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Control charts for arithmetic average with warning limits

Cartes de contrôle de la moyenne arithmétique à limites de surveillance

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 7873 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Sub-Committee SC 4, *Statistical process control*.

Annex A forms an integral part of this International Standard. Annexes B, C and D are for information only.

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Introduction

The statistical control of processes using arithmetic average control charts with warning limits is a modification of Shewhart control charts. Control charts for the arithmetic average using both warning and action limits are characterized by higher sensitivity to a process level shift.

Arithmetic average control charts with warning limits are able to reveal smaller shifts of the mean value of the controlled quality measure because of additional information obtained from the points being accumulated in the warning zone. In addition, sudden large shifts in process level are detectable if sample average values fall beyond action limits. In comparison with Shewhart control charts, they are more sensitive in the case of minor and slowly forming biases of the quality measure (that is, shifts not exceeding $2,5\sigma/\sqrt{n}$, where σ is the standard deviation of the quality measure and n is the sample size).

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Control charts for arithmetic average with warning limits

1 Scope

This International Standard specifies procedures for the statistical control of processes by using control charts based on calculating the arithmetic average of a sample and using warning limits and action limits. It is assumed that for large lots and for the mass output of piece and batch production, such a measure of quality is a random variable following a normal distribution. However, when averages of four or more items are plotted, this assumption of a normal distribution is not necessary for control purposes (see 4.2).

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 3534-1:1993, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms.*

ISO 3534-2:1993, *Statistics — Vocabulary and symbols — Part 2: Statistical quality control.*

3 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534-1 and ISO 3534-2 apply.

4 Conditions of application

4.1 The implementation of the statistical methods of process control should be preceded by statistical analysis during a base period of the quality measure to be controlled in order to provide a basis for constructing relationships between a process (the operations) and product quality, as well as for producing recommendations for the adjustment of the process.

If the statistical analysis shows that the process is out of control, and the process capability does not meet specified requirements, it is necessary to determine the causes of level shifts[1] and ways of adjusting the process.

4.2 In order to apply the rules of this International Standard, it is necessary first to establish the following.

a) The arithmetic average $\bar{X}$ approximates a normal distribution. Except for extremely unusual circumstances, averages of samples of four or more items will, under the Central Limit Theorem, follow approximately a normal distribution even though the individual observations may not.

b) For best results, the individual observations averaged to obtain $\bar{X}$ are made by scale-measuring instruments with scale divisions not exceeding $\sigma/2$.

c) The underlying, but unknown, mean value μ of the sample $\bar{X}$ values defines the current process level. If the process level shifts, then so will μ. The process level should then be adjusted.

d) The target level μ_0 corresponds to the value of the middle line of the tolerance zone of the quality measure specified in the documents when the two-sided criterion is used.

1) "Level shifts" means a case in which μ becomes equal to μ_1 or μ_{-1}.

e) The standard deviation σ of the quality measure is assumed to remain constant and acceptable. This assumption must be verified by using sample standard deviation or range control charts.

f) In the case of the one-sided criterion $\mu_1 > \mu_0$ or $\mu_{-1} < \mu_0$, the target level is assumed to be μ_0, but only the direction of concern is of interest. When a process is considered to be out of control in the direction of interest, it requires correction. Values of μ_1 or μ_{-1} are selected to indicate process shifts $\Delta = |\mu_1 - \mu_0|$ or $|\mu_{-1} - \mu_0|$ that should be discovered quickly and are called "highly undesirable" levels. This value shall correspond to a fraction of rejection (see annex A).

g) In the case of the two-sided criterion $\mu_1 > \mu_0$ and $\mu_{-1} < \mu_0$, interest lies on either side of μ_0. When the process is out of control in either direction, it requires correction.

Proceeding from the values μ_0, σ, μ_1 and/or μ_{-1} the value δ, which characterizes the standardized form of the mean value in the case where the process is out of control, is determined, i.e.

$$\delta = \frac{\mu_1 - \mu_0}{\sigma}$$

$$= \frac{\mu_0 - \mu_{-1}}{\sigma}$$

When the value σ is constant, the process may go out of control owing to the change of μ under the influence of assignable causes.

5 Description of the method

5.1 The statistical control of a process is monitored using control charts for the arithmetic average with warning limits.

The control chart is used to show graphically the level and the variability of the process; the current sample averages of the measure of quality $\overline{X}$ are plotted on the charts, as shown in figure B.1.

5.2 The control chart for arithmetic average with warning limits has a target line (central line) corresponding to the mean value of the quality measure for the adjusted process. This line corresponds to μ_0, the warning limits to

$$\mu_0 \pm B_2\sigma/\sqrt{n}$$

and the action limits to

$$\mu_0 \pm B_1\sigma/\sqrt{n}$$

where n is the sample size. An underlying assumption is that the individual observations used to compute $\overline{X}$ are statistically independent.

B_1 and B_2 are the values determining the position of action and warning limits on the control charts. The principle of the selection of B_1 and B_2 is described in clause 6 .

5.3 The control chart may be located on a printed form, on an illuminated indicator board, in computer memory in coded form, or displayed in other appropriate ways.

5.4 Control charts should be located as close to the working areas as practical and data entry and chart plotting should be clear and explicit.

5.5 A standard operating procedure for the definition, preparation, application, maintenance and use of a control chart as a method of measuring the variation of the process should be prepared and data as collected should be promptly entered on the chart.

5.6 Control charts for arithmetic average with warning limits may be used both for one-sided and for two-sided criteria of statistical process control. However, it is usual to use two-sided criteria.

5.6.1 When a process is statistically controlled by means of a two-sided criterion, five quality zones are used (see figure 1), as follows.

a) Zone T (target): the sample average value is located between the upper warning and lower warning limits.

b) Zones W_+ and W_- (warning): the sample average value is located between the upper warning and upper action limits, or between the lower warning and lower action limits, respectively.

c) Zones A_+ and A_- (action): the sample average value is located beyond the upper action limit, or the lower action limit, respectively.

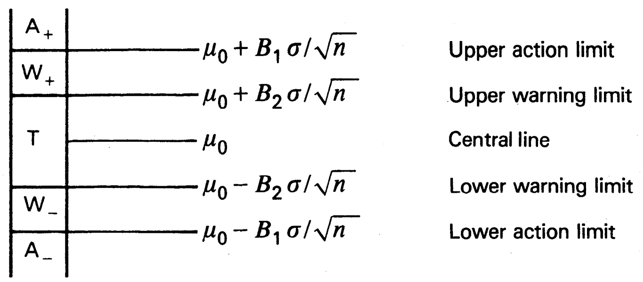


Figure 1 — Quality zones for statistical control with a two-sided criterion

5.6.2 When the process is statistically controlled by means of a one-sided criterion, three quality zones are used (see figures 2 and 3), as follows.

a) Zone T (target): the sample average value is located below the upper or above the lower warning limits as the case may be.

b) Zone *W* (warning): the sample average value is located between the warning and action limits.

c) Zone *A* (action): the sample average value is located beyond the action limit.

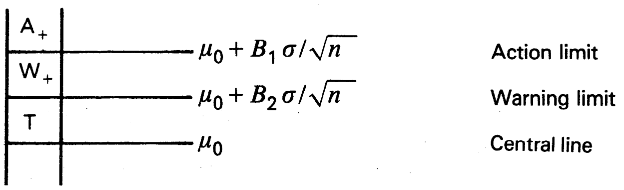


Figure 2 — Quality zones for statistical control with a one-sided criterion — Upper limits

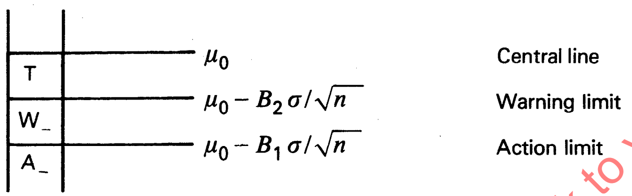


Figure 3 — Quality zones for statistical control with a one-sided criterion — Lower limits

Figure 2 shows the case when concern for a level shift is connected with an increase of the mean value of the measure of quality.

Figure 3 shows the case when concern for a level shift is connected with a decrease of the mean value of the measure of quality.

5.7 The sample average value of the measure of quality is plotted on control charts with warning limits in the following way.

A point is plotted on the chart for each sample with an identification number (numerical order, time order, etc.) as abscissa and the corresponding sample average as ordinate (see figure B.1).

2) Values for t and n are specified beforehand.

6 Statistical control of a process

6.1 A single point falling in the upper action zone A_+ or the lower action zone A_- is an out-of-control signal. When an out-of-control signal occurs, the cause of the out-of-control condition should be determined and corrected so as to obtain control of the process at the appropriate level.

6.2 When the selected number of successive points, K, fall into one of the warning zones, upper W_+ or lower W_-, this is an out-of-control signal and the process needs to be adjusted.

The value of the various parameters are chosen in accordance with the procedures shown in clause 7.

7 Choice of values of parameters for a statistical control plan of a process

7.1 When choosing a plan for statistical process control it is necessary to establish the following values:

a) sample size[2)], n (see 7.3);

b) sampling period[2)], t (see 7.3);

c) number of successive points, K (see 6.2);

d) values determining the positions of action and warning limits on control charts, B_1 and B_2 (see 7.2.2 and 7.4.1);

e) decision-making rules for process correction.

Initial values for choosing a plan for statistical process control are as follows:

μ_0, σ, μ_1 and/or μ_{-1} (see clause 4);

L_0 and L_1 [the average run lengths (ARL) of a process in and out of control, respectively (see 7.2 and annex C)].

7.2 The efficiency of a statistical process control plan can be described in terms of average run lengths.

7.2.1 The average run length (ARL) of a process is the average number of sample averages that will be plotted before an out-of-control signal is obtained, with the process average constant. The ARL is a maximum when the process level is at the target level (μ_0), and decreases progressively as the process deviates from target. The design of the control chart should provide a large ARL, L_0, when the process average is on target; this provides a low rate of false alarms. The design of the control chart should also provide a small ARL, L_1, when the process average is at μ_{+1} or μ_{-1}; this provides rapid detection of an unsatisfactory situation.

7.2.2 For the case of the one-sided criterion of the process statistical control, tables 1 to 3 give L_0 values (on the line $\delta\sqrt{n} = 0$) and L_1 values (on the line corresponding to the established value $\delta\sqrt{n}$) as a function of K, B_1, B_2 and $\delta\sqrt{n}$. When choosing L_0 and L_1, it is necessary to specify a few variants of B_1 and B_2 and choose, as far as possible, those which provide the highest L_0/L_1 ratio.

7.2.3 For the case of the two-sided criterion of the process statistical control, one should use tables 1 to 4. In this case the ARL of the process in control L_0 is determined from table 4 with $\delta\sqrt{n} = 0$. The ARL of the process out of control L_1 is determined using table 4 with $\delta\sqrt{n} < 1$ and tables 1 to 3 with $\delta\sqrt{n} \geqslant 1$, since at $\delta\sqrt{n} \geqslant 1$ the ARL for the two-sided criterion coincides numerically with the ARL for the one-sided criterion (see table C.1).

7.2.4 For the values of $\delta\sqrt{n}$ missing from tables 1 to 4, the corresponding L_1 values are obtained through linear interpolation.

7.3 The sample size n affects the ARL curves, as shown through the formulae in annex C, as well as the parameters μ_0, μ_{+1} and/or μ_{-1}, σ and K. Moreover, for the same total number of observations or measurements the control chart can be designed with a long sampling period, t, and a small sample size n, or conversely.

In each specific application, various trial combinations of n and t should be investigated during design of the control chart to determine the resulting values of L_0 and L_1. The design should be evaluated in terms of the elapsed process time associated with the resulting values of L_0 and L_1.

In most cases, the pre-existing sampling plan (n, t) will be the "base" trial combination, and other trial designs should be compared with the base design with respect to performance (L_0 and L_1) and cost.

7.4 Tables 1 to 4 are used for choosing a plan of the statistical process control.

7.4.1 If the values δ and n, as well as L_0 and L_1 (and their restrictions), are predetermined, then B_1, B_2 and K are found under the given value $\delta\sqrt{n}$ in tables 1 to 4 (interpolating, if necessary) (see clause B.2).

If there are several variants of the statistical process control plan that meet specified requirements (see clause B.2), a variant which provides the maximum L_0/L_1 ratio should be chosen, with regard to 7.2.1. In this case, if the ratio is high (is greater than or equal to 40) it is recommended that the variant giving the smaller value of L_1 be chosen.

7.4.2 If the sample size n is not predetermined, its possible values can be found using tables 1 to 4. Values are found by choosing those columns in tables 1 to 4 for which L_0 values satisfy given conditions and then the first number smaller than or equal to the given L_1 value is taken. Then, from the corresponding value of $\delta\sqrt{n}$, δ given, the sample size n is obtained by rounding the calculated number to the nearest integer (see clause B.4).

In this case, many variants of the statistical control plan are obtained; often it would be appropriate to choose that plan (with regard to points 7.2 and 7.4.1) which provides the smallest sample size. This is especially important when the process to improve control is rather expensive.

7.5 Various changes may take place in production technique manufacturing conditions, for example the skill of operators, materials supplied, the narrowing or widening of action limits because of some technological or economic reasons, etc. All these changes should be immediately taken note of in the plans for process statistical control.

To this end, it should be recorded in the documents that in some specified time periods (a month, a quarter, a year etc.) control charts and other documents shall be subject to statistical analysis in order to update them. The frequency of such an analysis shall be determined by production necessity.

Table 1 — ARL values for $B_1 = 2{,}75$ (One-sided criterion)

$\delta\sqrt{n}$	ARL with $B_1 = 2{,}75$ and B_2														
	$K = 2$					$K = 3$					$K = 4$				
	B_2					B_2					B_2				
	1,0	1,25	1,5	1,75	2,0	1,0	1,25	1,5	1,75	2,0	1,0	1,25	1,5	1,75	2,0
0,0	41,7	79,8	146,8	232,8	297,4	161,8	253,0	310,2	330,6	334,5	287,4	324,6	333,6	335,1	335,4
0,2	24,5	43,6	76,7	120,9	158,9	80,4	126,3	161,7	180,3	184,7	146,4	166,6	185,2	185,3	185,6
0,4	15,3	25,4	42,3	65,8	88,0	42,4	66,9	88,2	101,5	105,5	69,1	96,0	104,1	106,1	106,4
0,6	10,3	15,9	25,0	32,2	50,5	24,6	37,4	50,5	56,0	62,4	40,8	54,2	60,6	62,9	63,3
0,8	7,3	10,5	15,0	22,7	30,3	15,3	22,1	29,7	35,2	38,0	24,4	31,8	36,7	38,4	39,1
1,0	5,4	7,3	10,3	14,4	19,0	9,6	14,0	18,3	22,0	23,9	15,7	19,6	22,8	24,3	24,8
1,2	4,2	5,4	7,2	9,7	12,6	7,2	8,9	12,1	14,5	16,0	10,3	12,7	15,0	16,2	16,6
1,4	3,4	4,2	5,3	6,8	8,5	5,4	6,7	8,2	9,6	10,7	7,2	8,6	9,9	10,8	11,2
1,6	2,8	3,3	3,9	4,7	5,6	4,0	5,0	5,4	6,0	6,5	5,0	6,2	6,2	6,6	7,9
1,8	2,4	2,8	3,2	4,1	4,5	3,5	3,9	4,4	5,2	5,4	4,2	4,7	5,2	5,6	5,6
2,0	2,2	2,4	2,7	3,1	3,5	2,9	3,5	3,4	3,8	4,1	3,4	3,7	4,0	4,8	4,3
2,2	1,9	2,1	2,3	2,5	2,8	2,5	2,7	2,8	3,1	3,2	2,9	3,0	3,1	3,3	3,4
2,4	1,8	1,9	2,0	2,1	2,3	2,2	2,3	2,4	2,5	2,6	2,4	2,5	2,6	2,6	2,7
2,6	1,6	1,7	1,8	1,9	2,0	1,9	2,0	2,0	2,2	2,2	2,1	2,1	2,2	2,3	2,3
2,8	1,6	1,6	1,7	1,7	1,7	1,8	1,8	1,9	2,0	1,9	2,0	1,9	2,0	2,0	2,0
3,0	1,4	1,4	1,5	1,5	1,5	1,6	1,6	1,6	1,6	1,6	1,7	1,6	1,6	1,7	1,7
3,2	1,3	1,3	1,4	1,4	1,4	1,4	1,4	1,4	1,5	1,5	1,5	1,5	1,5	1,5	1,5
3,4	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3
3,6	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2
3,8	1,1	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2

Table 2 — ARL values for $B_1 = 3$ (One-sided criterion)

$\delta\sqrt{n}$	ARL with $B_1 = 3$ and B_2														
	$K = 2$					$K = 3$					$K = 4$				
	B_2					B_2					B_2				
	1,0	1,25	1,5	1,75	2,0	1,0	1,25	1,5	1,75	2,0	1,0	1,25	1,5	1,75	2,0
0,0	43,8	83,5	186,1	346,2	556,0	215,1	422,5	620,1	711,0	734,6	535,4	624,1	730,9	738,3	739,4
0,2	25,7	48,1	92,7	151,0	275,2	101,3	194,0	301,7	365,0	385,9	245,4	341,6	380,6	389,6	391,0
0,4	16,1	27,9	50,5	89,6	141,9	51,8	95,6	159,4	192,1	210,5	117,1	174,6	203,3	212,4	214,2
0,6	10,8	17,2	26,4	39,8	76,0	28,6	49,7	78,4	87,7	115,9	59,5	89,7	111,0	117,6	121,9
0,8	8,1	11,3	17,7	28,4	43,0	19,2	28,1	43,1	55,2	66,9	35,4	48,8	62,3	69,4	71,4
1,0	5,6	7,9	11,6	17,4	25,5	11,6	17,1	25,0	33,7	39,9	19,5	40,3	36,3	41,3	43,3
1,2	4,2	5,8	8,0	11,4	16,1	7,7	11,2	14,9	20,6	24,7	11,9	17,1	22,0	25,6	27,2
1,4	3,6	4,5	5,8	7,8	11,2	6,0	7,8	10,3	13,2	15,8	8,7	11,2	15,0	16,4	17,6
1,6	3,0	3,5	4,4	5,7	7,4	4,7	5,8	7,2	8,9	10,6	6,5	7,8	9,4	10,9	11,3
1,8	2,6	2,9	3,5	4,7	5,4	3,9	4,5	5,3	6,8	7,4	5,0	5,8	6,7	7,9	8,3
2,0	2,3	2,5	2,9	3,4	4,1	3,4	3,6	4,1	4,7	5,4	4,0	4,5	5,0	5,5	6,0
2,2	2,1	2,2	2,5	2,8	3,2	2,8	2,8	3,3	3,7	4,1	3,4	3,6	3,9	4,2	4,5
2,4	1,9	2,0	2,2	2,4	2,6	2,5	2,6	2,8	3,0	3,2	2,9	3,0	3,1	3,3	3,5
2,6	1,7	1,8	1,9	2,0	2,2	2,2	2,3	2,3	2,5	2,6	2,5	2,5	2,7	2,7	2,8
2,8	1,6	1,7	1,8	1,8	1,9	2,0	2,1	2,1	2,1	2,2	2,2	2,3	2,3	2,3	2,4
3,0	1,5	1,6	1,6	1,6	1,7	1,8	1,8	1,8	1,9	1,9	1,9	1,9	1,7	1,9	2,0
3,2	1,4	1,4	1,4	1,5	1,5	1,6	1,6	1,6	1,6	1,7	1,6	1,6	1,6	1,6	1,7
3,4	1,3	1,3	1,4	1,4	1,4	1,5	1,5	1,5	1,5	1,5	1,5	1,5	1,5	1,5	1,5
3,6	1,3	1,3	1,3	1,3	1,3	1,3	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4
3,8	1,2	1,2	1,2	1,2	1,2	1,2	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3

Table 3 — ARL values for $B_1 = 3{,}25$ (One-sided criterion)

$\delta\sqrt{n}$	ARL with $B_1 = 3{,}25$ and B_2														
	$K = 2$					$K = 3$					$K = 4$				
	B_2					B_2					B_2				
	1,0	1,25	1,5	1,75	2,0	1,0	1,25	1,5	1,75	2,0	1,0	1,25	1,5	1,75	2,0
0,0	45,1	94,7	212,0	481,5	987,8	448,7	618,6	1 176,0	1 567,8	1 698,7	904,8	1 454,7	1 675,9	1 720,8	1 730,4
0,2	26,4	50,7	105,3	223,3	432,2	116,2	263,9	469,5	744,9	843,6	369,7	653,0	819,6	864,3	872,9
0,4	16,6	29,2	55,6	110,2	207,6	58,0	121,2	230,3	360,3	430,2	161,3	299,9	392,2	446,9	455,8
0,6	11,0	18,0	31,7	58,4	105,3	32,1	60,7	112,3	178,9	225,1	99,5	140,1	204,6	216,4	235,7
0,8	7,8	11,8	19,3	30,2	56,5	19,3	33,3	58,1	92,1	117,2	40,3	69,9	104,2	118,1	133,2
1,0	6,8	8,2	12,5	20,0	32,3	12,6	19,8	32,2	49,8	67,1	23,3	37,5	56,2	71,4	95,1
1,4	3,7	4,7	6,2	6,7	12,6	6,6	8,8	12,4	17,4	23,2	10,1	13,9	19,2	24,7	28,6
1,6	3,3	3,7	4,7	6,3	8,6	5,1	6,5	8,5	11,3	14,7	7,4	9,5	12,4	15,6	18,2
1,9	2,7	3,1	3,8	4,7	6,1	4,2	4,9	6,2	7,8	9,8	5,7	6,9	8,5	10,4	12,0
2,0	2,4	2,7	3,1	3,7	4,5	3,5	3,9	4,7	5,5	6,6	4,6	5,2	6,0	6,9	7,9
2,2	2,1	2,4	2,6	3,1	3,6	3,1	3,4	3,8	4,4	5,1	3,9	4,3	4,8	5,4	6,0
2,4	2,0	2,1	2,3	2,6	2,9	2,7	2,9	3,2	3,5	3,9	3,3	3,5	3,8	4,1	4,4
2,6	1,9	1,9	2,1	2,2	2,5	2,4	2,5	2,7	2,9	3,1	2,9	3,0	3,1	3,3	3,5
2,8	1,9	1,8	1,9	2,0	2,1	2,2	2,3	2,3	2,5	2,6	2,5	2,6	2,6	2,7	2,8
3,0	1,6	1,7	1,7	1,8	1,9	2,0	2,0	2,1	2,1	2,2	2,2	2,2	2,3	2,3	2,4
3,2	1,5	1,6	1,6	1,6	1,8	1,8	1,8	1,8	1,9	1,9	1,9	1,9	2,0	2,0	2,0
3,4	1,4	1,4	1,5	1,5	1,5	1,6	1,6	1,7	1,7	1,7	1,7	1,7	1,7	1,7	1,8
3,6	1,4	1,4	1,4	1,4	1,4	1,5	1,5	1,5	1,5	1,5	1,5	1,5	1,5	1,6	1,6
3,8	1,3	1,3	1,3	1,3	1,3	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4

Table 4 — ARL values (Two-sided criterion)

B_1	$\delta\sqrt{n}$	ARL with B_2														
		$K = 2$					$K = 3$					$K = 4$				
		B_2					B_2					B_2				
		1,0	1,25	1,5	1,75	2,0	1,0	1,25	1,5	1,75	2,0	1,0	1,25	1,5	1,75	2,0
2,75	0,0	20,8	39,9	73,4	116,4	148,7	80,9	126,5	155,1	165,3	167,2	143,7	162,3	166,8	167,5	167,7
	0,2	18,6	33,0	61,0	95,2	123,5	65,8	100,0	126,6	140,8	142,9	117,6	131,6	142,9	143,0	143,1
	0,4	13,9	23,5	39,7	61,7	81,3	40,3	63,3	82,6	93,5	97,4	65,8	89,3	96,2	98,0	98,1
	0,6	10,0	15,6	24,6	31,7	49,5	24,3	36,9	49,5	54,6	60,9	40,2	53,2	59,2	61,3	61,8
	0,8	7,2	10,4	14,9	22,6	30,1	15,2	22,0	29,5	35,0	37,7	24,3	31,6	36,5	38,2	38,8
3,0	0,0	21,9	41,7	93,0	173,1	278,0	107,5	211,2	310,0	355,5	367,3	267,7	312,0	325,4	329,1	329,7
	0,2	19,5	37,9	75,2	126,6	222,6	84,0	159,9	243,9	294,1	303,0	201,0	277,8	303,0	306,2	307,0
	0,4	14,6	26,0	47,8	85,5	134,2	49,7	91,7	151,5	181,8	198,0	113,1	166,7	192,3	200,0	201,3
	0,6	10,5	16,9	26,0	39,4	75,3	28,2	49,3	77,2	86,2	113,3	58,5	88,1	108,7	114,9	119,0
	0,8	8,0	11,2	17,6	28,3	42,8	19,2	28,0	42,9	54,9	66,5	35,3	48,5	61,9	68,9	70,9
3,25	0,0	22,5	47,3	106,0	240,7	493,9	224,3	309,3	588,0	783,9	849,3	452,4	727,3	837,9	860,4	865,2
	0,2	20,0	40,0	87,7	184,5	357,1	98,0	222,2	395,4	609,0	673,4	312,5	555,5	657,9	686,3	692,0
	0,4	15,1	27,7	52,9	106,2	200,8	56,5	119,0	225,1	347,8	416,7	158,7	294,1	377,8	427,9	434,8
	0,6	10,7	17,6	31,3	58,0	104,0	31,8	60,2	111,1	175,4	220,2	98,5	138,9	200,5	212,8	230,3
	0,8	7,7	11,7	19,3	30,2	56,5	19,3	33,3	58,1	92,1	117,2	40,3	69,9	104,2	118,1	133,2

Annex A
(normative)

Determination of the mean value to be considered highly undesirable on the basis of a nonconforming fraction

A.1 One-sided criterion

The upper deviation of the process mean is to be controlled. The upper tolerance T_+ of the variable X is given. In this case, the nonconforming fraction for the process in control, q_0, is given by the formula

$$q_0 = 1 - \Phi\left(\frac{T_+ - \mu_0}{\sigma}\right) \quad \text{. . . (A.1)}$$

The nonconforming fraction for a process out of control, q_1, is given by the formula

$$q_1 = 1 - \Phi\left(\frac{T_+ - \mu_1}{\sigma}\right) \quad \text{. . . (A.2)}$$

where Φ is a standard normal distribution function.

Therefore, if T_+ and q_1 are known, then μ_1 can be determined by the formula

$$\mu_1 = T_+ - \sigma Z_{1-q_1} \quad \text{. . . (A.3)}$$

where Z is the $(1 - q_1)$ quantile of the standard normal distribution.

Similarly, if the lower deviation is controlled and the lower tolerance T_- is given, then

$$q_0 = 1 - \Phi\left(\frac{\mu_0 - T_-}{\sigma}\right) \quad \text{. . . (A.4)}$$

$$q_1 = 1 - \Phi\left(\frac{\mu_{-1} - T_-}{\sigma}\right) \quad \text{. . . (A.5)}$$

$$\mu_{-1} = T_- + \sigma Z_{1-q_1} \quad \text{. . . (A.6)}$$

where q_0 and q_1 are determined as above.

A.2 Two-sided control

This is the same case as when $T_+ - \mu_0 = \mu_0 - T_-$. Using the same designation, we obtain

$$q_0 = 2\left[1 - \Phi\left(\frac{T_+ - \mu_0}{\sigma}\right)\right] \quad \text{. . . (A.7)}$$

$$q_1 = 1 - \Phi\left(\frac{T_+ - \mu_1}{\sigma}\right) + 1 - \Phi\left(\frac{\mu_1 - T_-}{\sigma}\right) =$$

$$= 1 - \Phi\left(\frac{\mu_{-1} - T_-}{\sigma}\right)$$

$$+ 1 - \Phi\left(\frac{T_+ - \mu_{-1}}{\sigma}\right) \quad \text{. . . (A.8)}$$

Since usually

$$\frac{\mu_1 - T_-}{\sigma} = \frac{T_+ - \mu_{-1}}{\sigma} > 3$$

then

$$1 - \Phi\left(\frac{\mu_1 - T_-}{\sigma}\right) = 1 - \Phi\left(\frac{T_+ - \mu_{-1}}{\sigma}\right)$$

can be ignored. Then μ_1 and μ_{-1} are determined by formulae (A.3) and (A.6), respectively.

Annex B
(informative)

Example of application of this International Standard

B.1 Control charts for arithmetic average with warning limits are used for statistical control of the process of producing gas-washed nitrogen. The concentration of nitrogen in ammonia should be 25 % for the process in control.

Limits are given for the concentration of nitrogen:

$T_+ = 27,5$ %

$T_- = 22,5$ %

The highly undesirable nonconforming level is 3 %.

On the basis of previous experience, it is known that $\sigma = 1$ %.

The values μ_1 and μ_{-1} are to be determined.

According to formulae (A.3) and (A.6):

$$\mu_1 = 27,50\ \% - 1\ \% \times Z_{0,97}$$
$$= 27,50\ \% - 1,88\ \%$$
$$= 25,62\ \%$$
$$\mu_{-1} = 22,50\ + 1,88\ \%$$
$$= 24,38\ \%$$

B.2 Under the conditions given in clause B.1, $n = 5$ is taken as the number of sample measurements. Control lines in the control chart should be plotted so that the ARL for the process in control would be at least 300 and the ARL for a process at the highly undesirable level would not exceed 12.

We have:

$$\delta = \frac{25,62 - 25}{1}$$
$$= \frac{25 - 24,38}{1}$$
$$= 0,62$$

and

$$\delta\sqrt{n} = 0,62\sqrt{5}$$
$$= 1,39$$

A combination of B_1, B_2 and K is found from tables 1 to 4 (interpolating for $\delta\sqrt{n} = 1,39$) such that $L_0 \geqslant 300$ and $L_1 \leqslant 12$ (see table 4); i.e. $L_0 \geqslant 600$ and $L_1 \leqslant 12$ if tables 1 to 3 are used (see C.3).

The results are given in tabular form as follows:

No.	K	B_1	B_2	L_0	L_1
1	3	3,0	1,5	620,1	10,3
2	4	3,0	1,25	624,1	11,2
3	3	3,25	1,25	618,6	8,8
4	4	3,25	1,0	904,0	10,1

The imposed conditions are seen to specify an ambiguous control plan (there are four possible versions). According to 7.4.1 (since the ratio $L_0/L_1 \geqslant 50$), the version which yields the minimum value L_1 is chosen, i.e. version 3.

We have $K = 3$; $B_1 = 3,25$; $B_2 = 1,25$.

In accordance with clause 5, action limits are plotted:

$$25 + 3,25 \times \frac{1}{\sqrt{5}} = 26,45$$

$$25 - 3,25 \times \frac{1}{\sqrt{5}} = 23,55$$

and warning limits:

$$25 + 1,25 \times \frac{1}{\sqrt{5}} = 25,56$$

$$25 - 1,25 \times \frac{1}{\sqrt{5}} = 24,44$$

B.3 Under the conditions specified in clause B.1 and clause B.2, the following mean values of successive samples were obtained: 25,1 %; 25,2 %, 24,2 %, 25,6 %, 24,1 %, 24,3 %, 25,0 %, 25,3 %, 25,9 %, 24,7 %, 25,1 %, 25,3 %, 24,9 %, 25,4 %, 24,8 %, 24,7 %, 25,9 %, 25,6 %, 25,7 % (see figure B.1).

After the 19th sample, the decision is taken to adjust the process since the last three points (25,9; 25,6; 25,7) happened to be in zone W_+ between the warning and action limits.

To make it clear it should be mentioned that two adjacent points, 24,1 and 24,3, were found in zone W_-, but the adjustment was not made, since according to the accepted procedure there must be three of them. Finally, correction would have been made immediately if a single value had been more than 26,45 or less than 23,55.

NOTE 1 The 3σ limits are shown by dotted lines in figure B.1. As seen from this figure, the Shewhart chart gives no indication to adjust the process in this case.

B.4 For σ, μ_0, μ_1 and μ_{-1} given above and the conditions imposed on L_0 and L_1, the control plan yielding the smallest sample size n is to be found. From the columns corresponding to $L_0 \geqslant 600$, it is found that the minimum value out of all $\delta\sqrt{n}$ for which $L_1 \leqslant 12$ is 1,4 (for example, the plan with $B_1 = 3{,}0$, $B_2 = 1{,}5$, $K = 3$ gives $L_1 = 10{,}3$; a plan with $B_1 = 3{,}25$, $B_2 = 1{,}25$, $K = 3$ gives $L_1 = 8{,}8$).

Hence

$$\sqrt{n} = \frac{1{,}4}{0{,}62} = 2{,}26$$

$$n = 5$$

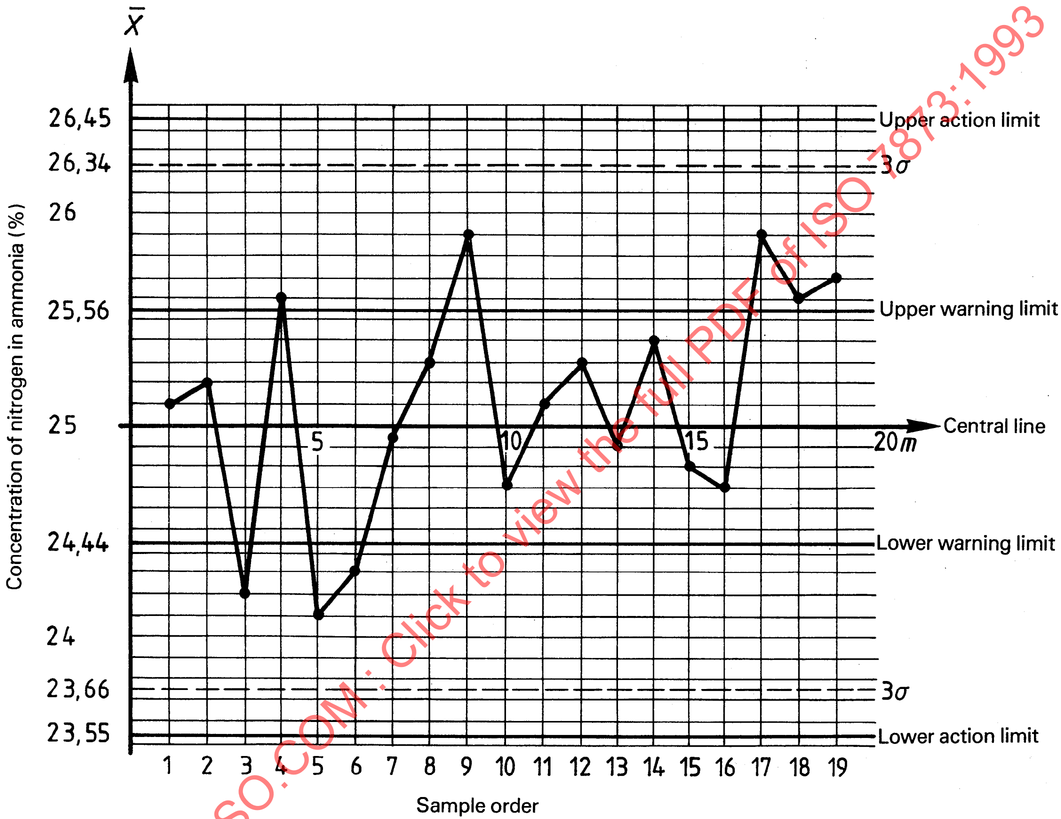


Figure B.1 — Example of application

Annex C
(informative)

Theoretical principles of this International Standard

C.1 General

Control charts showing only action limits are not always sufficiently sensitive to a maladjustment of the process. Average run lengths (ARL), i.e. the average number of samples, after which the decision about the adjustment is made, serve as a criterion of the sensitivity of the control chart to the shift of the process. If the process is in control, then such a decision is erroneous and here the average run length L_0 shall be as great as possible. If the process is out of control, then the decision to correct it shall follow the maladjustment as closely as possible. Here the average run length L_1 shall be as small as possible.

Warning limits have been introduced in addition to action limits to improve the sensitivity of control charts to maladjustment of the controlled process.

When comparing the control chart with warning limits to the Shewhart control chart which yields the same L_0, it can be seen that if values of $\delta\sqrt{n}$ do not exceed 2,5, then the first control chart yields significantly smaller values of L_1.

In figure C.1, the solid line shows the values of the ARL for the one-sided control chart with quality groups defined by conditions $B_1 = 3{,}00$, $B_2 = 1{,}75$, $K = 2$ (see table 2). The dotted line shows the values of ARL for a one-sided ordinary Shewhart chart with limits chosen so that it would yield the same value $L_0 = 346{,}2$ as the first chart (these limits will be $2{,}76\sigma/\sqrt{n}$ from the central line).

The example is given for the one-sided criterion. For the two-sided criterion the curves are constructed in a similar way (see 7.2.3).

C.2 Formulae for the ARL in the one-sided control

A point in the control chart can fall within the zone T with the probability p, and into the zone A with probability $1 - p - q$, where probabilities p and q (see figure 2) are given by the formulae

$$p = \Phi(B_2 - \delta\sqrt{n}) \quad \ldots (C.1)$$

$$q = \Phi(B_1 - \delta\sqrt{n}) - \Phi(B_2 - \delta\sqrt{n}) \quad \ldots (C.2)$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\xi^2/2} d\xi$$

and $\delta = 0$ for the process in control.

The average run length L is given by the formula

$$L = \frac{1 - q^K}{1 - p - q + pq^K} \quad \ldots (C.3)$$

When $K = 2$ it is convenient to use formula (C.3) in the following form:

$$L = \frac{1 + q}{1 - p - pq} \quad \ldots (C.4)$$

Tables 1 to 3 are calculated using these formulae.

C.3 Formula for the ARL in the two-sided control

For the two-sided control, formula (C.3) becomes (when $K = 2$):

$$L' = \frac{(1 + q_1)(1 + q_2)}{1 - q_1 q_2 - p'(1 + q_1)(1 + q_2)} \quad \ldots (C.5)$$

where

q_1 and q_2 are the probabilities of falling within zones W_+ and W_- respectively;

p' is the probability of falling within zone T.

Obviously, $p' = 2p - 1$, where p is as defined in formula (C.1).

When $\delta\sqrt{n} = 0$ then $q_1 = q_2 = q$ and formula (C.5) has the following form:

$$L' = \frac{(1 + q)^2}{1 - q^2 - (2p - 1)(1 + q)^2}$$

$$= \frac{1 + q}{1 - q - (2p - 1)(1 + q)}$$

$$= \frac{1 + q}{2(1 - p - pq)} \quad \ldots (C.6)$$

$$L_0' = \frac{1}{2} L_0$$

When $\delta\sqrt{n} \neq 0$, the smaller of the probabilities q_1 and q_2 (for example q_2) becomes so small that it can be ignored.

Therefore, for small values of $\delta\sqrt{n}$ (0,2; 0,4), the difference in ARL of one-sided and two-sided cases should be taken into account and when $\delta\sqrt{n} \geqslant 0,6$ it becomes so small as to be negligible (see table C.1).

Table C.1 — ARL for two-sided and one-sided cases when $B_1 = 3$, $B_2 = 2$ and $K = 2$

$\delta\sqrt{n}$	Two-sided case	One-sided case
0,0	278,0	556,0
0,2	222,6	275,2
0,4	134,2	141,9
0,6	75,3	76,0
0,8	42,8	43,0
1,0	25,5	25,5

Then, if $q_1 = q$, $q_2 = 0$, formula (C.5) becomes formula (C.4), i.e. the ARL of the process out of control will be the same as the ARL in the one-sided case:

$$L_1' = L_1 \quad \text{. . . (C.7)}$$

This can easily be explained. If, for example, the maladjustment consists in an increase in the mean value, then the possibility of overstepping the lower warning limit can be ignored as for the one-sided control.

Formulae (C.6) and (C.7) are also valid for $K = 3$ and $K = 4$.

These considerations form the basis of 7.2.3 and table 4.

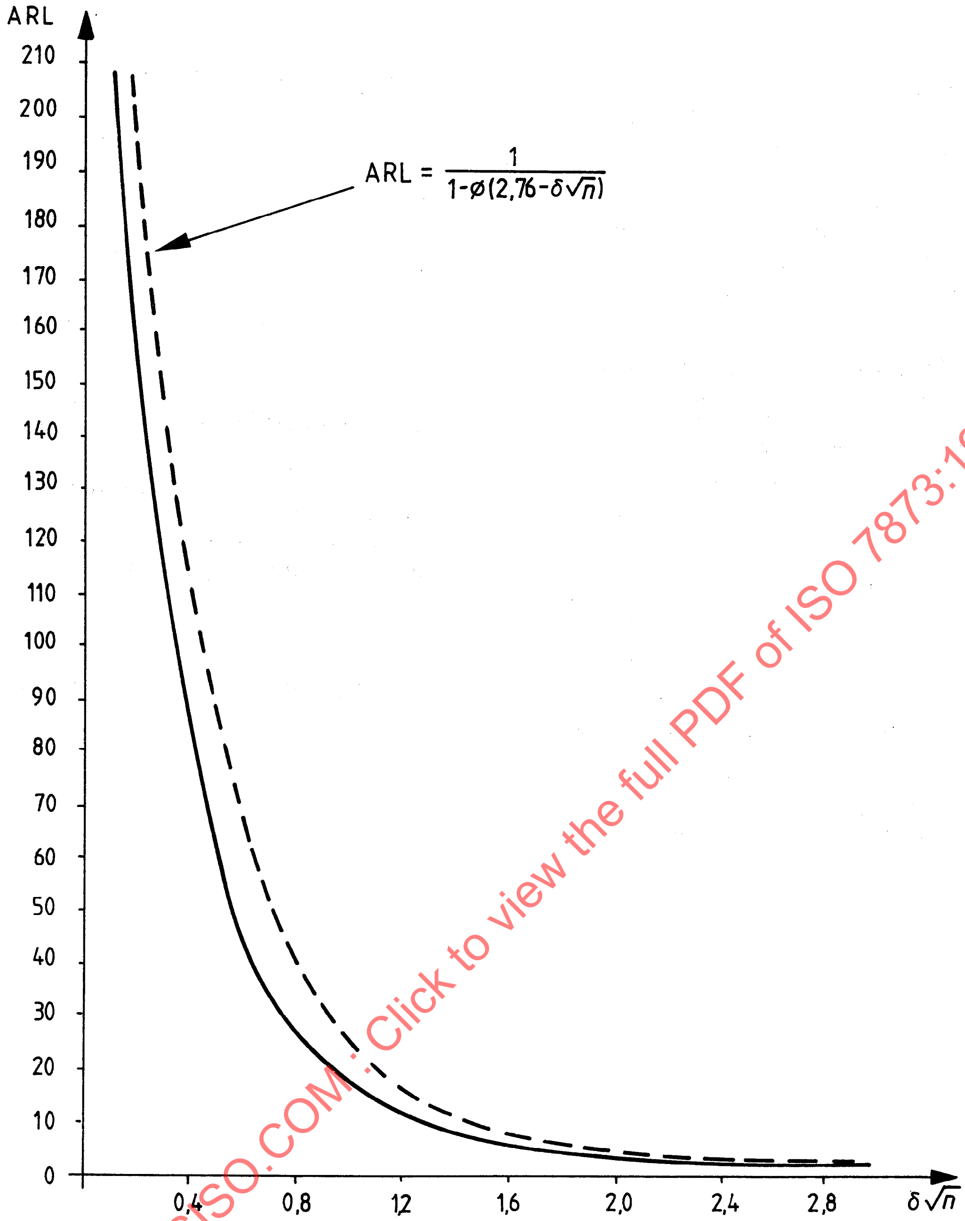
C.4 Relation with Markov chain theory

It should be noted that formulae (C.3), (C.4) and (C.5) can be easily deduced from the Markov chain theory. Thus, when treating one-sided control with $K = 2$, the chain with three states should be examined:

a) the point is in zone T;

b) the point is in zone W, while the previous one is in zone T;

c) the point is in zone A or in zone W together with the previous one.



NOTE — Φ is the distribution function.

Figure C.1 — Comparison of ARL as function of the value of maladjustment for ordinary control charts (dotted line) and control charts with warning limits (solid line)