
**Petroleum and liquid petroleum
products — Calibration of vertical
cylindrical tanks —**

**Part 4:
Internal electro-optical distance-ranging
method**

*Pétrole et produits pétroliers liquides — Jaugeage des réservoirs
cylindriques verticaux —*

Partie 4: Méthode par mesurage électro-optique interne de la distance



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Contents

Page

Foreword	iv
1 Scope	1
2 Normative references	1
3 Terms and definitions	1
4 Precautions	2
5 Equipment	2
6 General considerations	3
7 EODR instrument set-up within the tank	4
8 Selection of target points	4
9 Calibration procedure	5
10 Other measurements	7
11 Calculation and development of capacity tables	7
Annex A (normative) Procedure for verifying EODR instruments	8
Annex B (normative) Calculation of internal radii from the measured parameters	11
Annex C (informative) Calibration uncertainties	12
Bibliography	20

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 7507-4 was prepared by Technical Committee ISO/TC 28, *Petroleum products and lubricants*, Subcommittee SC 2, *Measurement of petroleum and related products*.

This second edition cancels and replaces the first edition (ISO 7507-4:1995), which has been technically revised.

ISO 7507 consists of the following parts, under the general title *Petroleum and liquid petroleum products — Calibration of vertical cylindrical tanks*:

- *Part 1: Strapping method*
- *Part 2: Optical-reference-line method*
- *Part 3: Optical-triangulation method*
- *Part 4: Internal electro-optical distance-ranging method*
- *Part 5: External electro-optical distance-ranging method*

Petroleum and liquid petroleum products — Calibration of vertical cylindrical tanks —

Part 4: Internal electro-optical distance-ranging method

1 Scope

This part of ISO 7507 specifies a method for the calibration of vertical cylindrical tanks having diameters greater than 5 m by means of internal measurements using an electro-optical distance-ranging (EODR) instrument, and for the subsequent compilation of tank capacity tables.

The method is suitable for tanks tilted up to a 3 % deviation from the vertical, provided that a correction is applied for the measured tilt as described in ISO 7507-1:2003, Clause 11.

This part of ISO 7507 also applies to tanks with floating roofs or internal floating blankets.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 7507-1:2003, *Petroleum and liquid petroleum products — Calibration of vertical cylindrical tanks — Part 1: Strapping method*

IEC 60825-1:2007, *Safety of laser products — Part 1: Equipment classification and requirements*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 7507-1 and the following apply.

3.1 reference target point

fixed point clearly marked on a fixed point within the tank, e.g. on the inside surface of the tank shell wall

3.2 slope distance

distance measured from the electro-optical distance-ranging instrument to a target point on any given course of the tank shell wall

3.3 target point

one of a series of points on the inside surface of the tank shell wall to which slope distance, and vertical and horizontal angles are measured by use of the electro-optical ranging instrument

4 Precautions

The general precautions and safety precautions specified in ISO 7507-1 shall apply to this part of ISO 7507. In addition, the inside of the tank shall be clean. The laser beam emitted by the distance-ranging unit shall conform to IEC 60825-1.

5 Equipment

5.1 Electro-optical distance-ranging instrument

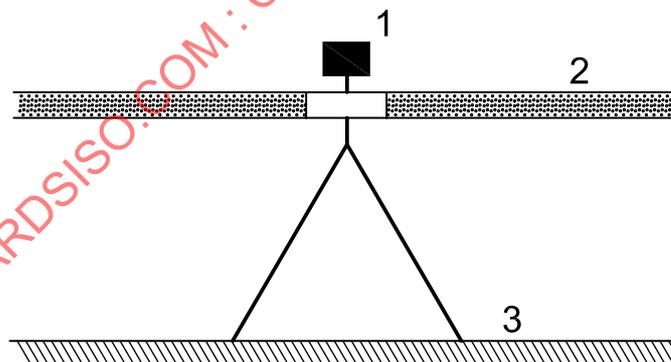
To achieve uncertainties of tank volumes acceptable in legal metrology, the angular measuring part of the instrument should have an angular graduation and resolution equal to or better than $3,142 \times 10^{-6}$ rad [0,2 mgon¹⁾] and the distance-measuring part of the instrument, used for direct determination of distances, should have a graduation and resolution equal to or better than 1 mm.

NOTE For analysis of the impact of uncertainty of the equipment on the uncertainty of the calibration table, refer to the examples in Annex C.

The accuracy of EODR equipment can be affected by variations of temperature. The manufacturer's guidance should be followed.

5.2 Instrument mounting

Instrument mounting shall be firm and stable. If a tripod is used, its legs shall be held firm, and steadied, by suitable devices such as magnetic bearers or an equivalent system. On floating-roof tanks or tanks with internal floating blankets, the supports of the EODR instrument should preferably rest on the tank bottom, while access to the instrument is through the roof or the blanket (see Figure 1). If this is not possible, mounting directly on the floating roof or blanket can be possible, but only if the stability of the instrument during the calibration procedure can be guaranteed.



Key

- 1 EODR instrument
- 2 floating roof
- 3 tank bottom

Figure 1 — EODR installation on floating roof

1) 2π radians = 400 000 mgon.

EODR measurements are restricted by

- a) minimum measurable distance;
- b) minimum incidence angle that still gives expected measurement uncertainty.

These limits are normally provided by the EODR equipment manufacturer.

If, for the given shape of the tank (height and diameter), the limits of incidence angles given by the manufacturer cannot be met from one position, either a higher uncertainty shall be accepted or the EODR equipment shall be placed in several positions. If this is not possible, this method might not be suitable for the given application.

NOTE Integration of measurements from multiple positions of the EODR, if required, is usually performed using procedures published by the instrument manufacturer. There are no standard calculations and, therefore, this part of ISO 7507 does not include equations for this case.

5.3 Laser beam emitter

The laser beam emitter used to position target points on the tank shell shall be either an integral part of the EODR instrument or a separate device whose axis is parallel to the axis of the EODR instrument.

5.4 Stadia

Stadia shall be a bar at least 2 m long with two marks. The graduated length between these marks shall be calibrated. Extended calibration uncertainty should be of the order of 0,05 mm. The stadia shall be made of a material whose thermal expansion is known.

5.5 Equipment for bottom calibration

Equipment for calibration of the tank bottom shall be as specified in ISO 7507-1:2003, Clause 10.

5.6 Auxiliary equipment

Auxiliary equipment shall include devices to steady the EODR unit, lighting, etc.

6 General considerations

6.1 The EODR instrument shall be maintained so that its performance can be traced to national and/or international measurement standards. The accuracy of the angular measuring unit shall be verified using the procedures described in Annex A. The distance-measuring unit is expected to hold its calibration and it is not normally necessary to verify it in the field. If verification is performed, the procedure given in Annex A shall be followed.

6.2 Tanks shall be calibrated only after they have been filled at least once with a liquid of density equal to or greater than that of the liquid that they will hold when in use.

NOTE The hydrostatic test applied to new tanks can satisfy this requirement in most cases.

6.3 The tank shall be free from vibration and air-borne dust particles. The floor of the tank should be as free as possible from debris, dust and loose scale.

6.4 Encrustation of the inner walls of the tank should be checked. If the estimated thickness, nature and distribution of the encrustation are deemed unacceptable, the tank should not be calibrated using this method.

6.5 Lighting, when required, shall be placed within the tank so as not to interfere with the operation of the EODR instrument.

6.6 Calibration should be carried out without interruption.

7 EODR instrument set-up within the tank

7.1 The instrument shall be set up with care, in accordance with the procedure and instructions given by the manufacturer.

7.2 The instrument shall be set up so as to be stable.

If necessary, the tank bottom in the vicinity of the instrument shall be made firm and steady by placing heavy weights in the area.

The legs of the tripod on which the instrument is mounted shall be steadied by use of suitable devices, such as magnetic bearers, to prevent slippage on the tank bottom.

7.3 The instrument should be located at, or near, the centre of the tank to ensure that the measured slope distances, at any one horizontal level, do not vary significantly and to minimize the overall uncertainty of slope distance determination.

NOTE Stability of the EODR instrument is likely to be more important than its position in the centre of the tank.

7.4 The instrument shall be free from external vibration.

7.5 The sighting lines from the instrument to the tank shell wall shall not be obstructed.

7.6 Select and clearly mark on fixed points inside the tank, e.g. on the shell wall, two reference target points. These should be approximately 1,571 rad (90 gon) apart and as near to the horizontal plane of the instrument as possible.

7.7 At least the minimum settling time recommended by the manufacturer should be allowed before the instrument is used.

7.8 The instrument shall be set horizontal, thus ensuring that the vertical axis (standing axis) is vertical.

8 Selection of target points

8.1 Two sets of target points per course shall be selected, one at approximately 1/4 of the course height above the lower horizontal seam, the other at approximately 1/4 of the course height below the upper horizontal seam.

The number of target points per set on each course of the tank shell wall is dependent on tank circumference. The minimum number of target points per set as a function of tank circumference shall be in accordance with Table 1, and is illustrated in Figure 2.

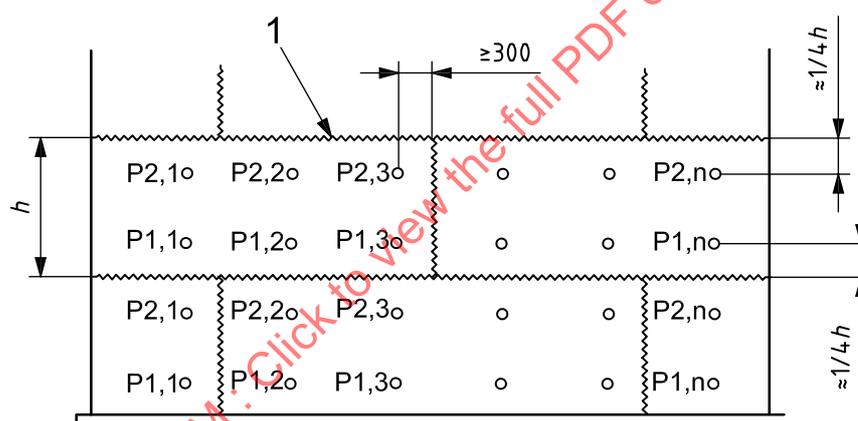
Where tank walls are deformed, the number of target points can be greater than the minimum given by this part of ISO 7507.

8.2 The target points shall be at least 300 mm from any vertical welded seam.

Table 1 — Minimum number of target points

Tank circumference m	Minimum number of points
≤ 50	10
$> 50, \leq 100$	12
$> 100, \leq 150$	16
$> 150, \leq 200$	20
$> 200, \leq 250$	24
$> 250, \leq 300$	30
> 300	36

A number of target points greater than the minimum number of points in Table 1 may be chosen depending on specific circumstances and tank conditions.

**Key**

1 seam

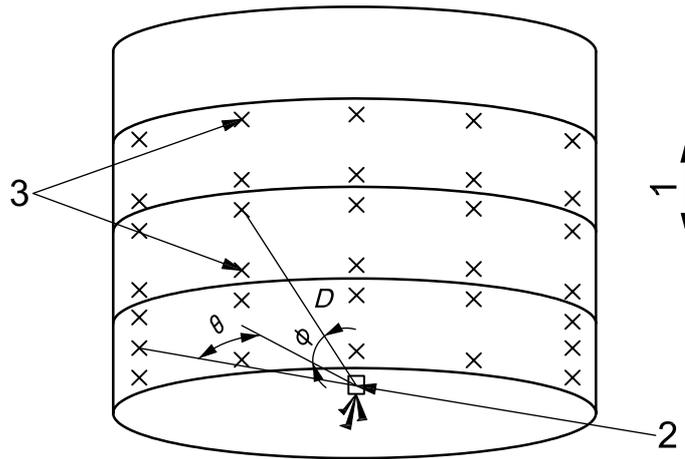
Figure 2 — Illustration of target positioning on tank shell wall

9 Calibration procedure

9.1 All measurements should be carried out without interruption and as quickly as possible.

9.2 Measure and record the slope distance, horizontal angle and vertical angle to each of the reference target points.

9.3 Sight all of the target points along the horizontal plane at each course location, and measure the slope distance, horizontal angle and vertical angle to each, as illustrated in Figure 3.



- Key**
- 1 course height
 - 2 EODR instrument
 - 3 target points on shell wall
 - θ horizontal angle
 - ϕ vertical angle
 - D slope distance

Figure 3 — Illustration of calibration procedure

9.4 After all measurements on a course have been completed, repeat the measurements to the reference target points.

9.5 If the repeated slope distances to the reference target points do not agree with the measurements taken during the setting up of the instrument, within 2 mm, repeat the procedure given in 9.1 to 9.4. Record the difference of the two measurements as uncorrected instrument drift, $e(R_{dr})$. Add the corresponding standard uncertainty, $u(R_{dr})$, calculated using Equation (1), to the resulting uncertainty of the tank radius (see Annex C):

$$u(R_{dr}) = \frac{e(R_{dr})}{2\sqrt{3}} \tag{1}$$

9.6 If the horizontal or vertical angles to the reference target points do not agree with the measurements taken during the setting up of the instrument, to within $1,571 \times 10^{-4}$ rad (0,01 gon), repeat the procedure given in 9.1 to 9.4. Record the difference of the two measurements as uncorrected instrument drift, $e(\theta_{dr})$ (horizontal angles) and $e(\phi_{dr})$ (vertical angles). Add the corresponding standard uncertainty, $u(\theta_{dr})$, calculated by using Equation (2) for horizontal angles, or Equation (3) for vertical angles, to the resulting uncertainty of the angular measurements (see Annex C):

$$u(\theta_{dr}) = \frac{e(\theta_{dr})}{2\sqrt{3}} \tag{2}$$

$$u(\phi_{dr}) = \frac{e(\phi_{dr})}{2\sqrt{3}} \tag{3}$$

10 Other measurements

10.1 Tank-bottom calibration and the overall height of the reference point at each dip-hatch shall be carried out in accordance with ISO 7507-1:2003, Clause 10 and 6.3, respectively.

10.2 The following data shall be determined and recorded:

- a) ranges of densities and the working temperatures of the liquids being stored in the tank;
- b) height of each course, in accordance with ISO 7507-1:2003, 8.2;
- c) thickness of each course of plating, not including the thickness of paint, in accordance with ISO 7507-1:2003, 8.1;
- d) safe filling height and maximum filling height, as determined by local practices;
- e) deadwood, in accordance with ISO 7507-1:2003, Clause 9;
- f) tilt of the tank as shown by the deviation from a vertical line, in accordance with ISO 7507-1:2003, Clause 9;
- g) average temperature of the tank shell. This should be measured around the tank (at least at four points), near the bottom of the shell and again as near to the top of the shell as possible. Average temperature shall be used to correct the measured radii.

10.3 The vertical height difference between the datum-point and the dip-point, if any, shall be measured by normal surveying methods and recorded.

NOTE In use, each measured tank dip is referred to as a dip-point; the position of the dip-point can differ from the datum-point used for the purpose of tank calibration (e.g. on the intersection of the tank shell and the tank bottom plate).

11 Calculation and development of capacity tables

11.1 The internal radii of the tank shall be calculated in accordance with Annex B.

11.2 Once the internal radii have been computed, the development of the capacity table shall be carried out in accordance with ISO 7507-1:2003, Clause 14. The following corrections shall be applied in the development of the capacity tables:

- a) correction for hydrostatic head effect, in accordance with ISO 7507-1:2003, Annex A;
- b) correction to the certified tank shell temperature, in accordance with ISO 7507-1:2003, 16.1;
- c) correction for deadwood, in accordance with ISO 7507-1:2003, 17.1;
- d) correction for tilt, in accordance with ISO 7507-1:2003, 16.2.

Annex A (normative)

Procedure for verifying EODR instruments

A.1 General

The EODR instrument used for measurement according to this part of ISO 7507 shall be verified according to the procedure in this annex.

This procedure shall be carried out to verify the measurement of horizontal and vertical angles by EODR. The verification should be carried out under as stable environmental conditions as possible. The verification shall be carried out on a regular basis, or in case of one-time use of the instrument, just prior to the calibration of the tank.

The procedure for verification of distance measurements may be omitted; instead, the user may rely on periodic calibration in a laboratory.

NOTE Unstable set-up, bad lighting and varying ambient conditions during the actual calibration of the tank influence the result and increase the measurement uncertainty.

A.2 Angular measurement verification

A.2.1 Procedure

A.2.1.1 Set up the EODR instrument with care in accordance with the procedure and instructions given by the manufacturer.

NOTE It is not necessary to carry out the verification in a tank.

A.2.1.2 Point the telescope at a clearly defined point that lies as close to the horizontal plane of the EODR instrument as possible. Measure and record the horizontal and vertical angles.

A.2.1.3 Make a two-face measurement by turning the telescope 3,142 rad (200 gon) in, for example, first the vertical and then the horizontal direction, and reposition it at the same point as in A.2.1.2. Measure and record the horizontal and vertical angles.

A.2.1.4 Repeat A.2.1.2 and A.2.1.3 four times to obtain a set of five collimation checks.

A.2.1.5 Repeat A.2.1.2 to A.2.1.4 at two other clearly defined points, with approximately 1,037 rad (66 gon) horizontally apart, to obtain three separate sets of collimation checks.

NOTE This covers the entire angular range of the device [6,184 rad (400 gon)].

A.2.2 Handling of the results

A.2.2.1 Calculate the worst-case difference between any two measurements at each measured point and the standard deviation of each set. Because the EODR instrument can be used around its full axis, the set that resulted in the greatest standard deviation shall be used to calculate the uncertainties of measured angles.

A.2.2.2 The standard uncertainty, $u(\theta_{\text{th}})$, of the measurement of horizontal and vertical angles by the EODR instrument can be estimated as given in Equation (A.1):

$$u(\theta_{\text{th}}) = \sqrt{\left(\frac{\Delta\varepsilon}{\sqrt{3}}\right)^2 + \left(\frac{s}{\sqrt{5}}\right)^2} \quad (\text{A.1})$$

where

$\Delta\varepsilon$ is the worst-case difference in two-face measurements at all measured points, expressed in radians;

s is the worst-case standard deviation of the measurements at each measured set, expressed in radians;

$\sqrt{3}$ is a factor corresponding to a rectangular distribution;

$\sqrt{5}$ is a factor derived from the number of measurements, i.e. five, in each set.

A.2.2.3 The result of the EODR instrument verification procedure shall be used to

- a) estimate the uncertainty of angular measurements used in Annex C;
- b) reject the EODR instrument if the angular uncertainty calculated in accordance with C.3 is greater than $7,854 \times 10^{-5}$ rad (5 mgon).

A.3 Distance measurement verification

A.3.1 Complete the procedures given in A.2 before verifying the distance/measurement component of the instrument.

A.3.2 Place the tripod at a distance less than 15 m from the instrument.

A.3.3 Mount the stadia on the tripod with care in accordance with the procedure and instructions given by the manufacturer. Mount the stadia horizontally and normal to an imaginary line between the EODR instrument and the centre mark of the stadia by adjusting the device on the stadia as illustrated in Figure A.1. Lock the stadia in position and verify that it is horizontal and perpendicular.

A.3.4 Using the EODR instrument, measure the horizontal angle, 2θ , subtended at the instrument by the two marks on the stadia.

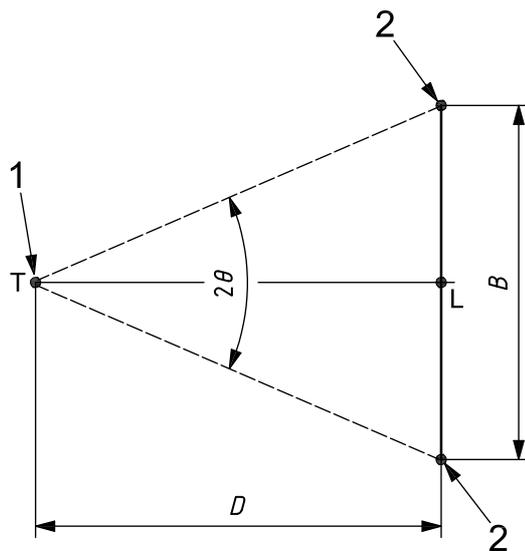
A.3.5 Calculate the horizontal distance, D , (see Figure A.1) from Equation (A.2):

$$D = \frac{B}{2} \times \cot \theta \quad (\text{A.2})$$

where B is the length, expressed in metres, between the two reference marks on the stadia.

A.3.6 Carry out the measurement of the angle and the computation of the distance, D , at least five times. The variations in the calculated distance, D , shall be within 2 mm, or the entire procedure shall be repeated. Calculate and record the average values.

A.3.7 The slope distance, D , measured by the EODR instrument and the average computed distance, D , shall agree to within ± 2 mm.



Key

- 1 EODR instrument
- 2 stadia reference mark
- T location of the tripod
- L location of the centre of the stadia

Figure A.1 — Illustration of the distance measurement set-up

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Annex B (normative)

Calculation of internal radii from the measured parameters

B.1 The dimensional coordinates of each target point (see Figure 3) shall be converted to Cartesian coordinates using Equations (B.1) to (B.3):

$$X = D \times \cos \theta \times \cos \phi \quad (\text{B.1})$$

$$Y = D \times \sin \theta \times \cos \phi \quad (\text{B.2})$$

$$Z = D \times \sin \phi \quad (\text{B.3})$$

where

D is the measured slope distance;

θ is the measured horizontal angle;

ϕ is the measured vertical angle.

B.2 The calculation of the positions of the various target points at any one height is simplified if the vertical height coordinates are all reduced to the same value. Care shall be exercised in this reduction to ensure that the plane to which the adjustment is made is normal to the vertical axis of symmetry of the tank. It should not be assumed that the tank is truly vertical.

Alterations of the value of a Z coordinate require a compensating alteration in the values of the X and Y coordinates of that point.

Such adjustments shall be carried out by established mathematical techniques.

B.3 Reduction of the values of the X and Y coordinates to the internal radius at each level at which measurements are taken shall be carried out by established mathematical techniques.

Annex C (informative)

Calibration uncertainties

C.1 Introduction

This annex describes calculations used in the estimation of measurement uncertainties using the electro-optical distance measurement method for calibrating vertical cylindrical tanks.

The calculations follow the guidelines set out in ISO Guide 98 (all parts)^[1].

NOTE ISO Guide 98 (all parts) is the revision of the *Guide to the expression of uncertainty in measurement (GUM)*.

It describes uncertainty components that derive from the measurement and calculation of the tank radius. The final tank table is also based upon other measurements and calculations. The uncertainty components of these measurements and calculations are described in ISO 7507-1:2003, Annex D.

The uncertainty components described in C.3 can vary depending on the equipment and measurement conditions. Each measurement and its object are unique and it is necessary to evaluate the uncertainty components for each measurement separately.

This part of ISO 7507 describes two-dimensional measurements. The procedures do not give full three-dimensional information. Uncertainties due to the fact that the measurements are not performed in a perfect plane (differences in height when sighting the points) are not considered in this evaluation.

C.2 Symbols

The following symbols and their definitions have been used in this annex.

k	Coverage factor, used for conversions between standard and expanded uncertainties	—
$e(\alpha_{tk})$	Maximum estimated error of coefficient of linear expansion of the tank shell	per degree Celsius
$u(\alpha_{tk})$	Standard uncertainty of coefficient of linear expansion of the tank shell	per degree Celsius
θ_r	Resolution of the EODR instrument	radian (gon)
$u(\theta_r)$	Standard uncertainty corresponding to the resolution of the instrument	radian (gon)
$e(\theta_{pi})$	Estimated error of misalignment of laser to the EODR instrument	radian (gon)
$e(\theta_{dr})$	Estimated error due to drift of the EODR instrument	radian (gon)
$u(\theta_{pi})$	Standard uncertainty due to misalignment of laser to the EODR instrument	radian (gon)
$u(\theta_{th})$	Standard uncertainty of measured angles due to non-linearity of the EODR instrument	radian (gon)
θ	Horizontal angle	radian (gon)

$u(\theta)$	Total standard uncertainty of horizontal angles	radian (gon)
ϕ	Vertical angle	radian (gon)
$u(\phi)$	Total standard uncertainty of vertical angles	radian (gon)
D	Slope distance	metre
$U_m(D)$	Expanded uncertainty of the distance meter received from calibration certificate	metre
U_a	Additional uncertainty of distance measurements	metre
$u(D)$	Standard uncertainty of the distance meter	metre
$u(X)$	Standard uncertainty of the coordinate X	metre
$u(Y)$	Standard uncertainty of the coordinate Y	metre
$e(T_{tk})$	Estimated range of temperatures of the tank shell	degree Celsius
$u(T_{tk})$	Standard uncertainty of the temperature of the tank shell	degree Celsius
$u(R_i)$	Standard uncertainty of radii of individual points of the fitted circle	metre
$u(R_{LS})$	Standard uncertainty of radius of fitted circles due to tank deformation	metre
$u(R_{th})$	Standard uncertainty of radius of the fitted circle corrected for temperature	metre
$u(R_{ang})$	Standard uncertainty of fitting circles to measured co-ordinates of the tank shell	metre
$e(R_{dr})$	Estimated error due to drift of the EODR instrument	metre
$u(R_{dr})$	Standard uncertainty due to alignment of the EODR instrument	metre
$u(R_{total})$	Total standard uncertainty of the radius	metre
R	Radius calculated from measurement	metre
$u(A)$	Standard uncertainty of the area	square metre

C.3 Measurement uncertainties

C.3.1 Uncertainty of slope distance

The distance meter is calibrated traceable to an (inter)nationally acceptable standard. The expanded uncertainty, $U_m(D)$, given by the calibration certificate {typically, $U_m(D)$, expressed in metres, is equal to $[5 \times 10^{-4} + (2 \times 10^{-5} \times D_m)]$, where D_m is the measured distance, expressed in metres, with a coverage factor k (usually, $k = 2$, corresponding to 95 % confidence level)}, yields the standard uncertainty, expressed in metres, as given in Equation (C.1):

$$u(D) = \frac{\sqrt{[U_m(D)]^2 + U_a^2}}{k} \quad (\text{C.1})$$

where U_a is additional uncertainty of distance measurements due to variations of vertical angles, quality of the reflections from tank walls, etc. Its value is typically 0,002 m at a confidence level of 95 % (with coverage factor $k = 2$), for good quality EODR instruments and good conditions of the tank walls.

C.3.2 Uncertainties of angles

C.3.2.1 Angular linearity of EODR instrument

The estimated standard uncertainty, $u(\theta_{tr})$, due to non-linearity of the angular part of the EODR instrument is given by the procedure in Annex A.

C.3.2.2 Angular resolution of EODR instrument

If the resolution of the instrument is θ_{tr} , expressed in radians (mgon) [typically $\theta_{tr} = 3,142 \times 10^{-6}$ rad ($\theta_{tr} = 0,2$ mgon)], then the corresponding standard uncertainty, expressed in radians (mgon), is given by Equation (C.2):

$$u(\theta_{tr}) = \frac{\theta_{tr}}{2\sqrt{3}} \quad (C.2)$$

where the factor $\sqrt{3}$ corresponds to a rectangular distribution.

C.3.2.3 Misalignment of lasers within EODR instrument

Misalignment of distance-measuring and angle-measuring lasers results in systematic error. The estimated value of this error {typically $e(\theta_{pi}) = 3,142 \times 10^{-5}$ rad [$e(\theta_{pi}) = 2$ mgon]} may be used, resulting in an uncertainty, expressed in radians (mgon), as given in Equation (C.3):

$$u(\theta_{pi}) = \frac{e(\theta_{pi})}{2\sqrt{3}} \quad (C.3)$$

where the factor $\sqrt{3}$ corresponds to a rectangular distribution.

C.3.2.4 Total uncertainty of angles

The total uncertainty of angles is obtained as the root mean square (RMS) of its components and is identical for horizontal and vertical angles, θ and ϕ , expressed in radians, as given in Equation (C.4):

$$u(\theta) = u(\phi) = \sqrt{u(\theta_{pi})^2 + u(\theta_{tr})^2 + u(\theta_{th})^2 + u(\theta_{dr})^2} \quad (C.4)$$

where all components are expressed in radians and $u(\theta_{dr})$ is the standard uncertainty due to instrument drift recorded in 9.6.

C.3.3 Uncertainty of coordinates

The total uncertainties of the co-ordinates, $u(X)$ and $u(Y)$, are calculated using partial derivatives of the different components, as given in Equations (C.5) to (C.12).

$$u(X) = \sqrt{u(X_D)^2 + u(X_\theta)^2 + u(X_\phi)^2} \quad (C.5)$$

where

$$u(X_D) = u(D) \times \cos \theta \times \cos \phi = u(D) \times \frac{X}{D} \quad (C.6)$$

$$u(X_\theta) = -u(\theta) \times D \times \sin \theta \times \cos \phi \quad (C.7)$$

$$u(X_\phi) = -u(\phi) \times D \times \cos \theta \times \sin \phi \quad (\text{C.8})$$

$$u(Y) = \sqrt{u(Y_D)^2 + u(Y_\theta)^2 + u(Y_\phi)^2} \quad (\text{C.9})$$

where

$$u(Y_D) = u(D) \times \sin \theta \times \cos \phi = u(D) \times \frac{Y}{D} \quad (\text{C.10})$$

$$u(Y_\theta) = u(\theta) \times D \times \cos \theta \times \cos \phi \quad (\text{C.11})$$

$$u(Y_\phi) = -u(\phi) \times D \times \sin \theta \times \sin \phi \quad (\text{C.12})$$

All uncertainties of X and Y are expressed in metres while those of θ and ϕ are expressed in radians. If the latter were previously calculated in gon or mgon, the conversion into radians should be made using relation: $2\pi \text{ rad} = 400 \text{ gon} = 400\,000 \text{ mgon}$.

C.3.4 Uncertainty of radius

C.3.4.1 Fitted radius

A circle with radius, R , is fitted to the measured coordinates of the tank shell using Equation (C.13):

$$R = \frac{\sum_{i=1}^n R_i}{n} = \frac{\sum_{i=1}^n \sqrt{(x_i - A_x)^2 + (y_i - B_y)^2}}{n} \quad (\text{C.13})$$

where

n is the total number of target points at one height;

A_x and B_y are co-ordinates of the centre of the fitted circle at this height.

Considering that errors in X and Y are correlated (they both depend on errors in D that are not random), the uncertainty, $u(R_{\text{ang}})$, of fitting circles to measured coordinates of the tank shell is calculated using Equation (C.14):

$$u(R_{\text{ang}}) = \frac{\sqrt{\sum \left[u(X_i) \times \frac{X_i - A_x}{R_i} \right]^2 + \sum \left[u(Y_i) \times \frac{Y_i - B_y}{R_i} \right]^2}}{n} \quad (\text{C.14})$$

where

X_i and Y_i are the co-ordinates for each target point as above, expressed in metres;

$u(X_i)$ and $u(Y_i)$ are uncertainties of the co-ordinates for each target point as above, expressed in metres.

By setting the values of A_x and B_y equal to zero, Equation (C.14) is reduced to the simplified form as given by Equation (C.15):

$$u(R_{\text{ang}}) = \sqrt{\left\{ [u(D) \times \cos \phi]^2 + [u(\phi) \times D \times \sin \phi]^2 \right\}} \quad (\text{C.15})$$

where

D is the (averaged) near-constant measured slope distance, with uncertainty $u(D)$;

ϕ is the (averaged) near-constant angle of vertical elevation, with uncertainty $u(\phi)$.

C.3.4.2 Fitting of circles to deformed tanks

If the tank is more or less deformed, i.e. the tank is not a perfect cylinder, this often has a great impact on the uncertainty of the radius. The procedure for fitting circles to sets of measured points is described in ISO 7507-3:2006, Annex B^[4]. To estimate the standard uncertainty of the fitting, $u(R_{\text{LS}})$, the standard deviation of the residuals, s , expressed in metres, is used as given in Equation (C.16):

$$u(R_{\text{LS}}) = \frac{s}{\sqrt{n}} \quad (\text{C.16})$$

NOTE 1 The factor \sqrt{n} is due to an average value of n readings used to calculate the radius.

NOTE 2 Standard deviation, s , of the residuals of the fitted radius is normally obtained by dividing the sum of squares of the deviations of the radii from their average by $(n - 3)$, where three points are considered to have been used to fit the circle.

C.3.4.3 Uncertainties due to temperature

The standard uncertainty of tank radii corrected for differential thermal expansion of the tank shell includes the following:

- standard uncertainty of the coefficients of expansion of the tank;
- standard uncertainty of the tank shell temperature.

The maximum range of the tank shell temperatures is $e(T_{\text{tk}})$, based on the following variables:

α_{tk} the coefficient of linear expansion of the tank shell material, expressed in reciprocal degrees Celsius;

T_{ref} the reference temperature of the tank (zero uncertainty), expressed in degrees Celsius;

T_{tk} the tank shell temperature at calibration, expressed in degrees Celsius.

This should be estimated from temperature measurements (hottest to coldest points on the tank shell) or estimates of thermal gradients at the time of calibration; assuming rectangular distribution, the standard uncertainty, expressed in degrees Celsius, is given by Equation (C.17):

$$u(T_{\text{tk}}) = \frac{e(T_{\text{tk}})}{2 \times \sqrt{3}} \quad (\text{C.17})$$

Maximum error of estimate of the linear expansion coefficient, $e(\alpha_{\text{tk}})$, is typically equal to $2 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$; assuming rectangular distribution, the standard uncertainty, expressed in reciprocal degrees Celsius, is given by Equation (C.18):

$$u(\alpha_{\text{tk}}) = \frac{e(\alpha_{\text{tk}})}{2 \times \sqrt{3}} \quad (\text{C.18})$$