General

The surface factor, $Y_{\text{R rel T}}$, accounts for the influence on tooth-root stress of the surface condition in the tooth-roots. Primarily, this is dependent on surface roughness in the tooth-root fillets.

The influence of surface condition on tooth-root bending strength does not depend solely on the surface roughness in the tooth-root fillets, but also on the size and shape (the problem of 'notches within a notch'). This subject has not to date been sufficiently well studied for it to be taken into account in this International Standard. The method applied here is only valid when scratches or similar defects deeper than $2 \times Rz$ are not present.

NOTE $2 \times Rz$ is a preliminary estimated value.

Besides surface texture, other influences on tooth bending strength are known and include: residual compressive stresses (shot peening), grain boundary oxidation, chemical effects etc. When fillets are shot-peened and/or are perfectly shaped, a value slightly greater than that obtained from the graph should be substituted for $Y_{\text{R rel T}}$. When grain boundary oxidation or chemical effects are present, a smaller value than that indicated by the graph should be substituted for $Y_{\text{R rel T}}$.

In this International Standard, Method C of ISO 6336-3:1996 is used.

7.7.2 $Y_{\text{R rel T}}$ for limited life, reference and long life stresses

a) For all materials, if $Rz < 1$ µm then:

$$Y_{\text{R rel T}} = 1{,}0 \tag{116}$$

b) For V, Eh and IF gears, if $Rz \geqslant 1$ µm then:

$$Y_{\text{R rel T}} = 1{,}674 - 0{,}529\,(Rz + 1)^{0{,}1} \tag{117}$$

c) For NT, NV gears, if $Rz \geqslant 1$ µm then:

$$Y_{\text{R rel T}} = 4{,}299 - 3{,}259\,(Rz + 1)^{0{,}005} \tag{118}$$

7.8 Size factor, Y_{X}

Y_{X} is used to allow for the influence of size on:

— the probable distribution of weak points in the material structure;

— the stress gradients, which in materials theory decrease with increasing dimensions;

— material quality;

— as regards quality of forging, presence of defects, etc.

Method B of ISO 6336-3:1996 is used in this International Standard.

a) For V gears:

$$Y_{\text{X}} = 1{,}03 - 0{,}006\,m_{\text{n}} \tag{119}$$

with the restriction: $0{,}85 \leqslant Y_{\text{X}} \leqslant 1{,}0$

b) For Eh, IF, NT, NV gears:

$$Y_{\text{X}} = 1{,}05 - 0{,}01\,m_{\text{n}} \tag{120}$$

with the restriction: $0{,}80 \leqslant Y_{\text{X}} \leqslant 1{,}0$

Annex A
(normative)

Tooth stiffness parameters c' and c_γ

A.1 General

A tooth stiffness parameter represents the requisite load over 1 mm facewidth, directed along the line of action[1] to produce in line with the load, the deformation amounting to 1 µm, of one or more pairs of deviation-free teeth in contact.

Single stiffness c' is the maximum stiffness of a single pair of a spur gear teeth. It is approximately equal to the maximum stiffness of a tooth pair in single pair contact[2]. c' for helical gears is the maximum stiffness normal to the helix of one tooth pair.

Mesh stiffness c_γ is the mean value of stiffness of all the teeth in a mesh.

Method B of ISO 6336-1:1996, used in this International Standard, is applicable in the range $x_1 \geqslant x_2 \leqslant 2$.

A.2 Single stiffness c'

A.2.1 Calculation of c'

For specific loading $(F_t\, K_A)/b \geqslant 100$ N/mm:

$$c' = 0{,}8\, c'_{th}\, C_R\, C_B \cos\beta \qquad \text{(A.1)}$$

A.2.2 Theoretical single stiffness, c'_{th}

$$c'_{th} = \frac{1}{q'} \qquad \text{(A.2)}$$

where

$$q' = C_1 + \frac{C_2}{z_{n1}} + \frac{C_3}{z_{n2}} + (C_4\, x_1) + \frac{(C_5\, x_1)}{z_{n1}} + (C_6\, x_2) + \frac{(C_7\, x_2)}{z_{n2}} + (C_8\, x_1^2) + (C_9\, x_2^2) \qquad \text{(A.3)}$$

Table A.1 — Constants for equation (A.3)

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
0,047 23	0,155 51	0,257 91	−0,006 35	−0,116 54	−0,001 93	−0,241 88	0,005 29	0,001 82

1) The tooth deflection may be determined approximately using F_t (F_m, F_{tH}, ...) instead of F_{bt}. Conversion from F_t to F_{bt} (load tangent to the base cylinder) is covered by the relevant factors, or the modifications resulting from this conversion may be ignored when compared with other uncertainties (e.g. tolerances on the measured values).

2) c' at the outer limit of single pair tooth contact can be assumed to approximate the maximum value of single stiffness when $\epsilon_\alpha > 1{,}2$.

A.2.3 Gear blank factor, C_R

$C_R = 1$ for gears made from solid disc blanks.

For other gears:

$$C_R = 1 + \frac{\ln (b_s/b)}{5\, e^{s_R/(5 m_n)}} \tag{A.4}$$

Boundary conditions:

when $b_s/b < 0{,}2$ substitute $b_s/b = 0{,}2$

when $b_s/b > 1{,}2$ substitute $b_s/b = 1{,}2$

when $s_R/m_n < 1$ substitute $s_R/s_n = 1$

See Figure A.1 for symbols.

A.2.4 Basic rack factor, C_B

For the specified basic rack[3], $C_{BS} = 1$ (standard value)

For other basic racks, C_{BD} (different value) can be obtained from equation (A.5)

$$c_{BD} = [1 + 0{,}5\,(1{,}2 - h_{fP}/m_n)]\ \ [1 - 0{,}02\,(20^\circ - \alpha_P)] \tag{A.5}$$

A.2.5 Additional information

a) Internal gearing: approximate values of the theoretical single stiffnesses of internal gear teeth can be determined from equations (A.2), (A.3), by the substitution of infinity for z_{n2}.

b) Specific loading $(F_t K_A)/b < 100$ N/mm

$$c' = 0{,}8\, c'_{th}\, C_R\, C_B \cos\beta \left[\frac{F_t\, K_A}{100\, b}\right]^{0{,}25} \tag{A.6}$$

c) The above is based on steel gear pairs; for other materials and material combinations, refer to ISO 6336-1:1996, clause 9.

A.2.6 Mesh stiffness, c_γ

For spur gears with $\epsilon_\alpha \geqslant 1{,}2$ and helical gears with $\beta \leqslant 30^\circ$, the mesh stiffness is given by:

$$c_\gamma = c'\ (0{,}75\,\epsilon_\alpha + 0{,}25) \tag{A.7}$$

with c' according to equation (A.1).

3) Series progression for gears with basic rack profile: $\alpha_P = 20^\circ$, $h_{aP} = m_n$, $h_{fP} = 1{,}2\, m_n$, and $\rho_{fP} = 0{,}2\, m_n$; equations (A.2) and (A.3) apply for the range $x_1 \geqslant x_2$; $-0{,}5 \leqslant x_1 + x_2 \leqslant 2{,}0$. Deviations of actual values from calculated values in the range $100 \leqslant F_{bt}/b \leqslant 1\,600$ N/mm are between $+5$ % and -8 %.

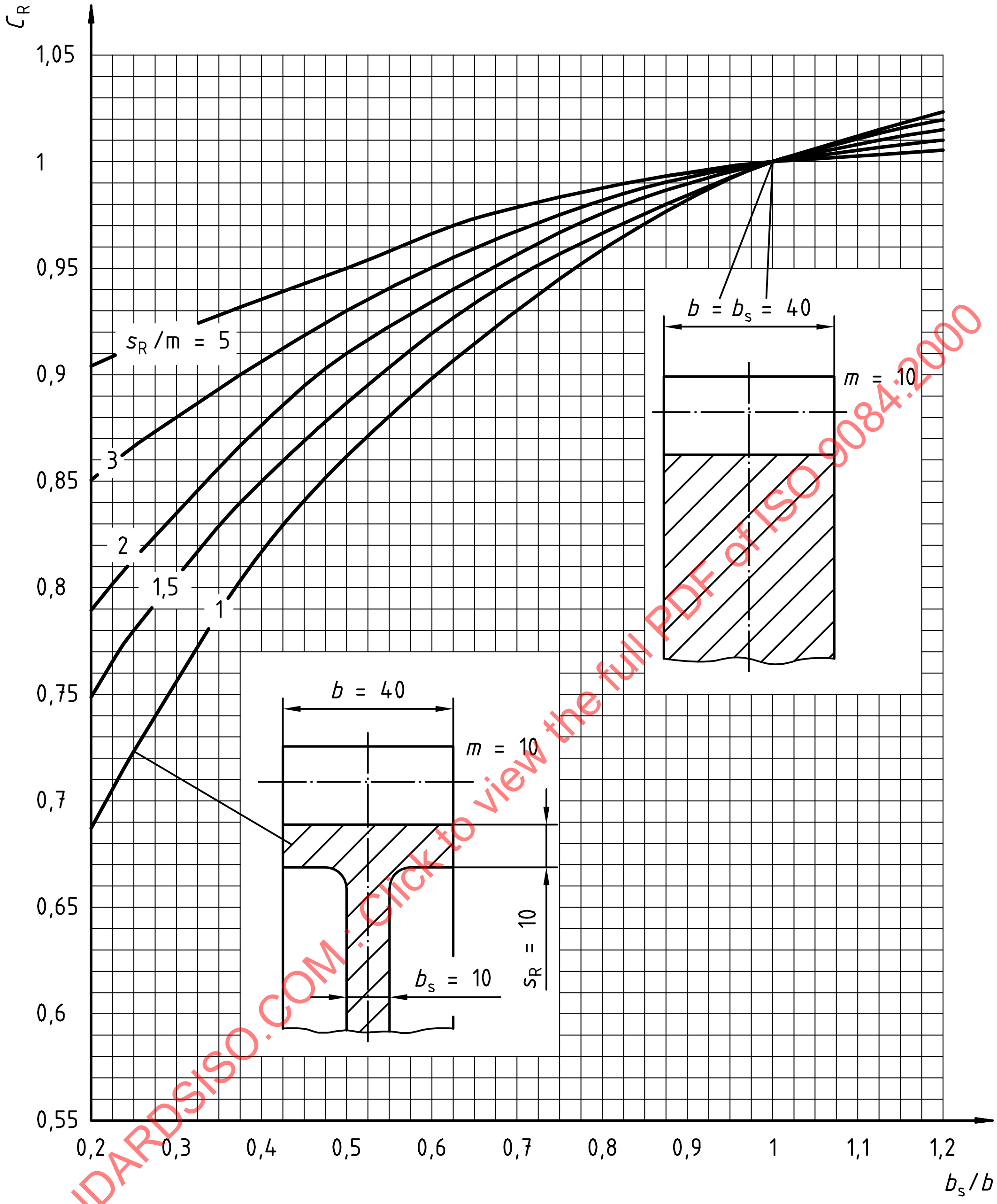


Figure A.1 — Wheel blank factor, C_R, mean values for mating gears of similar or stiffer wheel blank design

Annex B
(normative)

Special features of less common gear designs

B.1 Dynamic factor K_v for planetary gears

B.1.1 General

In gear trains which include multiple mesh gears such as idler gears and in epicyclic gearing, planet and sun gears, there are several natural frequencies. These can be higher or lower than the natural frequency of a single gear pair which has only one mesh.

Although values of K_v determined using the formulae in this International Standard shall be considered as unreliable, nevertheless they can be useful as preliminary assessments. It is recommended that, if possible, they should be re-assessed using a more accurate procedure.

Method A of ISO 6336-1:1996 is preferred for the analysis of less common transmission designs. Refer to 6.1.1 of ISO 6336-1:1996 for further information.

B.1.2 Calculation of the relative mass of a gear pair with external teeth

Refer to 5.6.2.

B.1.3 Resonance speed determination for less common gear designs

The resonance speed determination for less common gear designs should be made with the use of Method A of ISO 6336-1:1996. However, other methods may be used to approximate the effects. Some examples are as follows.

a) Pinion shaft diameter about equal to diameter at mid-tooth depth d_{m1}

The high torsional stiffness of the pinion shaft is to a great extent compensated by the shaft mass. Thus the resonance speed can be calculated in the normal way, using the mass of the pinion (toothed portion) and the normal mesh stiffness c_γ.

b) Two rigidly connected, coaxial gears

The mass of the larger of the connected gears is to be included.

c) One large wheel driven by two pinions

As the mass of the wheel is normally much greater than the masses of the pinions, each mesh can be considered separately, i.e.:

— as a pair comprising the first pinion and the wheel;

— as a pair comprising the second pinion and the wheel.

d) Planetary gears

Because of the many transmission paths which include stiffnesses other than mesh stiffness, the vibratory behaviour of planetary gears is very complex. Thus factors derived from simple formulae are generally inaccurate and values obtained from the method in this International Standard should be verified by means of a subsequent