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Quantities and units —

Part 2: **Mathematics**

Grandeurs et unités —

Partie 2: Mathématiques

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 12, *Quantities and units*, in collaboration with Technical Committee IEC/TC 25, *Quantities and units*.

This second edition cancels and replaces the first edition (ISO 80000-2:2009), which has been technically revised.

The main changes compared to the previous edition are as follows:

— Clause 4 revised to add clarification about writing of font types; revised rule for splitting equations over two or more lines;

— Clause 18 revised to include clarification on scalars, vectors and tensors;

— missing symbols and expressions added in the second column "Symbol, expression" of the tables, and additional clarifications given in the fourth column “Remarks and examples” when necessary;

— Annex A deleted.

NOTE Although missing symbols and expressions have been added in this second edition of ISO 80000-1, the document remains non exhaustive.

A list of all parts in the ISO 80000 and IEC 80000 series can be found on the ISO and IEC websites.

Any feedback or questions on this document should be directed to the user’s national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

Arrangement of the tables

Each table of symbols and expressions (except Table 13) gives hints (in the third column) about the meaning or how the expression may be read for each item (numbered in the first column) of the symbol under consideration, usually in the context of a typical expression (second column). If more than one symbol or expression is given for the same item, they are on an equal footing. In some cases, e.g. for exponentiation, there is only a typical expression and no symbol. The purpose of the entries is identification of each concept and is not intended to be a complete mathematical definition. The fourth column "Remarks and examples" gives further information and is not normative.

Table 13 has a different format. It gives the symbols of coordinates, as well as the position vectors and their differentials, for coordinate systems in three-dimensional spaces.

Quantities and units —

Part 2: **Mathematics**

1 Scope

This document specifies mathematical symbols, explains their meanings, and gives verbal equivalents and applications.

This document is intended mainly for use in the natural sciences and technology, but also applies to other areas where mathematics is used.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 80000-1, *Quantities and units — Part 1: General*

3 Terms and definitions

Tables 1 to 16 give the symbols and expressions used in the different fields of mathematics.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

4 Variables, functions and operators

It is customary to use different sorts of letters for different sorts of entities, e.g. x, y, ... for numbers or elements of some given set, f, g for functions, etc. This makes formulas more readable and helps in setting up an appropriate context.

Variables such as x, y, etc., and running numbers, such as i in $\sum_i x_i$ are printed in italic type. Parameters, such as a, b, etc., which may be considered as constant in a particular context, are printed in italic type. The same applies to functions in general, e.g. f, g.

An explicitly defined function not depending on the context is, however, printed in upright type, e.g. sin, exp, ln, Γ. Mathematical constants, the values of which never change, are printed in upright type, e.g. $\mathrm{e} = 2{,}718\ 281\ 828\ ...$; $\pi = 3{,}141\ 592\ ...$; $\mathrm{i}^2 = -1$. Well-defined operators are also printed in upright type, e.g. **div**, δ in δx and each d in df/dx. Some transforms use special capital letters (see Clause 19, Transforms).

Numbers expressed in the form of digits are always printed in upright type, e.g. 351 204; 1,32; 7/8.

Binary operators, for example +, −, /, shall be preceded and followed by thin spaces. This rule does not apply in case of unary operators, as in −17,3.

The argument of a function is written in parentheses after the symbol for the function, without a space between the symbol for the function and the first parenthesis, e.g. $f(x)$, $\cos(\omega\, t + \varphi)$. If the symbol for the function consists of two or more letters and the argument contains no operation symbol, such as +, −, × , or /, the parentheses around the argument may be omitted. In these cases, there shall be a thin space between the symbol for the function and the argument, e.g. int 2,4; sin $n\pi$; arcosh $2A$; Ei x.

If there is any risk of confusion, parentheses should always be inserted. For example, write $\cos(x) + y$; do not write $\cos x + y$, which could be mistaken for $\cos(x + y)$.

A comma, semicolon or other appropriate symbol can be used as a separator between numbers or expressions. The comma is generally preferred, except when numbers with a decimal comma are used.

If an expression or equation must be split into two or more lines, the following method shall be used:

— Place the line breaks immediately before one of the symbols =, +, −, ±, or ∓, or, if necessary, immediately before one of the symbols ×, ·, or /.

The symbol shall not be given twice around the line break; two minus signs could for example give rise to sign errors. If possible, the line break should not be inside of an expression in parentheses.

5 Mathematical logic

Table 1 — Symbols and expressions in mathematical logic

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-5.1	$p \wedge q$	conjunction of p and q, p and q	
2-5.2	$p \vee q$	disjunction of p and q, p or q	This “or” is inclusive, i.e. $p \vee q$ is true, if either p or q, or both are true.
2-5.3	$\neg\, p$	negation of p, not p	
2-5.4	$p \Rightarrow q$	p implies q, if p, then q	$q \Leftarrow p$ has the same meaning as $p \Rightarrow q$. $\Rightarrow$ is the implication symbol. $\rightarrow$ is also used as implication symbol.
2-5.5	$p \Leftrightarrow q$	p is equivalent to q	$(p \Rightarrow q) \wedge (q \Rightarrow p)$ has the same meaning as $p \Leftrightarrow q$. $\Leftrightarrow$ is the equivalence symbol. $\leftrightarrow$ is also used as equivalence symbol.
2-5.6	$\forall x \in A\ \ p(x)$	for every x belonging to A, the proposition $p(x)$ is true	If it is clear from the context which set A is considered, the notation $\forall x\, p(x)$ can be used. $\forall$ is the universal quantifier. For $x \in A$, see 2-6.1.
2-5.7	$\exists x \in A\ \ p(x)$	there exists an x belonging to A for which $p(x)$ is true	If it is clear from the context which set A is considered, the notation $\exists x\, p(x)$ can be used. $\exists$ is the existential quantifier. For $x \in A$, see 2-6.1. $\exists^1 x\, p(x)$ is used to indicate that there is exactly one element for which $p(x)$ is true. $\exists!$ is also used for $\exists^1$.

6 Sets

Table 2 — Symbols and expressions for sets

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.1	$x \in A$	x belongs to A, x is an element of the set A	$A \ni x$ has the same meaning as $x \in A$.
2-6.2	$y \notin A$	y does not belong to A, y is not an element of the set A	$A \not\ni y$ has the same meaning as $y \notin A$. The negating stroke may also be vertical.
2-6.3	$\{x_1, x_2, ..., x_n\}$	set with elements $x_1, x_2, ..., x_n$	Also $\{x_i \mid i \in I\}$, where I denotes a set of subscripts.
2-6.4	$\{x \in A \mid p(x)\}$	set of those elements of A for which the proposition $p(x)$ is true	EXAMPLE $\{x \in \mathbf{R} \mid x \geq 5\}$ If it is clear from the context which set A is considered, the notation $\{x \mid p(x)\}$ can be used (for example $\{x \mid x \geq 5\}$, if it is clear that real numbers are considered). Instead of the vertical line often a colon is used as separator: $\{x \in A : p(x)\}$.
2-6.5	card A $\lvert A \rvert$	number of elements in A, cardinality of A	The cardinality can be a transfinite number. The symbol $\lvert\,\rvert$ is also used for absolute value of a real number (see 2-10.16), modulus of a complex number (see 2-15.4) and magnitude of a vector (see 2-18.4).
2-6.6	$\varnothing$ $\{\}$	the empty set	
2-6.7	$B \subseteq A$	B is included in A, B is a subset of A	Every element of B belongs to A. $\subset$ is also used, but see remark to 2-6.8. $A \supseteq B$ has the same meaning as $B \subseteq A$.
2-6.8	$B \subset A$	B is properly included in A, B is a proper subset of A	Every element of B belongs to A, but at least one element of A does not belong to B. If $\subset$ is used for 2-6.7, then $\subsetneq$ shall be used for 2-6.8. $A \supset B$ has the same meaning as $B \subset A$.
2-6.9	$A \cup B$	union of A and B	The set of elements which belong to at least one of the sets A and B. $A \cup B = \{x \mid x \in A \lor x \in B\}$
2-6.10	$A \cap B$	intersection of A and B	The set of elements which belong to both sets A and B. $A \cap B = \{x \mid x \in A \land x \in B\}$
2-6.11	$\bigcup_{i=1}^{n} A_i$	union of the sets $A_1, A_2, ..., A_n$ $\bigcup_{i=1}^{n} A_i = A_1 \cup ... \cup A_n$	The set of elements belonging to at least one of the sets $A_1, A_2, ..., A_n$ $\bigcup_{i=1}^{n}$, $\bigcup_{i \in I}$ and $\bigcup_{i \in I}$ are also used, where I denotes a set of subscripts.

Table 2 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-6.12	$\bigcap_{i=1}^{n} A_i$	intersection of the sets $A_1, A_2, \ldots, A_n$ $\bigcap_{i=1}^{n} A_i = A_1 \cap \ldots \cap A_n$	The set of elements belonging to all sets A_1, A_2, …, A_n $\bigcap_{i=1}^{n}$, $\bigcap_{i \in I}$ and $\bigcap_{i \in I}$ are also used, where I denotes a set of subscripts.
2-6.13	$A \setminus B$	difference of A and B, A minus B	The set of elements which belong to A but not to B. $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$ The notation $A - B$ should not be used. $C_A B$ is also used. $C_A B$ is mainly used when B is a subset of A, and the symbol A may be omitted if it is clear from the context which set A is considered.
2-6.14	(a, b)	ordered pair a, b, couple a, b	$(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. If the comma can be mistaken as the decimal sign, then the semicolon (;) or a stroke (|) may be used as separator.
2-6.15	$(a_1, a_2, \ldots, a_n)$	ordered n-tuple	See remark to 2-6.14.
2-6.16	$A \times B$	Cartesian product of the sets A and B	The set of ordered pairs (a, b) such that $a \in A$ and $b \in B$. $A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$
2-6.17	$\prod_{i=1}^{n} A_i$	Cartesian product of the sets $A_1, A_2, \ldots, A_n$ $\prod_{i=1}^{n} A_i = A_1 \times \ldots \times A_n$	The set of ordered n-tuples $(x_1, x_2, \ldots, x_n)$ such that $x_1 \in A_1, x_2 \in A_2, \ldots, x_n \in A_n$. $A \times A \times \ldots \times A$ is denoted by A^n, where n is the number of factors in the product.
2-6.18	id_A	identity relation on set A, diagonal of $A \times A$	id_A is the set of all pairs (x, x) where $x \in A$. If the set A is clear from the context, the subscript A can be omitted.

7 Standard number sets and intervals

Table 3 — Symbols and expressions for standard number sets and intervals

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-7.1	**N**	the set of natural numbers, the set of positive integers and zero	**N** = {0, 1, 2, 3, …} **N*** = {1, 2, 3, …} Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{N}_{>5} = \{n \in \mathbf{N} \mid n > 5\}$ The symbols $\mathbb{N}$ and ℕ are also used.

Table 3 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-7.2	$\mathbf{Z}$	the set of integers	$\mathbf{Z} = \{..., -2, -1, 0, 1, 2, ...\}$ $\mathbf{Z}^* = \{n \in \mathbf{Z} \mid n \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Z}_{>-3} = \{n \in \mathbf{Z} \mid n > -3\}$ The symbol $\mathbb{Z}$ is also used.
2-7.3	$\mathbf{Q}$	the set of rational numbers	$\mathbf{Q}^* = \{r \in \mathbf{Q} \mid r \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{Q}_{<0} = \{r \in \mathbf{Q} \mid r < 0\}$ The symbols $\mathbb{Q}$ and $\mathbb{Q}$ are also used.
2-7.4	$\mathbf{R}$	the set of real numbers	$\mathbf{R}^* = \{x \in \mathbf{R} \mid x \neq 0\}$ Other restrictions can be indicated in an obvious way, as shown below. $\mathbf{R}_{>0} = \{x \in \mathbf{R} \mid x > 0\}$ The symbols $\mathbb{R}$ and $\mathbb{R}$ are also used.
2-7.5	$\mathbf{C}$	the set of complex numbers	$\mathbf{C}^* = \{z \in \mathbf{C} \mid z \neq 0\}$ The symbol $\mathbb{C}$ is also used.
2-7.6	$\mathbf{P}$	the set of prime numbers	$\mathbf{P} = \{2, 3, 5, 7, 11, 13, 17, ...\}$ The symbol $\mathbb{P}$ is also used.
2-7.7	$[a, b]$	closed interval from *a* included to *b* included	$[a, b] = \{x \in \mathbf{R} \mid a \leq x \leq b\}$
2-7.8	$(a, b]$	left half-open interval from *a* excluded to *b* included	$(a, b] = \{x \in \mathbf{R} \mid a < x \leq b\}$ The notation $]a, b]$ is also used.
2-7.9	$[a, b)$	right half-open interval from *a* included to *b* excluded	$[a, b) = \{x \in \mathbf{R} \mid a \leq x < b\}$ The notation $[a, b[$ is also used.
2-7.10	(a, b)	open interval from *a* excluded to *b* excluded	$(a, b) = \{x \in \mathbf{R} \mid a < x < b\}$ The notation $]a, b[$ is also used.
2-7.11	$(-\infty, b]$	closed unbounded interval up to *b* included	$(-\infty, b] = \{x \in \mathbf{R} \mid x \leq b\}$ The notation $]-\infty, b]$ is also used.
2-7.12	$(-\infty, b)$	open unbounded interval up to *b* excluded	$(-\infty, b) = \{x \in \mathbf{R} \mid x < b\}$ The notation $]-\infty, b[$ is also used.
2-7.13	$[a, +\infty)$	closed unbounded interval onward from *a* included	$[a, +\infty) = \{x \in \mathbf{R} \mid a \leq x\}$ The notations $[a, \infty)$, $[a, +\infty[$ and $[a, \infty[$ are also used.

Table 3 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-7.14	$(a, +\infty)$	open unbounded interval onward from a excluded	$(a, +\infty) = \{x \in \mathbf{R} \mid a < x\}$ The notations (a, ∞), $]a, +\infty[$ and $]a, \infty[$ are also used.

8 Miscellaneous symbols

Table 4 — Miscellaneous symbols and expressions

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-8.1	$a = b$	a is equal to b a equals b	The symbol $\equiv$ may be used to emphasize that a particular equality is an identity, i.e. holds universally. But see 2-8.18 for another meaning.
2-8.2	$a \neq b$	a is not equal to b	The negating stroke may also be vertical.
2-8.3	$a := b$	a is by definition equal to b	EXAMPLE $\boldsymbol{p} := m\boldsymbol{v}$, where $\boldsymbol{p}$ is momentum, m is mass and $\boldsymbol{v}$ is velocity. The symbols $=_{\text{def}}$ and $\stackrel{\text{def}}{=}$ are also used.
2-8.4	$a \triangleq b$	a corresponds to b	EXAMPLES When $E = kT$, then $1\ \text{eV} \triangleq 11\ 604{,}5\ \text{K}$. When 1 cm on a map corresponds to a length of 10 km, one may write $1\ \text{cm} \triangleq 10\ \text{km}$. The correspondence is not symmetric.
2-8.5	$a \approx b$	a is approximately equal to b	It depends on the user whether an approximation is sufficiently good. Equality is not excluded.
2-8.6	$a \simeq b$	a is asymptotically equal to b	EXAMPLE $\frac{1}{\sin(x-a)} \simeq \frac{1}{x-a}$ as $x \to a$ (For $x \to a$, see 2-8.16.)
2-8.7	$a \sim b$	a is proportional to b	The symbol $\sim$ is also used for equivalence relations. The notation $a \propto b$ is also used.
2-8.8	$M \cong N$	M is congruent to N, M is isomorphic to N	M and N are point sets (geometrical figures). This symbol is also used for isomorphisms of mathematical structures.
2-8.9	$a < b$	a is less than b	
2-8.10	$b > a$	b is greater than a	
2-8.11	$a \leq b$	a is less than or equal to b	
2-8.12	$b \geq a$	b is greater than or equal to a	
2-8.13	$a \ll b$	a is much less than b	It depends on the situation whether a is sufficiently small as compared to b.

Table 4 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-8.14	$b \gg a$	b is much greater than a	It depends on the situation whether b is sufficiently great as compared to a.
2-8.15	∞	infinity	This symbol does *not* denote a number but is often part of various expressions dealing with limits. The notations $+\infty$, $-\infty$ are also used.
2-8.16	$x \to a$	x tends to a	This symbol occurs as part of various expressions dealing with limits. a may be also ∞, $+\infty$, or $-\infty$.
2-8.17	$m \mid n$	m divides n	For integers m and n: $\exists\, k \in \mathbf{Z} \quad m \cdot k = n$
2-8.18	$n \equiv k \bmod m$	n is congruent to k modulo m	For integers n, k and m: $m \mid (n - k)$ This concept of number theory must not be confused with identity of an equation, mentioned in 2-8.1, column 4.
2-8.19	$(a + b)$ $[a + b]$ $\{a + b\}$ $\langle a + b \rangle$	parentheses square brackets braces angle brackets	It is recommended to use only parentheses for grouping, since brackets and braces often have a specific meaning in particular fields. Parentheses can be nested without ambiguity.

9 Elementary geometry

Table 5 — Symbols and expressions in elementary geometry

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-9.1	AB∥CD	the straight line AB is parallel to the straight line CD	It is written g ∥ h if g and h are the straight lines determined by the points A and B, and the points C and D, respectively.
2-9.2	AB⊥CD	the straight line AB is perpendicular to the straight line CD	It is written g ⊥ h if g and h are the straight lines determined by the points A and B, and the points C and D, respectively. In a plane, the straight lines intersect.
2-9.3	∢ABC	angle at vertex B in the triangle ABC	The angle is not oriented, it holds that ∢ABC = ∢CBA and 0 ≤ ∢ABC ≤ π rad. For a more general definition including rotation angles see ISO 80000-3.
2-9.4	$\overline{\mathrm{AB}}$	line segment from A to B	The line segment is the set of points between A and B on the straight line AB including the end points A and B.
2-9.5	$\overrightarrow{\mathrm{AB}}$	vector from A to B	If $\overrightarrow{\mathrm{AB}} = \overrightarrow{\mathrm{CD}}$ then B, seen from A, is in the same direction and distance as D is, seen from C. It does not follow that A = C and B = D.

Table 5 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-9.6	$d(\mathrm{A}, \mathrm{B})$	distance between points A and B	The distance is the length of the line segment $\overline{\mathrm{AB}}$ and also the magnitude of the vector $\overrightarrow{\mathrm{AB}}$.

10 Operations

Table 6 — Symbols and expressions for mathematical operations

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-10.1	$a + b$	*a* plus *b*	This operation is named addition. The symbol + is the addition symbol.
2-10.2	$a - b$	*a* minus *b*	This operation is named subtraction. The symbol − is the subtraction symbol.
2-10.3	$a \pm b$	*a* plus or minus *b*	This is a combination of two values into one expression.
2-10.4	$a \mp b$	*a* minus or plus *b*	$-(a \pm b) = -a \mp b$
2-10.5	$a \cdot b$ $a \times b$ $a\ b$ ab	*a* multiplied by *b*, *a* times *b*	This operation is named multiplication. The symbol for multiplication is a half-high dot (·) or a cross (×). Either symbol may be omitted if no misunderstanding is possible. See also 2-6.16, 2-6.17, 2-18.11, 2-18.12, 2-18.23 and 2-18.24 for the use of the dot and cross in various products.
2-10.6	$\frac{a}{b}$ a/b $a : b$	*a* divided by *b*	$\frac{a}{b} = a \cdot b^{-1}$ The symbol : is often used for ratios of quantity values of the same dimension. The symbol ÷ should not be used.
2-10.7	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$, sum of $a_1, a_2, \ldots, a_n$	The notations $\sum_{i=1}^{n} a_i$, $\sum_{i} a_i$, $\sum_i a_i$ and $\sum a_i$ are also used.
2-10.8	$\prod_{i=1}^{n} a_i$	$a_1 \cdot a_2 \cdot \ldots \cdot a_n$, product of $a_1, a_2, \ldots, a_n$	The notations $\prod_{i=1}^{n} a_i$, $\prod_{i} a_i$, $\prod_i a_i$ and $\prod a_i$ are also used.
2-10.9	a^p	*a* to the power *p*	The verbal equivalent of a^2 is *a* squared; the verbal equivalent of a^3 is *a* cubed.
2-10.10	$a^{1/2}$ $\sqrt{a}$	*a* to the power 1/2, square root of *a*	If $a \geq 0$, then $\sqrt{a} \geq 0$. The symbol $\surd a$ should be avoided. See remark to 2-10.11.

Table 6 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-10.11	$a^{1/n}$ $\sqrt[n]{a}$	a to the power $1/n$, n^{th} root of a	If $a \geq 0$, then $\sqrt[n]{a} \geq 0$. The symbol without the upper line ${}^n\surd a$ should be avoided. If however the symbol ${}^n\surd$ or $\surd$ is used acting on a composite expression, parentheses shall be used to avoid ambiguity.
2-10.12	$\bar{x}$ $\langle x \rangle$ $\bar{x}_a$	mean value of x, arithmetic mean of x	Mean values obtained by other methods are the — harmonic mean denoted by subscript h, — geometric mean denoted by subscript g, — quadratic mean, often called “root mean square”, denoted by subscript q or rms. The subscript may only be omitted for the arithmetic mean. In mathematics, $\bar{x}$ is also used for the complex conjugate of x; see 2-15.6.
2-10.13	sgn a	signum a	For real a: $\operatorname{sgn} a = \begin{cases} 1 \text{ if } a>0 \\ 0 \text{ if } a=0 \\ -1 \text{ if } a<0 \end{cases}$ See also item 2-15.7. Sometimes sgn 0 is left undefined.
2-10.14	inf M	infimum of M	Greatest lower bound of a non-empty set of numbers bounded from below.
2-10.15	sup M	supremum of M	Smallest upper bound of a non-empty set of numbers bounded from above.
2-10.16	$\|a\|$	absolute value of a, modulus of a, magnitude of a	The notation abs a is also used. The symbol || is also used for cardinality of a set (see 2-6.5), modulus of a complex number (2-15.4) and magnitude of a vector (see 2-18.4).
2-10.17	$\lfloor a \rfloor$	floor a, the greatest integer less than or equal to the real number a	The notation ent a is also used. EXAMPLES $\lfloor 2,4 \rfloor = 2$ $\lfloor -2,4 \rfloor = -3$
2-10.18	$\lceil a \rceil$	ceil a, the least integer greater than or equal to the real number a	“ceil” is an abbreviation of the English word “ceiling”. EXAMPLES $\lceil 2,4 \rceil = 3$ $\lceil -2,4 \rceil = -2$

Table 6 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-10.19	int a	integer part of the real number a	$\operatorname{int} a = \operatorname{sgn} a \cdot \lfloor \lvert a \rvert \rfloor$ EXAMPLES int(2,4) = 2 int(−2,4) = −2
2-10.20	frac a	fractional part of the real number a	frac a = a − int a EXAMPLES frac(2,4) = 0,4 frac(−2,4) = −0,4
2-10.21	min(a, b)	minimum of a and b	The operation generalizes to more than two numbers and to sets of numbers. However, an infinite set of numbers need not have a smallest element, in this case use inf (see 2-10.14).
2-10.22	max(a, b)	maximum of a and b	The operation generalizes to more than two numbers and to sets of numbers. However, an infinite set of numbers need not have a greatest element, in this case use sup (see 2-10.15).

11 Combinatorics

In this clause, n and k are natural numbers, with $k \leq n$.

Table 7 — Symbols and expressions in combinatorics

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-11.1	$n!$	factorial	$n! = \prod_{k=1}^{n} k = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$ (for $n > 0$) $0! = 1$
2-11.2	$a^{\underline{k}}$	falling factorial	$a^{\underline{k}} = a \cdot (a-1) \cdot \ldots \cdot (a-k+1)$ (for $k > 0$) $a^{\underline{0}} = 1$ a may be a complex number. For a natural number n: $n^{\underline{k}} = \frac{n!}{(n-k)!}$ In combinatorics and statistics, the symbol $(a)_k$ is often used for the falling factorial. In the theory of special functions, however, the same symbol is often used for the rising factorial and called Pochhammer symbol.

Table 7 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-11.3	$a^{\overline{k}}$	rising factorial	$a^{\overline{k}} = a\cdot(a+1)\cdot\ldots\cdot(a+k-1)$ (for $k > 0$) $a^{\overline{0}} = 1$ a may be a complex number. For a natural number n: $n^{\overline{k}} = \frac{(n+k-1)!}{(n-1)!}$ In the theory of special functions, the symbol $(a)_k$ is often used for the rising factorial and called Pochhammer symbol. In combinatorics and statistics, however, the same symbol is often used for the falling factorial.
2-11.4	$\binom{n}{k}$	binomial coefficient	$\binom{n}{k} = \frac{n!}{k!(n-k)!}$ (for $0 \le k \le n$)
2-11.5	B_n	Bernoulli numbers	$\mathrm{B}_n = -\frac{1}{n+1}\sum_{k=0}^{n-1}\binom{n+1}{k}\mathrm{B}_k$ (for $n > 0$) $\mathrm{B}_0 = 1$ $\mathrm{B}_1 = -1/2$, $\mathrm{B}_{2n+3} = 0$
2-11.6	C_n^k	number of combinations without repetition	$\mathrm{C}_n^k = \binom{n}{k} = \frac{n!}{k!(n-k)!}$
2-11.7	${}^{\mathrm{R}}\mathrm{C}_n^k$	number of combinations with repetition	${}^{\mathrm{R}}\mathrm{C}_n^k = \binom{n+k-1}{k}$
2-11.8	V_n^k	number of variations without repetition	$\mathrm{V}_n^k = n^{\underline{k}} = \frac{n!}{(n-k)!}$ The term “permutation” is used when $n = k$.
2-11.9	${}^{\mathrm{R}}\mathrm{V}_n^k$	number of variations with repetition	${}^{\mathrm{R}}\mathrm{V}_n^k = n^k$

12 Functions

Items 2-12.1 up to 2-12.13 concern functions in general, items 2-12.14 to 2-12.27 concern functions with numbers as values as used in calculus.

Table 8 — Symbols and expressions for functions

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-12.1	$f, g, h, \ldots$	functions	A function assigns to any argument in its domain a unique value in its range. The arguments are said to be *mapped* by the function onto the values, which are called *images* of the arguments under the function.
2-12.2	$f(x)$ $f(x_1, \ldots, x_n)$	value of function f for argument x or for argument $(x_1, \ldots, x_n)$, respectively	A function having a set of n-tuples as its domain is an n-place function.
2-12.3	dom f	domain of f	Set of objects to which f assigns a value. $\mathrm{D}(f)$ is also used.
2-12.4	ran f	range of f	Set of values of the function f. $\mathrm{R}(f)$ is also used.
2-12.5	$f: A \to B$	f maps A into B	$\mathrm{dom}\, f = A$ and $\mathrm{ran}\, f \subseteq B$ It is not necessary that all elements of B are values of the function f.
2-12.6	$f: A \twoheadrightarrow B$	f maps A surjectively onto B	$\mathrm{dom}\, f = A$ and $\mathrm{ran}\, f = B$
2-12.7	$f: A \rightarrowtail B$	f maps A injectively into B	$f: A \to B$ and for all $x, y \in A$ if $x \neq y$ then $f(x) \neq f(y)$. The function f is then said to be injective or one-one.
2-12.8	$f: A \rightarrowtail\!\!\!\!\twoheadrightarrow B$	f maps A bijectively onto B	$f: A \twoheadrightarrow B$ and $f: A \rightarrowtail B$
2-12.9	$x \mapsto T(x), x \in A$	function that maps any $x \in A$ onto $T(x)$	$T(x)$ is a defining term denoting the values of some function for the arguments $x \in A$. If this function is called f, then it holds $f(x) = T(x)$ for any $x \in A$. Therefore the defining term $T(x)$ is often used to denote the function f. EXAMPLE $x \mapsto 3x^2y, x \in [0, 2]$ This is the quadratic function (of x depending on the parameter y) defined on the stated interval by the term $3x^2y$. If no function symbol is introduced, the term $3x^2y$ is used to denote this function.
2-12.10	f^{-1}	inverse function of f	The inverse function f^{-1} of a function f is only defined if f is injective. If f is injective then $\mathrm{dom}(f^{-1}) = \mathrm{ran}(f)$, $\mathrm{ran}(f^{-1}) = \mathrm{dom}(f)$, and $f^{-1}(f(x)) = x$ for $x \in \mathrm{dom}\, f$. The inverse function f^{-1} should not be confused with the pointwise reciprocal function $x \mapsto f(x)^{-1}$.
2-12.11	$g \circ f$	composite function of f and g, g circle f	$(g \circ f)(x) = g(f(x))$ In the composite $g \circ f$, the function g is applied after function f has been applied.

Table 8 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-12.12	$x \xrightarrow{f} y$ $f: x \mapsto y$	$f(x) = y$, f maps x onto y	EXAMPLE $\pi \xrightarrow{\cos} -1$
2-12.13	$f\vert_a^b$ $f(\ldots, u, \ldots)\vert_{u=a}^{u=b}$	$f(b) - f(a)$ $f(\ldots, b, \ldots) - f(\ldots, a, \ldots)$	This notation is used mainly when evaluating definite integrals.
2-12.14	$\lim\limits_{x \to a} f(x)$ $\lim_{x \to a} f(x)$	limit of $f(x)$ as x tends to a	$f(x) \to b$ as $x \to a$ may be written for $\lim_{x \to a} f(x) = b$. Limits "from the right" ($x > a$) and "from the left" ($x < a$) are denoted by $\lim_{x \to a+} f(x)$ and $\lim_{x \to a-} f(x)$, respectively.
2-12.15	$f(x) = \mathrm{O}(g(x))$	$f(x)$ is upper case O of $g(x)$, $\lvert f(x)/g(x) \rvert$ is bounded from above in the limit implied by the context, $f(x)$ is of the order comparable with or inferior to $g(x)$	The symbol "=" here is used for historical reasons and does not have the meaning of equality, because transitivity does not apply. EXAMPLE $\sin x = \mathrm{O}(x)$, when $x \to 0$
2-12.16	$f(x) = \mathrm{o}(g(x))$	$f(x)$ is lower case o of $g(x)$, $f(x)/g(x) \to 0$ in the limit implied by the context, $f(x)$ is of the order inferior to $g(x)$	The symbol "=" here is used for historical reasons and does not have the meaning of equality, because transitivity does not apply. EXAMPLE $\cos x = 1 + \mathrm{o}(x)$, when $x \to 0$
2-12.17	Δf	delta f, finite increment of f	Difference of two function values implied by the context. EXAMPLES $\Delta x = x_2 - x_1$ $\Delta f(x) = f(x_2) - f(x_1)$
2-12.18	$\dfrac{\mathrm{d}f}{\mathrm{d}x}$ $\mathrm{d}f/\mathrm{d}x$ f' $\mathrm{D}f$	derivative of f with respect to x	Only to be used for functions of one variable. The independent variable may also be indicated, for example $\dfrac{\mathrm{d}f(x)}{\mathrm{d}x}$, $\mathrm{d}f(x)/\mathrm{d}x$, $f'(x)$ and $\mathrm{D}f(x)$. If the independent variable is time t, $\dot{f}$ is also used for f'.

Table 8 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-12.19	$\left(\frac{\mathrm{d}f}{\mathrm{d}x}\right)_{x=a}$ $(\mathrm{d}f/\mathrm{d}x)_{x=a}$ $f'(a)$ $\mathrm{D}f(a)$	value of the derivative of *f* for $x = a$	See also 2-12.18
2-12.20	$\frac{\mathrm{d}^n f}{\mathrm{d}x^n}$ $\mathrm{d}^n f/\mathrm{d}x^n$ $f^{(n)}$ $\mathrm{D}^n f$	n^{th} derivative of *f* with respect to *x*	Only to be used for functions of one variable. $\frac{\mathrm{d}^n f(x)}{\mathrm{d}x^n}$, $\mathrm{d}^n f(x)/\mathrm{d}x^n$, $f^{(n)}(x)$ and $\mathrm{D}^n f$ are also used. f'' and f''' are also used for $f^{(2)}$ and $f^{(3)}$, respectively. If the independent variable is time *t*, $\ddot{f}$ is also used for f''.
2-12.21	$\frac{\partial f}{\partial x}$ $\partial f/\partial x$ $\partial_x f$	partial derivative of *f* with respect to *x*	Only to be used for functions of several variables. $\frac{\partial f(x,y,...)}{\partial x}$, $\partial f(x, y, ...)/\partial x$, $\partial_x f(x, y, ...)$ and $\mathrm{D}_x f(x,y,...)$ are also used. The other independent variables may be shown as subscripts, e.g. $\left(\frac{\partial f}{\partial x}\right)_{y...}$. This partial derivative notation is extended to derivatives of higher order, e.g. $\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$ $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$ Other notations, e.g. $f_{xy} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$, are also used.
2-12.22	$\mathrm{d}f$	total differential of *f*	$\mathrm{d}f(x,y,...) = \frac{\partial f}{\partial x}\mathrm{d}x + \frac{\partial f}{\partial y}\mathrm{d}y + ...$
2-12.23	δf	(infinitesimal) variation of *f*	This symbol is used in variational calculus.
2-12.24	$\int f(x)\mathrm{d}x$	indefinite integral of *f*	

Table 8 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-12.25	$\int_a^b f(x)\mathrm{d}x$	definite integral of f from a to b	This is the simple case of a function defined on an interval. Integration of functions defined on more general domains may also be defined. Special notations, e.g. $\int_C, \int_S, \int_V, \oint$, are used for integration over a curve C, a surface S, a three-dimensional domain V, and a closed curve or surface, respectively. Multiple integrals are also denoted $\iint, \oiint$, etc.
2-12.26	$⨍_a^b f(x)\,\mathrm{d}x$	Cauchy principal value of the integral of f with singularity at c, where $a<c<b$	$\lim_{\delta\to 0+}\left(\int_a^{c-\delta} f(x)\mathrm{d}x + \int_{c+\delta}^b f(x)\mathrm{d}x\right)$
2-12.27	$⨍_{-\infty}^{\infty} f(x)\,\mathrm{d}x$	Cauchy principal value of the integral of f	$\lim_{a\to\infty} ⨍_{-a}^{a} f(x)\,\mathrm{d}x$ See 2-12.26.

13 Exponential and logarithmic functions

Complex arguments can be used, in particular for the base e.

Table 9 — Symbols and expressions for exponential and logarithmic functions

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-13.1	e	base of natural logarithm	$\mathrm{e} := \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2{,}718\ 81\ 28\ ...$ This number is called Euler number.
2-13.2	a^x	a to the power of x, exponential function to the base a of argument x	See also 2-10.9.
2-13.3	e^x exp x	e to the power of x, exponential function to the base e of argument x	See 2-15.5.
2-13.4	$\log_a x$	logarithm to the base a of argument x	log x is used when the base does not need to be specified.
2-13.5	ln x	natural logarithm of x	$\ln x = \log_\mathrm{e} x$ log x shall not be used in place of ln x, lg x, lb x, or $\log_\mathrm{e} x$, $\log_{10} x$, $\log_2 x$.
2-13.6	lg x	decimal logarithm of x, common logarithm of x	$\lg x = \log_{10} x$ See remark to 2-13.5.
2-13.7	lb x	binary logarithm of x	$\mathrm{lb}\, x = \log_2 x$ See remark to 2-13.5.

14 Circular and hyperbolic functions

Table 10 — Symbols and expressions for circular and hyperbolic functions

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-14.1	π	ratio of the circumference of a circle to its diameter	$\pi = 3{,}141\ 592\ 6\ldots$
2-14.2	$\sin x$	sine of x	$\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$ $\sin x = x - x^3/3! + x^5/5! - \ldots$ $(\sin x)^n$, $(\cos x)^n$, etc. (for $n \geq 2$), are often written $\sin^n x$, $\cos^n x$, etc.
2-14.3	$\cos x$	cosine of x	$\cos x = \sin(x + \pi/2)$
2-14.4	$\tan x$	tangent of x	$\tan x = \sin x/\cos x$ $\text{tg}\, x$ should not be used.
2-14.5	$\cot x$	cotangent of x	$\cot x = 1/\tan x$ $\text{ctg}\, x$ should not be used.
2-14.6	$\sec x$	secant of x	$\sec x = 1/\cos x$
2-14.7	$\csc x$	cosecant of x	$\csc x = 1/\sin x$ $\text{cosec}\, x$ is also used.
2-14.8	$\arcsin x$	arcus sine of x	$y = \arcsin x \Leftrightarrow x = \sin y$ (for $-\pi/2 \leq y \leq \pi/2$) The function arcsin is the inverse of the function sin with the restriction mentioned above.
2-14.9	$\arccos x$	arcus cosine of x	$y = \arccos x \Leftrightarrow x = \cos y$ (for $0 \leq y \leq \pi$) The function arccos is the inverse of the function cos with the restriction mentioned above.
2-14.10	$\arctan x$	arcus tangent of x	$y = \arctan x \Leftrightarrow x = \tan y$ (for $-\pi/2 \leq y \leq \pi/2$) The function arctan is the inverse of the function tan with the restriction mentioned above. $\text{arctg}\, x$ should not be used.
2-14.11	$\text{arccot}\, x$	arcus cotangent of x	$y = \text{arccot}\, x \Leftrightarrow x = \cot y$ (for $0 \leq y \leq \pi$) The function arccot is the inverse of the function cot with the restriction mentioned above. $\text{arcctg}\, x$ should not be used.
2-14.12	$\text{arcsec}\, x$	arcus secant of x	$y = \text{arcsec}\, x \Leftrightarrow x = \sec y$ (for $0 \leq y \leq \pi, y \neq \pi/2$) The function arcsec is the inverse of the function sec with the restriction mentioned above.

Table 10 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-14.13	arccsc x	arcus cosecant of x	$y = \text{arccsc}\, x \Leftrightarrow x = \csc y$ (for $-\pi/2 \le y \le \pi/2, y \ne 0$) The function arccsc is the inverse of the function csc with the restriction mentioned above. arccosec x should be avoided.
2-14.14	sinh x	hyperbolic sine of x	$\sinh x = \frac{e^x - e^{-x}}{2}$ $\sinh x = x + x^3/3! + \ldots$ sh x should be avoided.
2-14.15	cosh x	hyperbolic cosine of x	$\cosh^2 x = \sinh^2 x + 1$ ch x should be avoided.
2-14.16	tanh x	hyperbolic tangent of x	$\tanh x = \sinh x/\cosh x$ th x should be avoided.
2-14.17	coth x	hyperbolic cotangent of x	$\coth x = 1/\tanh x$
2-14.18	sech x	hyperbolic secant of x	$\text{sech}\, x = 1/\cosh x$
2-14.19	csch x	hyperbolic cosecant of x	$\text{csch}\, x = 1/\sinh x$ cosech x should be avoided.
2-14.20	arsinh x	inverse hyperbolic sine of x, area hyperbolic sine of x	$y = \text{arsinh}\, x \Leftrightarrow x = \sinh y$ The function arsinh is the inverse of the function sinh. arsh x should be avoided.
2-14.21	arcosh x	inverse hyperbolic cosine of x, area hyperbolic cosine of x	$y = \text{arcosh}\, x \Leftrightarrow x = \cosh y$ (for $y \ge 0$) The function arcosh is the inverse of the function cosh with the restriction mentioned above. arch x should be avoided.
2-14.22	artanh x	inverse hyperbolic tangent of x, area hyperbolic tangent of x	$y = \text{artanh}\, x \Leftrightarrow x = \tanh y$ The function artanh is the inverse of the function tanh. arth x should be avoided.
2-14.23	arcoth x	inverse hyperbolic cotangent of x, area hyperbolic cotangent of x	$y = \text{arcoth}\, x \Leftrightarrow x = \coth y$ (for $y \ne 0$) The function arcoth is the inverse of the function coth with the restriction mentioned above.
2-14.24	arsech x	inverse hyperbolic secant of x, area hyperbolic secant of x	$y = \text{arsech}\, x \Leftrightarrow x = \text{sech}\, y$ (for $y \ge 0$) The function arsech is the inverse of the function sech with the restriction mentioned above.
2-14.25	arcsch x	inverse hyperbolic cosecant of x, area hyperbolic cosecant of x	$y = \text{arcsch}\, x \Leftrightarrow x = \text{csch}\, y$ (for $y \ge 0$) The function arcsch is the inverse of the function csch with the restriction mentioned above. arcosech x should be avoided.

15 Complex numbers

Table 11 — Symbols and expressions for complex numbers

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-15.1	i j	imaginary unit	$i^2 = j^2 = -1$ i is used in mathematics and in physics, j is used in electrotechnology.
2-15.2	Re *z*	real part of *z*	If $z = x + \mathrm{i}\,y$, where *x* and *y* are real numbers, then $x = \mathrm{Re}\,z$ and $y = \mathrm{Im}\,z$.
2-15.3	Im *z*	imaginary part of *z*	See 2-15.2.
2-15.4	$\lvert z \rvert$	modulus of *z*	$\lvert z \rvert = \sqrt{x^2 + y^2}$ where $x = \mathrm{Re}\,z$ and $y = \mathrm{Im}\,z$. The symbol || is also used for cardinality of a set (see 2-6.5), absolute value of a real number (see 2-10.16), and magnitude of a vector (see 2-18.4).
2-15.5	arg *z*	argument of *z*	If $z = r\,\mathrm{e}^{\mathrm{i}\varphi}$, where $r = \lvert z \rvert$ and $-\pi < \varphi \leq \pi$, then $\varphi = \arg z$. It holds $\mathrm{Re}\,z = r \cos \varphi$ and $\mathrm{Im}\,z = r \sin \varphi$.
2-15.6	$\bar{z}$ z^*	complex conjugate of *z*	$\bar{z} = \mathrm{Re}\,z - \mathrm{i}\,\mathrm{Im}\,z$ $\bar{z}$ is mainly used in mathematics, z^* is mainly used in physics and engineering.
2-15.7	sgn *z*	signum *z*	$\mathrm{sgn}\,z = z / \lvert z \rvert = \exp(\mathrm{i}\arg z)\ (z \neq 0)$ $\mathrm{sgn}\,z = 0$ for $z = 0$ See also item 2-10.13. Sometimes sgn 0 is left undefined.

16 Matrices

Matrices are usually written with boldface italic capital letters and their elements with thin italic lower case letters, but other typefaces may also be used.

Table 12 — Symbols and expressions for matrices

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-16.1	$\boldsymbol{A}$ $\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$	matrix $\boldsymbol{A}$ of type *m* by *n*	$\boldsymbol{A}$ is the matrix with the elements $a_{ij} = (\boldsymbol{A})_{ij}$, *m* is the number of rows and *n* is the number of columns. $\boldsymbol{A} = (a_{ij})$ is also used. Square brackets are also used instead of parentheses.

Table 12 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-16.2	$\boldsymbol{A} + \boldsymbol{B}$	sum of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$	$(\boldsymbol{A} + \boldsymbol{B})_{ij} = (\boldsymbol{A})_{ij} + (\boldsymbol{B})_{ij}$ The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ must have the same number of columns and the same number of rows.
2-16.3	$x\boldsymbol{A}$	product of scalar x and matrix $\boldsymbol{A}$	$(x\boldsymbol{A})_{ij} = x(\boldsymbol{A})_{ij}$
2-16.4	$\boldsymbol{AB}$	product of matrices $\boldsymbol{A}$ and $\boldsymbol{B}$	$(\boldsymbol{AB})_{ik} = \sum_j (\boldsymbol{A})_{ij}(\boldsymbol{B})_{jk}$ The number of columns of $\boldsymbol{A}$ must be equal to the number of rows of $\boldsymbol{B}$.
2-16.5	$\boldsymbol{I}$ $\boldsymbol{E}$	identity matrix	A square matrix for which $(\boldsymbol{I})_{ik} = \delta_{ik}$ For the definition of δ_{ik} see 2-18.9.
2-16.6	$\boldsymbol{A}^{-1}$	inverse of a square matrix $\boldsymbol{A}$	$\boldsymbol{AA}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}$, when $\det \boldsymbol{A} \neq 0$, for the definition of $\det \boldsymbol{A}$ see 2-16.10.
2-16.7	$\boldsymbol{A}^{\mathrm{T}}$	transpose matrix of $\boldsymbol{A}$	$(\boldsymbol{A}^{\mathrm{T}})_{ik} = (\boldsymbol{A})_{ki}$
2-16.8	$\bar{\boldsymbol{A}}$ $\boldsymbol{A}^*$	complex conjugate matrix of $\boldsymbol{A}$	$(\bar{\boldsymbol{A}})_{ik} = \overline{(\boldsymbol{A})_{ik}}$ $\bar{\boldsymbol{A}}$ is used in mathematics, $\boldsymbol{A}^*$ in physics and electrotechnology.
2-16.9	$\boldsymbol{A}^{\mathrm{H}}$	Hermitian conjugate matrix of $\boldsymbol{A}$	$\boldsymbol{A}^{\mathrm{H}} = (\bar{\boldsymbol{A}})^{\mathrm{T}}$ The term "adjoint matrix" is also used. $\boldsymbol{A}^*$ and $\boldsymbol{A}^+$ are also used for $\boldsymbol{A}^{\mathrm{H}}$.
2-16.10	$\det \boldsymbol{A}$ $\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix}$	determinant of a square matrix $\boldsymbol{A}$	Sometimes $\lvert\boldsymbol{A}\rvert$ is used.
2-16.11	$\operatorname{rank} \boldsymbol{A}$	rank of matrix $\boldsymbol{A}$	The rank of matrix $\boldsymbol{A}$ is the number of the linearly independent rows of $\boldsymbol{A}$. It is also equal to the number of linearly independent columns.
2-16.12	$\operatorname{tr} \boldsymbol{A}$	trace of a square matrix $\boldsymbol{A}$	$\operatorname{tr} \boldsymbol{A} = \sum_i (\boldsymbol{A})_{ii}$
2-16.13	$\lVert\boldsymbol{A}\rVert$	norm of matrix $\boldsymbol{A}$	The norm of matrix $\boldsymbol{A}$ is a number characterizing this matrix and satisfying the triangle inequality: if $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{C}$, then $\lVert\boldsymbol{A}\rVert + \lVert\boldsymbol{B}\rVert \geq \lVert\boldsymbol{C}\rVert$. Different matrix norms are used.

17 Coordinate systems

In this section, some coordinate systems in three-dimensional space of classical physics are considered. A point O has to be fixed as *origin* of the coordinate system. Any point P is determined by the *position vector* from the origin O to the point P.

Table 13 — Coordinate systems in three-dimensional space

Item No.	Coordinates	Position vector and its differential	Name of coordinates	Remarks
2-17.1	x, y, z	$\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$ $\mathrm{d}\boldsymbol{r} = \mathrm{d}x\,\boldsymbol{e}_x + \mathrm{d}y\,\boldsymbol{e}_y + \mathrm{d}z\,\boldsymbol{e}_z$	Cartesian coordinates	The base vectors $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ form an orthonormal system (i.e. the vectors are unit vectors and ortho-gonal) which is right-handed, see Figures 1 and 4. $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$ or $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are also used for the base vectors and x_1, x_2, x_3 or i, j, k for the coordinates.
2-17.2	ρ, φ, z	$\boldsymbol{r} = \rho\,\boldsymbol{e}_\rho + z\,\boldsymbol{e}_z$ $\mathrm{d}\boldsymbol{r} = \mathrm{d}\rho\,\boldsymbol{e}_\rho + \rho\,\mathrm{d}\varphi\,\boldsymbol{e}_\varphi + \mathrm{d}z\,\boldsymbol{e}_z$	cylindrical coordinates	$\boldsymbol{e}_\rho(\varphi), \boldsymbol{e}_\varphi(\varphi), \boldsymbol{e}_z$ form an orthonormal right-handed system. See Figure 2. If $z = 0$, then ρ and φ are polar coordinates in the plane.
2-17.3	r, ϑ, φ	$\boldsymbol{r} = r\,\boldsymbol{e}_r$ $\mathrm{d}\boldsymbol{r} = \mathrm{d}r\,\boldsymbol{e}_r + r\,\mathrm{d}\vartheta\,\boldsymbol{e}_\vartheta + r\sin\vartheta\,\mathrm{d}\varphi\,\boldsymbol{e}_\varphi$	spherical coordinates	$\boldsymbol{e}_r(\vartheta,\varphi), \boldsymbol{e}_\vartheta(\vartheta, \varphi), \boldsymbol{e}_\varphi(\varphi)$ form an orthonormal right-handed system. See Figure 3.
NOTE If, exceptionally, instead of a right-handed system (see Figure 4), a left-handed system (see Figure 5) is used for certain purposes, this shall be clearly stated to avoid the risk of sign errors.				

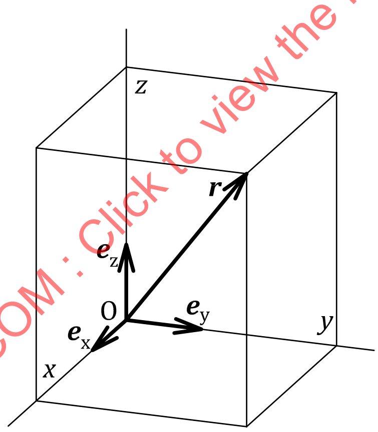

Figure 1 — Right-handed Cartesian coordinate system

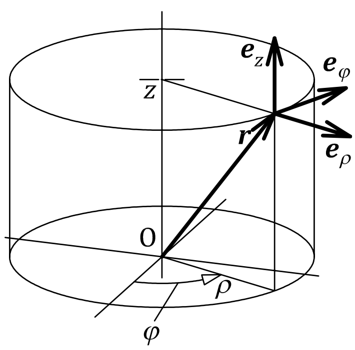

Figure 2 — Right-handed cylindrical coordinate system

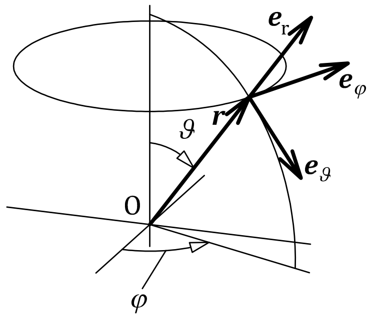

Figure 3 — Right-handed spherical coordinate system

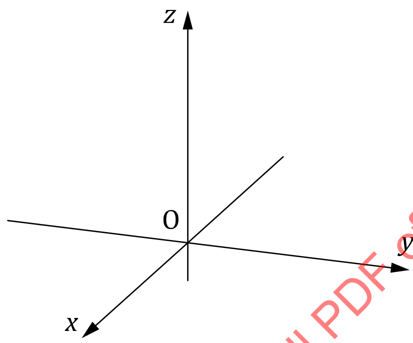

Figure 4 — Right-handed coordinate system

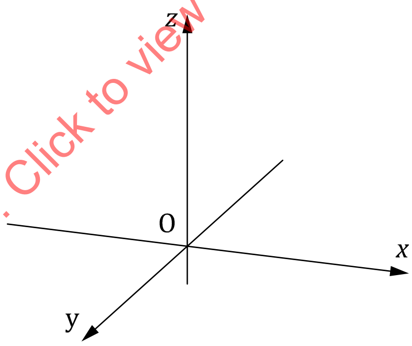

Figure 5 — Left-handed coordinate system

18 Scalars, vectors and tensors

In this section, $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ are used for base vectors. This notation easily generalizes to n-dimensional space (n a natural number, not necessarily 3), which is indispensable in mathematics and also important in physics. Many concepts in this section apply to this more general case. The mathematical concept of dimensionality of a space is a more general concept than the concept of quantity dimension, which is explained in ISO 80000-1.

Scalars, vectors and tensors are mathematical objects that can be used to denote certain physical quantities and their values. They are as such independent of the particular choice of a coordinate system, whereas each scalar component of a vector or a tensor and each component vector and component tensor depend on that choice. A vector is a tensor of the first order and a scalar is a tensor of order zero.

With respect to some base vectors $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, $\boldsymbol{e}_3$ each vector $\boldsymbol{a}$ has a representation $\boldsymbol{a} = a_1\boldsymbol{e}_1 + a_2\boldsymbol{e}_2 + a_3\boldsymbol{e}_3$ where a_1, a_2 and a_3 are uniquely determined scalar quantity values called “coordinates” of the vector with respect to the base vectors, the vectors $a_1\boldsymbol{e}_1$, $a_2\boldsymbol{e}_2$ and $a_3\boldsymbol{e}_3$ are called “component vectors” of the vector with respect to the base vectors.

The word “component” shall not be used alone, since it might be confused with “component vector”.

Instead of treating each coordinate of a vector as a physical quantity value (i.e. a number multiplied by a unit), the vector could be written as a numerical vector multiplied by a unit. All units are scalars.

EXAMPLE Force acting on a given particle, e.g. in Cartesian components $(F_x; F_y; F_z) = (-31{,}5; 43{,}2; 17{,}0)$ N.

The same considerations apply to tensors of second and higher orders.

In this section, only Cartesian (orthonormal) coordinates in ordinary space are considered. The more general cases requiring covariant and contravariant representations are not treated here. The Cartesian coordinates are denoted by x, y, z; by a_1, a_2, a_3; or by x_1, x_2, x_3.

If subscripts i, j, k, l, each ranging from 1 to 3, are used, then the following summation convention is sometimes used: If a subscript appears twice in a term, summation over the range of this subscript is understood, and the sign Σ may be omitted.

Vectors and tensors are often represented by general symbols for their scalar components, e.g. a_i for a vector, T_{ij} for a tensor of the second order, and a_ib_j for a dyadic product.

Table 14 — Symbols and expressions for scalars, vectors and tensors

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-18.1	$\boldsymbol{a}$ $\vec{a}$	vector $\boldsymbol{a}$	An arrow above the letter symbol can be used instead of bold face type to indicate a vector.
2-18.2	$\boldsymbol{a} + \boldsymbol{b}$	sum of vectors $\boldsymbol{a}$ and $\boldsymbol{b}$	$(\boldsymbol{a} + \boldsymbol{b})_i = a_i + b_i$
2-18.3	$x\boldsymbol{a}$	product of a number, scalar, or component x and vector $\boldsymbol{a}$	$(x\boldsymbol{a})_i = xa_i$
2-18.4	$\lvert\boldsymbol{a}\rvert$	magnitude of the vector $\boldsymbol{a}$, norm of the vector $\boldsymbol{a}$	$\lvert\boldsymbol{a}\rvert = \sqrt{a_x^2 + a_y^2 + a_z^2}$ $\lVert\boldsymbol{a}\rVert$ is also used. The symbol $\lvert\,\rvert$ is also used for cardinality of a set (see 2-6.5), absolute value of a real number (see 2-10.16), modulus of a complex number (2-15.4).
2-18.5	$\boldsymbol{0}$ $\vec{0}$	zero vector	The zero vector has magnitude 0.
2-18.6	$\boldsymbol{e}_{\boldsymbol{a}}$ $\hat{\boldsymbol{a}}$	unit vector in the direction of $\boldsymbol{a}$	$\boldsymbol{e}_{\boldsymbol{a}} = \hat{\boldsymbol{a}} = \boldsymbol{a}/\lvert\boldsymbol{a}\rvert$, (for $\boldsymbol{a} \neq \boldsymbol{0}$)
2-18.7	$\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$	unit vectors in the directions of the Cartesian coordinate axes	$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are also used.
2-18.8	a_x, a_y, a_z a_i	Cartesian coordinates of vector $\boldsymbol{a}$, Cartesian components of vector $\boldsymbol{a}$	$\boldsymbol{a} = a_x\,\boldsymbol{e}_x + a_y\,\boldsymbol{e}_y + a_z\,\boldsymbol{e}_z$ $a_1\,\boldsymbol{e}_1, a_2\,\boldsymbol{e}_2, a_3\,\boldsymbol{e}_3$ are the component vectors. If it is clear from the context which are the base vectors, the vector can be written $\boldsymbol{a} = (a_x, a_y, a_z)$. $a_x = \boldsymbol{a}\cdot\boldsymbol{e}_x$, $a_y = \boldsymbol{a}\cdot\boldsymbol{e}_y$, $a_z = \boldsymbol{a}\cdot\boldsymbol{e}_z$ $\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$ is the position vector (radius vector) of the point with coordinates x, y, z.

Table 14 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-18.9	δ_{ik}	Kronecker delta symbol	$\delta_{ik} = \begin{cases} 1 & \text{for} \quad i=k \\ 0 & \text{for} \quad i \neq k \end{cases}$
2-18.10	ε_{ijk}	Levi-Civita symbol	$\varepsilon_{123} = \varepsilon_{231} = \varepsilon_{312} = 1$ $\varepsilon_{132} = \varepsilon_{321} = \varepsilon_{213} = -1$ All other ε_{ijk} are equal to 0.
2-18.11	$\boldsymbol{a} \cdot \boldsymbol{b}$	scalar product of ***a*** and ***b***	$\boldsymbol{a} \cdot \boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z$ $\boldsymbol{a} \cdot \boldsymbol{b} = \sum_i a_i b_i$ $\boldsymbol{a} \cdot \boldsymbol{a} = \boldsymbol{a}^2 = \lvert\boldsymbol{a}\rvert^2 = a^2$ In special fields, $(\boldsymbol{a}, \boldsymbol{b})$ or $< a, b >$ is also used.
2-18.12	$\boldsymbol{a} \times \boldsymbol{b}$	vector product of ***a*** and ***b***	The coordinates, in a right-handed Cartesian coordinate system, are $(\boldsymbol{a} \times \boldsymbol{b})_x = a_y b_z - a_z b_y$ $(\boldsymbol{a} \times \boldsymbol{b})_y = a_z b_x - a_x b_z$ $(\boldsymbol{a} \times \boldsymbol{b})_z = a_x b_y - a_y b_x$ $(\boldsymbol{a} \times \boldsymbol{b})_i = \sum_j \sum_k \varepsilon_{ijk} a_j b_k$ See 2-18.10.
2-18.13	∇ $\vec{\nabla}$	nabla operator	$\nabla = \boldsymbol{e}_x \frac{\partial}{\partial x} + \boldsymbol{e}_y \frac{\partial}{\partial y} + \boldsymbol{e}_z \frac{\partial}{\partial z} = \sum_i \boldsymbol{e}_i \frac{\partial}{\partial x_i}$ The operator is also called “del operator”.
2-18.14	$\nabla \varphi$ **grad** φ	gradient of φ	$\nabla \varphi = \sum_i \boldsymbol{e}_i \frac{\partial \varphi}{\partial x_i}$ The operator **grad** should be written in bold face.
2-18.15	$\nabla \cdot \boldsymbol{a}$ **div** ***a***	divergence of ***a***	$\nabla \cdot \boldsymbol{a} = \sum_i \frac{\partial a_i}{\partial x_i}$ The operator **div** should be written in bold face.

Table 14 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-18.16	$\nabla \times \boldsymbol{a}$ **rot** $\boldsymbol{a}$	rotation of $\boldsymbol{a}$	The coordinates are $(\nabla \times \boldsymbol{a})_x = \frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}$ $(\nabla \times \boldsymbol{a})_y = \frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}$ $(\nabla \times \boldsymbol{a})_z = \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}$ The operator **curl** should be avoided and the operator **rot** should be written in bold face. $(\nabla \times \boldsymbol{a})_i = \sum_j \sum_k \varepsilon_{ijk} \frac{\partial a_k}{\partial x_j}$ See 2-18.10.
2-18.17	∇^2 Δ	Laplacian	$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$
2-18.18	$\square$	d'Alembertian	$\square = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ where the independent variable t is time (ISO 80000-3) and c is the speed (ISO 80000-3) of light
2-18.19	$\boldsymbol{\mathsf{T}}$ $\overset{\Rightarrow}{T}$	tensor $\boldsymbol{\mathsf{T}}$ of the second order	Two arrows above the letter symbol can be used instead of bold face sans serif type to indicate a tensor of the second order.
2-18.20	$T_{xx}, T_{xy}, \ldots, T_{zz}$ $T_{11}, T_{12}, \ldots, T_{33}$	Cartesian coordinates of tensor $\boldsymbol{\mathsf{T}}$, Cartesian components of tensor $\boldsymbol{\mathsf{T}}$	The scalars T_{xx}, T_{xy}, ..., T_{zz} in the representation $\boldsymbol{\mathsf{T}} = T_{xx}\boldsymbol{e}_x\boldsymbol{e}_x + T_{xy}\boldsymbol{e}_x\boldsymbol{e}_y + \ldots + T_{zz}\boldsymbol{e}_z\boldsymbol{e}_z$ of the tensor $\boldsymbol{\mathsf{T}}$ by the base tensors $\boldsymbol{e}_x\boldsymbol{e}_x, \boldsymbol{e}_x\boldsymbol{e}_y, \ldots, \boldsymbol{e}_z\boldsymbol{e}_z$. $T_{xx}\boldsymbol{e}_x\boldsymbol{e}_x$, $T_{xy}\boldsymbol{e}_x\boldsymbol{e}_y$, ... , $T_{zz}\boldsymbol{e}_z\boldsymbol{e}_z$, are the component tensors. If it is clear from the context which are the base vectors, the tensor can be written $\boldsymbol{\mathsf{T}} = \begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$.
2-18.21	$\boldsymbol{ab}$ $\boldsymbol{a} \otimes \boldsymbol{b}$	dyadic product, tensor product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$	tensor of the second order with coordinates $(\boldsymbol{ab})_{ij} = a_i b_j$
2-18.22	$\boldsymbol{\mathsf{T}} \otimes \boldsymbol{\mathsf{S}}$	tensor product of two tensors $\boldsymbol{\mathsf{T}}$ and $\boldsymbol{\mathsf{S}}$ of the second order	tensor of the fourth order with coordinates $(\boldsymbol{\mathsf{T}} \otimes \boldsymbol{\mathsf{S}})_{ijkl} = T_{ij}S_{kl}$

Table 14 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-18.23	$\boldsymbol{T} \cdot \boldsymbol{S}$	inner product of two tensors $\boldsymbol{T}$ and $\boldsymbol{S}$ of the second order	tensor of the second order with coordinates $(\boldsymbol{T} \cdot \boldsymbol{S})_{ik} = \sum_j T_{ij} S_{jk}$
2-18.24	$\boldsymbol{T} \cdot \boldsymbol{a}$	inner product of a tensor $\boldsymbol{T}$ of the second order and a vector $\boldsymbol{a}$	vector with coordinates $(\boldsymbol{T} \cdot \boldsymbol{a})_i = \sum_j T_{ij} a_j$
2-18.25	$\boldsymbol{T} : \boldsymbol{S}$	scalar product of two tensors $\boldsymbol{T}$ and $\boldsymbol{S}$ of the second order	scalar quantity $\boldsymbol{T} : \boldsymbol{S} = \sum_i \sum_j T_{ij} S_{ij}$

19 Transforms

Table 15 — Symbols and expressions for transforms

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-19.1	$\mathcal{F}f$	Fourier transform of f	$(\mathcal{F}f)(\omega) = \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\omega t} f(t)\,\mathrm{d}t$ (where $\omega \in \mathbf{R}$) This is often denoted by $\mathcal{F}(\omega)$. $(\mathcal{F}f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\omega t} f(t)\,\mathrm{d}t$ is also used.
2-19.2	$\mathcal{L}f$	Laplace transform of f	$(\mathcal{L}f)(s) = \int_{0}^{\infty} \mathrm{e}^{-st} f(t)\,\mathrm{d}t$ (where $s \in \mathbf{C}$) This is often denoted by $\mathcal{L}(s)$. The two-sided Laplace transform is also used, defined by the same formula, but with minus infinity instead of zero.

Table 15 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-19.3	$\mathfrak{Z}(a_n)$	Z transform of (a_n)	$\mathfrak{Z}(a_n) = \sum_{n=0}^{\infty} a_n z^{-n}$ (where $z \in \mathbf{C}$) $\mathfrak{Z}$ is an operator operating on a sequence (a_n) and not a function of a_n. The two-sided Z transform is also used, defined by the same formula, but with minus infinity instead of zero.
2-19.4	$\mathrm{H}(x)$ $\vartheta(x)$	Heaviside function, unit step function	$\mathrm{H}(x)=\begin{cases}1 & \text{for } x>0\\ 0 & \text{for } x<0\end{cases}$ $\mathrm{H}(0) \in [0,1]$, depending on the field of application. $\mathrm{U}(x)$ is also used. $\vartheta(t)$ is used for the unit step function of time. EXAMPLE $(\mathcal{L}\mathrm{H})(s) = 1/s$ (for $\mathrm{Re}\, s > 0$)
2-19.5	$\delta(x)$	Dirac delta distribution, Dirac delta function	The Dirac delta distribution is defined by $\int_{-\infty}^{\infty} \varphi(t)\delta(t-x)\mathrm{d}t=\varphi(x)$ The Dirac delta distribution can be interpreted as derivative of the Heaviside function. The name "unit pulse" is also used. EXAMPLE $\mathcal{L}\delta=1$ See also item 2-19.6 and IEC 60027-6:2006, item 2.01.
2-19.6	$f*g$	convolution of f and g	$f*g(x)=\int_{-\infty}^{\infty} f(y)g(x-y)\mathrm{d}y$

20 Special functions

In this Clause, a, b, c, z, w, v are complex numbers, x is a real number, and k, l, m, n, are natural numbers.

Table 16 — Symbols and expressions for special functions

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-20.1	γ C	Euler-Mascheroni constant	$\gamma=\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{k}-\ln n\right)= 0{,}577\ 215\ 6\ \ldots$

Table 16 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-20.2	$\Gamma(z)$	gamma function	$\Gamma(z)$ is a meromorphic function with poles at 0, −1, −2, −3, ... $\Gamma(z) = \int_0^\infty t^{z-1} \mathrm{e}^{-t} \mathrm{d}t$ (for Re $z > 0$) $\Gamma(n+1) = n!$ (for $n \in \mathbf{N}$)
2-20.3	$\zeta(z)$	Riemann zeta function	$\zeta(z)$ is a meromorphic function with one pole at $z = 1$. $\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^z}$ (for Re $z > 1$)
2-20.4	$\mathrm{B}(z, w)$	beta function	$\mathrm{B}(z,w) = \int_0^1 t^{z-1} (1-t)^{w-1} \mathrm{d}t$ (for Re $z > 0$, Re $w > 0$) $\mathrm{B}(z,w) = \Gamma(z)\Gamma(w)/\Gamma(z+w)$ $\frac{1}{(n+1)\mathrm{B}(k+1, n-k+1)} = \binom{n}{k}$ (for $k \le n$)
2-20.5	Ei x	exponential integral	$\mathrm{Ei}\, x = \fint_{-\infty}^{x} \frac{\mathrm{e}^t}{t} \mathrm{d}t$ For $\fint$, see 2-12.26.
2-20.6	li x	logarithmic integral	$\mathrm{li}\, x = \int_0^x \frac{1}{\ln t} \mathrm{d}t$ (for $0 < x < 1$) $\mathrm{li}\, x = \fint_0^x \frac{1}{\ln t} \mathrm{d}t$ (for $x > 1$) For $\fint$, see 2-12.26.
2-20.7	Si z	sine integral	$\mathrm{Si}\, z = \int_0^z \frac{\sin t}{t} \mathrm{d}t$ $\mathrm{si}\, z = -\frac{\pi}{2} + \mathrm{Si}\, z$ is called the complementary sine integral.

Table 16 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-20.8	$\mathrm{S}(z)$ $\mathrm{C}(z)$	Fresnel integrals	$\mathrm{S}(z)=\int_0^z \sin\left(\frac{\pi}{2}t^2\right)\mathrm{d}t$ $\mathrm{C}(z)=\int_0^z \cos\left(\frac{\pi}{2}t^2\right)\mathrm{d}t$
2-20.9	$\mathrm{erf}\, x$	error function	$\mathrm{erf}\, x=\frac{2}{\sqrt{\pi}}\int_0^x \mathrm{e}^{-t^2}\mathrm{d}t$ $\mathrm{erfc}\, x=1-\mathrm{erf}\, x$ is called the complementary error function. In statistics, the distribution function $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^x e^{-t^2/2}\mathrm{d}t$ is used.
2-20.10	$\mathrm{F}(\varphi,k)$	incomplete elliptic integral of the first kind	$\mathrm{F}(\varphi,k)=\int_0^\varphi \frac{\mathrm{d}\sigma}{\sqrt{1-k^2\sin^2\sigma}}$ $k\in\mathbf{R}$, $0<k<1$ $\mathrm{K}(k)=\mathrm{F}\left(\frac{\pi}{2},k\right)$ is the complete elliptic integral of the first kind.
2-20.11	$\mathrm{E}(\varphi,k)$	incomplete elliptic integral of the second kind	$\mathrm{E}(\varphi,k)=\int_0^\varphi \sqrt{1-k^2\sin^2\sigma}\,\mathrm{d}\sigma$ $k\in\mathbf{R}$, $0<k<1$ $\mathrm{E}(k)=E\left(\frac{\pi}{2},k\right)$ is the complete elliptic integral of the second kind.
2-20.12	$\Pi(n,\varphi,k)$	incomplete elliptic integral of the third kind	$\Pi(n,\varphi,k)=\int_0^\varphi \frac{\mathrm{d}\vartheta}{(1-n\sin^2\vartheta)\sqrt{1-k^2\sin^2\vartheta}}$ $n, k\in\mathbf{R}$, $0<k<1$. $\Pi(n,k)=\Pi\left(n,\frac{\pi}{2},k\right)$ is the complete elliptic integral of the third kind. (Sometimes the elliptic integral of the third kind is defined with an inverse sign for the *characteristic n*.)

Table 16 *(continued)*

Item No.	Symbol, expression	Meaning, verbal equivalent	Remarks and examples
2-20.13	$\mathrm{F}(a,b,c;z)$	hypergeometric function	$\mathrm{F}(a,b,c;z)=\sum_{n=0}^{\infty}\frac{(a)_n (b)_n z^n}{(c)_n n!}$ (for $-c \notin \mathbf{N}$) For $(a)_n$, $(b)_n$ and $(c)_n$, see 2-11.3. Solution of $z(1-z)y''+[c-(a+b+1)z]y'-aby=0$
2-20.14	$\mathrm{F}(a;c;z)$	confluent hypergeometric function	$\mathrm{F}(a;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n}{(c)_n n!}z^n$ (for $-c \notin \mathbf{N}$) For $(a)_n$ and $(c)_n$, see 2-11.3. Solution of $zy''+(c-z)y'-ay=0$
2-20.15	$\mathrm{P}_n(z)$	Legendre polynomial	$\mathrm{P}_n(z)=\frac{1}{2^n n!}\frac{\mathrm{d}^n}{\mathrm{d}z^n}(z^2-1)^n$ (for $n \in \mathbf{N}$) Solution of $(1-z^2)y''-2zy'+n(n+1)y=0$
2-20.16	$\mathrm{P}_n^m(z)$	associated Legendre function	$\mathrm{P}_n^m(z)=(-1)^m(1-z^2)^{m/2}\frac{\mathrm{d}^m}{\mathrm{d}z^m}\mathrm{P}_n(z)$ (for $m, n \in \mathbf{N}$, $m \le n$) Solution of $(1-z^2)y''-2zy'+\left[n(n+1)-\frac{m^2}{1-z^2}\right]y=0$ The factor $(-1)^m$ follows from the general theory of spherical functions.
2-20.17	$\mathrm{Y}_l^m(\vartheta,\varphi)$	spherical harmonic	$\mathrm{Y}_l^m(\vartheta,\varphi)=\left[\frac{(2l+1)}{4\pi}\frac{(l-\lvert m\rvert)!}{(l+\lvert m\rvert)!}\right]^{1/2} P_l^{\lvert m\rvert}(\cos\vartheta)\mathrm{e}^{\mathrm{i}m\varphi}$ (for $l, \lvert m\rvert \in \mathbf{N}$; $\lvert m\rvert \le l$) Solution of $\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial y}{\partial\vartheta}\right)+\frac{1}{\sin^2\vartheta}\frac{\partial^2 y}{\partial\varphi^2}+l(l+1)y=0$
2-20.18	$\mathrm{H}_n(z)$	Hermite polynomial	$\mathrm{H}_n(z)=(-1)^n\mathrm{e}^{z^2}\frac{\mathrm{d}^n}{\mathrm{d}z^n}\mathrm{e}^{-z^2}$ (for $n \in \mathbf{N}$) Solution of $y''-2zy'+2ny=0$