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Foreword

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International Standard ISO 8485 was prepared by Technical Committee ISO/TC 97, *Information processing systems.*

Users should note that all International Standards undergo revision from time to time and that any reference made herein to any other International Standard implies its latest edition, unless otherwise stated.

Annexes A and B are for information only.

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CONTENTS

0 Introduction . . . 3

1 Scope and Field of Application . . . 5

2 References . . . 7

3 Form of the Standard . . . 9
3.1 Form of Definitions . . . 9
3.2 Named Arrays in Examples . . . 10
3.3 Notes . . . 10
3.4 Cross-References . . . 10
3.5 General Definitions . . . 11

4 Compliance . . . 13
4.1 Conforming Implementations . . . 13
4.1.1 Required Behaviour for Conforming Implementations . . . 13
4.1.2 Required Documentation for Conforming Implementations . . . 14
4.1.2.1 Documentation of Optional-Facilities . . . 14
4.1.2.2 Documentation of Implementation-Defined-Facilities . . . 14
4.1.2.3 Consistent Extensions . . . 14
4.2 Conforming Programs . . . 15
4.2.1 Required Behaviour for Conforming Programs . . . 15
4.2.2 Required Documentation for Conforming Programs . . . 15

5 Definitions . . . 17
5.1 Characters . . . 17
5.2 Numbers . . . 19
5.2.1 Elementary Operations . . . 19
5.2.2 Number Constants . . . 20
5.2.3 Subsets of the Set of Numbers . . . 21
5.2.4 Implementation Algorithms . . . 23
5.2.5 Defined Operations . . . 24
5.3 Objects . . . 26
5.3.1 Lists . . . 26
5.3.2 Arrays . . . 27
5.3.3 Defined-Functions . . . 29
5.3.4 Tokens . . . 30
5.3.4.1 Metaclasses . . . 32
5.3.4.2 Index-List . . . 33
5.3.5 Symbols . . . 33
5.3.6 Contexts . . . 34
5.3.7 Workspaces . . . 34
5.3.8 Sessions . . . 35
5.3.9 Shared-Variables . . . 36
5.3.10 Systems . . . 37
5.4 Evaluation Sequences . . . 39
5.4.1 Evaluation Sequence Phrases . . . 39
5.4.2 Diagrams . . . 41
5.5 Other Terms . . . 42

6 Syntax and Evaluation 43
6.1 Introduction 43
6.1.1 Evaluate-Line 43
6.1.2 Character-Diagrams 45
6.1.3 Evaluate-Statement 52
6.1.4 Bind-Token-Class 53
6.1.5 Literal-Conversion 54
6.1.6 Statement-Analysis Token-Diagrams 55
6.2 Reduce-Statement 60
6.3 The Phrase Evaluators 63
6.3.1 Diagrams 64
6.3.2 Remove-Parentheses 65
6.3.3 Evaluate-Niladic-Function 65
6.3.4 Evaluate-Monadic-Function 66
6.3.5 Evaluate-Monadic-Operator 68
6.3.6 Evaluate-Dyadic-Function 69
6.3.7 Evaluate-Dyadic-Operator 71
6.3.8 Evaluate-Indexed-Reference 72
6.3.9 Evaluate-Assignment 73
6.3.10 Evaluate-Indexed-Assignment 74
6.3.11 Evaluate-Variable 75
6.3.12 Build-Index-List 76
6.3.13 Process-End-of-Statement 77
6.4 The Form Table 78

7 Scalar Functions 83
7.1 Monadic Scalar Functions 84
7.1.1 Conjugate *Z* ← + *B* 84
7.1.2 Negative *Z* ← - *B* 84
7.1.3 Signum *Z* ← × *B* 85
7.1.4 Reciprocal *Z* ← ÷ *B* 85
7.1.5 Floor *Z* ← ⌊ *B* 86
7.1.6 Ceiling *Z* ← ⌈ *B* 86
7.1.7 Exponential *Z* ← ⋆ *B* 87
7.1.8 Natural Logarithm *Z* ← ⍟ *B* 87
7.1.9 Magnitude *Z* ← | *B* 88
7.1.10 Factorial *Z* ← ! *B* 88
7.1.11 Pi times *Z* ← ○ *B* 89
7.1.12 Not *Z* ← ~ *B* 89
7.2 Dyadic Scalar Functions 90
7.2.1 Plus *Z* ← *A* + *B* 91
7.2.2 Minus *Z* ← *A* - *B* 91
7.2.3 Times *Z* ← *A* × *B* 92
7.2.4 Divide *Z* ← *A* ÷ *B* 92
7.2.5 Maximum *Z* ← *A* ⌈ *B* 93
7.2.6 Minimum *Z* ← *A* ⌊ *B* 93
7.2.7 Power *Z* ← *A* ⋆ *B* 94
7.2.8 Logarithm *Z* ← *A* ⍟ *B* 94
7.2.9 Residue *Z* ← *A* | *B* 95
7.2.10 Binomial *Z* ← *A* ! *B* 96
7.2.11 Circular Functions *Z* ← *A* ○ *B* 97
7.2.12 And *Z* ← *A* ∧ *B* 99
7.2.13 Or *Z* ← *A* ∨ *B* 99
7.2.14 Nand *Z* ← *A* ⍲ *B* 100
7.2.15 Nor *Z* ← *A* ⍱ *B* 100
7.2.16 Equal *Z* ← *A* = *B* 101
7.2.17 Less than *Z* ← *A* < *B* 103
7.2.18 Less than or equal to *Z* ← *A* ≤ *B* 103
7.2.19 Not equal *Z* ← *A* ≠ *B* 104
7.2.20 Greater than or equal to *Z* ← *A* ≥ *B* 104
7.2.21 Greater than *Z* ← *A* > *B* 105

8 Structural Primitive Functions **107**

8.1 Introduction 107

8.2 Monadic Structural Primitive Functions 107

8.2.1 Ravel .... *Z* ← , *B* 107

8.2.2 Shape .... *Z* ← ρ *B* 108

8.2.3 Index Generator .... *Z* ← ι *B* 109

8.3 Dyadic Structural Primitive Functions 110

8.3.1 Reshape .... *Z* ← *A* ρ *B* 110

8.3.2 Join .... *Z* ← *A* , *B* 111

9 Operators **113**

9.1 Introduction 113

9.2 Monadic Operators 114

9.2.1 Reduction .... *Z* ← f/ *B* 114

9.2.1 Reduction .... *Z* ← f/[*K*] *B* 114

9.2.1 Reduction .... *Z* ← f⌿ *B* 114

9.2.1 Reduction .... *Z* ← f⌿[*K*] *B* 114

9.2.2 Scan .... *Z* ← f\ *B* 116

9.2.2 Scan .... *Z* ← f\[*K*] *B* 116

9.2.2 Scan .... *Z* ← f⍀ *B* 116

9.2.2 Scan .... *Z* ← f⍀[*K*] *B* 116

9.3 Dyadic Operators 117

9.3.1 Outer Product .... *Z* ← *A* ∘ . f *B* 117

9.3.2 Inner Product .... *Z* ← *A* f . g *B* 118

10 Mixed Functions **119**

10.1 Monadic Mixed Functions 119

10.1.1 Roll .... *Z* ← ? *B* 119

10.1.2 Grade Up .... *Z* ← ⍋ *B* 121

10.1.3 Grade Down .... *Z* ← ⍒ *B* 122

10.1.4 Reverse .... *Z* ← ⌽ *B* 123

10.1.4 Reverse .... *Z* ← ⊖ *B* 123

10.1.4 Reverse .... *Z* ← ⌽[*K*] *B* 123

10.1.4 Reverse .... *Z* ← ⊖[*K*] *B* 123

10.1.5 Monadic Transpose .... *Z* ← ⍉ *B* 124

10.1.6 Matrix Inverse .... *Z* ← ⌹ *B* 125

10.1.7 Execute .... *Z* ← ⍎ *B* 126

10.2 Dyadic Mixed Functions 127

10.2.1 Join Along an Axis .... *Z* ← *A* ,[*K*] *B* 127

10.2.2 Index of .... *Z* ← *A* ι *B* 130

10.2.3 Member of .... *Z* ← *A* ∊ *B* 131

10.2.4 Deal .... *Z* ← *A* ? *B* 132

10.2.5 Compress .... *Z* ← *A* / *B* 133

10.2.5 Compress .... *Z* ← *A* ⌿ *B* 133

10.2.5 Compress .... *Z* ← *A* /[*K*] *B* 133

10.2.5 Compress .... *Z* ← *A* ⌿[*K*] *B* 133

10.2.6 Expand .... *Z* ← *A* \ *B* 135

10.2.6 Expand .... *Z* ← *A* ⍀ *B* 135

10.2.6 Expand .... *Z* ← *A* \[*K*] *B* 135

10.2.6 Expand .... *Z* ← *A* ⍀[*K*] *B* 135

10.2.7 Rotate .... *Z* ← *A* ⌽ *B* 137

10.2.7 Rotate .... *Z* ← *A* ⊖ *B* 137

10.2.7 Rotate .... *Z* ← *A* ⌽[*K*] *B* 137

10.2.7 Rotate .... *Z* ← *A* ⊖[*K*] *B* 137

10.2.8 Base Value .... *Z* ← *A* ⊥ *B* 139

10.2.9 Representation .... *Z* ← *A* ⊤ *B* 140

10.2.10 Dyadic Transpose .... *Z* ← *A* ⍉ *B* 142

10.2.11 Take .... *Z* ← *A* ↑ *B* 144

10.2.12 Drop .... *Z* ← *A* ↓ *B* 145

10.2.13 Matrix Divide .... *Z* ← *A* ⌹ *B* 146

10.2.14 Indexed Reference .... *Z* ← *A*[*I*] 147

10.2.15 Indexed Assignment .... *Z* ← *V*[*I*] ← *B* 148

11 System Functions **151**
11.1 Introduction 151
11.2 Definition 151
11.3 Diagram 151
11.4 Niladic System Functions 152
11.4.1 Time Stamp . . . . Z ← ⎕TS 152
11.4.2 Atomic Vector . . . . Z ← ⎕AV 153
11.4.3 Line Counter . . . . Z ← ⎕LC 153
11.5 Monadic System Functions 154
11.5.1 Delay . . . . Z ← ⎕DL B 154
11.5.2 Name Class . . . . Z ← ⎕NC B 155
11.5.3 Expunge . . . . Z ← ⎕EX B 156
11.5.4 Name List . . . . Z ← ⎕NL B 156
11.5.5 Query Stop . . . . Z ← ⎕STOP B 157
11.5.6 Query Trace . . . . Z ← ⎕TRACE B 157
11.6 Dyadic System Functions 158
11.6.1 Name List . . . . Z ← A ⎕NL B 158
11.6.2 Set Stop . . . . Z ← A ⎕STOP B 158
11.6.3 Set Trace . . . . Z ← A ⎕TRACE B 159
12 System Variables **161**
12.1 Definitions 161
12.2 Evaluation Sequences 162
12.2.1 Comparison Tolerance . . . . Z ← ⎕CT ← B 162
12.2.1 Comparison Tolerance . . . . Z ← ⎕CT 162
12.2.2 Random Link . . . . Z ← ⎕RL ← B 163
12.2.2 Random Link . . . . Z ← ⎕RL 163
12.2.3 Print Precision . . . . Z ← ⎕PP ← B 164
12.2.3 Print Precision . . . . Z ← ⎕PP 164
12.2.4 Index Origin . . . . Z ← ⎕IO ← B 165
12.2.4 Index Origin . . . . Z ← ⎕IO 165
12.2.5 Latent Expression . . . . Z ← ⎕LX ← B 166
12.2.5 Latent Expression . . . . Z ← ⎕LX 166
12.2.5 Latent Expression . . . . Z ← ⎕LX[I] ← B 166
13 Defined Functions **167**
13.1 Introduction 167
13.2 Definitions 168
13.3 Diagrams 171
13.4 Operations 174
13.4.1 Call-Defined-Function . . . . Z ← DFN 174
13.4.1 Call-Defined-Function . . . . Z ← DFN B 174
13.4.1 Call-Defined-Function . . . . Z ← A DFN B 174
13.4.2 Defined-Function-Control 175
13.4.3 Function Fix . . . . Z ← ⎕FX B 176
13.4.4 Character Representation . . . . Z ← ⎕CR B 177
13.5 Function Editing 178
13.5.1 Evaluate-Function-Definition-Request 178
13.5.2 Evaluate-Editing-Request 180
13.5.3 Diagrams 181

14 Shared Variables **183**
14.1 Informal Introduction 183
14.2 Definitions 186
14.3 Diagrams 186
14.4 Operations 186
14.4.1 Primary-Name 186
14.4.2 Surrogate-Name 186
14.4.3 Degree-of-Coupling 186
14.4.4 Access-Control-Vector 186
14.4.5 Offer 187
14.4.6 Retract 188
14.4.7 Shared-Variable-Reset 188

14.5 Shared Variable Forms 189
14.5.1 Shared Variable Reference . . . . . Z ← SHV 189
14.5.2 Shared Variable Assignment . . . . . Z ← SHV ← B 189
14.5.3 Shared Variable Indexed Assignment . . . . . Z ← SHV[I] ← B 190
14.6 Shared Variable System Functions 191
14.6.1 Shared Variable Access Control Inquiry . . . . . Z ← ⎕SVC B 191
14.6.2 Shared Variable Query . . . . . Z ← ⎕SVQ B 192
14.6.3 Shared Variable Degree of Coupling . . . . . Z ← ⎕SVO B 193
14.6.4 Shared Variable Offer . . . . . Z ← A ⎕SVO B 194
14.6.5 Shared Variable Retraction . . . . . Z ← ⎕SVR B 195
14.6.6 Shared Variable Access Control Set . . . . . Z ← A ⎕SVC B 196

15 Formatting and Numeric Conversion 197
15.1 Introduction 197
15.2 Numeric Conversion 197
15.2.1 Numeric-Input-Conversion 198
15.2.2 Numeric-Output-Conversion 199
15.3 Diagrams 199
15.4 Operations 203
15.4.1 Monadic Format . . . . . Z ← ⍕ B 203
15.4.2 Dyadic Format . . . . . Z ← A ⍕ B 206

16 Input and Output 209
16.1 Introduction 209
16.2 Definitions 209
16.2.1 User Facilities 209
16.2.2 Implementation Algorithms 210
16.2.3 Prompts 211
16.3 Diagrams 211
16.4 Operations 212
16.4.1 Immediate-Execution 212
16.4.2 Quad Input . . . . . Z ← ⎕ 213
16.4.3 Quote Quad Input . . . . . Z ← ⍞ 214
16.4.4 Quad Output . . . . . Z ← ⎕ ← B 214
16.4.5 Quote Quad Output . . . . . Z ← ⍞ ← B 215

17 System Commands 217
17.1 Introduction 217
17.2 Definitions 217
17.3 Diagrams 218
17.4 Operations 219
17.4.1 Evaluate-System-Command 219
17.5 Diagrams and Evaluation Sequences 220

Annex A. The APL Character Set for Workspace Interchange 225

Annex B: The Workspace Interchange Convention 233
B.1 Introduction 233
B.1.1 Purpose of the Convention 233
B.1.2 Organisation of the Convention 233
B.2 Level 1 Description 234
B.2.1 Canonical Representation Vectors 234
B.2.2 Representation of Variables 235
B.2.3 Representation of Functions 236
B.2.4 Representation of Executable Expressions 237
B.3 Level 2 Description 238
B.3.1 Multiple Canonical Representation Vectors 238
B.3.2 Workspace Environment 238
B.3.3 Pseudovariables 239
B.3.4 Interchange Convention Identifier 239
B.3.5 Escape 239
B.3.6 Bits 239
B.3.7 Translate 239
B.3.8 Note 240
B.3.9 Workspace Identifier 240
B.3.10 End of the Stream 240

B.3.11 Inclusion of Nonstandard Material 240
B.3.12 Use of Other Types 241
B.4 Level 3 Description 242
B.4.1 Atoms 242
B.4.2 Registered character sets 242
B.4.3 Escape sequences 242
B.4.4 Registered characters 242
B.4.5 ISO-APL characters 242
B.4.6 Characters 243
B.4.7 The Translate Table 243
B.4.8 Example 244
B.4.9 Escape 244
B.4.10 Bits 244
B.4.11 Translate 244
B.4.12 Example 245
B.4.13 Mapping of Characters to Bits 245
B.4.14 Stage 1 Translation 245
B.4.15 Stage 2 Translation 245
B.4.16 Example 246
B.4.17 Summary 246
B.5 Level 4 Description 247
B.5.1 Physical Media 247
B.5.2 Nine-track Magnetic Tape 247
B.6 Extended Example 248

Bibliography **253**

INDEX **255**

Programming languages — APL

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0 INTRODUCTION

APL stands for **A Programming Language**. It is a notation invented by K. E. Iverson in the late 1950s for the description of algorithms, and expanded on and made into the programming system *APL*\360 by Iverson and his colleagues Adin Falkoff, Larry Breed, Dick Lathwell, and Roger Moore in the mid-1960s.

Throughout this document

— the term "this standard" is understood to mean "this International Standard";

— the words "chapter", "section" and "subsection" are understood to mean "clause", "subclause" and "subsubclause", respectively.

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1 SCOPE AND FIELD OF APPLICATION

This standard defines the programming language APL and the environment in which APL programs are executed. Its purpose is to facilitate interchange and promote portability of APL programs and programming skills.

This standard specifies the syntax and semantics of APL programs and the characteristics of the environment in which APL programs are executed.

It also specifies requirements for conformance to this standard, including the publication of values and characteristics of implementation properties so that conforming implementations can be meaningfully compared.

This standard does not specify:

implementation properties that are likely to vary with the particular equipment or operating system used;

required values for implementation limits such as APL workspace size or numeric precision;

the data structures used to represent APL objects;

the facilities available through shared variables.

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2 REFERENCES

ISO 2375 : 1985, *Data processing — Procedure for registration of escape sequences.*

ISO 2382-15 : 1985, *Data processing — Vocabulary — Part 15: Programming languages.*

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3 FORM OF THE STANDARD

This standard is a formal model of an APL machine, specified as a collection of finite sets, diagrams, and evaluation sequences, and objects constructed from finite sets, diagrams, and evaluation sequences.

The finite sets are the **implementation-defined character-set**, the **implementation-defined** set of **numbers**, and the enumerated sets **array-type**, **class-names**, **keyboard-states**, **mode-names**, **required-character-set**, and **workspace-presence**.

Diagrams are directed graphs used to designate syntactic forms.

Evaluation sequences are formal procedures that operate on finite sets, diagrams, other evaluation sequences and objects defined in the standard.

Objects are entities consisting of enumerated set members and other objects. The objects are **list**, **array**, **defined-function**, **token**, **symbol**, **context**, **workspace**, **session**, **shared-variable**, and **system**.

Each object has attributes describing its state. The attributes of an array, for example, are its typical element, its shape, and its ravel.

Objects often have defined properties derived from their attributes. The rank of an array, for example, is the shape of the shape of the array.

3.1 FORM OF DEFINITIONS

Defined terms in this standard are always set in **bold** and indexed. The index entry begins with the page number of the definition followed by the page numbers of all references to the term. If the definition and a use of a term occur on the same page, that page number will occur twice in the Index.

The following terms occur throughout the document and are not cross-indexed: **character**, **content**, **class**, **item**, and **number**.

Note: *Terms in this document include both phrases such as* ***implementation-parameter*** *and words such as* ***nil.***

Each definition in this standard takes one of four forms.

(1) Regular definitions consist of the term being defined followed by a colon and the body of the definition. The term **Boolean** is defined in this way.

(2) Members of an enumerated set are defined simply by being listed in the definition of the enumerated set. The term **nil** is defined in this way, as a member of the enumerated set **class-names**.

(3) Diagrams are defined by directed graphs. The term **expression** is defined in this way.

(4) Definitions of terms that designate evaluation sequences take the following form:

The term being defined, such as **scan**.

The forms that the evaluation mechanism used in the standard recognises as designating this term, such as *Z* ← **f**\ *B*.

An informal introduction indicating the purpose of the procedure. The informal introduction is considered commentary on the standard.

An evaluation sequence expressed in a formal, though English-like, language defined in the subsection **Evaluation Sequences**. A **conforming-implementation** is required to emulate the behaviour described in the evaluation sequence as modified by the **additional-requirements**, if any.

Examples, which show effects of the procedure specified by the evaluation sequence. Examples are considered commentary on the standard.

Additional Requirements, giving aspects of the behaviour required of this operation that cannot conveniently be expressed in the evaluation sequence.

3.2 NAMED ARRAYS IN EXAMPLES

In the examples in this standard, APL identifiers beginning with *N*, such as *N*234 represent numeric arrays whose shape and content are specified by the digits in the identifier. Each digit in the identifier specifies an element of the shape vector; each element of the array, when broken down into digits, gives the index of that element.

For example,

```
      N4
1 2 3 4
      N23
11 12 13
21 22 23
      N234
111 112 113 114
121 122 123 124
131 132 133 134

211 212 213 214
221 222 223 224
231 232 233 234
      N234[2;3;1]
231
```

3.3 NOTES

This standard contains notes that comment on the text of the standard, pointing out the significance of definitions, noting relationships between definitions, and otherwise making the text approachable. These notes are set in a different type style than the text of the standard, and are prefixed with the word "Note:". The following is an example of a note.

Note: This is an example of a note.

Notes never set requirements for conformance. They may suggest desired properties, but such suggestions are not mandatory for conformance.

3.4 CROSS-REFERENCES

The heading levels in this standard are chapter, section, and subsection. When cross-references are given, they are always to a subsection title. In the Index, subsections are treated like definitions: the page on which a subsection begins is always the first entry in the Index; subsequent page numbers in the index show where references are made to the subsection.

3.5 GENERAL DEFINITIONS

For the purpose of this standard, the definitions given in ISO 2382-15 and the following definitions apply:

3.5.1 **Program**: An application.

***Note:** The term is used in this standard to include everything from an APL expression to a collection of workspaces communicating via shared variables.*

3.5.2 **Implementation**: A combination of computer hardware and software that processes (APL) **programs**.

***Note:** An implementation is an instance of the object **system** specified by this standard.*

3.5.3 **Facility** (of an implementation): A unit of behaviour. Every **facility** is one of:

Defined-Facility: A **facility** fully specified in this standard and not designated **optional** or **implementation-defined**.

Optional-Facility: A **facility** fully specified in this standard and designated **optional**.

Implementation-Defined-Facility: A **facility** not fully specified by this standard that is designated **implementation-defined**.

Consistent-Extension: A **facility** not specified in this standard that, for a construct this standard specifies as producing an error, gives some effect other than signalling the specified error.

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4 COMPLIANCE

4.1 CONFORMING IMPLEMENTATIONS

An APL **implementation** conforms to this standard if it meets the following requirements for both behaviour and documentation.

4.1.1 Required Behaviour for Conforming Implementations

A **conforming-implementation** shall provide all **defined-facilities** and **implementation-defined-facilities**. Each such **facility** shall behave as specified by this standard.

A **conforming-implementation** may provide **optional-facilities**. If provided, an **optional-facility** shall behave as specified by this standard. Attempted use of an **optional-facility** that is not provided shall cause the **conforming-implementation** to signal an error. A **conforming-implementation** shall not replace the error signalled by a missing **optional-facility** with other behaviour.

A **conforming-implementation** may provide **consistent-extensions**. The presence of a **consistent-extension** shall not affect the behaviour of a **conforming-program**.

A **conforming-implementation** shall use algorithms that produce results that are the same as those produced by the evaluation sequences. Mathematical function algorithms shall have at least the accuracy that the algorithms given in the evaluation sequences would produce.

Note: *The evaluation sequences used in this standard are intended to specify results, not implementation techniques.*

The errors produced by the absence of an ***optional-facility*** *cannot be replaced by* ***consistent-extensions*** *in a* ***conforming-implementation****, since this would affect the behaviour of* ***conforming-programs*** *that use the* ***optional-facility****.*

4.1.2 Required Documentation for Conforming Implementations

A **conforming-implementation** shall provide a reference document that satisfies the following requirements for the documentation of **optional-facilities**, **implementation-defined-facilities**, and **consistent-extensions**:

4.1.2.1 Documentation of Optional-Facilities

A **conforming-implementation** shall document the presence or absence of each of the following **optional-facilities**:

Shared-Variable-Protocol. A mechanism that permits one session to exchange data with other autonomous sessions.

Statement-Separator-Facility. A mechanism for placing more than one statement on a line.

Trace-and-Stop-Control. A mechanism that assists the testing and correction of defined functions.

4.1.2.2 Documentation of Implementation-Defined-Facilities

A **conforming-implementation** shall document the following aspects of **implementation-defined-facilities**:

A description of the **character-set**. This shall include a table showing the correspondence between index positions in the **atomic-vector** and the members of the **required-character-set**. If the graphic symbols used in a **conforming-implementation** are dissimilar to those in **Table 1**, the correspondence between the graphic symbols used in the implementation and those in **Table 1** shall be given.

A description of the **numbers**. This shall include a characterisation of the internal representation used for **numbers**.

Descriptions of the characteristics of each **implementation-algorithm**.

The value of each **implementation-parameter**.

A description of each **internal-value-set**.

4.1.2.3 Consistent Extensions

A **conforming-implementation** shall document all **consistent-extensions** it provides. The documentation shall clearly indicate that the use of a **consistent-extension** prevents a **program** from conforming with this standard.

Note: *Implementers of* ***conforming-implementations*** *should, in general, be wary of replacing* ***limit-errors*** *with* ***consistent-extensions****, since these errors are the only safeguards a* ***conforming-program*** *has when attempting to operate in a* ***conforming-implementation*** *whose* ***implementation-parameters*** *are inadequate to support it.*

For example, if the ***limit-error*** *on* ***identifier-length-limit*** *were not signalled, a* ***conforming-program*** *with identifiers longer than the local* ***identifier-length-limit*** *would malfunction without warning.*

4.2 CONFORMING PROGRAMS

A **program** conforms to this standard if it meets the following requirements for both behaviour and documentation.

4.2.1 Required Behaviour for Conforming Programs

A **conforming-program** shall use only those **facilities** specified in this standard.

A **conforming-program** shall not use **consistent-extensions**.

A **conforming-program** shall not depend on the signalling of any error by a **conforming-implementation**.

*Note: **Conforming-programs** cannot depend on error behaviour, since **consistent-extensions** that replace errors are permitted in **conforming-implementations**.*

4.2.2 Required Documentation for Conforming Programs

A **conforming-program** shall document which **optional-facilities** it requires.

A **conforming-program** shall document any specific requirements it has for **implementation-parameters**.

*Note: A **conforming-program** may or may not work, and may or may not produce identical results on all **conforming-implementations**, because of inherent dependencies on **implementation-parameters** or **implementation-algorithms** such as the algorithm used for matrix inversion.*

*It is suggested that **conforming-programs** provide documentation detailing the values of **implementation-parameters** they require so that their suitability for a given **conforming-implementation** can be determined readily.*

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5 DEFINITIONS

5.1 CHARACTERS

Character-Set: An **implementation-defined** finite set.

Character: A member of the **implementation-defined** finite set **character-set**.

Required-Character-Set: An enumerated set whose members are designated by the graphic symbols in **Table 1**. The **character-set** provided by a **conforming-implementation** shall contain all members of the **required-character-set**.

Additional Requirement:

Members of the **required-character-set** are represented in this standard by graphic symbols in a particular typeface, as shown in **Table 1**. The graphic symbols associated with the **required-character-set** in a **conforming-implementation** are **implementation-defined**.

Note: There are conformance requirements associated with the ***required-character-set.*** *A* ***conforming-implementation*** *must publish, as part of its required documentation, a table giving the correspondence between the graphic symbols in* ***Table 1*** *and the* ***atomic-vector.*** *Where the graphic symbols provided are not similar in appearance to those used in this standard, the correspondence between the graphic symbols in this standard and those provided by the implementation must also be provided.*

Some of the graphic symbols given in Table 1 are not used to designate APL constructs in this standard. They are present to provide all keys on common terminal keyboards with corresponding symbols.

Table 1 — The Required Character Set

Alphabetic Characters

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

A̲ B̲ C̲ D̲ E̲ F̲ G̲ H̲ I̲ J̲ K̲ L̲ M̲ N̲ O̲ P̲ Q̲ R̲ S̲ T̲ U̲ V̲ W̲ X̲ Y̲ Z̲

Numeric Characters

0 1 2 3 4 5 6 7 8 9

Special Characters

⍺	alpha
↓	down arrow
←	left arrow
→	right arrow
↑	up arrow
-	bar
	blank
{	left brace
}	right brace
[	left bracket
]	right bracket
∨	down caret
⍱	down caret tilde
<	left caret
>	right caret
∧	up caret
⍲	up caret tilde
○	circle
⍉	circle backslash
⊖	circle bar
⍟	circle star
⌽	circle stile
:	colon
,	comma
∇	del
⍒	del stile
⍫	del tilde
∆	delta
⍋	delta stile
⍙	delta underbar
¨	diaeresis
⋄	diamond
÷	divide
$	dollar sign
.	dot
∊	epsilon
=	equal
≥	greater-than or equal
⍳	iota
∘	jot
≤	less-than or equal
×	multiply
≠	not equal
¯	overbar
(	left parenthesis
)	right parenthesis
+	plus
⎕	quad
⌹	quad divide
?	query
'	quote
!	quote dot
⍞	quote quad
⍴	rho
;	semicolon
∪	down shoe
⊂	left shoe
⊃	right shoe
∩	up shoe
⍝	up shoe jot
/	slash
\	back slash
⌿	slash bar
⍀	back slash bar
⋆	star
∣	stile
⌊	down stile
⌈	up stile
⊥	down tack
⍎	down tack jot
⊢	left tack
⊣	right tack
⊤	up tack
⍕	up tack jot
∼	tilde
_	underbar
⍵	omega

The names in **Table 1** are not to be considered part of this standard.

5.2 NUMBERS

Number-Set: An **implementation-defined** finite set used to represent arithmetic quantities.

Number: A member of the **number-set**.

*Note: The **number-set** is an abstraction used in this document to represent the floating-point arithmetic quantities of an arbitrary computer.*

5.2.1 Elementary Operations

Elementary-Operation: One of the following four **implementation-algorithms**.

A **Plus** B.

*Note: **Plus** implements addition.*

A **Minus** B.

*Note: **Minus** implements subtraction.*

A **Times** B.

*Note: **Times** implements multiplication.*

A **Divided-by** B.

*Note: **Divided-by** implements division.*

*Note: Each **elementary-operation** is a mapping from the Cartesian product of the **number-set** with itself back onto the union of the **number-set** and the **metaclass error**, as described in the subsection **Implementation-Algorithms**. It is assumed that members of the **metaclass error** are returned by **elementary-operations** when results cannot be represented as **numbers**. In particular, **exponent-overflow** is returned when the result of an **elementary-operation** is too large in magnitude to be represented by a **number**, **exponent-underflow** is returned when the result of an **elementary-operation** is non-zero but too small in magnitude to be represented by a **number** other than **zero**, and **domain-error** is returned by an **elementary-operation** called with arguments for which its mathematical counterpart is undefined, such as **one divided-by zero**.*

*The treatment of **exponent-overflow** and **exponent-underflow** is not specified by this standard, except as suggestions to the implementer. Implementers should avoid having **exponent-overflow** and **exponent-underflow** occur in the intermediate results of **implementation-algorithms** wherever possible. In any case, **exponent-underflow** should not cause a **limit-error**.*

*The fact that these fundamental algorithms are effectively undefined in the standard is intentional. A definition general enough to cover all known and possible floating-point systems was felt to be less useful than the requirement that the internal representation for **numbers** in a **conforming-implementation** be described in the **required-documentation** for the implementation.*

5.2.2 Number Constants

*Note: The following **numbers**, used in this standard, are defined here as terms to make clear the distinction between APL constant arrays represented as strings of APL digits, such as* 1*, and members of the the **implementation-defined** finite set containing **numbers**, such as **one**.*

*These terms are always set in **bold** when they are used in this formal sense. They are not cross-indexed.*

Zero: A **number** such that, for any **number** *A*, *A* **plus zero** is *A*.

One: A **number** such that, for any **number** *A*, *A* **times one** is *A*.

Negative-One: **Zero minus one.**

Two: **One plus one.**

Three: **Two plus one.**

Four: **Three plus one.**

One-half: **One divided-by two.**

5.2.3 Subsets of the Set of Numbers

Note: *The following subsets of the numbers are used to define the domains of operations.*

Boolean: **zero** or **one**.

Note: *False is represented by* ***zero****, true by* ***one****.*

Note: *A* ***counting-number*** *is a member of the subset of the* ***numbers*** *accessible under a successor function.*

Positive-Counting-Number: **one** or any **number** that can be generated by A **plus one**, where A is a **positive-counting-number**.

Negative-Counting-Number: The **number negative-one** or any **number** that can be generated by N **plus negative-one**, where N is a **negative-counting-number**.

Nonnegative-Counting-Number: A **positive-counting-number** or **zero**.

Counting-Number: A **negative-counting-number** or a **nonnegative-counting-number**.

Note: *An* ***integer*** *is a member of the subset of the* ***numbers*** *accessible from* ***zero*** *and* ***one*** *through addition and subtraction.*

Positive-Integer: A **positive-counting-number** or a **number** that can be generated by A **plus** B, where A and B are **positive-integers**.

Negative-Integer: A **negative-counting-number** or a **number** that can be generated by A **plus** B, where A and B are **negative-integers**.

Nonnegative-Integer: A **positive-integer** or **zero**.

Integer: A **negative-integer** or a **nonnegative-integer**.

Positive-Number: A **positive-integer**, or a **number** other than **zero** that can be generated by A **times** B, by A **divided-by** B, or by A **plus** B, where A and B are **positive-numbers**.

Negative-Number: A **negative-integer**, or a **number** other than **zero** that can be generated by A **times** B, by A **divided-by** B, or by C **plus** D, where A is a **positive-number** and B, C and D are **negative-numbers**.

Nonnegative-Number: A **positive-number** or **zero**.

Note: *The* ***numbers*** *are assumed to comprise the* ***negative-numbers*** *and the* ***nonnegative-numbers****.*

Example

Consider a normalised base-2 floating point number system with a two position exponent field and a two position mantissa field with the radix point between the digits. The decimal values representable with this system and the categories into which each falls are as follows:

Exponent in Base 2	*Mantissa in Base 2*	*Value in Base 10*	*Boolean*	*Counting-Number*	*Integer*
00	*00*	*0*	*Y*	*Y*	*Y*
00	*10*	*1*	*Y*	*Y*	*Y*
00	*11*	*1.5*			
01	*10*	*2*		*Y*	*Y*
01	*11*	*3*		*Y*	*Y*
10	*10*	*4*		*Y*	*Y*
10	*11*	*6*			*Y*
11	*10*	*8*			*Y*
11	*11*	*12*			*Y*

As can be seen in this example, ***counting-numbers*** *form a dense subset of the* ***integers****; the largest* ***counting-number*** *is typically a power of the number system base.*

A given number, such as a ***counting-number****, may have several hardware representations. Except for their effect on system resources, these representations should be indistinguishable to a* ***conforming-program****.*

5.2.4 Implementation Algorithms

An **Implementation-Algorithm** is an algorithm used in this standard whose behaviour is **implementation-defined**. The following implementation algorithms are used in this standard.

Cosine
Current-Time
Deal
Display
Divided-by
Exponential
Function-Display
Gamma-Function
Hyperbolic-Cosine
Hyperbolic-Sine
Hyperbolic-Tangent
Inverse-Cosine
Inverse-Hyperbolic-Cosine
Inverse-Hyperbolic-Sine
Inverse-Hyperbolic-Tangent
Inverse-Sine
Inverse-Tangent
Matrix-Divide
Minus
Modulo
Natural-Logarithm
Next-Definition-Line
Numeric-Input-Conversion
Numeric-Output-Conversion
Pi-Times
Plus
Pseudorandom-Number-Generator
Read-Keyboard
Sine
Tangent
Times
Time-Stamp
To-the-Power
Trace-Display

The following **implementation-algorithms** take a **number** as an argument and return either a **number** or an **error**: **Cosine Exponential Gamma-Function Hyperbolic-Cosine Hyperbolic-Sine Hyperbolic-Tangent Inverse-Cosine Inverse-Hyperbolic-Cosine Inverse-Hyperbolic-Sine Inverse-Hyperbolic-Tangent Inverse-Sine Inverse-Tangent Natural-Logarithm Pi-Times Sine Tangent**.

The following **implementation-algorithms** take two **numbers** as arguments and return either a **number** or an **error**: **Divided-by Minus Modulo Plus Times To-the-Power**.

The properties of the remaining **implementation-algorithms** are described in the chapters in which they are used.

5.2.5 Defined Operations

A **Equals** B: An operation that, for A and B **numbers**, returns **one** if A and B are the same **number**, and **zero** otherwise.

Sign of A: An operation that, for A a **number**, returns **one** if A is a **positive-number**, **zero** if A is **zero**, and **negative-one** if A is a **negative-number**.

A is **Greater-Than** B: An operation that, for A and B **numbers**, returns **one** if A is a **positive-number** and B is a **negative-number**, returns **one** if A **minus** B is a **positive-number**, and returns **zero** otherwise.

A is **Less-Than** B: An operation that, for A and B **numbers**, returns B **greater-than** A.

Negation of A: An operation that, for any **number** A, returns **zero minus** A.

Absolute-Value of A: An operation that, for any **number** A, returns A if A is a **nonnegative-number**, and the **negation** of A otherwise.

Exact-Floor of A: An operation that, for any **number** A, returns the highest **integer** B such that B **equals** A or B is **less-than** A.

Exact-Ceiling of A: An operation that, for any **number** A, returns the lowest **integer** B, such that B **equals** A or B is **greater-than** A.

Note: *The two preceding definitions assume that the highest and lowest numbers are integers (as is in practice the case in computers).*

Open-Interval-Between A and B: An operation that, for any **numbers** A and B, returns a subset of the **numbers** as follows: if A is not **greater-than** B, the set of all **numbers greater-than** A and **less-than** B; otherwise, the **open-interval-between** B and A.

Closed-Interval-Between A and B: An operation that, for any **numbers** A and B, returns a subset of the **numbers** consisting of A, B, and the **open-interval-between** A and B.

Left-Neighbourhood of A **Determined-by** B: An operation that, for any **numbers** A and B, returns the **closed-interval-between** A and A **minus** the **absolute-value** of (A **times** B).

Distance-Between A and B: An operation that, for any two **numbers** A and B, returns the **absolute-value** of A **minus** B.

A is **Tolerantly-Equal** to B **Within** C: An operation that, given three **numbers** A, B, and C, returns a **Boolean** Z determined as follows:

If A **equals** B, then Z is **one**.
If the **sign of** A differs from the **sign of** B, then Z is **zero**.
If the **absolute-value** of B is **greater-than** that of A, and the **absolute-value** of A is in the **left-neighbourhood** of the **absolute-value** of B **determined-by** C, then Z is **one**.
If the **absolute-value** of A is **greater-than** that of B, and the **absolute-value** of B is in the **left-neighbourhood** of the **absolute-value** of A **determined-by** C, then Z is **one**.
Otherwise, Z is **zero**.

Tolerant-Floor of A **Within** B: An operation that, for A a **number** and B a **nonnegative-number**, returns an **integer** Z determined as follows:

Let C stand for the **integer** for which the **distance-between** A and C is the smallest. If there are two such **integers**, let C be the one that has the larger **absolute-value**.

If C is **less-than** A, then Z is C.

Otherwise,

If C is not **zero**, then Z is C if A is **tolerantly-equal** to C **within** B, and Z is C **minus one** otherwise.
If C is **zero**, then Z is **zero** if A is in the **closed-interval-between** the **negation** of B and **zero**, and **negative-one** otherwise.

A is **Integral-Within** B: An operation that, for two **numbers** A and B, returns a **Boolean** Z determined as follows:

Let C stand for the **negation** of A.
Z is **one** if the **tolerant-floor** of C **within** B **equals** the **negation** of the **tolerant-floor** of A **within** B, and **zero** otherwise.

A is a **Near-Integer**: An operation that, for a **number** A, returns **one** if A is **integral-within integer-tolerance**, and **zero** otherwise.

Note: *This definition contains a forward reference to* ***integer-tolerance****.*

Integer-Nearest-to A: An operation that, for a **near-integer** A, returns the **tolerant-floor** of A **within integer-tolerance**.

A **is Near-Boolean**: An operation that, for a **near-integer** A, returns **one** if the **integer-nearest-to** A is a **Boolean**, and **zero** otherwise.

Note: *Note that* ***near-integers*** *and* ***near-Booleans*** *include numbers whose* ***absolute-value*** *is* ***less-than integer-tolerance****. The operation* ***integer-nearest-to*** *maps such numbers to* ***zero****.*

5.3 OBJECTS

Note: *The description of APL in this document is based on objects – abstract data structures that are described in terms of characters, numbers, and elementary set theory.*

Each object has a small set of named attributes that take on values that are characters, numbers, or other objects.

Each object has additionally a small set of operations that can be performed upon it.

All the objects in the description are defined here, as are all the attributes of each object. The operations are introduced as they are needed. All objects, attributes, and operations are cross-referenced in the index.

5.3.1 Lists

Index: A **nonnegative-counting-number** less than or equal to **index-limit**.

List: An object with the following attributes:

Index-Set: A finite set I of **positive-counting-numbers** chosen so that for every subset of I, except the empty set, the cardinality of that subset is in I.

Value-Set: A finite set in a specific correspondence with the **index-set** of the **list**.

Empty-List: A **list**, the cardinality of whose **index-set** is **zero**.

Nonempty-List: A **list**, the cardinality of whose **index-set** is **greater-than zero**.

Number-of-Items in L: The cardinality of the **index-set** of the **list** L.

Item X of L: An operation that, for X a member of the **index-set** of the **list** L, returns the member of the **value-set** of L associated with X by the correspondence between the **value-set** of L and the **index-set** of L.

Note: *Note that this form of indexing is always in origin one.*

First-Item in L: An operation that for a **nonempty-list** L returns **item one** of L.

Last-Item in L: An operation that for a **nonempty-list** L, and for C the **number-of-items** in L, returns **item** C of L.

Rest-of L: An operation that, for a **nonempty-list** L whose **index-set** has cardinality C, returns a second **list** R whose **index-set** has cardinality C **minus one**, such that for each **item** J in the **index-set** of R, **item** J of R is **item** (J **plus one**) of L.

Product-of L: An operation defined on a **list** of **numbers** L as follows:

If the **number-of-items** in L is **zero**, **one**.
If the **number-of-items** in L is **one**, the **first-item** in L.
If the **number-of-items** in L is **greater-than one**, **first-item** in L **times** the **product-of** the **rest-of** L.

Prefix: A (possibly empty) **list** P is a **prefix** of a **list** L if the **index-set** of P is a subset of the **index-set** of L, and each **item** of P is the same as the corresponding **item** of L.

5.3.2 Arrays

Array-Type: An enumerated set containing the members **character** and **numeric**.

Array: An object with the following attributes:

Shape-list: A **list** of **nonnegative-counting-numbers**.

Ravel-list: A **list**. The **items** of the **list** may be either **characters** or **numbers**.

Type: A member of the enumerated set **array-type**.

The **number-of-items** in the **ravel-list** of an **array** A is the same as the **product-of** the **shape-list** of A.

If the **type** of A is **character**, the **ravel-list** of A is a **list** of **characters**; if the **type** of A is **numeric**, the **ravel-list** of A is a **list** of **numbers**.

Rank of A: An operation that, for an **array** or **array-of-vectors** A, returns the **number-of-items** in the **shape-list** of A.

Scalar: An **array** whose **rank** is **zero**.

Numeric-Scalar with value I: For I, a **number**, the **scalar** Z such that the **type** of Z is **numeric** and the **ravel-list** of Z is the **list** L such that the **number-of-items** in L is **one** and the **first-item** of L is I.

Count of A: For A an **array**, the **product-of** the **shape-list** of A.

Vector: An **array** whose **rank** is **one**.

Length of A: For A a **vector**, the **count** of A.

Typical-Element of A: For A an **array**, if the **type** of A is **character**, the **character** blank; if the **type** of A is **numeric**, **zero**.

Empty: Of an **array** A, the **count** of A is **zero**.

One-Element-Vector: A **vector** A is a **one-element-vector** if the **length** of A is **one**.

K is a **Valid-Axis** of A: An operation that, for A and K **arrays**, returns **one** if K is a **scalar** or **one-element-vector**, and the **first-item** in the **ravel-list** of K is a **near-integer**, the **integer-nearest-to** which is a member of the **index-set** of the **shape-list** of A; and returns **zero** otherwise.

Axis K of A: An operation that, for A an **array** and K a **valid-axis** of A, returns **item** K of the **shape-list** of A.

Array-of-vectors: An object with the following attributes:

Shape-list: A **list** of **nonnegative-counting-numbers**.

Ravel-list: A **list**. The **items** of the **list** are **vectors**, all of the same **type**.

Type: A member of the enumerated set **array-type**.

Along-Axis K of A: An operation that, for an **array** A with non-zero **rank** N, produces Z, an **array-of-vectors** of **rank** $N-1$ such that the **shape-list** of Z is the **shape-list** of A with **item** K omitted. Each **item** of the **ravel-list** of Z is a **vector** whose **length** is the same as **axis** K of A.

Note: Some operations on vectors extend to arrays of greater rank in a manner similar to ***scalar-extension****.* ***Along-axis*** *is an expository device used in the definition of these operations.*

Vector-Item *I* of *A*: An operation that, for an **array-of-vectors** *A*, returns **item** *I* of the **ravel-list** of *A*.

Ravel-Along-Axis *K* of *A*: An operation that, for an **array** *A* with non-zero **rank** *N*, produces an **array-of-vectors** *Z* such that the **shape-list** of *Z* is the **product-of** the **shape-list** of *A* with **item** *K* omitted, and the **ravel-list** of *Z* is the **ravel-list** of **along-axis** *K* of *A*.

First-Scalar in *A*: An operation that, for *A* a nonempty **array**, returns a **scalar** *Z* such that the **type** of *Z* is the **type** of *A* and the **ravel-list** of *Z* is the **first-item** in the **ravel-list** of *A*.

Remainder-of *A*: An operation that, for *A* a non-empty **vector**, returns a **vector** *Z* such that the **type** of *Z* is the **type** of *A*, the **length** of *Z* is **negative-one plus** the **length** of *A*, and the **ravel-list** of *Z* is the **rest-of** the **ravel-list** of *A*.

Row *I* of *A*: An operation that, for *A* an **array** of **rank two** and *I* a **number**, returns **vector-item** *I* **along-axis two** of *A*.

Number-of-Rows in *A*: An operation that, for *A* an **array** of **rank two**, returns **item one** of the **shape-list** of *A*.

Integer-Array-Nearest-to *A*: An operation that, for *A* an **array** having the property that each **item** of the **ravel-list** of *A* is a **near-integer**, returns a **numeric array** *Z* such that the **shape-list** of *Z* is the same as the **shape-list** of *A* and each **item** of the **ravel-list** of *Z* is the **integer-nearest-to** the corresponding **item** of the **ravel-list** of *A*.

Boolean-Array-Nearest-to *A*: An operation that, for *A* an **array** having the property that each **item** of the **ravel-list** of *A* is a **near-Boolean**, returns the **integer-array-nearest-to** *A*.

5.3.3 Defined-Functions

Note: A ***defined-function*** *represents a function defined by a user. The attributes of a* ***defined-function*** *are given here. Operations are described in the* ***Defined Functions*** *chapter.*

Defined-Function: An object with the following attributes:

Canonical-Representation: A **character array** whose **rank** is **two**.

Stop-Vector: A **numeric vector**.

Trace-Vector: A **numeric vector**.

5.3.4 Tokens

Class-Names: A set containing the following members:

- **Assignment-Arrow**
- **Axis-Error**
- **Branch**
- **Branch-Arrow**
- **Character-Literal**
- **Clear-State-Indicator**
- **Colon**
- **Command-Complete**
- **Committed-Value**
- **Complete-Index-List**
- **Constant**
- **Defined-Function**
- **Defined-Function-Name**
- **Definition-Error**
- **Distinguished-Identifier**
- **Domain-Error**
- **Dyadic-Operator**
- **Elided-Index-Marker**
- **Escape**
- **Implicit-Error**
- **Incorrect-Command**
- **Index-Error**
- **Index-Separator**
- **Interrupt**
- **Label**
- **Label-Name**
- **Left-Argument-Name**
- **Left-Axis-Bracket**
- **Left-End-of-Statement**
- **Left-Index-Bracket**
- **Left-Parenthesis**
- **Length-Error**
- **Limit-Error**
- **Local-Name**
- **Monadic-Operator**
- **Nil**
- **Niladic-Defined-Function**
- **Niladic-Defined-Function-Name**
- **Niladic-System-Function-Name**
- **Not-Copied**
- **Not-Erased**
- **Not-Found**
- **Not-Saved**
- **Numeric-Literal**
- **Partial-Index-List**
- **Primitive**
- **Primitive-Function**
- **Rank-Error**
- **Result-Name**
- **Right-Argument-Name**
- **Right-Axis-Bracket**
- **Right-End-of-Statement**
- **Right-Index-Bracket**
- **Right-Parenthesis**
- **Semicolon**
- **Shared-Variable**
- **Shared-Variable-Name**
- **Simple-Identifier**
- **Small-Circle**
- **Syntax-Error**
- **System-Function-Name**
- **System-Variable-Name**
- **Unwind**
- **Value-Error**
- **Variable**
- **Variable-Name**

Token: An object with the following attributes:

Class: A member of the set **Class-Names**.

Content: A **number**, a **character**, or an object, according to the **class** of the **token**, as indicated in Table 2.

Note: Tokens represent the internal objects manipulated by an implementation of APL.

*The **class** of a **token** is a straightforward indication of the sort of object it represents. A **token** of **class nil**, for example, is used to return a **value-error** from a defined function that does not set its result name. Note that **class-names** is an enumerated set. This means that nil has no definition other than its literal appearance on this page. Its significance, like the significance of APL characters in this document, lies in the use of its name in evaluation sequences.*

*The **content** of a **token** varies with the **class** of the **token**. If specified, the content is a **character**, a **list** of **characters**, a **list** of **numbers**, an array, an **index-list**, a **shared-variable**, or a **defined-function**.*

Table 2 — Relationship between Class-Name and Content

Class-Name	Content	Class-Name	Content
Assignment-Arrow		Local-Name	A list of characters
Axis-Error		Monadic-Operator	A character
Branch	An array	Nil	
Branch-Arrow		Niladic-Defined-Function	A defined-function
Character-Literal	A list of characters	Niladic-Defined-Function-Name	A list of characters
Clear-State-Indicator		Niladic-System-Function-Name	A list of characters
Colon		Not-Copied	
Command-Complete		Not-Erased	
Committed-Value	An array	Not-Found	
Complete-Index-List	An index-list	Not-Saved	
Constant	An array	Numeric-Literal	A list of numbers
Defined-Function	A defined-function	Partial-Index-List	An index-list
Defined-Function-Name	A list of characters	Primitive	A character
Definition-Error		Primitive-Function	A character
Distinguished-Identifier	A list of characters	Rank-Error	
Domain-Error		Result-Name	A list of characters
Dyadic-Operator	A character	Right-Argument-Name	A list of characters
Elided-Index-Marker		Right-Axis-Bracket	
Escape		Right-End-of-Statement	
Implicit-Error		Right-Index-Bracket	
Incorrect-Command		Right-Parenthesis	
Index-Error		Semicolon	
Index-Separator		Shared-Variable	A shared-variable
Interrupt		Shared-Variable-Name	A list of characters
Label	An array	Simple-Identifier	A list of characters
Label-Name	A list of characters	Small-Circle	
Left-Argument-Name	A list of characters	Syntax-Error	
Left-Axis-Bracket		System-Function-Name	A list of characters
Left-End-of-Statement		System-Variable-Name	A list of characters
Left-Index-Bracket		Unwind	
Left-Parenthesis		Value-Error	
Length-Error		Variable	An array
Limit-Error		Variable-Name	A list of characters

5.3.4.1 Metaclasses

*Note: Metaclasses are subsets of the enumerated set **class-names**. They are used to shorten evaluation sequences by abstracting a sequence of tests on the **class** of a **token** into a single test for membership in a metaclass. "If B is an **error**," is equivalent to "If the **class** of B is in the metaclass **error**."*

Metaclass: A subset of the enumerated set **class-names**.

Identifier: A metaclass containing **simple-identifier** and **distinguished-identifier**.

Literal: A metaclass containing **character-literal** and **numeric-literal**.

Lexical-Unit: A metaclass containing **primitive**, **literal**, and **identifier**.

Value: A metaclass containing **committed-value** and **constant**.

Delimiter: A metaclass containing **primitive-function**, **branch-arrow**, **assignment-arrow**, **left-end-of-statement**, **right-end-of-statement**, **left-index-bracket**, **right-index-bracket**, **elided-index-marker**, **left-axis-bracket**, **right-axis-bracket**, **left-parenthesis**, **right-parenthesis**, **small-circle**, and **semicolon**.

Defined-Name: A metaclass containing **shared-variable-name**, **variable-name**, **defined-function-name**, **niladic-defined-function-name**, **result-name**, **left-argument-name**, **right-argument-name**, **label-name**, and **local-name**.

System-Name: A metaclass containing **system-variable-name**, **system-function-name**, and **niladic-system-function-name**.

Classified-Name: A metaclass containing the members of **system-name** and **defined-name**.

Syntactic-Unit: A metaclass containing the members of **classified-name** and **delimiter**.

Error: A metaclass containing **axis-error**, **domain-error**, **implicit-error**, **index-error**, **length-error**, **limit-error**, **rank-error**, **syntax-error**, **value-error**, and **interrupt**.

*Note: This **metaclass** includes only errors that occur in the evaluation of APL statements.*

Report: A metaclass containing **incorrect-command**, **not-copied**, **not-erased**, **not-found**, and **not-saved**.

Exception: A metaclass containing **branch**, **escape**, **clear-state-indicator**, **unwind**, and the members of **error** and **report**.

Result: A metaclass containing **nil** and the members of **exception** and **value**.

5.3.4.2 Index-List

Index-List: A (possibly empty) **list** consisting of **tokens** whose **class** is either **constant** or **elided-index-marker**.

5.3.5 Symbols

Symbol: An object with the following attributes:

Name: A **list** of **characters**.

Referent-List: A **list** of **tokens**.

5.3.6 Contexts

Mode-Names: An enumerated set containing the members **immediate-execution**, **execute**, **function-definition**, **quad-input**, and **defined-function**.

Context: An object with the following attributes:

Mode: An element of the enumerated set **mode-names**.

Stack: A **list** of **tokens**.

Current-Line: A **list** of **characters**.

Current-Statement: A **list** of **tokens**.

Current-Function: If **mode** is **defined-function**, a **defined-function**; otherwise undefined.

Current-Line-Number: If **mode** is **defined-function**, an **index**; otherwise undefined.

5.3.7 Workspaces

Note: The workspace is the basic organisational unit in an APL system. A workspace contains data, programs, execution status, and environmental information.

Workspace-Presence: An enumerated set containing **absent** and **present**.

Workspace: An object with the following attributes:

Owner: A **user-identification**.

Workspace-Name: A **list** of **characters**.

Symbol-Table: A **list** of all **symbols** whose **names** are unique.

State-Indicator: A **list** of **contexts**.

Existential-Property: A member of the enumerated set **workspace-presence**.

Clear-Workspace: A **workspace** with the following values:

Owner: **this-owner**.

Workspace-Name: The **clear-workspace-identifier**.

Symbol-Table: A **list** of all **symbols** whose **names** are unique and whose **referent-lists** each consist of the **list** whose only member is **nil**.

State-Indicator: An empty **list** of **contexts**.

Existential-Property: **absent**.

5.3.8 Sessions

Note: *A user interacts with an APL system through a session, an abstraction that represents a hypothetical machine capable of carrying out the evaluation sequences in the standard.*

The attributes of a ***session*** *are given here.*

Session-Identification: Either a **number** or a **list** of **characters**, depending upon the **implementation-parameter session-identification-type**. **User-Identification**: Either a **number** or a **list** of **characters**, depending upon the **implementation-parameter user-identification-type**.

Keyboard-States: An enumerated set containing the members **open-keyboard** and **locked-keyboard**.

Session: An object with the following attributes:

Active-Workspace: A **workspace**.

This-Session: A **session-identification**.

This-Owner: A **user-identification**.

Attention-Flag: A member of the enumerated set **Boolean**.

Keyboard-State: A member of the enumerated set **keyboard-states**.

Current-Prompt: A **character vector**.

Quote-Quad-Prompt: A **character vector**.

Current-Context: The **first-item** in the **state-indicator** of the **active-workspace**.

Current-Stack: The **stack** of the **current-context**.

Symbol-Named-By *T*: The **symbol** in the **symbol-table** of the **active-workspace** whose **name** is the same as the **content** of the **token** *T*.

Current-Referent of *T*: The **first-item** in the **referent-list** of the **symbol-named-by** *T*.

Current-Class of *T*: The **class** of the **current-referent** of *T*.

Current-Content of *T*: The **content** of the **current-referent** of *T*.

Comparison-Tolerance: The **current-content** of ⎕*CT*.

Random-Link: The **current-content** of ⎕*RL*.

Print-Precision: The **current-content** of ⎕*PP*.

Index-Origin: The **current-content** of ⎕*IO*.

Latent-Expression: The **current-content** of ⎕*LX*.

System-Parameter: Any of **comparison-tolerance** , **random-link** , **print-precision** , **index-origin** , or **latent-expression** .

The initial values of **system-parameters** in a **clear-workspace** are **implementation-defined**.

5.3.9 Shared-Variables

*Note: A **shared-variable** is a variable shared between two **sessions**.*

Shared Variables are an **optional-facility**.

Shared-Variable: An object with the following attributes:

Session-A: A **session-identification**.

Session-A-Active: A **Boolean**.

Session-A-ACV: A **Boolean vector** of **length four**.

Session-B: A **session-identification**.

Session-B-Active: A **Boolean**.

Session-B-ACV: A **Boolean vector** of **length four**.

Shared-Name: An **identifier**.

Shared-Value: A **token**, either a **constant** or **nil**.

State: An **integer**, either **zero**, **one**, or **two**.

*Note: The operations in the chapter **Shared Variables** use the **shared-variable** attributes as follows:*

*Session-A: The **session-identification** of the first **session** to offer the **shared-variable**.*

*Session-A-Active: A **Boolean**; **one** if **session-A** is currently sharing this **shared-variable**, **zero** if not.*

*Session-A-ACV: The contribution of **session-A** to the ACV.*

*Session-B: The **session-identification** of the **session** with which **session-A** offered to share the **shared-variable**; this may be the **general-offer** while **state** is **one**.*

*Session-B-Active: A **Boolean**; **one** if **session-B** is currently sharing this **shared-variable**, **zero** if not.*

*Session-B-ACV: The contribution of **session-B** to the ACV.*

*Shared-Name: An **identifier**, the **name** **session-A** designated for this **shared-variable**.*

*Shared-Value: A **token** of class **constant** or **nil**; the current value of this **shared-variable**.*

*State: An **integer**, either **zero**, **one**, or **two**, used in combination with the ACV to determine which operations must be delayed.*

5.3.10 Systems

Note: A system represents a set of active APL users, a library, and, optionally, a shared variable facility.

System: An object with the following attributes:

Library: A **list** of **workspaces** in which each combination of possible values for the attributes **owner** and **workspace-name** occurs exactly once.

Active-Users: A **list** of **sessions**.

Shared-Variable-List: A **list** of **shared-variables**.

Implementation-Parameters: The following quantities, referred to in this standard by name, whose values are **implementation-defined**:

Atomic-Vector: An **implementation-defined character vector** containing every member of the **character-set** exactly once.

Initial-Comparison-Tolerance: A member of the **internal-value-set** for **comparison-tolerance** that is the value of **comparison-tolerance** in a **clear-workspace**.

Initial-Index-Origin: A member of the **internal-value-set** for **index-origin** that is the value of **index-origin** in a **clear-workspace**.

Initial-Latent-Expression: A member of the **internal-value-set** for **latent-expression** that is the value of **latent-expression** in a **clear-workspace**.

Initial-Print-Precision: A member of the **internal-value-set** for **print-precision** that is the value of **print-precision** in a **clear-workspace**.

Initial-Random-Link: A member of the **internal-value-set** for **random-link** that is the value of **random-link** in a **clear-workspace**.

Clear-Workspace-Identifier: A **list** of **characters**.

Positive-Number-Limit: The **number** **greater-than** all other **numbers**.

Negative-Number-Limit: The **number** **less-than** all other **numbers**.

Positive-Counting-Number-Limit: The **counting-number** greater than all other **counting-numbers**.

Negative-Counting-Number-Limit: The **counting-number** **less-than** all other **counting-numbers**.

Index-Limit: The **index** greater than all other indices. This value specifies the maximum value of any **item** of the **shape-list** of any **array**, ignoring storage limitations.

Count-Limit: An **index** not greater than **index-limit** that specifies the maximum value for the **count** of an array, ignoring storage limitations.

Rank-Limit: An **index** not greater than **count-limit** that specifies the maximum value for the **rank** of any array, ignoring storage limitations.

Workspace-Name-Length-Limit: A **positive-counting-number** that specifies the maximum number of **characters** in a **workspace-name**.

Identifier-Length-Limit: A **positive-counting-number** not greater than **count-limit** that specifies the maximum number of **characters** in an **identifier**.

Quote-Quad-Output-Limit: A **nonnegative-counting-number** that specifies the maximum number of **characters** that can be used in a prompt set by **Quote Quad Output**.

Comparison-Tolerance-Limit: The largest **number** permitted by the implementation for the system parameter **comparison-tolerance** .

Integer-Tolerance: A **number** not greater than **comparison-tolerance-limit** used to determine whether a given number is to be considered integral.

Full-Print-Precision: The smallest **positive-counting-number** which, when used as the value of **print-precision** , causes **numeric-output-conversion** to produce a unique **numeric-scalar-literal** for every **number**.

Print-Precision-Limit: The largest **positive-counting-number** permitted by the implementation for the system parameter **print-precision** .

Note: *Print-precision-limit must be at least* ***full-print-precision*** *if every unique* ***number*** *is to have a unique decimal representation.*

Exponent-Field-Width: A **positive-counting-number** giving the number of places, including sign and trailing blanks, used for the exponent field in the result of **dyadic format**.

Session-Identification-Type: A member of the enumerated set **array-type**, either **character** or **numeric.**

User-Identification-Type: A member of the enumerated set **array-type**, either **character** or **numeric**.

Indent-Prompt: A **list** of **characters** used to indicate to the user that the **session** is in **immediate-execution** mode.

Quad-Prompt: A **list** of **characters** used to indicate to the user that the **session** is in **quad-input** mode.

Function-Definition-Prompt: A **list** of **characters** used to indicate to the user that the **session** is in **function-definition** mode.

Line-Limit: A **positive-counting-number** that specifies the maximum number of lines permitted in a defined function, ignoring storage limitations.

Definition-Line-Limit: A **positive-number** that specifies the maximum value of a line number in **function-definition** mode.

General-Offer: A reserved **session-identification** used to indicate that the offerer of a **shared-variable** is willing to share the proffered variable with any other **session**. This **implementation-parameter** is required only if the **optional-facility** **shared-variable-protocol** is present.

Any action that would cause a limit specified by an implementation parameter to be exceeded shall signal a **limit-error**.

5.4 EVALUATION SEQUENCES

The evaluation sequences that define APL operations in the remainder of this standard are written in English in the imperative mood. The English phrases used in evaluation sequences are restricted to the set specified in this subsection.

Indention is used in evaluation sequences to indicate scope, typically of the consequent of an implication.

For example, in the evaluation sequence fragment below, the indented text is evaluated only if both A and B are ***vectors****; the "otherwise" clause is evaluated only if at least one of A or B is not a* ***vector****.*

```
If    A is a vector and B is a vector,
      If    A is empty and ...
Otherwise, return ...
```

Expressions in APL are used in evaluation sequences. A given evaluation sequence uses only APL operations that have been specified earlier in the standard. Where indices are generated or used by APL expressions in evaluation sequences, they are evaluated with origin **one**. The APL relational operations are never used in evaluation sequences unless they are qualified with the value to be used for **comparison-tolerance** .

5.4.1 Evaluation Sequence Phrases

Note: *The following phrases are used in evaluation sequences. They are not set in* ***bold*** *type nor cross-referenced when employed in evaluation sequences.*

For all A, C: An evaluation sequence phrase used to specify that the action or condition specified by the consequent C is to be performed or checked for every value in the antecedent A.

Example:

```
For all    I in the index-set of A,
      Set item I of the ravel-list of A to zero.
```

For form *F*, *C*:

For pattern *F*, *C*:

An evaluation sequence phrase used to specify that the actions listed in the consequent *C* are to be performed only if the pattern or form *F* is the one that caused this evaluation sequence to be selected.

Example:

For form *A* ⌽ *B*

If *B* *is a* ***scalar*** ...

For form *A* ⊖ *B*

If ...

If *A*, *C*: An evaluation sequence phrase used to specify that the actions listed in the consequent *C* are to be performed only if the value of the antecedent *A* is **one**.

If *T* is an **mc**, *C*: For *T* a **token** and **mc** a **metaclass**, an evaluation sequence phrase equivalent to "If the **class** of *T* is in the **metaclass mc**, *C*."

Let *A* **stand for** *B*: An evaluation sequence phrase used to indicate that the name *A* is to be an abbreviation for the phrase *B* in subsequent evaluation sequence lines.

Otherwise, *C*: An evaluation sequence phrase used to indicate that the actions listed in consequent *C* are to be performed only if the antecedent in the immediately preceding **if** phrase was **zero**. If a consequent is an indented paragraph, the immediately preceding **if** statement is the one at the same level of indention as the **otherwise** phrase.

Repeat: An evaluation sequence phrase used to indicate that the block of text indented after the **repeat** is to be executed repeatedly until a **return** or **signal** phrase is encountered. The end of a repeated block is indicated by the parenthetic remark "(End of repeated block)."

Return *X*: An evaluation sequence phrase used to specify that evaluation of this evaluation sequence is to stop and that a **token** is to be returned to the caller of the evaluation sequence. If *X* is a **token**, then *X* is returned; if *X* is an **array**, a **token** of class **constant** and content *X* is returned.

Set *A* **to** *B*: An evaluation sequence phrase used to specify that the referent of *A* is to be assigned the value *B*.

Signal *X*: An evaluation sequence phrase used to specify that evaluation of this evaluation sequence is to stop and that a **token** whose **class** is *X*, where *X* is an **error**, is to be returned.

Using *O*, *C*: An evaluation sequence phrase used to indicate that the consequent *C* is to be evaluated against the specific object *O*. This construct is used, for example, in the description of shared variables to indicate which shared variable is to be changed.

Wait until: An evaluation sequence phrase that indicates that the **session** is waiting for a condition to hold before continuing.

5.4.2 Diagrams

*Note: **Diagrams** are used in this standard to indicate permissible sequences of **characters** or of **tokens**.*

Character-Diagram: A graph that designates a subset of the set of all **lists** of **characters**.

Token-Diagram: A graph that designates a subset of the set of all **lists** of **tokens**.

Thread D **with** A: An evaluation sequence phrase used to indicate that a search is to be made for a route through the diagram D that corresponds to the list or part of a list A.

A **character-diagram** is **threaded** with a **list** of **characters** by setting a list-cursor to the first item in the **list**, and a diagram-cursor to the arrow-tail '>>——', in the diagram, then progressing along paths in the diagram to the arrow-head '——>>'. The diagram-cursor advances to the left only through a left rectifier '—<—'. It advances upward only after it has just passed a right rectifier '—>—' or a left rectifier '—<—'. Otherwise, the diagram-cursor advances only rightward and downward. At a junction, any unlabelled path may be taken only if all labelled paths have already been tried.

For the diagram-cursor to advance along a path labelled with a graphic symbol such as ÷, that graphic symbol must be pointed to by the list-cursor, which then also advances. For the diagram-cursor to advance along a path labelled with a diagram name, that diagram must itself be threaded, using the same list-cursor and a new diagram-cursor. If in either case the diagram-cursor cannot advance, the diagram-cursor is set back to the previous junction, the list-cursor is set back correspondingly, and a new path is tried; or if the diagram-cursor is now at the arrow-tail, the diagram cannot be threaded with the list.

If a route is found through a diagram for a given **list** of **characters**, it is unique. In this case, the diagram can be **rethreaded**, following the same route without entering blind alleys. For example, characters are collected into identifier tokens through actions performed while a **character-diagram** is being **rethreaded**.

Token-diagrams are **threaded** exactly like **character-diagrams**, except that the items pointed to by the list-cursor are **tokens** and are matched either by their **class** or by both their **class** and their **content**.

Rethread D **with** A: An evaluation sequence phrase used to indicate that the route found by a previous **threaded** phrase is to be threaded again, in order to effect certain actions through **when** phrases.

When A, C: An evaluation sequence phrase used during a **rethread** phrase to indicate that when the antecedent A is true, the consequent C is to be performed.

A **Matches** D: An operation that, for diagram D and **list** A, is true if A is a member of the set of **lists** designated by D, and false otherwise.

A **list** matches a diagram if it can be **threaded** in such a way that there are no items remaining in the **list** when the final exit path is taken.

*Note: The question of whether a **list matches** a diagram is different from the question of whether a **list** can **thread** a diagram, since there will normally be items left over in a **list** once a diagram has been **threaded** -- in collecting the digits in a number, for example, the **digit** diagram removes only one digit at a time from a list of **characters**.*

5.5 OTHER TERMS

Side-effect: Any effect an operation has other than returning a result.

Atomic: The property of an operation with **side-effects** to produce its **side-effects** only if it completes without error.

Note: *For example, the specification of items 3, 4, and 5 of A in the APL line*

```
B ← A[3 4 5] ← 2
```

is a side effect, since the result placed in B is the scalar 2; further, since ***indexed assignment*** *is specified to be* ***atomic****, no change to A will occur if the indexed assignment fails with, for example, an index error.*

6 SYNTAX AND EVALUATION

6.1 INTRODUCTION

Note: *This chapter specifies the rules for evaluating lines. These rules are used by the subsections* ***execute, quad input, immediate-execution****, and* ***defined-function-control****.*

The rules are described in three subsections, ***evaluate-line, evaluate-statement****, and* ***reduce-statement****.*

The data structures and procedures used to describe syntax and evaluation in this standard are strictly expository devices; they are not intended to represent required or desirable implementation techniques. The order of reporting errors implied by the diagram-matching actions in the model is not required.

6.1.1 Evaluate-Line

Form: **Evaluate-Line**

Informal Description: Evaluate-line is the principal procedure in the evaluation of APL lines. It decomposes the **list of characters** named **current-line** into **lexical-units** according to the **character-diagram** named **line**.

Some diagrams referred to in **line** have their exit paths flagged with asterisks. Once the diagram **line** has been threaded, the rethreading pass is used to gather certain character sequences (diagrams ending in two asterisks), and either create tokens (diagrams ending in one asterisk) or discard the character sequences gathered (diagrams ending in three asterisks).

Evaluate-line calls **evaluate-statement** to convert the **lexical-units** into a **result**.

Note that the **result** can be a **constant** (from, for example, 2+2), **committed-value** (from *A*←1), **nil** (⍎''), **branch** (⍎'→3'), **escape** (→), **unwind** (→), **error** (2−), **clear-state-indicator** (from)*SIC*) or **interrupt** (from **signal-interrupt**).

Note: *The handling of these result classes by the different callers of* ***evaluate-line*** *specifies the treatment of exceptions, and should be analysed carefully.*

Evaluate-line is called from evaluation sequences in the subsections **defined-function-control**, **execute**, **immediate-execution**, and **quad-input**. If the **optional-facility statement-separator-facility** is implemented, **evaluate-line** evaluates successive statements in **current-line** beginning with the leftmost statement and continuing until evaluation produces an **exception** or the rightmost statement has been evaluated.

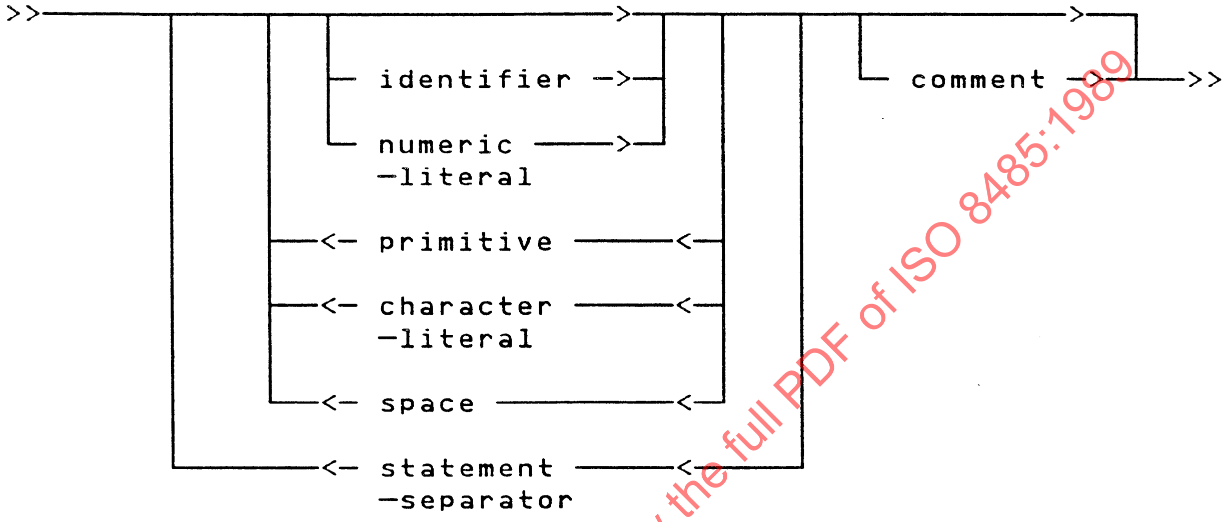
Evaluation Sequence:

```
Set   C to the empty-list of characters.
Set  current-statement to the empty-list of tokens.
Thread line with current-line; if current-line does not match line, signal syntax-error.
Rethread  line with current-line, taking the following actions:

    When a  character-diagram  ending in '—***>>' is recognised, set C to the empty-list of
      characters.
    When a  character-diagram ending in '—**>>' is recognised, append to C as a new last item
      the character just passed.
    When a  character-diagram ending in '—*>>' is recognised,

        Append to  current-statement as a new last item a token  with class given by the name of
          the character-diagram and with content C.
        Set   C to the empty-list of characters.

    When the  optional-facility statement-separator-facility  is implemented and the
      character-diagram  statement-separator is recognised,

        Set   Z to evaluate-statement.
        If   Z is an exception, return Z.
        If   Z is a constant, display Z.
        Set  current-statement to the empty-list of tokens.
        Set   C to the empty-list of characters.

    When the  character-diagram line is matched,

        Set   Z to evaluate-statement.
        Return   Z.
```

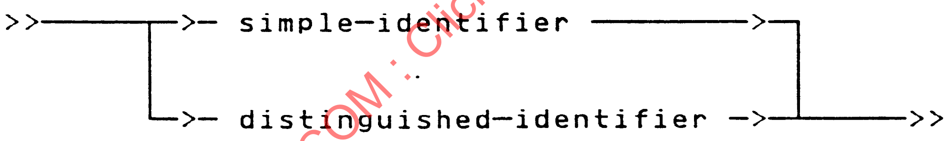
6.1.2 Character-Diagrams

The diagrams in this subsection are **character-diagrams**: the APL Graphic Symbols such as ∇ in these **character-diagrams** match corresponding **characters** in the **required-character-set**.

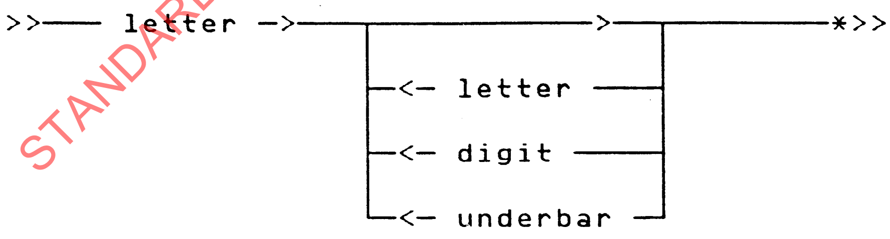
Line



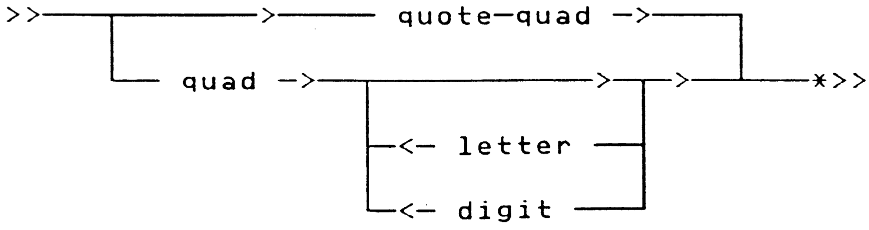
Identifier



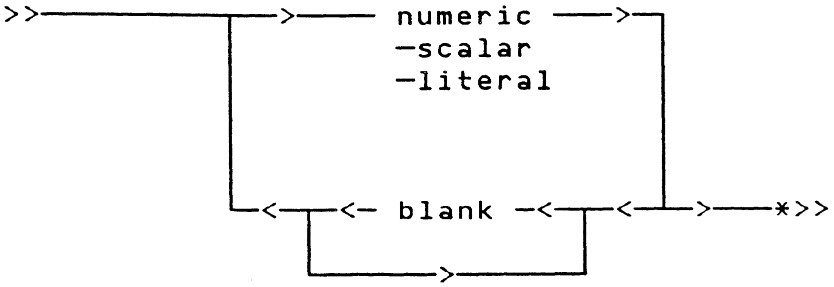
Simple-Identifier



Distinguished-Identifier

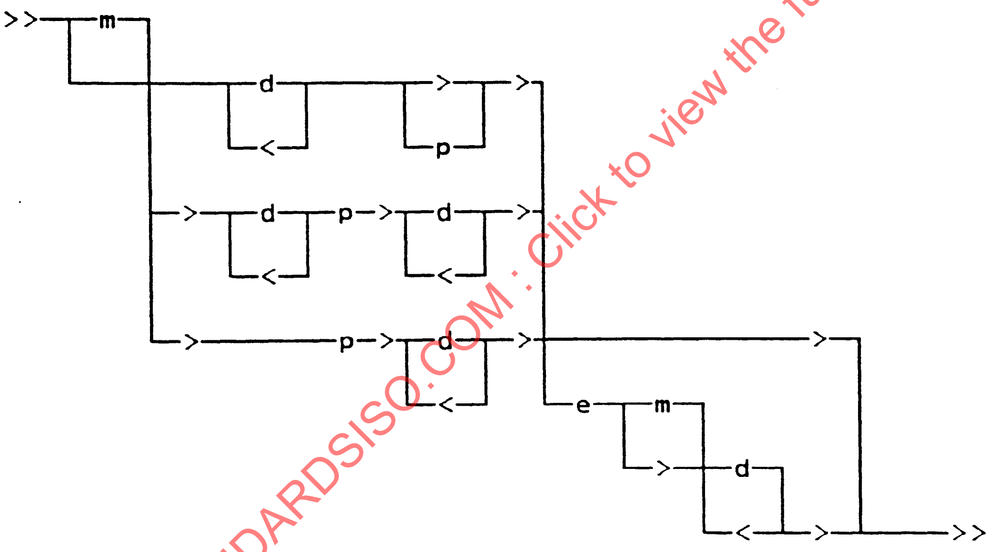


Numeric-Literal



Numeric-Scalar-Literal

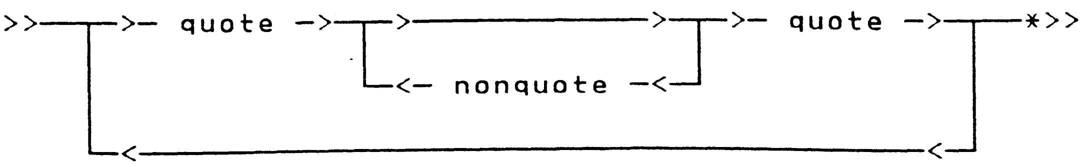
d stands for **digit**.
e stands for **exponent-marker**.
m stands for **overbar**.
p stands for **dot**.



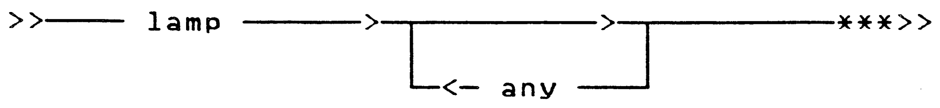
Example:

```
¯12.34567E¯890
```

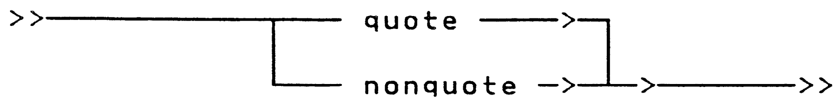
Character-Literal



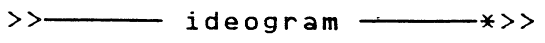
Comment



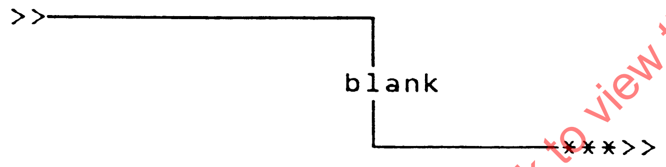
Any



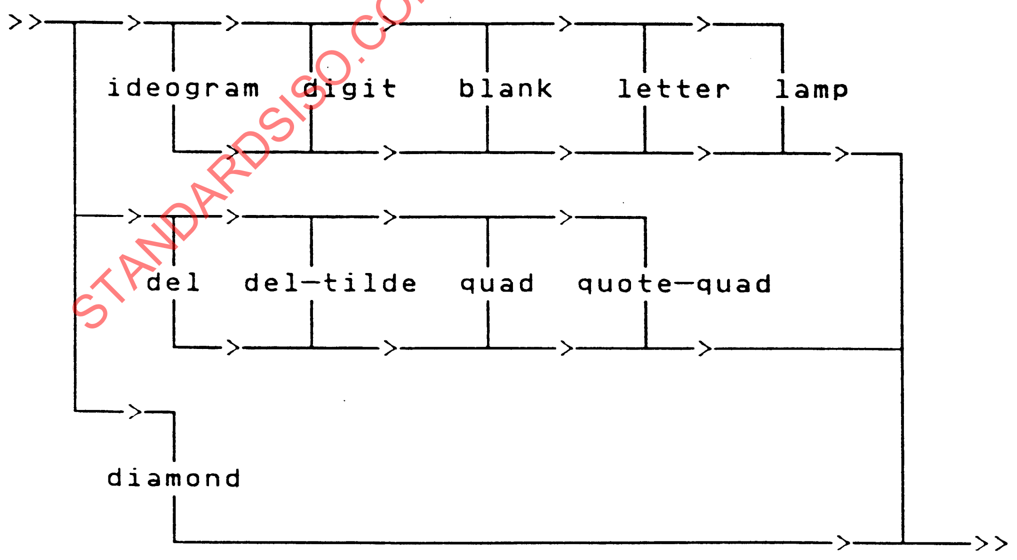
Primitive



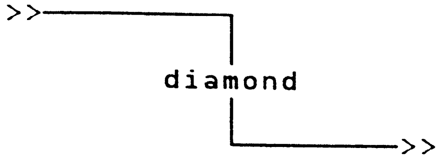
Space



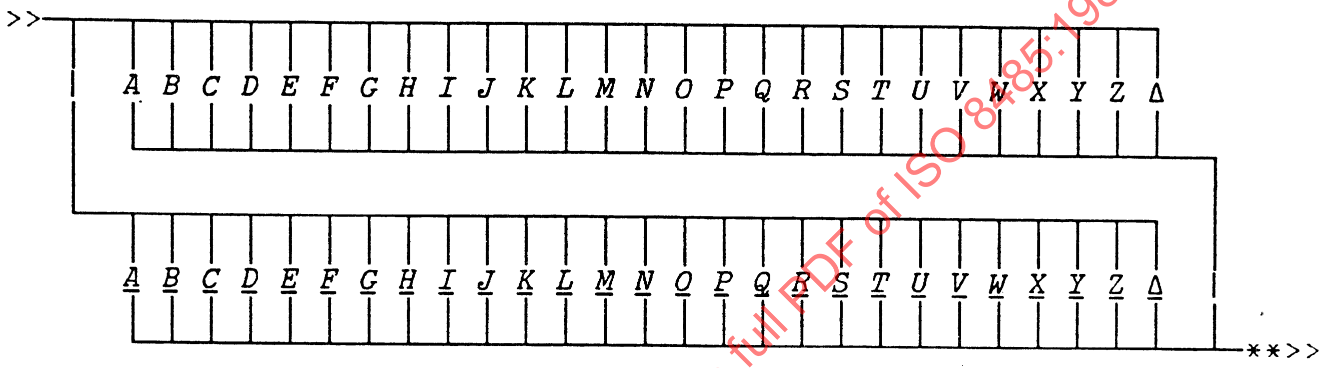
Nonquote



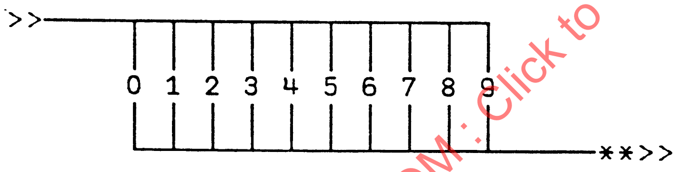
Statement-Separator



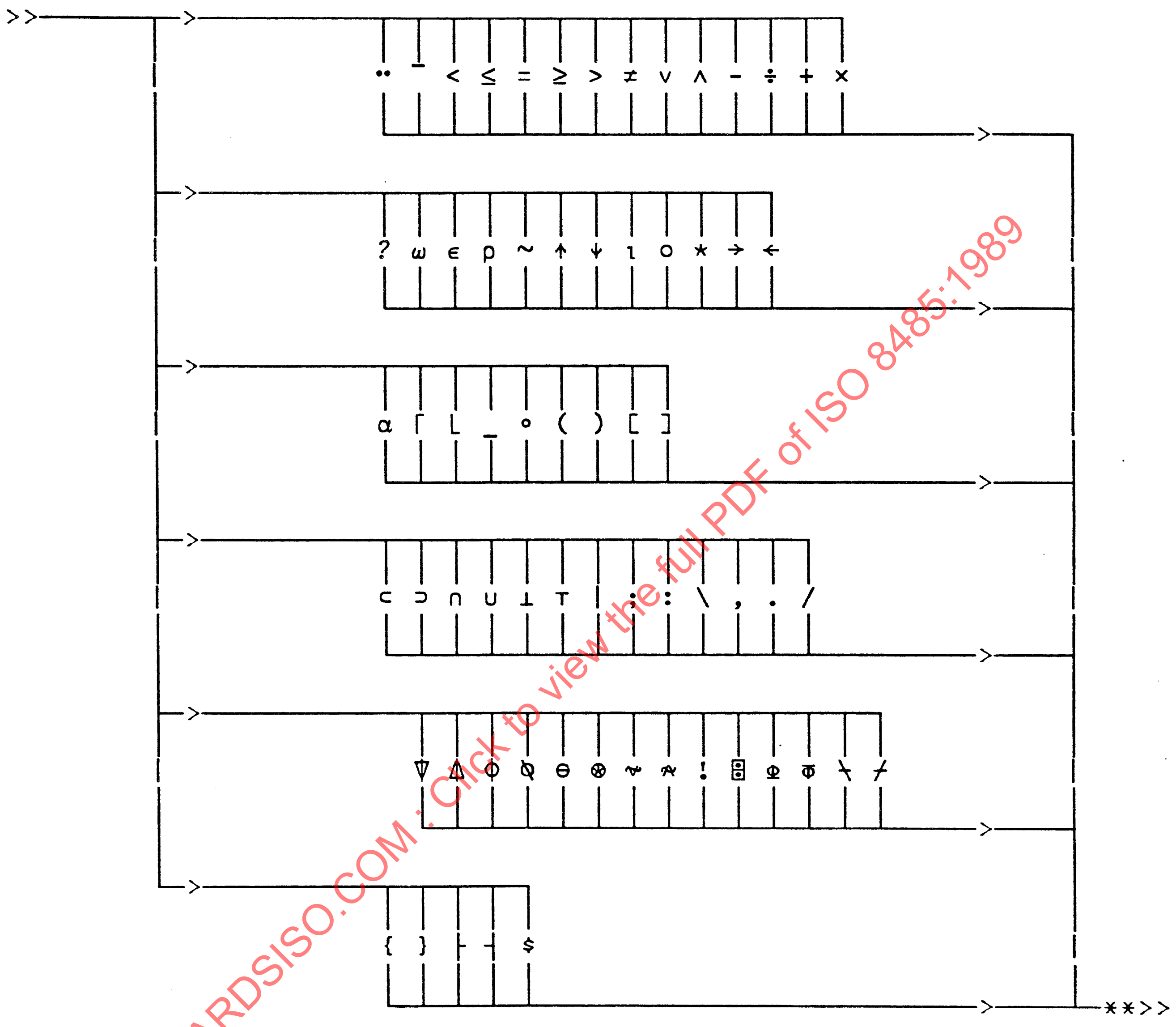
Letter


>>
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z Δ
A B C D E F G H I J K L M N O P Q R S T U V W X Y Z Δ
**>>

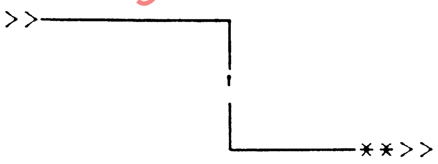
Digit



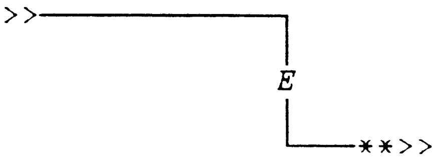
Ideogram



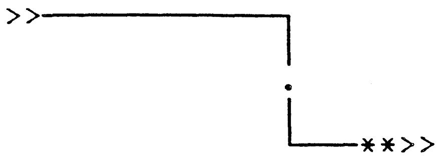
Quote



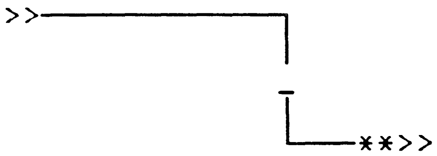
Exponent-Marker



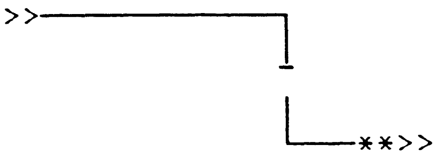
Dot



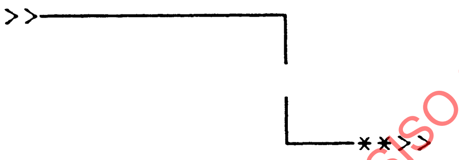
Underbar



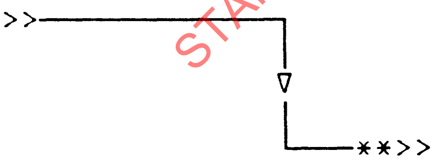
Overbar



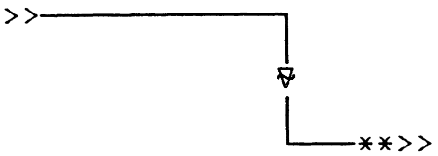
Blank



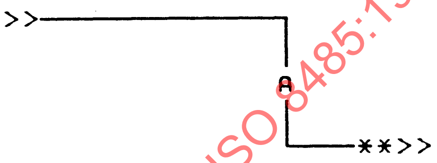
Del



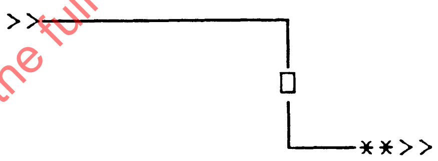
Del-Tilde



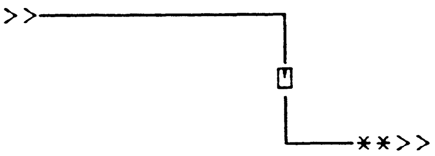
Lamp



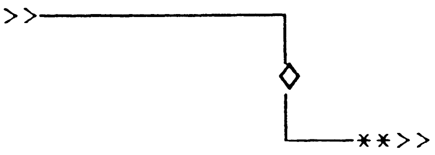
Quad



Quote-Quad



Diamond



Example

The example introduced in this subsection is continued through the syntax analysis portion of this standard.

*After **evaluate-line** has processed the **current-line***

```
ABC←FN ⎕⌽[1+0] DEF[1;5 6]×3.45E4,ρ'ABC'  ⍝COMMENT
```

***current-statement** looks like this:*

Identification	Content	Class
T01	'ABC'	simple-identifier
T02	'←'	primitive
T03	'FN'	simple-identifier
T04	'⎕'	distinguished-identifier
T05	'⌽'	primitive
T06	'['	primitive
T07	'1'	numeric-literal
T08	'+'	primitive
T09	'0'	numeric-literal
T10	']'	primitive
T11	'DEF'	simple-identifier
T12	'['	primitive
T13	'1'	numeric-literal
T14	';'	primitive
T15	'5 6'	numeric-literal
T16	']'	primitive
T17	'×'	primitive
T18	'3.45E4'	numeric-literal
T19	','	primitive
T20	'ρ'	primitive
T21	''ABC''	character-literal

*Each **token** has a **content**, which is a list of **characters**, and a **class**, which is in the metaclass **lexical-unit**. The **tokens** produced are numbered* T01 *to* T21 *for later reference. Note that comments and blanks between tokens are discarded during this tokenisation process.*

***Note:** As the diagram **line** shows, identifiers and numeric literals are separated by one or more spaces, character-literals, or primitives.* 1 ABC'A' *is a line,* 1ABC'A' *is not. No such separation rule applies to primitive function symbols.* 1ρ'AB' *is a line.*

The sequence 3+.4 *is parsed **numeric-literal, primitive, numeric-literal** but the sequence* 3+.× *is parsed **numeric-literal, primitive, primitive, primitive**.*

A comment may appear at the end of a line or alone on a line.

The **statement-separator-facility** is an **optional-facility**.

Parsing an **identifier token** signals a **limit-error** if the **number-of-items** in the **list** of **characters** is greater than **identifier-length-limit**.

6.1.3 Evaluate-Statement

Evaluate-Statement

Informal Description: **Evaluate-statement** is performed on **current-statement**, a **list** of **tokens** found in the **current-context**.

It uses **bind-token-class** to convert **identifiers** to **classified-names** and **constants**.
It uses **literal-conversion** to convert **literals** to **constants**.
It uses the **token-diagram** named **statement** to verify that the statement is properly formed and to resolve certain ambiguous **tokens**. For example, the **token** containing] is resolved to either **right-axis-bracket** or **right-index-bracket**.
It calls the evaluation sequence in the subsection **reduce-statement** to convert the **current-statement** to a **result**, which it returns to its caller.

Evaluate-statement is called by **evaluate-line**.

Evaluation Sequence:

```
For every index $I$ in the index-set of current-statement,

    Let    $T$ stand for item $I$ of current-statement.
    If     $T$ is an identifier, set $Q$ to bind-token-class of $T$
    If     $T$ is a literal, set $Q$ to the literal-conversion of $T$.
    If     $T$ is a primitive, set $Q$ to $T$.
    If     $Q$ is an exception, return $Q$.
    Otherwise, set    $T$ to $Q$.

Thread statement with current-statement; if statement cannot be matched, signal syntax-error.
Rethread statement with current-statement, taking the following action:

    When any token-diagram ending in '—*>>' is threaded,

        Replace the token in current-statement that matched the diagram with a token having the
          same content, but having a class given by the name of the token-diagram.

Append to current-statement as a new first item a left-end-of-statement token.
Append to current-statement as a new last item a right-end-of-statement token.

Set    $Z$ to reduce-statement.
If mode is defined-function and current-line-number is in current-trace-vector, set $Z$ to
  trace-display of $Z$.
Return    $Z$.
```

Additional Requirement:

This standard permits two orders of evaluation for expressions in brackets, as follows:

The syntax evaluator used here distinguishes axis brackets from index brackets in order to classify a function immediately to the right as monadic or dyadic. For example, the evaluation of ⌽[⍴X]+ X is specified to proceed by first calling monadic plus, then evaluating monadic rho.

The permitted alternative is to evaluate any expression in brackets before determining whether a function immediately to the right is monadic or dyadic. In the example given, this alternative behaviour would evaluate monadic rho before discovering that plus is used monadically.

Note: *The distinction can be observed only through side effects.*

6.1.4 Bind-Token-Class

Bind-Token-Class of *T*

Informal Description: Bind-token-class is used to bind each **identifier** in **current-statement** to its current **syntactic-unit**; if the **class** of a **token** changes between this point and the time the **token** is used (as it would for example in *F* ⎕*EX*'*F*', assuming *F* were initially a **defined-function**), the change will be detected and reported as a **syntax-error** in the appropriate **phrase-evaluator**.

This prebinding limits the standard to defining the meaning of statements only when the syntax class of their tokens does not change in mid-statement. **Conforming-implementations** may, of course, relax these rules. **Conforming-programs** must abide by them.

Evaluation Sequence:

Let **f** stand for the **content** of *T*.
If *T* is a **simple-identifier**,

If the **current-class** of *T* is

defined-function, return a **token** of **class defined-function-name** and **content f**.
niladic-defined-function, return a **token** of **class niladic-defined-function-name** and **content f**.
nil or **variable**, return a **token** of **class variable-name** and **content f**.
shared-variable, return a **token** of **class shared-variable-name** and **content f**.
label, return a **token** of **class constant** and **content** the **current-content** of *T*.

Note: Other cases cannot occur.

If *T* is a **distinguished-identifier**,

If both forms *Z* ← **f** and *Z* ← **f** ← *B* occur in the **form-table**, return a **token** of **class system-variable-name** and **content f**.
If the form *Z* ← **f** occurs in the **form-table**, but the form *Z* ← **f** ← *B* does not, return a **token** of **class niladic-system-function-name** and **content f**.
If either form *Z* ← **f** *B* or form *Z* ← *A* **f** *B* occurs in the **form-table**, return a **token** of **class system-function-name** with **content f**.

Otherwise, signal **syntax-error**.

6.1.5 Literal-Conversion

Literal-Conversion of T.

Informal Description: Literal-conversion converts T, a **token** of **class literal**, to a **token** of **class constant**. The **content** of such a **token** is converted from a **list** of **characters** to an **array**.

Evaluation Sequence:

If T is a **character-literal**, generate Z, a **character vector** such that the **ravel-list** of Z is the **content** of T with the initial and final quotes removed, and each pair of adjacent quotes in T replaced by a single quote.

If T is a **numeric-literal**, generate Z, a **numeric vector** such that the **ravel-list** of Z is a **list** of **numbers**, each of which is obtained by calling the **implementation-algorithm numeric-input-conversion** for the corresponding **numeric-scalar-literal** in the **numeric-literal**.

If the **length** of Z is **greater-than one**, return Z.
Otherwise, return **first-scalar** in Z.

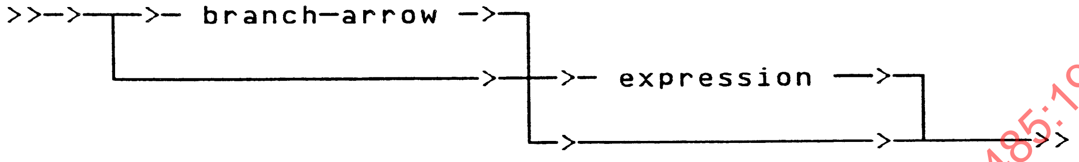
*Note: A quote character is represented in a **character-literal** by two adjacent quote characters. A single character between quotes is a scalar. All other cases are **character** vector literals. For example, the **character** literal* '' *is the empty **character** vector literal and the character literal* '''' *is the **character** scalar "quote".*

A numeric literal containing a single number is a scalar. A numeric literal containing two or more numbers is a vector.

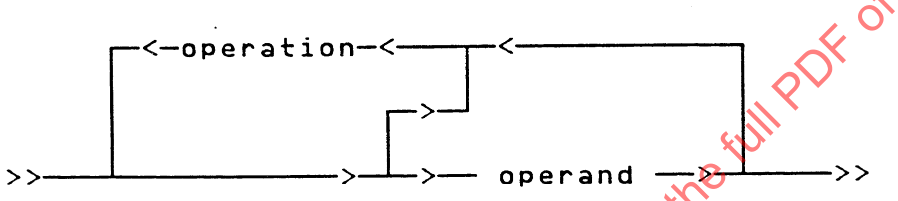
6.1.6 Statement-Analysis Token-Diagrams

Paths containing **ideograms** such as ⊗ in these **token-diagrams** match **tokens** whose **class** is **primitive** and whose **content** is the ideogram. Paths containing the word **token**, such as **shared-variable-name token** in **operand**, match **tokens** with the given **class**.

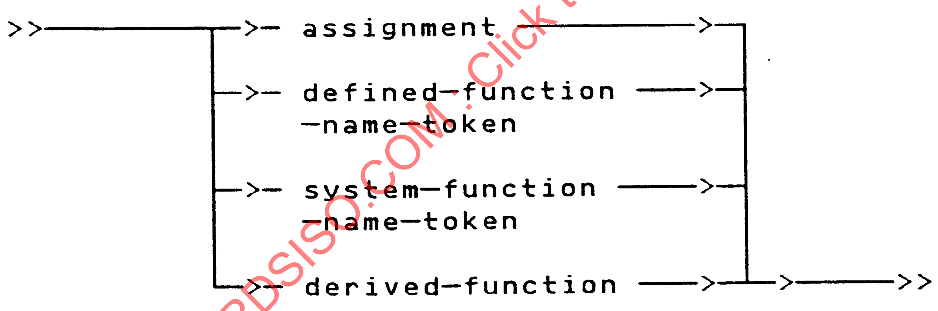
Statement



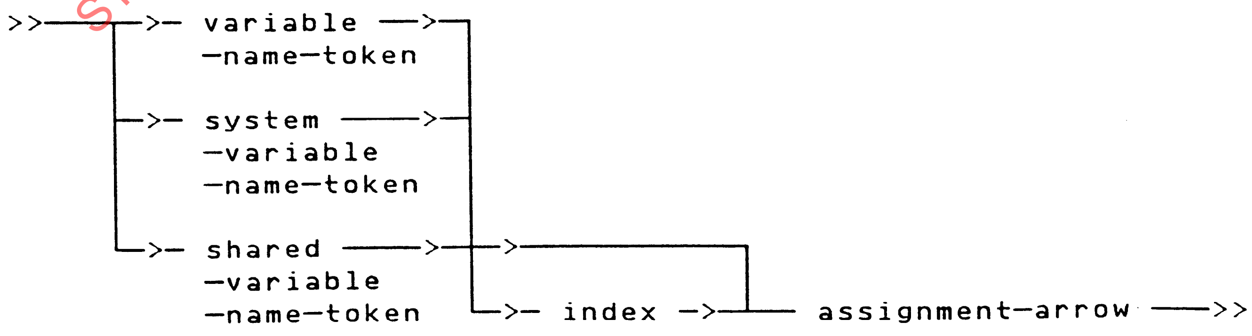
Expression



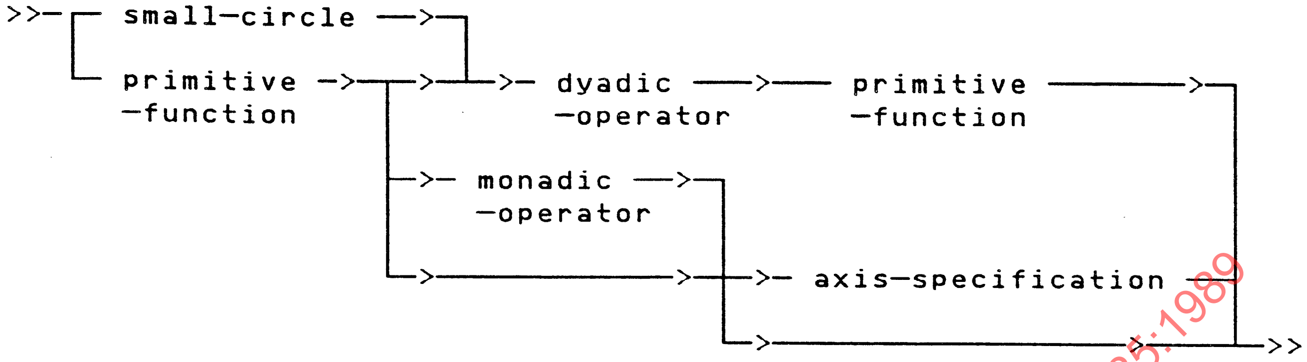
Operation



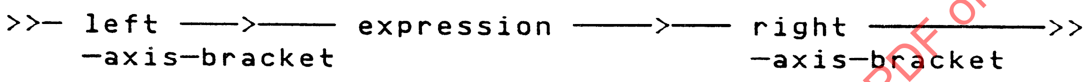
Assignment



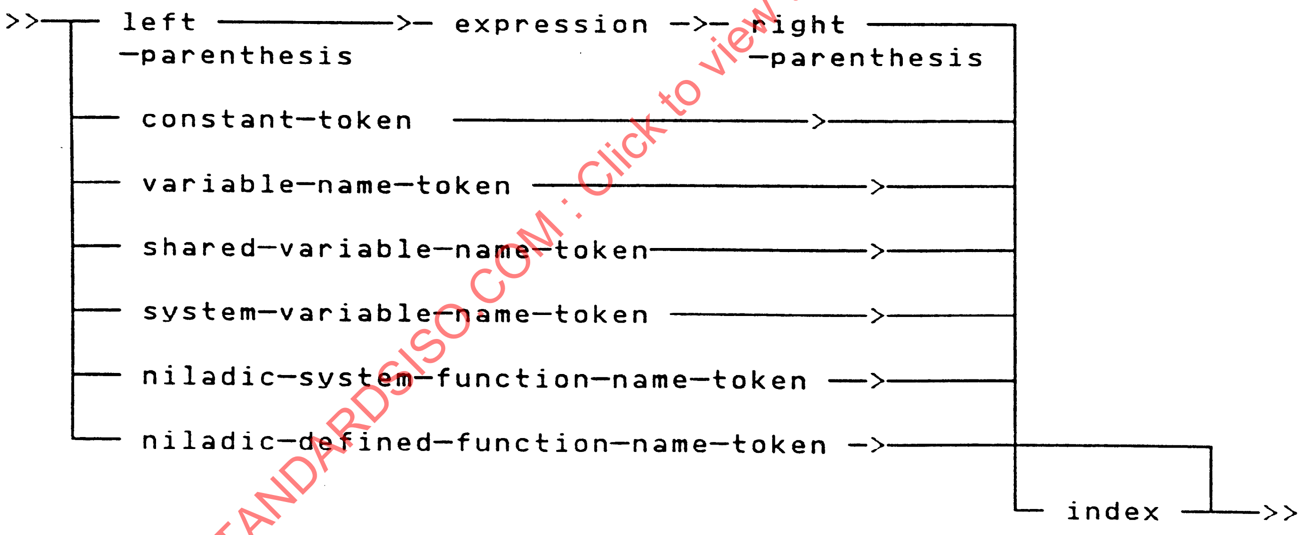
Derived-Function



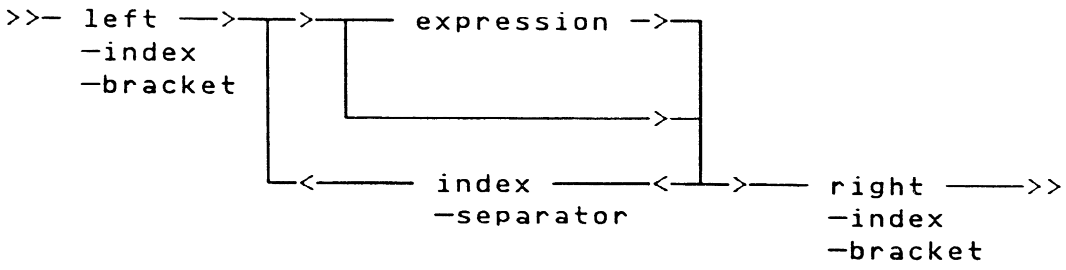
Axis-Specification



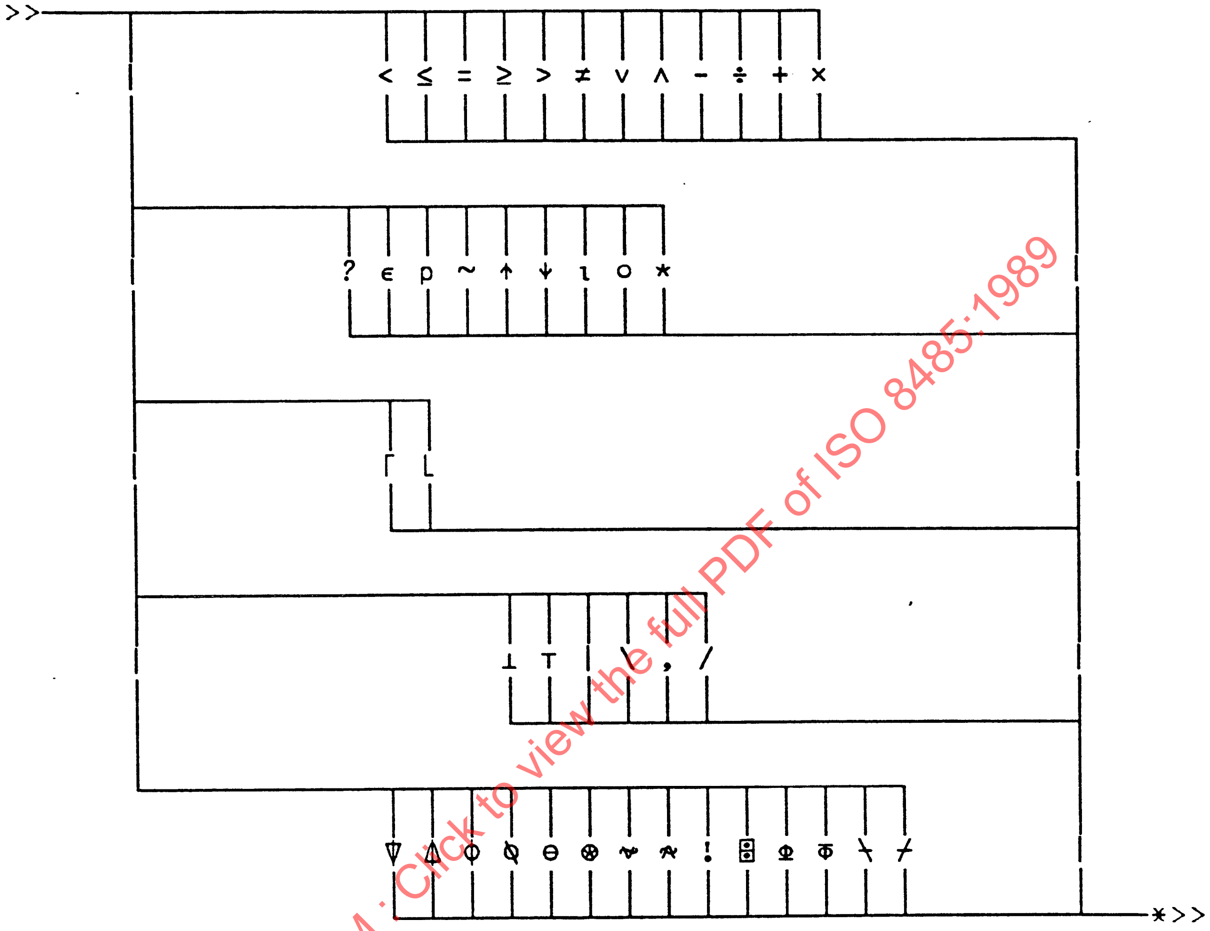
Operand


left-parenthesis
expression
right-parenthesis
constant-token
variable-name-token
shared-variable-name-token
system-variable-name-token
niladic-system-function-name-token
niladic-defined-function-name-token
index

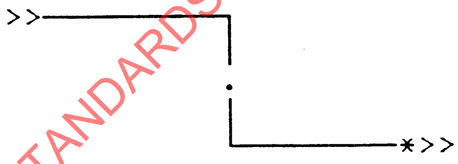
Index



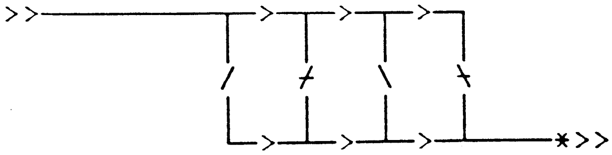
Primitive-Function



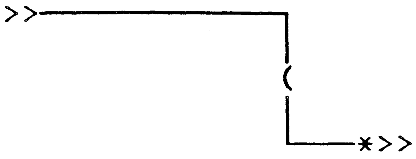
Dyadic-Operator



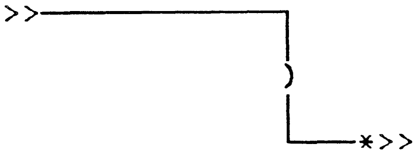
Monadic-Operator



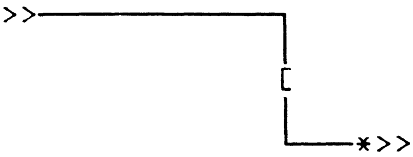
Left-Parenthesis



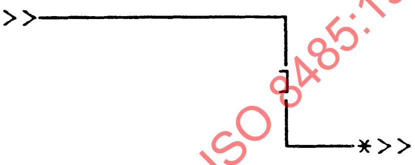
Right-Parenthesis



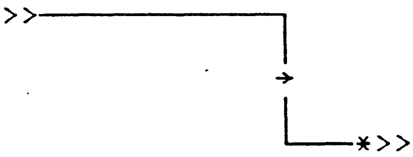
Left-Axis-Bracket



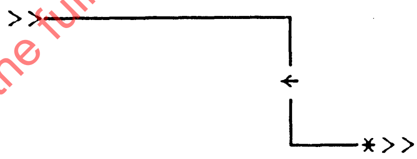
Right-Axis-Bracket



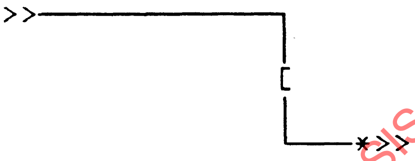
Branch-Arrow



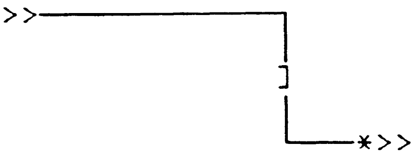
Assignment-Arrow



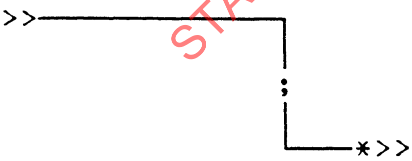
Left-Index-Bracket



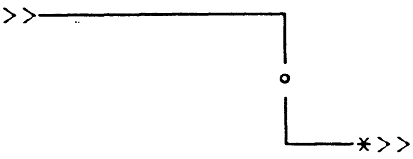
Right-Index-Bracket



Index-Separator



Small-Circle



Example

*From the list of **characters** in **current-line**,*

```
ABC←FN ⎕⌽[1+0] DEF[1;5 6]×3.45E4,ρ'ABC'   ⍝COMMENT
```

***evaluate-line** generated a list of **tokens** and stored it in **current-statement**. Here, **evaluate-statement** has replaced that list with a new list of **tokens**.*

Identification	Content	Class
*T*00		**left-end-of-statement**
*T*01	'*ABC*'	**variable-name**
*T*02		**assignment-arrow**
*T*03	'*FN*'	**defined-function-name**
*T*04	'⎕'	**system-variable-name**
*T*05	'⌽'	**primitive-function**
*T*06		**left-axis-bracket**
*T*07	1	**constant**
*T*08	'+'	**primitive-function**
*T*09	0	**constant**
*T*10		**right-axis-bracket**
*T*11	'*DEF*'	**variable-name**
*T*12		**left-index-bracket**
*T*13	1	**constant**
*T*14		**index-separator**
*T*15	5 6	**constant**
*T*16		**right-index-bracket**
*T*17	'×'	**primitive-function**
*T*18	34500	**constant**
*T*19	','	**primitive-function**
*T*20	'ρ'	**primitive-function**
*T*21	'*ABC*'	**constant**
*T*22		**right-end-of-statement**

*The new list, shown above, begins with a **left-end-of-statement token** and ends with a **right-end-of-statement token**. Old **identifier tokens** have a new **class** and the same **content**; old **literal tokens** are now **constants** whose **content** is an appropriate **array**; old **primitive tokens** are now either **primitive-functions** whose **content** is the **character** identifying the primitive function, or are grouping signs such as **right-axis-bracket** with no specified **content**.*

6.2 REDUCE-STATEMENT

Reduce-Statement

Informal Description: **Reduce-statement** converts **current-statement**, a **list** of **syntactic-units**, to a **result** by decomposing it into shorter lists of **syntactic-units** called **phrases**, then calling procedures termed **phrase-evaluators** to convert these phrases into **tokens**.

The letters *Z*, *K*, and *J* in this evaluation sequence refer to the graphic symbols found in the **resultant-prefix** column of **Table 3**.

Evaluation Sequence:

```
Set current-stack to the empty-list of tokens.
Repeat:

    Find the first entry in the phrase-table whose pattern matches a prefix of current-stack.
    If there is no matching entry,

        If current-statement is empty, signal syntax-error.
        Otherwise,

            Remove the last token from current-statement.
            Append it to current-stack as a new first item.

    If there is a matching entry,

        Let p stand for the prefix of the current-stack that matched the entry.
        Let r stand for the resultant-prefix of the entry.
        Let s stand for the phrase-evaluator of the entry.

        Call s and set y to the token it returns.
        If s is process-end-of-statement, return y.
        If y is an exception, return y.
        Otherwise, replace p with r in which y has been substituted for $Z$, $K$ or $J$ according to
          whether y is a result, a complete-index-list or a partial-index-list.

(End of repeated block)
```

Table 3 — The Phrase Table

Pattern	Phrase Evaluator	Resultant-Prefix
(*B*)	**Remove-Parentheses**	*Z*
N	**Evaluate-Niladic-Function**	*Z*
X F B	**Evaluate-Monadic-Function**	*X Z*
X F [*C*] *B*	**Evaluate-Monadic-Function**	*X Z*
X F M B	**Evaluate-Monadic-Operator**	*X Z*
X F M [*C*] *B*	**Evaluate-Monadic-Operator**	*X Z*
A F B	**Evaluate-Dyadic-Function**	*Z*
A F [*C*] *B*	**Evaluate-Dyadic-Function**	*Z*
A F D G B	**Evaluate-Dyadic-Operator**	*Z*
A ∘ *D G B*	**Evaluate-Dyadic-Operator**	*Z*
A [*K*]	**Evaluate-Indexed-Reference**	*Z*
V [*K*] ← *B*	**Evaluate-Indexed-Assignment**	*Z*
V ← *B*	**Evaluate-Assignment**	*Z*
V	**Evaluate-Variable**	*Z*
]	**Build-Index-List**	*J*]
; *I*	**Build-Index-List**	*J*
; *B I*	**Build-Index-List**	*J*
[*I*	**Build-Index-List**	[*K*
[*B I*	**Build-Index-List**	[*K*
L R	**Process-End-of-Statement**	-
L B R	**Process-End-of-Statement**	
L → *B R*	**Process-End-of-Statement**	
L → *R*	**Process-End-of-Statement**	

Legend:

A , *B* , *Z* match **result**.
D matches **dyadic-operator**.
F , *G* match **defined-function-name**, **primitive-function**, or **system-function-name**.
I , *J* match **partial-index-list**.
C , *K* match **complete-index-list**.
L matches **left-end-of-statement**.
M matches **monadic-operator**.
N matches **niladic-defined-function-name** or **niladic-system-function-name**.
R matches **right-end-of-statement**.
V matches **variable-name**, **system-variable-name**, or **shared-variable-name**.

X matches **assignment-arrow**, **branch-arrow**, **defined-function-name**, **index-separator**, **left-axis-bracket**, **left-end-of-statement**, **left-index-bracket**, **left-parenthesis**, **primitive-function**, **system-function-name**, or **right-axis-bracket**.
(matches **left-parenthesis**.
) matches **right-parenthesis**.
[matches **left-axis-bracket** or **left-index-bracket**.
] matches **right-axis-bracket** or **right-index-bracket**.
∘ matches **small-circle**.
; matches **index-separator**.
← matches **assignment-arrow**.
→ matches **branch-arrow**.

The graphic symbols in the **Pattern** and **Resultant-Prefix** columns of **Table 3** designate lists of **syntactic-units**. Each graphic symbol matches **tokens** of the designated **classes** or **metaclasses**.

*Note: The graphic symbols employed are chosen to be suggestive of the **list of characters** that give rise to such phrases. The **build-index-list** entries are called with B bound to a **value** and I bound to an **partial-index-list**; they return either another **partial-index-list** (J) or a **complete-index-list** (K). Brackets are not passed as arguments or returned by **build-index-list**; they are preserved by **reduce-statement** to make the **patterns** more obvious.*

Example

This example is continued from **evaluate-statement**.

The line being evaluated is

```
ABC ← FN ⎕ ⌽ [ 1 + 0 ] DEF [ 1 ; 5 6 ] × 34500  , ρ 'ABC'
↑   ↑ ↑  ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑   ↑ ↑ ↑ ↑   ↑ ↑ ↑      ↑ ↑ ↑
T   T T  T T T T T T T T   T T T T   T T T      T T T
0   0 0  0 0 0 0 0 0 1 1   1 1 1 1   1 1 1      1 2 2
1   2 3  4 5 6 7 8 9 0 1   2 3 4 5   6 7 8      9 0 1
```

T00 is **left-end-of-statement**, *and T22 is* **right-end-of-statement**.

The **tokens** *T00 through T22 initially form the columns of* **Figure 1**. *The rows of the figure show the actions taken because of the pattern or lack of pattern in* **current-stack**.

Consider step 5: the line fragment ,ρ'ABC' *matches the phrase X F B as follows: X matches* **token** *T19 because* , *is a* **primitive-function**. *F matches* **token** *T20 because* ρ *is a* **primitive-function**. *B matches* **token** *T21 because* 'ABC' *is a* **constant**. **Token** *T22,* **right-end-of-statement**, *is not considered because the pattern X F B is concerned with only the first three* **tokens** *in* **current-stack**.

The **phrase-evaluator** *associated with X F B is* **evaluate-monadic-function**. *This* **phrase-evaluator**, *seeing that* ρ *is a primitive function, searches the* **form-table** *for Z ← ρ B, and calls the corresponding evaluation sequence,* **shape**. *Shape returns a* **constant**, *the one-element-vector containing* **three**.

This becomes Z in the **resultant-prefix** *column for X F B.* **Current-stack** *now holds three* **tokens**: *(***primitive-function***;* , *), (***constant** *;* ,3*) and (***right-end-of-statement** *;). Since no entry in the* **phrase-table** *has an entry whose* **pattern** *matches a* **prefix** *of* **current-stack**, *the* **token** *T18 (***constant** *; 34500) is added to* **current-stack**. *A* **prefix** *of* **current-stack** *now matches a pattern (A F B matches (***constant** *; 34500), (***primitive-function** *;* , *), (***constant** *;* ,3*)), so the corresponding* **phrase-evaluator** *(***evaluate-dyadic-function***) is called.*

```
 1 GET NEXT TOKEN T22                                   ()
 2 GET NEXT TOKEN T21                                  (R)
 3 GET NEXT TOKEN T20                                (B R)
 4 GET NEXT TOKEN T19                              (F B R)
 5 EVALUATE MONADIC FUNCTION                    (<X F B>R)
 6 GET NEXT TOKEN T18                           (<X Z>  R)
 7 EVALUATE DYADIC FUNCTION                   (<A F B>  R)
 8 GET NEXT TOKEN T17                          (Z       R)
 9 GET NEXT TOKEN T16                        (F B       R)
10 BUILD INDEX LIST                       (<]>F B       R)
11 GET NEXT TOKEN T15                    (J ] F B       R)
12 GET NEXT TOKEN T14                  (B I ] F B       R)
13 BUILD INDEX LIST                 (<; B I>] F B       R)
14 GET NEXT TOKEN T13                (J     ] F B       R)
15 GET NEXT TOKEN T12              (B I     ] F B       R)
16 BUILD INDEX LIST             (<[ B I>    ] F B       R)
17 GET NEXT TOKEN T11             ([ K       ] F B       R)
18 EVALUATE VARIABLE          (<V>[ K       ] F B       R)
19 EVALUATE INDEXED REFERENCE (<A [ K       ]>F B       R)
20 EVALUATE DYADIC FUNCTION   (<A             F B>      R)
21 GET NEXT TOKEN T10          (Z                       R)
22 BUILD INDEX LIST         (<]>B                       R)
23 GET NEXT TOKEN T09      (I ] B                       R)
24 GET NEXT TOKEN T08    (B I ] B                       R)
25 GET NEXT TOKEN T07  (F B I ] B                       R)
26 EVALUATE DYADIC  (<A F B>I ] B   FUNCTION            R)
27 GET NEXT          (Z     I ] B   TOKEN T06           R)
28 BUILD INDEX    (<[ B     I>] B   LIST                R)
29 GET NEXT        ([ C       ] B   TOKEN T05           R)
30 GET NEXT      (F [ C       ] B   TOKEN T04           R)
31 EVALUATE   (<V>F [ C       ] B   VARIABLE            R)
32 EVALUATE   (<A F [ C       ] B>  DYADIC FUNCTION     R)
33 GET NEXT    (Z                   TOKEN T03           R)
34           (F B       GET NEXT TOKEN T02              R)
35        (<X F B>      EVALUATE MONADIC FUNCTION       R)
36         (X Z         GET NEXT TOKEN T01              R)
37      (<V ← B>        EVALUATE ASSIGNMENT             R)
38       (Z             GET NEXT TOKEN T00              R)
39 *  (<L B             END OF STATEMENT                R>)
```

Figure 1 — Statement Evaluation

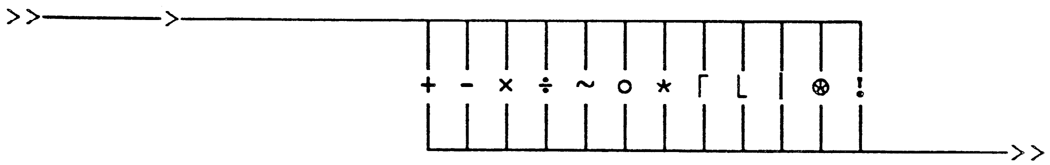
Extent of **current-stack** shown by (and). **Prefix** (when matched) shown by < and >.

6.3 THE PHRASE EVALUATORS

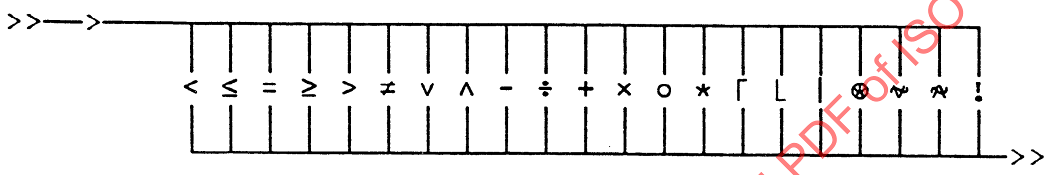
Informal Description: Each **phrase-evaluator** takes a **phrase** and reduces it to a single **token**. The **form-table** used by the **phrase-evaluators** is given as **Table 4**.

6.3.1 Diagrams

Primitive-Monadic-Scalar-Function



Primitive-Dyadic-Scalar-Function



6.3.2 Remove-Parentheses

Pattern (*B*)

Evaluation Sequence:

```
If    B is nil, signal value-error.
If    B is a branch, signal value-error.
Otherwise, return B.
```

Note: *(⍎'') and (⍎'→3') fail with a* ***value-error****. (A←3) and (⍎'A←3') do not display, since the* ***token*** *returned by* ***remove-parentheses*** *is a* ***committed-value****, not a* ***constant*** *; for the same reason, (⎕←3) displays the value* ***three*** *only once. (→3) fails with a* ***syntax-error*** *in* ***evaluate-statement****.*

(→) also fails in ***evaluate-statement****, but (⍎'→') succeeds;* ***evaluate-statement*** *threads* ***line*** *successfully, and* ***remove-parentheses*** *receives and returns an* ***escape token****.*

6.3.3 Evaluate-Niladic-Function

Pattern *N*

Evaluation Sequence:

```
Let  n stand for the content of N.
If   N is a niladic-defined-function-name,

    If the  current-class of N is niladic-defined-function,

        Search the  form-table for Z ← DFN.
        Call the corresponding evaluation sequence, passing n as the value of DFN.
        Return the  token it returns.

    Otherwise, signal  syntax-error.

If   N is a niladic-system-function-name,

    Search the  form-table for  Z  ← n.
    If it is not found, signal syntax-error.
    Otherwise, call the corresponding evaluation sequence.
    Return the  token it returns.
```

Note: *This phrase evaluator checks for a change in syntax class.*

6.3.4 Evaluate-Monadic-Function

Pattern   *X F B*

Pattern   *X F[C] B*

Evaluation Sequence:

```
If  B is not a value, signal value-error.

Let  f stand for the content of F.

For pattern   X  F  B

    If  F is a defined-function-name,

        If the  current-class of F is defined-function,

            Search the  form-table for Z  ←  DFN  B.
            Call the corresponding evaluation sequence, passing f as the value of DFN.
            Return the token it returns.

        Otherwise, signal  syntax-error.

    If  F is a primitive-function  or a system-function-name,

        If   F matches primitive-monadic-scalar-function and B is not a scalar, perform
          monadic-scalar-extension  as follows:

            Return a  numeric array Z  such that the shape-list of  Z is the shape-list of B and for
              all I in the index-set of the ravel-list of Z, item I of the ravel-list of  Z is f (item I of
              the ravel-list of B).

        Otherwise,

            Search the  form-table for  Z  ← f  B.
            If it is not found, signal syntax-error.
            Otherwise, call the corresponding evaluation sequence.
            Return the  token it returns.
```

For pattern *X F*[*C*] *B*

```
    If F is not a primitive-function, signal syntax-error.
    If index-origin is nil, signal implicit-error.
    Let C1 stand for the first-item in the index-list C.
    If C1 is not a scalar or one-element-vector, signal axis-error.
    If any item of the ravel-list of C1 is not a number, signal axis-error.
    Set K to C1 plus (one minus index-origin ).
    Search the form-table for Z ←f[K] B.
    If it is not found, signal syntax-error.
    Otherwise, call the corresponding evaluation sequence and return the token it returns.
```

Additional Requirements:

The order in which the individual items of *Z* are produced during **monadic-scalar-extension** is not specified by this standard.

If any call of **f** signals an error during **monadic-scalar-extension**, that error is returned as the result of **evaluate-monadic-function**.

Note: *This phrase evaluator checks for a change in syntax class.*

6.3.5 Evaluate-Monadic-Operator

Pattern *X F M B*

Pattern *X F M*[*C*] *B*

Evaluation Sequence:

```
If B is not a value, signal value-error.

Let m stand for the content of M.
Let f stand for the content of F.

For pattern X F M B

    If F is not a primitive-function, signal syntax-error.
    Otherwise,

        Search the form-table for Z ← f m B.
        If it is not found, signal syntax-error.
        Otherwise, call the corresponding evaluation sequence, passing token F as the value of f.
        Return the token it returns.

For pattern X F M[C] B

    If F is not a primitive-function, signal syntax-error.
    If index-origin is nil, signal implicit-error.
    Let C1 stand for the first-item in the index-list C.

    If C1 is not a scalar or one-element-vector, signal axis-error.
    If any item of the ravel-list of C1 is not a number, axis-error.
    Set K to C1 plus (one minus index-origin ).
    Search the form-table for Z ← f m[K] B.
    If it is not found, signal syntax-error.
    Otherwise, call the corresponding evaluation sequence, passing token F as the value of f.
    Return the token it returns.
```

6.3.6 Evaluate-Dyadic-Function

Pattern *A F B*

Pattern *A F*[*C*] *B*

Evaluation Sequence:

```
If  A is not a value, signal value-error.
If  B is not a value, signal value-error.
Let f stand for the content of F.

For pattern  A F B

    If  F is a defined-function-name,

        If the current-class of F is defined-function,

            Search the form-table for Z ← A DFN B.
            Call the corresponding evaluation sequence, passing f as the value of DFN.
            Return the token it returns.

        Otherwise, signal syntax-error.

    If F is a primitive-function or a system-function-name,

        If  F matches primitive-dyadic-scalar-function and A and B are not both scalars, perform
        dyadic-scalar-extension as follows:

            If the rank of A differs from the rank of B,

                If  A is a scalar or one-element-vector and B is not a scalar, return ((ρB)ρA)
                  f B.
                If  B is a scalar or one-element-vector, return A f (ρA)ρB.
                Otherwise, signal rank-error.

            If the shape-list of A differs from the shape-list of B, signal length-error.
            Return  Z, an array such that the shape-list of Z is the same as the shape-list of A,
              the type of Z is numeric, and the ravel-list of Z is such that, for all I in the index-set
              of the ravel-list of Z, item I of the ravel-list of Z is (item I of the ravel-list of A) f
              (item I of the ravel-list of B).

        Otherwise, search the form-table for Z ← A f B.
        If it is not found, signal syntax-error.
        Call the corresponding evaluation sequence and return the token it returns.

For pattern  A F[C] B

    If F is not a primitive-function, signal syntax-error.
    If index-origin is nil, signal implicit-error.
    Let  C1 stand for the first-item in the index-list C.
    If  C1 is not a scalar or one-element-vector, signal axis-error.
    If any item of the ravel-list of C1 is not a number, signal axis-error.
    Set  K to C1 plus (one minus index-origin ).
    Search the form-table for Z ← A f[K] B.
    If it is not found, signal syntax-error.
    Otherwise, call the corresponding evaluation sequence, passing token F as the value of f.
    Return the token it returns.
```

Additional Requirements:

There is an intentional forward reference to **shape** and **reshape** in the description of **dyadic-scalar-extension**.

The order in which the individual items of *Z* are produced during **dyadic-scalar-extension** is not specified by this standard.

If any call of **f** signals an error during **dyadic-scalar-extension**, that error is returned as the result of **evaluate-dyadic-function**.

Note: ***Dyadic-scalar-extension*** *is intentionally stricter than it is in existing systems. For example,* `(1 1⍴1)+⍳3` *signals a* ***rank-error*** *and* `1 2 + ,1` *signals a* ***length-error***.

This phrase evaluator checks for a change in syntax class.

6.3.7 Evaluate-Dyadic-Operator

Pattern $A\ F\ D\ G\ B$

Pattern $A\ \circ\ D\ G\ B$

Evaluation Sequence:

```
If A is not a value, signal value-error.
If B is not a value, signal value-error.
Let d stand for the content of D.

For pattern A F D G B

    If F or G is not a primitive-function, signal syntax-error.
    Otherwise,

        Search the form-table for Z ← A f d g B.
        If it is not found, signal syntax-error.
        If it is found, call the corresponding evaluation sequence, passing token f as the value of
          F and token g as the value of G.
        Return the token it returns.

For pattern A ∘ D G B

    If G is not a primitive-function, signal syntax-error.
    Otherwise,

        Search the form-table for Z ← A ∘ d g B.
        If it is not found, signal syntax-error.
        If it is found, call the corresponding evaluation sequence, passing token g as the value of
          G.
        Return the token it returns.
```

6.3.8 Evaluate-Indexed-Reference

Pattern A[K]

Evaluation Sequence:

```
If  A is not a value, signal value-error.
If  index-origin is nil, signal implicit-error.

If the  number-of-items in the index-list  K differs from the rank of A, signal rank-error.

If the  rank of A is greater-than one,

    Search the  form-table for Z ← A[I].
    Call the corresponding evaluation sequence, passing  K as the value of I.
    Return the  token it returns.

Otherwise,

    If  first-item in K is an elided-index-marker, return A.
    Otherwise,

        Set  X to first-item in the index-list  K.
        If any  item of the ravel-list of X is not a  near-integer, signal domain-error.
        Generate  X1, a numeric  array with the shape-list of X such that each item of the
          ravel-list of X1 is (one minus index-origin ) plus the integer-nearest-to  X.
        If any  item of the ravel-list of  X1 is not in the index-set of  A, signal index-error.
        Return  Z, an array with the type of A and the shape-list of X1, such that for each
          integer  I in the index-set of Z, item I of the ravel-list of Z is item J of the ravel-list of
          A, where  J is item I of the ravel-list of X1.
```

Note: *Since an* ***index-list*** *will never have zero* ***items****, indexing will always signal a* ***rank-error*** *when argument* A *is a* ***scalar****.*

6.3.9 Evaluate-Assignment

Pattern *V* ← *B*

Evaluation Sequence:

```
If  B is not a value, signal value-error.
If  V is a shared-variable-name,

    If the current-class of V is shared-variable,

        Search the form-table for Z ← SHV ← B.
        Call the corresponding evaluation sequence, passing token V as the value of SHV.
        Return the token it returns.

    Otherwise, signal syntax-error.

If  V is a system-variable-name,

    Search the form-table for Z ← q ← B, where q is the content of V.
    If it is not found, signal syntax-error.
    Otherwise,

        Call the corresponding evaluation sequence
        Return the token it returns

If  V is a variable-name,

    If the current-class of V is nil or variable,

        Set the current-referent of V to a token whose class is variable and whose content is the
        content of B.
        Return a token whose class is committed-value and whose content is B.

    Otherwise, signal syntax-error.
```

Note: *The phrase ABC←⍎'→3' yields **value-error**.*

*The phrase V←⎕SVR 'V' where V was a **shared-variable** yields **syntax-error**.*

6.3.10 Evaluate-Indexed-Assignment

Pattern V[K] ← B

Evaluation Sequence:

```
If index-origin is nil, signal implicit-error.

If B is not a value, signal value-error.
If V is a shared-variable-name,

    If the current-class of V is shared-variable,

        Search the form-table for Z ← SHV[I] ← B.
        Call the corresponding evaluation sequence, passing token V as the value of SHV, and K
          as the value of I.
        Return the token it returns.

    Otherwise, signal syntax-error.

If V is a system-variable-name,

    Search the form-table for Z ← q[I] ← B, where q is the content of V.
    If it is not found, signal syntax-error.
    Otherwise,

        Call the corresponding evaluation sequence, passing K as the value of I.
        Return the token it returns.

If V is a variable-name,

    If the current-class of V is nil, signal value-error.
    If the current-class of V is a variable,

        Search the form-table for Z ← V[I] ← B.
        Call the corresponding evaluation sequence, passing K as the value of I.
        Return the token it returns.

    Otherwise, signal syntax-error.
```

6.3.11 Evaluate-Variable

Pattern V

Evaluation Sequence:

```
If  $V$ is a shared-variable-name,

    If the current-class of $V$ is shared-variable,

        Search the form-table for $Z \leftarrow SHV$.
        Call the corresponding evaluation sequence, passing token $V$ as the value of $SHV$.
        Return the token it returns.

    Otherwise, signal syntax-error.

If  $V$ is a system-variable-name,

    Search the form-table for $Z \leftarrow$ q, where q is the content of $V$.
    Call the corresponding evaluation sequence.
    Return the token it returns.

If  $V$ is a variable-name,

    If the current-class of $V$ is nil, signal value-error.
    If the current-class of $V$ is variable, return the current-content of $V$.
    Otherwise, signal syntax-error.
```

6.3.12 Build-Index-List

Pattern]

Pattern ; *I*

Pattern ; *B I*

Pattern [*I*

Pattern [*B I*

Evaluation Sequence:

For pattern]

Return *J*, a **partial-index-list** with **content** the **index-list** of length **zero**.

For pattern ; *I*

Return *J*, a **partial-index-list** with **content** *Z*, an **index-list** such that the **first-item** in *Z* is an **elided-index-marker** and the **rest-of** *Z* is *I*.

For pattern ; *B I*

Return *J*, a **partial-index-list** with **content** *Z*, an **index-list** such that the **first-item** in *Z* is *B* and the **rest-of** *Z* is *I*.

For pattern [*I*

Return *J*, a **complete-index-list** with **content** *Z*, an **index-list** such that the **first-item** in *Z* is an **elided-index-marker** and the **rest-of** *Z* is *I*.

For pattern [*B I*

Return *J*, a **complete-index-list** with **content** *Z*, an **index-list** such that the **first-item** in *Z* is *B* and the **rest-of** *Z* is *I*.

6.3.13 Process-End-of-Statement

Pattern *L R*

Pattern *L A R*

Pattern *L* → *R*

Pattern *L* → *A R*

Evaluation Sequence:

For pattern *L R*

Return a **token** whose **class** is **nil**.

For pattern *L A R*

Return *A*.

For pattern *L* → *R*

Return a **token** whose **class** is **escape**.

For pattern *L* → *A R*

```
If the rank of A is greater-than one, signal rank-error.
If A is empty, return a token whose class is nil.
Otherwise, set A1 to the first-scalar in A.
If A1 is not a near-integer, signal domain-error.
Return a token whose class is branch and whose content is the numeric-scalar with value
  the integer-nearest-to A1.
```

6.4 THE FORM TABLE

The **form-table** is the **list** of all **lists** of **syntactic-units** for which evaluation sequences exist.

The following matching rules apply in the **form-table**.

A, *B*, *Z* match **constant**.
I,*K* match **complete-index-list**.
f, **g** match **primitive-function**.
A given ideogram, such as ⍟, matches a **primitive-function token** that contains it.
A given **distinguished-identifier**, such as ⎕*IO*, matches any **system-variable-name token** or **system-function-name token** that contains it.

The behaviour of operations in the **form-table** that do not create new **contexts** is **atomic**.

This behaviour is observable only for those operations that have **side-effects**. For example, if any of the elements of an argument array is not in the domain of **roll**, the value of the system parameter **random-link** following execution will be as it was when **roll** was called.

Table 4 — The Form Table

Form	Operation Name	Page
Z ← + *B*	Conjugate	84
Z ← – *B*	Negative	84
Z ← × *B*	Signum	85
Z ← ÷ *B*	Reciprocal	85
Z ← ⌊ *B*	Floor	86
Z ← ⌈ *B*	Ceiling	86
Z ← ⋆ *B*	Exponential	87
Z ← ⍟ *B*	Natural Logarithm	87
Z ← | *B*	Magnitude	88
Z ← ! *B*	Factorial	88
Z ← ○ *B*	Pi times	89
Z ← ~ *B*	Not	89
Z ← *A* + *B*	Plus	91
Z ← *A* – *B*	Minus	91
Z ← *A* × *B*	Times	92
Z ← *A* ÷ *B*	Divide	92
Z ← *A* ⌈ *B*	Maximum	93
Z ← *A* ⌊ *B*	Minimum	93
Z ← *A* ⋆ *B*	Power	94
Z ← *A* ⍟ *B*	Logarithm	94
Z ← *A* | *B*	Residue	95
Z ← *A* ! *B*	Binomial	96
Z ← *A* ○ *B*	Circular Functions	97
Z ← *A* ∧ *B*	And	99
Z ← *A* ∨ *B*	Or	99
Z ← *A* ⍲ *B*	Nand	100
Z ← *A* ⍱ *B*	Nor	100
Z ← *A* = *B*	Equal	101
Z ← *A* < *B*	Less than	103
Z ← *A* ≤ *B*	Less than or equal to	103
Z ← *A* ≠ *B*	Not equal	104
Z ← *A* ≥ *B*	Greater than or equal to	104
Z ← *A* > *B*	Greater than	105
Z ← , *B*	Ravel	107
Z ← ⍴ *B*	Shape	108
Z ← ⍳ *B*	Index Generator	109
Z ← *A* ⍴ *B*	Reshape	110
Z ← *A* , *B*	Join	111
Z ← f/ *B*	Reduction	114
Z ← f/[*K*] *B*	Reduction	114
Z ← f⌿ *B*	Reduction	114
Z ← f⌿[*K*] *B*	Reduction	114
Z ← f\ *B*	Scan	116
Z ← f\[*K*] *B*	Scan	116
Z ← f⍀ *B*	Scan	116
Z ← f⍀[*K*] *B*	Scan	116
Z ← *A* ∘ . f *B*	Outer Product	117
Z ← *A* f . g *B*	Inner Product	118
Z ← ? *B*	Roll	119
Z ← ⍋ *B*	Grade Up	121
Z ← ⍒ *B*	Grade Down	122
Z ← ⌽ *B*	Reverse	123

Table 4 — The Form Table (continued)

Form	Operation Name	Page
Z ← ⊖ B	Reverse	123
Z ← ⌽[K] B	Reverse	123
Z ← ⊖[K] B	Reverse	123
Z ← ⍉ B	Monadic Transpose	124
Z ← ⌹ B	Matrix Inverse	125
Z ← ⍎ B	Execute	126
Z ← A ,[K] B	Join Along an Axis	127
Z ← A ⍳ B	Index of	130
Z ← A ∊ B	Member of	131
Z ← A ? B	Deal	132
Z ← A / B	Compress	133
Z ← A ⌿ B	Compress	133
Z ← A /[K] B	Compress	133
Z ← A ⌿[K] B	Compress	133
Z ← A \ B	Expand	135
Z ← A ⍀ B	Expand	135
Z ← A \[K] B	Expand	135
Z ← A ⍀[K] B	Expand	135
Z ← A ⌽ B	Rotate	137
Z ← A ⊖ B	Rotate	137
Z ← A ⌽[K] B	Rotate	137
Z ← A ⊖[K] B	Rotate	137
Z ← A ⊥ B	Base Value	139
Z ← A ⊤ B	Representation	140
Z ← A ⍉ B	Dyadic Transpose	142
Z ← A ↑ B	Take	144
Z ← A ↓ B	Drop	145
Z ← A ⌹ B	Matrix Divide	146
Z ← A[I]	Indexed Reference	147
Z ← V[I] ← B	Indexed Assignment	148
Z ← ⎕TS	Time Stamp	152
Z ← ⎕AV	Atomic Vector	153
Z ← ⎕LC	Line Counter	153
Z ← ⎕DL B	Delay	154
Z ← ⎕NC B	Name Class	155
Z ← ⎕EX B	Expunge	156
Z ← ⎕NL B	Name List	156
Z ← ⎕STOP B	Query Stop	157
Z ← ⎕TRACE B	Query Trace	157
Z ← A ⎕NL B	Name List	158
Z ← A ⎕STOP B	Set Stop	158
Z ← A ⎕TRACE B	Set Trace	159
Z ← ⎕CT ← B	Comparison Tolerance	162
Z ← ⎕CT	Comparison Tolerance	162
Z ← ⎕RL ← B	Random Link	163
Z ← ⎕RL	Random Link	163
Z ← ⎕PP ← B	Print Precision	164
Z ← ⎕PP	Print Precision	164
Z ← ⎕IO ← B	Index Origin	165
Z ← ⎕IO	Index Origin	165
Z ← ⎕LX ← B	Latent Expression	166
Z ← ⎕LX	Latent Expression	166

Table 4 — The Form Table (continued)

Form	Operation Name	Page
Z ← ⎕*LX*[*I*] ← *B*	Latent Expression	166
Z ← *DFN*	Call-Defined-Function	174
Z ← *DFN B*	Call-Defined-Function	174
Z ← *A DFN B*	Call-Defined-Function	174
Z ← ⎕*FX B*	Function Fix	176
Z ← ⎕*CR B*	Character Representation	177
Z ← *SHV*	Shared Variable Reference	189
Z ← *SHV* ← *B*	Shared Variable Assignment	189
Z ← *SHV*[*I*] ← *B*	Shared Variable Indexed Assignment	190
Z ← ⎕*SVC B*	Shared Variable Access Control Inquiry	191
Z ← ⎕*SVQ B*	Shared Variable Query	192
Z ← ⎕*SVO B*	Shared Variable Degree of Coupling	193
Z ← *A* ⎕*SVO B*	Shared Variable Offer	194
Z ← ⎕*SVR B*	Shared Variable Retraction.	195
Z ← *A* ⎕*SVC B*	Shared Variable Access Control Set	196
Z ← ⍕ *B*	Monadic Format	203
Z ← *A* ⍕ *B*	Dyadic Format	206
Z ← ⎕	Quad Input	213
Z ← ⍞	Quote Quad Input	214
Z ← ⎕ ← *B*	Quad Output	214
Z ← ⍞ ← *B*	Quote Quad Output	215

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7 SCALAR FUNCTIONS

Note: The primitive functions described in this chapter are called ***scalar-functions****. All* ***scalar-functions*** *have uniform behaviour with respect to the structure of their argument arrays. The shape of the result of a* ***scalar-function*** *is determined solely by the shapes of its arguments.*

This section defines ***scalar-functions*** *individually for scalar arguments. Their common behaviour is described in this informal description by the expository device of an implicit operator, called the* ***scalar-extension-operator****.*

If the argument to a monadic scalar function is not a scalar, a monadic scalar function extension operator can be thought of as being invoked to produce a derived function which, in turn, applies the monadic scalar function to every element of the argument array, producing a result array of the same shape as the argument. The order in which the elements of the argument array are presented to the monadic scalar function is not specified by this standard. Monadic scalar functions never signal ***rank-error*** *or* ***length-error****.*

If either of the arguments of a dyadic scalar function is not a scalar, a dyadic scalar extension operator can be thought of as being invoked to produce a derived function, which provides pairs of scalars to the scalar function as follows:

The dyadic scalar extension operator first tests whether the two argument arrays have the same shape. Arguments to a dyadic scalar function must have the same shape. If they do not, and the argument of lesser rank is a scalar or one-element vector, the argument of lesser rank is reshaped to the shape of the argument of greater rank.

If the arguments cannot be made to have the same shape, the dyadic scalar extension operator signals a ***rank-error*** *if the arguments are of different ranks and a* ***length-error*** *otherwise.*

When the dyadic scalar extension operator succeeds, it produces a derived function which generates a scalar for each position in its result ***array*** *by applying the subject* ***scalar-function*** *to pairs of scalars selected from corresponding positions in the argument arrays. The order in which the elements of the result* ***array*** *are produced is not specified by this standard.*

Because the derived function produced by either scalar extension operator never calls its ***scalar-function*** *argument for empty arrays,* ***domain-error*** *can never be signalled for empty array arguments or for arrays reshaped by scalar extension to empty.*

The type of all empty results produced by the functions derived from monadic and dyadic scalar extension is a property of the function argument to scalar extension. Since all scalar functions specified in this standard produce ***numeric*** *results, the type of all empty results produced by scalar extension is specified as* ***numeric****.*

For example, `''∧5` *and* `-''` *return empty* ***numeric*** *results rather than signalling an error.*

7.1 MONADIC SCALAR FUNCTIONS

Note: The definitions in this section cover only scalar arguments. The ***phrase-evaluator evaluate-monadic-function*** *handles non-scalar cases. All* ***scalar-functions*** *yield scalar results when applied to scalar arguments.*

Note that, in this standard, ***roll*** *is not a* ***scalar-function****.*

7.1.1 Conjugate

```
Z ← + B
```

Informal Description: *Z* is *B*.

Evaluation Sequence:

If *B* is not a **number**, signal **domain-error**.
Return *B*.

Example:

```
      + 3 ¯4 0 0.5
3 ¯4 0 0.5
```

Note: This operation is named "conjugate" in accordance with its intended use in complex arithmetic.

7.1.2 Negative

```
Z ← - B
```

Informal Description: *Z* is the **negation** of *B*.

Evaluation Sequence:

If *B* is not a **number**, signal **domain-error**.
Return **zero minus** *B*.

Example:

```
      - 7 0 ¯7
¯7 0 7
```

7.1.3 Signum

```
Z ← × B
```

Informal Description: *Z* is ¯1, 0, or 1, according to whether *B* is negative, zero, or positive, respectively.

Evaluation Sequence:

```
If   B is not a number, signal domain-error.
If   B is zero, return zero.
If   B is greater-than zero, return one.
If   B is less-than zero, return negative-one.
```

Example:

```
      × 1 ¯.5 .33 0 ¯1E¯20
1 ¯1 1 0 ¯1
```

7.1.4 Reciprocal

```
Z ← ÷ B
```

Informal Description: *Z* is 1÷*B*.

Evaluation Sequence:

```
If   B is not a number, signal domain-error.
If   B is zero, signal domain-error.
Return  one divided-by B.
```

Examples:

```
      ÷ ¯.25 .5 1 2 ¯4
¯4 2 1 0.5 ¯0.25

      ÷0
 domain-error
```

7.1.5 Floor

```
Z ← ⌊ B
```

Informal Description: *Z* is the greatest **integer** tolerantly less than or equal to *B*. Uses **comparison-tolerance** .

Evaluation Sequence:

```
If comparison-tolerance is nil, signal implicit-error.
If B is not a number, signal domain-error.
Return the tolerant-floor of B within comparison-tolerance .
```

Example:

In the following, **comparison-tolerance** is 1*E*¯10.

```
        ⌊ ¯3.1416 3.1416 .99999999999 5E20 ¯0.5E¯10
¯4 3 1 5E20 0
```

7.1.6 Ceiling

```
Z ← ⌈ B
```

Informal Description: *Z* is the least **integer** tolerantly greater than or equal to *B*. Uses **comparison-tolerance** .

Evaluation Sequence:

```
If comparison-tolerance is nil, signal implicit-error.
If B is not a number, signal domain-error.
Return -⌊-B.
```

Example:

In the following, **comparison-tolerance** is 1*E*¯10.

```
        ⌈ ¯3.1416 3.1416 5.00000000001
¯3 4 5
```

7.1.7 Exponential

Z ← ⋆ *B*

Informal Description: *Z* is e raised to the power *B*, where e is the base of the natural logarithms.

Evaluation Sequence:

If *B* is not a **number**, signal **domain-error**.
Return the **exponential** of *B*.

Examples:

```
      ⋆ ¯1E50 ¯2 ¯1 0 1 2
0 0.135335 0.367879 1 2.71828 7.38906
      ⋆ .693147
2
```

7.1.8 Natural Logarithm

Z ← ⍟ *B*

Informal Description: *Z* is the natural logarithm of *B*.

Evaluation Sequence:

If *B* is not a **positive-number**, signal **domain-error**.
Return the **natural-logarithm** of *B*.

Examples:

```
      ⍟ 2.718281828459045 2 1E¯50 1E50
1 0.693147 ¯115.129 115.129

      ⍟⋆1
1
```

7.1.9 Magnitude

```
Z ← | B
```

Informal Description: *Z* is the absolute value of *B*.

Evaluation Sequence:

```
If    B is not a number, signal domain-error.
Return the  absolute-value of B.
```

Example:

```
      | 1 ¯0.5 0.33 ¯0.25 0 1E¯20
1 0.5 0.33 0.25 0 1E¯20
```

7.1.10 Factorial

```
Z ← ! B
```

Informal Description: *Z* is the **gamma-function** of *B*+1. If *B* is a **nonnegative-integer**, this is factorial *B*.

Evaluation Sequence:

```
If    B is not a number, signal domain-error.
If    B is a negative-integer, signal domain-error.
Set   B1 to B plus one.
Return  gamma-function of B1.
```

Examples:

```
      ! 0 1 2 3 4 5 6 7 8 9
1 1 2 6 24 120 720 5040 40320 362880

      !¯.5
1.77245
      5 1 ⍴ ! - 1.502 1.503 1.504 1.505 1.506
¯3.54471
¯3.54466
¯3.54464
¯3.54466
¯3.5447
```

Note: The ***gamma-function*** *is defined in, for example, the* ***National Bureau of Standards Handbook of Mathematical Functions****, U.S.Government Printing Office, Washington D.C., 1964.*

See also Hart, J. F., ***Computer Approximations****, Robert C. Krieger Publishing Company, Huntington, NY, 1978.*

7.1.11 Pi times

Z ← ○ *B*

Informal Description: *Z* is π times *B*.

Evaluation Sequence:

If *B* is not a **number**, signal **domain-error**.
Return **pi-times** *B*.

Example:

```
        ○ 1 10 100
3.14159 31.4159 314.159
```

7.1.12 Not

Z ← ~ *B*

Informal Description: *Z* is the Boolean complement of *B*.

Evaluation Sequence:

If *B* is not **near-Boolean**, signal **domain-error**.
If the **integer-nearest-to** *B* is **one**, return **zero**.
Otherwise, return **one**.

Example:

For the following, the **implementation-parameter integer-tolerance** is 1E¯10.

```
   ~ 0 1 1E¯11 .999999999999
1 0 1 0
```

7.2 DYADIC SCALAR FUNCTIONS

__Note:__ The definitions in this section cover only scalar arguments. The __phrase-evaluator evaluate-dyadic-function__ handles non-scalar cases. All __scalar-functions__ yield scalar results when applied to scalar arguments.

The outer product operator, which has not yet been formally introduced at this point in the document, is used in the examples in this section as a convenient way of generating tables. The use of outer product in this section is limited to vector arguments. The same results could be obtained from each example, although not so compactly, by supplying the elements of the left argument one at a time, starting from the leftmost, as left arguments to the scalar function.

For example,

```
      0 1∘.=0 1 2
1 0 0
0 1 0
```

is equivalent to

```
      0 = 0 1 2
1 0 0
      1 = 0 1 2
0 1 0
```

7.2.1 Plus

```
Z ← A + B
```

Informal Description: *Z* is *A* plus *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
Return *A* **plus** *B*.

Example:

```
      ¯2 ¯1 0 1 ∘.+ ¯2 ¯1 0 1
¯4 ¯3 ¯2 ¯1
¯3 ¯2 ¯1  0
¯2 ¯1  0  1
¯1  0  1  2
```

7.2.2 Minus

```
Z ← A - B
```

Informal Description: *Z* is *A* minus *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
Return *A* **minus** *B*.

Example:

```
      ¯2 ¯1 0 1 ∘.- ¯2 ¯1 0 1
0 ¯1 ¯2 ¯3
1  0 ¯1 ¯2
2  1  0 ¯1
3  2  1  0
```

7.2.3 Times

Z ← *A* × *B*

Informal Description: *Z* is *A* times *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
Return *A* **times** *B*.

Example:

```
      ¯2 ¯1 0 1 ∘.× ¯2 ¯1 0 1
 4  2 0 ¯2
 2  1 0 ¯1
 0  0 0  0
¯2 ¯1 0  1
```

7.2.4 Divide

Z ← *A* ÷ *B*

Informal Description: *Z* is *A* divided by *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
If *B* is **zero** and *A* is not **zero**, signal **domain-error**.
If *B* is **zero** and *A* is **zero**, return **one**.
Otherwise, return *A* **divided-by** *B*.

Example:

```
      0 1 2 3 4 ∘.÷ 1 2 3 4
0 0   0        0
1 0.5 0.333333 0.25
2 1   0.666666 0.5
3 1.5 1        0.75
4 2   1.33333  1
```

7.2.5 Maximum

```
Z ← A ⌈ B
```

Informal Description: *Z* is the larger of *A* and *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
If *A* is **greater-than** *B*, return *A*.
Otherwise, return *B*.

Example:

```
        ¯2 ¯1 0 1 ∘.⌈ ¯2 ¯1 0 1
¯2 ¯1 0 1
¯1 ¯1 0 1
 0  0 0 1
 1  1 1 1
```

7.2.6 Minimum

```
Z ← A ⌊ B
```

Informal Description: *Z* is the smaller of *A* and *B*.

Evaluation Sequence:

If either of *A* or *B* is not a **number**, signal **domain-error**.
If *A* is **greater-than** *B*, return *B*.
Otherwise, return *A*.

Example:

```
        ¯2 ¯1 0 1 ∘.⌊ ¯2 ¯1 0 1
¯2 ¯2 ¯2 ¯2
¯2 ¯1 ¯1 ¯1
¯2 ¯1  0  0
¯2 ¯1  0  1
```

7.2.7 Power

```
Z ← A ⋆ B
```

Informal Description: *Z* is *A* raised to the *B*th power.

Evaluation Sequence:

```
If either of  A or B is not a number, signal domain-error.
If  A is a negative-number and B is not an integer, signal domain-error.
If  A is zero and  B is a negative-number, signal domain-error.
If  A is zero and B is zero, return one.
Return   A to-the-power B.
```

Examples:

In the following, **print-precision** is 12.

```
      2⋆32
4294967296

      4⋆0.5
2

      ¯8⋆÷3
domain-error
```

Note: *The distinction made in some APL implementations between rational and irrational powers (to permit expressions such as* ¯8⋆÷3 *to have real results) has been eliminated in this standard to allow complex arithmetic to be a* ***consistent-extension.***

7.2.8 Logarithm

```
Z ← A ⍟ B
```

Informal Description: *Z* is the logarithm of *B* to the base *A*.

Evaluation Sequence:

```
If  A is not a positive-number, signal domain-error.
If  B is not a positive-number, signal domain-error.
If  A is one and B is one, return one.
If  A is one, signal domain-error.
Set   A1 to the natural-logarithm of A.
Set   B1 to the natural-logarithm of B.
Return   B1 divided-by A1.
```

Example:

```
      10 2 10 0.1 ⍟ 2 65536 1E15 1E15
0.30103 16 15 ¯15
```

7.2.9 Residue

```
Z ← A | B
```

Informal Description: *Z* is *B* modulo *A*. Uses **comparison-tolerance**.

Evaluation Sequence:

```
If comparison-tolerance is nil, signal implicit-error.
If either of A or B is not a number, signal domain-error.
If A is zero, return B.
If comparison-tolerance is not zero, and B divided-by A is integral-within comparison-tolerance,
  return zero.
Otherwise set Z to B modulo A.
If Z is A, return zero.
Otherwise, return Z.
```

Examples:

In the following, **print-precision** is 16 and **comparison-tolerance** is 1E¯10.

```
      7 ¯7 ∘.| 31 28 ¯30
 3 0  5
¯4 0 ¯2

      0.2 | 1.4 1.5 1.6
0 0.1 0

      1 | 1E30 1E¯30 ¯1E¯30 .99999999999
0 0 0 0
```

In the following, **comparison-tolerance** is **zero**.

```
      1 | 1E30 1E¯30 ¯1E¯30 .99999999999
0 1E¯30 0 0.99999999999
```

Additional Requirements:

The range of residue is the union of **zero** and the **open-interval-between** **zero** and *A*, except when *A* is **zero**, in which case the range is the single **number** *B*.

Note: The implementation-algorithm P ***modulo*** *Q provides an exact modulo operation. It evaluates R←P-(×P)×|Q×⌊|P÷Q exactly, and returns R if (×R)=×Q, or R+Q otherwise.*

The definition of "mod" in the proposed IEEE standard for Binary Floating-Point Arithmetic (P754) provides an example of this exact evaluation.

Implementations should avoid signalling ***limit-error*** *in* ***residue****. If the operation B* ***divided-by*** *A causes* ***exponent-overflow****, return* ***zero****. If it causes* ***exponent-underflow****, and if A and B have the same signs, return B. If they have different signs, return* ***zero****.*

7.2.10 Binomial

```
Z ← A ! B
```

Informal Description: Z is (**gamma**(1+B))÷((**gamma**(1+A))×**gamma**(1+B-A))

If A and B are **nonnegative-integers**, Z is the number of combinations of B things taken A at a time.

Evaluation Sequence:

If either of A or B is not a **number**, signal **domain-error**.
Determine if each of A, B, and $B-A$ is a **negative-integer**.
Select the appropriate case from the following table, where a **one** indicates that the corresponding value is a **negative-integer** and a **zero** indicates that it is not.

Case			**Rule**
A	*B*	*B-A*	
0	0	0	Return (!*B*)÷(!*A*)×!*B*-*A*.
0	0	1	Return **zero**.
0	1	0	Signal **domain-error**.
0	1	1	Return (¯1**A*)×*A*!*A*-*B*+1.
1	0	0	Return **zero**.
1	0	1	(Case cannot arise.)
1	1	0	Return (¯1**B*-*A*)×(|*B*+1)!(|*A*+1).
1	1	1	Return **zero**.

Example:

```
      ¯4 ¯3 ¯2 ¯1 0 1 2 3 4 ∘.! ¯4 ¯3 ¯2 ¯1 0 1 2 3 4

   1  ¯3  3 ¯1 0 0 0 0 0
   0   1 ¯2  1 0 0 0 0 0
   0   0  1 ¯1 0 0 0 0 0
   0   0  0  1 0 0 0 0 0
   1   1  1  1 1 1 1 1 1
  ¯4  ¯3 ¯2 ¯1 0 1 2 3 4
  10   6  3  1 0 0 1 3 6
 ¯20 ¯10 ¯4 ¯1 0 0 0 1 4
  35  15  5  1 0 0 0 0 1
```

Note: *The APL expressions in the **rule** column indicate the result required, not the algorithm to be used. For example,* 64!65 *should be* 65 *even if* !65 *signals **limit-error**.*

7.2.11 Circular Functions

Z ← *A* ○ *B*

Informal Description: *Z* is the result of applying a function designated by *A* to *B*.

Evaluation Sequence:

If *A* is not a **near-integer**, signal **domain-error**.
If *B* is not a **number**, signal **domain-error**.
Set *A*1 to the **integer-nearest-to** *A*.
If *A*1 is not in the **closed-interval-between** ¯7 and 7, signal **domain-error**.

If *A*1 is ¯7,

- If *B* is not in the **open-interval-between negative-one** and **one**, signal **domain-error**.
- Return the **inverse-hyperbolic-tangent** of *B*.

If *A*1 is ¯6,

- If *B* is **less-than one**, signal **domain-error**.
- Return *Z*, the principal value of the **inverse-hyperbolic-cosine** of *B*, where *Z* is a **nonnegative-number**.

If *A*1 is ¯5, return the **inverse-hyperbolic-sine** of *B*.

If *A*1 is ¯4,

- If *B* is in the **open-interval-between negative-one** and **one**, signal **domain-error**.
- Return *B*×(1-*B**¯2)*0.5.

If *A*1 is ¯3, return *Z*, the principal value in radians of the **inverse-tangent** of *B*, where *Z* is in the **open-interval-between** -π/2 and π/2.

If *A*1 is ¯2,

- If *B* is not in the **closed-interval-between negative-one** and **one**, signal **domain-error**.
- Return *Z*, the principal value in radians of the **inverse-cosine** of *B*, where *Z* is either **zero** or a **number** in the **open-interval-between zero** and π.

If *A*1 is ¯1,

- If *B* is not in the **closed-interval-between negative-one** and **one**, signal **domain-error**.
- Return *Z*, the principal value in radians of the **inverse-sine** of *B*, where *Z* is either π/2, or a **number** in the **open-interval-between** -π/2 and π/2.

If *A*1 is 0,

If *B* is not in the **closed-interval-between negative-one** and **one**, signal **domain-error**.
Return (1-*B**2)*0.5.

If *A*1 is 1, return the **sine** of *B* radians.
If *A*1 is 2, return the **cosine** of *B* radians.
If *A*1 is 3,

If *B* is an odd multiple of π/2, signal **domain-error**.
Return the **tangent** of *B* radians.

If *A*1 is 4, return (1+*B**2)*0.5.
If *A*1 is 5, return the **hyperbolic-sine** of *B*.
If *A*1 is 6, return the **hyperbolic-cosine** of *B*.
If *A*1 is 7, return the **hyperbolic-tangent** of *B*.

Examples:

```
      2 ○ ¯1 ○ .6
.8
      2 ○ 0
1
      3 ○○ ÷4
1
      6 ○ 0
1
```

Note: The APL expressions used for 0○*X*, ¯4○*X*, *and* 4○*X above indicate the result desired, not the algorithm to be used.*

The definition of ¯4○*B has been changed to allow complex arithmetic to be a* ***consistent-extension****. The previous definition was* (¯1+*B**2)*0.5.

7.2.12 And

```
Z ← A ∧ B
```

Informal Description: *Z* is the Boolean product of *A* and *B*.

Evaluation Sequence:

If either *A* or *B* is not **near-Boolean**, signal **domain-error**.
Set *A*1 to the **integer-nearest-to** *A*.
Set *B*1 to the **integer-nearest-to** *B*.
If either *A*1 or *B*1 is **zero**, return **zero**.
Otherwise, return **one**.

Example:

```
          0 1 ∘.∧ 0 1
0 0
0 1
```

7.2.13 Or

```
Z ← A ∨ B
```

Informal Description: *Z* is the Boolean sum of *A* and *B*.

Evaluation Sequence:

If either *A* or *B* is not **near-Boolean**, signal **domain-error**.
Set *A*1 to the **integer-nearest-to** *A*.
Set *B*1 to the **integer-nearest-to** *B*.
If either *A*1 or *B*1 is **one**, return **one**.
Otherwise, return **zero**.

Example:

```
          0 1 ∘.∨ 0 1
0 1
1 1
```

7.2.14 Nand

```
Z ← A ⍲ B
```

Informal Description: *Z* is the Boolean complement of the Boolean product of *A* and *B*.

Evaluation Sequence:

If either *A* or *B* is not **near-Boolean**, signal **domain-error**.
Otherwise, return ~*A*∧*B*.

Example:

```
      0 1 ∘.⍲ 0 1
1 1
1 0
```

7.2.15 Nor

```
Z ← A ⍱ B
```

Informal Description: *Z* is the Boolean complement of the Boolean sum of *A* and *B*.

Evaluation Sequence:

If either *A* or *B* is not **near-Boolean**, signal **domain-error**.
Otherwise, return ~*A*∨*B*.

Example:

```
      0 1 ∘.⍱ 0 1
1 0
0 0
```

7.2.16 Equal

```
    Z ← A = B
```

Informal Description: *Z* is **one** if *A* and *B* are considered equal and **zero** otherwise. *A* and *B* are equal if they are the same **character**, or if they are both **numeric** and *A* is tolerantly equal to *B* within **comparison-tolerance** . Uses **comparison-tolerance** .

Evaluation Sequence:

```
If comparison-tolerance is nil, signal implicit-error.
If the type of A is not the same as the type of B, return zero.
If both A and B are characters,
    If A is the same character as B, return one.
    Otherwise, return zero.
If both A and B are numbers,
    If A is tolerantly-equal to B within comparison-tolerance , return one.
    Otherwise, return zero.
```

Example:

```
      1 2 3 ∘.= 1 2 3
1 0 0
0 1 0
0 0 1
```

In the following, **comparison-tolerance** is 1E¯13.

```
      4 = 4 + 5E¯13 2E¯13 ¯2E¯13 ¯5E¯13
0 1 1 0

      0 = ¯1E¯20 1E¯20 0
0 0 1
      3 = 'A3'
0 0
```

Note: Comparisons of **numbers** whose signs differ are not affected by **comparison-tolerance** .

For any value of **comparison-tolerance** and any two values A and B exactly one of the expressions $A<B$, $A=B$, and $A>B$ is **one**.

Equal should not signal a **limit-error**. For example, the result of **positive-number-limit** = **negative-number-limit** is **zero**. The following is a sample technique for handling exponent-overflow and exponent-underflow when scaling **comparison-tolerance** .

```
Set $C$ to the larger of the absolute values of $A$ and $B$.
Set $D$ to comparison-tolerance times $C$.
If exponent-underflow occurs,
    Set $A1$ to $A$ divided-by $C$.
    Set $B1$ to $B$ divided-by $C$.
    Set $C1$ to the absolute value of $A1$ minus $B1$.
    If $C1$ is greater-than comparison-tolerance , return zero.
    Otherwise, return one.
Set $E$ to the absolute value of $A$ minus $B$.
If exponent-overflow occurs, return zero.
If exponent-underflow occurs,
    Set $A1$ to $A$ divided-by $C$.
    Set $B1$ to $B$ divided-by $C$.
    Set $C1$ to the absolute value of $A1$ minus $B1$.
    If $C1$ is greater-than comparison-tolerance , return zero.
    Otherwise, return one.
If $E$ is not greater-than $D$, return one.
Otherwise, return zero.
```

7.2.17 Less than

```
Z ← A < B
```

Informal Description: *Z* is **one** if *A* is tolerantly less-than *B*, and **zero** otherwise. Uses **comparison-tolerance** .

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of *A* or *B* is not a **number**, signal **domain-error**.
If *A*=*B*, evaluated with the current value of **comparison-tolerance** , is **one**, return **zero**.
If *A* is **less-than** *B*, return **one**.
Otherwise, return **zero**.

Examples:

```
      1 2 3 ∘.< 1 2 3
0 1 1
0 0 1
0 0 0
      0 1 ∘.< 0 1
0 1
0 0
```

7.2.18 Less than or equal to

```
Z ← A ≤ B
```

Informal Description: *Z* is **one** if *A* is less than or tolerantly equal to *B*, and **zero** otherwise. Uses **comparison-tolerance** .

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of *A* or *B* is not a number, signal **domain-error**.
If *A*=*B*, evaluated with the current value of **comparison-tolerance** , is **one**, return **one**.
If *A* is **less-than** *B*, return **one**.
Otherwise, return **zero**.

Examples:

```
      1 2 3 ∘.≤ 1 2 3
1 1 1
0 1 1
0 0 1

      0 1 ∘.≤ 0 1
1 1
0 1
```

7.2.19 Not equal

```
Z ← A ≠ B
```

Informal Description: *Z* is **one** if *A* does not equal *B*, and **zero** otherwise. Uses **comparison-tolerance** .

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
Return ~*A*=*B*, evaluated with the current value of **comparison-tolerance** .

Examples:

```
          'A' ≠ 41
1
          1 2 3 ∘.≠ 1 2 3
0 1 1
1 0 1
1 1 0

          0 1 ∘.≠ 0 1
0 1
1 0
```

7.2.20 Greater than or equal to

```
Z ← A ≥ B
```

Informal Description: *Z* is **one** if *A* is greater than or tolerantly equal to *B*, and **zero** otherwise. Uses **comparison-tolerance** .

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of *A* or *B* is not a **number**, signal **domain-error**.
If *A*=*B*, evaluated with the current value of **comparison-tolerance** , is **one**, return **one**.
If *A* is **greater-than** *B*, return **one**.
Otherwise, return **zero**.

Examples:

```
          1 2 3 ∘.≥ 1 2 3
1 0 0
1 1 0
1 1 1
          0 1 ∘.≥ 0 1
1 0
1 1
```

7.2.21 Greater than

Z ← *A* > *B*

Informal Description: *Z* is **one** if *A* is tolerantly greater than *B*, and **zero** otherwise. Uses **comparison-tolerance** .

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
If either of *A* or *B* is not a **number**, signal **domain-error**.
If *A*=*B*, evaluated with the current value of **comparison-tolerance** , is **one**, return **zero**.
If *A* is **greater-than** *B*, return **one**.
Otherwise, return **zero**.

Examples:

```
      1 2 3 ∘.> 1 2 3
0 0 0
1 0 0
1 1 0

      0 1 ∘.> 0 1
0 0
1 0
```

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8 STRUCTURAL PRIMITIVE FUNCTIONS

8.1 INTRODUCTION

***Note:** The functions in this chapter are used in the evaluation sequences of many non-scalar operations. They are defined here to avoid forward references.*

8.2 MONADIC STRUCTURAL PRIMITIVE FUNCTIONS

8.2.1 Ravel

```
Z ← , B
```

Informal Description: *Z* is a vector containing the elements of *B* in row-major order.

Evaluation Sequence:

Return *Z*, a **vector** such that the **ravel-list** of *Z* is the same as the **ravel-list** of *B*, the **type** of *Z* is the same as the the **type** of *B*, and the **shape-list** of *Z* is a **list** of length **one** containing the **count** of *B* as its only **item**.

Examples:

```
        ,N22
11 12 21 22
        ,N222
111 112 121 122 211 212 221 222
        ,N2221
1111 1121 1211 1221 2111 2121 2211 2221
```

***Note:** Ravel always produces a vector result. The expressions **ravel-list** of Z, **type** of Z, and **shape-list** of Z refer to attributes of an array object.*

8.2.2 Shape

```
Z ← ⍴ B
```

Informal Description: Z is a **numeric** vector containing the shape of the array B.

Evaluation Sequence:

Return Z, an **array** such that the **type** of Z is **numeric**, the **ravel-list** of Z is the **shape-list** of B, and the **shape-list** of Z is a **list** whose only **item** contains the **number-of-items** in the **shape-list** of B.

Examples:

```
        ⍴N

        ⍴,N
1
        ⍴⍴N
0
        ⍴N3
3
        ⍴⍴N3
1
        ⍴N34
3  4
```

Note: Shape always produces a vector result. The expression ***shape-list*** *of* Z *refers to an attribute of an* ***array*** *object.*

8.2.3 Index Generator

```
Z ← ⍳ B
```

Informal Description: *Z* is a **numeric** vector of *B* consecutive ascending **integers**, the first of which is **index-origin** . Uses **index-origin** .

Evaluation Sequence:

```
If index-origin is nil, signal implicit-error.
If the rank of B is greater-than one, signal rank-error.
If the count of B is not one, signal length-error.
If  B is not a near-integer, signal domain-error.
Set  B1 to the integer-nearest-to B.
If  B1 is not a nonnegative-integer, signal domain-error.
If  B1 is zero, return an empty numeric vector.
Generate a numeric vector Z1 of length B1 such that the ravel-list of Z1 consists of the
  integers in the closed-interval-between one and B1 in ascending order.
Generate  Z, a numeric array with the shape-list of Z1 such that each item of the ravel-list of Z
  is (index-origin minus one) plus the corresponding element of Z1.
Return  Z.
```

Examples:

In the following, **index-origin** is **zero**.

```
      ⍳4
0 1 2 3
```

In the following, **index-origin** is **one**.

```
      ⍳4
1 2 3 4
```

8.3 DYADIC STRUCTURAL PRIMITIVE FUNCTIONS

8.3.1 Reshape

```
Z ← A ρ B
```

Informal Description: *Z* is an array of shape ,*A* whose elements are taken sequentially from ,*B* repeated cyclically as required.

Evaluation Sequence:

If the **rank** of *A* is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of *A* is not a **near-integer**, signal **domain-error**.
Set *A*1 to the **integer-array-nearest-to** ,*A*.
If any **item** of the **ravel-list** of *A*1 is not a **nonnegative-counting-number**, signal **domain-error**.
Let *RA* stand for the **product-of** the **ravel-list** of *A*1.
Let *CB* stand for the **count** of *B*.
If *RA* is not **zero** and *CB* is **zero**, signal **length-error**.
Return an **array** *Z* such that the **type** of *Z* is the same as the **type** of *B*, the **shape-list** of *Z* is the same as the **ravel-list** of *A*1, and the **ravel-list** of *Z* is a **list** with *RA* **items** such that for all *I* in the **index-set** of *Z*, **item** *I* of the **ravel-list** of *Z* is **item** 1+*CB*|*I*-1 of the **ravel-list** of *B*.

Examples:

```
        2 4ρN213
111 112 113 211
212 213 111 112
        ρ0ρ'ABCD'
0
        B ← 1E7 1E7 1E7 0 1E7 1E7 1E7 ρ 42
        ρB
1E7 1E7 1E7 0 1E7 1E7 1E7
```

Note: *For any X that is not* ***empty***, *''ρX and (ι0)ρX produce the same result:* *a* ***scalar*** *whose value is that of the* ***first-scalar*** *in X.*

8.3.2 Join

```
Z ← A , B
```

Informal Description: If *A* and *B* are scalars or vectors, *Z* is the vector of length (ρ,*A*)+ρ,*B* whose first ρ,*A* elements are ,*A* and whose last ρ,*B* elements are ,*B*.

If either *A* or *B* has rank greater than one, *Z* is *A*,[(ρρ*A*)⌈ρρ*B*]*B*, as defined under **Join Along an Axis**.

Evaluation Sequence:

If *A* is a **scalar** and *B* is a **scalar**, return (,*A*),,*B*.
If *A* is a **scalar** and *B* is a **vector**, return (,*A*),*B*.
If *A* is a **vector** and *B* is a **scalar**, return *A*,,*B*.
If *A* is a **vector** and *B* is a **vector**,

> If *A* is **empty** and *B* is **empty** and the **type** of *A* differs from the **type** of *B*, signal **domain-error**.
> If *B* is **empty**, return *A*.
> If *A* is **empty**, return *B*.
> If the **type** of *A* differs from the **type** of *B*, signal **domain-error**.
> Otherwise, return a **vector** *Z*, such that the **shape-list** of *Z* is (ρ*A*)+ρ*B*, the **ravel-list** of *Z* is a **list** whose first ρ*A* **items** are the **ravel-list** of *A* and whose last ρ*B* **items** are the **ravel-list** of *B*, and the **type** of *Z* is the same as the **type** of *A*.

Otherwise, return *A*,[(ρρ*A*)⌈ρρ*B*]*B*.

Example:

```
      '',0
0
```

*Note: This subsection intentionally contains a forward reference to **Join Along an Axis**. **Join** and **Join Along an Axis** are defined separately because the description of **Join Along an Axis** requires APL operations that depend for their definitions in turn upon **Join**.*

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9 OPERATORS

9.1 INTRODUCTION

Note: *The forms in this chapter are referred to as* ***operators****.*

Operators take scalar functions as arguments and produce functions, called derived functions, as results. For example, the operator ***reduction*** *takes a single dyadic* ***scalar-function*** *as an argument and produces a monadic function as a result.*

Because f is restricted to scalar functions, the derived functions produced by scan and reduction may be viewed either as being applied between subarrays of rank 0⌈¯1+⍴⍴*B* *or as being repeated on vector subarrays. As an expository device, the evaluation sequences in this document use the vector definition.*

9.2 MONADIC OPERATORS

9.2.1 Reduction

```
Z ← f/ B
Z ← f/[K] B
Z ← f⌿ B
Z ← f⌿[K] B
```

Informal Description: *Z* is the value produced by placing the primitive scalar dyadic function **f** between subarrays of *B* and evaluating the resulting expression. The axis designated determines how the subarrays are chosen. Uses **index-origin** .

Evaluation Sequence:

```
If f is not a primitive-dyadic-scalar-function, signal syntax-error.

For form f/B

    If B is a scalar, return B.
    Otherwise, return f/[ρρB] B evaluated with index-origin set to one.

For form f⌿B

    If B is a scalar, return B.
    Otherwise, return f/[1] B evaluated with index-origin set to one.

For forms f/[K] B and f⌿[K] B

    If K is not a valid-axis for B, signal axis-error.
    Otherwise, set K1 to the integer-nearest-to K.
    If B is a vector,

        If the length of B is zero, take the action designated in Table 5 for f.
        If the length of B is one, return a scalar Z such that the type of Z is the type of B and the
          ravel-list of Z is the ravel-list of B.
        If the length of B is greater-than one,

            Set B1 to the first-scalar in B.
            Set B2 to the remainder-of B.
            Return B1 f f/B2.

    If the rank of B is greater-than one, return an array Z such that the shape-list of Z is the
      shape-list of B with item K1 omitted, and the ravel-list of Z has the property that if Z1 is
      an item of Z and B3 is the corresponding vector-item along-axis K1 of B, then Z1 is f/B3.
```

Examples:

```
      +/1 2 3
6
      ×/1 2
2
      =/'A'
A
      =/'AA'
1
      =/'AAA'
```

```
0
```

Additional Requirements:

If Z is **empty**, the **type** of Z is determined by the argument function **f**; since **f** is a scalar function in this standard, the **type** of Z when **empty** is always **numeric**.

The **scalar-function f** may signal **domain-error**.

When applied to a rank N array, the derived function produces a result array of rank `0⌈`N`-1`. When applied along an empty **axis** in an array, reduction produces an array whose shape is that of the argument array with the designated **axis** deleted. The elements of the array, if any, are determined by the argument function and **Table 5**. In some cases, an error is signalled.

For example,

```
      +/2 0 ρ5.1
0 0
      ρ+/2 0ρ5.1
2
```

Table 5 — Actions for the Reduction of an Empty Vector

Dyadic Function		Action
Plus	+	Return **zero**.
Minus	-	Return **zero**.
Times	×	Return **one**.
Divide	÷	Return **one**.
Residue	|	Return **zero**.
Minimum	⌊	Return **positive-number-limit**.
Maximum	⌈	Return **negative-number-limit**.
Power	*	Return **one**.
Logarithm	⍟	Signal **domain-error**.
Circular	○	Signal **domain-error**.
Binomial	!	Return **one**.
And	∧	Return **one**.
Or	∨	Return **zero**.
Nand	⍲	Signal **domain-error**.
Nor	⍱	Signal **domain-error**.
Less	<	Return **zero**.
Not greater	≤	Return **one**.
Equal	=	Return **one**.
Not less	≥	Return **one**.
Greater	>	Return **zero**.
Not equal	≠	Return **zero**.

9.2.2 Scan

```
Z ← f\ B
Z ← f\[K] B
Z ← f⍀ B
Z ← f⍀[K] B
```

Informal Description: *Z* is an array having the same shape as *B* and containing the results produced by **f** reduction over all prefixes of a designated axis of *B*. Uses **index-origin** .

Evaluation Sequence:

If **f** is not a **primitive-dyadic-scalar-function**, signal **syntax-error**.

For form **f***B*

If *B* is a **scalar**, return *B*.
Otherwise, return **f**\[ρρ*B*] *B* evaluated with **index-origin** set to **one**.

For form **f**⍀ *B*

If *B* is a **scalar**, return *B*.
Otherwise, return **f**\[1] *B* evaluated with **index-origin** set to **one**.

For forms **f**\[*K*]*B* and **f**⍀[*K*]*B*

If *K* is not a **valid-axis** for *B*, signal **axis-error**.
Otherwise, set *K*1 to the **integer-nearest-to** *K*.

If *B* is a **vector**,

If the **count** of *B* is **less-than two**, return *B*.
If the **type** of *B* differs from the **type** of **f**/*B*[ι2], signal **domain-error**.
Otherwise return *Z*, a **vector** such that the **type** of *Z* is the **type** of **f**/*B*[ι2], the **shape-list** of *Z* is the **shape-list** of *B*, and the **ravel-list** of *Z* is such that **item** *I* of the **ravel-list** of *Z* is **f**/*B*[ι*I*] for all *I* in the **index-set** of the **ravel-list** of *B*.

If the **rank** of *B* is **greater-than one**, each **vector-item along-axis** *K*1 of *Z* is **f***B*1, where *B*1 is the corresponding **vector-item along-axis** *K*1 of *B*.

Examples:

```
      +\1 1 1
1 2 3
      ∧\1 1 1 0 0 0 1 1 1
1 1 1 0 0 0 0 0 0
      -\'A'
A
      =\'AB'
domain-error
```

Additional Requirements:

If the operator **reduction** signals an error when called by **scan**, **scan** returns the resultant error **token**.

The evaluation sequence above describes a quadratic algorithm for **scan**. If the function **f** is associative, **scan** may be implemented with a linear algorithm.

Note: Various error checks have been performed on K by the phrase evaluators, and ***index-origin*** *is* ***one****, before this evaluation sequence is called.*

9.3 DYADIC OPERATORS

9.3.1 Outer Product

Z ← *A* ∘ . **f** *B*

Informal Description: *Z* is an array of shape (ρ*A*),ρ*B*. The elements of *Z* are the result of applying the **primitive-dyadic-scalar-function f** to every possible combination of scalar arguments where the left argument is an element of *A* and the right an element of *B*.

Z is such that if *I* is an **index-list** that selects a single element of *Z*, the first ρ*A* **items** of *I* are the **index-list** that would select from *A* the element used as the left argument to **f** and the last ρ*B* **items** of *I* are the **index-list** that would select from *B* the element used as the right argument to **f**.

Evaluation Sequence:

If **f** is not a **primitive-dyadic-scalar-function**, signal **syntax-error**.
Return *Z*, an **array** such that the **type** of *Z* is **numeric**, the **shape-list** of *Z* is (ρ*A*),ρ*B*, and the **ravel-list** of *Z* has the following property:

Let *I* stand for an **item** of the **index-set** of the **ravel-list** of *A*.
Let *J* stand for an **item** of the **index-set** of the **ravel-list** of *B*.
Let *X* stand for **item** *I* of the **ravel-list** of *A*.
Let *Y* stand for **item** *J* of the **ravel-list** of *B*.
Let *N* stand for the **count** of *B*.
Let *P* stand for *J*+(*N*×(*I*-1)).

Then, **item** *P* of the **ravel-list** of *Z* is *X* **f** *Y*.

Example:

```
      10 20 30 ∘.+ 1 2 3
11 12 13
21 22 23
31 32 33
```

Additional Requirements:

If the **scalar-function f** signals an error, outer product returns the resulting error **token**.

Note: The type of the result of outer product is numeric only because all permitted argument functions return numeric results.

9.3.2 Inner Product

Z ← *A* **f** **.** **g** *B*

Informal Description: *Z* is an array of shape (ρ*A*)[ι0⌈¯1+ρρ*A*],(ρ*B*)[1+ι0⌈¯1+ρρ*B*]. The elements of *Z* are the results obtained from evaluating the expression **f**/*X***g***Y* for all possible combinations of *X* and *Y*, where *X* is a **vector-item along-axis** ρρ*A* of *A* and *Y* a **vector-item along-axis one** of *B*.

Evaluation Sequence:

If either **f** or **g** is not a **primitive-dyadic-scalar-function**, signal **syntax-error**.

If *A* is a **scalar** or **one-element-vector** and *B* is not, set *A*1 to (1ρρ*B*)ρ*A*.
If *A* and *B* are **scalars** or **one-element-vectors**, set *A*1 to ,*A*.
Otherwise, set *A*1 to *A*.

If *B* is a **scalar** or **one-element-vector** and *A* is not, set *B*1 to (ρ*A*)[ρρ*A*]ρ*B*.
If *A* and *B* are **scalars** or **one-element-vectors**, set *B*1 to ,*B*.
Otherwise, set *B*1 to *B*.

If the **last-item** in the **shape-list** of *A*1 is not the same as the **first-item** in the **shape-list** of *B*1, signal **length-error**.
If *A*1 and *B*1 are both **vectors**, return **f**/*A*1 **g** *B*1.
Otherwise, return *Z*, an **array** such that the **type** of *Z* is **numeric**, the **shape-list** of *Z* is (ρ*A*1)[ι0⌈¯1+ρρ*A*1],(ρ*B*1)[1+ι0⌈¯1+ρρ*B*1] and the **ravel-list** of *Z* has the following property:

Let *I* stand for an **item** of the **index-set** of the **ravel-along-axis** (ρρ*A*1) of *A*1.
Let *X* stand for **vector-item** *I* of the **ravel-along-axis** (ρρ*A*1) of *A*1.
Let *J* stand for an **item** of the **index-set** of the **ravel-along-axis one** of *B*1.
Let *Y* stand for **vector-item** *J* of the **ravel-along-axis one** of *B*1.
Let *N* stand for the **number-of-items** in the **ravel-along-axis one** of *B*1.
Let *P* stand for (*N*×(*I*-1))+*J*.
Then, **item** *P* of the **ravel-list** of *Z* is **f** / *X* **g** *Y*.

Examples:

```
      4 2 1+.×1 0 1
5
      N22+.×0 1
12 22
      N22+.×1 0
11 21
      N22+.×2 2ρ0 1 1 0
12 11
22 21
```

Additional Requirements:

If **scalar-function f** or **g** signals an error, inner product returns the resulting error **token**.

*Note: The evaluation sequence rule for when A1 and B1 are **vectors** holds if A1 or B1 is the empty vector. The result returned is f/ι0.*

The type of the result of inner product is numeric only because all permitted argument functions return numeric results.

10 MIXED FUNCTIONS

10.1 MONADIC MIXED FUNCTIONS

10.1.1 Roll

Z ← ? *B*

Informal Description: *Z*, for *B* a scalar integer, is **index-origin** plus *R*, where *R* is an integer selected pseudorandomly from the set of all nonnegative integers less than *B*. Each integer in the set has an equal chance of being selected. Uses **index-origin** . Uses and sets **random-link** .

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If **random-link** is **nil**, signal **implicit-error**.
Let *J* stand for **item** *I* of the **ravel-list** of *B*.
If *B* is not **scalar**, return a **numeric array** *Z* such that the **shape-list** of *Z* is the **shape-list** of *B* and, for all *I* in the **index-set** of the **ravel-list** of *Z*, **item** *I* of the **ravel-list** of *Z* is ?*J*.
If *B* is not a **near-integer**, signal **domain-error**.
Set *B*1 to the **integer-nearest-to** *B*.
If *B*1 is not a **positive-integer**, signal **domain-error**.
Set *Z*0 to a **nonnegative-integer less-than** *B*1 and set **random-link** to a new value resulting from the **implementation-algorithm pseudorandom-number-generator** whose only inputs are *B*1 and **random-link** .
Return *Z*0 **plus index-origin** .

Examples:

```
      ?1 1 1
1 1 1

      S←⎕RL
      ?4⍴1E30
1.69584E29 2.5949E29 5.38382E29 8.38274E29
      ⎕RL←S
      ?4⍴1E30
1.69584E29 2.5949E29 5.38382E29 8.38274E29

      (?1E20 1E20)÷1E20
0.218959   0.678865
```

Additional Requirements:

The operation of **roll** is **atomic**: If **roll** signals an error, **random-link** shall be unchanged.

The result of *?B*, where *B* is an **array**, shall be reproducible.

A **conforming-implementation** shall provide documentation describing the properties of its **pseudorandom-number-generator**.

Note: *One class of appropriate algorithms is Lehmer's linear congruential method, described in Knuth, D. E.,* ***Seminumerical Algorithms****, page 9.*

Roll is often considered a scalar function. However, it does not have the property that the elements of its result array can be produced in parallel.

10.1.2 Grade Up

```
Z ← ⍋ B
```

Informal Description: Z is, for B a vector, a permutation of ιρB for which B[Z] is a monotone increasing sequence. The indices of identical elements of B occur in Z in ascending order. Uses **index-origin** .

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If the **rank** of B is not **one**, signal **rank-error**.
If ρB is **zero**, return ι0.
If any **item** of the **ravel-list** of B is not a **number**, signal **domain-error**.
If ρB is **one**, return a **one-element-vector** Z such that the **type** of Z is **numeric** and the **ravel-list** of Z contains **index-origin** .
Otherwise, generate Z1, a permutation of ιρB such that for I and J, arbitrary elements of ιρB for which I is **less-than** J,

B[Z1[I]] is not **greater-than** B[Z1[J]] and

B[Z1[I]] **equals** B[Z1[J]] implies that Z1[I] is **less-than** Z1[J].

Generate Z, a **numeric array** with the **shape-list** of Z1 such that each **item** of the **ravel-list** of Z is (**index-origin minus one**) **plus** the corresponding element of Z1.
Return Z.

Examples:

```
         ⍋V ← 1.1   3.1   1.1   2.1   5.1
1  3  4  2  5
         ⍋⍋V
1  4  2  3  5
         V[⍋V]
1.1  1.1  2.1  3.1  5.1
```

In the following, **index-origin** is **zero**.

```
         ⍋V
0  2  3  1  4
```

Additional Requirements:

The **system-parameter comparison-tolerance** is not an implicit argument of grade up.

10.1.3 Grade Down

```
Z ← ⍒ B
```

Informal Description: *Z* is, for a vector *B*, a permutation of ⍳⍴*B* for which *B*[*Z*] is a monotone decreasing sequence. The indices of identical elements of *B* occur in *Z* in ascending order. Uses **index-origin** .

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If the **rank** of *B* is not **one**, signal **rank-error**.
Return ⍋-*B*.

Examples:

```
       V ← 1.1  3.1  1.1  2.1  5.1
       ⍒V
5 2 4 1 3
       V[⍒V]
5.1 3.1 2.1 1.1 1.1
```

In the following, **index-origin** is **zero**.

```
       ⍒V
4 1 3 0 2
```

Additional Requirements:

The **system-parameter comparison-tolerance** is not an implicit argument of grade down.

Note: *Grade up and grade down are* ***stable sort algorithms*** *because they preserve the relative order of identical elements of B.*

One appropriate algorithm for grade up and down appears in Woodrum, L. J., ***Internal Sorting with Minimal Comparing****, IBM System Journal, Vol. 8, No. 3, p.189, 1969.*

10.1.4 Reverse

```
Z ← ⌽ B
Z ← ⊖ B
Z ← ⌽[K] B
Z ← ⊖[K] B
```

Informal Description: Z is an array whose elements are those of B taken in reverse order along a specified axis. Uses **index-origin** .

Evaluation Sequence:

```
For form   ⌽B

    If  B is scalar, return B.
    Otherwise, return  ⌽[⍴⍴B] B, evaluated with index-origin set to one.

For form   ⊖B

    If  B is scalar, return B.
    Otherwise, return  ⌽[1] B, evaluated with index-origin set to one.

For forms   ⌽[K] B and ⊖[K] B

    If  K is not a valid-axis for B, signal axis-error.
    Otherwise, set  K1 to the integer-nearest-to K.
    If  B is a vector, return B[(1+⍴B)-⍳⍴B], evaluated with index-origin set to one.
    Otherwise, return an array  Z, such that the type of Z is the type of B, the shape-list of Z is
      the shape-list of B, and the ravel-list of Z has the property that each vector-item along-axis
      K1 of Z is ⌽ applied to the corresponding vector-item along-axis K1 of B.
```

Examples:

```
        ⌽N23
13  12  11
23  22  21
        ⌽[2] N224

        121 122 123 124
        111 112 113 114

        221 222 223 224
        211 212 213 214
```

Note: Various error checks have been performed on K by the phrase evaluators, and ***index-origin*** *is* ***one****, before this evaluation sequence is called.*

10.1.5 Monadic Transpose

Z ← ⍉ B

Informal Description: Z is B with the order of the axes reversed.

Evaluation Sequence:

Return (⌽⍳⍴⍴B)⍉B, evaluated with **index-origin** set to **one**.

Examples:

```
      ⍉3
3
      ⍴⍴⍉3
0

      ⍉ N23
11 21
12 22
13 23

      ⍉ N234
111 211
121 221
131 231

112 212
122 222
132 232

113 213
123 223
133 233

114 214
124 224
134 234
```

Note: This subsection contains a forward reference to ***dyadic transpose.***

10.1.6 Matrix Inverse

Z ← ⌹ *B*

Informal Description: *Z* is the result of applying a generalisation of the matrix inverse function to *B*. **Matrix inverse** is **matrix divide** with an appropriate identity matrix as a left argument.

Evaluation Sequence:

If the **rank** of *B* is **greater-than two**, signal **rank-error**.
Return ((2⍴1⍴⍴*B*)⍴1,(1⍴⍴*B*)⍴0) ⌹ *B*.

Note: *This subsection contains a forward reference to* ***matrix divide.***

The following article, Martir, G. A., The Solutions of Linear Systems in APL: Towards An Extension of Matrix Divide APL80 Noordwijkerhout June 24-26, 1980 — Published by Gijsbert van der Linden, North-Holland Publishing Company Amsterdam. New York. Oxford, describes the motivation for matrix inverse and an acceptable algorithm.

The following document also deals with this subject: Jenkins, M. A., Domino — An APL Primitive Function for Matrix Inversion — Its Implementation and Applications. APL Quote-Quad Vol III No. 4, February 1972, pp. 4-15.

10.1.7 Execute

Z ← ⍎ B

Informal Description: Z is the result of evaluating the **character** scalar or vector B as a line of APL.

Evaluation Sequence:

```
If the rank of B is greater-than one, signal rank-error.
If any item of the ravel-list of B is not a character, signal domain-error.
Generate a new context in which

    mode is execute,
    current-line is the ravel-list of B,
    current-function is 0 0⍴' ',
    current-line-number is one.
    current-statement is the empty list of tokens, and
    stack is the empty list of tokens.

Append the new context to the state-indicator of the active-workspace as a new first item.
Set Z to evaluate-line.
Remove the first context from the state-indicator.
Return Z.
```

Examples:

```
      ⍎'T←3'

      T
3
      ⎕←⍎'T←3'
3
      A←⍎''
value-error
```

*Note: If an error is signalled during **execute**, the user should be able to determine from information provided by the system where the error occurred in the argument of **execute** as well as where the failing execute primitive occurred in the **immediate-execution** or **defined-function** line.*

10.2 DYADIC MIXED FUNCTIONS

10.2.1 Join Along an Axis

Z ← *A* ,[*K*] *B*

Informal Description: *Z* is formed by joining *A* and *B* along a designated axis. There are two suboperations, catenate and laminate. The catenate suboperation joins the arrays along an existing axis, the laminate suboperation along a new axis. The choice of axis and the choice of operation is determined by *K*: if *K* is a **near-integer**, the operation is catenate and the axis is *K*; if *K* is not a **near-integer**, the operation is laminate, the new axis is ⌈*K*, and the axes greater than or equal to ⌈*K* are renumbered. Uses **index-origin** .

Evaluation Sequence:

If *K* is not a **near-integer**,

If *A* is a **scalar**, set *A*1 to (ρ*B*)ρ*A*.
Otherwise, set *A*1 to *A*.

If *B* is a **scalar**, set *B*1 to (ρ*A*)ρ*B*.
Otherwise, set *B*1 to *B*.

If the **rank** of *A*1 differs from the **rank** of *B*1, signal **rank-error**.

If *K* is not in the **open-interval-between zero** and (**one plus** the **rank** of *A*1), signal **axis-error**.

If the **shape-list** of *A*1 differs from the **shape-list** of *B*1, signal **length-error**.
Set *T* to (1,ρ*A*1)[⍋*K*,⍳ρρ*A*1].
Set *A*2 to *T*ρ*A*1.
Set *B*2 to *T*ρ*B*1.

Return *A*2,[⌈*K*] *B*2, evaluated with **index-origin** set to **one** and with **comparison-tolerance** set to **integer-tolerance**.

10.2 DYADIC MIXED FUNCTIONS

10.2.1 Join Along an Axis

Z ← *A* ,[*K*] *B*

Informal Description: *Z* is formed by joining *A* and *B* along a designated axis. There are two suboperations, catenate and laminate. The catenate suboperation joins the arrays along an existing axis, the laminate suboperation along a new axis. The choice of axis and the choice of operation is determined by *K*: if *K* is a **near-integer**, the operation is catenate and the axis is *K*; if *K* is not a **near-integer**, the operation is laminate, the new axis is ⌈*K*, and the axes greater than or equal to ⌈*K* are renumbered. Uses **index-origin** .

Evaluation Sequence:

If *K* is not a **near-integer**,

If *A* is a **scalar**, set *A*1 to (⍴*B*)⍴*A*.
Otherwise, set *A*1 to *A*.

If *B* is a **scalar**, set *B*1 to (⍴*A*)⍴*B*.
Otherwise, set *B*1 to *B*.

If the **rank** of *A*1 differs from the **rank** of *B*1, signal **rank-error**.

If *K* is not in the **open-interval-between zero** and (**one plus** the **rank** of *A*1), signal **axis-error**.

If the **shape-list** of *A*1 differs from the **shape-list** of *B*1, signal **length-error**.
Set *T* to (1,⍴*A*1)[⍋*K*,⍳⍴⍴*A*1].
Set *A*2 to *T*⍴*A*1.
Set *B*2 to *T*⍴*B*1.

Return *A*2,[⌈*K*] *B*2, evaluated with **index-origin** set to **one** and with **comparison-tolerance** set to **integer-tolerance**.

Examples:

```
      ⎕←M←2 3⍴'Δ'
ΔΔΔ
ΔΔΔ
      ⎕←H←3 3⍴'○'
○○○
○○○
○○○
      M,[1]H
ΔΔΔ
ΔΔΔ
○○○
○○○
○○○
      ⎕←L←2 4⍴'⎕'
⎕⎕⎕⎕
⎕⎕⎕⎕
      M,L
ΔΔΔ⎕⎕⎕⎕
ΔΔΔ⎕⎕⎕⎕
      M,'+'
ΔΔΔ+
ΔΔΔ+
      M,'34'
ΔΔΔ3
ΔΔΔ4
      M,[1]'345'
ΔΔΔ
ΔΔΔ
345
      1 2 3,[.5] 4 5 6
1 2 3
4 5 6
      1 2 3,[1.5]4 5 6
1 4
2 5
3 6
      1 2 3,[1.5]4
1 4
2 4
3 4
      (2 0⍴5),'A'
A
A
      ⍴3,[.5]''
2 0
```

Note: *Various error checks have been performed on K by the phrase evaluators, and* ***index-origin*** *is* ***one****, before this evaluation sequence is called.*

10.2.2 Index of

```
Z ← A ι B
```

Informal Description: *Z* is a numeric array of shape ρ*B*. Each element of *Z* is the least index in *A* of a value **tolerantly-equal** within **comparison-tolerance** to the corresponding item of *B*. Uses **index-origin** and **comparison-tolerance**.

Evaluation Sequence:

```
If   index-origin is nil, signal implicit-error.
If   comparison-tolerance is nil, signal implicit-error.
If   A is not a vector, signal rank-error.
Set  Z to +/∧\B∘.≠A, evaluated with the current value of comparison-tolerance .
Return   Z plus index-origin .
```

Examples:

In the following, **index-origin** is **zero**.

```
      ⎕←A←2 2ρ1.1 3.1 5.1 4.1
1.1 3.1
5.1 4.1
      3.1 4.1 5.1ιA
3 0
2 1
      'ABC'ι3
3
```

In the following, **index-origin** is **one**.

```
      '123ABC'ι'3BD'
3 5 7
      '123ABC'ι3
7
```

10.2.3 Member of

```
Z ← A ∊ B
```

Informal Description: Z is a Boolean array with the shape of A. An element of Z is **one** if the corresponding element of A is **tolerantly-equal** to some element of B; otherwise, it is **zero**. Uses **comparison-tolerance** .

Evaluation Sequence:

If **comparison-tolerance** is **nil**, signal **implicit-error**.
Return ∨/A∘.=,B, evaluated with the current value of **comparison-tolerance** .

Examples:

```
      ⎕←B←2 2⍴1.1 3.1 5.1 4.1
1.1 3,1
5.1 4.1
      3.1 5.1 7.1 ∊ B
1 1 0
      19∊'CLUB'
0
      'BE' ∊ 'BOP'
1 0
      'NADA'∊⍳0
0 0 0 0
       (⌈/⍳0)∊⌊/⍳0
0
```

***Note:** If B is empty, the result of A∊B is* (⍴A)⍴0.

10.2.4 Deal

```
Z ← A ? B
```

Informal Description: *Z* is **index-origin** plus *R*, where *R* is a vector of shape *A* obtained by making *A* pseudorandom selections without replacement from the set of nonnegative integers less than *B*. Uses **index-origin** . Uses and sets **random-link** .

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If **random-link** is **nil**, signal **implicit-error**.
If the **rank** of *A* or the **rank** of *B* is **greater-than one**, signal **rank-error**.
If either of *A* or *B* is neither a **scalar** nor a **one-element-vector**, signal **length-error**.
If either of *A* or *B* is not a **near-integer**, signal **domain-error**.

Set *A*1 to the **integer-nearest-to** *A*.
Set *B*1 to the **integer-nearest-to** *B*.
If either of *A*1 or *B*1 is not a **nonnegative-number**, signal **domain-error**.

If *A*1 is **greater-than** *B*1, signal **domain-error**.
If *A*1 is **zero**, return ⍳0.

Using the **implementation-algorithm deal**, generate a **numeric vector** *Z*0 of **length** *A*1 whose elements are selected in a pseudorandom fashion without duplication from the **integers** in the **closed-interval-between zero** and *B*1 **minus one**, then **set random-link** to a new value.
Return *Z*0 **plus index-origin** .

Example:

```
      12?300
2 3 42 94 70 105 215 9 110 298 201 5
```

Additional Requirements:

The selection of elements in *Z* is pseudorandom (see the operation **roll**). The elements of *Z*, their order, and the new value of **random-link** are determined by an **implementation-algorithm** whose only inputs are *A*1, *B*1 and **random-link** . The operation of **deal** is **atomic**: if **deal** signals an error, **random-link** shall be unchanged. The result of *A*?*B*, where *A* and *B* are **arrays**, shall be reproducible.

10.2.5 Compress

```
Z ← A / B
Z ← A ⌿ B
Z ← A /[K] B
Z ← A ⌿[K] B
```

Informal Description: Z is an array formed by selecting those subarrays, across a specified axis of B, for which the corresponding element of the **near-Boolean** vector or scalar A is tolerantly equal to **one** within **integer-tolerance**. Uses **index-origin** .

Evaluation Sequence:

```
If A is a scalar, set A1 to ,A.
Otherwise, set A1 to A.
If B is a scalar, set B1 to (ρA1)ρB.
Otherwise, set B1 to B.

For form A/B

    Return A1/[ρρB1] B1 evaluated with index-origin set to one.

For form A⌿B

    Return A1/[1] B1 evaluated with index-origin set to one.

For forms A/[K] B and A⌿[K] B

    If K is not a valid-axis for B1, signal axis-error.
    Otherwise, set K1 to the integer-nearest-to K.

    If the rank of A1 is greater-than one, signal rank-error.

    If the count of A1 is one, set A2 to (ρB1)[K1]ρA1.
    Otherwise, set A2 to A1.
    If ρA2 is not the same as (ρB1)[K1], signal length-error.
    If any item of the ravel-list of A2 is not a near-Boolean, signal domain-error.
    Set A3 to the Boolean-array-nearest-to A2.
    If B1 is a vector, return B1[(+/A3)ρ⍒A3].

    Otherwise, return an array Z such that

        The type of Z is the same as the type of B,
        the shape-list of Z is ((K1≠ιρρB1)×ρB1)+(K1=ιρρB1)×+/A3, evaluated with
          comparison-tolerance set to zero, and
        the ravel-list of Z has the property that if Z2 and B2 are corresponding vector-items
          along-axis K1 of Z and B1 respectively, then Z2 is A3/B2.
```

Examples:

```
      1 0 1 / 1 2 3
1 3
      1 / 1 2 3
1 2 3
      1 0 1/ 2
2 2
      0 0 1 0 0 1 0 /[2] N2714
1311 1312 1313 1314

1611 1612 1613 1614


2311 2312 2313 2314

2611 2612 2613 2614
      ρ1/1
1
      ρρ (,1)/2
1
```

Note: Various error checks have been performed on K by the phrase evaluators, and ***index-origin*** *is* ***one****, before this evaluation sequence is called.*

10.2.6 Expand

```
Z ← A \ B
Z ← A ⍀ B
Z ← A \[K] B
Z ← A ⍀[K] B
```

Informal Description: Z is, for a **near-Boolean** scalar or vector A, an array of the same type as B containing subarrays of B along a specified axis.

For vector B, Z is such that A/A\B is B and, if P is the **typical-element** of B, (~A)/A\B is (+/~A)⍴P; similarly, A/[K]A\[K]B is B, A⌿A⍀B is B, and (~A)/[K]A\[K]B and (~A)⌿A⍀B are arrays of the **typical-element** of B. Uses **index-origin** .

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of A is not a **near-Boolean**, signal **domain-error**.
Set A1 to the **Boolean-array-nearest-to** ,A.
If B is a **scalar**, set B1 to (+/A1)⍴B.
Otherwise, set B1 to B.

For form A\B

> Return A1\[⍴⍴B1] B1 evaluated with **index-origin** set to **one**.

For form A⍀B

> Return A1\[1] B1 evaluated with **index-origin** set to **one**.

For forms A\[K] B and A⍀[K] B

> If K is not a **valid-axis** for B1, signal **axis-error**.
> Otherwise, set K1 to the **integer-nearest-to** K.
>
> If (⍴B1)[K1] differs from +/A1, signal **length-error**.
> Return an **array** Z such that
>
> > the **type** of Z is the **type** of B,
> > the **shape-list** of Z is ((K1≠⍳⍴⍴B1)×⍴B1)+(K1=⍳⍴⍴B1)×⍴A1, evaluated with **comparison-tolerance** set to **zero**, and
> > the **ravel-list** of Z has the property that A1/[K1]Z is B1 and (~A1)/[K1]Z is an **array** consisting entirely of the **typical-element** of B.

Examples:

```
      1 0 1 \ 1 3
1 0 3
      1 1 1 0 1 \ 'ABCD'
ABC D
      1 0 1\ 2
2 0 2
      1 0 1\[2] 2 2 ρ 'ABCD'
A B
C D
      1 0 1 1\1 2 3
1 0 2 3
      1 0 1 1⍀3
3 0 3 3
      0 1 \ 3 1 ρ 3.14 2E17 ¯47
0  3.14E0
0  2.00E17
0 ¯4.70E1

      1 0 1 0\[2] N224
111 112 113 114
  0   0   0   0
121 122 123 124
  0   0   0   0

211 212 213 214
  0   0   0   0
221 222 223 224
  0   0   0   0
```

Note: Various error checks have been performed on K by the phrase evaluators, and ***index-origin*** *is* ***one****, before this evaluation sequence is called.*

10.2.7 Rotate

```
Z ← A ⌽ B
Z ← A ⊖ B
Z ← A ⌽[K] B
Z ← A ⊖[K] B
```

Informal Description: Z is an array of the same type and shape as B in which elements have been shifted cyclically along a specified axis. The amount and direction of shift is controlled by A. Uses **index-origin** .

Evaluation Sequence:

For form A ⌽ B

If B is a **scalar** and A is either a **scalar** or a **vector** of **length one**,

If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
Otherwise, return B.

Otherwise, return A ⌽[⍴⍴B] B evaluated with **index-origin** set to **one**.

For form A ⊖ B

If B is a **scalar** and A is either a **scalar** or a **vector** of **length one**,

If any **item** of the **ravel-list** of A is not a **near-integer**, signal **domain-error**.
Otherwise, return B.

Otherwise, return A ⌽[1] B evaluated with **index-origin** set to **one**.

For forms A ⌽[K] B and A ⊖[K] B

If K is not a **valid-axis** for B, signal **axis-error**.
Otherwise, set K1 to the **integer-nearest-to** K.

If A is a **scalar**, set A1 to ((K1≠⍳⍴⍴B)/⍴B)⍴A, evaluated with **comparison-tolerance** set to **zero**.
Otherwise, set A1 to A.

If A1 is a **one-element-vector** and B is a **vector**, set A2 to (⍳0)⍴A1.
Otherwise, set A2 to A1.

If the **rank** of B minus the **rank** of A2 is not **one**, signal **rank-error**.
If the **shape-list** of A2 is not the same as the **shape-list** of B with **axis** K1 omitted, signal **length-error**.
If any **item** of the **ravel-list** of A2 is not a **near-integer**, signal **domain-error**.
Set A3 to the **integer-array-nearest-to** A2.

If A3 is a **scalar** and B is a vector, return B[1+(⍴B)|¯1+A3+⍳⍴B] evaluated with **comparison-tolerance** set to **zero**.

Otherwise return Z, an **array** such that the **shape-list** of Z is the same as the **shape-list** of B, the **type** of Z is the same as the **type** of B, and the **ravel-list** of Z has the property that if Z0 is a **vector-item along-axis** K1 of Z, A0 is the corresponding **item** of A3, and B0 is the corresponding **vector-item along-axis** K1 of B, then Z0 is A0 ⌽ B0.

Examples:

```
      3⌽1 2 3 4 5
4 5 1 2 3
      ¯1⌽1 2 3 4 5
5 1 2 3 4
      ¯7⌽'ABCDEF'
FABCDE
      1⊖N33
21 22 23
31 32 33
11 12 13
      1⌽[1]N33
21 22 23
31 32 33
11 12 13
      1 2 3⌽N34
12 13 14 11
23 24 21 22
34 31 32 33
      N23⌽[2]N243
141 112 123
111 122 133
121 132 143
131 142 113

221 232 243
231 242 213
241 212 223
211 222 233
```

Note: Various error checks have been performed on K by the phrase evaluators, and ***index-origin*** *is* ***one****, before this evaluation sequence is called.*

10.2.8 Base Value

```
Z ← A ⊥ B
```

Informal Description: *Z* is, for *A* and *B* numeric vectors, a number produced by regarding *B* as the representation of a number in the mixed radix number system specified by *A*.

If *A* or *B* is an array of rank greater than **one**, base value is like inner product: each **vector-item** along the last **axis** of *A* is applied to each **vector-item** along the first axis of *B*.

Evaluation Sequence:

```
Set  A0 to 1ρ⌽1,ρA.
Set  B0 to 1ρ(ρB),1.
If   A0 differs from B0,

    If  A is a scalar or one-element-vector, return (B0ρA)⊥B.
    If  B is a scalar or one-element-vector, return A⊥A0ρB.
    Otherwise, signal length-error.

If   A0 is the same as B0,

    If any item of the ravel-list of A is not a number, signal domain-error.
    If any item of the ravel-list of B is not a number, signal domain-error.
    Return  (⌽⍉(⌽ρA)ρ⍉×\⌽A,1)+.×B.
```

Examples:

```
      10⊥1 2 3
123
      24 60 60 ⊥ 1 2 3
3723
      ⎕←A←2 3ρ10 10 10 12 60 60
10 10 10
12 60 60
      ⎕←B←3 2ρ1 4 2 5 3 6
1 4
2 5
3 6
      A⊥B
 123  456
3723 14706
      ¯.001 10 10⊥1 2 3
123
      60 ⊥ 1 2 3
3723
      ''⊥3
0
      'A'⊥⍳0
0
```

Note: The shape requirements of A and B are intentionally stricter in this definition than in several existing systems. Specifically, this standard does not require that if the last ***axis*** *of A is* ***one*** *it be replicated to match the first* ***axis*** *of B, or that if the first* ***axis*** *of B is* ***one*** *it be replicated to match the last* ***axis*** *of A.*

The first element of each left argument has no actual effect on the result of base value, but permits use of the same left argument for base value and representation. If A is a positive numeric vector and B a non-negative numeric scalar such that B<×/A, *then* A⊥A⊤B *is* B.

10.2.9 Representation

Z ← *A* ⊤ *B*

Informal Description: *Z* is the representation of *B* in mixed radix number system *A*.

Evaluation Sequence:

If *A* or *B* is **empty**, return ((ρ*A*),ρ*B*)ρ0.
If any **item** of the **ravel-list** of *A* is a **character**, signal **domain-error**.
If any **item** of the **ravel-list** of *B* is a **character**, signal **domain-error**.
If *A* is **scalar**, return *A*|*B*, evaluated with **comparison-tolerance** set to **zero**.
If *A* is a **vector** and *B* is a **scalar**,

> Generate two **numeric vectors** *Z* and *C* that satisfy the following constraints:
>
> > The **length** of *Z* is ρ*A*.
> > The **length** of *C* is 1+ρ*A*.
> > *C*[1+ρ*A*] is *B*.
> > For all **scalar** indices *I* in ιρ*A*:
> >
> > > *Z*[*I*] is *A*[*I*]⊤*C*[*I*+1].
> > > If *A*[*I*] is **zero**, *C*[*I*] is **zero**;
> > > Otherwise, *C*[*I*] is (*C*[*I*+1]-*Z*[*I*])÷*A*[*I*].
>
> Return *Z*.

Otherwise, return *Z*1, an **array** such that the **type** of *Z*1 is **numeric**, the **shape-list** of *Z*1 is (ρ*A*),ρ*B* and the **ravel-list** of *Z*1 has the following property:

> Let *I* stand for an **item** of the **index-set** of the **ravel-along-axis one** of *A*.
> Let *A*1 stand for **vector-item** *I* of the **ravel-along-axis one** of *A*.
> Let *J* stand for an **item** of the **index-set** of the **ravel-list** of *B*.
> Let *B*1 stand for **item** *J* of the **ravel-list** of *B*.
> Let *N* stand for the **count** of *B*.
> Let *P* stand for *J*+(*N*×(*I*-1)).
>
> Then, **vector-item** *P* of the **ravel-list** of *Z*1 **along-axis one** is *A*1⊤*B*1.

Examples:

```
      10 10 10⊤123
1 2 3
      10 10 10⊤123 456
1 4
2 5
3 6
      A←11 3⍴10 16 2
      '0123456789ABCDEF'[⍉ 1+A⊤1000 1024]
00000001000
000000003E8
01111101000

00000001024
00000000400
10000000000
      2 2 2 ⊤ ¯1
1 1 1
      0 2 2 ⊤ ¯1
¯1 1 1
      0 1 ⊤ 3.75 ¯3.75
3    ¯4
0.75  0.25
```

Note: *The shape of the result of* ***representation*** *is always* (⍴*A*),⍴*B*.

The ***system-parameter comparison-tolerance*** *is not an implicit argument of representation.*

10.2.10 Dyadic Transpose

```
Z ← A ⍉ B
```

Informal Description: *Z* is an array formed by rearranging and possibly coalescing the axes of *B* according to the vector *A*. Each element of *A* corresponds to an axis of *B* by position and to an axis of *Z* by value. The largest value in *A* determines the rank of *Z*. All axes of *Z* must be present in *A*. If *A* contains no repeated elements, the shape of *Z* is (⍴*B*)[⍋*A*]. Repeated elements in *A* select diagonals from *B*. The length of the corresponding axis of *Z* is the shortest of the lengths of the designated axes of *B*. Uses **index-origin** .

Evaluation Sequence:

If **index-origin** is **nil**, signal **implicit-error**.
If *A* is a **scalar**, set *A*1 to ,*A*.
Otherwise, set *A*1 to *A*.

If *A*1 is not a **vector**, signal **rank-error**.
If the **length** of *A*1 is not the same as the **rank** of *B*, signal **length-error**.
If any **item** of the **ravel-list** of *A*1 is not a **near-integer**, signal **domain-error**.
Set *A*2 to the **integer-array-nearest-to** *A*1.
Generate *A*3, a **numeric array** with the **shape-list** of *A*2 such that each **item** of the **ravel-list** of *A*3 is (**one minus index-origin**) **plus** the corresponding element of *A*2.
If ∧/*A*3∊⍳⌈/0,*A*3 evaluated with **comparison-tolerance** set to **zero** is not **one**, signal **domain-error**.
If ∧/(⍳⌈/0,*A*3)∊*A*3 evaluated with **comparison-tolerance** set to **zero** is not **one**, signal **domain-error**.

Return an **array** *Z* having the following properties:

The **rank** of *Z* is ⌈/0,*A*3.
The **type** of *Z* is the same as the **type** of *B*.
The **shape-list** of *Z* is such that for all *I* in the **index-set** of the **shape-list** of *Z*, **item** *I* of the **shape-list** of *Z* is ⌊/(*A*3=*I*)/⍴*B*, evaluated with **comparison-tolerance** set to **zero**.
The **ravel-list** of *Z* is such that, for all *J* in the **index-set** of the **ravel-list** of *Z*, **item** *J* of the **ravel-list** of *Z* is **item** 1+(⍴*B*)⊥((⍴*Z*)⊤*J*-1)[*A*3] of the **ravel-list** of *B*.

Examples:

```
      1 3 2⍉N234
111 121 131
112 122 132
113 123 133
114 124 134

211 221 231
212 222 232
213 223 233
214 224 234
      1 1⍉N34
11 22 33
```

```
      3 1 2⍉N234
111 211
112 212
113 213
114 214

121 221
122 222
123 223
124 224

131 231
132 232
133 233
134 234
      ⍴⍴(⍳0)⍉5
0
```

10.2.11 Take

```
Z ← A ↑ B
```

Informal Description: Z is an array having shape ,|A. Informally, Z is a corner of B. The choice of corner is based on the signs of the elements of A. If the absolute values of elements of A are greater than corresponding elements of ρB, Z is padded with the **typical-element** of B.

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
Set A1 to ,A.

If B is a **scalar**, set B1 to ((ρA1)ρ1)ρB.
Otherwise, set B1 to B.

If the **shape-list** of A1 does not match the **rank** of B1, signal **length-error**.
If any **item** of the **ravel-list** of A1 is not a **near-integer**, signal **domain-error**.
Set A2 to the **integer-array-nearest-to** A1.

Return an **array** Z such that the **type** of Z is the same as the **type** of B, the **shape-list** of Z is |A2, and the **ravel-list** of Z has the property that for each **scalar** I in the **index-set** of the **ravel-list** of Z, **item** I of the **ravel-list** of Z is determined as follows:

> Let J stand for ((|A2)⊤I−1)+(A2<0)×A2+ρB1 evaluated with **comparison-tolerance** set to **zero**.
> If each element of J is in the **open-interval-between negative-one** and the corresponding element of ρB1, **item** I of the **ravel-list** of Z is (,B1)[1+(ρB1)⊥J].
> Otherwise, **item** I of the **ravel-list** of Z is the **typical-element** of B.

Examples:

```
          2↑N5
1  2
         ¯2↑N5
4  5
         ¯2 6↑N44
31  32  33  34  0  0
41  42  43  44  0  0
         ¯4 ¯4↑99
0  0  0   0
0  0  0   0
0  0  0   0
0  0  0  99
```

10.2.12 Drop

```
Z ← A ↓ B
```

Informal Description: Z is a corner of B with shape 0⌈(ρB)-|A. The choice of corner is based on the signs of the elements of A.

Evaluation Sequence:

If the **rank** of A is **greater-than one**, signal **rank-error**.
Set A1 to ,A.

If B is a **scalar**, set B1 to ((ρA1)ρ1)ρB.
Otherwise, set B1 to B.

If the **shape-list** of A1 does not match the **rank** of B1, signal **length-error**.
If any **item** of the **ravel-list** of A1 is not a **near-integer**, **near-integer**, signal **domain-error**.
Set A2 to the **integer-array-nearest-to** A1.

Return (((A2<0)×0⌈A2+ρB1)+(A2≥0)×0⌊A2-ρB1) ↑ B1 evaluated with **comparison-tolerance** set to **zero**.

Examples:

```
      1↓N5
2 3 4 5
      2 ¯1↓N44
31 32 33
41 42 43
      ρ1↓5
0
      ρ0↓5
1
      ρ1 2 3↓4
0 0 0
      ''↓5
5
      ρρ''↓5
0
```

10.2.13 Matrix Divide

Z ← *A* ⌹ *B*

Informal Description: *Z* is the least squares solution to a linear system specified by *A* and *B*.

Evaluation Sequence:

If the **rank** of *A* or the **rank** of *B* is **greater-than two**, signal **rank-error**.
Set *A*1 to (2↑(⍴*A*),1 1)⍴*A*.
Set *B*1 to (2↑(⍴*B*),1 1)⍴*B*.
If 1↑⍴*A*1 is not the same as 1↑⍴*B*1, signal **length-error**.
If 1↑⍴*B*1 is **less-than** ¯1↑⍴*B*1, signal **length-error**.
If any **item** of the **ravel-list** of *A* is not a **number**, signal **domain-error**.
If any **item** of the **ravel-list** of *B* is not a **number**, signal **domain-error**.
Use the **implementation-algorithm matrix-divide** to generate an **array** *Z*, such that the **type** of *Z* is **numeric**, the **shape-list** of *Z* is (¯1↑⍴*B*1),¯1↑⍴*A*1, and the **ravel-list** of *Z* has the property that for each **scalar** *I* in ⍳¯1↑⍴*A*1, *Z*[;*I*] is such that it minimises +/(*A*1[;*I*]-*B*1+.×*Z*[;*I*])*2.
If *Z* cannot be uniquely determined within a reasonable round-off criterion by the above constraints, signal **domain-error**.
Return ((1↓⍴*B*),1↓⍴*A*)⍴*Z*.

Additional Requirements:

The round-off criterion discussed above is not specified by this standard.

Note: *The following article, Martin, G. A., The Solutions of Linear Systems in APL: Towards An Extension of Matrix Divide APL80 Noordwijkerhout June 24-26, 1980 — Published by Gijsbert van der Linden, North-Holland Publishing Company Amsterdam. New York. Oxford, describes an acceptable algorithm for matrix division.*

The following document also deals with this subject: Jenkins, M. A., Domino — An APL Primitive Function for Matrix Inversion — Its Implementation and Applications. APL Quote-Quad Vol III No. 4, February 1972, pp. 4-15.

10.2.14 Indexed Reference

```
Z ← A[I]
```

Informal Description: *Z* is an array consisting of elements of the array *A* selected and structured by the position and values of the arrays in the **index-list** *I*. Uses **index-origin** .

Evaluation Sequence:

```
If the number-of-items in the index-list I is not the same as the rank of A, signal rank-error.
In the following,

    Let  IX stand for (array) item X of I.
    Let  JX stand for (array) item X of J.
    Let  LX stand for (array) item X of L.
    Let  LY stand for (array) item X+1 of L.

Generate J, an list of arrays having the count of I, such that for every member X of the
  index-set of I,

    If IX is an elided-index-marker, set JX to ¯1+ι(ρA)[X].
    Otherwise,

        If any item of the ravel-list of IX is not a near-integer, signal domain-error.
        Set JX to the integer-array-nearest-to IX.
        Set JX to JX minus index-origin .
        If any element of JX is not an element of ¯1+ι(ρA)[X], signal index-error.

    Set JX to JX × ×/ X↓ρA.

Generate L, a list of arrays with 1+ρρA (array) items such that

    Item 1+ρρA of L is one.
    For all X in ιρρA, LX is JX ∘.+ LY.

Let K stand for first-item in the list of arrays L.
Return (,A)[K].
```

Examples:

```
      1 2 3[2]
2
      N222[2 1;;2]
212 222
112 122
```

*Note: Since an **index-list** can never have zero **items**, indexing will always signal **rank-error** when argument A is a scalar.*

*The vector indexing on which this subsection is based is defined in the subsection **evaluate-indexed-reference**.*

10.2.15 Indexed Assignment

```
Z ← V[I] ← B
```

Informal Description: *Z* is *B*. As a side effect, indexed assignment sets elements of the array *V* selected by the position and values of the arrays in the **index-list** *I* to corresponding elements of *B*. Note that, at this stage, *V* is known to be a **variable-name**, not a **value**. Uses **index-origin** .

Evaluation Sequence:

```
If index-origin is nil, signal implicit-error.
Set A to the current-content of V.

If the number-of-items in the index-list I does not match the rank of A, signal rank-error.

In the following,

    Let IX stand for (array) item X of I.
    Let JX stand for (array) item X of J.
    Let LX stand for (array) item X of L.
    Let LY stand for (array) item X+1 of L.

Generate J, an index-list having the same count as I such that for every item X of the
  index-set of I,

    If IX is an elided-index-marker, set JX to ¯1+⍳(⍴A)[X].
    Otherwise,

        If any item of the ravel-list of IX is not a near-integer, signal domain-error.
        Set JX to the integer-array-nearest-to IX.
        Set JX to JX minus index-origin.
        If any element of JX is not an element of ¯1+⍳(⍴A)[X], signal index-error.

    Set JX to JX × ×/ X↓⍴A.

Generate L, an index-list with 1+⍴⍴A (array) items such that

    Item 1+⍴⍴A of L is one.
    For all X in ⍳⍴⍴A, LX is JX ∘.+ LY.

Let K stand for first-item in the list of arrays L.

Set K1 to ((1≠⍴K)/⍴K)⍴K evaluated with comparison-tolerance set to zero.
Set B1 to ((1≠⍴B)/⍴B)⍴B evaluated with comparison-tolerance set to zero.

If B1 is a scalar, set B2 to (⍴K1)⍴B1.
Otherwise, set B2 to B1.

If the rank of K1 is not the same as the rank of B2, signal rank-error.
If any item of the shape-list of K1 is not the same as the corresponding item of the shape-list of
  B2, signal length-error.
If K1 is not empty and the type of A differs from the type of B, signal domain-error.
Set M to one.
Repeat:

    If M is not greater-than the count of K1,

        Let K2 stand for item M of the ravel-list of K1.
        Set item K2 of the ravel-list of A to item M of the ravel-list of B.
        Set M to M plus one.
```

Otherwise,
Set the **current-referent** of V to a **token** whose **class** is **variable** and whose **content** is A.
Return a **token** whose **class** is **committed-value** and whose **content** is B.

(End of repeated block)

Examples:

```
      X←1 2 3
      ⎕←X[3 2]←4 5
4 5
      X
1 5 4
      ⎕←Y←N222
 111 112
 121 122

 211 212
 221 222
      ⎕←Y[2 1;;1]←N12121
 11111
 11121


 12111
 12121
      Y
 12111   112
 12121   122

 11111   212
 11121   222
```

Additional Requirements:

The evaluation sequence requires that the value of `A[1]` after the statement `A[1 1 1]←1 2 3` has been evaluated be `3`.

Indexed assignment exhibits **atomic** behaviour. If **indexed assignment** signals an error, the value of V shall be unchanged.

*Note: Since an **index-list** can never have zero items, indexing will always signal **rank-error** when argument A is a scalar.*

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11 SYSTEM FUNCTIONS

11.1 INTRODUCTION

Note: *System functions are primitive functions whose names are* ***distinguished-identifiers*** *rather than* ***primitives****. System function names are not permitted in the* ***locals-list*** *of a* ***defined-function header-line****.*

The system functions related to ***shared-variables*** *and* ***defined-functions*** *are specified in those chapters.*

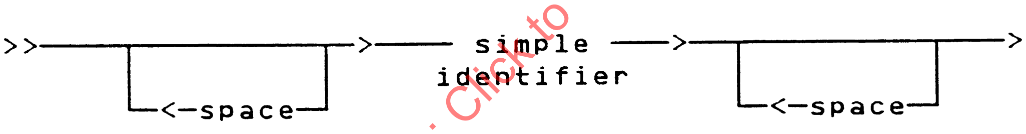
11.2 DEFINITION

Note: *Some system functions take an argument which is treated as an* ***identifier-array****.*

Identifier-Array: A **character array** of **rank two** each of whose **rows** matches the **character-diagram identifier-row**.

11.3 DIAGRAM

Identifier-Row



11.4 NILADIC SYSTEM FUNCTIONS

*Note: A **niladic system function** is an implementation-provided facility associated with a **distinguished-identifier**. The primary difference between a **niladic system function** and a **system variable** is that a **niladic system function** causes an error to be signalled if its name is included in the list of locals of a defined function header.*

11.4.1 Time Stamp

```
Z ← ⎕TS
```

Informal Description: Z is a seven-element numeric vector representing the current date and time relative to an epoch and a time zone. The choice of epoch and time zone are not specified by this standard.

Evaluation Sequence:

Return the seven-element **numeric vector** result of the **implementation-algorithm time-stamp**.

Example:

```
      ⎕TS
1789 7 14 11 14 45 586.4
```

Additional Requirements:

The first through sixth elements of the result returned by **time-stamp** are integral and represent respectively the current year, month, day, hour, minute, and second relative to the underlying epoch and time zone. The hour corresponds to the number of integral hours that have elapsed within the current day.

The seventh and last element is a quantity, not necessarily integral, representing the milliseconds that have elapsed within the current second.

The accuracy, resolution, epoch, and time zone for **time-stamp** are not specified by this standard.

11.4.2 Atomic Vector

Z ← ⎕*AV*

Informal Description: *Z* is an **implementation-defined character** vector containing every element of the **character-set** exactly once.

Evaluation Sequence:

Return **atomic-vector**.

Note: *The elements of the* ***atomic-vector*** *are often ordered so that, in* ***index-origin zero,*** ((⌈2⍟⍴⎕*AV*)⍴2)⊤⎕*AV*⍳'*X*' *gives the bit representation of* '*X*'*. This behaviour is not required.*

11.4.3 Line Counter

Z ← ⎕*LC*

Informal Description: *Z* is a vector of statement numbers of active functions ordered so that the most recently called function has the lowest index.

Evaluation Sequence:

Set *Y* to a **numeric vector** whose **length** is the same as the **number-of-items** in the **state-indicator**, such that for every *I* in ⍳⍴*Y*, *Y*[*I*] is the **current-line-number** of **item** *I* of the **state-indicator**, if the **mode** of **item** *I* of the **state-indicator** is **defined-function**, and **zero** otherwise.
Set *M* to a **Boolean vector** whose **length** is the same as the **number-of-items** in the **state-indicator**, such that for every *I* in ⍳⍴*M*, *M*[*I*] is **one** if the **mode** of **item** *I* of the **state-indicator** is **defined-function**, and **zero** otherwise.
Return *M*/*Y*.

Note: *The operations that affect the value of* ⎕*LC* *are discussed in the subsection* ***defined-function-control.***

11.5 MONADIC SYSTEM FUNCTIONS

Note: Refer to the ***Defined Functions*** *chapter for definitions of* ⎕*FX and* ⎕*CR.*

11.5.1 Delay

```
Z ← ⎕DL B
```

Informal Description: *Z* is the time, in seconds, for completion of this operation. As a side effect, ⎕*DL* causes a delay in execution of at least *B* seconds.

Evaluation Sequence:

```
Set  T0 to current-time seconds.
If  B is not a scalar, signal rank-error.

If  B is not a number, signal domain-error.

Set  T1 to T0 plus B.
Wait until current-time is not less-than T1.
Return  Z, a numeric scalar such that the first-item in the ravel-list of Z is current-time minus
  T0.
```

Note: B may be fractional or negative.

11.5.2 Name Class

```
Z ← ⎕NC B
```

Informal Description: Z is a vector of name classes giving the usage of the identifier in the corresponding row of B. 0 means an identifier is available without current referent; 1 a label; 2, a variable or a shared variable; 3, a defined function.

Evaluation Sequence:

If the **rank** of B is **greater-than two**, signal **rank-error**.
If B is **empty**, return ⍳0.
Set B1 to (¯2↑1 1,ρB)ρB.
If any row of B1 does not match **identifier-row**, signal **domain-error**.
Generate Z, a **numeric vector** such that the **length** of Z is the same as the **number-of-rows** in B1, and the **ravel-list** of Z has the property that for every I in ⍳1↑ρB, the following holds:

> Let ZI stand for **item** I of the **ravel-list** of Z.
> Let N stand for the **token** that matched the **character-diagram simple-identifier** in the **identifier-row**.
> If the **current-class** of N is **nil**, ZI is **zero**.
> If the **current-class** of N is **label**, ZI is **one**.
> If the **current-class** of N is **variable** or **shared-variable**, ZI is **two**.
> If the **current-class** of N is **defined-function** or **niladic-defined-function**, ZI is **three**.
> Otherwise, signal **domain-error**.

Return Z.

Example:

```
     ∇ Z←TEST;R0;R21;R22;R31;R32
 [1]   R1:R21←1 ⎕SVO 'R22'
 [2]   Z←⎕FX 1 3ρ'R31'
 [3]   Z←⎕FX 1 5ρ'R32 X'
 [4]   Z←⎕NC 6 3ρ'R0 R1 R21R22R31R32'
     ∇
         TEST
0 1 2 2 3 3
```

11.5.3 Expunge

Z ← ⎕*EX* *B*

Informal Description: *Z* is a Boolean vector such that an element of *Z* is **one** if the identifier in the corresponding row of *B* is available for use when the operation completes. Expunge changes the **current-referent** of **symbols** whose **current-class** is **variable**, **shared-variable**, **defined-function**, or **niladic-defined-function** to **nil**, making them available for redefinition. The **symbols** to be changed are named by the rows of *B*, an **identifier-array**.

Evaluation Sequence:

If the **rank** of *B* is **greater-than two**, signal **rank-error**.
Set *B*1 to (¯2↑1 1,⍴*B*)⍴*B* .

For each **row** *J* of *B*1,

> If **row** *J* of *B*1 does not match **identifier-row**, signal **domain-error**.
> Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
> If the **current-class** of *N* is **shared-variable**, **retract** *N*.
> If the **current-class** of *N* is **variable**, set the **current-referent** of *N* to a **token** whose **class** is **nil**.
> If the **current-class** of *N* is **defined-function** or **niladic-defined-function**, and if the **current-referent** of *N* is **locally-erasable**, set the **current-referent** of *N* to a **token** whose **class** is **nil**.

Return ~×⎕*NC* *B*.

Additional Requirement:

The operation of Expunge is **atomic**: if Expunge signals an error, **current-referents** shall be unchanged.

Note: *Because of the definition of* ***locally-erasable***, *a* ***defined-function*** *cannot be expunged by a* ***conforming-program*** *while it is* ***pendent*** *or* ***waiting***. *This does not prevent a* ***conforming-implementation*** *from permitting such behaviour; it simply leaves the consequences to a* ***conforming-program*** *undefined until some future standard.*

11.5.4 Name List

Z ← ⎕*NL* *B*

Informal Description: *Z* is an **identifier-array**. An identifier occurs in *Z* if its **name-class** is in *B*. Elements of *B* can be: 1, for labels; 2, for variables or shared variables; 3, for user defined functions.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of *B* is not a **near-integer**, signal **domain-error**.
Set *B*1 to the **integer-array-nearest-to** *B*.
If *B*1 contains any **numbers** other than **one**, **two**, or **three**, signal **domain-error**.
Generate *Z*, an **identifier-array** consisting of the **names** of **symbols** whose **current-class** is **label**, **variable**, **shared-variable**, **niladic-defined-function**, or **defined-function**. The **symbol names** are left-justified in the rows of *Z*, and *Z* has no all-blank trailing columns.
Return ((⎕*NC* *Z*)∊*B*1)⌿*Z*.

Note: *The* ***rank*** *of Z is always* ***two***. *If there are no* ***symbols*** *with a* ***name-class*** *designated in B, Z is* 0 0⍴''.

11.5.5 Query Stop

```
Z ← ⎕STOP B
```

Informal Description: *Z* is an integer vector of line numbers in the function whose name is *B* that have stop control set.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If *B* does not match **identifier-row**, signal **domain-error**.
Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
If the **current-class** of *N* is not **defined-function** or **niladic-defined-function**, signal **domain-error**.
Return the **stop-vector** of the **current-content** of *N*.

Additional Requirement:

Query Stop is part of the **optional-facility Trace-and-Stop-Control**.

Note: *Query Stop is not origin sensitive.*

If no lines have stop control set, the result is ι0.

11.5.6 Query Trace

```
Z ← ⎕TRACE B
```

Informal Description: *Z* is an integer vector of line numbers in the function whose name is *B* that are being traced.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If *B* does not match **identifier-row**, signal **domain-error**.
Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
If the **current-class** of *N* is not **defined-function** or **niladic-defined-function**, signal **domain-error**.
Return the **trace-vector** of the **current-content** of *N*.

Additional Requirement:

Query Trace is part of the **optional-facility Trace-and-Stop-Control**.

Note: *Query Trace is not origin sensitive.*

If no lines are being traced, the result is ι0.

11.6 DYADIC SYSTEM FUNCTIONS

11.6.1 Name List

Z ← *A* ⎕*NL* *B*

Informal Description: *Z* is an **identifier-array** of all names having a **name-class** in *B* that begin with a letter in *A*.

Evaluation Sequence:

If the **rank** of *A* is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of *A* does not match **letter**, signal **domain-error**.
Set *Z*1 to ⎕*NL* *B*.
If 0∊ρ*Z*1, return *Z*1.
Return (*Z*1[;1]∊*A*)⌿*Z*1.

11.6.2 Set Stop

Z ← *A* ⎕*STOP* *B*

Informal Description: *Z* is a numeric vector representing the stop controls in effect for the function whose name is in *B* before ⎕*STOP* began evaluation. As a side effect, ⎕*STOP* sets stop controls for the function whose name is in *B* at lines specified by *A*.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If *B* does not match **identifier-row**, signal **domain-error**.
Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
If the **current-class** of *N* is not **defined-function** or **niladic-defined-function**, signal **domain-error**.

If the **rank** of *A* is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of *A* is not a **near-integer**, signal **domain-error**.
Set *A*1 to the **integer-array-nearest-to** *A*.
If any **item** of the **ravel-list** of *A*1 is not a **positive-integer**, signal **domain-error**.
Set *Z* to ⎕*STOP* *B*.
Let *M* stand for the **last-line-number** of the **current-content** of *N*.
Set the **stop-vector** of the **current-content** of *N* to ((⍳*M*)∊*A*1)/⍳*M*, evaluated with **index-origin one**.
Return *Z*.

Additional Requirement:

Set Stop is part of the **optional-facility Trace-and-Stop-Control**.

*Note: Stop controls are an attribute of **defined-function** objects.* ⎕*STOP* *affects the **current-referent** of B only.*

11.6.3 Set Trace

```
Z ← A ⎕TRACE B
```

Informal Description: *Z* is a numeric vector representing the lines being traced in the function whose name is in *B* before ⎕*TRACE* began evaluation. As a side effect, ⎕*TRACE* begins tracing the lines specified by *A* in the function whose name is in *B*.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If *B* does not match **identifier-row**, signal **domain-error**.
Let *N* stand for the **token** that matched the **character-diagram simple-identifier** in **identifier-row**.
If the **current-class** of *N* is not **defined-function** or **niladic-defined-function**, signal **domain-error**.

If the **rank** of *A* is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of *A* is not a **near-integer**, signal **domain-error**.
Set *A*1 to the **integer-array-nearest-to** *A*.
If any **item** of the **ravel-list** of *A*1 is not a **positive-integer**, signal **domain-error**.
Set *Z* to ⎕*TRACE B*.
Let *M* stand for the **last-line-number** of the **current-content** of *N*.
Set the **trace-vector** of the **current-content** of *N* to ((⍳*M*)∊*A*1)/⍳*M*, evaluated with **index-origin one**.
Return *Z*.

Additional Requirement:

Set Trace is part of the **optional-facility Trace-and-Stop-Control.**

Note: Trace controls are an attribute of ***defined-function*** *objects.* ⎕*TRACE affects the* ***current-referent*** *of B only.*

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12 SYSTEM VARIABLES

12.1 DEFINITIONS

System-Variable-Symbol: A **symbol**, having as its name a **distinguished-identifier**, and as its **referent-list** a **list** of **tokens** representing the value assigned to an associated **system-parameter** in each **context** of the **state-indicator**.

Note: *System parameters are values used implicitly and set as side effects by primitive operations.*

Internal-Value-Set: An **implementation-defined** set of values that the **tokens** in the **referent-list** of a **system-variable-symbol** can take on.

Note: *When a **system-variable-symbol** is **localised** the initial **class** of its associated **system-parameter** is **nil**. If a primitive operation is invoked that requires a system parameter whose class is currently **nil**, the primitive operation signals **implicit-error**. Therefore, a **conforming-program** that **localises** **system-variable-symbols** should assign them values from their **internal-value-set** before calling primitive operations that require them.*

*The assignment operation for a system variable rejects attempts to set its associated system parameter to a value outside its **internal-value-set**.*

12.2 EVALUATION SEQUENCES

12.2.1 Comparison Tolerance

```
Z ← ⎕CT ← B
Z ← ⎕CT
```

Informal Description: *Z* is the current value of **comparison-tolerance** .

The distance within which numbers are to be considered equal by certain primitives is a function of **comparison-tolerance** .

Evaluation Sequence:

```
For form   ⎕CT

    Return  comparison-tolerance .

For form   ⎕CT ← B

    If the  rank of B is greater-than one, signal rank-error.
    If the  count of B is not one, signal length-error.
    Let  B1 be the first-scalar in B.
    If  B1 is not a nonnegative-number, signal domain-error.
    If  B1 is not in the internal-value-set of comparison-tolerance , signal limit-error.
    Set  comparison-tolerance to B1.
    Return a  token whose class is committed-value and whose content is B.
```

Additional Requirements:

The **internal-value-set** for **comparison-tolerance** consists of **nonnegative-numbers** not greater than the **implementation-parameter comparison-tolerance-limit.**

The initial value of **comparison-tolerance** in a **clear-workspace** is that member of the **internal-value-set** for **comparison-tolerance** given by the **implementation-parameter initial-comparison-tolerance.**

Note: The following subsections reference ***comparison-tolerance*** *:* ***ceiling, equal, floor, greater than or equal to, greater than, index of, less than or equal to, less than, member of, not equal, and residue.***

12.2.2 Random Link

```
Z ← ⎕RL ← B
Z ← ⎕RL
```

Informal Description: *Z* is the current value of **random-link** ; **random-link** is the current seed of the pseudorandom number generator.

Evaluation Sequence:

```
For form ⎕RL

    Return random-link .

For form  ⎕RL ← B

    If the rank of B is greater-than one, signal rank-error.
    If the count of B is not one, signal length-error.
    Set B1 to the first-scalar in B.
    If B1 is not a near-integer, signal domain-error.
    Set B2 to the integer-nearest-to B1.
    If B2 is less-than one, signal domain-error.
    If B2 is not in the internal-value-set for random-link , signal limit-error.
    Set random-link to B2.
    Return a token whose class is committed-value and whose content is B.
```

Additional Requirements:

The **internal-value-set** of **random-link** is **implementation-defined**.

The initial value of **random-link** in a **clear-workspace** is that member of the **internal-value-set** for **random-link** given by the **implementation-parameter initial-random-link**.

Note: *The system parameter* ***random-link*** *is used and set by* ***roll*** *and* ***deal***. ***Roll*** *gives a reference for a suitable algorithm.*

12.2.3 Print Precision

```
Z ← ⎕PP ← B
Z ← ⎕PP
```

Informal Description: *Z* is the current value of **print-precision** , which controls the number of significant positions in the output form produced by **monadic format** and **numeric-output-conversion.**

Evaluation Sequence:

```
For form  ⎕PP

    Return  print-precision .

For form  ⎕PP ← B

    If the rank of B is greater-than one, signal rank-error.
    If the count of B is not one, signal length-error.
    If B is not near-integer, signal domain-error.
    Set B1 to the integer-nearest-to the first-scalar in B.
    If B1 is less-than one, signal domain-error.
    If B1 is not in the internal-value-set of print-precision , signal limit-error.
    Set print-precision to B1.
    Return a token whose class is committed-value and whose content is B.
```

Additional Requirements:

The **internal-value-set** of **print-precision** consists of the **integer** scalars in the **closed-interval-between one** and **print-precision-limit.**

The initial value of **print-precision** in a **clear-workspace** is that member of the **internal-value-set** for **print-precision** given by the **implementation-parameter initial-print-precision.**

Note: *The system parameter print-precision is used by monadic format and numeric-output-conversion. Print-precision-limit should be greater than or equal to full-print-precision.*

12.2.4 Index Origin

```
Z ← ⎕IO ← B
Z ← ⎕IO
```

Informal Description: *Z* is the current value of **index-origin** , which is the index associated with the first position of any axis of non-zero length.

Evaluation Sequence:

```
For form   ⎕IO

    Return index-origin .

For form   ⎕IO ← B

    If the rank of B is greater-than one, signal rank-error.
    If the count of B is not one, signal length-error.
    If B is not a near-integer, signal domain-error.
    Set B1 to the integer-nearest-to the first-scalar in B.
    If B1 is not in the internal-value-set of index-origin , signal limit-error.
    Set index-origin to B1.
    Return a token whose class is committed-value and whose content is B.
```

Additional Requirements:

The **internal-value-set** for **index-origin** is the **scalar integer** values **zero** and **one**.

The initial value of **index-origin** in a **clear-workspace** is that member of the **internal-value-set** for **index-origin** given by the **implementation-parameter initial-index-origin**.

Note: *The following primitive operations refer to* ***index-origin*** *:* ***deal, dyadic transpose, grade down, grade up, index generator, index of, indexed assignment, indexed reference, roll,*** *and all functions that provide for axis specification when the form containing an axis is used.*

12.2.5 Latent Expression

```
Z ← ⎕LX ← B
Z ← ⎕LX
Z ← ⎕LX[I] ← B
```

Informal Description: *Z* is the current value of **latent-expression** , which is a **character** vector that is executed when a workspace is activated.

Evaluation Sequence:

For form ⎕*LX*

Return **latent-expression** .

For form ⎕*LX* ← *B*

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If any **item** of the **ravel-list** of *B* is not a **character**, signal **domain-error**.
If *B* is not in the **internal-value-set** of **latent-expression** , signal **limit-error**.
Set **latent-expression** to **,***B*.
Return a **token** whose **class** is **committed-value** and whose **content** is *B*.

For form ⎕*LX*[*I*] ← *B*

Set *LX* to **latent-expression** .
Evaluate *LX*[*I*] ← *B*.
Set **latent-expression** to *LX*.
Return a **token** whose **class** is **committed-value** and whose **content** is *B*.

Additional Requirements:

The **internal-value-set** for **latent-expression** is an **implementation-defined** subset of the set of all character vectors which includes the empty vector.

The initial value of **latent-expression** in a **clear-workspace** is that member of the **internal-value-set** for **latent-expression** given by the **implementation-parameter initial-latent-expression**.

13 DEFINED FUNCTIONS

13.1 INTRODUCTION

Note: *Algorithms written in APL are called defined functions. A defined function consists of a header line and zero or more body lines. The header line indicates the name and syntax class of the function and gives a list of names to be localised and, in the case of argument names, initialised. Each body line consists of an optional label followed by an APL line to be evaluated. Body lines are evaluated in the order in which they occur in the defined function unless a statement beginning with a right arrow is evaluated.*

A defined function is established in a workspace by the system function ⎕FX, by actions in function definition mode, or by the system command)COPY .

In this standard, a ***defined-function*** *is represented as an object with three attributes:* ***canonical-representation, trace-vector,*** *and* ***stop-vector.*** *A* ***defined-function*** *is called when its* ***name*** *occurs in a* ***prefix*** *of the* ***current-stack*** *matching one of the* ***patterns*** *in the* ***phrase-table.***

A ***defined-function*** *is called by the operation* ***call-defined-function.*** *This operation prefixes the* ***state-indicator*** *with a new* ***context, localises*** *and initialises any local names in the* ***header-line,*** *and calls the evaluation sequence in the subsection* ***defined-function-control.***

A ***defined-function*** *ends the evaluation if* ***defined-function-control*** *finds* ***current-line-number*** *set to a value not in the* ***closed-interval-between one*** *and* ***last-line-number.*** *At this point, the first* ***item*** *of the* ***state-indicator*** *is discarded and a* ***token*** *of class* ***result*** *is returned, via* ***call-defined-function,*** *to the* ***phrase-evaluator*** *that called it.*

The returned ***token*** *may be* ***constant*** *(for functions returning a value),* ***nil*** *(for functions that return no value),* ***unwind*** *(for functions that end through the evaluation of an escape arrow), or* ***clear-state-indicator*** *(as the result of an)SIC command).*

If a line in a ***defined-function*** *signals an error,* ***immediate-execution*** *is called to report the error and suspend the* ***defined-function.*** ***Immediate-execution*** *may return either a* ***branch*** *to indicate that evaluation of the* ***defined-function*** *should continue, or an* ***escape*** *or* ***clear-state-indicator*** *to indicate that the* ***defined-function context*** *should be removed from the* ***state-indicator.***

13.2 DEFINITIONS

Note: *The following term defines the subset of* ***arrays*** *that can be used to create* ***defined-functions****.*

Proper-Defined-Function: An **array** *A* is a **proper-defined-function** if

The **type** of *A* is **character**.
The **rank** of *A* is **two**.
A has at least one **row**.
The first **row** of *A* matches the **character-diagram header-line**.
No **identifier** in the first row of *A* is the **distinguished-identifier** ⎕ or ⍞ or is a **system-function-name** or **niladic-system-function-name**.
All **rows** after the first in *A* match the **character-diagram body-line**.

Note: *The following definitions give properties of arrays that are* ***proper-defined-functions****.*

Niladic: *A*, an **array** that is a **proper-defined-function**, is **niladic** if the **form** of the first **row** of *A* is **niladic-function-header**.

Monadic: *A*, an **array** that is a **proper-defined-function**, is **monadic** if the **form** of the first **row** of *A* is **monadic-function-header**.

Dyadic: *A*, an **array** that is a **proper-defined-function**, is **dyadic** if the **form** of the first **row** of *A* is **dyadic-function-header**.

Value-Returning: *A*, an **array** that is a **proper-defined-function**, is **value-returning** if, when **header-line** is threaded with the first **row** of *A*, the diagram **result** is threaded.

Note: *The following terms categorise* ***defined-functions*** *according to their presence on or absence from the* ***state-indicator****.*

Suspended: *N*, a **simple-identifier**, is **suspended** if it is the **function-name** of a **context** that immediately follows an **immediate-execution context**.

Pendent: *N*, a **simple-identifier**, is **pendent** if it is the **function-name** of a **context** that does not immediately follow an **immediate-execution context**.

Waiting: *N*, a **simple-identifier**, is **waiting** if a **token** whose **class** is **defined-function-name** and whose **content** is the **content** of *N* occurs in the **stack** of some **context** in the **state-indicator** of the **active-workspace**, and is not the first **defined-function-name** in that **stack**.

Note: *For example, if F and G are defined function names, G is waiting during the evaluation of* (*F* 2) *in the line* (*F* 2) *G* 3.

Editable: *N*, a **simple-identifier**, is **editable** if the **current-referent** of *N* is **nil**, or if all the following are true:

The **current-class** of *N* is **defined-function** or **niladic-defined-function**.
The **number-of-items** in its **referent-list** is **one**.
N is neither **pendent** nor **waiting**.
N, if **suspended**, is **suspended** in only the **current-context**.

Note: *This definition requires that a* ***conforming-implementation*** *be capable of editing the top function on the state indicator as long as it has not been* ***localised*** *and occurs nowhere else on the state indicator. This minimal definition is included to be certain that all* ***conforming-implementations*** *be capable of interactive program correction.*

Globally-Erasable: **Globally-Erasable** is defined for *N*, a **simple-identifier**, as follows:

If no **context** of the **state-indicator** contains, in its **current-statement**, a **defined-name** with **content** that of *N*, then *N* is **globally-erasable**.
Otherwise, let *L* be the last (most global) **context** in whose **local-name-list** *N* appears.
If there is no such **context** then *N* is not **globally-erasable**.
Otherwise, let *C* be the last (most global) **context** in whose **current-statement** a **defined-name** with **content** that of *N* appears.

If the index of L is greater-than or equal to that of C then N is **globally-erasable**.

Note: *Informally,* N *is* ***globally-erasable*** *if it was localised before it was made waiting or pendent.*

Locally-Erasable: **Locally-Erasable** is defined for N, a **simple-identifier**, as follows:

If no **context** of the **state-indicator** contains, in its **current-statement**, a **defined-name** with **content** that of N, then N is **locally-erasable**.
Otherwise, let L be the first (most local) **context** in whose **local-name-list** N appears.
If there is no such **context** then N is not **locally-erasable**.
Otherwise, let C be the first (most local) **context** in whose **current-statement** a **defined-name** with **content** that of N appears.
If the index of C is greater than that of L, then N is **locally-erasable**.

Note: *Informally,* N *is* ***locally-erasable*** *if it has been localised since the last time it was made waiting or pendent. This definition is used in* ⎕EX *to specify that the expression* (⎕EX'F') F 1 *be the same as* 0 F 1*; it is used in* ⎕FX *to specify that the expression* (⎕FX 1 1⍴'F') F 1 *signal a* ***domain-error***.

Note: *The following operations provide information about arrays that are* ***proper-defined-functions***.

Function-Name of A: For A a **proper-defined-function**, the **simple-identifier** that matches **function-name** when the **character-diagram header-line** is threaded with the first **row** of A.

Last-Line-Number of A: For A a **proper-defined-function**, **negative-one plus** the **number-of-rows** in A.

Header-Name-List of A: For A a **proper-defined-function**, a **list** of **identifiers** developed as follows:

Thread the **character-diagram header-line** with the first **row** of A.
Return a **list** of **tokens** consisting of **identifiers** that matched any of the **character-diagrams local-name**, **result-name**, **left-argument-name**, or **right-argument-name**.

Label-Name-List of A: For A a **proper-defined-function**, a **list** of **simple-identifiers** developed as follows:

Set Z to the **empty-list** of **tokens**.
For each **index** I in the **closed-interval-between one** and **last-line-number** of A,

> Thread the **character-diagram body-line** with **row** I **plus one** of A.
> If **labelled-line** was matched, append the **simple-identifier token** that matched the **character-diagram label-name** to Z as a new last **item**.

Return Z.

Local-Name-List of A: For A a **proper-defined-function**, a **list** of **identifiers** consisting of the **label-name-list** followed by the **header-name-list**.

Identifier-Matching D **in** A: For A a **proper-defined-function** and D a **character-diagram**, a **simple-identifier token** developed as follows:

Thread the **character-diagram header-line** with the first **row** of A.
Return a **token** of **class simple-identifier** whose **content** is the **list** of **characters** that matched D.

Note: ***Identifier-Matching*** *is used to refer to the arguments and result of an arbitrary defined function.*

Function-Line I of A: For A a **proper-defined-function**, a **list** of **characters** developed as follows:

Thread the **character-diagram body-line** with **row** I **plus one** of A.
Return the **list** of **characters** that matched the **character-diagram line** in **body-line**.

Note: *The following operations affect* ***symbols***.

Localise *N*:

Append a new first **item**, a **token** of **class** **nil**, to the **referent-list** of the **symbol-named-by** *N*.

Delocalise *N*:

If the **current-class** of *N* is **shared-variable**, **retract** *N*.
Remove the first **item** from the **referent-list** of the **symbol-named-by** *N*.

Current-Canonical-Representation: The **canonical-representation** of the **current-function** of the **current-context** in the **active-workspace**.

Current-Function-Line *I*: **Function-Line** *I* of the **current-canonical-representation**.

Current-Last-Line-Number: The **last-line-number** of the **current-canonical-representation**.

Current-Stop-Vector: The **stop-vector** of the **current-function** of the **current-context** in the **active-workspace**.

Current-Trace-Vector: The **trace-vector** of the **current-function** of the **current-context** in the **active-workspace**.

Current-Local-Names: The **local-name-list** of the **current-canonical-representation**.

Current-Right-Argument-Name: The **identifier-matching** **right-argument-name** in the **current-canonical-representation**.

Current-Left-Argument-Name: The **identifier-matching** **left-argument-name** in the **current-canonical-representation**.

Current-Result-Name: The **identifier-matching** **result-name** in the **current-canonical-representation**.

13.3 DIAGRAMS

In these diagrams,

b stands for **permitted-blanks**.
r stands for **required-blanks**.

Header-Line

```
>>─ b ─┬── result ──┬─ b ─ form ─ b ─┬── locals─list ─┬─>>
       └─────>──────┘                └───────>────────┘
```

Example:

```
        Z ← A F B;C;⎕IO
```

Note: The header line indicates the name and syntactic class of the ***defined-function****. It also gives a list of identifiers to be* ***localised*** *when the* ***defined-function*** *is called.*

This standard does not provide for end-of-line comments in header lines.

Result

```
>>──────── result─name ── b ── assignment─arrow ──>>
```

Result-Name

```
>>──── simple─identifier ──>>
```

Form

```
>>──────────┬>─ niladic─function─header  ─>─┐
            ├─>─ monadic─function─header ─>─┤
            └─>─ dyadic─function─header  ─>─┴─────>>
```

Niladic-Function-Header

```
>>─────── function ────>>
          ─name
```

Monadic-Function-Header

```
>>─────── function ───── r ── right ───────>>
          ─name                ─argument
                               ─name
```

Dyadic-Function-Header

```
>>———— left ———— r — function ———— r — right ————>>
       -argument     -name              -argument
       -name                            -name
```

Right-Argument-Name

```
>>———— simple-identifier ——>>
```

Left-Argument-Name

```
>>———— simple-identifier ——>>
```

Locals-List

```
    ┌<———————————————————————————————<┐
>>——┴—>— local —— b — local —— b ————┴——>>
         -marker      -name
```

Colon

```
>>—————————————┐
               :
               └——>>
```

Local-Marker

```
>>—————————————┐
               ;
               └——>>
```

Function-Name

```
>>—— simple-identifier ——>>
```

Local-Name

```
>>—— identifier ————————>>
```

Label-Name

```
>>—— simple—identifier ——>>
```

Body-Line

```
>>— b ——┬—>— labelled—line —>—┬——————>>
        │                      │
        └—>————— line ————————>┘
```

Labelled-Line

```
>>————— label—name — b — colon — b — line ——>>
```

Permitted-Blanks

```
>>————┬—————————>—————————┬————>>
      │                   │
      └—<— blank —<———————┘
```

Required-Blanks

```
>>————┬—>— blank —>———┬————>>
      │               │
      └———————<———————┘
```

13.4 OPERATIONS

13.4.1 Call-Defined-Function

```
Z ← DFN
Z ← DFN B
Z ← A DFN B
```

Informal Description: Z is the value of the variable given as the result name in the header line of the defined function whose name is DFN. If there is no result name, Z is **nil**.

Evaluation Sequence:

Generate a new **context** in which

> **mode** is **defined-function**,
> **current-line** is the empty **list** of **characters**,
> **current-function** is the **current-referent** of DFN,
> **current-line-number** is **one**,
> **current-statement** is the empty **list** of **tokens**, and
> **stack** is the empty **list** of **tokens**.

Append the new **context** to the **state-indicator** of the **active-workspace** as a new first **item**.
Localise the **current-local-names**.

For form $Z ← A\ DFN\ B$

> If **current-function** is not **dyadic**, signal **syntax-error**.
> Set the **symbol-named-by** **current-left-argument-name** to A.
> Set the **symbol-named-by** **current-right-argument-name** to B.

For form $Z ← DFN\ B$

> If **current-function** is not **monadic**, signal **syntax-error**.
> Set the **symbol-named-by** **current-right-argument-name** to B.

For all **indices** I in the **closed-interval-between** **one** and **current-last-line-number**,

> If **row** I **plus** **one** of **canonical-representation** matches the diagram **labelled-line**, set the **current-referent** of the **simple-identifier** that matches **label-name** to a **token** of **class** **label** and **content** the **numeric-scalar** with value I.

Set Z to **defined-function-control**.
Delocalise the **current-local-names**.
Remove the **first-item** in the **state-indicator** of the **active-workspace**.
If Z is **escape**, signal **unwind**.
Otherwise, return Z.

*Note: **Tokens** of **class** **escape** are converted to **tokens** of **class** **unwind** so that **immediate-execution** can distinguish an escape issued in **immediate-execution** mode from one issued in a **defined-function**.*

*The **context** attribute **current-function** includes the function itself plus its trace and stop vectors.*

13.4.2 Defined-Function-Control

Form: Defined-Function-Control.

Informal Description: This procedure controls the execution of a user defined function.

Evaluation Sequence:

```
Set   Z to a token whose class is nil.
Let   I stand for current-line-number.
Set   I to one.

Repeat:

    If   I is in the current-stop-vector, set attention-flag to one.
    If   Z is an error or attention-flag is one,

        Set   N to immediate-execution with Z.
        If    N is escape or clear-state-indicator, return N.
        If    N is branch, set I to the content of N.

    If   I is not in the closed-interval-between one and current-last-line-number,

        If the  current-function is not value-returning, return nil.
        If the  current-class of the current-result-name is variable or nil, return the current-referent
          of the current-result-name.
        Otherwise, signal  value-error.

    Set   Z to evaluate-line of current-function-line I.
    If    Z is escape, unwind, or clear-state-indicator, return Z.
    If    Z is branch, set I to the content  of Z.
    If    Z is value or nil, set I to I plus one.

(End of repeated block)
```

*Note: The **attention-flag** is reset by the user facility **enter**.*

*The **class** of the **token** returned by **evaluate-line** for →ι0 is nil, not **branch**.*

*The **value-error** signalled from **defined-function-control** is signalled in the line that invoked the defined function; the error is introduced to prevent **conforming-programs** from exiting a **defined-function** with the **current-class** of the **result-name** set to **defined-function**, **niladic-defined-function**, or **shared-variable**.*

13.4.3 Function Fix

$Z \leftarrow \square FX\ B$

Informal Description: Z is, for B the **canonical-representation** of a defined function having **function-name** N, a **character** vector holding N. As a side effect, the defined function that B represents is established as the current referent of N. Rows of B that are all blanks give rise to blank lines in the established function.

Evaluation Sequence:

If the **rank** of B is not **two**, signal **rank-error**.
If B is an **empty array**, signal **length-error**.
If any **item** of the **ravel-list** of B is not a **character**, signal **domain-error**.
If B is not a **proper-defined-function**, signal **domain-error**.
If **identifiers** are duplicated in the **local-name-list** of B, signal **domain-error**.
Let N stand for the **simple-identifier** that matches **function-name** in the first **row** of B.
If the **current-class** of N is not **defined-function**, **niladic-defined-function**, or **nil**, signal **domain-error**.
If N is not **locally-erasable**, signal **domain-error**.
Set the **current-referent** of N to a **defined-function** for which

Canonical-Representation is B.
Stop-Vector is $\iota 0$.
Trace-Vector is $\iota 0$.

Return a **character vector** Z such that the **length** of Z is the **number-of-items** in the **content** of N and the **ravel-list** of Z is the **content** of N.

13.4.4 Character Representation

Z ← ⎕*CR* *B*

Informal Description: *Z* is a **character** matrix representation of the defined function named by *B*.

Evaluation Sequence:

If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If the **length** of *B* is **zero**, signal **length-error**.
If *B* does not match the **character-diagram identifier-row**, signal **domain-error**.
If the **current-class** of *B* is not **defined-function** or **niladic-defined-function**, signal **domain-error**.
Otherwise return a **token** whose **class** is **constant** and whose **content** is the **canonical-representation** of the **current-content** of *B*.

Additional Requirements:

The formal model of APL used in this standard assumes that the internal representation of a defined function is precisely the **character** matrix satisfying the requirements for a **proper-defined-function** that was supplied as an argument to **function fix**. Representation of defined functions in this way in an implementation is neither required nor suggested.

The preservation of numeric literals and blanks as entered is desirable in the character representation, but it is not required by this standard. ⎕*CR* ⎕*FX* ⎕*CR* ⎕*FX* *X* shall be the same as ⎕*CR* ⎕*FX* *X*, if *X* is a **proper-defined-function**.

If **numeric-literals** are stored as **numbers** in **defined-functions**, they are to be formatted with **print-precision** set to **full-print-precision**.

13.5 FUNCTION EDITING

Note: The function editing facilities described here are included to provide the APL programmer with a standard method for entering and correcting defined functions.

13.5.1 Evaluate-Function-Definition-Request

Evaluate-Function-Definition-Request Q.

*Note: This subsection provides minimal facilities for the establishment and alteration of **defined-functions**.*

*An identifier can be edited by the function editing facilities only if it is is **editable**. The header line of a **defined-function** can be altered only if the **function-name** is **locally-erasable**.*

*In **function-definition mode**, lines of the function being edited are temporarily associated with numbers in **decimal-rational** form by an unspecified mechanism. When a function is closed, the order of the lines given by the line number associated with each line is maintained.*

*The display of a function should be the same as the entries required to add the function to a **clear-workspace**.*

*This operation is called by **immediate-execution** when it recognises a **function-definition-line**.*

Evaluation Sequence:

```
Thread  opening-request with $Q$; if $Q$ does not match opening-request, signal definition-error.
Rethread  opening-request with $Q$, taking the following actions:

    When  creation-request is matched,

        Set   $G$ to the list of characters that matched creation-request.
        Set   $N$ to the simple-identifier that matched function-name in $G$.
        If the  current-class of $N$ is not nil and  $G$ does not match identifier-row, signal
          definition-error.

    When  change-request is matched,

        Set   $N$ to the simple-identifier that matched the diagram subject-function in
          change-request.
        Set   $G$ to the list of characters that matched initial-request.

If the  current-class of $N$ is nil, set $M$ to a new defined-function for which

    Canonical-Representation is 0  0ρ' '
    Stop-Vector is ι0
    Trace-Vector is ι0.

Otherwise,

    If   $N$ is not editable, signal definition-error.
    Set   $M$ to the current-referent of  $N$.

Generate a new  context in which

    mode is function-definition,
    current-line is $G$,
    current-function is $M$,
        current-line-number is the number-of-rows in the canonical-representation of $M$,
        current-statement is the empty list of tokens, and
        stack is the empty list of tokens.

    Append the context as a new first item to the state-indicator.
    Repeat:

        Set $Z$ to evaluate-editing-request.
        If $Z$ is command-complete,

            Remove the first-item in the state-indicator of the active-workspace.
            Return $Z$.

        If $Z$ is an error, display $Z$.
        Set current-prompt to function-definition-prompt of current-line-number.
        Set current-line to read-keyboard.

    (End of repeated block)
```

Additional Requirements:

The prompt in definition mode is a line number enclosed in brackets. Line numbers are always displayed in **decimal-rational** form. The line number of the header is always **zero**; it is not affected by **index-origin** .

13.5.2 Evaluate-Editing-Request

Evaluate-Editing-Request

Informal Description: This subsection acts on a single line of input to the editor.

Evaluation Sequence:

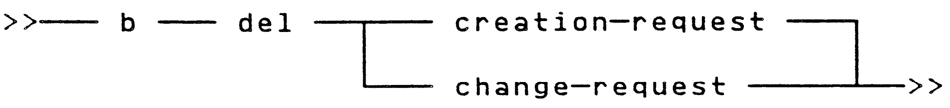
```
Let   C stand for the current-canonical-representation.
Let   N stand for the simple-identifier that matches function-name in function-line.
Let   S stand for the current-stop-vector.
Let   T stand for the current-trace-vector.
Thread  general-request with current-line; if general-request does not match current-line, signal
  definition-error.
Otherwise, rethread  general-request with current-line, taking the following actions:

    When  positioning-request is threaded,

        Set   L to the numeric-input-conversion of the list of characters that matched line-number
          in positioning-request.
        If   L is a negative-number, signal definition-error.
        If   L is greater-than  definition-line-limit, signal limit-error.
        Set  current-line-number to L.

    When  deletion-request is threaded,

        Set   L to the numeric-input-conversion of the list of characters that matched line-number
          in deletion-request.
        If   L is not a positive-number, signal definition-error.
        If   L is greater-than  definition-line-limit, signal limit-error.
        Set  current-line-number to L.
        Delete from    C the row associated with  L, if it exists.
        Delete from    S and T the item associated with  L, if it exists.

    When  display-request is threaded, display function-display of C.

    When  function-line is threaded, if function-line does not match permitted-blanks,

        If  current-line-number is zero,

            If  function-line does not match header-line, signal definition-error.
            If   N is not locally-erasable, signal definition-error.

        Set the row of C associated with current-line-number  to the list of characters  that
          matched function-line.
        If  current-line-number  is not an integer, or is greater-than  current-last-line-number, set
          the items of S and T associated with current-line-number to zero.
        Set  current-line-number to next-definition-line of current-line-number.

    When  end-definition is matched,

        If   C is not a proper-defined-function, signal definition-error.
        If  identifiers are duplicated in the local-name-list  of C, signal definition-error.
        Set the  current-referent of N to a defined-function for which

            Canonical-Representation is C.
            Stop-Vector is S.
            Trace-Vector is T.

    Return  command-complete.
```

13.5.3 Diagrams

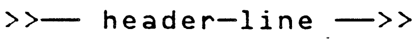
In these diagrams,

b stands for **permitted-blanks.**
r stands for **required-blanks.**

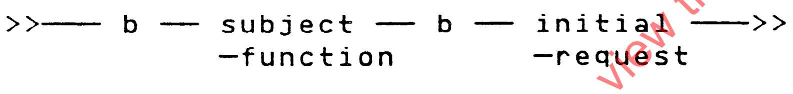
Opening-Request



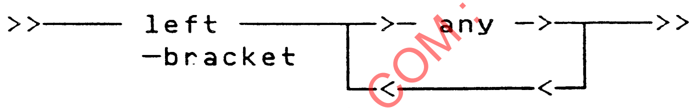
Creation-Request



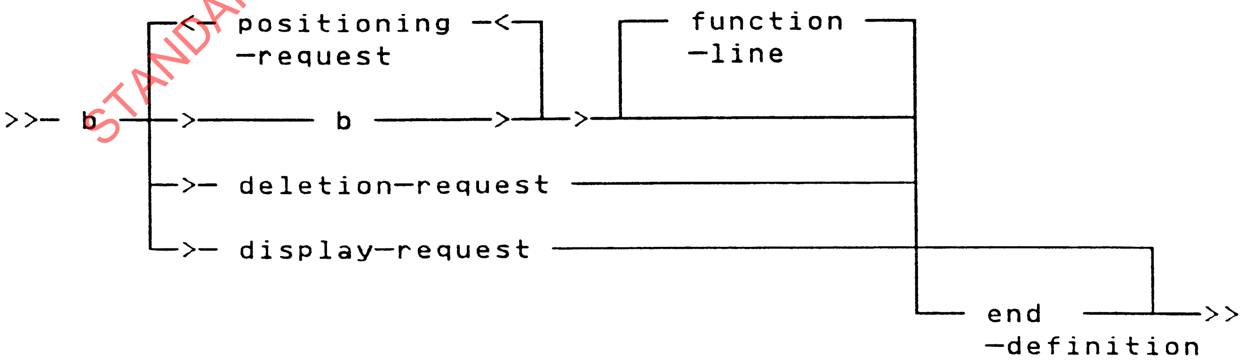
Change-Request



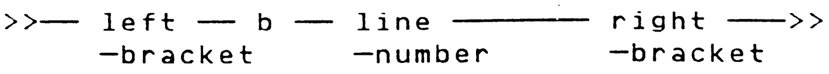
Initial-Request



General-Request



Positioning-Request



Deletion-Request

```
>>— left — b — delta — b — line ——— right ——>>
    -bracket                  -number    -bracket
```

Display-Request

```
>>— left — b — quad — b — right——>>
    -bracket               -bracket
```

Function-Line

```
>>— body-line ——>>
```

End-Definition

```
>>— b — del — b ——>>
```

Line-Number

```
>>— decimal-rational — b ——>>
```

Subject-Function

```
>>— simple-identifier ——>>
```

Delta

```
>>——————————┐
            │
            Δ
            │
            └——>>
```

Right-Bracket

```
>>——————————┐
            │
            ]
            │
            └——>>
```

Left-Bracket

```
>>——————————┐
            │
            [
            │
            └——>>
```

14 SHARED VARIABLES

14.1 INFORMAL INTRODUCTION

This section gives an informal introduction to the **Shared-Variable-Protocol**. The evaluation sequences which follow this introduction determine the required behaviour of **shared-variables** exactly.

The **Shared-Variable-Protocol** is an **optional-facility**.

*The **Shared-Variable-Protocol** is described in this document as an interface between cooperating **sessions**, but it may also be used to provide an interface between a **session** and an **auxiliary processor**. An **auxiliary processor** is a program, written in any programming language, that typically provides access to facilities in the underlying operating system, such as file systems.*

*A **shared-variable** can be shared by precisely two partners, each either a **session** or an **auxiliary processor**. **Conforming-programs** that use the **Shared-Variable-Protocol** should document this fact, since the **Shared-Variable-Protocol** is not a **defined-facility** of this standard.*

Operations

*There are conceptually four operations involved in the **Shared-Variable-Protocol**: **offer, retract, set,** and **reference**.*

Sharing

***Offer** is an operation provided to permit **sessions** to share **shared-variables**. A given **shared-variable** can be shared between at most two **sessions**. Once a **shared-variable** has been shared, assignments made to it in one **session** are visible in the other **session**.*

*In the evaluation sequences for the **Shared-Variable-Protocol**, sharing is accomplished by appending **tokens** of **class shared-variable** to the **shared-variable-list** which can be accessed by all **active-workspaces** in the **system**.*

*Any **Shared-Variable-Protocol** operation may cause a **session** to be delayed while another **session** changes a **shared-variable** or adds an item to the **shared-variable-list**.*

*Sharing begins when one **session** offers to share a **symbol** whose **current-class** is **nil** or **variable**. The **system** adds a new **shared-variable** to the **shared-variable-list** and returns the **shared-variable** to the offering **session**. The **shared-variable** then replaces the **current-referent** of the offered **symbol**.*

*There are two forms of offers. A **specific-offer** identifies a particular **session** as the partner; the **general-offer** does not.*

*Two offers match if both **sessions** use the same **shared-name** and:*

> *At least one of the offers is a **specific-offer**.*
> *Either **specific-offer** identifies the other **session**.*

*Consequently, two **general-offers** do not match.*

*Once both **sessions** have shared the **shared-variable**, the **shared-variable** is the **current-referent** of a **symbol** in both **active-workspaces**, and a change made to the **shared-variable** by one **session** will be visible to the other **session**.*

*Note that two **sessions** can share more than one **shared-variable**, that a given **session** can share **shared-variables** with more that one other **session**, and that each **shared-variable** can be shared by only two **sessions**.*

Retracting

*Two **sessions**, coupled by a **shared-variable**, decouple when either **retracts** the **shared-variable**. This leaves the **shared-variable** in the same state it would have been had the remaining **session** made a **specific-offer** to the session that **retracted**. If the remaining **session** **retracts**, the **shared-variable** becomes inaccessible to the **sessions**.*

Degree of Coupling

*All **symbols** in the **symbol-table** of an **active-workspace** have a **degree of coupling**. This is **zero** if the **current-referent** is not a **shared-variable**, **one** if the **current-referent** is a **shared-variable** that is offered to another **session**, and **two** if **current-referent** is a **shared-variable** shared with another **session**.*

Naming

*For any session, there are two **identifiers** associated with a given **shared-variable**. The first is the **name** of the **symbol** whose **current-referent** is the **shared-variable**; the second is the **shared-name** of the **shared-variable**. The **shared-name** is the name used in matching offers to share.*

*Only the **name** of the **symbol** is required for invoking any of the **Shared-Variable-Protocol** system functions other than **Shared-Variable Offer**.*

Setting, Referencing, and Using

Each partner can ***set*** *and* ***reference*** *the* ***shared-variable****.*

A ***shared-variable*** *is* ***set*** *when the operation* ***shared-variable-assignment*** *or* ***shared-variable-indexed-assignment*** *is called. A* ***shared-variable*** *is* ***referenced*** *when the operation* ***shared-variable-reference*** *is called. A* ***shared-variable*** *is used when it is either* ***set*** *or* ***referenced****. Both indexed assignment and indexed reference of* ***shared-variables*** *are possible.*

Synchronisation

Synchronisation of ***sessions*** *sharing a* ***shared-variable*** *is achieved by* ***access control****. With each* ***shared-variable*** *is associated an* ***access-control-vector****, which is designated here as the* ACV*. The* ACV *is a four element Boolean vector that is accessible to a* ***session*** *through the* ***Shared-Variable-Access-Control-Inquiry*** *operation.*

The order of the elements in the ACV *is relative to the viewing* ***session****. When* ***session*** *A, sharing a* ***shared-variable*** *with* ***session*** *B, views the* ACV*, its elements have the following meanings:*

If ACV[1] *is* ***one****, once A has set the* ***shared-variable*** *B must use it before A can set it again.*

If ACV[2] *is* ***one****, once B has set the* ***shared-variable*** *A must use it before B can set it again.*

If ACV[3] *is* ***one****, once A has used the* ***shared-variable*** *B must set it before A can reference it again.*

If ACV[4] *is* ***one****, once B has used the* ***shared-variable*** *A must set it before B can reference it again.*

If a ***session*** *attempts to use a* ***shared-variable*** *when not permitted to by the above rules, it is delayed until the partner has performed the required intermediate operation.*

The rules can be summarised in a three-state state diagram. Let A be the ***session*** *that made the initial offer. Then the states are as follows:*

State 0: The ***shared-variable*** *has been* ***referenced*** *by the partner of the* ***session*** *that last set it.*

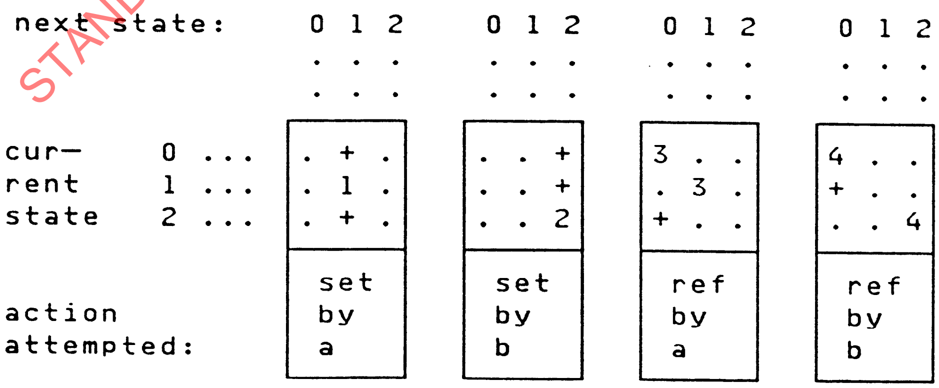
State 1: The ***shared-variable*** *was last* ***set*** *by A and has not been* ***referenced*** *by B since then.*

State 2: The ***shared-variable*** *was last* ***set*** *by B and has not been* ***referenced*** *by A since then.*

State 0 is the state entered when the ***shared-variable*** *is initially* ***offered****. Note that only an explicit assignment is considered a* ***set*** *to the* ***shared-variable****. The act of transferring the* ***current-content*** *of a* ***symbol*** *to the* ***shared-value*** *of a* ***shared-variable*** *when an offer is made is not a* ***set****. The state is not affected by retraction and re-sharing.* ***Figure 2*** *summarises the state transitions permitted. In the figure, the action is always permitted if there is a + in the row corresponding to the input state. The column in which the + appears determines the output state. A digit in the row indicates that the corresponding element in the* ACV *as seen by A must be zero.*

For example, if the state of a ***shared-variable*** *is* 0 *and the* ACV *is* 0 0 1 0*,* ***session*** *A can assign a new value to the variable (SET by A), but must wait until the variable is in state* 2 *to refer to it (REF by A).*

Note that all digits lie along the diagonal in these matrices, indicating that only actions that do not change the state can be blocked by the ACV*. That is, the* ACV *can prevent only a given session from referencing or setting a* ***shared-variable*** *twice in a row.*



```
next state:          0 1 2     0 1 2     0 1 2     0 1 2
                     . . .     . . .     . . .     . . .
                     . . .     . . .     . . .     . . .

cur-       0 ...     . + .     . . +     3 . .     4 . .
rent       1 ...     . 1 .     . . +     . 3 .     + . .
state      2 ...     . + .     . . 2     + . .     . . 4

                      set       set       ref       ref
action                by        by        by        by
attempted:            a         b         a         b
```

Figure 2 — Shared Variable Access Rules

Other Rules

***Retraction** of all the **shared-variables** in a **session** takes place when a new **workspace** is loaded, the **active-workspace** is cleared, or the **session** ends. A **shared-variable** is automatically **retracted** when the **shared-variable** is erased, expunged, or replaced by a)COPY command, or, if it is local, when the function to which it is local terminates execution. The state of sharing is not saved in a library workspace.*

*The shared variable mechanism must appear to the user to be consistent with one in which there exists a single copy of each **shared-variable**. Specifically, this means:*

> *When a **shared-variable** is initially offered, its value, if any, becomes the shared-value of the **shared-variable**.*
>
> *When an offered **shared-variable** is matched, it retains its value if it has one. If it does not have a value, it takes on the value (if any) in the matcher's workspace.*
>
> *When a **shared-variable** is retracted, its value in the retractor's workspace is the value last set by either partner before the retraction.*

*Note that, since there is conceptually a single copy across a period of sharing, no use is implied in the above rules for making the value of a **shared-variable** available.*

*It must appear to the APL user that **sessions** communicating via **shared-variables** are independently scheduled. Any synchronisation between the **sessions** must appear to be by means of the delays caused by accessing **shared-variables**. The procedures used to describe the **Shared-Variable-Protocol** are written as though synchronisation conflicts do not occur when two **sessions** are accessing the **shared-variable-list** or the same **shared-variable**.*

*A **limit-error** may occur during the retraction of an active **shared-variable** if there is not enough room in a **workspace** for the **shared-value** of the **shared-variable**. When the expected system action is to ignore the variable (as in the case of)CLEAR or delocalisation), a **limit-error** is not signalled.*

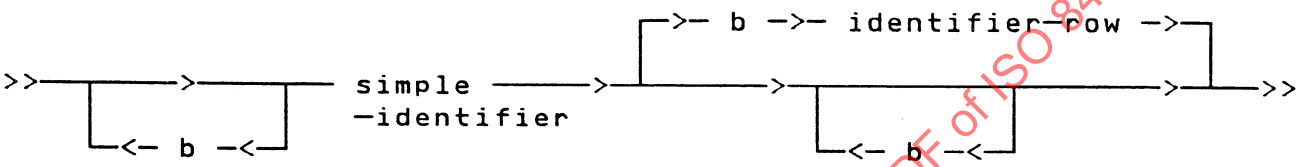
14.2 DEFINITIONS

Identifier-Pair-Array: A **character array** of **rank two** in which each **row** matches the **character-diagram identifier-pair-row**.

14.3 DIAGRAMS

In the following, b stands for **blank**.

Identifier-Pair-Row



14.4 OPERATIONS

14.4.1 Primary-Name

Primary-Name in *A*: For *A*, a **character vector** that matches **identifier-pair-row**, the first **simple-identifier** matched in **identifier-pair-row**.

14.4.2 Surrogate-Name

Surrogate-Name in *A*: For *A*, a **character vector** that matches **identifier-pair-row**, the last **simple-identifier** matched in **identifier-pair-row**.

14.4.3 Degree-of-Coupling

Degree-of-Coupling of *N*: For **symbol** *N*, a **number** defined as follows:

If the **current-class** of *N* is not **shared-variable**, **zero**.
Otherwise, using the **current-referent** of *N*, the sum of **session-A-active** and **session-B-active**.

14.4.4 Access-Control-Vector

Access-Control-Vector of *N*: An operation that, for *N* a **shared-variable**, is defined as follows:

Using *N*,

Let *ACVA* stand for **session-A-ACV** and *ACVB* stand for **session-B-ACV**.
If **this-session** is **session-A**, return *ACVA*∨*ACVB*.
If **this-session** is **session-B**, return (*ACVA*∨*ACVB*)[2 1 4 3].

14.4.5 Offer

Offer S **to** P **with value** V: An operation defined as follows:

```
Let $X$ stand for the item in the shared-variable-list with the smallest index such that

    shared-name is $S$.
    session-A is this-session or, if $P$ is not general-offer, general-offer.
    session-A-active is zero.
    session-B is $P$.
    session-B-active is one.

or

    shared-name is $S$.
    session-A is $P$.
    session-A-active is one.
    session-B is this-session or, if $P$ is not general-offer, general-offer.
    session-B-active is zero.

If such a shared-variable exists,

    If shared-value is nil, set shared-value to $V$.

        If session-A-active is zero,

            Set session-A-active to one.
            Set session-A to this-session.

        If session-B-active is zero,

            Set session-B-active to one.
            Set session-B to this-session.

    Return $X$.

Otherwise, create a new shared-variable, $Y$, as follows:

    shared-name is $S$.
    session-A is this-session.
    session-A-active is one.
    session-A-ACV is 0 0 0 0.
    session-B is $P$.
    session-B-active is zero.
    session-B-ACV is 0 0 0 0.
    shared-value is $V$.
    state is zero.

Append $Y$ to the shared-variable-list as a new last item.
Return $Y$.
```

14.4.6 Retract

Retract N: An operation that, for N a **shared-variable**, is defined as follows:

```
Using $N$,
    If session-A is this-session, set session-A-active to zero.
    If session-B is this-session, set session-B-active to zero.
```

14.4.7 Shared-Variable-Reset

Shared-Variable-Reset

```
For each item in the shared-variable-list,
    If session-A is this-session, set session-A-active to zero.
    If session-B is this-session, set session-B-active to zero.
For each symbol $N$ of the symbol-table of the active-workspace,
    For each token $T$ in the referent-list of $N$,
        If $T$ is a shared-variable, set $T$ to a token whose class is nil.
```

Note: In the model, a ***shared-variable*** *is never reused. It is effectively discarded once* ***session-A-active*** *and* ***session-B-active*** *are both* ***zero****.*

14.5 SHARED VARIABLE FORMS

14.5.1 Shared Variable Reference

```
Z ← SHV
```

Informal Description: Return the value of a shared variable.

Evaluation Sequence:

```
Using the current-content of SHV,

    Let ACV stand for access-control-vector.
    Wait until at least one of the following is true:

        ACV[3] is zero.
        State is two and session-A is this-session.
        State is one and session-B is this-session.

    If either of the following is true, set State to zero:

        State is two and session-A is this-session.
        State is one and session-B is this-session.

    Return Shared-Value.
```

*Note: This operation is all that is required for **shared-variable-indexed-reference**, since the **phrase-evaluator** calls **indexed reference** with a value, not a name.*

14.5.2 Shared Variable Assignment

```
Z ← SHV ← B
```

Informal Description: Z is B. As a side effect, the value B is assigned to the shared variable SHV.

Evaluation Sequence:

```
Using the current-content of SHV,

    Let ACV stand for access-control-vector.
    Wait until at least one of the following is true:

        ACV[1] is zero.
        State is not one and session-A is this-session.
        State is not two and session-B is this-session.

    Set shared-value to B.
    If session-A is this-session, set state to one.
    If session-B is this-session, set state to two.
    Return a token whose class is committed-value and whose content is B.
```

14.5.3 Shared Variable Indexed Assignment

$Z \leftarrow SHV[I] \leftarrow B$

Informal Description: Z is B. As a side effect, elements of the array SHV selected by the position and values of the arrays in the **index-list** I are replaced by elements of the array B.

Evaluation Sequence:

```
Using the current-content of SHV,

    Let ACV stand for access-control-vector.
    Wait until at least one of the following is true:

        ACV[1] is zero.
        session-A is this-session and state is not one.
        session-B is this-session and state is not two.

    If the current-class of shared-value is nil, signal value-error.
    Set C to shared-value.
    Search the form-table for Z ← V[I] ← B.
    Call the corresponding evaluation sequence, passing C as the value of V.
    Set Z to the token it returns.
    If Z is an exception, return Z.
    Otherwise,

        Set shared-value to C.
        If session-A is this-session, set state to one.
        If session-B is this-session, set state to two.
        Return Z.
```

*Note: Indexed assignment of a shared variable constitutes a **set**, not a **reference**, of the **shared-variable**.*

14.6 SHARED VARIABLE SYSTEM FUNCTIONS

14.6.1 Shared Variable Access Control Inquiry

Z ← ⎕*SVC B*

Informal Description: Each row of *Z* is the **access-control-vector** of the shared variable named by the corresponding row of *B*.

Evaluation Sequence:

```
If the rank of B is greater-than two, signal rank-error.
Set B1 to (¯2↑1 1,⍴B)⍴B.
If any item of the ravel-list of B1 is not a character, signal domain-error.

Set Z to ((1↑⍴B1),4)⍴0.

For every I in ⍳1↑⍴B1, evaluated with index-origin set to one,

    Let B2 stand for row I of B1.
    If B2 matches identifier-row,

        Set N to the simple-identifier in B2.
        If the current-class of N is shared-variable,

            Set row I of Z to access-control-vector of the current-referent of N.

    Otherwise, signal domain-error.

Return ((¯1↓⍴B),4)⍴Z.
```

14.6.2 Shared Variable Query

Z ← ⎕*SVQ B*

Informal Description: *Z* is one of the following:

An array representing sessions offering to share variables with this session.
An array representing the names of shared variables offered to this session by another session.

Evaluation Sequence:

```
If session-identification-type is character, and B is the general-offer,

    Set  Z to 0 0⍴' '.
    Using each item of the shared-variable-list,

        If

            session-A is this-session and
            session-A-active is zero and
            session-B-active is one,

                Set  S to session-B
                Set  Z to ((⍴Z)⌈0,⍴S),[1]((¯1↑⍴Z)⌈⍴S)↑S, evaluated with
                  index-origin set to one.

        If

            session-A-active is one and
            session-B is this-session and
            session-B-active is zero,

                Set  S to session-B
                Set  Z to ((⍴Z)⌈0,⍴S),[1]((¯1↑⍴Z)⌈⍴S)↑S, evaluated with
                  index-origin set to one.

    Return  Z.

If session-identification-type is numeric, and B is zero, signal domain-error.
If session-identification-type is numeric, and B is ⍳0,

    Set  Z to ⍳0.
    Using each item of the shared-variable-list,

        If all of the following are true,

            session-A is this-session
            session-A-active is zero
            session-B-active is one

        append session-B as a new last element to the vector Z.
        If all of the following are true,

            session-A-active is one
            session-B is this-session
            session-B-active is zero

        append session-A as a new last element to the vector Z.

    Return  ((Z⍳Z)=⍳⍴Z)/Z.

Otherwise,
If    B is not a session-identification, signal domain-error.
If    B is this-session, signal domain-error.
Set    Z to the empty character array of rank two.
Using each item of the shared-variable-list,

    If all of the following are true,

        session-A is this-session
        session-A-active is zero
        session-B is B
        session-B-active is one

    append shared-name as a new last row to the identifier-array Z.

    If all of the following are true,

        session-A is B
        session-A-active is one
        session-B is this-session
        session-B-active is zero

    append shared-name as a new last row to the identifier-array Z.

Return    Z.
```

14.6.3 Shared Variable Degree of Coupling

```
Z ← ⎕SVO B
```

Informal Description: *Z* is a numeric vector in which each element is the degree of coupling of the symbol named by the corresponding row of *B*.

Evaluation Sequence:

```
If the rank of B is greater-than two, signal rank-error.
Set    B1 to (¯2↑1 1,ρB)ρB.
If any item of the ravel-list of B1 is not a character, signal domain-error.
Set    Z to (1↑ρB1)ρ0.

For every I in ι1↑ρB1,

    Let    B2 stand for row I of B1.
    If    B2 matches identifier-row,

        Set    N to the primary-name in B2.
        Set item I of the ravel-list of Z to degree-of-coupling of the symbol-named-by N.

    Otherwise, signal domain-error.

Return    Z.
```

*Note: **Degree-of-coupling** is a property of **symbols**, not **shared-variables**.*

14.6.4 Shared Variable Offer

Z ← A ⎕SVO B

Informal Description: Z is a numeric vector representing the **degree-of-coupling** of the identifiers in the rows of the **identifier-pair-array** B after the operation. As a side effect, dyadic ⎕SVO shares variables with another **session**. B is an **identifier-pair-array** representing the names of variables to be shared. A is a **session-identification**, representing the **session** with which the names in B are to be shared.

Evaluation Sequence:

```
If the rank of B is greater-than two, signal rank-error.
If any item of the ravel-list of B is not a character, signal domain-error.
If A is not a valid session-identification, signal domain-error.
If this-session is A, signal domain-error.
Set B1 to (¯2↑1 1,⍴B)⍴B.
For every I in ⍳1↑⍴B1,

    Let B2 stand for row I of B1.
    If B2 does not match identifier-pair-row, signal domain-error.
    Otherwise,

        Set N to the primary-name in B2.
        Set S to the surrogate-name in B2.
        If the current-class of N is neither shared-variable, variable nor nil, signal domain-error.
        If the current-class of N is variable or nil,

            Set V to the current-referent of N.
            Set Y to offer S to A with value V.
            Set the current-referent of N to Y.

Return ⎕SVO B.
```

Note: This operation does not allow for multiple ***session-identifications.***

14.6.5 Shared Variable Retraction

Z ← ⎕*SVR B*

Informal Description: Elements of *Z* hold the degree of coupling that the identifier represented in the corresponding row of the **identifier-array** *B* had, before this retraction. As a side effect, ⎕*SVR* retracts any shared-variables named in *B*.

Evaluation Sequence:

```
If the rank of B is greater-than two, signal rank-error.
Set  B1 to (¯2↑1 1,ρB)ρB.
Set  Z to ⎕SVO B1.
If  Z is an error, return Z.
For every  I in ⍳1↑ρB1,

    Let  B2 stand for row I of B1.
    If  B2 matches identifier-row,

        Set  N to the simple-identifier in B2.
        If the current-class of N is shared-variable,

            Set  SV to the current-referent of N.
            Set the current-referent of N to shared-value of SV.
            Retract SV.

    Otherwise, signal domain-error.
    Return  Z.
```

14.6.6 Shared Variable Access Control Set

```
Z ← A ⎕SVC B
```

Informal Description: *Z* is the **access-control-vector** of each of the shared variables represented by the **identifier-array** *B* when ⎕*SVC* completes. As a side effect, ⎕*SVC* sets to *A* the contribution this session makes to the combined access control vector of the shared variables.

Evaluation Sequence:

```
If the rank of B is greater-than two, signal rank-error.
Set B1 to (¯2↑1 1,ρB)ρB.
If the rank of A is greater-than two, signal rank-error.
If the first item of (¯1↑4,ρA) is not four, signal length-error.
If A is a scalar or a vector, set A1 to ((1↑ρB1),4)ρA.
Otherwise, set A1 to A.
If 1↑ρA1 is not the same as 1↑ρB1, signal length-error.
If any item of the ravel-list of B1 is not a character, signal domain-error.
If any item of the ravel-list of A1 is not a near-Boolean, signal domain-error.
Set A2 to the Boolean-array-nearest-to A1.
For every I in ι1↑ρB1,

    Let B2 stand for row I of B1.
    If B2 matches identifier-row,

        Set N to the simple-identifier in B2.
        If the current-class of N is shared-variable,

            Let ACV stand for row I of A2.
            Using the current-content of N,

                If session-A is this-session, set session-A-ACV to ACV.
                If session-B is this-session, set session-B-ACV to ACV[2 1 4 3].

    Otherwise, signal domain-error.

Return ⎕SVC B.
```

15 FORMATTING AND NUMERIC CONVERSION

15.1 INTRODUCTION

The mapping of arrays of numbers into arrays of characters is referred to as formatting.

Note: *The form, field width, and number of digits in a formatted number can be specified explicitly through the use of* ***dyadic format****, or left to be chosen by the system.* ***Conforming-programs*** *that depend upon a particular selection of formatting parameters should employ* ***dyadic format****.*

15.2 NUMERIC CONVERSION

Note: *Conversion involves the transformation of lists of* ***characters*** *to and from* ***numbers****. This conversion is performed by the* ***implementation-algorithms*** ***numeric-input-conversion*** *and* ***numeric-output-conversion****.*

The accuracy and rounding properties of **numeric-input-conversion** and **numeric-output-conversion** are interdependent. The most significant property of these **implementation-algorithms** is the following:

> For any **numeric scalar** X, when **print-precision** is set to **full-print-precision**, X shall be the same as ⍎⍕X.

15.2.1 Numeric-Input-Conversion

Numeric-Input-Conversion of C.

Informal Description: Numeric input conversion converts numeric values represented in decimal notation as **lists** of **characters** into equivalent **numbers** – numeric quantities whose format is **implementation-defined**.

Evaluation Sequence:

Let L and G be respectively the most negative and the most positive abstract numeric values represented by all **numeric-scalar-literals** producible by **numeric-output-conversion** for all **numbers** and all valid values of **print-precision** .

Let T be the set of all **lists of characters** that match the diagram **numeric-scalar-literal** and have abstract numeric values lying in the closed interval between L and G.

If C is not an element of T, signal **limit-error**.

Otherwise,

Let **c** stand for the abstract numeric value of C.

If **c** is the same as some **number** N, return N.

If **c** is greater than **positive-number-limit**, return **positive-number-limit**.

If **c** is less-than **negative-number-limit**, return **negative-number-limit**.

Otherwise, **c** lies between two **numbers**. Return one that permits **numeric-input-conversion** to satisfy the following property:

For any two **numeric-scalar-literals** $C1$ and $C2$, if the abstract numeric value of $C1$ is less than the abstract numeric value of $C2$, then **numeric-input-conversion** of $C1$ is not **greater-than numeric-input-conversion** of $C2$.

Examples:

```
      1-1.000000000000000000000000000000000001
0
      1-0.999999999999999999999999999999999999
0
      0.00000000000000000001E20
1
      0E999
0
      1E¯9999 = 0
1
      2.5E4-25000
0
```

Note: In many programming languages, the syntactic form of a literal is used to determine the datatype of the corresponding number. This is not so in APL. Therefore, it is important that the input conversion routine not make distinctions between different forms of the same abstract numeric value. 2.5E4 *should produce the same* ***number*** *as* 25000 *does and* 002 *should produce the same number as* 2 *does.*

Every ***number*** *can be obtained by converting some* ***numeric-scalar-literal.***

The set T *in the evaluation sequence above may include members with abstract numeric values less than* ***negative-number-limit*** *or greater than* ***positive-number-limit****, due to rounding by* ***numeric-output-conversion****.*

There is no ***number*** N *for which the expression* ⍎⍕N *signals a* ***limit-error****.*

15.2.2 Numeric-Output-Conversion

E **Numeric-Output-Conversion** of *N*.
E **Numeric-Output-Conversion** of *N* **to** *P* **places**.

Informal Description: Numeric output conversion converts numeric values represented as **numbers** -- numeric quantities whose format is **implementation-defined** -- into the same numeric quantities represented in decimal notation as **lists** of **characters**.

For *E* a **character-diagram**, and *N* and *P* **numbers**, numeric output conversion returns a **numeric-scalar-literal** that matches *E* and represents the abstract numeric value of *N* either to *P* decimal places, if *P* is specified, or else to **print-precision** places.

Evaluation Sequence:

If *E* is **minimal-decimal-exponential**,

Return a **list** of **characters** whose abstract numeric value is a good approximation of *N*.

Note: *The choice of approximation is* ***implementation-defined****, but can be characterised as follows:*

Let DS be the set whose members D are ***lists*** *of* ***characters*** *that match* ***minimal-decimal-exponential*** *and have at most* ***print-precision*** *digits to the left of the* ***exponent-marker****.*
Let DT be the subset of DS for which |*N*−⍎*D* *is minimal.*
Let DM be the subset of DT whose members have the fewest digits.
Choose the member of DM whose abstract numeric value has the largest magnitude.

If *E* is **fixed-decimal**,

Let *DF* be the set whose members are **lists** of **characters** that match **fixed-decimal** and have *P* digits to the right of the units digit.
Return a member *D* of *DF* for which |*N*−⍎*D* is minimal.

If *E* is **decimal-exponential**,

Let *DE* be the set whose members are **lists** of **characters** that match diagram **decimal-exponential** and have *P* digits to the left of the **exponent-marker**.
Select a member *D* of *DE* for which |*N*−⍎*D* is minimal.
Let *W* stand for **exponent-field-width**.
Return (*W*+*D*⍳'*E*')↑*D*.

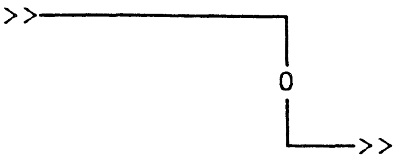
Note: *When P is not specified, E is always* ***minimal-decimal-exponential****.*

Fixed-decimal *and* ***decimal-exponential*** *cases are used only by* ***dyadic format****. They are permitted, but not required, to return results of arbitrarily high precision. They should, however, return accurate results if no more than* ***full-print-precision*** *significant digits are requested.*

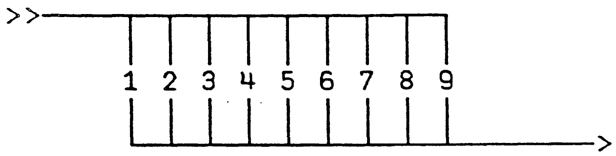
15.3 DIAGRAMS

Note: *The following* ***character-diagrams*** *characterise the outputs of both monadic and dyadic format as subsets of the set of* ***lists*** *of* ***characters*** *that match* ***numeric-scalar-literal****.*

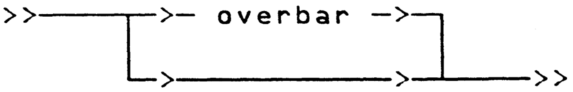
Zero-Digit



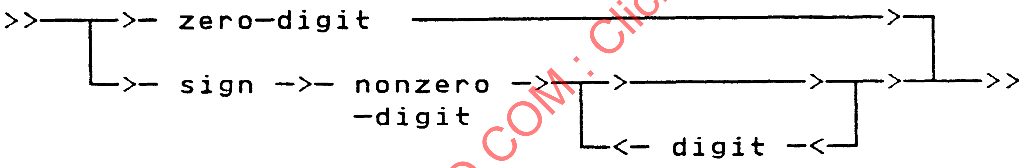
Nonzero-Digit



Sign



Decimal-Integer

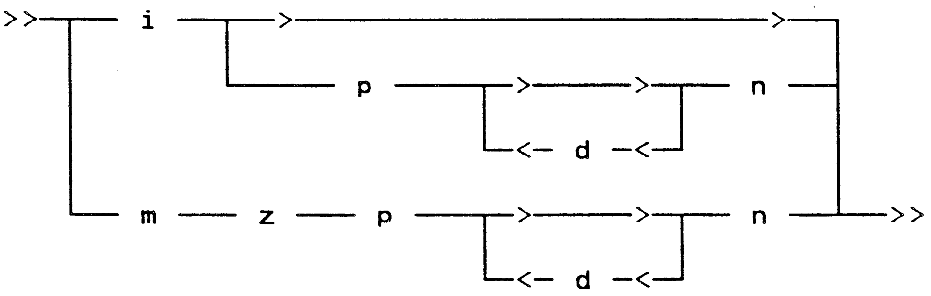


Examples:

```
¯44 0 622 1066 1215 1415 1685 1750 1776 1812 1867 1944
```

In the following diagrams,
b stands for **blank**.
d stands for **digit**.
e stands for **exponent-marker**.
i stands for **decimal-integer**.
m stands for **overbar**.
n stands for **nonzero-digit**.
p stands for **dot**.
s stands for **sign**.
z stands for **zero-digit**.

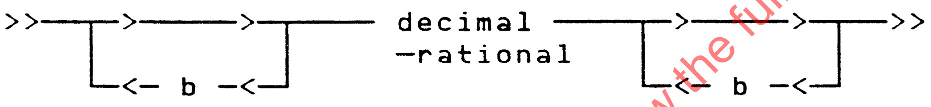
Decimal-Rational



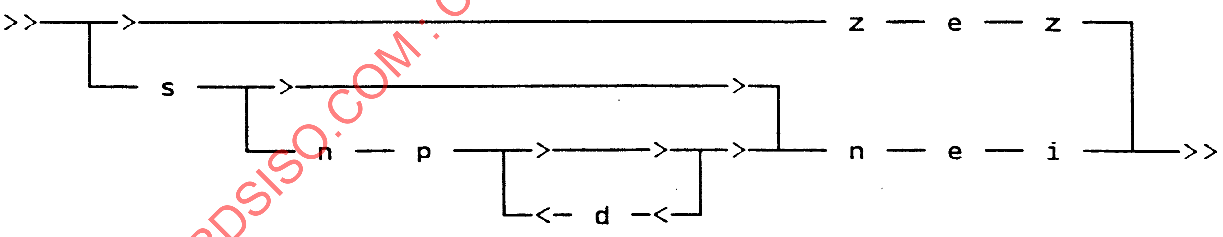
Examples:

98.6 212 ¯2.001 ¯0.00000000001

Decimal-Rational-Row



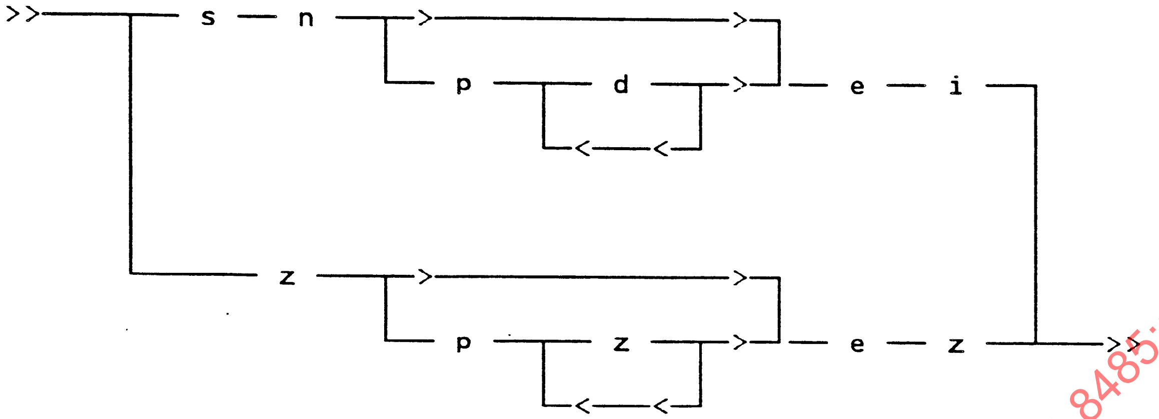
Minimal-Decimal-Exponential



Examples:

¯1.234567*E*¯890   1*E*1 ¯1.1111*E*1 1*E*¯11111 0*E*0

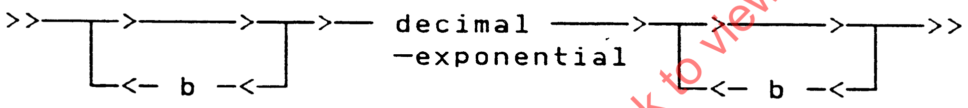
Decimal-Exponential



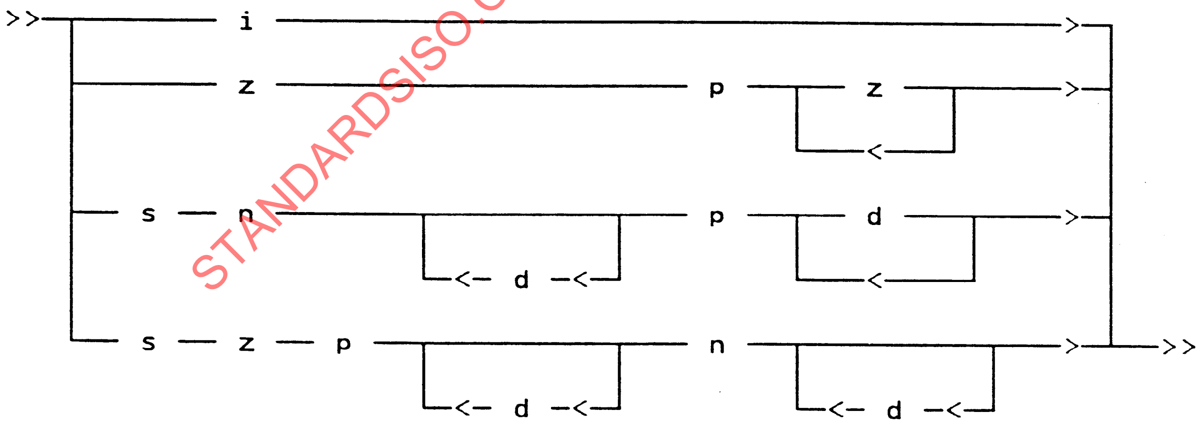
Examples:

```
¯1.23456700000E¯890   0.00000000E0   ¯1.0000E¯10
```

Decimal-Exponential-Row



Fixed-Decimal



Examples:

```
0 ¯1 12 0.00000000 12.30 0.1235 ¯0.1235 0.1 ¯0.00010000
```

15.4 OPERATIONS

15.4.1 Monadic Format

Z ← ⍕ *B*

Informal Description: *Z* is a **character** array. If *B* is a **character** array, then *Z* is *B*. If *B* is a numeric array, then *Z* is a **character** array arranged so that each row of *Z* is a decimal representation of the corresponding row of *B*. The output form chosen for a column of *B* is determined by **print-precision** and the values of elements in that column. Uses **print-precision** .

Evaluation Sequence:

If **print-precision** is **nil**, signal **implicit-error**.
If the **type** of *B* is **character**, return *B*.

If *B* is **empty**, return an **array** *Z*0 such that the **type** of *Z*0 is **character**, the **shape-list** of *Z*0 is the **shape-list** of *B*, and the **ravel-list** of *Z*0 is the **empty-list**.

If the **rank** of *B* is **less-than two**,
return `,⍕(¯2↑1 1,⍴B)⍴B`.

If the last **item** of the **shape-list** of *B* is not **one**,
return `(⍕((¯1↓⍴B),1)↑B),' ',⍕((-⍴⍴B)↑1)↓B`.

Otherwise, choose *U*, *E*, and *W*, and return an **array** *Z* for which the **type** of *Z* is **character**, the **shape-list** of *Z* is `(¯1↓⍴B),W`, and one of the following statements is true:

Every **row** of *Z* is the **decimal-exponential-row** equivalent of the **minimal-decimal-exponential numeric-output-conversion** of the corresponding **row** of *B*, and has its **exponent-marker** at position *E* and its first digit at position *U*.

Every **row** of *Z* is the **decimal-rational-row** equivalent of the **minimal-decimal-exponential numeric-output-conversion** of the corresponding **row** of *B*, and has its units digit at position *U*.

Note: *The quantities W, U and E and the choice of formatting form are not standardised here, since this is currently impractical. However, the following algorithm is recommended for all future implementations of APL:*

```
If any of the following conditions holds for any element X of B

    X is greater-than positive-counting-number-limit.
    X is less-than negative-counting-number-limit.
    The decimal-rational equivalent of the minimal-decimal-exponential numeric-output-conversion of X would have
      more than print-precision significant digits to the left of the decimal point.
    The decimal-rational equivalent of the minimal-decimal-exponential numeric-output-conversion of X would have
      more than five zero digits to the right of the decimal point and to the left of the first nonzero digit.

then select decimal-exponential form.

Otherwise, select decimal-rational form.

If decimal-exponential form is selected,

    Let S be one if any element of B is a negative-number.
    Let M be the maximum number of characters to the left of the exponent-marker in the minimal-decimal-exponential
      numeric-output-conversion of any element of B.
    Let N be the maximum number of characters to the right of the exponent-marker in the minimal-decimal-exponential
      numeric-output-conversion of any element of B.
    Set U to S+1.
    Set E to M+1.
    Set W to M+1+N.

If decimal-rational form is selected,

    Let M be the maximum number of characters to the left of the units position in the decimal-rational equivalent of
      the minimal-decimal-exponential numeric-output-conversion of any number in B.
    Let N be the maximum number of characters to the right of the units position in the decimal-rational equivalent of the
      minimal-decimal-exponential numeric-output-conversion of any number in B.
    Set U to M+1.
    Set W to M+1+N.
```

Examples:

```
      ⎕PP←10

      '|',(⍕ 5 ¯6 7 8 9 10∘.÷⍳5),'|'
| 5  2.5  1.666666667  1.25  1  |
|¯6 ¯3   ¯2           ¯1.5  ¯1.2|
| 7  3.5  2.333333333  1.75  1.4|
| 8  4    2.666666667  2     1.6|
| 9  4.5  3            2.25  1.8|
|10  5    3.333333333  2.5   2  |

      D←0.7 0.8 0.9, 7 8 9÷10
      ⍕ D
0.7 0.8 0.9 0.7 0.8 0.9
      D-⍎⍕ D
0 0 0 0 ¯1.387778781E¯17 0

      ⎕PP←999
      ⍕ D
0.7 0.8 0.9 0.7 0.79999999999999999 0.9
      D-⍎⍕ D
0 0 0 0 0 0

      ⎕PP←4
      M←0.78901×(10*⍳6)∘.×1 1E¯10 1E¯6 0.01 0.1
      ⍕ M
7.89E0 7.89E¯10 0.00000789    0.0789 7.89E¯1
7.89E1 7.89E¯9  0.0000789     0.789  7.89E0
7.89E2 7.89E¯8  0.000789      7.89   7.89E1
7.89E3 7.89E¯7  0.00789      78.9    7.89E2
7.89E4 7.89E¯6  0.0789      789      7.89E3
7.89E5 7.89E¯5  0.789      7890      7.89E4

      ⍕ 5⍴M
7.89 7.89E¯10 0.00000789 0.0789 0.789
      ⍕ 1 5⍴M
7.89 7.89E¯10 0.00000789 0.0789 0.789
```

Note: When ***print-precision*** *has a value in its* ***internal-value-set*** *greater than or equal to* ***full-print-precision****, the result produced for N is unique and is unaffected by increases in the value of* ***print-precision*** *. At such settings of* ***print-precision*** *, N is the same as ⍎⍕N for any* ***numeric scalar*** *N.*

15.4.2 Dyadic Format

Z ← A ⍕ B

Informal Description: Z is a **character** array representing the numbers in B formatted according to the specifications in A. A consists of pairs of integers. One pair is associated with each column of B. The first integer in the pair specifies the field width – the number of columns in the character result – for the formatted numeric values.

If A has only two elements then all elements of B are formatted according to A.

A non-negative second integer indicates that the field is to have **fixed-decimal** form and gives the number of digits to the right of the decimal point, with zero indicating no decimal places and no decimal point. A negative second integer indicates that the field is to have decimal exponential form; the absolute value specifies the number of digits in the mantissa.

Evaluation Sequence:

```
If the rank of A is greater-than one, signal rank-error.
If ρ,A is not 2×¯1↑1,ρB

    If ρA is two, return ((2×¯1↑ρB)ρA)⍕B.
    Otherwise, signal length-error.

If any item of the ravel-list of A is not a near-integer, signal domain-error.
If any item of the ravel-list of B is not a number, signal domain-error.
Set A1 to the integer-array-nearest-to A.
If B is empty,

    Set W to +/((ρA1)ρ1 0)/A1.
    Return ((¯1↓ρB),W)ρ' '.

If ρA1 is not two, return ((2↑A1)⍕((¯1↓ρB),1)↑B),(2↓A1)⍕((-ρρB)↑1)↓B.
If B is not a scalar,

    If A1[1] is not a positive-integer, signal domain-error.
    Return a character array Z such that the shape-list of Z is (¯1↓ρB),A1[1] and the
      ravel-list of Z has the property that each vector-item along-axis ρρZ of Z is A1 ⍕ B0,
      where B0 is the corresponding (scalar) item of (¯1↓ρB)ρB.

If B is a scalar,

    If A1[2] is not less-than zero, set R to the
      fixed-decimal numeric-output-conversion of B to A1[2] places.
    Otherwise, set R to the decimal-exponential numeric-output-conversion of B to |A1[2]
      places.

    If A1[1] is less-than ρR, signal domain-error.
    Otherwise, return (-A1[1])↑R.
```

Examples:

In the following examples, **exponent-field-width** is 3.

```
      D ← ¯1 ¯0.1 0 0.1 1 ∘.× 5⍴0.5
      '|',(9 ¯3 7 ¯1 6 3 5 1 3 0 ⍕ D),'|'
|¯5.00E¯1  ¯5E¯1 ¯0.500 ¯0.5 ¯1|
|¯5.00E¯2  ¯5E¯2 ¯0.050 ¯0.1  0|
| 0.00E0    0E0   0.000  0.0  0|
| 5.00E¯2   5E¯2  0.050  0.1  0|
| 5.00E¯1   5E¯1  0.500  0.5  1|

      ⍴ 1 0 2 0 4 0 8 0⍕0 4⍴1
0 15
      31 0 ⍕ 2*100
1267650600228229401496703205376
      31 20 ⍕ .07
         0.07000000000000000666

      4 1 ⍕ ¯.99 ¯.89 0 7.5 11.5
¯1.0¯0.9 0.0 7.511.5
```

Note: *Like other axis lengths, the field width* A[1] *is limited only by* ***index-limit.***

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16 INPUT AND OUTPUT

16.1 INTRODUCTION

Note: *A user interacts with an APL system through a* ***session****, an abstraction that represents a hypothetical machine capable of carrying out the evaluation sequences in this standard.*

The protocol for the use of a session takes the form of a dialogue: the user makes an ***entry*** *and passes control to the APL system. The system processes the entry, produces a* ***response****, and returns control to the user.*

The user makes entries on a keyboard, and obtains responses by seeing them presented on a display-device. The combination of a keyboard and a display-device is intended to represent, abstractly, a terminal. This standard does not require the use of any particular terminal, keyboard, display or method of encoding ***characters****. The ISO 2022.2 APL character encoding reproduced in Appendix A is widely used.*

The display-device is considered a window into a ***presentation-space****, an unbounded sequence of past entries and responses. The mapping between the* ***presentation-space*** *and the display-device is not specified by this standard.*

The system obtains entries as ***tokens*** *of* ***class constant*** *or* ***interrupt*** *by calling the session operation* ***read-keyboard****. It presents responses by calling the session operation* ***display*** *with a* ***token*** *of class* ***result****. The transformations from entry to token and from token to response are not specified by this standard, but certain desirable characteristics of these transformations are given as comments.*

Because the dialogue protocol described here alternates between user and system, the session is always in one of two logical ***keyboard-states****,* ***open-keyboard*** *or* ***locked-keyboard****, depending upon whether the user is making an entry or the system is producing a response. The user can request a change in* ***keyboard-state****. from* ***locked-keyboard*** *to* ***open-keyboard*** *by using the* ***signal-attention*** *facility. In addition, the user can force a change in* ***keyboard-state*** *by signalling* ***interrupt****.*

16.2 DEFINITIONS

16.2.1 User Facilities

A user should have the following facilities.

Edit-Actions: A collection of **implementation-defined** facilities that permit the user to make entries.

Note: *Typical* ***edit-actions*** *include inserting, deleting, replacing and superimposing graphic symbols within the current entry and combining display-lines from the presentation-space with the current entry.*

Enter: A facility, available to the user when the terminal is in **open-keyboard** state, for changing the **keyboard-state** to **locked-keyboard** and returning to the caller of **read-keyboard** either an **interrupt** or a **character vector**. **Enter** sets the value of the session attribute **attention-flag** to **zero**.

Signal-Attention: A facility, available to the user when the terminal is in **locked-keyboard** state, for setting the value of the session attribute **attention-flag** to **one**.

Signal-Interrupt: A facility, available to the user at all times, for interrupting any evaluation sequence, causing it to return an **interrupt token** to the current caller of **evaluate-line**. Any **atomic** operation so interrupted has no effect on the state of the **active-workspace**.

Note: *Signal-interrupt can be issued in* ***open-keyboard*** *state and in* ***locked-keyboard*** *state, although the particular means of invoking the operation may be different for the two states. Transmitting the superimposition of the graphics for* *O* *U* *and* *T* *is a typical means of signalling* ***interrupt*** *in* ***open-keyboard*** *state.*

16.2.2 Implementation Algorithms

Read-Keyboard: An operation, available to the system when the session is in **locked-keyboard** state, that returns either an **interrupt token** or a **token** of **class constant** and **content** a **character vector**.

*Note: **Read-keyboard** should **display current-prompt**, then change the **keyboard-state** to **open-keyboard** to permit the user to use **edit-actions** to enter a list of **characters** or to signal an **interrupt**.*

*A concept long honoured in APL is that of visual fidelity: the graphic symbols that appear on the display as the result of **edit-actions** should correspond exactly to those returned to the system as the result of **read-keyboard**.*

Display T: An operation, available when the session is in **locked-keyboard** state, for presenting the **token** T on the **display-device**. T is either a **value** or an **error**.

*Note: The following is a suggested, but not required, evaluation sequence for **display** T:*

```
If quote-quad-prompt is not the empty character vector,

    Append to the presentation-space as a new last item a display-line whose graphic symbols represent the
      characters in quote-quad-prompt.
    Set quote-quad-prompt to the empty character vector.

If  T is an error, append to the presentation-space as a new last item an indication of

    The class of the error.
    The point in current-line at which evaluation was stopped.
    If the mode is defined-function, the function-name and current-line-number of the current-context.

If  T is a value,

    Let  A stand for the content of T.
    If  A is numeric, display ⍕A.
    Otherwise,
    If  A is a scalar or vector, append to the presentation-space as a new last item a display-line whose graphic
      symbols represent the characters in A.
    Otherwise,

        Set  I to zero.
        Set  M to ((×/¯1↓⍴A),¯1↑⍴A)⍴A.
        Repeat the following  1↑⍴M times:

            Set  I to I+1.
            Display row I of M.
            Append  (¯1+⍴⍴A)|+/∧\0=⌽(¯1↓⍴A)⊤I blank display-lines to the end of the
              presentation-space.

        (End of repeated block)
```

16.2.3 Prompts

Note: *The following implementation-defined arrays are used to indicate to the user the type of entry required.*

Indent-Prompt: An **implementation-defined character array**, typically a **vector** of six **blanks**.

Quad-Prompt: An **implementation-defined character array**, whose first row typically begins with `'⎕: '` and whose second row is the **indent-prompt**.

Function-Definition-Prompt: An **implementation-defined character array**, typically a **vector** beginning with `[nn]` where **nn** is a **line-number**.

16.3 DIAGRAMS

System-Command-Line

```
>>────── right─parenthesis ──>─┬─>──────────>─┬──────────>>
                               │              │
                               └─<─ any ──<───┘
```

Function-Definition-Line

```
>>────── del ──────────────────>─┬─>──────────>─┬──────────>>
                                 │              │
                                 └─<─ any ──<───┘
```

16.4 OPERATIONS

16.4.1 Immediate-Execution

Immediate-Execution with X.

Informal Description: **Immediate-execution** mode is the primal mode, the first evaluation sequence processed by the system to enter into a dialogue with a user. X is either an **error token** or **nil**.

The system also calls **immediate-execution** when an **error** or the **attention-flag** causes a defined function to be suspended. In this case, **immediate-execution** displays a status indication before prompting for input.

Evaluation Sequence:

```
Display $X$.
Repeat:

    Set current-prompt to indent-prompt.
    Set $E$ to read-keyboard.
    If $E$ is a constant,

        Let $Q$ stand for the content of $E$.
        If $Q$ matches system-command-line, set $T$ to evaluate-system-command $Q$.
        If $Q$ matches function-definition-line, set $T$ to evaluate-function-definition-request $Q$.
        Otherwise,

            Generate a new context in which

                mode is immediate-execution,
                current-line is $Q$,
                current-function is 0 0ρ'  ',
                current-line-number is one,
                current-statement is the empty list of tokens, and
                stack is the empty list of tokens.

            Append the new context to the state-indicator of the active-workspace as a new first
              item.
            Set $T$ to evaluate-line.
            Remove the first-item in the state-indicator of the active-workspace.

        If $T$ is a branch, a clear-state-indicator, or an escape and the state-indicator of the
          active-workspace is not an empty-list, return $T$.
        If $T$ is an error or a constant, display $T$.

    (End of repeated block)
```

16.4.2 Quad Input

Z ← ⎕

Informal Description: Z is an array provided by the user in response to a prompt.

Evaluation Sequence:

```
Repeat:

    Set current-prompt to quad-prompt.
    Set  E to read-keyboard.
    If  E is an interrupt, return E.
    Let  Q stand for the content of E.
    If some element of  Q is not blank,

        Generate a new context in which

            mode is quad-input,
            current-line is Q,
            current-function is 0  0ρ' ',
            current-line-number is one,
            current-statement is the empty list of tokens, and
            stack is the empty list of tokens.

        Append the new context to the state-indicator of the active-workspace as a new first
          item.
        Set  Z to evaluate-line.
        Remove the first-item in the state-indicator.
        If  Z is escape, signal unwind.
        If  Z is unwind or clear-state-indicator, return Z.
        If  Z is a value, return a token with class constant and content that of Z.
        If  Z is an error, display Z.
        Otherwise, display value-error. (see note)

(End of repeated block)
```

Note: The ***display value-error*** *line in the above evaluation sequence is introduced to permit certain* ***consistent-extensions****. In the evaluation sequence,* ***errors*** *are* ***displayed*** *rather than* ***signalled****, since signalling would terminate the Repeat loop and force Return.*

Quad-input returns only ***tokens*** *of class* ***escape*** *and* ***unwind*** *(from →),* ***clear-state-indicator*** *(from*)SIC *),* ***constant*** *(from* 2⋆.5*) and* ***interrupt****. It reports an error and reprompts for all other* ***results****. Note that it reprompts without an error report if the input was empty or all blank.*

16.4.3 Quote Quad Input

Z ← ⍞

Informal Description: Z is a **character vector**. Input in response to ⍞ is treated as a **character** value. Z is a vector whose length is the number of positions from the left margin up to the rightmost **character** of the input, including explicitly entered trailing blanks.

Evaluation Sequence:

```
Set current-prompt to quote-quad-prompt.
Set quote-quad-prompt to the empty character vector.
Set E to read-keyboard.
Return E.
```

Note: The behaviour required here differs from that of some existing systems, in which a single character response to ⍞ is returned as a scalar.

16.4.4 Quad Output

Z ← ⎕ ← B

Informal Description: Z is B. As a side effect, the array B is displayed on the terminal.

Evaluation Sequence:

```
Display B.
Return a committed-value with content B.
```

16.4.5 Quote Quad Output

Z ← ⍞ ← *B*

Informal Description: *Z* is *B*, and **quote-quad-prompt** is set to *B*.

Evaluation Sequence:

If any **item** of the **ravel-list** of *B* is not a **character**, signal **domain-error**.
If the **rank** of *B* is **greater-than one**, signal **rank-error**.
If the **count** of *B* is **greater-than quote-quad-output-limit**, signal **limit-error**.
If **quote-quad-prompt** is not empty, signal **limit-error**.
Set **quote-quad-prompt** to *B*.
Return a **committed-value** with **content** *B*.

Example:

```
     ∇ Z←PROMPT X
[1]    ⍞←X←,⍕X
[2]    Z←(⍴X)↓⍞
     ∇
        PROMPT '1 CLEANSPACE DATE? '
1 CLEANSPACE DATE? 1966-11-27
1966-11-27
```

Note: Quote-quad output is difficult to standardise because it is implemented in several different ways in existing systems, and, in each system, is heavily used. There is, however, sufficient commonality to permit the inclusion of this restricted form of quote-quad output. Quote-quad output on a ***conforming-implementation*** *should support the prompt function shown in the example.*

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17 SYSTEM COMMANDS

17.1 INTRODUCTION

Note: Any line of input in ***immediate-execution*** *whose first non-blank* ***character*** *is a right parenthesis is considered to be a system command.*

17.2 DEFINITIONS

Global-Referent of T: For T a **classified-name**, the **last-item** in the **referent-list** of the **symbol-named-by** T.

Global-Context: The last **context** in the **state-indicator** of the **active-workspace**.

Library-Workspace-Named A: For **workspace-identifier** A, the **item** of the **library** whose **owner** is **this-owner**, and whose **workspace-name** is A.

Already-Exists: A **workspace already-exists** if its **existential-property** is **present**.

Does-Not-Exist: A **workspace does-not-exist** if its **existential-property** is **absent**.

Note: The model of libraries used in this standard assumes that a ***workspace*** *object exists in the* ***library*** *for all possible* ***workspace-identifiers. Workspaces*** *that have been dropped or that have never been saved are distinguished from those that have been saved by the setting of their* ***existential-property***; *the* ***existential-property*** *of saved* ***workspaces*** *is* ***present***, *while that of unsaved* ***workspaces*** *is* ***absent***.

Attempt-to-Erase A: For A, a **simple-identifier**, an operation defined as follows:

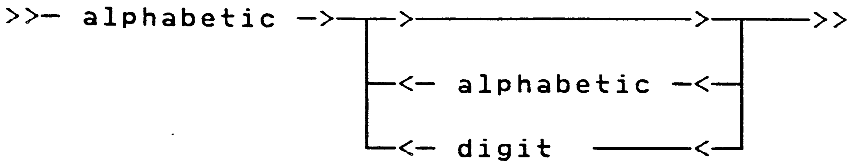
```
If   $A$ is globally-erasable,

    If the  current-class of $A$ is shared-variable, retract $A$.
    Set    $A$ to nil.
    Return  command-complete.

Otherwise, signal  not-erased.
```

Copy A **from** W: For **simple-identifier** A, and **workspace-identifier** W, an operation defined as follows:

```
Let $GA$ stand for the global-referent of $A$ in the active-workspace.
Let $GB$ stand for the global-referent of $A$ in the library-workspace-named $W$.
Set    $Z$ to attempt-to-erase $GA$.
If     $Z$ is an exception, signal not-copied.
If     $GB$ is a shared-variable, set $GA$ to the shared-value of $GB$.
Otherwise, set $GA$ to $GB$.
Return  command-complete.
```

17.3 DIAGRAMS

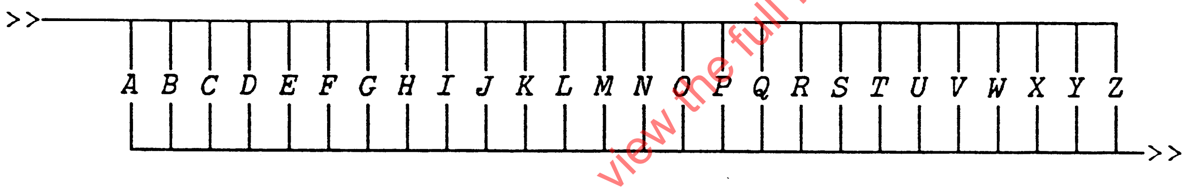
Workspace-Identifier



Example:

CLEANSPACE

Alphabetic



17.4 OPERATIONS

17.4.1 Evaluate-System-Command

Evaluate-System-Command Q.

Note: *The forms of system commands are not regular enough to permit much generalisation. Therefore, the forms themselves are used as the key to the evaluation sequences.*

Evaluation Sequence:

```
Find an entry that matches Q in the system-command evaluation sequences.
If no such entry is found, signal incorrect-command.
Otherwise, call the corresponding evaluation sequence.
Return the token it returns.
```

17.5 DIAGRAMS AND EVALUATION SEQUENCES

In the following,
p stands for **permitted-blanks,**
r stands for **required-blanks,**
w stands for **workspace-identifier**, and
a stands for **simple-identifier**.

Clear Active Workspace

```
>>—     )    — p —     CLEAR     — p —>>
```

```
Call  Shared-Variable-Reset.
Set the  active-workspace  to the clear-workspace.
Set  comparison-tolerance to initial-comparison-tolerance.
Set  index-origin to initial-index-origin.
Set  latent-expression to initial-latent-expression.
Set  print-precision to initial-print-precision.
Set  quote-quad-prompt to the empty character vector.
Set  random-link to initial-random-link.
Return  command-complete.
```

Copy Library Workspace Object

```
>>—     )    — p —     COPY   — r — w — r — a — p —>>
```

```
If the library-workspace-named w  does-not-exist, signal not-found.
If the  global-referent of a in w is not a defined-function, niladic-defined-function, variable, or
  shared-variable, signal incorrect-command.
Set    Z to copy a from w.
Return    Z.
```

Copy Library Workspace

```
>>—     )    — p —     COPY   — r — w — p —>>
```

```
If the library-workspace-named w does-not-exist, signal not-found.
For each  symbol S of the library-workspace-named    w

    Let    A stand for the name of S.
    If the  global-referent of  A in w is a defined-function, niladic-defined-function, variable, or
      shared-variable,

        Set    Z to copy A from w.
        If    Z is an exception, return Z.

Return  command-complete.
```