

# INTERNATIONAL STANDARD

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## **Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)**

*Plans d'échantillonnage progressif pour le contrôle par mesures des  
pourcentages de non conformes (écart-type connu)*



Reference number  
ISO 8423:1991(E)

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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 8423 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Sub-Committee SC 5, *Acceptance sampling*.

Annexes A, B and C form an integral part of this International Standard. Annex D is for information only.

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# Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)

## Section 1: General

### 1.1 Scope

1.1.1 This International Standard specifies sequential sampling plans and procedures for inspection by variables of discrete items. It is complementary to ISO 8422. The plans in the main body of the standard are indexed in terms of the producer's risk point and the consumer's risk point.

Annex A specifies sequential sampling plans and procedures indexed in terms of the acceptable quality level (AQL) to supplement the system of sampling plans in ISO 3951.

The purpose of this International Standard is to provide procedures based on a sequential assessment of inspection results, that may be used to induce the supplier through the economic and psychological pressure of non-acceptance of lots of inferior quality to supply lots of a quality having a high probability of acceptance. At the same time the consumer is protected by a prescribed upper limit to the probability of accepting lots of poor quality.

1.1.2 The sampling plans in this International Standard are primarily designed for use when all of the following conditions are satisfied:

- a) where the inspection procedure is to be applied to a *continuing series of lots* of discrete items all supplied by one producer using one production process. If there are different producers, this International Standard shall be applied to each one separately;
- b) where only a *single quality characteristic*  $x$  of these items is taken into consideration, which must be *measurable on a continuous scale*. If several such characteristics are of importance, this International Standard does not apply;

- c) where production is stable (under statistical control) and the quality characteristic  $x$  has a known standard deviation and is distributed according to a normal distribution or a close approximation to the normal distribution;

- d) where a contract or standard defines an *upper specification limit*  $U$ , a *lower specification limit*  $L$ , or both; an item is qualified as nonconforming when its measured quality characteristic  $x$  satisfies one of the following inequalities:

$$x > U \quad \dots (1.1)$$

$$x < L \quad \dots (1.2)$$

either

$$x > U$$

or

$$x < L \quad \dots (1.3)$$

Inequalities (1.1) and (1.2) are called cases with a *single specification limit*, and (1.3) a case with *double specification limits*. In this last situation a further distinction is made between *separate* or *combined double specification limits* according to whether the risks are considered for each limit separately or for both limits combined (see 2.3.3).

### 1.2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of ap-

plying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 2854:1976, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances.*

ISO 2859-1:1989, *Sampling procedures for inspection by attributes — Part 1: Sampling plans indexed by acceptable quality level (AQL) for lot-by-lot inspection.*

ISO 3534-1:—<sup>1)</sup>, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

ISO 3534-2:—<sup>1)</sup>, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control.*

ISO 3951:1989, *Sampling procedures and charts for inspection by variables for percent nonconforming.*

ISO 8422:1991, *Sequential sampling plans for inspection by attributes.*

## 1.3 Definitions and symbols

### 1.3.1 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534-1, ISO 3534-2 and ISO 3951, together with the following definitions, apply.

**1.3.1.1 separate double specification limits:** The term used when both upper and lower limits are specified and separate AQLs [or separate producer's and consumer's risks] apply to each limit. (See 2.3.3.)

NOTE 1 The words in brackets have been added to the definition given in ISO 3951 as a broader definition is required in this International Standard.

**1.3.1.2 combined double specification limits:** The term used when both upper and lower limits are specified and an AQL is given [or the producer's and consumer's risks] which applies to the combined percent nonconforming at the two limits. (See 2.3.3.)

**1.3.1.3 limiting process standard deviation for combined double specification limits [LPSD (com.):]** The upper limit to those values of the process standard deviation for which sequential sampling is applicable in the case of combined double specification limits. (See 2.4.3.1.)

**1.3.1.4 maximum process standard deviation for separate double specification limits [MPSD (sep.):]** The upper limit to those values of the process standard deviation for which acceptance sampling is applicable in the case of separate double specification limits. (See 2.4.3.2.)

**1.3.1.5 cumulative sample size ( $n_{cum}$ ):** When sampling inspection of items from a lot is performed sequentially, the total number of inspected items, counting from the start of the inspection up to, and including, the item last inspected.

**1.3.1.6 least assessable quality level (LAQ):** For a given sequential sampling plan, the process quality level which entails the largest average sample size.

**1.3.1.7 leeway ( $\nu$ ):** A quantity derived from the measured value on an item. In the case of a single lower specification limit and in the case of double specification limits, the leeway is obtained by subtracting the numerical value of the lower specification limit from the measured value. In the case of an upper specification limit, the leeway is obtained by subtracting the measured value from the numerical value of the upper specification limit.

**1.3.1.8 cumulative leeway ( $Y$ ):** When sampling inspection of items from a lot is performed sequentially, the value calculated by summing the leeways obtained from the start of the inspection up to, and including, that of the item last inspected.

**1.3.1.9 acceptance value for sequential sampling ( $A$ ):** A value derived from the specified parameters of the sampling plan and the cumulative sample size. Whether the lot may yet be accepted is determined by comparing the cumulative leeway with the acceptance value.

**1.3.1.10 rejection value for sequential sampling ( $R$ ):** A value derived from the specified parameters of the sampling plan and the cumulative sample size. Whether the lot may yet be considered unacceptable is determined by comparing the cumulative leeway with the rejection value.

### 1.3.2 Symbols

The symbols used in this International Standard are as follows:

$A$	Acceptance value for sequential sampling.
$A_t$	Acceptance value corresponding to the curtailed value of the cumulative sample size.

1) To be published.

CRQ	Consumer's risk quality level (in percent nonconforming).	$p_A$	Producer's risk quality level (in proportion nonconforming). $P_a = 1 - \alpha$ when $p = p_A$ .
$f$	In the case of separate double specification limits, a coefficient, which in conjunction with $\sigma$ and $(U - L)$ determines the applicability of acceptance sampling. (See 2.4.3.2.)	$p_g$	Least assessable quality level (in proportion nonconforming). $p_g = 1 - F(g)$ .
$F$	Standardized normal distribution function.	$p_R$	Consumer's risk quality level (in proportion nonconforming). $P_a = \beta$ when $p = p_R$ .
$g$	Multiplier of the cumulative sample size that is used to determine the acceptance values and the rejection values (slope of the acceptance and rejection lines).	$P_a$	The probability of acceptance.
$h_A$	Constant that is used to determine the acceptance values (intercept of the acceptance line).	PRQ	Producer's risk quality level (in percent nonconforming).
$h_R$	Constant that is used to determine the rejection values (intercept of the rejection line).	$R$	Rejection value for sequential sampling.
$L$	Lower specification limit (as a superscript to a parameter or a variable, denotes its value for $L$ ).	$U$	Upper specification limit (as a superscript to a parameter or a variable, denotes its value for $U$ ).
LAQ	Least assessable quality level (in percent nonconforming).	$x$	Measured value of the characteristic on an item of product.
LPSD (com.)	Limiting process standard deviation for combined double specification limits.	$y$	Leeway, defined as $y = U - x$ for a single upper specification limit $y = x - L$ for a single lower specification limit, and $y = x - L$ for double specification limits
MPSD (sep.)	Maximum process standard deviation for separate double specification limits.	$\gamma$	Cumulative leeway obtained by adding the leeways up to, and including, the item last inspected.
$n_0$	Sample size for a single sampling plan corresponding to the sequential sampling plan.	$z_p$	The $P$ -fractile of the standardized normal distribution: $z_{0,05} = -1,644\ 9$ since $F(-1,644\ 9) = 0,05$ $z_{0,10} = -1,281\ 6$ since $F(-1,281\ 6) = 0,10$
$n_{av}$	Average sample size.		
$n_{cum}$	Cumulative sample size.		
$n_t$	Curtailment value of the cumulative sample size.		
$p$	Process quality level (in proportion nonconforming).		
	NOTE 2 To convert $p$ to percent nonconforming, multiply by 100.		

$\alpha$	The producer's risk. <sup>2)</sup>
$\beta$	The consumer's risk. <sup>2)</sup>
$\sigma$	Standard deviation of $x$ in the process. ( $\sigma^2$ , the square of the standard deviation, is known as the variance.)
$\psi$	In the case of combined double specification limits, a coefficient which in conjunction with $\sigma$ and $(U - L)$ determines the applicability of sequential sampling. (See 2.4.3.1.)
$\lambda$	Index parameter that is used to determine approximations to the OC curve at general quality levels. (See C.2.2.)

#### 1.4 Principle of sequential sampling by variables

Under a sequential sampling plan by variables, items are selected at random and subjected to inspection one by one. After the inspection of each individual item, the cumulative leeway is calculated and used to assess whether there is sufficient information to sentence the lot at that stage of the inspection.

If, at a given stage the cumulative leeway is such that the risk of accepting a lot of unsatisfactory quality (the consumer's risk) is sufficiently low, the

lot is considered acceptable and sampling of that lot is terminated.

If, on the other hand, the cumulative leeway is such that the risk of non-acceptance for a lot of satisfactory quality (the producer's risk) is sufficiently low, the lot shall be considered not acceptable, and sampling of that lot is terminated.

If the cumulative leeway does not allow either of the above decisions to be taken, then an additional item is inspected. The process is continued until sufficient sample information has been accumulated to warrant a decision that the lot is acceptable or not acceptable.

NOTE 3 In the case of separate double specification limits, the assessment of the cumulative leeway is performed for each limit separately. If, at a given stage, the risk of judging a lot of satisfactory quality to be unacceptable is sufficiently low for one or other of the limits, inspection terminates and the lot is not accepted. Alternatively, if at a given stage the risk of judging a lot of unsatisfactory quality to be acceptable is sufficiently low for one of the limits, then the lot is considered to be acceptable with respect to that limit and inspection for that limit is terminated; inspection is continued until

- satisfactory results are obtained also for the other limit, in which case the lot is considered to be acceptable, or
- inspection for the other limit leads to non-acceptance of the lot.

2)  $\alpha$  and  $\beta$  may be considered to be the type I and type II risks, respectively, when testing the null hypothesis

$$H_0: p = p_A$$

against the alternative hypothesis

$$H_1: p = p_R$$

## Section 2: Choice of sampling plan

### 2.1 Factors determining the choice of a sequential sampling plan by variables

#### 2.1.1 Choice between variables and attributes

**2.1.1.1** The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

- a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items of product by an attributes plan with the generally more elaborate procedure required by a variables plan, which is usually more expensive in time and money per item.
- b) A variables plan can be less readily understood than an attributes plan; for example, it may at first be difficult to accept that, when inspecting by variables, a decision not to accept a lot may be based on measurements taken on a sample that does not contain any nonconforming items.
- c) A comparison of the size of the samples required for sampling plans for inspection by attributes and equivalent sampling plans for inspection by variables shows that a variables sampling plan requires a smaller sample size to give the same producer's and consumer's risks than an attributes sampling plan. A variables sampling plan therefore has a substantial advantage when the inspection process is expensive, for example in the case of destructive testing.
- d) The sampling plans in this International Standard only apply to the case of a single quality characteristic. If conformance to specification is to be assessed in terms of more than one quality characteristic, the standard shall be applied separately to each of these. In such a situation it is recommended that all the quality characteristics be treated as attributes and that attributes sampling plans from ISO 2859-1 or ISO 8422 be used.

**2.1.1.2** The sequential sampling plans in this International Standard may only be used when there is reason to believe that the distribution of measurements is normal and when there is valid evidence that the standard deviation of the process can be considered constant and taken to be  $\sigma$ .

If inspection is carried out on a continuing series of lots, the hypothesis of normality can be verified us-

ing results previously obtained using a single sampling plan, and the stability of the standard deviation may emerge from a control chart measuring process variability. If it appears that the standard deviation is in control, the (weighted) root mean square value of observed standard deviations may be presumed to be  $\sigma$ , the "known" standard deviation of the process. In order to verify that the variability remains under control, the standard deviation in subsequent samples should still be calculated.

If inspection is carried out on an isolated lot, there will be no evidence about the standard deviation of the process, and therefore this International Standard does not apply to the inspection of isolated lots.

#### NOTES

4 Tests for departure from normality are dealt with in section two of ISO 2854:1976, which provides examples of graphical methods which can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

5 A more comprehensive treatment of tests of normality is given in ISO 5479.

6 If  $k$  samples have given the estimates  $s_1, s_2, \dots, s_k$  of  $\sigma$  then the weighted root mean square estimate  $s$  is determined as

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \dots + (n_k - 1)s_k^2}{n_1 - 1 + n_2 - 1 + \dots + n_k - 1}}$$

where  $n_1, n_2, \dots, n_k$  denote the sizes of the  $k$  samples.

#### 2.1.2 Choice between sequential and single sampling plans

The average sample size is the average of the various sample sizes which may occur under a sampling plan for a given level of the process quality. The use of sequential sampling plans leads to a smaller average sample size than single sampling plans having the same operating characteristic. For good or very poor quality lots, the savings over the corresponding single sampling plans may reach, or even exceed, 50 %.

On the other hand, the actual number of items inspected for a particular lot when using a sequential sampling plan may considerably exceed that of the corresponding single sampling plan. For the sequential sampling plans in this International Standard, a curtailment rule (see 2.1.4) has been introduced in order to limit the potential number of inspected items. Annex C gives a method for determining approximate values of the average sample size.

As the ultimate sample size from a particular lot is not known in advance, the selection of the sample may present organizational difficulties when sequential sampling plans are used. Moreover, the scheduling of inspection operations may present difficulties when using a sequential sampling plan. A further disadvantage is that the execution of a sequential sampling plan is more easily misunderstood by the inspectors than the simpler rules for single sampling.

When the inspection procedure is applied to the case of separate double specification limits on a single quality characteristic, it may happen that inspection concerning one of the limits terminates favourably long before enough information has been accumulated to warrant a decision regarding the other limit. Therefore sampling has to continue for some time before an overall decision can be taken.

The balance between the advantages of a smaller average sample size and the organizational disadvantages associated with a fluctuating inspection load results in sequential sampling by variables being suitable when only a single quality characteristic is considered and when the inspection of individual items is costly in comparison with inspection overheads.

### 2.1.3 Caution

The choice between single and sequential sampling plans shall be made before the inspection of a lot is started. During the inspection of a lot, it is not permitted to switch between one type of sampling plan to another as the operating characteristic of the plan may be drastically changed if the actual inspection results influence the choice of acceptance criterion.

### 2.1.4 Curtailment of the sample size

Although a sequential sampling plan is on average much more economical than the equivalent single sampling plan, it may occur, during the inspection of a particular lot, that acceptance or non-acceptance comes at a very late stage because the cumulative leeway remains between the acceptance value and the rejection value for a long time. With the graphical method this corresponds to the random progress of the step curve remaining in the indecision zone. Such a situation is most likely to occur when the quality of the lot is close to LAQ.

In order to alleviate this disadvantage, a maximum cumulative sample size  $n_4$  is set before sampling begins, and inspection is stopped if the cumulative sample size reaches the curtailment value,  $n_4$ , with-

out a decision having been made. The acceptance or non-acceptance of the lot is then determined in accordance with a rule which is also agreed in advance of sampling. The curtailment rules of this International Standard have been determined in such a way that the producer's and consumer's risks are hardly affected by this deviation from the principles underlying the statistical theory of sequential sampling inspection. The curtailment rules to be used are given in 2.4.2.

## 2.2 Particular reservations on the inspection of small lots

The statistical theory underlying the sequential sampling plans in this International Standard is based on the assumption that the samples are taken from an infinitely large population. When sampling is carried out without replacement from a lot, the theory remains valid for all practical purposes if the cumulative sample size does not exceed one-tenth of the lot size  $N$ ; the theory remains approximately valid even for cumulative sample sizes up to one-seventh of  $N$ . Unfortunately, in contrast to the situation for single sampling plans, the actual cumulative sample size that is necessary in a sequential sampling plan will not be known in advance.

In the case of a small lot it is therefore advisable to ensure that the size of the lot is sufficiently large to allow a curtailed sequential sampling plan to operate under sampling without replacement, in accordance with the specified producer's and consumer's risks. For the general sequential sampling plans described in 2.3.2 and 2.4.1, it is therefore recommended that the lot size exceed  $7n_4$ , where  $n_4$  is the curtailment value of the sequential sampling plan.

NOTE 7 If the lot size is not sufficiently large to satisfy the above requirement, both the consumer's and the producer's risks will generally become less than their specified values.

## 2.3 Selection of a sampling plan

### 2.3.1 Plans matching those of ISO 3951

If it is required to find a sequential sampling plan matching a " $\sigma$ " method plan from ISO 3951:1989, then annex A may be used. Annex A contains sequential sampling plans indexed by acceptable quality level (AQL) and sample size code letter. The operating characteristic curves of these plans match, as closely as practicable, those of the corresponding " $\sigma$ " method plans in ISO 3951.

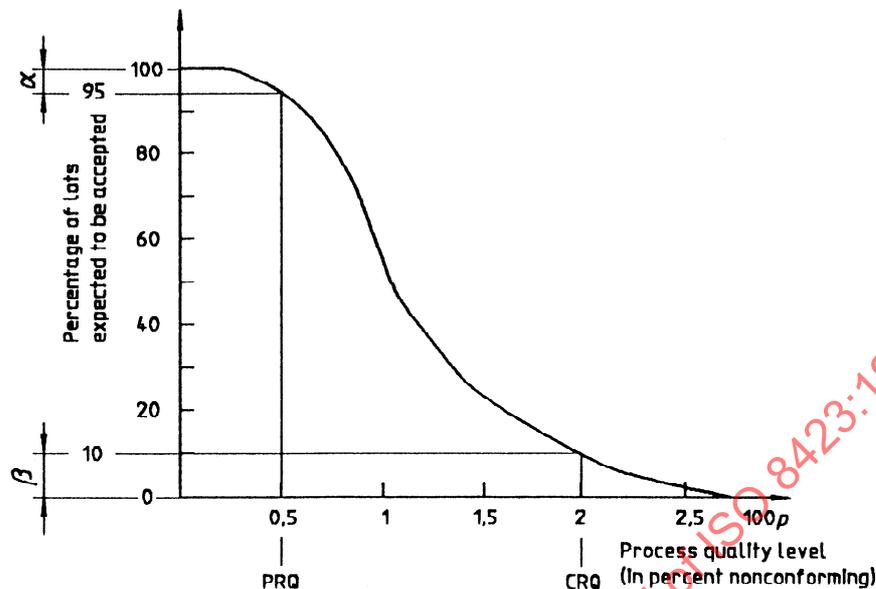


Figure 1 — Operating characteristic curve for a sampling plan with producer's risk  $\alpha = 0,05$  and consumer's risk  $\beta = 0,10$

### 2.3.2 General plans

The general method described in 2.3.2 and 2.4.1 is used when the requirements of the sequential sampling plan are specified in terms of two points on the operating characteristic curve of the plan. The point corresponding to the higher probability of acceptance shall be designated the *producer's risk point*; the other shall be designated the *consumer's risk point*.

The first step when designing a sequential sampling plan by the general method is to choose the producer's and consumer's risk points, if they have not already been dictated by circumstances. For this purpose, a producer's risk of  $\alpha = 0,05$  and a consumer's risk of  $\beta = 0,10$  are often used. (See figure 1.)

When the desired sequential sampling plan is required to have approximately the same operating characteristic as an existing sampling plan, the producer's risk point and the consumer's risk point may be read off from a graph or a table of the operating characteristic of that plan. When no such plan exists, the producer's and consumer's risk points have to be determined from direct considerations of the conditions under which the sampling plan will operate.

### 2.3.3 Specifying quality levels

When only a single specification limit is considered, the proportion nonconforming in incoming product relates to that limit.

When both upper and lower specification limits are given, it may be appropriate to consider quality levels specified separately for each limit, in which case the limits are known as "separate double specification limits". Alternatively, an overall quality level may be specified for the proportion nonconforming for the combined process at both the upper and lower limits, in which case the limits are known as "combined double specification limits".

## 2.4 Pre-operation preparations

### 2.4.1 Obtaining the parameters $h_A$ , $h_R$ and $g$

The criteria for acceptance and for non-acceptance of a lot that are invoked at each stage of the inspection are determined from the parameters  $h_A$ ,  $h_R$  and  $g$ .

The values of these parameters corresponding to a producer's risk of  $\alpha = 0,05$ , a consumer's risk of  $\beta = 0,10$  and preferred values of the producer's and consumer's risk qualities are given in table 1.

Annex B gives general procedures for determining  $h_A$ ,  $h_R$  and  $g$  for any combination of producer's and consumer's risk points.

In the case of a single specification limit or combined double specification limits, only one set of parameters  $h_A$ ,  $h_R$  and  $g$  shall be determined.

With separate double specification limits, two sets of parameters shall be determined:

$h_A^{(U)}$ ,  $h_R^{(U)}$  and  $g^{(U)}$ , corresponding to the upper limit, and

$h_A^{(L)}$ ,  $h_R^{(L)}$  and  $g^{(L)}$ , corresponding to the lower limit.

**2.4.2 Determination of the curtailment value of the sample size**

**2.4.2.1 Standard procedures**

a) If the sample size  $n_0$  of the single sampling plan that is equivalent to the sequential sampling plan under consideration is known, the curtailment value for the cumulative sample size is determined as  $n_t = 1,50n_0$ , rounded up to the nearest integer.

b) If the sample size of the equivalent single sampling plan is not known, the curtailment value is determined from the producer's and consumer's risk points.

The curtailment values corresponding to a producer's risk of  $\alpha = 0,05$ , a consumer's risk of  $\beta = 0,10$  and preferred values of the producer's and consumer's risk qualities are given in table 1.

Annex B gives general procedures for determining  $n_t$  for any combination of producer's and consumer's risk points.

NOTE 8 In the case of separate double specification limits, the curtailment value is determined for each of the limits separately according to the above rule. The curtailment value to be used for the curtailed sequential sampling plan is taken as the larger of the two values obtained.

**2.4.2.2 Truncation for small lots**

If the resulting value of  $n_t$  exceeds the lot size, then the sequential sampling plan shall be used with the curtailment value  $n_t$  of the sample size equal to the lot size.

**2.4.2.3 Example**

The specified minimum withstand voltage for certain insulators is 200 kV. Lots from a steady production are submitted for inspection. Production is stable and it has been verified that the withstand voltages vary within a lot in accordance with a normal distribution. It has further been documented that the standard deviation within a lot is stable and can be taken to be  $\sigma = 1,2$  kV.

It has been decided to use a sequential sampling plan with the following properties:

a) If submitted quality is 0,5 % nonconforming, then the probability of acceptance shall be 0,95.

b) If submitted quality is 2,0 % nonconforming, then the probability of acceptance shall be 0,10.

These requirements are achieved by fixing the producer's risk point at  $p_A = 0,005$ ,  $\alpha = 0,05$  and the consumer's risk point at  $p_R = 0,02$ ,  $\beta = 0,10$ .

The specification refers to a single, lower limit. From table 1 it is seen that the required sequential sampling plan has the parameters

$$h_A = 4,312$$

$$h_R = 5,536$$

$$g = 2,315$$

$$n_t = 49$$

The same values would have resulted from using the general procedure given in annex B.

**2.4.3 Maximum values of process standard deviation**

**2.4.3.1 Limiting process standard deviation for combined double specification limits, LPSD (com.)**

In the case of combined double specification limits, sequential sampling is only applicable if the process standard deviation  $\sigma$  is sufficiently small in relation to the specification interval  $(U - L)$ . The limiting value of the process standard deviation is given by

$$LPSD = (U - L)\psi$$

where  $\psi$  depends only on PRQ, the producer's risk quality.

Table 2 gives values of  $\psi$  corresponding to preferred values of PRQ.

**2.4.3.2 Maximum process standard deviation for separate double specification limits, MPSD (sep.)**

In the case of separate double specification limits, sampling by variables is only applicable if the process standard deviation  $\sigma$  is sufficiently small in relation to the specification interval  $(U - L)$ . The maximum tolerable value of the process standard deviation is given by

$$MPSD = (U - L)f$$

where  $f$  depends on  $PRQ^{(U)}$  and  $PRQ^{(L)}$ , the producer's risk qualities for the upper and the lower specification limit, respectively.

Table 3 gives values of  $f$  corresponding to preferred values of  $PRQ^{(U)}$  and  $PRQ^{(L)}$ .

If, in the case of separate double specification limits,  $\sigma$  exceeds MPSD, the lot shall immediately be judged not acceptable without a sample being drawn.

NOTE 9 When  $\sigma$  exceeds MPSD, no lot can satisfy both requirements

$$p^{(U)} < PRQ^{(U)}$$

and

$$p^{(L)} < PRQ^{(L)}$$

and therefore acceptance sampling is pointless.

#### 2.4.4 Choosing the form of the sampling plan

This International Standard gives two methods: a numerical method and a graphical method.

The numerical method has the advantage of being accurate, thereby avoiding disputes about acceptance or non-acceptance.

The graphical method is well suited to the inspection of series of lots, as the chart need only to be drawn once, but the method is less accurate due to the inaccuracy inherent in plotting points and in drawing straight lines. On the other hand, the method does have the advantage of displaying the increase in the information on the quality of the lot as additional items are inspected, information being represented by the progress of a broken line within the indecision zone until the line reaches, or crosses, one of the boundaries of that zone.

The numerical method is the standard method. The graphical method may be used only with the proviso that "close" decisions to accept or not to accept a lot shall always be resolved by the numerical method.

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**Table 1 — Sequential sampling plans by variables for percent nonconforming for producer's risk  $\alpha = 0,05$  and consumer's risk  $\beta = 0,10$**

PRQ and CRQ levels in percent nonconforming

PRQ	Par- am- eters	Consumer's risk quality level CRQ																
		0,80	1,00	1,25	1,60	2,00	2,50	3,15	4,00	5,00	6,30	8,00	10,00	12,50	16,00	20,00	25,00	31,50
0,100	$h_A$	3,304	2,947	2,652	2,380	2,172	1,992	1,829	1,681	1,558	1,443	1,336	1,245	1,161	1,074	1,001	0,932	0,863
	$h_R$	4,242	3,784	3,405	3,056	2,789	2,557	2,348	2,158	2,000	1,853	1,715	1,598	1,490	1,379	1,285	1,196	1,108
	$g$	2,750	2,708	2,666	2,617	2,572	2,525	2,475	2,420	2,368	2,310	2,248	2,186	2,120	2,042	1,966	1,882	1,786
	$n_t$	29	23	19	16	13	11	10	8	8	7	7	5	5	4	4	4	4
0,125	$h_A$	3,664	3,230	2,879	2,561	2,322	2,117	1,934	1,769	1,633	1,508	1,391	1,293	1,202	1,110	1,032	0,958	0,886
	$h_R$	4,704	4,147	3,696	3,288	2,981	2,718	2,483	2,271	2,097	1,936	1,786	1,659	1,543	1,425	1,325	1,231	1,137
	$g$	2,716	2,675	2,632	2,584	2,539	2,492	2,441	2,387	2,334	2,277	2,214	2,152	2,087	2,009	1,932	1,849	1,753
	$n_t$	35	28	23	19	16	13	11	10	8	7	7	5	5	5	4	4	4
0,160	$h_A$	4,177	3,622	3,187	2,802	2,518	2,279	2,068	1,881	1,728	1,588	1,459	1,351	1,252	1,153	1,069	0,990	0,913
	$h_R$	5,363	4,651	4,091	3,597	3,233	2,926	2,655	2,414	2,218	2,039	1,873	1,735	1,608	1,480	1,372	1,271	1,172
	$g$	2,678	2,637	2,595	2,546	2,501	2,454	2,404	2,349	2,296	2,239	2,176	2,115	2,049	1,971	1,895	1,811	1,715
	$n_t$	46	35	28	22	17	14	13	10	10	8	7	7	5	5	4	4	4
0,200	$h_A$	4,798	4,080	3,536	3,068	2,731	2,452	2,209	1,997	1,825	1,670	1,528	1,410	1,303	1,195	1,105	1,022	0,939
	$h_R$	6,160	5,238	4,539	3,939	3,506	3,148	2,837	2,564	2,344	2,144	1,962	1,810	1,673	1,534	1,419	1,312	1,206
	$g$	2,644	2,602	2,560	2,511	2,466	2,419	2,369	2,314	2,262	2,204	2,142	2,080	2,014	1,936	1,860	1,776	1,680
	$n_t$	59	44	34	25	20	17	14	11	10	8	7	7	5	5	5	4	4
0,250	$h_A$	5,655	4,683	3,980	3,398	2,989	2,658	2,375	2,131	1,937	1,763	1,606	1,476	1,359	1,242	1,145	1,056	0,968
	$h_R$	7,260	6,013	5,110	4,362	3,837	3,412	3,049	2,736	2,487	2,263	2,062	1,895	1,745	1,595	1,471	1,355	1,243
	$g$	2,608	2,567	2,524	2,476	2,430	2,384	2,333	2,279	2,226	2,169	2,106	2,044	1,979	1,901	1,824	1,741	1,644
	$n_t$	83	58	41	31	25	19	16	13	11	10	8	7	7	5	5	4	4
0,315	$h_A$	6,974	5,553	4,591	3,833	3,320	2,917	2,580	2,295	2,071	1,873	1,697	1,552	1,424	1,296	1,191	1,094	1,001
	$h_R$	8,953	7,130	5,895	4,921	4,263	3,745	3,313	2,946	2,659	2,405	2,179	1,993	1,828	1,664	1,529	1,405	1,285
	$g$	2,570	2,529	2,487	2,438	2,393	2,346	2,295	2,241	2,188	2,131	2,068	2,007	1,941	1,863	1,787	1,703	1,607
	$n_t$	125	80	55	38	29	23	19	14	13	10	8	8	7	5	5	5	4
0,40	$h_A$	9,259	6,912	5,482	4,435	3,763	3,253	2,839	2,498	2,235	2,006	1,805	1,643	1,499	1,358	1,244	1,138	1,037
	$h_R$	11,887	8,874	7,038	5,694	4,831	4,176	3,645	3,207	2,870	2,576	2,318	2,109	1,925	1,744	1,596	1,462	1,332
	$g$	2,530	2,489	2,447	2,398	2,353	2,306	2,256	2,201	2,148	2,091	2,029	1,967	1,901	1,823	1,747	1,663	1,567
	$n_t$	218	122	77	52	37	28	22	17	14	11	10	8	7	7	5	5	4
0,50	$h_A$	13,488	9,024	6,732	5,218	4,312	3,656	3,141	2,728	2,418	2,153	1,923	1,739	1,579	1,424	1,298	1,184	1,075
	$h_R$	17,317	11,586	8,643	6,700	5,536	4,693	4,033	3,503	3,105	2,764	2,469	2,233	2,028	1,828	1,667	1,520	1,380
	$g$	2,492	2,451	2,409	2,360	2,315	2,268	2,218	2,163	2,110	2,053	1,990	1,929	1,863	1,785	1,709	1,625	1,529
	$n_t$	463	208	116	71	49	35	26	20	16	13	11	10	8	7	5	5	4
0,63	$h_A$	26,190	13,358	8,882	6,424	5,103	4,209	3,542	3,025	2,649	2,333	2,066	1,855	1,674	1,500	1,362	1,237	1,118
	$h_R$	33,625	17,150	11,403	8,247	6,552	5,403	4,547	3,884	3,400	2,996	2,652	2,382	2,150	1,926	1,748	1,588	1,436
	$g$	2,452	2,411	2,368	2,320	2,274	2,227	2,177	2,123	2,070	2,012	1,950	1,888	1,823	1,745	1,668	1,585	1,488
	$n_t$	1,739	454	202	106	68	46	34	25	19	16	13	10	8	7	7	5	5
0,80	$h_A$		27,265	13,440	8,511	6,339	5,015	4,095	3,420	2,946	2,562	2,243	1,997	1,789	1,592	1,436	1,298	1,168
	$h_R$		35,005	17,255	10,927	8,138	6,438	5,258	4,391	3,783	3,289	2,879	2,564	2,297	2,043	1,844	1,666	1,500
	$g$		2,368	2,325	2,277	2,231	2,184	2,134	2,080	2,027	1,969	1,907	1,845	1,780	1,702	1,625	1,542	1,445
	$n_t$		1,886	460	185	103	65	44	31	23	19	14	11	10	8	7	5	5
1,00	$h_A$			26,505	12,374	8,259	6,145	4,819	3,911	3,303	2,827	2,444	2,155	1,914	1,690	1,516	1,363	1,220
	$h_R$			34,028	15,886	10,603	7,889	6,187	5,021	4,241	3,630	3,137	2,766	2,458	2,170	1,947	1,750	1,567
	$g$			2,284	2,235	2,190	2,143	2,093	2,039	1,986	1,928	1,866	1,804	1,738	1,660	1,584	1,500	1,404
	$n_t$			1,781	389	175	97	61	40	29	22	17	13	11	8	7	7	5

Table 1 — (concluded)

PRQ	Param- eters	Consumer's risk quality level CRQ													
		1,60	2,00	2,50	3,15	4,00	5,00	6,30	8,00	10,00	12,50	16,00	20,00	25,00	31,50
1,25	$h_A$	23,209	11,997	7,999	5,890	4,588	3,774	3,165	2,692	2,345	2,063	1,805	1,608	1,437	1,279
	$h_R$	29,798	15,402	10,270	7,562	5,890	4,845	4,063	3,456	3,011	2,649	2,318	2,065	1,845	1,643
	$g$	2,193	2,148	2,101	2,050	1,196	1,943	1,886	1,823	1,761	1,696	1,618	1,542	1,458	1,362
	$n_t$	1 367	367	164	89	55	38	26	20	16	13	10	8	7	5
1,60	$h_A$		24,832	12,206	7,893	5,718	4,507	3,665	3,045	2,609	2,265	1,958	1,728	1,532	1,354
	$h_R$		31,881	15,671	10,134	7,341	5,786	4,705	3,909	3,350	2,908	2,513	2,219	1,966	1,738
	$g$		2,099	2,052	2,002	1,948	1,895	1,837	1,775	1,713	1,647	1,569	1,493	1,409	1,313
	$n_t$		1 564	379	160	85	53	35	25	19	14	11	10	7	7
2,00	$h_A$			24,006	11,572	7,429	5,506	4,299	3,471	2,915	2,492	2,125	1,857	1,632	1,432
	$h_R$			30,821	14,857	9,537	7,069	5,519	4,456	3,743	3,199	2,729	2,385	2,096	1,839
	$g$			2,007	1,956	1,902	1,849	1,792	1,729	1,668	1,602	1,524	1,448	1,364	1,268
	$n_t$			1 462	341	142	79	49	32	23	17	13	10	8	7
2,50	$h_A$				22,341	10,757	7,144	5,237	4,057	3,318	2,781	2,332	2,013	1,751	1,523
	$h_R$				28,683	13,811	9,173	6,723	5,209	4,260	3,570	2,994	2,585	2,248	1,955
	$g$				1,910	1,855	1,802	1,745	1,683	1,621	1,555	1,477	1,401	1,317	1,221
	$n_t$				1 267	295	131	71	43	29	22	16	11	10	7
3,15	$h_A$					20,747	10,503	6,840	4,957	3,897	3,176	2,603	2,212	1,900	1,634
	$h_R$					26,637	13,485	8,782	6,365	5,004	4,078	3,342	2,840	2,440	2,098
	$g$					1,805	1,752	1,695	1,632	1,570	1,505	1,427	1,350	1,267	1,170
	$n_t$					1 093	281	121	64	40	28	19	14	11	8
4,00	$h_A$						21,273	10,204	6,514	4,799	3,750	2,977	2,476	2,092	1,774
	$h_R$						27,311	13,101	8,363	6,161	4,815	3,822	3,179	2,686	2,278
	$g$						1,698	1,640	1,578	1,516	1,451	1,373	1,296	1,213	1,116
	$n_t$						1 148	265	109	59	37	23	17	13	10
5,00	$h_A$							19,612	9,389	6,197	4,553	3,461	2,803	2,320	1,936
	$h_R$							25,180	12,054	7,956	5,845	4,444	3,598	2,979	2,485
	$g$							1,587	1,525	1,463	1,398	1,320	1,243	1,160	1,063
	$n_t$							976	224	98	55	32	22	16	11
6,30	$h_A$								18,010	9,059	5,929	4,203	3,270	2,631	2,147
	$h_R$								23,123	11,630	7,612	5,396	4,198	3,378	2,757
	$g$								1,468	1,406	1,340	1,262	1,186	1,102	1,006
	$n_t$								824	209	91	46	29	19	13
8,0	$h_A$									18,226	8,838	5,483	3,996	3,082	2,438
	$h_R$									23,400	11,347	7,039	5,130	3,956	3,130
	$g$									1,343	1,278	1,200	1,123	1,040	0,993 4
	$n_t$									844	199	77	41	26	17
10,0	$h_A$										17,159	7,842	5,117	3,709	2,815
	$h_R$										22,030	10,068	6,570	4,761	3,614
	$g$										1,216	1,138	1,062	0,978 0	0,881 6
	$n_t$										748	157	68	37	22

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**Table 2 — Values of  $\psi$  for limiting process standard deviation LPSD  
(combined double specification limits)**

PRQ (%)	0,10	0,125	0,16	0,20	0,25	0,315	0,40	0,50	0,63	0,80	1,00	1,25	1,60	2,00	2,50	3,15	4,00	5,00	6,30	8,00	10,00
$\psi$	0,143	0,146	0,149	0,152	0,155	0,158	0,161	0,165	0,169	0,174	0,178	0,183	0,189	0,194	0,201	0,208	0,216	0,225	0,235	0,246	0,259

NOTE — The limiting process standard deviation for sequential sampling, LPSD, is obtained by multiplying the standardized value,  $\psi$ , by the difference between the upper specification limit,  $U$ , and the lower specification limit,  $L$ , i.e.  $LPSD = (U - L)\psi$ .

The limiting process standard deviation, LPSD, indicates the greatest allowable magnitude of the process standard deviation when using sequential sampling plans for the combined double specification limit case. If the process standard deviation exceeds LPSD, no sequential sampling plan is available, but if  $\sigma$  is not too large, an appropriate single sampling plan may be found in ISO 3951.

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Table 3 — Values of  $f$  for maximum process standard deviation MPSD  
(separate double specification limits)

PRQ <sup>(L)</sup>	PRQ <sup>(U)</sup> and PRQ <sup>(U)</sup> in percent nonconforming														
	1,00	1,125	1,25	1,40	1,56	1,75	2,00	2,50	3,15	4,00	5,00	6,30	8,00	10,00	
0,100	0,162	0,164	0,166	0,168	0,170	0,172	0,174	0,176	0,178	0,180	0,182	0,184	0,186	0,188	
0,125	0,164	0,165	0,167	0,169	0,172	0,174	0,176	0,179	0,181	0,184	0,187	0,190	0,193	0,196	
0,160	0,166	0,167	0,170	0,172	0,174	0,176	0,179	0,181	0,183	0,186	0,189	0,192	0,195	0,198	
0,200	0,168	0,170	0,172	0,174	0,176	0,178	0,181	0,183	0,186	0,188	0,191	0,194	0,197	0,200	
0,250	0,170	0,172	0,174	0,176	0,178	0,181	0,183	0,186	0,188	0,191	0,194	0,197	0,200	0,204	
0,315	0,172	0,174	0,176	0,178	0,181	0,183	0,186	0,188	0,191	0,194	0,197	0,200	0,204	0,208	
0,400	0,174	0,176	0,178	0,181	0,183	0,186	0,188	0,191	0,194	0,197	0,200	0,204	0,208	0,212	
0,500	0,176	0,178	0,181	0,183	0,186	0,188	0,191	0,194	0,197	0,200	0,204	0,208	0,212	0,216	
0,630	0,179	0,181	0,184	0,186	0,189	0,191	0,194	0,197	0,200	0,204	0,208	0,212	0,216	0,220	
0,800	0,182	0,184	0,187	0,189	0,192	0,195	0,198	0,201	0,204	0,208	0,212	0,216	0,220	0,224	
1,000	0,185	0,187	0,190	0,192	0,195	0,198	0,201	0,204	0,208	0,212	0,216	0,220	0,224	0,228	
1,250	0,188	0,190	0,193	0,195	0,198	0,201	0,204	0,208	0,212	0,216	0,220	0,224	0,228	0,233	
1,600	0,191	0,194	0,196	0,199	0,202	0,205	0,208	0,212	0,216	0,220	0,224	0,228	0,233	0,238	
2,000	0,194	0,197	0,200	0,203	0,206	0,209	0,213	0,216	0,220	0,224	0,228	0,233	0,238	0,243	
2,500	0,198	0,201	0,204	0,207	0,210	0,213	0,217	0,220	0,224	0,228	0,233	0,238	0,244	0,249	
3,150	0,202	0,205	0,208	0,211	0,214	0,218	0,222	0,225	0,230	0,234	0,239	0,244	0,250	0,256	
4,000	0,207	0,209	0,213	0,216	0,219	0,223	0,227	0,231	0,236	0,240	0,245	0,250	0,256	0,263	
5,000	0,211	0,214	0,218	0,221	0,225	0,228	0,233	0,237	0,242	0,247	0,252	0,257	0,262	0,269	
6,300	0,216	0,220	0,223	0,227	0,231	0,235	0,239	0,244	0,248	0,254	0,259	0,265	0,271	0,277	
8,000	0,222	0,226	0,230	0,233	0,237	0,242	0,246	0,251	0,256	0,262	0,268	0,274	0,280	0,287	
10,000	0,229	0,232	0,236	0,240	0,245	0,249	0,254	0,259	0,265	0,271	0,277	0,283	0,289	0,296	

NOTE — The maximum process standard deviation, MPSD, is obtained by multiplying the standardized value,  $f$ , by the difference between the upper specification limit,  $U$ , and the lower specification limit,  $L$ , i.e.  $MPSD = (U - L)f$ .

The maximum process standard deviation, MPSD, indicates the greatest allowable magnitude of the process standard deviation when using acceptance sampling plans for the separate double specification limit case. If the process standard deviation exceeds the MPSD, all lots shall be considered not acceptable.

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## Section 3: Operation of a sequential sampling plan

### 3.1 Specification of the plan

The choice between single and sequential sampling shall be made before the inspection of a lot is started.

Before sequential sampling takes place, the inspector must record on the sampling document the specified values of  $h_A$ ,  $h_R$ ,  $g$ ,  $n_t$  and  $A_t$ .

In addition the inspector shall record whether the decision criteria are based on a single upper specification limit, a single lower specification limit, combined double specification limits or separate double specification limits. In this last case separate parameters must be supplied for the upper and lower decision criteria. The appropriate values shall be determined by reference to tables A.1 and A.2 if the sampling is required to be compatible with ISO 3951.

### 3.2 Preparing the sampling plan

#### 3.2.1 Single specification limit

##### 3.2.1.1 Acceptance and rejection values

For each value,  $n_{cum}$ , of the cumulative sample size that is less than the curtailment value  $n_t$  of the sample size, the corresponding acceptance value  $A$  is found as

$$A = g\sigma n_{cum} + h_A\sigma$$

The rejection value  $R$  is found as

$$R = g\sigma n_{cum} - h_R\sigma$$

The acceptance value  $A_t$  corresponding to the curtailment sample size is determined as

$$A_t = g\sigma n_t$$

The acceptance and rejection values shall be recorded with one decimal place more than the inspection results.

##### 3.2.1.2 Acceptance chart

Prepare a graph such as figure 3, with the cumulative sample size as the horizontal axis, the cumulative leeway as the vertical axis, and the formulae for the acceptance values and the rejection values re-

presented by two straight lines with the same slope,  $g\sigma$ . The line with intercept  $h_A\sigma$  is the acceptance line and the line with intercept  $-h_R\sigma$  is the rejection line.

Add a vertical line, the curtailment line, at a cumulative sample size  $n_t$ .

The lines define three zones on the chart.

- The *acceptance zone* is the zone above (and including) the acceptance line together with that part of the curtailment line which is above (and including) the point  $(n_t, A_t)$ .
- The *rejection zone* is the zone below (and including) the rejection line together with that part of the curtailment line which is below the point  $(n_t, A_t)$ .
- The *indecision zone* is the strip between the acceptance and rejection lines that is to the left of the curtailment line.

##### 3.2.1.3 Example

Consider the sequential sampling plan by variables with parameters  $L = 200$  kV;  $\sigma = 1,2$  kV;  $h_A = 4,312$ ;  $h_R = 5,536$ ;  $g = 2,315$  and  $n_t = 49$  that were determined in the example given in 2.4.2.3.

The formula for the acceptance value  $A$  becomes

$$A = 2,778n_{cum} + 5,174$$

and the formula for the rejection value  $R$  becomes

$$R = 2,778n_{cum} - 6,643$$

The acceptance and rejection values corresponding to the cumulative sample sizes  $n_{cum} = 1, 2, \dots, 48$  are determined by successively inserting the values of  $n_{cum}$  in these formulae. The acceptance value  $A_t$  corresponding to the curtailment sample size is determined from

$$A_t = 2,778n_t$$

with the curtailment sample size  $n_t = 49$ .

Since the withstand voltage of the insulators is determined to one decimal place, the acceptance and rejection values are rounded to two decimal places. The results are shown in figure 2.

Cumulative sample size $n_{cum}$	Inspection result $x$ kV	Leeway $y$	Rejection value $R$	Cumulative leeway $Y$	Acceptance value $A$
1	202,5	2,5	-3,86	2,5	7,95
2	203,8	3,8	-1,09	6,3	10,73
3	201,9	1,9	1,69	8,2	13,51
4	205,6	5,6	4,47	13,8	16,29
5	199,9	-0,1	7,25	13,7	19,06
6	202,7	2,7	10,02	16,4	21,84
7	203,2	3,2	12,80	19,6	24,62
8	203,6	3,6	15,58	23,2	27,40
9	204,0	4,0	18,36	27,2	30,18
10	203,6	3,6	21,14	30,8	32,95
11	203,3	3,3	23,91	34,1	35,73
12	204,7	4,7	26,69	38,8	38,51
13	—	—	29,47	The lot is accepted	41,29
14	—	—	32,25		44,07
.	.	.	.	.	.
.	.	.	.	.	.
48	.	.	126,70	.	138,58
49	.	.	.	.	136,12

Figure 2 — Inspection record sheet for the sequential sampling plan for a single specification limit considered in the example given in 3.2.1.3

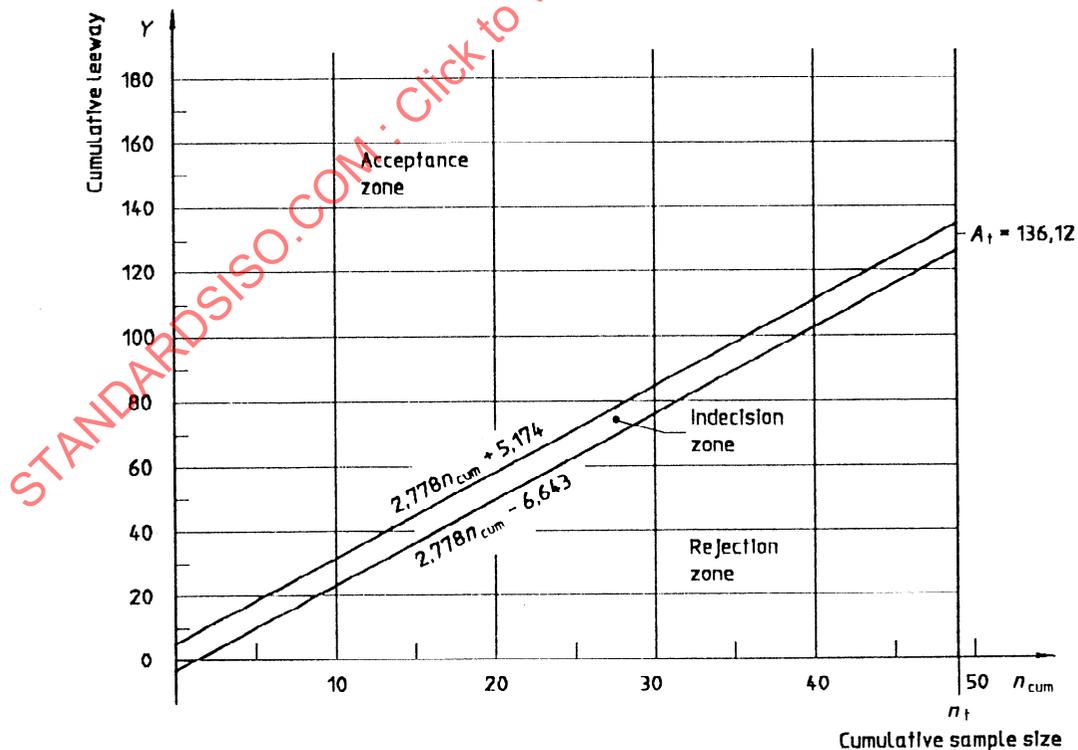


Figure 3 — Acceptance chart for the sequential sampling plan for a single specification limit considered in the example given in 3.2.1.3

Figure 3 shows an acceptance chart for this sequential sampling plan. To construct this chart a sheet of graph paper was prepared with the cumulative sample size  $n_{cum}$  as the horizontal axis and the cumulative leeway  $Y$  as the vertical axis. The acceptance line is the line passing through the point  $(0; h_A\sigma)$  and all the points  $(n_{cum}; g\sigma n_{cum} + h_A\sigma)$ . Choosing  $n_{cum} = 30$  we find  $g\sigma n_{cum} + h_A\sigma = 88,51$ . The line was constructed by plotting the points  $(0; 5,17)$  and  $(30; 88,51)$  on the graph paper and connecting the two points by a straight line. The rejection line was similarly constructed by connecting the points  $(0; -6,64)$  and  $(30; 76,70)$  corresponding to  $(0; h_R\sigma)$  and  $(n_{cum}; R)$  with  $n_{cum} = 30$ . Finally, the curtailment line was constructed as a vertical line through  $n_{cum} = 49$ .

**3.2.2 Combined double specification limits**

**3.2.2.1 Acceptance and rejection values**

For each value,  $n_{cum}$ , of the cumulative sample size that is less than the curtailment value  $n_t$  of the sample size, a pair of acceptance values and a pair of rejection values are determined.

The upper acceptance value  $A^{(U)}$  is found as

$$A^{(U)} = (U - L - g\sigma)n_{cum} - h_A\sigma$$

and the lower acceptance value  $A^{(L)}$  as

$$A^{(L)} = g\sigma n_{cum} + h_A\sigma$$

The upper rejection value  $R^{(U)}$  is found as

$$R^{(U)} = (U - L - g\sigma)n_{cum} + h_R\sigma$$

and the lower rejection value  $R^{(L)}$  as

$$R^{(L)} = g\sigma n_{cum} - h_R\sigma$$

Whenever the value of  $A^{(U)}$  is less than the corresponding value of  $A^{(L)}$ , the cumulative sample size is too small to allow acceptance of the lot.

The acceptance values  $A_t^{(U)}$  and  $A_t^{(L)}$  corresponding to the curtailment sample size are determined as

$$A_t^{(U)} = (U - L - g\sigma)n_t - h_A\sigma$$

and

$$A_t^{(L)} = g\sigma n_t + h_A\sigma$$

The acceptance and rejection values shall be recorded with one decimal place more than the inspection results.

**3.2.2.2 Acceptance chart**

Prepare a graph such as figure 5 with the cumulative sample size as the horizontal axis, the cumulative leeway as the vertical axis, a vertical line (the

curtailment line) at a cumulative sample size  $n_t$ , and the formulae for the acceptance and rejection values represented by four straight lines.

The uppermost line (the upper rejection line) has slope  $(U - L - g\sigma)$  and intercept  $h_R\sigma$ . The *upper rejection zone* is the zone above (and including) this line together with that part of the curtailment line which is above the point  $[n_t; A_t^{(U)}]$ .

The lower rejection line is the lowermost line, with slope  $g\sigma$  and intercept  $-h_R\sigma$ . The *lower rejection zone* is the zone below (and including) this line together with that part of the curtailment line which is below the point  $[n_t; A_t^{(L)}]$ .

The *rejection zone* consists of the upper and the lower rejection zones.

The upper acceptance line has slope  $(U - L - g\sigma)$  and intercept  $-h_A\sigma$ . The fourth line (the lower acceptance line) is the line with slope  $g\sigma$  and intercept  $h_A\sigma$ . The *acceptance zone* is the triangular sector on the chart which is bounded by the upper acceptance line, the lower acceptance line and the curtailment line. The acceptance zone includes the two acceptance lines; moreover, that part of the curtailment line which is between (and including) the points  $[n_t; A_t^{(U)}]$  and  $[n_t; A_t^{(L)}]$  belongs to the acceptance zone.

The V-shaped strip of the chart between the acceptance and rejection zones that is to the left of the curtailment line is termed the *indecision zone*.

**3.2.2.3 Example**

The specification for the dimension of an industrially manufactured mechanical part is  $205 \text{ mm} \pm 5 \text{ mm}$ . Production is stable and it has been verified that the dimension varies within a lot in accordance with a normal distribution. It has further been documented that the standard deviation within a lot is stable and can be taken to be  $\sigma = 1,2 \text{ mm}$ .

It has been decided to use a sequential sampling plan by variables with  $p_A = 0,005$ ,  $\alpha = 0,05$ ,  $p_R = 0,02$  and  $\beta = 0,10$  for both limits combined.

The parameters of the sampling plan are found from table 1 to be  $h_A = 4,312$ ,  $h_R = 5,536$ ,  $g = 2,315$  and  $n_t = 49$ .

The formulae for the upper and lower acceptance values  $A^{(U)}$  and  $A^{(L)}$  become

$$A^{(U)} = 7,222n_{cum} - 5,174$$

and

$$A^{(L)} = 2,778n_{cum} + 5,174$$

Similarly, the formulae for the upper and lower rejection values  $R^{(U)}$  and  $R^{(L)}$  become

$$R^{(U)} = 7,222n_{cum} + 6,643$$

for the upper rejection value, and

$$R^{(L)} = 2,778n_{cum} - 6,643$$

for the lower rejection value.

The acceptance and rejection values corresponding to the cumulative sample sizes  $n_{cum} = 1, 2, \dots, 48$  are determined by successively inserting the values of  $n_{cum}$  into these formulae.

The acceptance values  $A_t^{(U)}$  and  $A_t^{(L)}$  corresponding to the curtailment sample size are determined from

$$A_t^{(U)} = 7,222n_t$$

and

$$A_t^{(L)} = 2,778n_t$$

with the curtailment sample size  $n_t = 49$ .

Since the dimension is determined to one decimal place, the acceptance and rejection values are rounded to two decimal places. The result is shown in figure 4.

Cumulative sample size $n_{cum}$	Inspection result $x$ mm	Leeway $y$	Lower rejection value $R^{(L)}$	Lower acceptance value $A^{(L)}$	Cumulative leeway $Y$	Upper acceptance value $A^{(U)}$	Upper rejection value $R^{(U)}$
1	202,5	2,5	-3,86	7,95*	2,5	2,05*	13,87
2	203,8	3,8	-1,09	10,73*	6,3	9,27*	21,09
3	201,9	1,9	1,69	13,51	8,2	16,49	28,31
4	205,6	5,6	4,47	16,29	13,8	23,71	35,53
5	199,9	-0,1	7,25	19,06	13,7	30,94	42,75
6	202,7	2,7	10,02	21,84	16,4	38,16	49,98
7	203,2	3,2	12,80	24,62	19,6	45,38	57,20
8	203,6	3,6	15,58	27,40	23,2	52,60	64,42
9	204,0	4,0	18,36	30,18	27,2	59,83	71,64
10	203,6	3,6	21,14	32,95	30,8	67,05	78,87
11	203,3	3,3	23,91	35,73	34,1	74,27	86,09
12	204,7	4,7	26,69	38,51	38,8	81,49	93,31
13	—	—	29,47	41,29	The lot is accepted	88,71	100,53
14	—	—	32,25	44,07		95,94	107,75
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
48			126,70	138,52		341,48	353,30
49				136,12		353,88	

\* Acceptance is not permitted for this cumulative sample size since the lower acceptance value exceeds the upper acceptance value.

Figure 4 — Inspection record sheet for the sequential sampling plan for combined double specification limits considered in the example given in 3.2.2.3

Figure 5 shows an acceptance chart for this sequential sampling plan. To construct this chart a sheet of graph paper was prepared with cumulative sample size  $n_{cum}$  as the horizontal axis and the cumulative leeway  $Y$  as the vertical axis. The acceptance line for the lower specification limit was constructed by connecting the points (0; 5,17) and (30; 88,51) by a straight line, and the rejection line for the lower specification limit was constructed by connecting the points (0; - 6,64) and (30; 76,70).

The acceptance line for the upper specification limit was constructed as the line connecting the points (0; - 5,17) and (30; 211,49). Similarly, the rejection line for the upper specification limit was constructed by connecting the points (0; 6,64) and (30; 223,30). Finally the curtailment line was constructed as a vertical line through  $n_{cum} = 49$ .

### 3.2.3 Separate double specification limits

#### 3.2.3.1 Acceptance and rejection values

For each value,  $n_{cum}$ , of the cumulative sample size that is less than the curtailment value  $n_t$  of the sample size, two pairs of acceptance and rejection val-

ues are determined. Each pair of acceptance and rejection values refers to each specification limit separately.

The acceptance value  $A^{(U)}$  for the upper specification limit is found as

$$A^{(U)} = (U - L - g^{(U)}\sigma)n_{cum} - h_A^{(U)}\sigma$$

and the rejection value  $R^{(U)}$  for the upper specification limit as

$$R^{(U)} = (U - L - g^{(U)}\sigma)n_{cum} + h_R^{(U)}\sigma$$

The acceptance value  $A^{(L)}$  for the lower specification limit is found as

$$A^{(L)} = g^{(L)}\sigma n_{cum} + h_A^{(L)}\sigma$$

and the rejection value  $R^{(L)}$  for the lower specification limit as

$$R^{(L)} = g^{(L)}\sigma n_{cum} - h_R^{(L)}\sigma$$

The acceptance values  $A_t^{(U)}$  and  $A_t^{(L)}$  corresponding to the curtailment sample size are determined as

$$A_t^{(U)} = (U - L - g^{(U)}\sigma)n_t - h_A^{(U)}\sigma$$

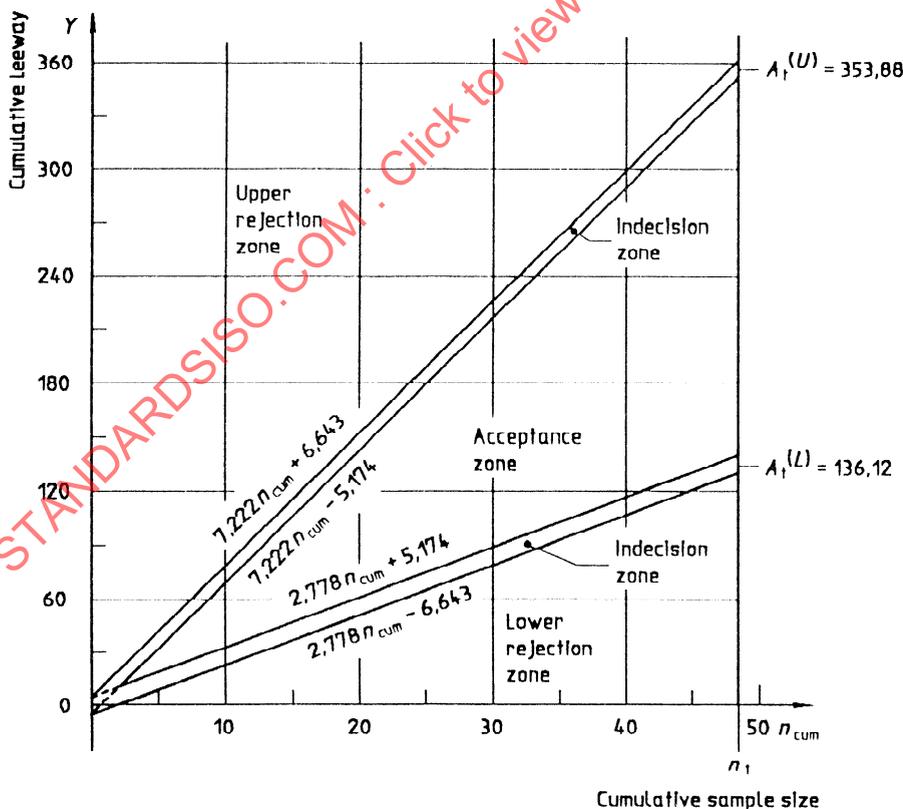


Figure 5 — Acceptance chart for the sequential sampling plan for combined double specification limits considered in the example given in 3.2.2.3

and

$$A_i^{(L)} = g^{(L)}\sigma n_i + h_A^{(L)}\sigma$$

The acceptance and rejection values shall be recorded with one decimal place more than the inspection results.

### 3.2.3.2 Acceptance chart

Prepare a graph such as figure 7 with the cumulative sample size as the horizontal axis, the cumulative leeway as the vertical axis, a vertical line (the curtailment line) at a cumulative sample size  $n_i$ , and the formulae for the acceptance and rejection values represented by two straight lines referring to the lower specification limit and two lines relating to the upper specification limit.

The two lower lines, with common slope  $g^{(L)}\sigma$ , refer to the lower specification limit. The lowermost line, with intercept  $-h_R^{(L)}\sigma$ , is designated the rejection line for the lower specification limit and the other line, with intercept  $h_A^{(L)}\sigma$ , is the acceptance line for the lower specification limit.

These lines define three zones on the chart.

- The *acceptance zone for the lower specification limit* is the zone above (and including) the acceptance line for the lower specification limit together with that part of the curtailment line which is above (and includes) the point  $[n_i; A_i^{(L)}]$ .
- The *rejection zone for the lower specification limit* is the zone below (and including) the rejection line for the lower specification limit together with that part of the curtailment line which is below the point  $[n_i; A_i^{(L)}]$ .
- The *indecision zone for the lower specification limit* is the strip between the acceptance and rejection lines for the lower specification limit which is to the left of the curtailment line.

Analogously, the two upper lines, with common slope  $[U - L]g^{(U)}\sigma$ , refer to the upper specification limit. The uppermost line, with intercept  $h_R^{(U)}\sigma$ , is the rejection line for the upper specification limit and the other line, with intercept  $-h_A^{(U)}\sigma$ , is the acceptance line for the upper specification limit.

These lines also define three zones on the acceptance chart.

- The *acceptance zone for the upper specification limit* is the zone below (and including) the acceptance line for the upper specification limit together with that part of the curtailment line which is below (and includes) the point  $[n_i; A_i^{(U)}]$ .

- The *rejection zone for the upper specification limit* is the zone above (and including) the rejection line for the upper specification limit together with that part of the curtailment line which is above the point  $[n_i; A_i^{(U)}]$ .
- The *indecision zone for the upper specification limit* is the strip between the acceptance and rejection lines for the upper specification limit which is to the left of the curtailment line.

The two sets of lines have been determined in such a way that they apply to the same cumulative leeway. Since the acceptability criteria in 3.5.3 are applied to each specification limit separately, the user may choose to prepare two charts, one corresponding to the lower specification limit, and another corresponding to the upper specification limit.

### 3.2.3.3 Example

The specification for the output voltage of an electronic component is  $5\,950\text{ mV} \pm 50\text{ mV}$ . Production is stable and it has been verified that the output voltage varies within a lot in accordance with a normal distribution. It has further been documented that the standard deviation within a lot is stable and can be taken to be  $\sigma = 12\text{ mV}$ .

It has been decided to use a sequential sampling plan by variables with  $p_A^{(U)} = 0,005$ ,  $\alpha^{(U)} = 0,05$ ,  $p_R^{(U)} = 0,02$  and  $\beta^{(U)} = 0,10$  for the upper specification limit  $U = 6\,000\text{ mV}$  and  $p_A^{(L)} = 0,025$ ,  $\alpha^{(L)} = 0,05$ ,  $p_R^{(L)} = 0,10$  and  $\beta^{(L)} = 0,10$  for the lower specification limit  $L = 5\,900\text{ mV}$ .

Since the quality levels have been specified for each of the limits separately, two sets of parameters are determined for the sequential sampling plan.

The parameters referring to the upper specification limit are found from table 1 to be  $h_A^{(U)} = 4,312$ ,  $h_R^{(U)} = 5,536$ ,  $g^{(U)} = 2,315$  and  $n_i^{(U)} = 49$ .

Similarly, the parameters referring to the lower specification limit are found to be  $h_A^{(L)} = 3,318$ ,  $h_R^{(L)} = 4,260$ ,  $g^{(L)} = 1,621$  and  $n_i^{(L)} = 29$ .

Since the larger of the two curtailment values is  $n_i^{(U)} = 49$ , the curtailment value that shall be used for the sequential sampling plan is  $n_i = 49$ .

The formulae for the acceptance value  $A^{(U)}$  and the rejection value  $R^{(U)}$  corresponding to the upper specification limit become

$$A^{(U)} = 72,22n_{\text{cum}} - 51,74$$

and

$$R^{(U)} = 72,22n_{\text{cum}} + 66,43$$

Similarly, the formulae for the acceptance value  $A^{(L)}$  and the rejection value  $R^{(L)}$  corresponding to the lower specification limit become

$$A^{(L)} = 19,45n_{cum} + 39,82$$

and

$$R^{(L)} = 19,45n_{cum} - 51,12$$

The acceptance and rejection values corresponding to the cumulative sample sizes  $n_{cum} = 1, 2, \dots, 48$  are determined by successively inserting the values of  $n$  into these formulae.

The acceptance values  $A_t^{(U)}$  and  $A_t^{(L)}$  corresponding to the curtailment sample size are determined from

$$A_t^{(U)} = 72,22n_t$$

and

$$A_t^{(L)} = 19,45n_t$$

with the curtailment sample size  $n_t = 49$ .

Since the output voltage is determined in millivolts without decimals, the acceptance and rejection values are rounded to one decimal place. The result is shown in figure 6.

Figure 7 shows an acceptance chart for this sequential sampling plan. To construct this chart a sheet of graph paper was prepared with the cumulative sample size  $n_{cum}$  as the horizontal axis and the cumulative leeway  $Y$  as the vertical axis. The acceptance line for the lower specification limit was constructed by connecting the points (0; 39,8) and (30; 623,3) by a straight line, and the rejection line for the lower specification limit was constructed by connecting the points (0; -51,1) and (30; 532,4).

The acceptance line for the upper specification limit was constructed as the line connecting the points (0; -51,7) and (30; 2 114,9). Similarly, the rejection line for the upper specification limit was constructed by connecting the points (0; 66,4) and (30; 2 233,0). Finally, the curtailment line was constructed as a vertical line through  $n_{cum} = 49$ .

Cumulative sample size $n_{cum}$	Inspection result $x$ mV	Leeway $y$	Rejection value for lower limit $R^{(L)}$	Acceptance value for lower limit $A^{(L)}$	Cumulative leeway $Y$	Acceptance value for upper limit $A^{(U)}$	Rejection value for upper limit $R^{(U)}$
1	5930	30	-31,7	59,3	30	20,5	138,7
2	5909	9	-12,2	78,7	39	92,7	210,9
3	5921	21	7,2	98,2	60	164,9	283,1
4	5924	24	26,7	117,6	84	237,1	355,3
5	5927	27	46,1	137,1	111	309,4	427,5
6	5939	39	65,5	156,5	150	381,6	499,8
7	5914	14	85,0	176,0	164	453,8	572,0
8	5916	16	104,5	195,4	180	526,0	644,2
9	5932	32	123,9	214,9	212	598,2	716,4
10	5918	18	143,4	234,3	230	670,5	788,6
11	5934	34	162,8	253,8	264	742,7	860,9
12	—	—	182,3	273,2	The lot is accepted	814,9	933,1
13	—	—	201,7	292,7		887,1	1 005,3
14	—	—	221,2	312,1		959,3	1 077,5
·	·	·	·	·		·	·
·	·	·	·	·	·	·	·
·	·	·	·	·	·	·	·
48	·	·	882,5	973,4	·	3 414,8	·
49	·	·	·	953,1	·	3 538,8	3 533,0

Figure 6 — Inspection record sheet for the sequential sampling plan for separate double specification limits considered in the example given in 3.2.3.3

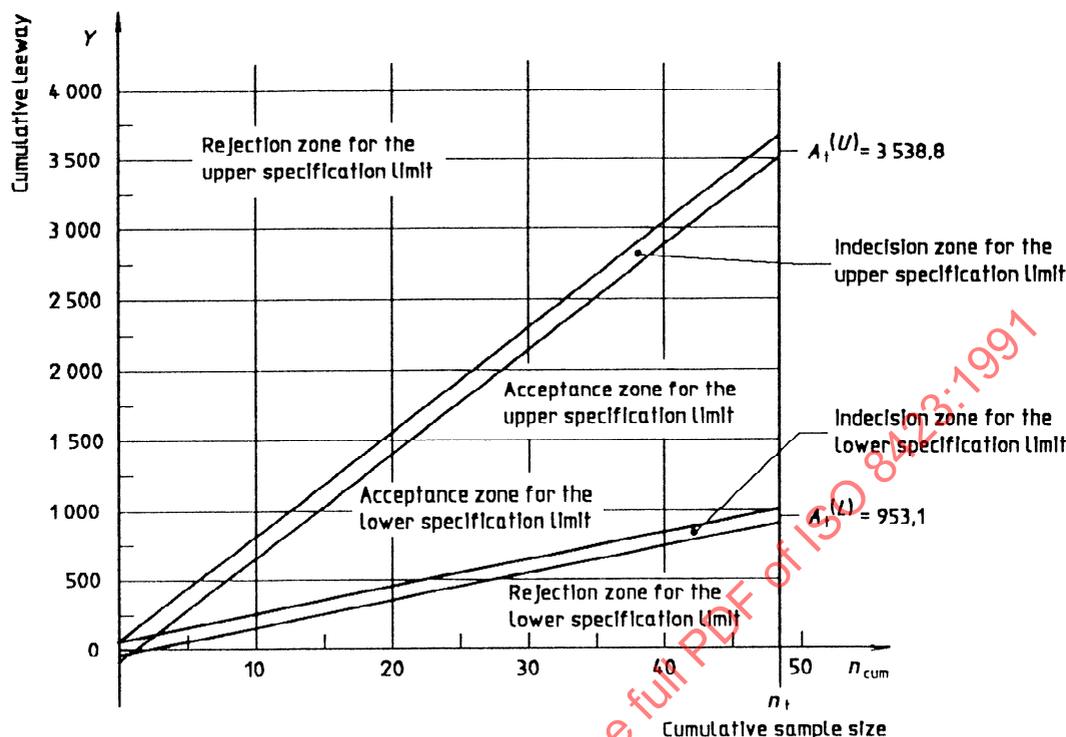


Figure 7 — Acceptance chart for the sequential sampling plan for separate double specification limits considered in the example given in 3.2.3.3

### 3.3 Drawing of the sample

The individual items in the sample shall be drawn at random from the lot and subjected to the inspection one by one in the order in which they were drawn. If, for convenience, successive samples of several items at a time are drawn, the order in which the items of each sample are inspected shall be independent of their original position in the lot.

### 3.4 The cumulative leeway

Following the inspection of each item, record the inspection result  $x$  against the current value,  $n_{cum}$ , of the cumulative sample size.

Calculate the leeway  $y$  for that item as

$$y = x - L \text{ in the case of double specification limits or a single lower specification limit}$$

$$y = U - x \text{ in the case of a single upper specification limit}$$

Record the cumulative leeway  $Y$  as the sum of the leeways found so far in the sample from the lot.

### 3.5 Determination of acceptability

#### 3.5.1 Single specification limit

##### 3.5.1.1 Numerical method

Compare the cumulative leeway  $Y$  with the corresponding acceptance value  $A$  and rejection value  $R$ .

- If  $Y \geq A$ , the lot shall be considered acceptable.
- If  $Y \leq R$ , the lot shall be considered not acceptable.
- If neither a) nor b) is satisfied, another item shall be sampled and inspected.

When the cumulative sample size reaches the curtailment value  $n_t$ , the lot shall be considered acceptable if  $Y \geq A_t$ , otherwise the lot shall be considered not acceptable.

##### 3.5.1.2 Graphical method

Plot the point  $(n_{cum}; Y)$  on the acceptance chart prepared in accordance with 3.2.1.2.

- a) If the point lies in the acceptance zone, the lot shall be considered acceptable.
- b) If the point lies in the rejection zone, the lot shall be considered not acceptable.
- c) If the point lies in the indecision zone, another item shall be sampled and inspected.

The successive points on the chart may be connected by a step curve to show up any trend in the inspection results.

**CAUTION — If the point is close to the acceptance or rejection lines, the numerical method shall be used to make the decision.**

**3.5.1.3 Example**

For the sequential sampling plan with  $L = 200$  kV,  $\sigma = 1,2$  kV,  $h_A = 4,312$ ,  $h_R = 5,536$ ,  $g = 2,315$  and  $n_t = 49$ , the acceptance and rejection values were determined in the example given in 3.2.1.3.

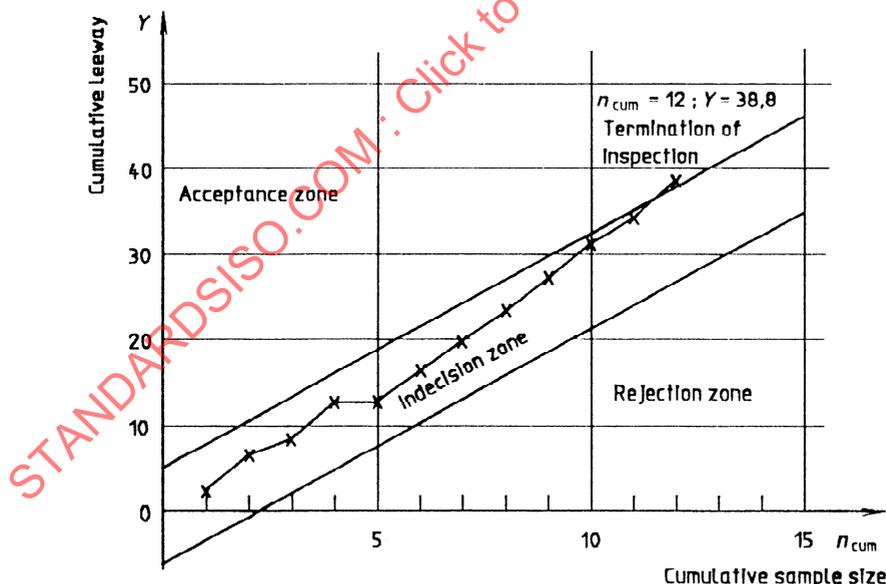
The inspection results  $x$  for the first 12 insulators sampled from a lot are given in the second column of figure 2. The third column gives the corresponding leeway  $y = x - 200$ , and in the fifth column the cumulative leeways are recorded.

Inspection is continued up to and including the 12th item, since the successive cumulative leeways after inspection of item 1, 2, ..., 11 are greater than the corresponding rejection values but less than the corresponding acceptance values. The cumulative leeway after inspection of the 12th item is greater than the corresponding acceptance value and therefore inspection is terminated and the lot is accepted.

Note that the lot is accepted although the withstand voltage of the fifth item inspected is below the specified value. It is an inherent feature of a sequential variables sampling plan that only the sample average is used to assess the quality of a lot, and therefore it may happen that a lot is judged acceptable even if some inspection results deviate from the specification by a small amount.

Figure 8 shows an acceptance chart for this sequential sampling plan. The preparation of this chart is described in the example given in 3.2.1.3.

The successive pairs of the cumulative sample size and the cumulative leeway ( $n_{cum}; Y$ ) from figure 2 have been plotted on the acceptance chart and connected by a step curve. Because the point (12; 38,8) is in the acceptance zone, inspection shall be terminated after inspection of the 12th item, and the lot shall be declared acceptable.



**Figure 8 — Acceptance chart for the sequential sampling plan for a single specification limit considered in the example given in 3.5.1.3**

### 3.5.2 Combined double specification limits

#### 3.5.2.1 Numerical method

Compare the cumulative leeway  $Y$  with the corresponding upper and lower acceptance values  $A^{(U)}$  and  $A^{(L)}$ , and the corresponding upper and lower rejection values  $R^{(U)}$  and  $R^{(L)}$ .

- If  $A^{(L)} \leq Y \leq A^{(U)}$ , the lot shall be considered acceptable.
- If either  $Y > R^{(U)}$  or  $Y < R^{(L)}$ , the lot shall be considered not acceptable.
- If neither a) nor b) is satisfied, another item shall be sampled and inspected.

When the cumulative sample size reaches the curtailment value  $n_t$ , the lot shall be considered acceptable if  $A_t^{(L)} \leq Y \leq A_t^{(U)}$ , otherwise the lot shall be considered not acceptable.

#### 3.5.2.2 Graphical method

Plot the point  $(n_{cum}; Y)$  on the acceptance chart prepared in accordance with 3.2.2.2.

- If the point lies in the acceptance zone, the lot shall be considered acceptable.
- If the point lies in the rejection zone, the lot shall be considered not acceptable.
- If the point lies in the indecision zone, another item shall be sampled and inspected.

The successive points on the chart may be connected by a step curve to show up any trend in the inspection results.

**CAUTION** — If the point is close to the acceptance or rejection lines, the numerical method shall be used to make the decision.

### 3.5.2.3 Example

For the sequential sampling plan with  $L = 200$  mm,  $U = 210$  mm,  $\sigma = 1,2$  mm,  $h_A = 4,312$ ,  $h_R = 5,536$ ,  $g = 2,315$  and  $n_t = 49$ , the acceptance and rejection values corresponding to combined double specification limits were determined in the example given in 3.2.2.3.

The inspection results  $x$  for the first 12 items sampled from a lot are given in the second column of figure 4. The third column gives the corresponding leeway,  $y = x - 200$ , and in the sixth column the cumulative leeway is recorded. Inspection is continued up to and including the 12th item, since the successive cumulative leeways after inspection of item 1, 2, ..., 11 satisfy neither a) nor b) in 3.5.2.2. The value of the cumulative leeway after inspection of the 12th item is between the corresponding lower and upper acceptance values and therefore inspection is terminated and the lot is accepted.

Figure 9 shows an acceptance chart for this sequential sampling plan. The preparation of this chart is described in the example given in 3.5.2.3.

The successive pairs of the cumulative sample size and the cumulative leeway  $(n_{cum}; Y)$  from figure 4 have been plotted on the acceptance chart and connected by a step curve. Since the point (12; 38,8) is in the acceptance zone, inspection shall be terminated after inspection of the 12th item, and the lot shall be declared acceptable.

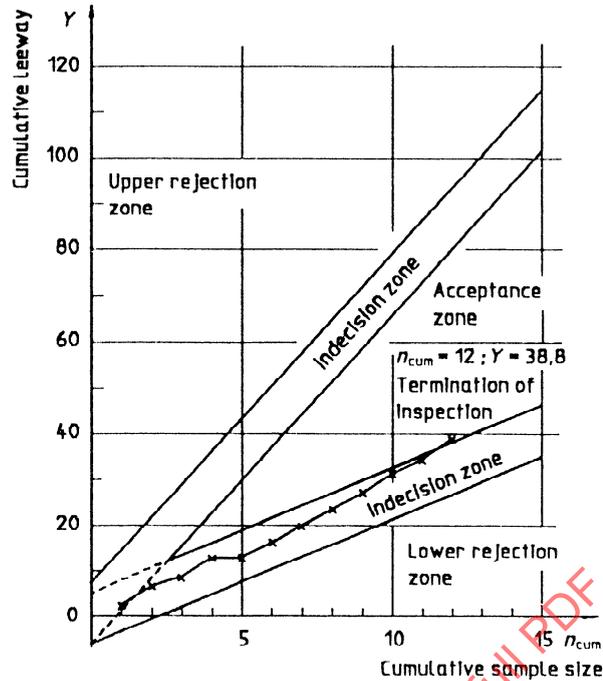


Figure 9 — Acceptance chart for the sequential sampling plan for combined double specification limits considered in the example given in 3.5.2.3

**3.5.3 Separate double specification limits**

**3.5.3.1 Numerical method**

The acceptability criteria in 3.5.3.1.1 and 3.5.3.1.2 shall be applied to determine the acceptability for each specification limit separately. The lot shall be considered acceptable and inspection shall terminate if the lot has been considered acceptable with respect to both limits according to 3.5.3.1.1 a) and 3.5.3.1.2 a).

**3.5.3.1.1 Acceptability criterion for the upper specification limit**

Compare the cumulative leeway  $Y$  with the corresponding acceptance value  $A^{(U)}$  and rejection value  $R^{(U)}$ .

- a) If  $Y \leq A^{(U)}$ , the lot shall be considered acceptable with respect to the upper specification limit and inspection with respect to that limit shall terminate.
- b) If  $Y \geq R^{(U)}$ , the lot shall be considered not acceptable and inspection with respect to both limits shall terminate.
- c) If neither a) nor b) is satisfied, another item shall be sampled and inspected with respect to the upper specification limit.

When the cumulative sample size reaches the curtailment value  $n_t$  and  $Y > A_t^{(U)}$ , the lot shall be considered not acceptable and inspection shall terminate.

When the cumulative sample size reaches the curtailment value  $n_t$  and  $Y \leq A_t^{(U)}$ , the lot shall be considered acceptable with respect to the upper limit. If the lot has already been considered acceptable with respect to the lower limit, or if  $Y \geq A_t^{(L)}$ , the lot shall be considered acceptable and inspection shall terminate, otherwise the lot shall be considered not acceptable, and inspection shall terminate.

**3.5.3.1.2 Acceptability criterion for the lower specification limit**

Compare the cumulative leeway  $Y$  with the corresponding acceptance value  $A^{(L)}$  and rejection value  $R^{(L)}$ .

- a) If  $Y \geq A^{(L)}$ , the lot shall be considered acceptable with respect to the lower specification limit and inspection with respect to that limit shall terminate.
- b) If  $Y < R^{(L)}$ , the lot shall be considered not acceptable and inspection with respect to both limits shall terminate.

- c) If neither a) nor b) is satisfied, another item shall be sampled and inspected with respect to the lower specification limit.

When the cumulative sample size reaches the curtailment value  $n_t$  and  $Y < A_t^{(L)}$ , the lot shall be considered not acceptable and inspection shall terminate.

When the cumulative sample size reaches the curtailment value  $n_t$  and  $Y \geq A_t^{(L)}$ , the lot shall be considered acceptable with respect to the lower limit. If the lot has already been considered acceptable with respect to the upper limit, or if  $Y \leq A_t^{(U)}$ , the lot shall be considered acceptable and inspection shall terminate, otherwise the lot shall be considered not acceptable, and inspection shall terminate.

### 3.5.3.2 Graphical method

Plot the point ( $n_{cum}; Y$ ) on the acceptance chart prepared in accordance with 3.2.3.2.

The acceptability criteria in 3.5.3.2.1 and 3.5.3.2.2 shall be applied to determine the acceptability for each specification limit separately. The lot shall be considered acceptable, and inspection shall terminate, if the lot has been considered acceptable with respect to both limits according to 3.5.3.2.1 a) and 3.5.3.2.2 a).

#### 3.5.3.2.1 Acceptability criterion for the upper specification limit

- a) If the point lies in the acceptance zone for the upper specification limit, the lot shall be considered acceptable with respect to the upper specification limit and inspection with respect to that limit shall terminate.
- b) If the point lies in the rejection zone for the upper specification limit, the lot shall be considered not acceptable and inspection with respect to both limits shall terminate.
- c) If the point lies in the indecision zone for the upper specification limit, another item shall be sampled and inspected with respect to the upper specification limit.

#### 3.5.3.2.2 Acceptability criterion for the lower specification limit

- a) If the point lies in the acceptance zone for the lower specification limit, the lot shall be considered acceptable with respect to the lower specification limit and inspection with respect to that limit shall terminate.
- b) If the point lies in the rejection zone for the lower specification limit, the lot shall be considered not acceptable and inspection with respect to both limits shall terminate.
- c) If the point lies in the indecision zone for the lower specification limit, another item shall be sampled and inspected with respect to the lower specification limit.

#### 3.5.3.3 Example

For the sequential sampling plan with  $L = 5900$  mV,  $U = 6000$  mV,  $\sigma = 12$  mV,  $h_A^{(L)} = 3,318$ ,  $h_R^{(L)} = 4,260$ ,

$g^{(L)} = 1,621$ ,  $h_A^{(U)} = 4,312$ ,  $h_R^{(U)} = 5,536$ ,  $g^{(U)} = 2,315$  and  $n_t = 49$ , the acceptance and rejection values corresponding to separate double specification limits were determined in the example given in 3.2.3.3.

The inspection results  $x$  for the first 11 items sampled from a lot are given in the second column of figure 6. The third column gives the corresponding leeway  $y = x - 5900$ , and in the sixth column the cumulative leeway is recorded.

After the inspection of the second item, the acceptance criterion for the upper specification limit [ 3.5.3.2.1 a)] is satisfied, and inspection for that limit is terminated. Inspection for the lower limit is continued until the 11th item is inspected. After inspection of that item, the acceptance criterion for the lower limit [ 3.5.3.2.2 a)] is satisfied and the lot is accepted.

Figure 10 shows an acceptance chart for this sequential sampling plan. The preparation of this chart is described in the example given in 3.2.3.3.

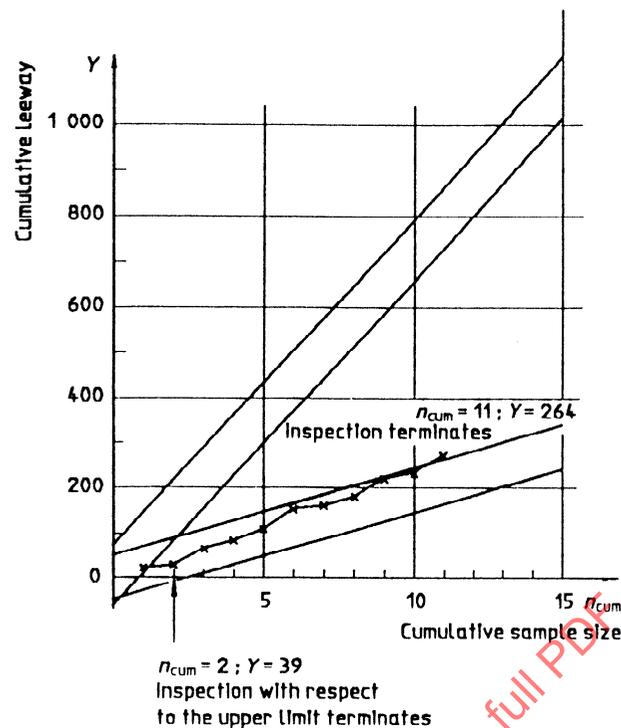


Figure 10 — Acceptance chart for the sequential sampling plan for separate double specification limits considered in the example given in 3.5.3.3

The successive pairs of the cumulative sample size and the cumulative leeway ( $n_{cum}; Y$ ) from figure 6 have been plotted on the acceptance chart and connected by a step curve. Since the point (2; 39) is in the acceptance zone for the upper limit, inspection with respect to that limit is terminated after inspection of the second item. Inspection with respect to the lower limit is continued until the cumulative sample size reaches 11. Because the point (11; 264) is in the acceptance zone for the lower limit, inspection is terminated and the lot is accepted after inspection of the 11th item.

### 3.6 Operating characteristic curves and average sample size

#### 3.6.1 Operating characteristic curves

The operating characteristic curve for a sampling plan indicates the fraction of lots which may be expected to be accepted by the sampling plan at any given process quality level.

When the method described in 2.3.2 and 2.4.1 has been used to determine the sampling plan, the probability of acceptance  $P_a$  will be approximately equal to  $1 - \alpha$  when the process quality level is PRQ and approximately equal to the consumer's risk  $\beta$  when the level of the process quality is CRQ. Annex C gives a method for determining intermediate points on the OC curve.

**CAUTION** — If the actual standard deviation of the process exceeds the value of  $\sigma$  used in the sampling plan, then the actual producer's risk will be smaller than the nominal value, but the actual consumer's risk will exceed the nominal value  $\beta$ .

#### 3.6.2 Average sample size

The average sample size is the average of the various sample sizes which may be expected to occur under a sampling plan for given process quality levels. Annex C gives a method for determining appropriate values of the average sample size.

## Annex A (normative)

### Sequential sampling plans corresponding to ISO 3951 single sampling plans

#### A.1 Introduction

This annex contains sequential sampling plans that supplement the system of sampling plans given in ISO 3951. The sequential sampling plans are indexed by the same values of AQL and the same sample size code letters as in ISO 3951.

The plans have been designed such that  $h_A = h_R$ . The common symbol  $h$  is therefore used to represent the parameters  $h_A$  and  $h_R$ .

#### A.2 Relation to ISO 3951

When the sampling plans in this annex are used to supplement the system of sampling plans in ISO 3951, all rules in ISO 3951 shall apply, the only exception being that the acceptance criteria in clause 15 of ISO 3951:1989 shall be replaced by the acceptance criteria for sequential sampling in 3.5 of this International Standard.

#### A.3 Choice between single and sequential sampling plans

See 2.1.

#### A.4 Obtaining a sampling plan

The AQL and the sample size code letter shall be used to obtain a plan from table A.1.

Plans for normal inspection are found by entering the table using the AQL values given at the top of the table and the sample size code letter in the left column of the table.

Plans for tightened inspection are found using the AQL values at the line next to the bottom line of the table and the sample size code letter in the left column of the table.

Plans for reduced inspection are found using the AQL values at the bottom line and the sample size code letters in the right-hand column of the table.

Table A.1 gives the parameters  $h$  and  $g$  for the sequential sampling plans (see 2.4.1).

Table A.2 gives the curtailment value,  $n_c$ , of the sample size and the corresponding acceptance

number  $A_1$  for the sequential sampling plans in this annex.

When no sampling plan is available for a given combination of AQL and sample size code letter, arrows in the table direct the user to a different code letter.

#### A.5 Determination of acceptability

To determine acceptability of a lot, the applicable sampling plan shall be used in accordance with the rules given in 3.5.

#### A.6 Switching rules for normal, tightened and reduced inspection

When the sequential sampling plans in this annex are substituted for plans in ISO 3951, the switching rules given in clause 19 of ISO 3951:1989 shall apply.

#### A.7 Maximum values of process standard deviation

##### A.7.1 Limiting process standard deviation for combined double specification limits, LPSD (com.)

In the case of combined double specification limits, a factor  $\psi$  is used to determine LPSD for the sequential sampling plans in this annex (see 2.4.3.1). The value of  $\psi$  depends only on the specified AQL.

Table A.3 gives values of  $\psi$  corresponding to AQL.

When  $\sigma$  exceeds  $LPSD = (U - L)\psi$ , no sequential sampling plan is available. If  $\sigma$  is between LPSD and MPSD (from ISO 3951), a single sampling plan is available in ISO 3951.

##### A.7.2 Maximum tolerable process standard deviation for separate double specification limits, MPSD

In the case of separate double specification limits, the maximum value of the process standard deviation for a sequential sampling plan in this annex is given by

$$MPSD = (U - L)f$$

where  $f$  depends on the values of the AQL specified for the upper and the lower limit.

Table A.4 gives values of  $f$  corresponding to combinations of AQL values. If, in the case of separate double specification limits,  $\sigma$  exceeds MPSD, the lot shall immediately be judged not acceptable without a sample being drawn.

NOTE 10 When  $\sigma$  exceeds MPSD, no lot can satisfy both requirements  $p^{(U)} < AQL^{(U)}$  and  $p^{(L)} < AQL^{(L)}$  and therefore acceptance sampling is pointless.

## A.8 Supplementary information

### A.8.1 Operating characteristic curves

The sampling plans in this annex have been determined such that their operating characteristic (OC) curves match as closely as practicable the OC curves of the corresponding single sampling plans in ISO 3951. The curves and tables in table V of ISO 3951:1989 may therefore be used to determine the OC curves for the plans in this annex.

### A.8.2 Average sample size tables

Values of the average sample size for the sequential sampling plans are given in table A.5.

For each sampling plan table A.5 gives the exact value of the average sample size corresponding to three levels of process quality  $p$ . The three process quality levels that have been chosen to illustrate the average sample size are

- a)  $p_A$ , the process quality level that would lead to 90 % of the lots being accepted by the sampling plan;
- b)  $p_R$ , the process quality level that would lead to 10 % of the lots being accepted by the sampling plan; and
- c)  $p_g$ , virtually the worst possible case, corresponding to the process quality level that would lead to 50 % of the lots being accepted by the sampling plan.

The values of  $p_A$ ,  $p_R$  and  $p_g$  for the individual single sampling plans are given in Table V of ISO 3951:1989.

Values of the average sample size corresponding to process quality levels that are not tabulated may be found by interpolation between the tabulated values for the sampling plan, or by calculation in accordance with C.2.2.

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**Table A.1 — Sequential sampling plans corresponding to ISO 3951 single sampling plans for normal, tightened and reduced inspection (master table)**

Code letter for normal inspection	Parameters	Acceptable quality level in percent nonconforming (normal inspection)												
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10		
C	h	↓	↓	↓	↓	↓	1,136	1,185	1,258	1,349	1,450	1,550	1,757	F
	g						1,360	1,250	1,090	0,936	0,755	0,573	0,344	
D	h	↓	↓	↓	↓	↓	1,190	1,244	1,307	1,390	1,485	1,595	1,706	G
	g						1,580	1,420	1,330	1,170	1,010	0,825	0,641	
E	h	↓	↓	↓	1,257	1,313	1,373	1,442	1,505	1,607	1,719	1,844	1,978	H
	g				1,940	1,810	1,690	1,560	1,440	1,280	1,110	0,919	0,728	
F	h	↓	↓	↓	↓	↓	1,408	1,470	1,533	1,610	1,678	1,765	1,877	I
	g						2,190	2,070	1,910	1,800	1,690	1,530	1,390	
G	h	1,579	1,632	1,701	1,755	1,838	1,938	2,034	2,119	2,260	2,406	2,596	2,792	J
	g	2,490	2,390	2,300	2,140	2,050	1,880	1,780	1,620	1,450	1,280	1,070	0,877	
H	h	1,819	1,866	1,950	2,019	2,095	2,215	2,316	2,432	2,595	2,766	2,977	3,198	K
	g	2,550	2,460	2,340	2,230	2,080	1,950	1,800	1,680	1,490	1,310	1,110	0,906	
I	h	2,041	2,074	2,142	2,252	2,338	2,459	2,569	2,686	2,878	3,077	3,317	3,547	L
	g	2,590	2,490	2,370	2,250	2,130	1,960	1,830	1,700	1,510	1,340	1,130	0,924	
J	h	2,391	2,462	2,494	2,618	2,732	2,868	3,015	3,147	3,382	3,592	3,871	4,164	M
	g	2,630	2,540	2,450	2,290	2,160	2,010	1,880	1,750	1,560	1,380	1,170	0,964	
K	h	2,801	2,897	2,959	3,090	3,234	3,384	3,562	3,709	3,965	4,244	4,588	4,941	N
	g	2,720	2,590	2,490	2,350	2,210	2,070	1,930	1,790	1,610	1,420	1,210	0,995	
L	h	3,373	3,492	3,587	3,747	3,919	4,094	4,299	4,491	4,804	5,145	5,584	5,997	P
	g	2,770	2,650	2,540	2,410	2,270	2,120	1,970	1,840	1,650	1,460	1,240	1,030	
M	h	3,774	3,938	4,071	4,322	4,492	4,688	4,937	5,164	5,518	5,893	6,367	6,926	
	g	2,800	2,690	2,570	2,430	2,290	2,140	2,000	1,860	1,670	1,480	1,260	1,050	
N	h	4,820	4,712	5,072	5,177	5,416	5,695	5,966	6,272	6,668	7,158	7,711	8,370	
	g	2,840	2,720	2,620	2,470	2,330	2,170	2,030	1,890	1,690	1,510	1,290	1,070	
P	h	5,273	5,519	5,795	5,960	6,298	6,545	6,866	7,246	7,736	8,252	8,904	9,494	
	g	2,850	2,730	2,620	2,480	2,340	2,180	2,040	1,890	1,700	1,510	1,290	1,070	
Code letter for tightened inspection		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10		
		Acceptable quality level in percent nonconforming (tightened inspection)												
			0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10	Code letter for reduced inspection
			Acceptable quality level in percent nonconforming (reduced inspection)											

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**Table A.2 — Curtailment values for the sequential sampling plans of table A.1**

Code letter for normal inspection	Parameters	Acceptable quality level in percent nonconforming (normal inspection)															
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10					
C	$n_1$ $A_1/\sigma$	↓	↓	↓	↓	↓	3	3	3	3	5	5	6	F			
D	$n_1$ $A_1/\sigma$						4,080	3,750	3,270	2,808	3,775	2,865	2,064				
E	$n_1$ $A_1/\sigma$						3	3	5	5	5	6	6	8	8	9	H
F	$n_1$ $A_1/\sigma$						5,820	5,430	8,450	7,800	7,200	7,620	6,660	7,352	5,824	4,635	
G	$n_1$ $A_1/\sigma$	5	6	6	6	8	8	9	9	11	12	14	17	18	J		
H	$n_1$ $A_1/\sigma$	12,45	14,34	13,80	12,84	16,40	15,04	16,02	14,58	15,95	15,36	14,98	14,91	11,68			
I	$n_1$ $A_1/\sigma$	6	8	8	9	9	11	11	12	14	15	18	21	24	K		
J	$n_1$ $A_1/\sigma$	15,30	19,68	18,72	20,07	18,72	21,45	19,80	20,16	20,86	19,65	19,98	19,03	16,44			
K	$n_1$ $A_1/\sigma$	9	9	9	11	12	12	14	15	17	20	23	26	30	L		
L	$n_1$ $A_1/\sigma$	23,31	22,41	21,33	24,75	25,56	23,52	25,62	25,50	25,67	26,80	25,99	24,02	21,18			
M	$n_1$ $A_1/\sigma$	11	12	14	14	15	17	18	21	23	27	30	36	41	M		
N	$n_1$ $A_1/\sigma$	28,93	30,48	34,30	32,06	32,40	34,17	33,84	36,75	35,88	37,26	35,10	34,70	30,22			
P	$n_1$ $A_1/\sigma$	17	17	18	20	21	24	26	29	33	28	44	50	57	N		
M	$n_1$ $A_1/\sigma$	46,24	44,03	44,82	47,00	46,41	49,68	50,18	51,91	53,13	53,96	53,24	49,75	43,89			
L	$n_1$ $A_1/\sigma$	23	24	26	29	32	35	38	42	48	54	63	74	84	P		
M	$n_1$ $A_1/\sigma$	63,71	63,60	66,04	69,89	72,64	74,20	74,86	77,28	79,20	78,84	78,12	76,22	67,45			
N	$n_1$ $A_1/\sigma$	30	33	35	38	41	45	50	54	63	72	83	96				
P	$n_1$ $A_1/\sigma$	84,00	88,77	89,95	92,34	93,89	96,30	100,0	100,4	105,2	106,6	104,6	100,8				
N	$n_1$ $A_1/\sigma$	45	47	51	56	60	66	74	81	92	105	123	143				
P	$n_1$ $A_1/\sigma$	127,8	127,8	133,6	138,3	139,8	143,2	150,2	153,1	155,5	158,6	158,7	153,0				
P	$n_1$ $A_1/\sigma$	60	63	68	74	81	89	98	107	122	140	164	191				
P	$n_1$ $A_1/\sigma$	171,0	172,0	178,2	183,5	189,5	194,0	199,9	202,2	207,4	211,4	211,6	204,4				
Code letter for tightened inspection		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10					
		Acceptable quality level in percent nonconforming (tightened inspection)															
				0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10	Code letter for reduced inspection		
		Acceptable quality level in percent nonconforming (reduced inspection)															

**Table A.3 — Values of  $\psi$  for limiting process standard deviation, LPSD (combined double specification limits), for the sequential sampling plans of table A.1**

AQL (%)	0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
$\psi$	0,143	0,148	0,155	0,161	0,170	0,178	0,187	0,201	0,216	0,236	0,259

NOTE — The limiting process standard deviation for sequential sampling, LPSD, is obtained by multiplying the standardized value,  $\psi$ , by the difference between the upper specification limit,  $U$ , and the lower specification limit,  $L$ , i.e.  $LPSD = (U - L)\psi$ .  
 The limiting process standard deviation, LPSD, indicates the greatest allowable magnitude of the process standard deviation when using sequential sampling plans for the combined double specification limit case. If the process standard deviation exceeds LPSD no sequential sampling plan is available, but if  $\sigma$  is not too large, a single sampling plan exists in ISO 3951.

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