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Control charts —

Part 9:
Control charts for stationary processes

Cartes de contrôle —

Partie 9: Cartes de contrôle de processus stationnaires

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Foreword

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A list of all parts in the ISO 7870 series can be found on the ISO website.

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Introduction

Statistical process control (SPC) techniques are widely used in industry for process monitoring and quality improvement. Various statistical control charts have been developed to monitor the process mean and variability. Traditional SPC methodology is based on a fundamental assumption that process data are statistically independent. Process data, however, are not always statistically independent from each other. In the industry for continuous productions such as the chemical industry, most process data on quality characteristics are self-correlated over time or autocorrelated. In general, autocorrelation can be caused by the measurement system, the dynamics of the process, or both. In many cases, the data can exhibit a drifting behaviour. In biology, random biological variation, for example the random burst in the secretion of some substance that influences the blood pressure, can have a sustained effect so that several consecutive measurements are all influenced by the same random phenomenon. In data collection, when the sampling interval is short, autocorrelation, especially the positive autocorrelation of the data, is a concern. Under such conditions, traditional SPC procedures are not effective and appropriate for monitoring, controlling and improving process quality.

Autocorrelated processes can be classified in two kinds of processes, based on whether they are stationary or nonstationary.

1) Stationary process – a direct extension of an independent and identically distributed (i.i.d.) sequence. An autocorrelated process is stationary if it is in a state of “statistical equilibrium”. This implies that the basic behaviour of the process does not change in time. In particular, a stationary process has identical means and variances.

2) Nonstationary process.

Detailed information about stochastic process and time series can be found in Annex A.

To accommodate autocorrelated data, some SPC methodologies have been developed. Mainly, there are two approaches. The first approach is to use a process residual chart after fitting a time series model or other mathematical model to the data. Another more direct approach is to modify the existing charts, for example by adjusting the control limits based on process autocorrelation.

The aim of this document is to outline the major process control charts for monitoring both of the process mean and the process variance when the process is autocorrelated.

Control charts —

Part 9:
Control charts for stationary processes

1 Scope

This document describes the construction and applications of control charts for stationary processes.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions, and abbreviated terms and symbols

3.1 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-2 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

3.1.1
autocovariance
internal covariance between members of series of observations ordered in time

3.1.2
control charts for autocorrelated processes
statistical process control charts applied to autocorrelated processes

3.2 Abbreviated terms and symbols

3.2.1 Abbreviated terms

ARL	average run length
i.i.d.	independent and identically distributed
SPC	statistical process control
ACF	autocorrelation function
AR(1)	first order autoregressive process
EWMA	exponentially weighted moving average
EWMAST	exponentially weighted moving average for a stationary process
EWMS	exponentially weighted mean squared deviation
CUSUM	cumulative sum

3.2.2 Symbols

T	index set for a stochastic process
μ	true process mean
σ	true process standard deviation
$N\left(\mu,\sigma^2\right)$	normal distribution with a mean of μ and variance of σ^2
γ	autocovariance
$\hat{\gamma}$	estimator of autocovariance
ρ	autocorrelation
$\hat{\rho}$	estimator of autocorrelation
ϕ	dependent parameter of an AR(1) process
λ	smoothing parameter for EWMA
r	smoothing parameter for EWMS
τ	time lag between two time points
S_t^2	EWMS at t
S_0^2	initial value of S_t^2
X_t	random variable X at t
a_t	random variable a at t in an AR(1) process
Δ	step mean change as a multiple of the process standard deviation
$\overline{x}$	arithmetic mean value of a sequence of x
s	standard deviation of a sequence of x
$\hat{X}_t$	prediction of X_t
R_t	residual at t
$\overline{R}$	arithmetic mean value of R_t
S_R	standard deviation of $\{R_t\}$
Z_t	EWMA statistic at t
Z_0	initial value of Z_t
L_Z	value of the control limit for Z_t (expresses in number of standard deviation of Z_t)

σ_Z	standard deviation of EWMA statistic
σ_a	standard deviation of the random variables a_t from white noise in an AR(1) process

4 Control charts for autocorrelated processes for monitoring process mean

4.1 General

Many statisticians and statistical process control practitioners have found that autocorrelation in process data has an impact on the performance of the traditional SPC charts. Similar to autocovariance (see 3.1.1), autocorrelation is internal correlation between members of a series of observations ordered in time. Autocorrelation can be caused by the measurement system, the dynamics of the process, or both. In Annex B, the impact of positive autocorrelation on the performance of various traditional control charts is demonstrated.

4.2 Residual charts

The residual charts have been used to monitor possible changes of the process mean. To construct a residual chart, time series or other mathematical modelling has to be applied to the process data.

The residual chart requires modelling the process data and to obtain the process residuals[1]. For a set of time series data, $\{x_t; t=1,2,...,N\}$, a time series or other mathematical model is established to fit the data. A residual at t is defined as:

$$R_t = x_t - \hat{x}_t$$

where $\hat{x}_t$ is the prediction of the time series at t based on a time series or other mathematical model.

Assuming that the model is true, the residuals are statistically uncorrelated to each other. Then, traditional SPC charts such as X charts, CUSUM charts and EWMA charts can be applied to the residuals. When an X chart is applied to the residuals, it is usually called an X residual chart. Once a change of the mean in the residual process is detected, it is concluded that the mean of the process itself has been out-of-control.

Similarly, the CUSUM residual chart and EWMA residual chart are proposed[2][3]. See Reference [4] for comparisons between residual charts and other control charts.

Advantage of the residual charts:

— a residual chart can be applied to any autocorrelated data, even if it is nonstationary. Usually, a model is established with time series or other model fitting software.

Disadvantages of the residual charts:

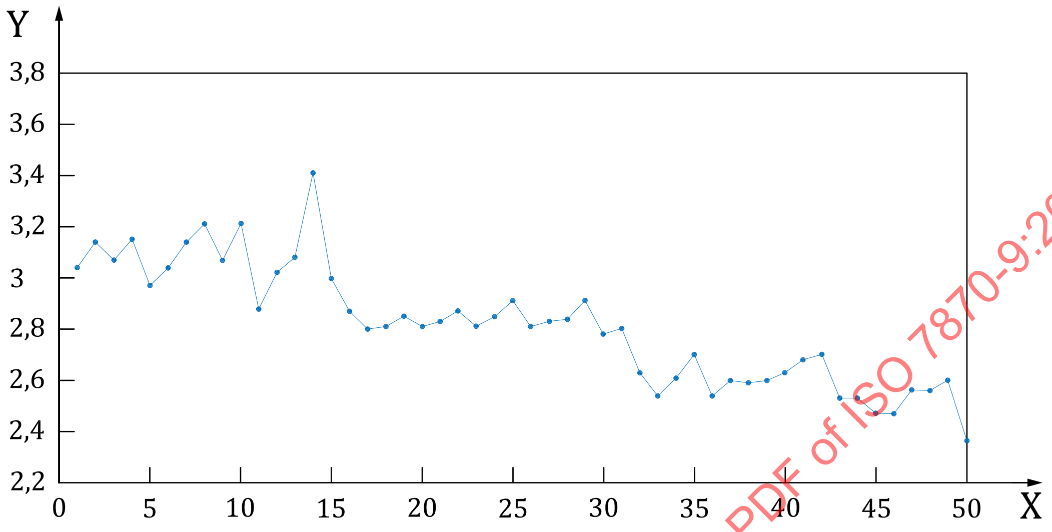
— the residual charts do not have the same properties as the traditional charts. The X residual chart for an AR(1) process (for an AR(1) process, see A.3.3) can have poor capability to detect a mean shift. Reference [5] shows that when the process is positively autocorrelated, the X residual chart does not perform well. Reference [6] shows that the detection capability of an X residual chart sometimes is small comparing to that of an X chart;

— the residual charts require time series or other modelling. The user of a residual chart shall check the validity of the model over time to reduce the mixed effect of modelling error and process change.

An example is illustrated in which the data, with a size of 50, are the daily measurements of the viscosity of a coolant in an aluminium cold rolling process[7]. Figure 1 shows the data with a decreasing trend. It is suspected that the measurements are not independent. Figure 2 shows the sample autocorrelation function (ACF) for lags from 0 to 12. For sample autocorrelation and ACF, see A.4.2 and A.5 in Annex A, and Reference [8]. As indicated in A.5, under the assumption for an i.i.d. normal sequence, approximately

95 % of the sample autocorrelations with a lag larger than one should fall between the bounds of $\pm 1{,}96/\sqrt{50}$. Based on that, the data are not independent. Reference [7] provides a model with the predicted viscosity at a period t given by:

$$\hat{x}_t = a + bx_{t-1} + cx_{t-2} + dx_{t-3} + ex_{t-4},\ t = 1,\ldots,50$$

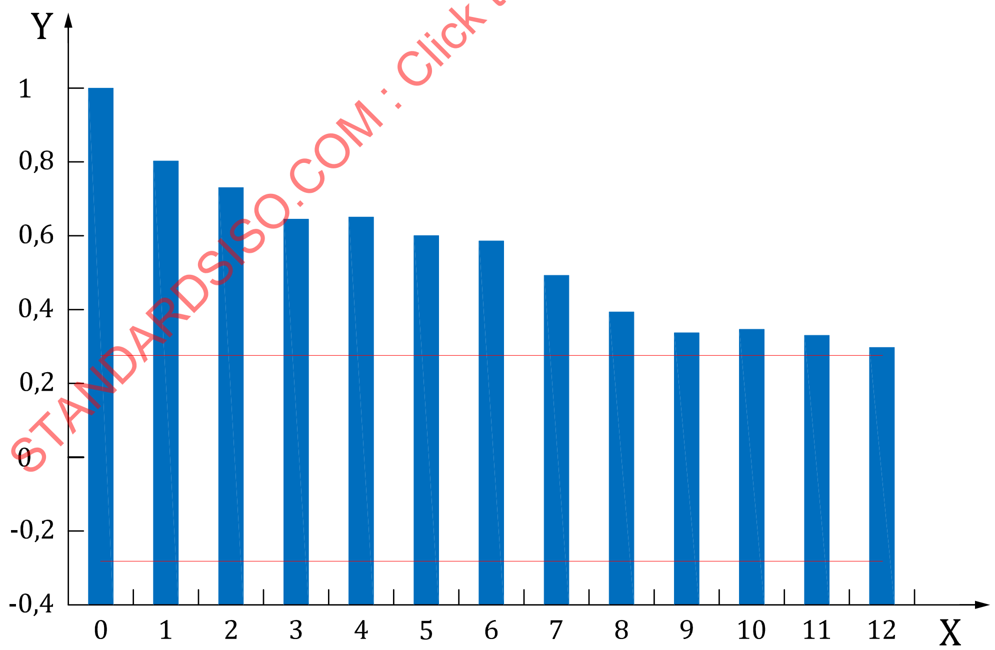


Key

X observation

Y viscosity

Figure 1 — Example



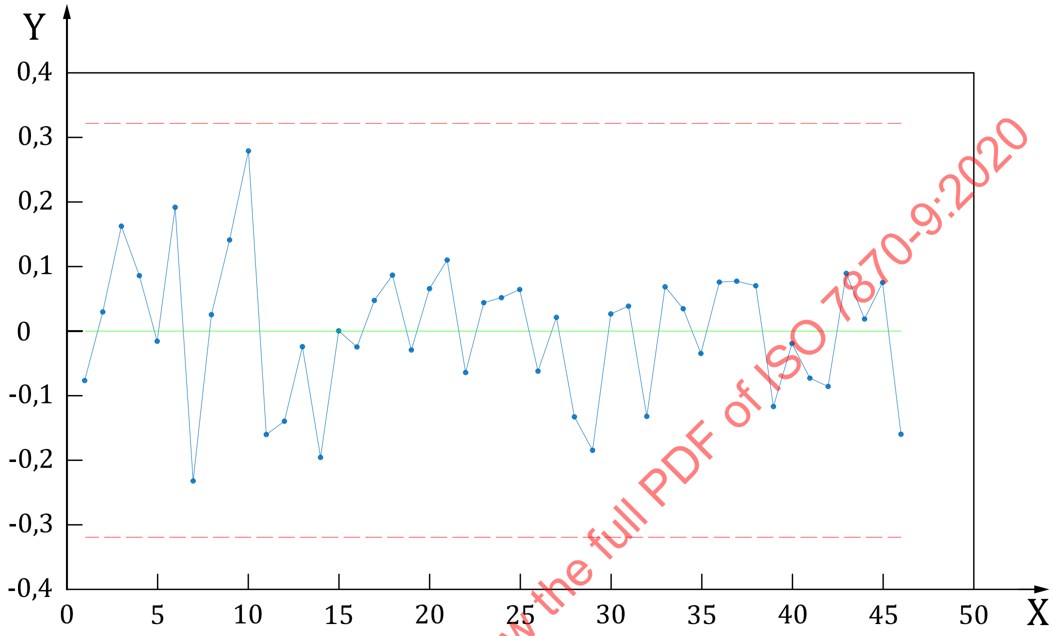
Key

X lag

Y autocorrelation

Figure 2 — Sample autocorrelations for the series of daily measurements of viscosity and an approximate 95 % confidence band

For the estimates of a, b, c, and d given in Reference [7], the residuals are calculated by $R_t = x_t - \hat{x}_t$, t = 1,..., 46 which are shown in Figure 3. To test whether the residuals are independent from each other, the ACF with a confidence band is again applied and shown in Figure 4. Since the residuals are determined to be not autocorrelated, a X chart with 3σ control limits ($\bar{R} \pm 3S_R$, where $\bar{R}$ is the average of $\{R_t\}$ and S_R is the standard deviation of $\{R_t\}$) applies to the residuals, as shown in Figure 3. It is concluded that the mean of the residuals, as well as the process, is in control.

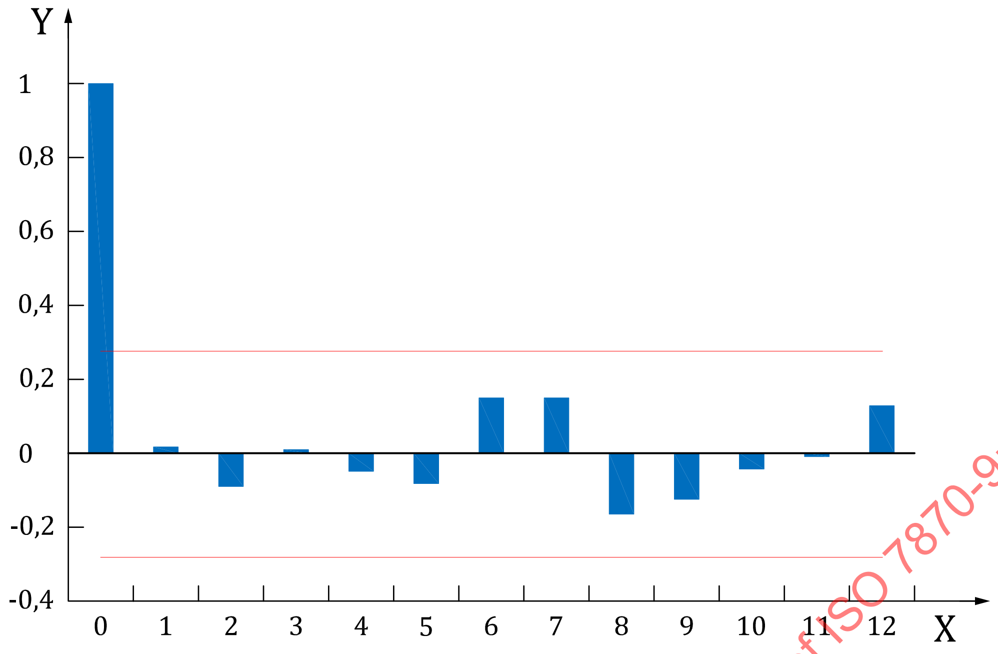


Key

X time

Y residual

Figure 3 — Residuals of the viscosity series and the *X* chart with 3*σ* control limits



Key

X lag

Y autocorrelation

Figure 4 — Sample autocorrelation of the residuals of viscosity series and an approximate 95 % confidence band

4.3 Traditional control charts with adjusted control limits

4.3.1 Modified EWMA chart

Comparing to the residual charts, a more direct approach is to modify the existing charts by adjusting the control limits without time series modelling. Some methods based on this approach, however, are restricted to specific processes, for example AR(1) processes[9]. Reference [10] proposes monitoring EWMA for a stationary process, an EWMAST chart, which can be applied to a stationary process in general. The chart is constructed by charting the EWMA statistic[10]:

$$Z_t = (1-\lambda) Z_{t-1} + \lambda X_t \tag{1}$$

where

$Z_0 = \mu$ is the process mean;

λ is the smoothing constant ($0 < \lambda \leq 1$).

Assume that the process $\{X_t; t = 1, 2, ..., N\}$ is stationary with mean μ and variance σ^2. When t is large, the variance of Z_t is approximated by:

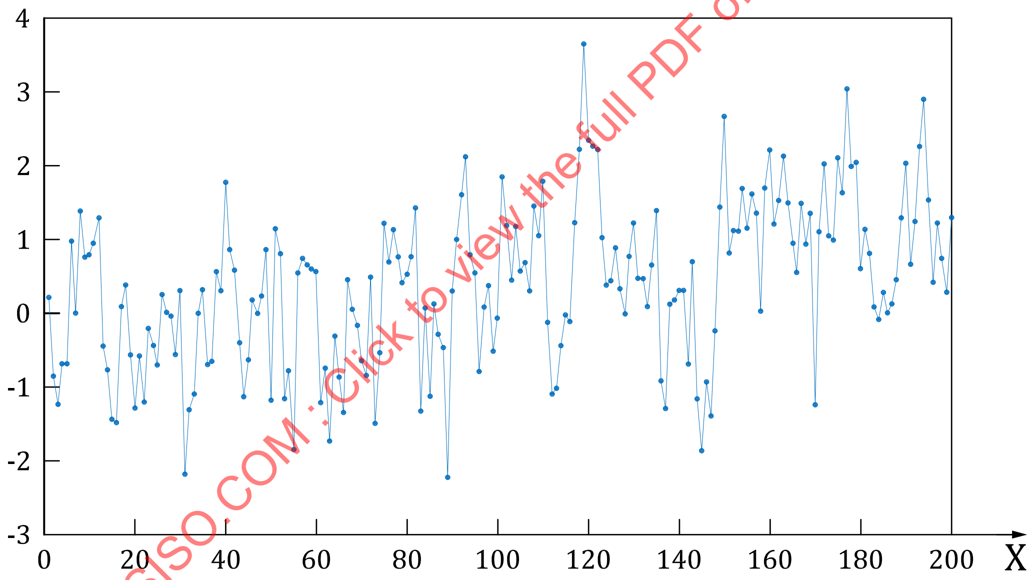
$$\sigma_z^2 \approx \left(\frac{\lambda}{2-\lambda}\right)\sigma^2 \left[1 + 2\sum_{k=1}^{M} \rho(k)(1-\lambda)^k \left[1-(1-\lambda)^{2(M-k)}\right]\right] \tag{2}$$

where M is an integer and $\rho(k)$ is the process autocorrelation at lag of k. Note that when the process is not autocorrelated, σ_z^2 is of the same form as that for the traditional EWMA chart. Assuming that X_t is normally distributed, Z_t is also normally distributed with a mean of μ. The EWMAST chart is constructed by charting Z_t. The centre line is at μ and the $L_Z\,\sigma$ control limits are given by:

$$\mu \pm L_z \sigma_z \,.$$

In general, λ = 0,2 is recommended[10], and L_Z usually equals two or three. When μ, σ and the autocorrelations are unknown, they are usually estimated by the arithmetic mean, $\bar{x}$, sample standard deviation, s, and sample autocorrelations, $\hat{\rho}\left(k\right)$, respectively based on some historical data of $\{X_t\}$ when the process is under control. When a set of historical data are used to estimate the autocorrelations, some rules of thumb can be followed. Reference [11] (p. 32) suggests that useful estimates of $\rho(k)$ can only be made if the data size N is roughly 50 or more and $k \le N/4$. Thus, M in Formula (2) should be large enough to make the approximation in Formula (2) usable and at the same time less than $N/4$ to avoid large estimation errors of autocorrelations. Based on simulation, when $N \ge 100$, M = 25 is recommended[10].

An example is illustrated, in which data from an AR(1) process with ϕ = 0,5, process variance σ^2 = 1, and length of 200 are simulated. The white noise (see A.3.2) is normally distributed. The process mean is zero for the first 100 observations. Beginning at the observation number 101, the process mean has a step mean change from 0 to 1 or 1σ. The plot of the simulated data is shown in Figure 5.

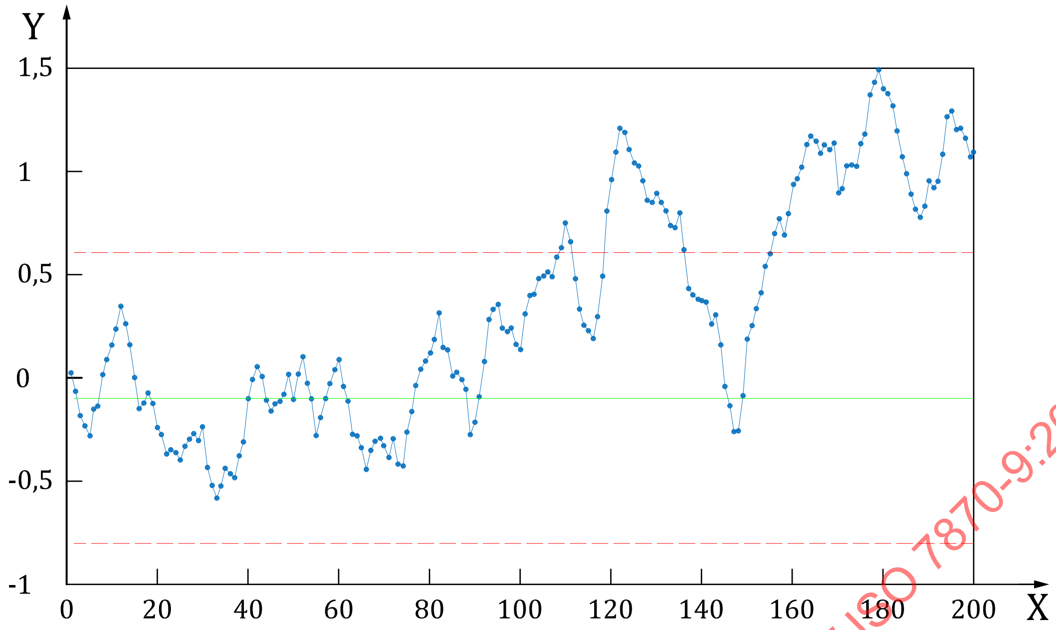


Key

X time

Figure 5 — Realization of the AR(1) process used to illustrate the EWMAST chart

Treating the period of the first 100 data points as stationary, the mean, the process standard deviation, and the sample autocorrelations are estimated. $\bar{x} = -0{,}10$, s = 0,91, and $\hat{\rho}(k)$, $(k = 1,\dots,25)$ are obtained. With M = 25 and λ = 0,2 in Formula (2), the standard deviation of Z_t is estimated by $\hat{\sigma}_z = 0{,}24$. Figure 6 shows the EWMAST chart with the centre line at $\bar{x} = -0{,}10$ and the 3σ control limits given by $\bar{x} \pm 3\hat{\sigma}_z = \left(-0{,}81; 0{,}60\right)$. The chart gives a signal indicating a mean increase starting at observation number 110.



Key

X time

Y EWMA

Figure 6 — EWMAST chart applies to the simulated data with a mean increase displayed in Figure 5

4.3.2 Modified CUSUM chart

Reference [12] considers charting the raw data directly by a CUSUM chart when the process autocorrelation is low. When the autocorrelation is high, the use of transformed observations is considered. Other approaches are proposed to apply modified CUSUM charts to AR(1) processes or some other time series[9][13].

4.4 Comparisons among charts for autocorrelated data

There are comparisons among some control charts for autocorrelated data. References [9] and [4] compare the *X* chart, *X* residual chart, CUSUM residual chart, EWMA residual chart, and EWMAST chart for stationary AR(1) processes by simulations. The EWMAST chart performs better than the CUSUM residual and EWMA residual charts. Overall, it also performs better than the *X* chart and *X* residual chart. The comparisons also show that the CUSUM residual and EWMA residual charts perform almost the same. The CUSUM residual and EWMA residual charts perform better than the *X* residual chart when the process autocorrelation is not strong. On the contrary, when the autocorrelation is strong, the *X* residual chart performs better than the other residual charts. When the process autocorrelation is very strong, i.e. the process is near nonstationary, the EWMAST chart still performs relatively better than other charts.

An obvious advantage of using EWMAST chart is that there is no need to build a time series model for stationary process data. The implementation of an EWMAST chart only requires the estimation of the process mean, standard deviation, and autocorrelations obtained when the process is under control. In summary, when the process is autocorrelated and stationary, it is recommended to use EWMAST chart to monitor the process mean.

5 Monitoring process variability for stationary processes

Reference [14] considers two control charts for monitoring the process variability: one is based on the exponentially weighted mean squared deviation from the target, called the exponentially weighted mean squared deviation (EWMS) chart, and the other is based on an exponentially weighted moving variance in which the process mean is estimated using an EWMA chart of the observations, called the exponentially weighted moving variance (EWMV) chart.

Assume that $\{X_t, t=1,2...\}$ is a process with mean μ and process variance σ^2 and jointly normally distributed. The exponential weighted moving mean square error is defined as:

$$S_t^2 = (1-r)S_{t-1}^2 + r(X_t - \mu)^2$$

where

t = 1,2,... ;

r is a smoothing parameter ($0 < r \leq 1$).

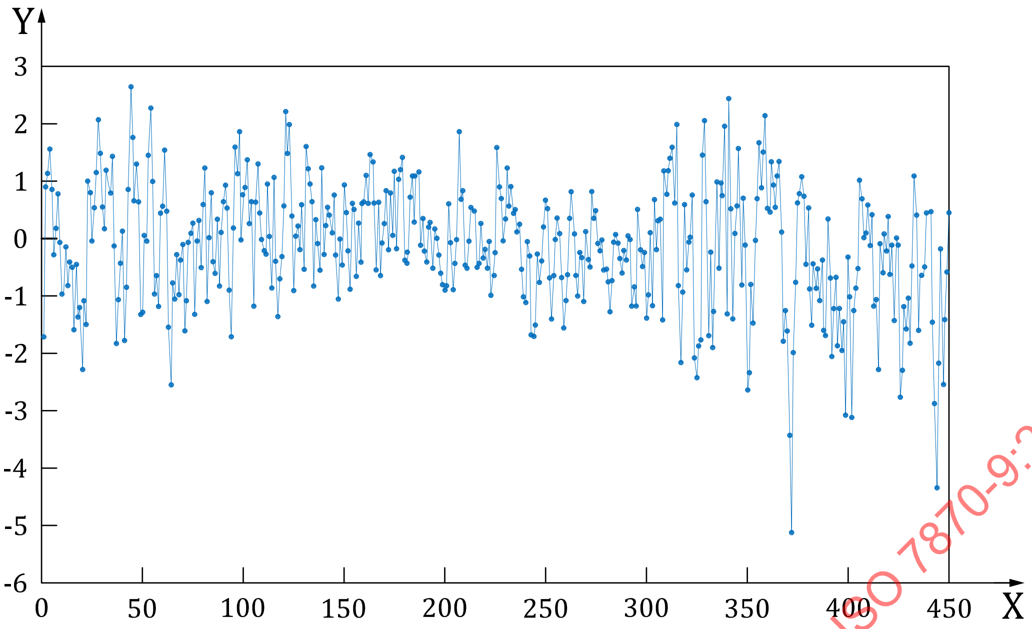
Usually, let $S_0^2 = \sigma^2$ be the process variance. From the above, S_t^2 is an estimator of the process mean squared error at time t. The EWMS chart is constructed by charting S_t^2 with the centre line at $S_0^2 = \sigma^2$, and the control limits are determined by σ^2 and a Chi-squared distribution with its degrees of freedom being a function of r for each t. Reference [14] proposes applying EWMS chart to an i.i.d. sequence and the processes, which can be represented as an AR(1) process plus white noise. Reference [15] proposes using residual chart to monitoring possible variance changes for a processes which is an AR(1) process plus white noise.

Reference [16] extends the EWMS chart to the case of stationary processes. Combining with the EWMAST chart, an EWMS chart can be used to detect possible variance change for a stationary process.

For illustration on the EWMS chart, a constructed example is presented. A realization from an AR(1) process is generated with mean $\mu = 0$ and the dependence parameter ϕ = 0,5. The process variance is σ^2 = 1 from t = 1 to t = 150, σ^2 = 0,5 from t = 151 to t = 300, and σ^2 = 2 from t = 301 to t = 450. The observed process is displayed in Figure 7.

An EWMAST chart is applied to the simulated data with the chart parameter λ = 0,2. The standard deviation of the EWMA statistic in the EWMAST chart based on Formula (2) is 0,51. The chart with 3σ control limits, shown in Figure 8, shows that although there are nine points between t from 372 to 448 out of the control limits, the process mean seems stable. Thus, the process is treated to have a constant mean.

For the EWMS chart, r = 0,05 and α = 0,05 are chosen, which give the asymptotic lower and upper control limits to be 0,52 and 1,64, respectively. Decreases in the mean squared error are detected from t = 158 and other points, and increases from t = 329 and other points, as shown in Figure 9. Since it is shown in Figure 8 that the process mean seems stable, it is concluded that the process variance changed.

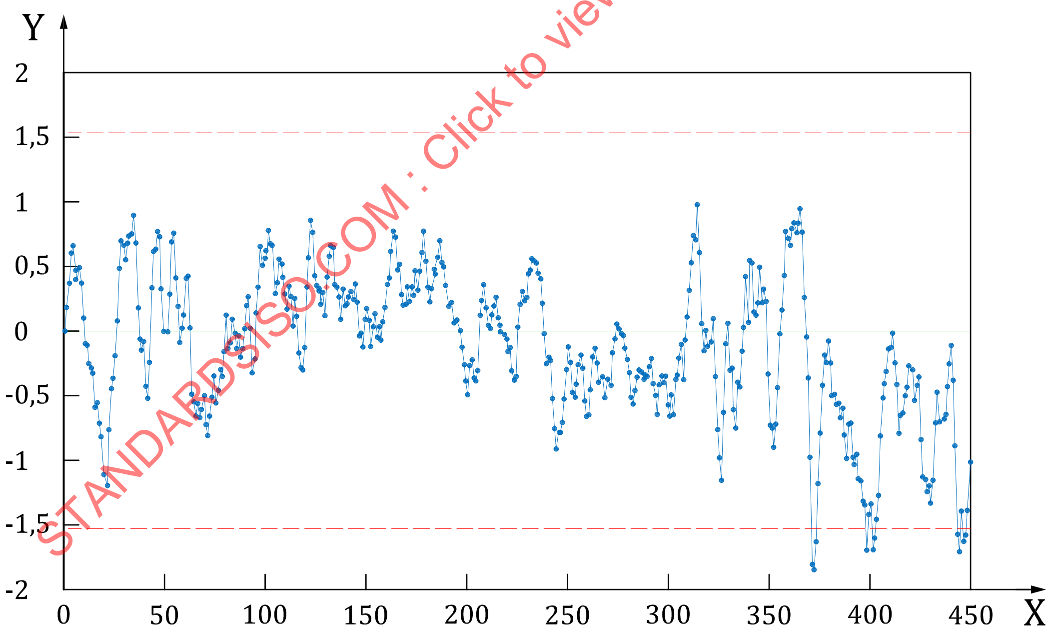


Key

X time

Y X

Figure 7 — Realization of the AR(1) process used to illustrate the EWMS procedure where the process mean is fixed at 0, but the process variance changes two times

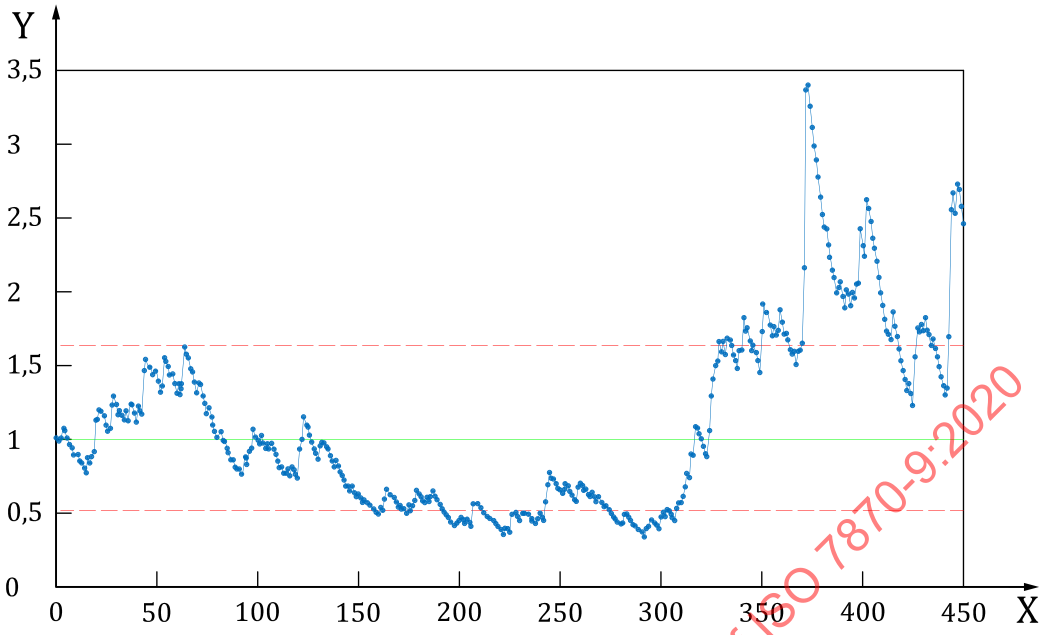


Key

X time

Y EWMA

Figure 8 — EWMAST chart with control limits for the time series displayed in Figure 7



Key
X time
Y EWMS

Figure 9 — EWMS chart with control limits for the time series displayed in Figure 7

6 Other approaches to deal with process autocorrelation

In Clause 4 and Clause 5, various process control charts to accommodate the autocorrelation of the process data are discussed. As an alternative to accommodating, the effect of the autocorrelation can be reduced by some data treatment mechanism. Reference [17] discusses the effects of the choice of the sampling interval on some process data. When the process is stationary and the samples are taken less frequently in time, the autocorrelation of the sampled data decreases. Thus, when the sampling interval is sufficiently large, the data appear to be uncorrelated. However, this approach discards the intermediate data and therefore increases the possibility of missing important events in the process. Instead of choosing a large sampling interval, moving averages of process with a fixed window size can be formed. Reference [18] shows that, when a process is stationary and satisfies some regularity conditions, the non-overlapping means or batch means are asymptotically independent and normally distributed. Thus, when the batch size is large enough, the batch means can be treated as white noise. For some specific stationary processes, numerous papers discuss the process behaviour of the subsample means or batch means, and the related charts for batch means. In Reference [19], the effect of using generalized moving averages of a stationary process to reduce its autocorrelation and its applications to process control charts are discussed.

Annex A
(informative)

Stochastic process and time series

A.1 General

A stochastic process $\{X_t; t \in T\}$ is a collection of random variables, where T is an index set[8]. When T represents time, the stochastic process is referred to as a time series. When T takes on a discrete set of values, e.g. $T = \{0, \pm 1, \pm 2, ...\}$, the process is said to be a discrete time series. In this document, only discrete time series with equal time space are considered. A discrete time series $x_1, x_2, ..., x_n$ can be viewed as the values taken by a sequence of random variables $X_1, X_2, ..., X_n$. The sequence of $x_1, x_2, ..., x_n$ is called a realization of $X_1, X_2, ..., X_n$.

A.2 Autocovariance and autocorrelation of a time series

If $\{X_t; t \in T\}$ is a time series with mean μ_t and standard deviation σ_t at time t,

1) for any $t_1, t_2 \in T$, the autocovariance function $\gamma(.)$ is: $\gamma(t_1, t_2) = \mathrm{E}\left[\left(X_{t_1} - \mu_{t_1}\right)\left(X_{t_2} - \mu_{t_2}\right)\right]$;

2) for any $t_1, t_2 \in T$, the autocorrelation function $\rho(.)$ is: $\rho(t_1, t_2) = \dfrac{\gamma(t_1, t_2)}{\sigma_{t_1}\sigma_{t_2}}$.

For a stochastic process or a time series, if there exist non-zero $\rho(t_1, t_2)$ for any $t_1 \neq t_2$, then the stochastic process or the time series is called autocorrelated.

A.3 Stationary time series and stationary time series models

A.3.1 General

A time series is said to be stationary if it is in a state of “statistical equilibrium.” Namely, the basic behaviour of such a time series does not change in time. The times series $\{X_t; t \in T\}$ is said to be covariance stationary, or stationary in this document, if:

1) $\mathrm{E}\left[X_t\right] = \mu$ (constant for all t);

2) the variance $\mathrm{V}\left[X_t\right] = \sigma^2 < \infty$ (i.e. a finite constant for all t);

3) $\gamma(t_1, t_2)$ depends only on lag $\tau = t_1 - t_2$. Then, $\gamma(t_1, t_2)$ is denoted by $\gamma(t_1, t_2) = \gamma(t_1 - t_2) = \gamma(\tau)$.

The first and second requirements are that the time series shall have identical means and identical variances. The third requirement is that the autocovariance function shall only depend on the time lag. If one or more requirements in the above are not met, the process is nonstationary. For a stationary time series, the autocovariance function at lag τ is often denoted by $\gamma(\tau)$.

The autocorrelation function (ACF) of a stationary times series at lag τ is given by:

$$\rho(\tau) = \frac{\gamma(\tau)}{\sigma^2}$$

It is obvious that $\rho(0) = 1$. Some simple stationary time series models are introduced in A.3.2 and A.3.3.

A.3.2 White noise

The time series is called white noise if:

1) X_t are identically distributed with a same mean and same finite variance;

2) the autocovariance $\gamma(t_1, t_2) = 0$ when $t_1 \neq t_2$.

It follows from 2) that the non-zero lag autocorrelations of white noise are all zero. When $\{X_t\}$ is white noise and each X_t is normally distributed, it is an i.i.d. sequence.

A.3.3 First order autoregressive [AR(1)] processes

The discrete time series $\{X_t; t \in (0, \pm 1, \pm 2, ...)\}$ is called a first order autoregressive [AR(1)] process if

$$X_t - \mu = \phi(X_{t-1} - \mu) + a_t \quad \text{(A.1)}$$

where

ϕ is a constant parameter;

$\{a_t\}$ are random variables from white noise with zero mean and variance σ_a^2.

When $|\phi| < 1$, $\{X_t\}$ is stationary with mean of μ and variance of $\sigma_a^2 / (1 - \phi^2)$ [20], Formula A.1 indicates the relationship between the term $X_{t-1} - \mu$, the deviation of the previous measurement from the mean and the deviation of the current value from the mean $X_t - \mu$. This models the dependency between the measurements. The magnitude of the dependency is determined by ϕ. When $|\phi| > 0$, it is often inferred that the process measurements are positively autocorrelated. Conversely, when $|\phi| < 0$, the process measurements are negatively autocorrelated. In particular, when $\phi = 0$ the process is white noise. Figures B.1 to B.5 demonstrates five realizations; each is generated from an AR(1) process with zero mean and ϕ from 0 to 0,9.

A.4 Estimation of the mean, autocovariance and autocorrelation for stationary time series

A.4.1 Estimation of μ

Given a realization $\{x_t; t = 1, 2, ..., N\}$, the process mean μ is often estimated by the arithmetic mean or sample mean $\bar{x} = \frac{\sum_{t=1}^{N} x_t}{N}$.

A.4.2 Estimation of $\gamma(\tau)$ and $\rho(\tau)$

For a stationary time series, the autocovariance at τ is often estimated by

$\hat{\gamma}(\tau) = \frac{\sum_{t=1}^{N-|\tau|} (X_t - \bar{X})(X_{t+|\tau|} - \bar{X})}{N}$ for $\tau = 0, \pm 1, ..., (N-1)$ and zero for $|\tau| \geq N$ [8]. In particular, when τ = 0, $\hat{\gamma}(0)$ is an estimator of the process variance. In practice, the traditional sample variance S^2, which uses

(N − 1) in the denominator instead of N, is often used in place of $\hat{\gamma}(0)$. The corresponding estimator of the autocorrelation, called the sample autocorrelation, is given by $\hat{\rho}(\tau)=\frac{\hat{\gamma}(\tau)}{\hat{\gamma}(0)}$.

A.5 Tests of autocorrelation of time series data

It is important to check whether the process data are autocorrelated. The two following simple tests are often used.

1) Use of sample autocorrelation function (ACF) plot with a confidence band:

   For a large N, the sample autocorrelations $\{\hat{\rho}(\tau)\}$ of an i.i.d. sequence $X_1,...,X_N$ with finite variance are approximated i.i.d. with distribution $N(0,1/N)$[20]. If $x_1,...,x_N$ is a realization of such a sequence, about 95 % of the sample autocorrelation with lag > 1 should fall between the bounds of $\pm 1{,}96/\sqrt{N}$. This is often used to check whether the process data are autocorrelated or not[21]. A practical example is presented in 4.2 to show the use of the sample ACF with a confidence band to test whether a set of process data are autocorrelated or not.

2) Runs test:

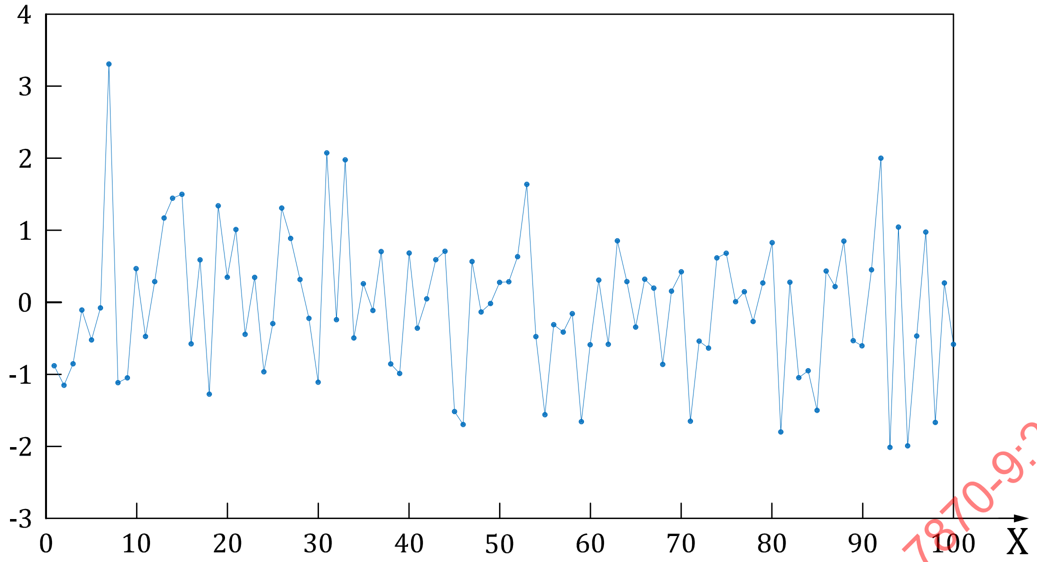
   The use of the sample ACF is based on the assumption that the underlying process is stationary, implicating a constant mean. However, the ACF plot can falsely show significant autocorrelations if the underlying random process does not have a constant mean. In that case, using ACF to check whether the process is autocorrelated or not can be misleading. A runs test can be used to supplement the ACF plot. The runs test of randomness is a nonparametric test based on runs up and down[21]. An example is presented in Reference [21] (pp. 99 – 101) to show the use of the runs test.

Annex B
(informative)

Performance of traditional control charts for autocorrelated data

The autocorrelation in process data has an impact on the performance of the traditional SPC charts. In the following example, the impact of positive autocorrelation on the performance of various traditional control charts is demonstrated. To produce autocorrelated data, a simulation of the stationary first order autoregressive [AR(1)] process [for an AR(1) process, see A.3.3] with $0 < \phi < 1$ and normally distributed random noise is used. To demonstrate the effect of process autocorrelation, five realizations of AR(1) processes with zero mean and white noise variance equal to 1, and length of 100 for each of ϕ = 0, 0,25, 0,5, 0,75, and 0,9 are generated and plotted in Figures B.1 to B.5, respectively.

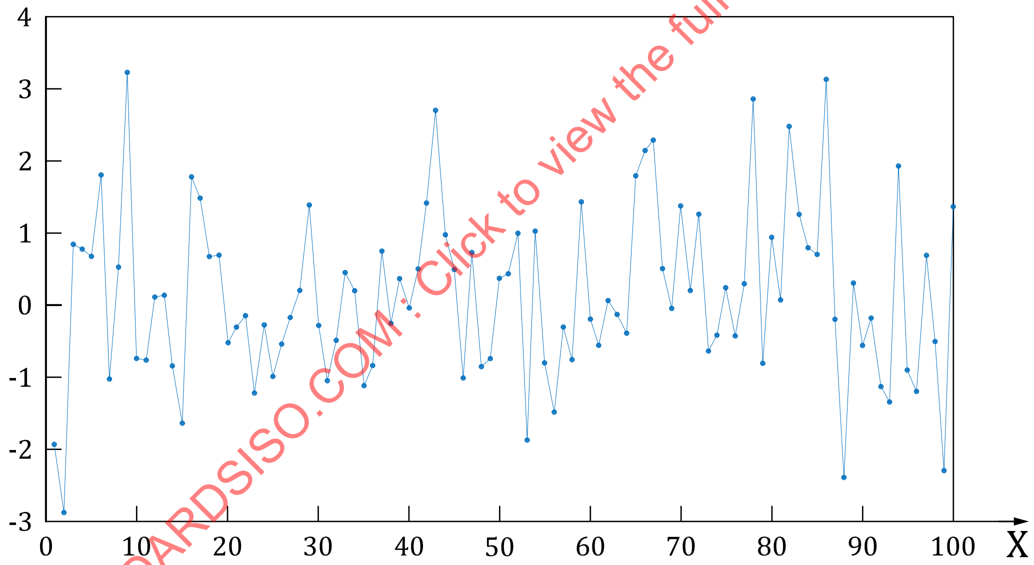
This part of the document is to show the impact of various magnitudes of autocorrelation on the type-1 error rate expressed as the in-control average run length (ARL) and on the type-2 error rate expressed as the out-of-control ARL for various changes in the process mean value. ARL is the average number of samples taken up to the point at which a signal occurs. See ISO 7870-4:2011, Clause 3.2[22]. A desired control chart should have large in-control ARL and small out-of-control ARL. That is, for a desired chart, when the process has no mean shift, the ARL should be large, and when a mean shift occurs, the ARL should be small to indicate the occurrence of the mean shift quickly. In the simulation, the magnitude of the autocorrelation varies, by changing the value of ϕ of the AR(1) process. For each value of ϕ, the ARL is measured when the underlying process is in statistical control and when it is not. Simulation is used to study the impact of each case of the AR(1) process. In each case, at least 2 000 time series are generated. For a process with no change in process mean, the run length is measured for each series and the average value is computed to obtain the in-control ARL. For an unstable process, the mean value is changed during the simulation of a series and the run length then is measured. Here, only step mean changes, Δ, as a multiple of the process standard deviation, are considered, where the mean changes instantaneously and then remains the same at the new level. The average of the run lengths of the 2 000 series is calculated to obtain the out-of-control ARL. The impact on the X chart, the CUSUM chart, and the EWMA chart is studied. For the X chart, 3σ control limits are used. For the EWMA chart, the smoothing parameter λ is 0,2 and the control limits are also 3σ limits. For the CUSUM chart, the ARLs reported in References [23] and [2] are used. The chart parameters used are, h = 5,0 and k = 0,5, for the tabular form of the CUSUM chart.



Key

X time

Figure B.1 — Realization of an AR(1) process with ϕ = 0



Key

X time

Figure B.2 — Realization of an AR(1) process with ϕ = 0,25