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Second edition
2024-07

Control charts —

Part 6: EWMA control charts for the process mean

Cartes de contrôle —

Partie 6: Cartes de contrôle EWMA pour la moyenne d'un processus

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Foreword

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This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in product and process management*.

This second edition cancels and replaces the first edition (ISO 7870-6:2016), which has been technically revised.

A list of all parts in the ISO 7870 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

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Introduction

Shewhart control charts are the most widespread statistical control methods used for controlling a process, but they are slow in signalling shifts of small magnitude in the process parameters. The exponentially weighted moving average[13] (EWMA) control chart makes possible faster detection of small to moderate shifts.

The Shewhart control chart is simple to implement and it rapidly detects shifts of major magnitude. However, it is fairly ineffective for detecting shifts of small or moderate magnitude. It happens quite often that the shift of the process is slow and progressive (in case of continuous processes in particular); this shift has to be detected very early in order to react before the process deviates seriously from its target value. There are two possibilities for improving the effectiveness of the Shewhart control charts with respect to small and moderate shifts.

— The simplest, but not the most economical possibility is to increase the subgroup size. This may not always be possible due to low production rate; time consuming or too costly testing. As a result, it may not be possible to draw samples of size more than 1.

— The second possibility is to take into account the results preceding the control under way in order to try to detect the existence of a shift in the production process. The Shewhart control chart takes into account only the information contained in the last sample observation and it ignores any information given by the entire sequence of points. This feature makes the Shewhart control chart relatively insensitive to small process shifts. Its effectiveness can be improved by taking into account the former results.

Where it is desired to detect slow, progressive shifts, it is preferable to use specific charts which take into account the past data and which are effective with a moderate control cost. Two very effective alternatives to the Shewhart control chart in such situations are

a) Cumulative sum (CUSUM) control chart. This chart is described in ISO 7870-4. The CUSUM control chart reacts more sensitively than the X-bar chart to a shift of the mean value in the range of half to two sigma. If one plots the cumulative sum of deviations of successive averages from a specified target, even minor, permanent shifts in the process mean will eventually lead to a sizable cumulative sum of deviations. Thus, this chart is particularly well-suited for detecting such small permanent shifts that may go undetected when using the X-bar chart.

b) Exponentially weighted moving average (EWMA) control chart which is covered by this document. This chart is presented like the Shewhart control chart; however, instead of placing on the chart the successive averages of the samples, one monitors a weighted average of the current average and of the previous averages.

EWMA control charts are generally used for detecting small shifts in the process mean. They will detect shifts of half sigma to two sigma much faster. They are, however, slower in detecting large shifts in the process mean. EWMA control charts can also be preferred when the subgroups are of size $n = 1$.

The joint use of an EWMA control chart with a small value of smoothing parameter (λ) and a Shewhart control chart has been recommended as a means of guaranteeing fast detection of both small and large shifts. The here considered EWMA control chart monitors only the process mean; monitoring the process variability requires the use of some other technique including special EWMA control charts.

The numbers in all tables and figures were calculated using the R-package SPC, (Knoth 2022), which makes use of the algorithm proposed by Crowder (1987).

The R-file containing the calculations can be downloaded on https://standards.iso.org/iso/7870/-6/ed-2/en.

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Control charts —

Part 6: EWMA control charts for the process mean

1 Scope

This document covers EWMA control charts, originally proposed by Roberts (1959)[16], as a statistical process control technique to detect small shifts in the process mean. It makes possible the faster detection of small to moderate shifts in the process mean. In this chart, the process mean is evaluated in terms of exponentially weighted moving average of all previous observations or averages.

The EWMA control chart's application is worthwhile in particular when

— production rate is slow,

— a minor or moderate shift in the process mean is vital to be detected,

— sampling and inspection procedure is complex and time consuming,

— testing is expensive, and

— it involves safety risks.

NOTE EWMA control charts are applicable for both variables and attributes data. The given examples illustrate both types (see 5.5, Annex A, Annex B and Annex C).

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-2 apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
run length
number of samples taken up to the point at which a signal occurs

3.2
average run length
ARL
Mathematical expectation of the run length

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3.3
maximum run length
MAXRL
95 %-percentile of the run length

4 Symbols and abbreviated terms

μ_0	Target value for the mean of the process
U_μ, L_μ	Upper rejectable value of the mean, lower rejectable value of the mean
$\bar{x}_i$	Average of the sample i
R_i	Range of the sample i
n	Number of items in a sample (sample size)
z_i	EWMA value for the sample i
z_0	Initial value of EWMA series $\{z_i\}$
λ	The smoothing parameter
L_z	Factor to establish the control limit for z_i
s	Estimate of the standard deviation σ
σ	Standard deviation of the distribution of X (the random variable to be monitored)
σ_0	In-control standard deviation
$\sigma_{\bar{x}}$	Standard deviation of the averages of n individual observations; $\sigma_{\bar{x}} = \sigma / \sqrt{n}$
σ_z	Standard deviation of z_i when i tends towards infinity
δ	Shift in the mean from the target value μ_0, expressed in number of standard deviations
δ_1	Maximum acceptable shift in the mean from μ_0, expressed in number of standard deviations
p	Proportion of nonconforming items of the process
p_0	Target value for the proportion of nonconforming items of the process
p_1	Upper refusable value of the proportion of nonconforming items
p_i	Proportion of nonconforming items in the i^{th} sample
c	Mean number of nonconformities
c_0	Target value for the mean number of nonconformities
c_1	Refusable mean of nonconformities
c_i	Number of nonconformities in the i^{th} sample
U_{CL}	Upper control limit
L_{CL}	Lower control limit

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ARL Average Run Length

ARL_0 Average Run Length of the process in control

ARL_1 Average Run Length of the process with shift

CL Centre line

MAXRL Maximum Run Length

5 EWMA for inspection by variables

5.1 General

An EWMA control chart plots exponentially weighted moving averages of past and current data in which the values being averaged are assigned weights that decrease exponentially from the present into the past, see Figure 1. Consequently, the average values are influenced more by recent process performance. The exponentially weighted moving average is defined as Formula (1):

$$z_i = \lambda x_i + (1 - \lambda) z_{i-1} \tag{1}$$

Where $0 < \lambda \leq 1$ is a constant and the starting value (required with the first sample at $i = 1$) is the process target, so that $z_0 = \mu_0$.

NOTE 1 When the EWMA control chart is used with rational subgroups of size $n > 1$ then x_i is simply replaced with $\bar{x}_i$.

NOTE 2 μ_0 can be estimated by the average of preliminary data.

The EWMA control chart becomes an $\bar{X}$ chart for $\lambda = 1$.

5.2 Weighted average explained

To demonstrate that the EWMA is a weighted average of all previous observations or averages, the right-hand side of Formula (1) in 5.1 can be substituted with z_{i-1} to obtain Formula (2):

$$\begin{aligned} z_i &= \lambda x_i + (1-\lambda)[\lambda x_{i-1} + (1-\lambda) z_{i-2}] \\ &= \lambda x_i + \lambda(1-\lambda) x_{i-1} + (1-\lambda)^2 z_{i-2} \end{aligned} \tag{2}$$

Continuing to substitute recursively for z_{i-j}, where $j = 2, 3, \ldots$, we obtain Formula (3):

$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \tag{3}$$

For $i = 1$, $z_1 = \lambda x_1 + (1 - \lambda)\mu_0$.

The weights, $\lambda(1 - \lambda)^j$, decrease geometrically with the age of the observation or average. Furthermore, the weights sum to unity:

$$\lambda \sum_{j=0}^{i-1} (1-\lambda)^j + (1-\lambda)^i = \lambda \left[\frac{1-(1-\lambda)^i}{1-(1-\lambda)} \right] + (1-\lambda)^i = 1 \tag{4}$$

If $\lambda = 0{,}25$, then the weight assigned to the current average is 0,25 and the weights given to the preceding averages are 0,187 5; 0,140 6; 0,105 5 and so forth. These weights are shown in Figure 1. Because these weights decline geometrically, the EWMA is sometimes called a geometric moving average (GMA), which is the original name of the control chart[16].

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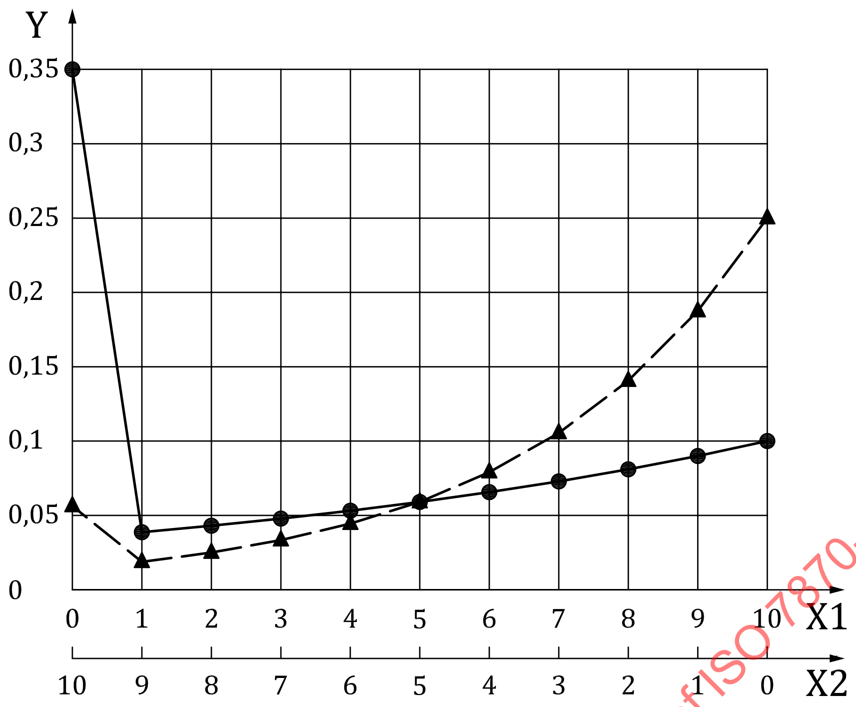

Key

X1 sample number

X2 age of the observation or average

Y weights $\lambda(1-\lambda)^{10-X1}$

—●— $\lambda = 0{,}1$

--▲-- $\lambda = 0{,}25$

Figure 1 — Weights of all 10 averages after having incorporated sample 10

Since the EWMA value can be viewed as a weighted average of all past and current observations, it is very insensitive to the normality assumption. It is, therefore an ideal control chart to use with individual observations.

5.3 Control limits for EWMA control chart

If the observations x_i are independent random variables with variance σ^2, then the variance of z_i is represented by Formula (5):

$$\sigma_{z_i}^2 = \frac{\sigma^2}{n}\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right] \quad (5)$$

Therefore, the EWMA control chart would be constructed by plotting z_i versus the sample number i (or time). The centre line and control limits for the EWMA control chart are as follows:

Centre line = μ_0

$$U_{CL} = \mu_0 + L_z \frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \quad (6)$$

$$L_{CL} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \quad (7)$$

The factor L_z refers to the distance of the control limits from the centre line and can be derived by setting an appropriate in-control ARL (average run length) value. It is well-known that the common Shewhart

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control chart with 3 sigma limits exhibits an in-control ARL of 370,4. To achieve the same value for an EWMA control chart with control limits, see Formula (6) and Formula (7), one chooses factor L_z =2,715 and 2,864 for λ = 0,1 and 0,2, respectively. For other choices of λ see Table 4. Note that using just the Shewhart factor 3 (or other values accordingly) provides a quick and dirty approach to set EWMA limits, where the detection performance is mostly better than that of the older Shewhart control chart.

No action is taken as long as z_i falls between these limits, and the process is considered to be out of control as soon as z_i overshoots the control limits. In this case, an investigation is initiated to locate the assignable cause, and the process can be stopped or adjusted. In the latter case, resume the EWMA control chart after reinitializing it, i.e. by not taking into account the results obtained prior to this resetting, but by taking z_0 as the initial value.

The term $[1 - (1 - \lambda)^{2i}]$ approaches unity as i gets larger. This means that after the EWMA control chart has been running for several time periods, the control limits will approach steady state values obtained using Formula (8) and Formula (9):

Centre line = μ_0

$$U_{\mathrm{CL}} = \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \tag{8}$$

$$L_{\mathrm{CL}} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \tag{9}$$

However, it is strongly recommended to use the exact control limits. This will greatly improve the performance of the control chart in detecting an off-target process immediately after the EWMA control chart is initiated.

NOTE For practical purposes, use the estimate of σ, denoted by s, estimated from the data.

5.4 Construction of EWMA control chart

To illustrate the construction of an EWMA control chart, a process with the following parameters calculated from historical data (individual observations, i.e. sample size n = 1) is considered:

μ_0 = 50

σ = 2,053 9

with λ chosen to be 0,3; so that

$$\sqrt{\frac{\lambda}{(2-\lambda)}} = \sqrt{\frac{0{,}3}{1{,}7}} = 0{,}420\,1 \tag{10}$$

The control limits at steady-state are given, obtained using Formula (11) and Formula (12):

$$U_{\mathrm{CL}} = 50 + 2{,}925\ (0{,}420\ 1)\ (2{,}053\ 9) = 52{,}523\ 8 \tag{11}$$

$$L_{\mathrm{CL}} = 50 - 2{,}925\ (0{,}420\ 1)\ (2{,}053\ 9) = 47{,}476\ 2 \tag{12}$$

The factor L_z is picked from Table 2. The data consisting of 20 points as given in Table 1 are considered.

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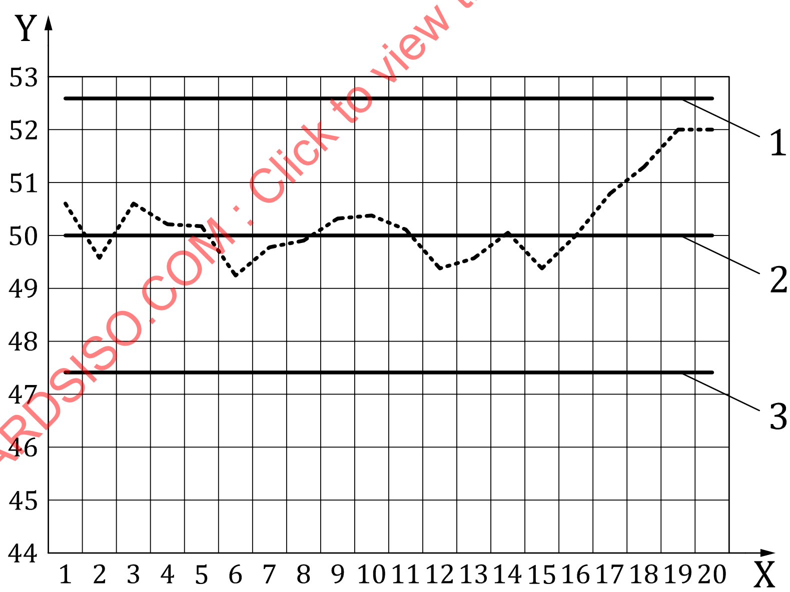
Table 1 — EWMA values

Sample	X_i	**EWMA value**, $z_i = \lambda X_i + (1 - \lambda) z_{i-1}, z_0 = \mu_0$
1	52,0	50,600 0
2	47,0	49,520 0
3	53,0	50,564 0
4	49,3	50,184 8
5	50,1	50,159 4
6	47,0	49,211 6
7	51,0	49,748 1
8	50,1	49,853 7
9	51,2	50,257 6
10	50,5	50,330 3
11	49,6	50,111 2
12	47,6	49,357 8
13	49,9	49,520 5
14	51,3	50,054 3
15	47,8	49,378 0
16	51,2	49,924 6
17	52,6	50,727 2
18	52,4	51,229 1
19	53,6	51,940 3
20	52,1	51,988 2



Key

X sample

Y EWMA value

1 U_{CL} = 52,523 8

2 CL = 50

3 L_{CL} = 47,476 2

Figure 2 — EWMA control chart plot

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The EWMA control chart in Figure 2 shows that the process is in control because all EWMA points lie between the control limits.

For convenience, we provide factor L_z values for several choices of in-control ARL_0 and of the smoothing constant λ in case of constant (steady-state) limits, see Formula (8) and Formula (9). The numbers in Table 2 were calculated using the R package SPC, (Knoth 2022), which makes use of the algorithm proposed by Crowder (1987).

Table 2 — L_z values for constant (steady-state) limits

λ	ARL_0 = 100	ARL_0 = 200	ARL_0 = 370,4	ARL_0 = 500
0,02	1,467	1,827	2,135	2,278
0,05	1,879	2,216	2,490	2,615
0,1	2,148	2,454	2,701	2,814
0,15	2,279	2,567	2,801	2,907
0,2	2,360	2,635	2,859	2,962
0,25	2,414	2,681	2,898	2,998
0,3	2,453	2,713	2,925	3,023
0,4	2,504	2,754	2,959	3,054
0,5	2,534	2,777	2,978	3,071
0,75	2,568	2,802	2,997	3,087
1	2,576	2,807	3,000	3,090

5.5 Example

Consider the data in Table 3 (observations x_i). The first 20 observations were drawn at random from a normal distribution with mean μ = 10 and standard deviation σ = 1. The last 10 observations were drawn from a normal distribution with mean μ = 11 and standard deviation σ = 1, i.e. after the process has experienced a shift in the mean of one sigma.

An EWMA control chart with λ = 0,10 and L_z = 2,715 is applied to the data in Table 3 [differently to 5.4, the control limits given in Formula (6) and Formula (7) are deployed].

The target value of the mean is μ = 10 and the standard deviation is σ = 1.

The calculations for EWMA control chart are summarized in Table 3 and the control chart is shown in Figure 3.

To illustrate the calculations, the first observation, x_i = 9,45 is considered.

The first value of the EWMA statistic is shown in Formula (13):

$$\begin{aligned} z_1 &= \lambda x_1 + (1-\lambda) z_0 = 0{,}1 \times 9{,}45 + 0{,}9 \times 10 \\ &= 9{,}945\,00 \end{aligned} \quad (13)$$

Therefore, z_1 = 9,945 00 is the first value plotted on the control chart in Figure 3.

The second value of the EWMA is shown in Formula (14):

$$\begin{aligned} z_2 &= \lambda x_2 + (1-\lambda) z_1 = 0{,}1 \times 7{,}99 + 0{,}9 \times 9{,}945 \\ &= 9{,}749\,50 \end{aligned} \quad (14)$$

The other values of the EWMA statistic are computed similarly.

The control limits are calculated following Formula (15) and Formula (16):

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For period i = 1:

$$U_{CL} = \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$= 10 + 2{,}715 \times 1 \times \sqrt{\frac{0{,}1}{(2-0{,}1)}\left[1-(1-0{,}1)^{2\times 1}\right]} \quad (15)$$

$$= 10{,}271\,50$$

and

$$L_{CL} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$= 10 - 2{,}715 \times 1 \times \sqrt{\frac{0{,}1}{(2-0{,}1)}\left[1-(1-0{,}1)^{2\times 1}\right]} \quad (16)$$

$$= 9{,}728\,50$$

For period i = 2, the limits are shown in Formula (17) and Formula (18):

$$U_{CL} = \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$= 10 + 2{,}715 \times 1 \times \sqrt{\frac{0{,}1}{(2-0{,}1)}\left[1-(1-0{,}1)^{2\times 2}\right]} \quad (17)$$

$$= 10{,}365\,27$$

and

$$L_{CL} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]}$$

$$= 10 - 2{,}715 \times 1 \times \sqrt{\frac{0{,}1}{(2-0{,}1)}\left[1-(1-0{,}1)^{2\times 2}\right]} \quad (18)$$

$$= 9{,}634\,73$$

The calculation of control limits are also summarized in Table 2 and plotted in Figure 3.

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Table 3 — EWMA calculations

Sample	x_i	EWMA z_i	U_{CL}	L_{CL}
1	9,45	9,945 00	10,271 50	9,728 50
2	7,99	9,749 50	10,365 27	9,634 73
3	9,29	9,703 55	10,426 36	9,573 64
4	11,66	9,899 20	10,470 06	9,529 94
5	12,16	10,125 28	10,502 68	9,497 32
6	10,18	10,130 75	10,527 62	9,472 38
7	8,04	9,921 67	10,547 00	9,453 00
8	11,46	10,075 51	10,562 20	9,437 80
9	9,20	9,987 96	10,574 22	9,425 78
10	10,34	10,023 16	10,583 77	9,416 23
11	9,03	9,923 84	10,591 40	9,408 60
12	11,47	10,078 46	10,597 51	9,402 49
13	10,51	10,121 61	10,602 41	9,397 59

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Table 3 *(continued)*

Sample	x_i	EWMA z_i	U_{CL}	L_{CL}
14	9,40	10,049 45	10,606 35	9,393 65
15	10,08	10,052 51	10,609 52	9,390 48
16	9,37	9,984 26	10,612 08	9,387 92
17	10,62	10,047 83	10,614 14	9,385 86
18	10,31	10,074 05	10,615 81	9,884 19
19	8,52	9,918 64	10,617 15	9,382 85
20	10,84	10,010 78	10,618 24	9,381 76
21	10,90	10,099 70	10,619 12	9,380 88
22	9,33	10,022 73	10,619 84	9,380 16
23	12,29	10,249 46	10,620 41	9,379 59
24	11,50	10,374 51	10,620 88	9,379 12
25	10,60	10,397 06	10,621 26	9,378 74
26	11,08	10,465 35	10,621 56	9,378 44
27	10,38	10,456 82	10,621 81	9,378 19
28	11,62	10,573 14	10,622 01	9,377 99
29	11,31	10,646 82	10,622 17	9,377 83
30	10,52	10,634 14	10,622 30	9,377 70

It can be noted from Figure 3 that the control limits increase in width as i increases from i = 1, 2, ..., until they stabilize at the steady-state values given in Formula (19) and Formula (20):

$$\begin{aligned} U_{CL} &= \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \\ &= 10 + 2{,}715 \times 1 \times \sqrt{\frac{0{,}1}{(2-0{,}1)}} \\ &= 10{,}619\,65 \end{aligned} \tag{19}$$

and

$$\begin{aligned} L_{CL} &= \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \\ &= 10 - 2{,}715 \times 1 \times \sqrt{\frac{0{,}1}{(2-0{,}1)}} \\ &= 9{,}380\,35 \end{aligned} \tag{20}$$

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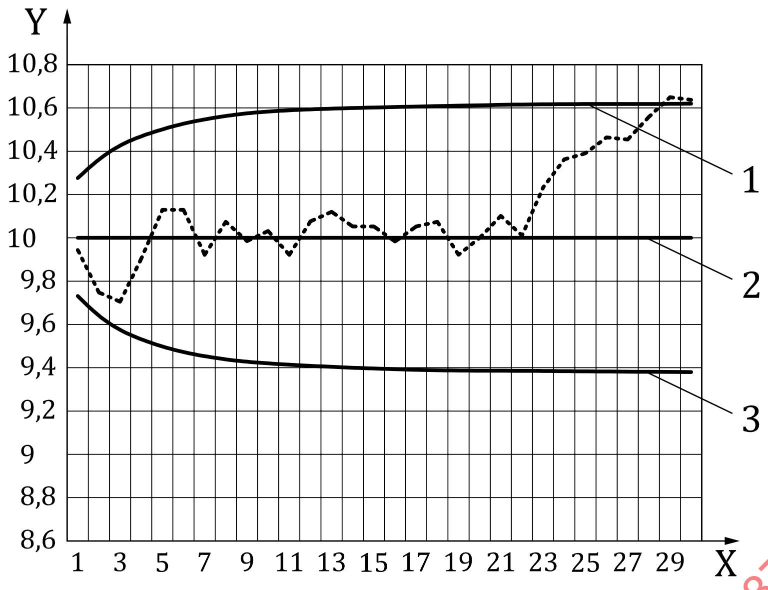

Key

X sample

Y EWMA value

1 U_{CL} = 10,62

2 CL = 10,00

3 L_{CL} = 9,38

Figure 3 — EWMA control chart

The EWMA control chart signals that observation 29 has gone beyond U_{CL}. Hence it is concluded that the process is out of control.

6 Choice of the control chart

6.1 Shewhart control chart versus EWMA control chart

Unlike the Shewhart control chart, it is not possible to find the probability of detecting a shift in the process on the basis of a sample because the probability is not constant. It depends on the number of the samples. It is possible calculate this probability for each sample, but these probabilities are too numerous to be used in practice.

The effectiveness of the EWMA technique is therefore judged according to the ARL, i.e. the average number of successive samples required for detecting a shift.

If the process is under control, it is expected that there can be few false alarms, i.e. that the average number of samples prior to a false alarm is high (in general ARL_0 is taken between 100 and 1 000).

On the other hand, in the event of a shift, it is expected that it can be detected as quickly as possible, i.e. that the expected number of samples between the moment the shift occurred and that of the first point outside the control limits be the lowest possible (low ARL_1).

Compared to the Shewhart control chart, the EWMA technique is extremely effective for minor or moderate shifts (because EWMA uses information from past samples): the lower λ is, the more effective the chart is. On the other hand, the Shewhart control chart is more effective for sudden high shifts.

The effectiveness of the chart depends on the size of the sample: the higher n is, the more effective the chart is.

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6.2 Average run length

Table 4 gives the ARL and the MAXRL of the chart as a function of the shift, $\delta\sqrt{n}$. Therefore, the effectiveness for any value of n can be obtained. All measures are given for the limits in Formula (6) and Formula (7). They were calculated by following Knoth (2003, 2004) and utilizing the R package "SPC" (Knoth, 2022).

For example, the EWMA control chart with λ = 0,5, L_z = 2,979 and n = 1 detects a shift of δ = 1 standard deviation in 14,9 samples on average because $\delta\sqrt{n} = 1$. Whereas, the same chart with n = 4 detects it in 3,2 samples, because $\delta\sqrt{n} = 2$.

In Table 4, the values of L_z – utilizing the exact limits in Formula (6) and Formula (7) – for the EWMA techniques have been chosen, so that the ARL (Average Run Length) = 370 approximately (i.e. the same as that of the Shewhart control chart, with control limits established at $\mu_0 \pm 3\sigma / \sqrt{n}$) when the shift, δ, is equal to 0. Hence, you can compare the figures in the six columns, because all EWMA control chart designs exhibit the same in control (δ =0) ARL. Table 3 shows that the effectiveness for detecting minor shifts is better for small values of λ (e.g. the ARL goes from 14,9 to 7,6 for $\delta\sqrt{n} = 1$).

The choice of λ and L_z is made so as to obtain an average run length which one sets in an a priori manner as the quality objective. One can therefore thus obtain charts which correspond to the practical requirements of industry or services.

Table 4 — Comparison of mean operational periods of EWMA and Shewhart control chart

Shift	Shewhart control chart		EWMA control charts									
	λ = 1,0 L_z = 3,0		λ = 0,5 L_z = 2,979		λ = 0,4 L_z = 2,961		λ = 0,3 L_z = 2,928		λ = 0,2 L_z = 2,864		λ = 0,1 L_z = 2,715	
$\delta\sqrt{n}$	ARL	MAXRL	ARL	MAXRL	ARL	MAXRL	ARL	MAXRL	ARL	MAXRL	ARL	MAXRL
0,00	370,4		370,6		371,0		370,8		370,1		370,8	
0,25	281,2	842	195,9	585	173,7	517	148,5	441	119,7	353	86,3	249
0,50	155,2	464	71,3	210	58,0	169	45,8	132	35,0	97	25,7	66
0,75	81,2	242	29,9	86	24,0	67	19,2	52	15,4	39	12,5	29
1,00	43,9	130	14,9	41	12,3	33	10,3	26	8,8	21	7,6	17
1,25	25,0	74	8,7	23	7,5	18	6,6	15	5,9	13	5,3	11
1,50	15,0	44	5,7	14	5,1	12	4,7	10	4,3	9	3,9	8
1,75	9,5	27	4,1	9	3,8	8	3,6	7	3,4	7	3,1	6
2,00	6,3	18	3,2	7	3,0	6	2,9	6	2,7	5	2,5	5
2,25	4,4	12	2,6	5	2,5	5	2,4	5	2,3	4	2,1	4
2,50	3,2	9	2,2	4	2,1	4	2,0	4	2,0	4	1,8	3
2,75	2,5	6	1,9	4	1,8	3	1,8	3	1,7	3	1,6	3
3,00	2,0	5	1,6	3	1,6	3	1,6	3	1,5	3	1,5	3

6.3 Choice of parameters for EWMA control chart

6.3.1 Choice of λ

The smaller λ is, the more the past is taken into account and the better minor shifts are detected; on the other hand, major shifts are rather less well detected.

The higher λ is, the less the past is taken into account and the better the reactivity to major shifts will be; on the other hand, minor shifts are less well detected.

The choice of λ shall be made on the basis of the experience. In general, 0,05 ≤ λ ≤ 0,50 works well in practice,

— if small shifts are expected, a value of λ between 0,05 to 0,25 should be used, and

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— if moderate magnitude shifts should be avoided/are feared, rather a value of λ close to 0,5 should be used.

The most commonly used values of λ are between 0,25 and 0,5. It is to be noted that one obtains the Shewhart control chart if one takes $\lambda = 1$.

6.3.2 Choice of L_z

The factor L_z is multiplied with the corresponding standard deviation to establish the control limits. It is typically set at 3 for simplicity, but it can be necessary to reduce it slightly for small values of λ. L_z between 2,6 and 2,8 is useful when $\lambda \leq 0{,}1$. Some SPC software systems allow explicit setting as function of λ and the in-control ARL target.

6.3.3 Calculation for n

Table 5 (following Table 4 in Lucas/Saccucci 1990 and augmented with R package “SPC”) gives the parameters of the EWMA control chart for a given effectiveness: one sets the ARL_0 when the process is controlled and ARL_1 when the process has shifted by a given value δ_1. Table 5 gives the values of L_z and λ which enable to obtain the desired effectiveness.

The maximum acceptable shift δ_1 of the mean is shown in Formula (21):

$$\delta_1 = \min\left\{\frac{U_\mu - \mu_0}{\sigma_0}, \frac{\mu_0 - L_\mu}{\sigma_0}\right\} \quad (21)$$

Proceed as follows:

— Stage 1: Select the average number of samples (ARL_0) desired between two false alarms (generally between 100 and 1 000); this determines the choice of the column of Table 4;

— Stage 2: Select the average number of samples (ARL_1) required in order to detect the maximum acceptable shift δ_1; then look for, within the table in the previously determined column, the value of ARL_1 which is closest to that sought after; read L_z and λ values associated with ARL_1; the corresponding line gives $\delta_1\sqrt{n}$; hence n;

— Stage 3: if n is too high for practical reasons (cost, feasibility, etc.) return to Stage 1 after reducing the requirements on the input parameters of Table 4 (ARL_0, ARL_1, δ_1).

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Table 5 — Determination of L_z and λ as a function of the controlled and uncontrolled run lengths (ARL_0 and ARL_1) and of the shift δ_1

$\delta_1\sqrt{n}$	ARL **when the process is controlled**, ARL_0			
	100	**370**	**500**	**1 000**
0,5	$\lambda = 0{,}07$ $L_z = 2{,}015$ $\mathrm{ARL}_1 = 17{,}3$	$\lambda = 0{,}06$ $L_z = 2{,}550$ $\mathrm{ARL}_1 = 26{,}6$	$\lambda = 0{,}05$ $L_z = 2{,}615$ $\mathrm{ARL}_1 = 28{,}8$	$\lambda = 0{,}04$ $L_z = 2{,}818$ $\mathrm{ARL}_1 = 34{,}2$
0,75	$\lambda = 0{,}12$ $L_z = 2{,}209$ $\mathrm{ARL}_1 = 10{,}3$	$\lambda = 0{,}10$ $L_z = 2{,}701$ $\mathrm{ARL}_1 = 14{,}7$	$\lambda = 0{,}09$ $L_z = 2{,}787$ $\mathrm{ARL}_1 = 15{,}8$	$\lambda = 0{,}07$ $L_z = 2{,}974$ $\mathrm{ARL}_1 = 18{,}4$
1,0	$\lambda = 0{,}19$ $L_z = 2{,}346$ $\mathrm{ARL}_1 = 6{,}96$	$\lambda = 0{,}15$ $L_z = 2{,}800$ $\mathrm{ARL}_1 = 9{,}58$	$\lambda = 0{,}15$ $L_z = 2{,}907$ $\mathrm{ARL}_1 = 10{,}2$	$\lambda = 0{,}13$ $L_z = 3{,}113$ $\mathrm{ARL}_1 = 11{,}7$
1,5	$\lambda = 0{,}33$ $L_z = 2{,}471$ $\mathrm{ARL}_1 = 3{,}95$	$\lambda = 0{,}26$ $L_z = 2{,}904$ $\mathrm{ARL}_1 = 5{,}17$	$\lambda = 0{,}24$ $L_z = 2{,}992$ $\mathrm{ARL}_1 = 5{,}46$	$\lambda = 0{,}22$ $L_z = 3{,}200$ $\mathrm{ARL}_1 = 6{,}14$
2,0	$\lambda = 0{,}52$ $L_z = 2{,}539$ $\mathrm{ARL}_1 = 2{,}63$	$\lambda = 0{,}40$ $L_z = 2{,}959$ $\mathrm{ARL}_1 = 3{,}35$	$\lambda = 0{,}37$ $L_z = 3{,}047$ $\mathrm{ARL}_1 = 3{,}51$	$\lambda = 0{,}35$ $L_z = 3{,}253$ $\mathrm{ARL}_1 = 3{,}90$
2,5	$\lambda = 0{,}66$ $L_z = 2{,}560$ $\mathrm{ARL}_1 = 1{,}89$	$\lambda = 0{,}54$ $L_z = 2{,}983$ $\mathrm{ARL}_1 = 2{,}38$	$\lambda = 0{,}52$ $L_z = 3{,}073$ $\mathrm{ARL}_1 = 2{,}50$	$\lambda = 0{,}46$ $L_z = 3{,}272$ $\mathrm{ARL}_1 = 2{,}76$
3,0	$\lambda = 0{,}81$ $L_z = 2{,}572$ $\mathrm{ARL}_1 = 1{,}45$	$\lambda = 0{,}70$ $L_z = 2{,}994$ $\mathrm{ARL}_1 = 1{,}78$	$\lambda = 0{,}70$ $L_z = 3{,}086$ $\mathrm{ARL}_1 = 1{,}86$	$\lambda = 0{,}66$ $L_z = 3{,}286$ $\mathrm{ARL}_1 = 2{,}06$
NOTE If ARL_1 is chosen below 1,40, use a Shewhart control chart.				

6.3.4 Example

For a process, the target mean value $\mu_0 = 100$ and a standard deviation of $\sigma_0 = 0{,}8$ is desired to get a maximum of one false alarm every 500 samples. It is desired to detect within three or four samples, on average, a shift of ±1 unit (rejectable means $U_\mu = 101$ and $L_\mu = 99$).

— Select from Table 5 the column which corresponds to $\mathrm{ARL}_0 = 500$.

— Look for, in this column, the ARL_1 which is closest to 3. We find 3,5 which corresponds to $L_z = 3{,}05$ and $\lambda = 0{,}37$ for $\delta_1\sqrt{n} = 2$.

— Calculate $\delta_1 = \min(1/0{,}8;1/0{,}8) = 1{,}25$. As $\delta_1\sqrt{n} = 2$ one deduces from this $n = (2/1{,}25)^2 = 2{,}56 \cong n = 3$, rounding off to the higher integer (which improves the detection effectiveness).

7 Procedure for implementing the EWMA control chart

The implementation of the EWMA control chart is the same as for any other type of control procedure. The procedure is built on the assumption that the “good” historical data are representative of the in-control process, with future data from the same process tested for agreement with the historical data. To start the procedure, a target value (mean) and process standard deviation are estimated from historical data. Then the procedure enters the monitoring stage with the EWMA statistics computed and tested against the control limits.

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8 Sensitivity of the EWMA to non-normality

For subgroup size one, both individual Shewhart control chart and EWMA control chart can be used. However, an individual Shewhart control chart is sensitive to non-normality, whereas a properly designed EWMA is less sensitive to the normality assumption.

9 Advantages and limitations

9.1 Advantages

a) The Shewhart control chart only uses the current observation or sample to monitor the process. EWMA control charts utilize all previous observations, but the weight attached to the data are exponentially declining as the observations get older and older. It gives more weight to the recent past value depending upon the value of λ. By varying the parameter of the EWMA statistic the 'memory' of the EWMA control chart can be influenced.

b) The EWMA control chart is relatively robust in the face of non-normally distributed quality characteristics.

c) Variables control charts can be constructed for individual observations taken from the production line, rather than samples of observations. This is sometimes necessary when testing samples of multiple observations would be too expensive, inconvenient, or impossible. For example, the number of customer complaints or product returns are only available on a monthly basis; yet, one would like to chart those numbers to detect quality problems. Another common application of these charts occurs in cases when automated testing devices inspect every single item that is produced. In that case, one is often primarily interested in detecting small shifts in the product quality (for example, gradual deterioration of quality due to machine wear).

9.2 Limitations

a) The EWMA control chart is sensitive to small shifts in the process mean, but does not reflect the out of control situations for larger shifts as quickly as the Shewhart control chart does. It is also recommended to superimpose the EWMA control chart on top of a suitable Shewhart control chart with widened control limits in order to detect both small and large shifts in the process mean.

b) When the EWMA control chart is used with a small value of the weight λ then at the beginning of the production, the EWMA control chart is more efficient in detecting the shift. As production proceeds, if a trend develops, the trend is unfortunately shown to be well within control of the EWMA chart's control limits.

c) In Figure 4, we give an example, where after 30 observations the mean shifts from 50 to 51. The standard deviation remains at the level 1,2. The smoothing constant is set to 0,1 for illustrating the potential inertia problems. We observe that right at the time of change the EWMA statistic is close to the lower boundary. This makes it more difficult to detect the increase in the mean.

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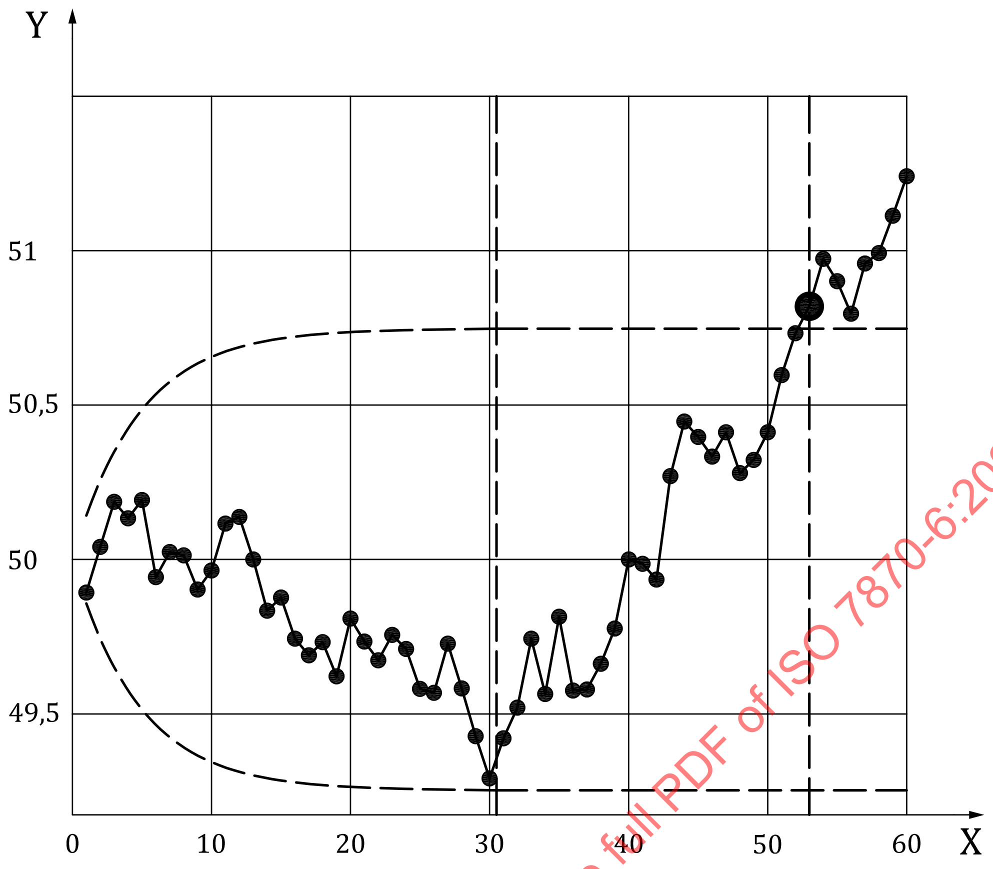

Key

X sample

Y EWMA value

Figure 4 — EWMA path with inertia problem

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Annex A
(informative)

Application of the EWMA control chart

A continuous production process involving the filling of μ_0 = 100 ml dosage bottles with a pharmaceutical product is considered. The target is for customers to have a low risk, about 0,135 %, of finding a bottle under the lower tolerance T_L = 99,5 ml. Over proportioning should be avoided for economic reasons and since customers use the bottle as a dose. The upper tolerance, T_U, for the individual values is fixed at 100,6 ml.

When the process is in control, the standard deviation of the individual measurements is assumed to be known as σ_0 = 0,1 ml and it was ascertained that the distribution could be considered normal. The mean can vary within limits set at 3 standard deviations above and below the tolerance limits. This ensures a probability of less than 0,135 % of out of tolerance values as shown in Formula (A.1) and Formula (A.2):

$$u_\mu = T_U - 3\sigma_0 = 100{,}6 - 3 \times 0{,}1 = 100{,}3 \tag{A.1}$$

$$l_\mu = T_L + 3\sigma_0 = 99{,}5 + 3 \times 0{,}1 = 99{,}8 \tag{A.2}$$

Hence, $\delta_1 = \min\left[(U_\mu - \mu_0)/\sigma_0;(\mu_0 - L_\mu)/\sigma_0\right] = \min\left[(100{,}3 - 100)/0{,}1;(100{,}0 - 99{,}8)/0{,}1\right] = 2{,}0$

An ARL_0 of 500 can be achieved when the process is properly centred and detected within two or three successive samples when the process has shifted by δ_1 = 2. Table 5 gives for ARL_0 = 500 and ARL_1 between 2 and 3 the following values:

ARL_1 = 2,50;

$\delta_1 \sqrt{n} = 2{,}5$;

L_z = 3,073;

λ = 0,52.

Hence, $n = (2{,}5/2)^2 = 1{,}562\,5$, where n = 2 (rounding off to the higher integer, which improves the detection effectiveness).

The steady-state control limits are shown in Formula (A.3) and Formula (A.4):

$$U_{CL} = 100 + \left(\frac{3{,}073 \times 0{,}1}{\sqrt{2}}\sqrt{0{,}52/(2-0{,}52)}\right) = 100{,}129 \tag{A.3}$$

$$L_{CL} = 100 - \left(\frac{3{,}073 \times 0{,}1}{\sqrt{2}}\sqrt{0{,}52/(2-0{,}52)}\right) = 99{,}871 \tag{A.4}$$

The initial and target values are μ_0 = 100.

The following individual values are obtained on conducting a control (Table A.1); one calculates their means, $\bar{x}_i$, their ranges, R_i, and the statistics, z_i:

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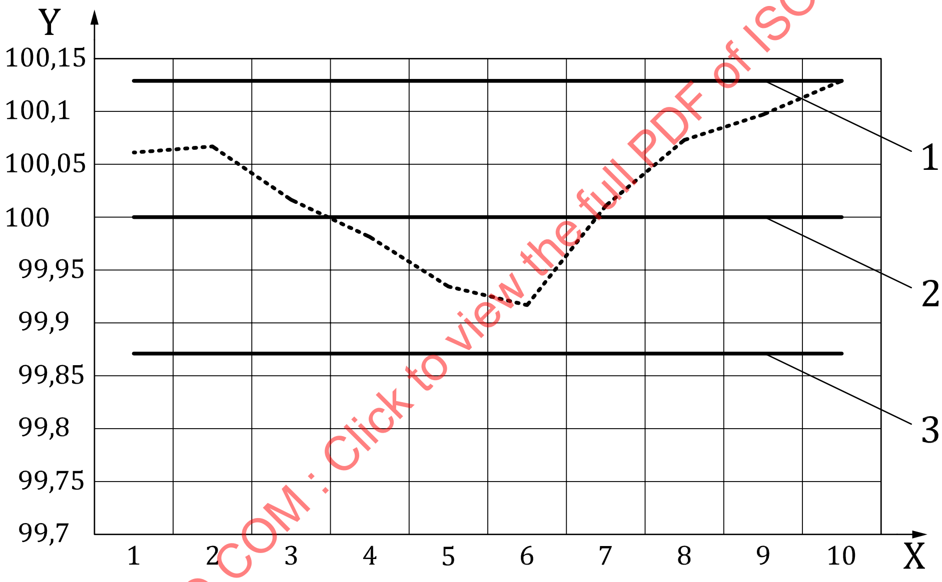
Table A.1 — Calculation of Shewhart control charts and EWMA values for n = 2

Sample	Individual values		$\overline{x}_i$	R_i	$z_i = 0{,}52\overline{x}_i + 0{,}48 z_{i-1}$
1	99,99	100,25	100,12	0,26	100,062
2	100,01	100,13	100,07	0,12	100,066
3	99,98	99,96	99,97	0,02	100,016
4	99,84	100,06	99,95	0,22	99,982
5	99,93	99,85	99,89	0,08	99,934
6	99,86	99,94	99,90	0,08	99,916
7	100,05	100,15	100,10	0,10	100,012
8	100,28	99,98	100,13	0,30	100,073
9	100,17	100,07	100,12	0,10	100,098
10	100,13	100,19	100,16	0,06	100,130

Figure A.1 shows that at the 10th sample, z_i, overshoots the upper control limit, indicating that the process has shifted and should be reset. After resetting, restart a new chart, replacing previous values with $z_0 = \mu_0 = 100$.



Key

X sample

Y EWMA value

1 U_{CL} = 100,129

2 CL = 100,000

3 L_{CL} = 99,871

Figure A.1 — EWMA control chart for the mean

The associated chart of the range R_i of the samples (Figure A.2) does not show any change in the dispersion. The mentioned shift corresponds to a shift in the mean and not to an increase in the dispersion of the process.

NOTE The calculations of the centre line and the control limit values for the dispersion (range) chart are defined in ISO 7870-2.

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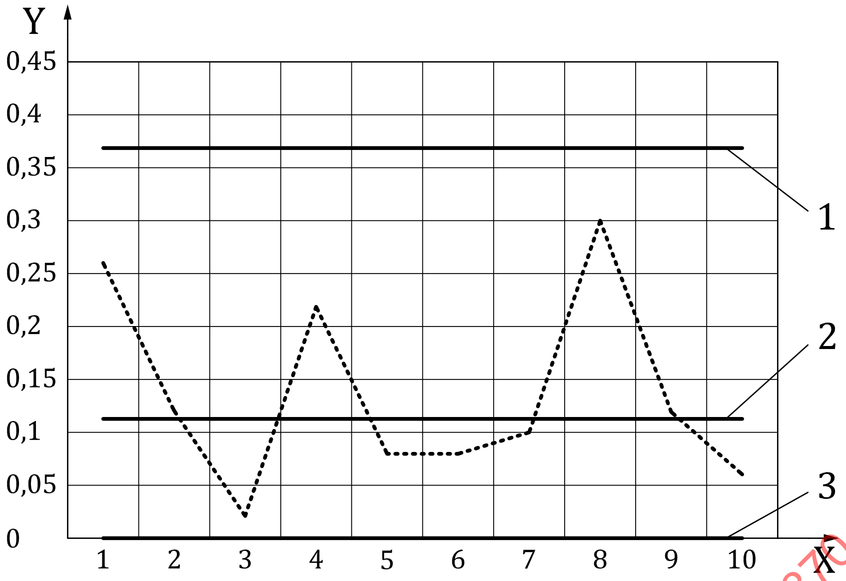

Key

X sample

Y range

1 upper control limit for the range

2 target range

3 lower control limit for the range

Figure A.2 — Shewhart range control chart

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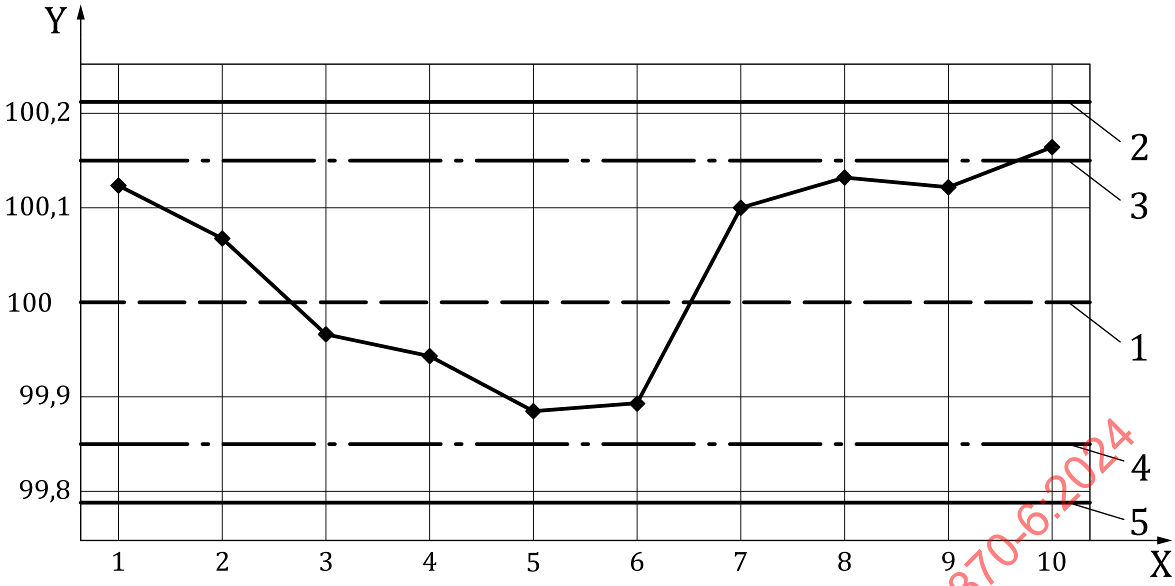
The control limits of the corresponding Shewhart X-bar control chart with n = 2 and μ_0 ±3,09 $\sigma_0/\sqrt{n}$ are located at 100,22 and 99,78: This chart does not detect any process shift. It would be necessary to double the size of samples (n = 4), i.e. the cost of the control, in order to detect a shift concerning these data (Figure A.3). For simplicity assume that the same data points stem now from samples of size n = 4. The control limits are, of course, tighter and trigger a signal at sample 10.



Key

X sample

Y X-bar

1 target

2 upper control limit for $n = 2$

3 upper control limit for $n = 4$

4 lower control limit for $n = 4$

5 lower control limit for $n = 2$

Figure A.3 — Shewhart X-bar control chart for $n = 2$ and $n = 4$

NOTE This example illustrates the fact that the EWMA control chart is more sensitive than the Shewhart control chart for a small shift of the mean. If the lag had been sudden and high, the Shewhart control chart would have been more rapid in pointing it out.

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