INTERNATIONAL STANDARD

ISO 7066-1

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Assessment of uncertainty in the calibration and use of flow measurement devices —

Part 1 :
Linear calibration relationships

Évaluation de l'incertitude dans l'étalonnage et l'utilisation des appareils de mesure du débit —

Partie 1 : Relations d'étalonnage linéaires

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Contents

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council. They are approved in accordance with ISO procedures requiring at least 75 % approval by the member bodies voting.

International Standard ISO 7066-1 was prepared by Technical Committee ISO/TC 30, *Measurement of fluid flow in closed conduits.*

ISO 7066 consists of the following parts, under the general title *Assessment of uncertainty in the calibration and use of flow measurement devices*:

- *Part 1: Linear calibration relationships*
- *Part 2: Non-linear calibration relationships*

Annexes A, B and C form an integral part of this part of ISO 7066. Annexes D, E, F and G are for information only.

Introduction

This International Standard has been drawn up according to the principles outlined in ISO 5168 and gives guidance on how the uncertainty in a calibration curve or in the mean of a number of measurements of the same flow-rate may be calculated. To achieve this it is assumed that the uncertainty in each individual measurement of flow-rate is calculated in accordance with ISO 5168.

This part of ISO 7066 deals only with calibration graphs which are linear or which can be linearized. ISO 7066-2 deals with non-linear calibration graphs.

Assessment of uncertainty in the calibration and use of flow measurement devices —

Part 1: Linear calibration relationships

1 Scope

This International Standard deals with methods of assessing the uncertainty in the calibration of any method of measuring flow-rate, either in closed conduits or in open channels. It also deals with the estimation of the uncertainty in one or more measurements which use the resulting calibration graph.

Only linear relations are considered in this part of ISO 7066; the uncertainty in non-linear relations is the subject of ISO 7066-2. Where a calibration curve is not linear, this part of ISO 7066 is therefore applicable only if

a) the variables may be transformed (for example by taking logarithms) to create a linear relationship between them;

b) the range over which the relationship is established may be subdivided in such a way that one variable varies linearly with the other within each subdivision; or

c) systematic deviations from linearity of the calibration graph are negligible in comparison with the uncertainty associated with the individual points forming the graph[1)].

Although it is assumed that the uncertainty in the independent and dependent variables for which the calibration graph is constructed is normally established prior to determining the calibration graph, consideration is given in 5.3 to how these uncertainties may sometimes be determined during the calibration procedure itself, when the uncertainty in an individual calibration point is not known.

For most of the calculations given in this part of ISO 7066, computer programs exist which are generally referred to in program libraries as "linear regression methods" or "linear curve fitting".

Examples are given in annexes A and B of how the principles in this part of ISO 7066 may be applied.

2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this part of ISO 7066. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this part of ISO 7066 are encouraged to investigate the possibility of applying the most recent editions of the standards listed below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 1100-2 : 1982, *Liquid flow measurement in open channels — Part 2 : Determination of the stage-discharge relation.*

ISO 5168 : 1978, *Measurement of fluid flow — Estimation of uncertainty of a flow-rate measurement.*

3 Symbols and definitions

The symbols and definitions used in this part of ISO 7066 have been taken from ISO 772 and ISO 4006.

The definitions given in 3.3 are only for terms used in some special sense or for terms the meaning of which it seems useful to emphasize.

3.1 Symbols

a intercept of the calibration graph on the ordinate

b gradient of the calibration graph

C discharge coefficient

d diameter of the orifice in an orifice plate flow-meter

D diameter of the pipe

1) For example, a turbine meter calibration graph may have a minimum value after the "hump" before rising asymptotically to become horizontal, but the linear calibration range is often assumed to extend down to the flow-rate at which the extrapolation of the horizontal part of the graph intercepts the graph as it rises towards the maximum peak.

$e(\ )$	uncertainty of variable contained in parentheses[1)]
$e_r(\ )$	random uncertainty of variable contained in parentheses[1)]
$e_s(\ )$	systematic uncertainty of variable contained in parentheses[1)]
K	calibration coefficient
M	number of repetitions of a measurement of flow-rate
n	number of measurement points used to establish calibration graph
N_p	number of pulses generated by a turbine meter per second
Q	flow-rate
Re_d	Reynolds number based on bore diameter
$s(\ )$	experimental standard deviation of variable contained in parentheses
s_R	standard deviation of points about the best straight line [see equation (17)]
$s(x, y)$	covariance of x and y [see equation (10)]
t	Student's t
x	independent variable
y	dependent variable
$\hat{y}$	the value of the dependent variable predicted by the calibration graph
ν	number of degrees of freedom

3.2 Subscripts and superscripts

i	ith value of a variable
k	a specific value of a variable
$\bar{\ }$	arithmetic mean value of a variable
$\hat{\ }$	the value of the variable predicted by an equation of a fitted curve

3.3 Definitions

3.3.1 (absolute) error of measurement: The result of a measurement minus the (conventional) true value of the measurand.

NOTES

1 The term relates equally to

- the indication,
- the uncorrected result,
- the corrected result.

2 The known parts of the error of measurement may be compensated by applying appropriate corrections. The error of the corrected result can only be characterized by an uncertainty.

3 The "absolute error", which has a sign, should not be confused with the absolute value of an error which is the modulus of an error.

3.3.2 random error: Component of the error of measurement which, in the course of a number of measurements of the same measurand, varies in an unpredictable way.

NOTE — It is not possible to correct for random error.

3.3.3 systematic error: Component of the error of measurement which, in the course of a number of measurements of the same measurand, remains constant or varies in a predictable way.

NOTE — Systematic errors and their causes may be known or unknown.

3.3.4 spurious errors: Errors which invalidate a measurement. They generally have a single cause such as the incorrect recording of one or more significant digits or malfunction of instruments.

3.3.5 uncertainty: An estimate characterizing the range of values within which the true value of a measurand lies.

3.3.6 random uncertainty: Component of uncertainty associated with a random error. Its effect on mean values can be reduced by taking many measurements.

3.3.7 systematic uncertainty: Component of uncertainty associated with a systematic error. Its effect cannot be reduced by taking many measurements.

3.3.8 experimental standard deviation: For a series of n measurements of the same measurand, the parameter s characterizing the dispersion of the results and given by the formula

$$s = \left[\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1} \right]^{1/2}$$

where

x_i is the result of the ith measurement;

$\bar{x}$ is the arithmetic mean of the n results considered.

NOTES

1 The experimental standard deviation should not be confused with the population standard deviation σ of a population of size N and of mean m, given by the formula

$$\sigma = \left[\frac{\sum_{i=1}^{N} (x_i - m)^2}{N} \right]^{1/2}$$

2 If the series of n measurements is considered to be a sample of a population, s is an estimate of the population standard deviation.

1) In some International Standards the symbols U and E have been used instead of e.

3.3.9 variance: The square of the standard deviation.

3.3.10 confidence limits: The lower and upper limits within which the true value is expected to lie, with a specified probability assuming negligible systematic error.

3.3.11 calibration graph: Locus of points obtained by plotting some index of the response of a flow-meter against some function of the flow-rate.

4 General

For a calibration to be meaningful, the systematic uncertainty in the calibrator shall be very much less than the systematic uncertainty in the device or system being calibrated. This is especially true when the procedures specified in 7.3 are used.

The calibration of a flow-metering device or system will result in a graph of the calibration coefficient which will subsequently be used to predict the flow-rate. As this subsequent flow-rate prediction has to have an uncertainty attached to it, then not only the functional relationship between calibration coefficient and flow-rate but also the uncertainty in the calibration coefficient shall be established during calibration.

There will exist a number of pairs of values (x, y) where the uncertainties in x and y [$e(x)$ and $e(y)$ respectively[1)]] are known from one of the methods given in clause 5. The choice of the procedure by which the coefficients and the uncertainty of the calibration equation are calculated is determined by the relative magnitudes of the random components of the uncertainties $e_r(x)$ and $e_r(y)$, as described in clause 7.

When $e_r(x)$ can be ignored (as, for example, is normally the case in the calibration of an orifice plate), the calibration equation and the uncertainty in the calibration coefficient are computed by the methods specified in 7.2 and 9.3 respectively. When, however, the random uncertainties in x and y are of similar magnitude, the methods specified in 7.3 and 9.4 should be used. When the uncertainties in x and y are both significant but cannot be regarded as approximately equal, then the calculation of the uncertainty in the calibration graph is outside the scope of this part of ISO 7066.

A special case is that where y is effectively independent of x; this is a common situation with flow-meters used in closed pipes, since there is an obvious advantage in having a calibration coefficient which is independent of flow-rate. In such cases, the method specified in 9.2 may be used for calculating the uncertainty.

In addition to determining the uncertainty in the coefficient or curve obtained during the calibration of a flow-meter or gauging station, it is necessary to determine the uncertainty in the particular value which is used as a coefficient or is read from the calibration curve when the flow-meter is used after having been calibrated. Where the value of the calibration coefficient to be used is determined completely independently of the measurement from which a flow-rate is to be obtained, then these two quantities are the same, provided that the conditions of use are identical with those of the calibration; if, however, some information from the test to measure the flow-rate is required before the calibration coefficient or curve can be used, then an additional uncertainty will be introduced. Annex C describes how this additional uncertainty is introduced.

The approaches to be used in these different circumstances are described in clause 9, and in clause 11 methods for assessing the uncertainty in the average of a number of measurements are described.

5 Uncertainties in individual calibration points

5.1 General

When a flow-meter is being calibrated, some function of its output may be plotted against either a reference measurement of the flow-rate or some function of this flow-rate, such as the Reynolds number.

In either case, it is necessary to establish the uncertainty in the coordinates of a single point in order to be able to compute the uncertainty in the calibration graph, and there are two ways in which this may be done:

- ISO 5168 may be used; or
- the information can sometimes be obtained from the calibration data directly.

It should be noted, however, that the uncertainty in a coordinate may vary with the value of the coordinate itself; thus, for example, where the reference flow-rate is measured by a diversion system involving the static weighing of a quantity of liquid collected over a measured period of time, the uncertainty due to the timing is usually less for long diversion periods (and consequently for low flow-rates if approximately the same weight of water is collected at each test point) than for short diversion periods.

1) The customary categories of independent and dependent variables, and of abscissae (horizontal) and ordinate (vertical) coordinates in a graph, are irrelevant for linear regressions in that the important distinction here is between variables that have significant uncertainties and variables that have negligible (or zero) uncertainties. When the uncertainty in one variable is significantly greater, in the manner described in clause 7, than the other, the former will be denoted by y and the latter by x. Thus the regressions studied are all for y on x irrespective of whether a variable is considered to be independent, and irrespective of which variable is plotted "horizontally".

Justification shall always be provided where it is assumed that the uncertainty is constant throughout the range of a co-ordinate. Where the uncertainty cannot be regarded as constant, it shall be estimated for sufficient values of the coordinate to give a clear idea of how it varies with the value of the coordinate.

5.2 Use of ISO 5168

ISO 5168 describes in detail how an estimation of the uncertainty in a single measurement of flow-rate may be arrived at, and the procedures described in ISO 5168 may be used to calculate the uncertainties of both the independent and the dependent variables in the calibration graph.

Is it important, however, that the random and systematic contributions to the uncertainty are calculated separately and not combined; the various formulae for calculating the uncertainty in a calibration graph are first used to calculate only the random component of the uncertainty, and so only the random component of the uncertainty in the individual points is used at that stage. Subsequently, the systematic uncertainty in the individual points is added by the root-sum-square method to the value obtained to give the final combined value. The random component of the uncertainty is required separately in any case, since it is the relative magnitudes of random uncertainties in x and y which determine the calculation procedures to be used.

This method of calculating the uncertainty in the coordinates of a single experimental point may be used only when previous investigations have been carried out to establish the uncertainties in the various subsidiary measurements which have to be made, or when this information is available from some other source.

5.3 Use of calibration data

Where the measurement conditions can be kept constant, it is possible to determine the random components of the uncertainties of the coordinates of the calibration graph during the calibration by repeating measurements. Thus, for example in the calibration of an orifice plate, it might be possible to keep the Reynolds number constant and to take a series of readings of the data necessary to compute the calibration coefficient, and it might conversely be possible to keep the differential pressure across the orifice plate constant, while making a number of determinations of the reference flow-rate (and consequently the Reynolds number).

The uncertainty (due to random effects) in the calibration coefficient or Reynolds number may then be calculated from the standard deviations of the resulting measurements.

There is no alternative to using the methods of ISO 5168 in assessing systematic uncertainties if the assessment of the uncertainty is to be in accordance with this part of ISO 7066.

6 Linearity of the calibration graph

6.1 General

In considering the shape of any calibration graph, previous knowledge or supporting information appropriate to the flow-metering method being used (for example if there is a relevant published standard) should always be taken into account to justify any assumption made about the shape of the graph. Where the shape of the calibration graph is unusual, a sufficient number of calibration points shall be repeated to verify the shape for that particular flow-meter.

In the absence of such information, or where there is no reason to believe in advance that the form of the curve should be linear, simple techniques are available to establish whether or not the graph can be treated as linear, but these are applicable only when the experimental points are not grouped into sets. When the experimental points are grouped into sets, only visual observations may be used, since the extensive statistical tests which are otherwise required are outside the scope of this part of ISO 7066.

When the data can be grouped into sets, the following test may be used, but the sets shall be such that each one consists of a number of measurements made at one of a number of fixed values for one of the coordinates. Figure 1 illustrates a case where there are five sets, the number of measurements within the sets varying from four to six.

The test consists of comparing the variance of the means of the groups about the fitted straight line with the variance within the groups.

The variance within the groups, s_g^2, is given by

$$s_g^2 = \frac{\sum_{i=1}^{q} \sum_{j=1}^{n_i} (y_{i,j} - \bar{y}_i)^2}{n - q} \qquad \ldots \quad (1)$$

where

n is the total number of measurements;

n_i is the number of measurements in the ith group;

q is the number of groups;

$y_{i,j}$ is the jth measurement in the ith group;

$$\bar{y}_i = \frac{\sum_{j=1}^{n_i} y_{i,j}}{n_i}$$

The variance of the means of the group about the fitted straight line, s_m^2, is given by

$$s_m^2 = \frac{\sum_{i=1}^{q} n_i (\hat{y}_i - \bar{y}_i)^2}{q - 2} \qquad \ldots \quad (2)$$

where

n_i and q have the same meaning as in equation (1);

$\hat{y}_i$ is the value of y obtained from the fitted straight line for a given value of x.

The test quotient is s_m^2/s_g^2, and if this number equals or exceeds the value given in table 1 for $\nu_1 = q - 2$ and $\nu_2 = n - q$ degrees of freedom, then the best-fit equation through the data points cannot be assumed to be linear. If, however, the test quotient is less than the corresponding value given in table 1, the best-fit equation may be assumed to be linear at the 95 % confidence level.

If the relationship between the calibration coefficient and the independent variable is not linear, there are two possible ways in which the data may be made suitable for analysis in accordance with this part of ISO 7066. The first, to linearize the curve, may, however, only be done on the basis of some physical model.

6.2 Linearization of curve

The coordinates may be transformed to give two new variables which, when plotted one against the other, produce a straight line, or a function of one of the coordinates may be used instead of the coordinate itself so as to produce this result, although it is of course essential that the transformation be capable of being used easily when the resulting graph is subsequently applied to the use of the flow-meter or the flow measurement method.

Table 1 — Values of the *F* distribution for selected degrees of freedom — Probability level 0,05

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	20	30	40	80	100
1	161,44	200	216	225	230	234	237	239	241	242	248	250	251	252	253
2	18,51	19	19,2	19,2	19,3	19,3	19,4	19,4	19,4	19,4	19,4	19,5	19,5	19,5	19,5
3	10,13	9,55	9,28	9,12	9,01	8,94	8,89	8,85	8,81	8,79	8,66	8,62	8,59	8,56	8,55
4	7,71	6,94	6,59	6,39	6,26	6,16	6,09	6,04	6	5,96	5,8	5,75	5,72	5,67	5,66
5	6,61	5,79	5,41	5,19	5,05	4,95	4,88	4,82	4,77	4,74	4,56	4,5	4,46	4,41	4,41
6	5,99	5,14	4,76	4,53	4,39	4,28	4,21	4,15	4,1	4,06	3,87	3,81	3,77	3,72	3,71
7	5,59	4,74	4,35	4,12	3,97	3,87	3,79	3,73	3,68	3,64	3,44	3,38	3,34	3,29	3,27
8	5,32	4,46	4,07	3,84	3,69	3,58	3,5	3,44	3,39	3,35	3,15	3,08	3,04	2,99	2,97
9	5,12	4,26	3,86	3,63	3,48	3,37	3,29	3,23	3,18	3,14	2,94	2,86	2,83	2,77	2,76
10	4,96	4,1	3,71	3,48	3,33	3,22	3,14	3,07	3,02	2,98	2,77	2,7	2,66	2,6	2,59
20	4,35	3,49	3,1	2,87	2,71	2,6	2,51	2,45	2,39	2,35	2,12	2,04	1,99	1,92	1,91
30	4,17	3,32	2,92	2,69	2,53	2,42	2,33	2,27	2,21	2,16	1,93	1,84	1,79	1,71	1,7
40	4,08	3,23	2,84	2,61	2,45	2,34	2,25	2,18	2,12	2,08	1,84	1,74	1,69	1,61	1,59
80	3,96	3,11	2,72	2,49	2,33	2,21	2,13	2,06	2	1,95	1,7	1,6	1,54	1,45	1,43
100	3,94	3,09	2,7	2,46	2,31	2,19	2,1	2,03	1,97	1,93	1,68	1,57	1,52	1,41	1,39

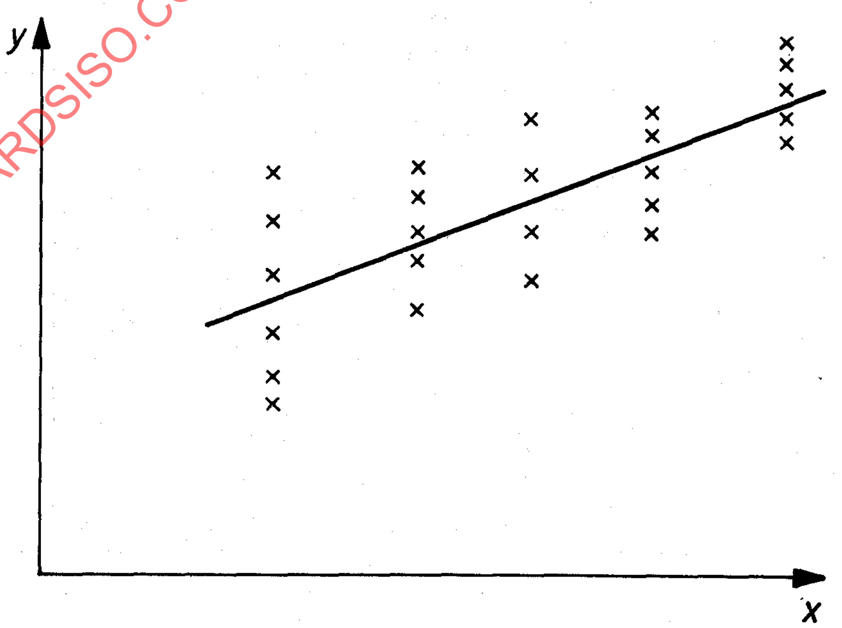

Figure 1 — Example of grouping data to establish linearity of best-fit straight line

An example of the first possibility would be to plot "log y" against "log x" where the basic relation is of the form "$y = ax^2$" ("a" being a constant).

The second method of linearizing a curve is commonly used when calibrating an orifice plate. In this case, it is known from experience that it is better to plot the discharge coefficient C against the Reynolds number than to plot the differential pressure against the Reynolds number, since this gives a graph which is linear over a fairly wide range. The graph is, however, non-linear at low Reynolds numbers, and it has been found from experience that plotting the discharge coefficient against some function of the Reynolds number (for example $Re_d^{-0,5}$ or $Re_d^{-0,75}$) extends the linear range.

6.3 Subdivision of the curve

Although a calibration graph may not be linear over the full range of the calibration, it may well be that by subdividing the curve into a number of parts, each part can be regarded as linear; in this case, a separate calibration equation and confidence limits shall be calculated for each portion of the graph. Where possible, the number of points in each sub-section of the graph should be such that they give approximately the same uncertainties for the line through each sub-section. Since calibration graphs should never be extrapolated beyond the extreme data points unless there is extremely good reason for doing so, there shall be at least three points common to adjacent portions of the calibration graph.

7 Fitting the best straight line

7.1 General

Before a best straight line and its associated uncertainty are calculated, the data available shall first be examined, since they can fall into one of several classes, and different formulae apply for different types of data. Some basic principles are illustrated in figure 2.

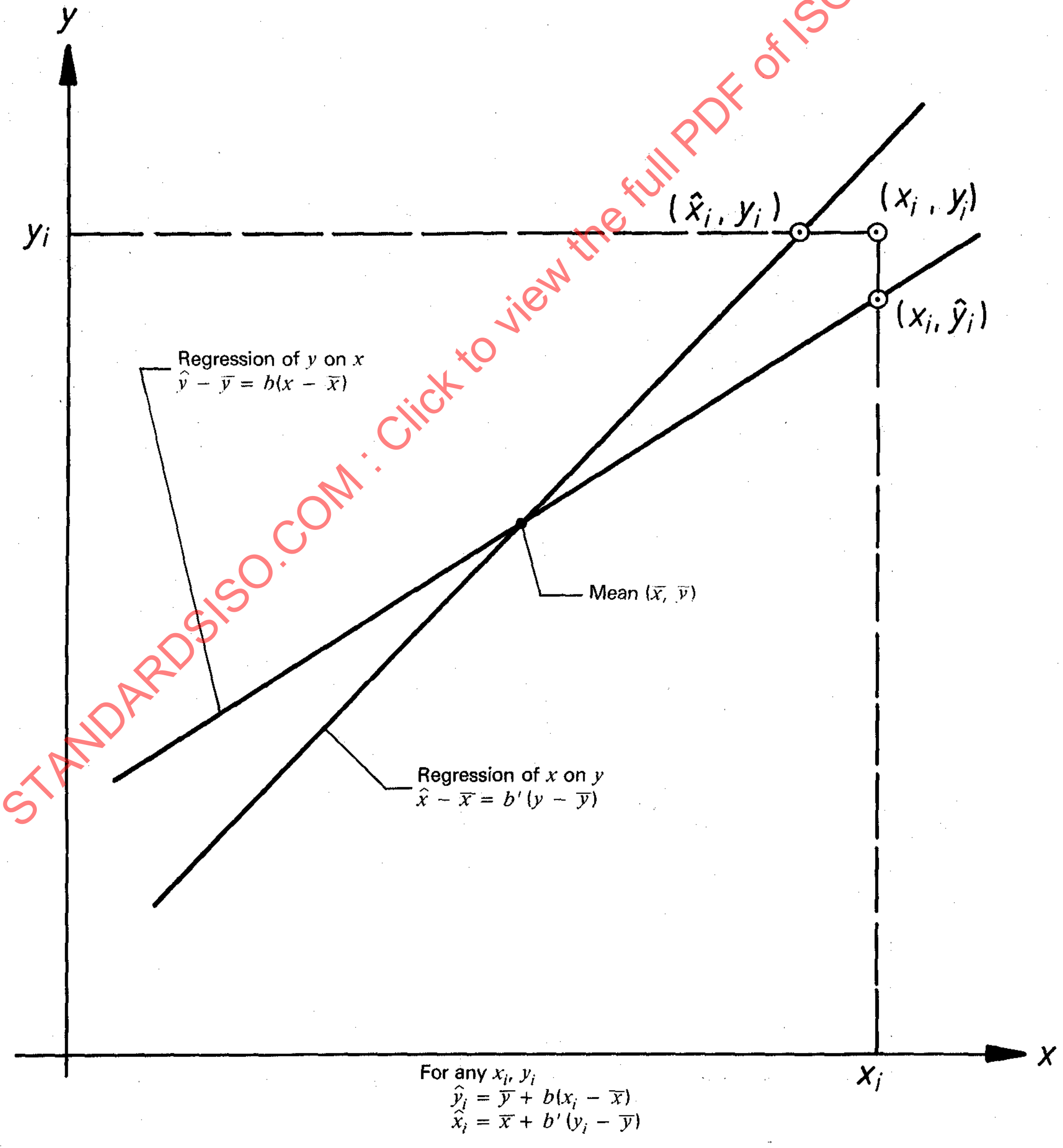

Figure 2 — Basic principles for best-fit straight lines

The best straight line fit to a sample of n points, (x_i, y_i), when $1 \leqslant i \leqslant n$, is given by the regression of y on x:

$$\hat{y}_i - \bar{y} = b(x_i - \bar{x}) \qquad \ldots (3)$$

where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad \ldots (4)$$

$\hat{y}_i$ is the value of y on the line for a measured value x_i;

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \qquad \ldots (5)$$

Equation (3) may also be written as

$$\hat{y}_i = a + bx_i \qquad \ldots (6)$$

where

$$a = \bar{y} - b\bar{x} \qquad \ldots (7)$$

The method to be used for computing the values of the coefficients a and b depends on the magnitude of the random uncertainties in x and y.

The most common case occurs when the random uncertainties in x and y are both significantly different from zero. Fortunately, if both variables have random uncertainties significantly different from zero, it is normally possible to assume that

a) one random uncertainty is negligible in comparison with the other, or

b) these random uncertainties are of approximately equal magnitude.

The former assumption leads to the same formulae as for the cases where only one variable has a random uncertainty significantly different from zero; this is the situation which normally applies.

To assess the relative magnitude of the random uncertainties in x and y, first calculate $e_r(x)$ and $e_r(y)$ according to the principles of ISO 5168.

Obtain an approximate value for the gradient b from a graph or from equation (8) or from equation (11). If the absolute value of $be_r(x)$ is less than approximately one-fifth of $e_r(y)$, the formulae given in 7.2 shall be used to fit a straight line; if not, the formulae given in 7.3 shall be used.

7.2 Random uncertainty negligible or small in one variable in comparison with the other

The gradient of the line is, in this case, given by

$$b = \frac{s(x, y)}{s^2(x)} \qquad \ldots (8)$$

where

$$s^2(x) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \qquad \ldots (9)$$

$$s(x, y) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y}) \qquad \ldots (10)$$

The intercept, a, is given by equation (7) :

$$a = \bar{y} - b\bar{x}$$

7.3 Random uncertainty in both variables of similar magnitude

The gradient is calculated from

$$b = \pm \left[\frac{s^2(y)}{s^2(x)} \right]^{1/2} \qquad \ldots (11)$$

where

$$s^2(y) = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2 \qquad \ldots (12)$$

$s^2(x)$ is given by equation (9);

the intercept, a, is again given by equation (7).

The sign of b is the same as that of $s(x, y)$.

NOTE — The following alternative formulae for $s^2(x)$, $s^2(y)$ and $s(x, y)$ are easier to use, if the computation is carried out manually, but great care should be taken since they are prone to rounding errors:

$$s^2(x) = \frac{1}{n(n-1)}\left[n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2\right]$$

$$s^2(y) = \frac{1}{n(n-1)}\left[n \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2\right]$$

$$s(x, y) = \frac{1}{n(n-1)}\left[n \sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)\right]$$

8 Detection of outliers

Spurious errors (see ISO 5168) are errors, such as human errors or instrument malfunction, which invalidate a measurement; they may be due to, for example, the transposing of numbers in recording data or the presence of pockets of air in leads from a water line to a manometer. Such errors cannot be incorporated into any statistical analysis and the measurement shall be discarded. Where the error is not large enough to make the result obviously invalid, some rejection criterion should be applied to decide whether the data point should be rejected or retained.

Whenever it is suspected that one or more results have been affected by errors of this nature, a statistical "outlier" test should be applied. For the purposes of this part of ISO 7066, either the Dixon test or the Grubbs extreme deviation outlier test may be used. The Dixon test is easy to use by hand, but when a set of values are being processed by computer, the Grubbs test is more suitable, since it is more reliable, easier to program and takes up less storage.

Details of these tests are given in annex E.

9 Uncertainty of calibration

9.1 General

In general, the uncertainty in the best straight line arises from uncertainties in the values for the intercept, a, and for the gradient, b.

From equation (3)

$$\hat{y}_i = \bar{y} + b(x_i - \bar{x}) \qquad \ldots (13)$$

Thus, combining the variances of $\bar{y}$, $\bar{x}$ and b by the root-sum-square method (see ISO 5168),

$$s^2(\hat{y}) = s^2(\bar{y}) + b^2 s^2(\bar{x}) + (x_k - \bar{x})^2 s^2(b) \qquad \ldots (14)$$

where x_k is the value of x at which the uncertainty in $\hat{y}$ is required.

Thus $s^2(\bar{y})$ is the variance of $\bar{y}$ which would be obtained from the scatter of several different determinations of $\bar{y}$ obtained using the same range and values of x, and with the same number of measurements of y, and is given by

$$s^2(\bar{y}) = \frac{\sum_{i=1}^{n} (\bar{y}_i - \bar{y})^2}{n - 1}$$

$s^2(\bar{x})$ is defined similarly.

Note that $s^2(\bar{x})$ and $s^2(\bar{y})$ are quite different from $s^2(x)$ and $s^2(y)$ defined in equations (9) and (12). $s^2(y)$, for example, is the variance of all the values of y over the range of the calibration graph relative to their mean value, and $s^2(x)$ has a similar meaning, whereas $s^2(\bar{y})$ is associated only with the scatter of different determinations of $\bar{y}$ and is essentially an indication of the random uncertainty in y.

From equation (7), it can be seen that the variance of a is given by

$$s^2(\bar{y}) + b^2 s^2(\bar{x})$$

and the contribution of the variance in b to the variance in $\hat{y}$ is

$$(x_k - \bar{x})^2 s^2(b)$$

Thus the random component of the uncertainty in $\hat{y}$, $e_r(\hat{y})$, is given at the 95 % confidence level by

$$e_r(\hat{y}) = \pm t\,[s^2(\bar{y}) + b^2 s^2(\bar{x}) + (x_k - \bar{x})^2 s^2(b)]^{1/2} \qquad \ldots (15)$$

where t is obtained from table 2 for $n - 2$ degrees of freedom.

Table 2 — Value of Student's t at 95 % confidence level

Number of degrees of freedom, ν	t
1	12,7
2	4,3
3	3,2
4	2,8
5	2,6
6	2,4
7	2,4
10	2,2
15	2,1
20	2,1
30	2
60	2
∞	1,96

Equation (15) is the basic equation for uncertainty which is used either directly or in a modified form in the various paragraphs below.

As noted in clause 4, the method to be used for calculating the uncertainty in a calibration depends on the magnitude of the random uncertainty in the variable x, and on whether or not the value of y is independent of the value of x, that is on whether or not b is zero. The first step is therefore to establish whether or not the gradient, b, of the line is significantly different from zero, and this is carried out as follows.

The standard deviation of b, $s(b)$, shall be calculated from

$$s^2(b) = \frac{s_R^2}{(n-1)s^2(x)} \qquad \ldots (16)$$

where s_R is the standard deviation of the points about the best straight line, i.e.

$$s_R = \left[\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n-2} \right]^{1/2} \qquad \ldots (17)$$

An alternative formula for $s(b)$ is

$$s(b) = \left[\frac{s^2(y)s^2(x) - s^2(x, y)}{(n-2)s^4(x)} \right]^{1/2} \qquad \ldots (18)$$

This is more convenient, but the calculation of the numerator is prone to rounding errors, so it is important to ensure that enough significant figures are used in the computation.

The 95 % confidence limits for b are then given by

$$\begin{matrix} b - ts(b) \\ b + ts(b) \end{matrix} \qquad \ldots (19)$$

where t is the value obtained from table 2 for $n - 2$ degrees of freedom.

If these limits include zero and there is independent evidence that the particular flow-meter or type of flow-meter is expected to have a constant coefficient (for example from the information in a relevant standard or a previous calibration), it can be assumed that the calibration coefficient has a constant value. If there is no independent evidence that a constant calibration coefficient is expected, then, whether or not the limits include zero, the formulae given in 9.3 (if the uncertainty in x can be ignored) or in 9.4 shall be used.

In order to calculate the uncertainty in the calibration coefficient the random and systematic components shall first be evaluated separately. Random components are calculated from the formulae given in 9.2 to 9.4 and shall include the contributions from the random uncertainties in the instruments used during the calibration, so that these do not have to be allowed for separately. In particular, hysteresis effects in instruments will contribute to the random uncertainty component obtained in this way. Systematic components are calculated in accordance with ISO 5168.

NOTE — Equation (16) is strictly valid only when the random uncertainty in x contributes significantly less to the calibration graph uncertainty than the random uncertainty in y does, since it is a simplification of a more general formula. The expansion of equation (17), used to develop equation (18), also makes this assumption. The use of equations (18) and (19) will, however, give a smaller uncertainty for b than the full formulae would, and so if they include zero, the more general formula would lead to the same conclusion. If they do not include zero, it is possible that the gradient will be treated as having a non-zero value, when it might have been acceptable to take its value as zero, but this would be a rare situation, and the only penalty incurred would be that the more complicated formulae given in 9.4 would be used instead of the simpler method given in 9.2. The results obtained for the uncertainty of the best-fit line would in such a case be virtually identical no matter whether the formulae given in 9.2 or 9.4 were used.

9.2 Calibration curve with zero gradient

In this case, the calibration coefficient has a single value, and all of the estimates of this value can be analysed together, irrespective of the value of the independent variable to which they correspond. An example of where this often occurs is in the calibration of a turbine meter for use in water.

The best estimate of the value of the calibration coefficient is given by equation (5):

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

In this case, $b = s(b) = 0$, and so, from equation (15), the random component of the uncertainty in $\hat{y}$ is given at the 95 % confidence level by

$$e_r(\hat{y}) = \pm\, ts(\bar{y}) = \pm\, t\,\frac{s(y)}{n^{1/2}} \qquad \ldots (20)$$

where

$$s(y) = \left[\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1} \right]^{1/2} \qquad \ldots (21)$$

and t is obtained from table 2 for $n - 1$ degrees of freedom.

The systematic component of the uncertainty in $\hat{y}$, $e_s(\hat{y})$, is calculated as described in clause 5 and the uncertainty in the calibration coefficient, $e(\hat{y}_c)$, is then given by

$$e(\hat{y}_c) = [e_r^2(\hat{y}) + e_s^2(\hat{y})]^{1/2} \qquad \ldots (22)$$

9.3 Random uncertainty negligible in one variable in comparison with the other

In general, the random component of the uncertainty in $\hat{y}$ is given by equation (15). However, when the gradient of the best straight line through the data is not zero, but the uncertainty in x may be ignored, it can be shown that the variance in $\overline{y}$ is given by

$$s^2(\overline{y}) = \frac{s_R^2}{n} \qquad \ldots (23)$$

Also, for ease of computation, s_R^2 may, for this case, be calculated from

$$s_R^2 = \frac{n-1}{n-2}\left[s^2(y) - \frac{s^2(x,y)}{s^2(x)}\right] \qquad \ldots (24)$$

For this particular case, $s^2(b)$ is also given by equation (16):

$$s^2(b) = \frac{s_R^2}{(n-1)s^2(x)}$$

Since the uncertainty in x is, in this case, being treated as negligible, $s^2(\overline{x}) = 0$, and so, from equations (15), (23) and (16), the random component of the uncertainty in $\hat{y}$ is given by

$$e_r(\hat{y}) = \pm\, t s_R \left[\frac{1}{n} + \frac{(x_k - \overline{x})^2}{(n-1)s^2(x)}\right]^{1/2} \qquad \ldots (25)$$

where

s_R may be calculated from either equation (17) or equation (24);

t is obtained from table 2 for $n - 2$ degrees of freedom;

x_k is the value of x for which the uncertainty in $\hat{y}$ is required.

The uncertainty in $\hat{y}$ is obviously a minimum at $x_k = \overline{x}$, and so any set of calibration points should be such that the value of x, for which the flow-meter is most commonly used, is close to the middle of the range of values of x over which measurements are made.

In the same way as in 9.2, the uncertainty in the calibration coefficient, $e(\hat{y}_c)$, is then given by equation (22):

$$e(\hat{y}_c) = [e_r^2(\hat{y}) + e_s^2(\hat{y})]^{1/2}$$

9.4 Random uncertainty in both variables of similar magnitude

When the uncertainties in x and y affect equally the uncertainty in the calibration graph, it can be shown that

$$s^2(\overline{y}) + b^2 s^2(\overline{x}) = \frac{s_R^2}{n} \qquad \ldots (26)$$

In this case, s_R^2 is given by

$$s_R^2 = \frac{n-1}{n-2}[s^2(y) - 2bs(x,y) + b^2 s^2(x)] \qquad \ldots (27)$$

Similarly, $s^2(b)$ shall be calculated from a more complicated formula than in the simpler case given in 9.3 and is given, in this instance, by

$$s^2(b) = \frac{4b}{n-2}\left[\frac{s^2(y) - bs(x,y)}{bs^2(x) + s(x,y)}\right] \qquad \ldots (28)$$

In this case, therefore, the random component of the uncertainty in $\hat{y}$ is given by

$$e_r(\hat{y}) = t\left[\frac{s_R^2}{n} + (x_k - \overline{x})^2 s^2(b)\right] \qquad \ldots (29)$$

where

s_R^2 is calculated from equations (17) or (27);

$s^2(b)$ is calculated from equation (28);

$s(x)$, $s(x, y)$, $s(y)$ and b are obtained for use in equation (28) from equations (9), (10), (11) and (12);

t is obtained from table 2 for $n - 2$ degrees of freedom.

As before, the uncertainty in the calibration coefficient, $e(\hat{y}_c)$, is then given by equation (22):

$$e(\hat{y}_c) = [e_r^2(\hat{y}) + e_s^2(\hat{y})]^{1/2}$$

9.5 Extrapolation

Values of calibration coefficients and uncertainties obtained by extrapolating a calibration graph cannot be said to be obtained in accordance with this part of ISO 7066, since unpredictable effects might render these values meaningless. Nevertheless, limitations in the calibration facilities available occasionally result in the calibration graph having to be extrapolated to higher flow-rates than were achieved during the calibration; annex D gives guidance on how extrapolation might be used.

10 Uncertainty in the use of the calibration graph for a single measurement of flow-rate

10.1 General

The uncertainty in a calibration graph will have a systematic effect on any estimate of flow-rate for which the graph is used. The value of the calibration coefficient which is used is the best estimate of that coefficient, but any error in its determination during a calibration will have the same effect on every value of flow-rate which the coefficient is subsequently used to compute.

Moreover, when a flow-meter is being used to measure a flow-rate under identical conditions to those experienced during the calibration, and the value of its calibration coefficient or the position to be used on its calibration graph is determined by a measurement made during the test in which the flow-rate is being measured (i.e. independently from the measurements which established the calibration graph), as is the case in 10.3, then the uncertainty in the value used for the calibration coefficient is greater than simply the uncertainty in the calibration graph. This increase is due to the uncertainty in the experimental observations which are required to locate the position to be used on the calibration graph (see annex C).

NOTE — A further additional uncertainty will be introduced if a flow-meter is used in conditions (for example the installation layout, the fluid used or the data processing methods) which are different from those under which it was calibrated and, generally, it is impossible to predict the magnitude of this so this source of uncertainty will have to be assessed separately in each particular case.

This additional uncertainty, denoted by $e(\hat{y}_o)$, will have both a random and a systematic component, and if the procedures laid down in ISO 5168 were to be followed rigidly, only the latter should be combined with the systematic uncertainty in the calibration graph described above. In practice, however, the systematic component will be at least as large as the random component, and will very often be much larger; in addition, the complicated calculations which are required if the uncertainties in the value used for the calibration coefficient are to be split up into their random and systematic components are not justified by any appreciable improvement in the validity of the result, and so the uncertainties associated with the use of the calibration graph or coefficient are treated as entirely systematic.

A more detailed explanation of how, in these situations, the uncertainty in the value of the calibration coefficient is increased beyond the value it had during the initial calibration is given in annex C.

In general, the uncertainty, $e(\hat{y})$, in the value used for the coefficient is, therefore, obtained by combining, by the root-sum-square method, the uncertainty in the calibration graph, $e(\hat{y}_c)$, with the additional uncertainty, $e(\hat{y}_o)$, and so is given by

$$e(\hat{y}) = [e^2(\hat{y}_c) + e^2(\hat{y}_o)]^{1/2} \quad \ldots (30)$$

The manner in which the use of the calibration contributes to the uncertainty in the flow-rate depends on the nature of the calibration graph. There are two broad groups.

10.2 Calibration coefficient with a constant value

If the calibration graph has a zero gradient, as described in 9.2, the flow-rate is obtained by multiplying some function of an output of the flow-meter (for example the output could be the differential pressure across an orifice plate or the height of water over a weir) by a coefficient which is independent of flow-rate. There is, therefore, no additional component $e(\hat{y}_o)$ and so the contribution of the calibration coefficient to the systematic uncertainty in the flow-rate estimation is, in this case, equal to the uncertainty in the calibration coefficient itself, i.e. it is given by

$$e(\hat{y}) = e(\hat{y}_c) = \left[\frac{t^2 s^2(y)}{n} + e_s^2(\hat{y})\right]^{1/2} \quad \ldots (31)$$

EXAMPLE

When a turbine meter is used over the flat part of its characteristic, the flow-rate Q is given by

$$Q = \frac{N_p}{K}$$

where

K is the calibration coefficient;

N_p is the number of pulses generated per second by the turbine meter.

The systematic uncertainty in Q is given by

$$e_s^2(Q) = \left(\frac{Q}{N_p}\right)^2 e_s^2(N_p) + \left(\frac{Q}{K}\right)^2 e_s^2(K) \quad \ldots (32)$$

In order to obtain an estimate of the uncertainty in Q it is then of course necessary to assess the random uncertainty in Q arising from N_p and to combine it with $e_s(Q)$ in accordance with ISO 5168.

10.3 Calibration coefficient determined by the iterative method

This subclause generally applies only to closed conduit flow-meters, since the case it describes does not arise in any open channel flow-metering method.

If the flow-rate is obtained by multiplying an output of the flow-meter by a coefficient, where the coefficient is, in some way, itself a function of flow-rate, then it is necessary to use an iterative method to determine the value of the calibration coefficient to be used. An estimate is made of the value of the calibration coefficient, which is used to give an approximate value for the flow-rate, and this, in turn, is used to give a more precise value for the calibration coefficient.

In this case, any error in the measurement of the flow-meter output will introduce an uncertainty, $e(\hat{y}_o)$, in the value used for the coefficient, as described in annex C. The uncertainty, $e(\hat{y})$, in the value used for the coefficient is, therefore, given by equation (30):

$$e(\hat{y}) = [e^2(\hat{y}_c) + e^2(\hat{y}_o)]^{1/2}$$

The method of evaluating $e^2(\hat{y}_c)$ depends on whether the flow-meter coefficient is obtained by calibration or taken from a relevant standard. If it is taken from a calibration graph, it also depends on whether the uncertainties in x and y are of approximately equal magnitude or the effect of the uncertainty in x is negligible in the calibration graph.

Thus when a calibrated flow-meter is used to measure flow-rate, the uncertainty in the calibration coefficient used is given by

$$e(\hat{y}) = \left\{ t^2 s_R^2 \left[\frac{1}{n} + \frac{(x_k - \bar{x})^2}{(n-1)s^2(x)} + e_s^2(\hat{y}) + e^2(\hat{y}_o) \right] \right\}^{1/2} \quad \ldots (33)$$

when the uncertainty in x is negligible [from equations (25) and (22)], or

$$e(\hat{y}) = \left\{ t^2 \left[\frac{s_R^2}{n} + (x_k - \bar{x})^2 s^2(b) \right] + e_s^2(\hat{y}) + e^2(\hat{y}_o) \right\}^{1/2} \quad \ldots (34)$$

when the uncertainties in x and y are approximately equal [from equations (29) and (22)].

Alternatively, if the flow-meter has not been calibrated, but the value for its calibration coefficient is taken from a relevant published standard, then the value to be used for $e^2(\hat{y}_c)$ is the uncertainty quoted in the relevant standard for the flow-meter.

The value to be used for $e^2(\hat{y}_o)$ is normally obtained by using ISO 5168 to calculate the effect which the uncertainties in the variables, such as pressure and temperature, have on the value read from the calibration graph for the calibration coefficient. In the particular case where the coefficient has been obtained from a published standard, the appropriate formulae in the relevant standard relating these variables to the calibration coefficient shall be used in accordance with ISO 5168 to calculate the contribution to the uncertainty in the calibration coefficient.

If, however, when a calibrated flow-meter is used, the secondary equipment used with it to measure variables, such as pressure and temperature, is the same equipment as was used during the calibration, and if all other conditions are also identical, then

$$e^2(\hat{y}_o) = t^2 s_R^2 \quad \ldots (35)$$

11 Uncertainty in the average of several flow-rate measurements

11.1 General

When a number of measurements of flow-rate are made, the uncertainty in the mean value is, of course, less than the uncertainty in any single measurement. However, the contribution of the uncertainty in the calibration coefficient to the uncertainty in the mean flow-rate may or may not be reduced by repeating the measurement. If the situation described in 10.2 applies, then the assumption made throughout clause 10 that the uncertainty contribution of the calibration coefficient is entirely systematic is valid, and it will take the same value for the mean of a number of measurements as it does for a single measurement, namely

$$e(\hat{y}_c) = \left[\frac{t^2 s^2(y)}{n} + e_s^2(\hat{y}) \right]^{1/2}$$

If the situation is that described in 10.3, then the uncertainty in the calibration coefficient has random and systematic components which have to be determined separately if any benefit is to be gained from repeating the measurements.

The formulae for the random and systematic components for the special case where the same secondary equipment is used in both the calibration and the subsequent tests are given in 11.2. If this is not the case, the formulae for the systematic components are still valid, but ISO 5168 would have to be used to calculate the random component applicable for each individual measurement.

11.2 Calibration coefficient determined by the iterative method

The random component in the uncertainty in the use of the calibration coefficient or curve, $e_r(\hat{y}_o)$, is given by

$$e_r(\hat{y}_o) = \frac{t s_R}{M^{1/2}} \quad \ldots (36)$$

where M is the number of times the flow-rate is measured.

The systematic component (which cannot be reduced by repeated measurements of the flow-rate) is given by equation (22) in conjunction with either equation (25) or equation (29), depending on the magnitude of the uncertainty in x relative to that in y.

Annex A
(normative)

Example for a closed conduit

A.0 Introduction

This example describes the calculation of the uncertainty in the calibration of an orifice plate with flange tappings using a gravimetric water calibration facility, and also illustrates how to calculate the uncertainty in a measurement of flow-rate using the orifice plate after it has been calibrated.

The calibration facility was one in which the water, after flowing through the orifice plate assembly in the test section of the circuit, normally passed into a sump from where it was pumped back round the circuit to the inlet of the test section. When flow conditions are steady, the flow is diverted for a measured time interval into a weigh-tank instead of into the sump. During the time of diversion, the differential head, H, across the orifice plate was measured using water/compressed air or mercury/water manometers. This procedure was repeated at 25 test points covering the flow-rate range over which the orifice plate calibration was required.

The temperature of the water in the test line and the ambient air temperature adjacent to the manometers were noted at each test point. The relative density, δ, of the water, defined as the ratio between the density of this water and that of distilled water at the same temperature, was obtained using a density bottle, and a value of 1,001 42 was used throughout the test.

A.1 Symbols, subscripts and superscripts

A.1.1 Symbols

a	intercept of the calibration graph on the ordinate
A_o	area of the orifice bore
b	gradient of the calibration graph
C	discharge coefficient
d	mean diameter of the orifice bore
D	mean diameter of the pipe
$e(C_c)$	uncertainty in C arising from the calibration
$E_r(\ )$	percentage random uncertainty of variable contained in parentheses
$E_s(\ )$	percentage systematic uncertainty of variable contained in parentheses
g	acceleration due to gravity
H, H'	differential head across the orifice plate
m_w	mass of water collected during the diversion period
n	number of test points in the calibration
p_s	absolute static pressure
Q	volumetric flow-rate
Re_d	Reynolds number based on the orifice bore diameter
$s(x)$	standard deviation of x
s_R	standard deviation of points about the best straight line [see equation (17)]
T	period of diversion
β	diameter ratio ($\beta = d/D$)
δ	relative density
θ	temperature
ν	kinematic viscosity
ϱ_w	density of distilled water

A.1.2 Subscripts and superscripts

k	a specific value of a variable
$\bar{\ }$	arithmetic mean value of a variable

A.2 Calculation of calibration coefficient

The mean diameter, d, of the bore of the orifice plate was 164,34 mm, and that of the upstream pipework, D, was 204,98 mm, giving a diameter ratio β of 0,801 7.

The volumetric flow-rate, Q, derived from the weighing method, was given by

$$Q = \frac{1{,}001\ 05 m_w}{\delta \varrho_w T} \qquad \text{(A.1)}$$

where

m_w is the mass of water collected during the diversion period T;

the constant 1,001 05 is a correction which allows for the effect of air buoyancy on the weigh-bridge reading.

The density, ϱ_w, of distilled water is obtained from

$$\varrho_w = 1\ 000{,}25 - 0{,}008\theta_w - 0{,}004\ 86\theta_w^2 + 0{,}46 \times 10^{-6} p_s \qquad \text{(A.2)}$$

where

θ_w is the temperature of the water in the test section;

p_s is the absolute static pressure of the water in the test section.

The Reynolds number, Re_d, based on the bore of the orifice, is given by

$$Re_d = \frac{4Q}{\pi \nu d} \qquad \ldots \text{(A.3)}$$

where ν is the kinematic viscosity of water, obtained from published tables.

The discharge coefficient, C, of the orifice plate is given by

$$C = \frac{Q(1-\beta^4)^{1/2}}{A_o (2gH)^{1/2}} \qquad \ldots \text{(A.4)}$$

where

A_o is the area of the bore of the orifice;

g is the acceleration due to gravity;

H is the differential head across the orifice plate.

The test results are given in table A.1.

The discharge coefficient is plotted against Re_d in figure A.1 and against $10^3/(Re_d)^{1/2}$ in figure A.2.

A.3 Linearity of calibration graph

It is obvious from figure A.1 that the relation between C and Re_d is not a straight line. It is known that for orifice plates such graphs may be transformed to give a linear relationship by plotting C against some function of the reciprocal of Re_d, and, in this case, the reciprocal of the square root of Re_d was chosen. The resulting graph is given in figure A.2. Since the experimental points were ungrouped, only visual observation may be used to decide whether it is justifiable to treat this graph as linear, and, in this case, there is no doubt that the relationship between C and $10^3/(Re_d)^{1/2}$ can be regarded as linear, both by inspecting the graph and from previous experience in plotting orifice plate calibrations.

A.4 Uncertainty of individual calibration points

From equation (A.4), C is a function of Q, d, D, A_o, g and H. The random and systematic uncertainties in these six variables were calculated in accordance with ISO 5168, and the results are given in table A.2.

The percentage random uncertainty in C, $E_r(C)$, is calculated in accordance with ISO 5168 from

$$E_r^2(C) = E_r^2(Q) + \frac{1}{4}E_r^2(g) + \frac{1}{4}E_r^2(H) + \left(\frac{2}{1-\beta^4}\right)^2 E^2(d) + \left(\frac{2\beta^4}{1-\beta^4}\right)^2 E_r^2(D) \qquad \ldots \text{(A.5)}$$

Substituting the values from table A.2 into equation (A.5) gives

$$E_r(C) = 0{,}16\ \%$$

Table A.1 — Calibration results

Test point No.	Flow-rate Q m³/s	Discharge coefficient C	Reynolds number based on orifice throat diameter $Re_d \times 10^{-6}$	$\frac{10^3}{(Re_d)^{1/2}}$
1	0,031 5	0,599 7	0,244 8	2,020 9
2	0,046	0,596 2	0,357 6	1,672 3
3	0,058 9	0,595 7	0,459 3	1,475 5
4	0,071 9	0,593 7	0,560 9	1,335 3
5	0,087 3	0,593 7	0,682 8	1,210 2
6	0,083 5	0,592 7	0,653 3	1,237 2
7	0,130 2	0,590 8	1,018 8	0,990 7
8	0,154	0,590 4	1,207 8	0,909 9
9	0,178 3	0,589 6	1,398 5	0,845 6
10	0,205 1	0,589 6	1,615 9	0,786 7
11	0,231	0,587 5	1,811 6	0,743
12	0,258 3	0,589 5	2,016	0,704 3
13	0,256 8	0,589 2	2,023 6	0,703
14	0,245 8	0,589	1,941 4	0,717 7
15	0,231 4	0,588 5	1,845 7	0,736 1
16	0,217 4	0,591 2	1,737 7	0,758 6
17	0,202 4	0,589 3	1,622	0,785 2
18	0,192 7	0,589 1	1,544 2	0,804 7
19	0,179 9	0,588 2	1,441 8	0,832 8
20	0,153 6	0,590 3	1,230 7	0,901 4
21	0,141 8	0,590 1	1,136 5	0,938
22	0,128 4	0,589 5	1,029	0,985 8
23	0,117 9	0,590 4	0,942 7	1,029 9
24	0,107 6	0,590 5	0,859 9	1,078 4
25	0,094 4	0,592 2	0,754 9	1,151

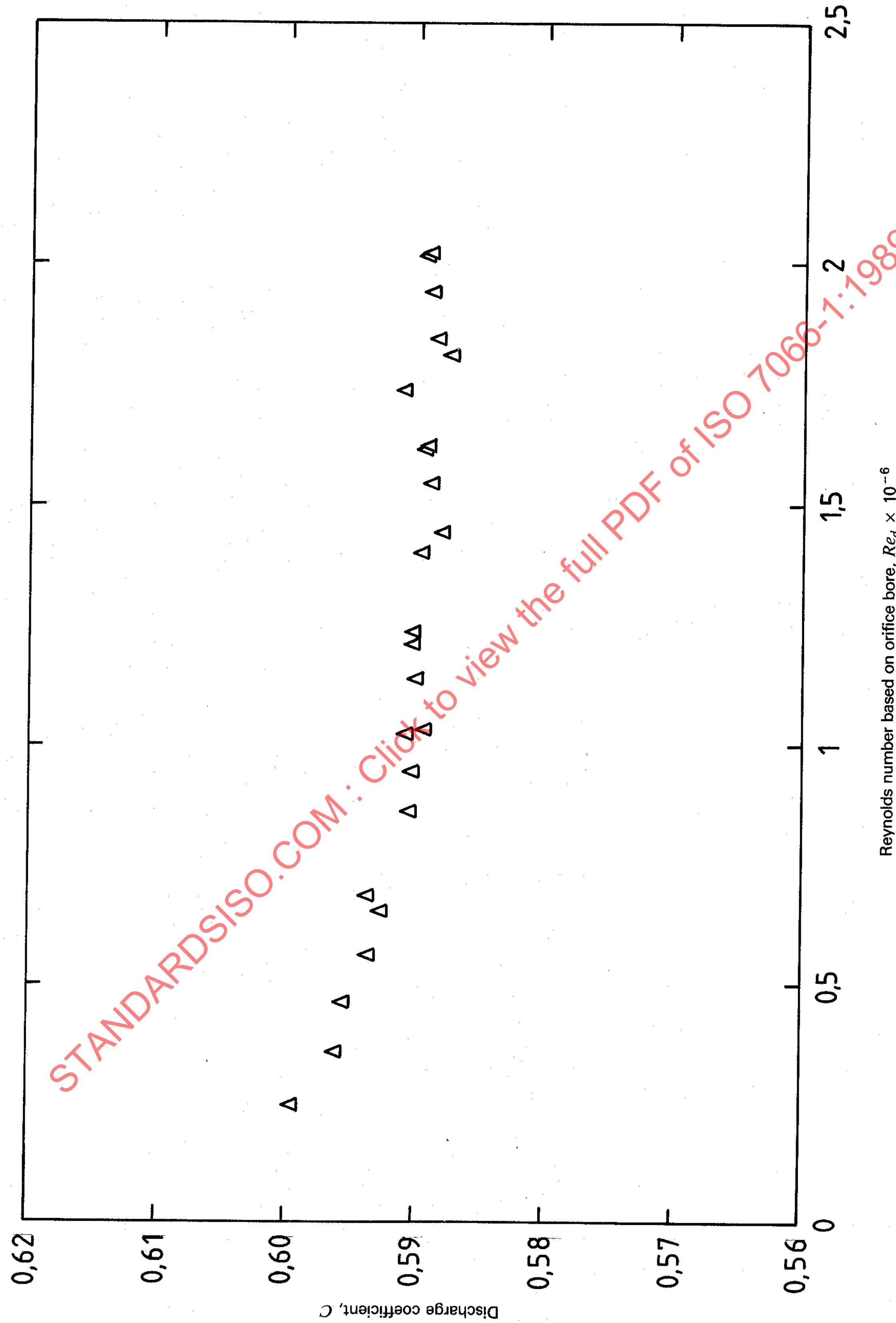

Figure A.1 — Discharge coefficient as a function of the Reynolds number

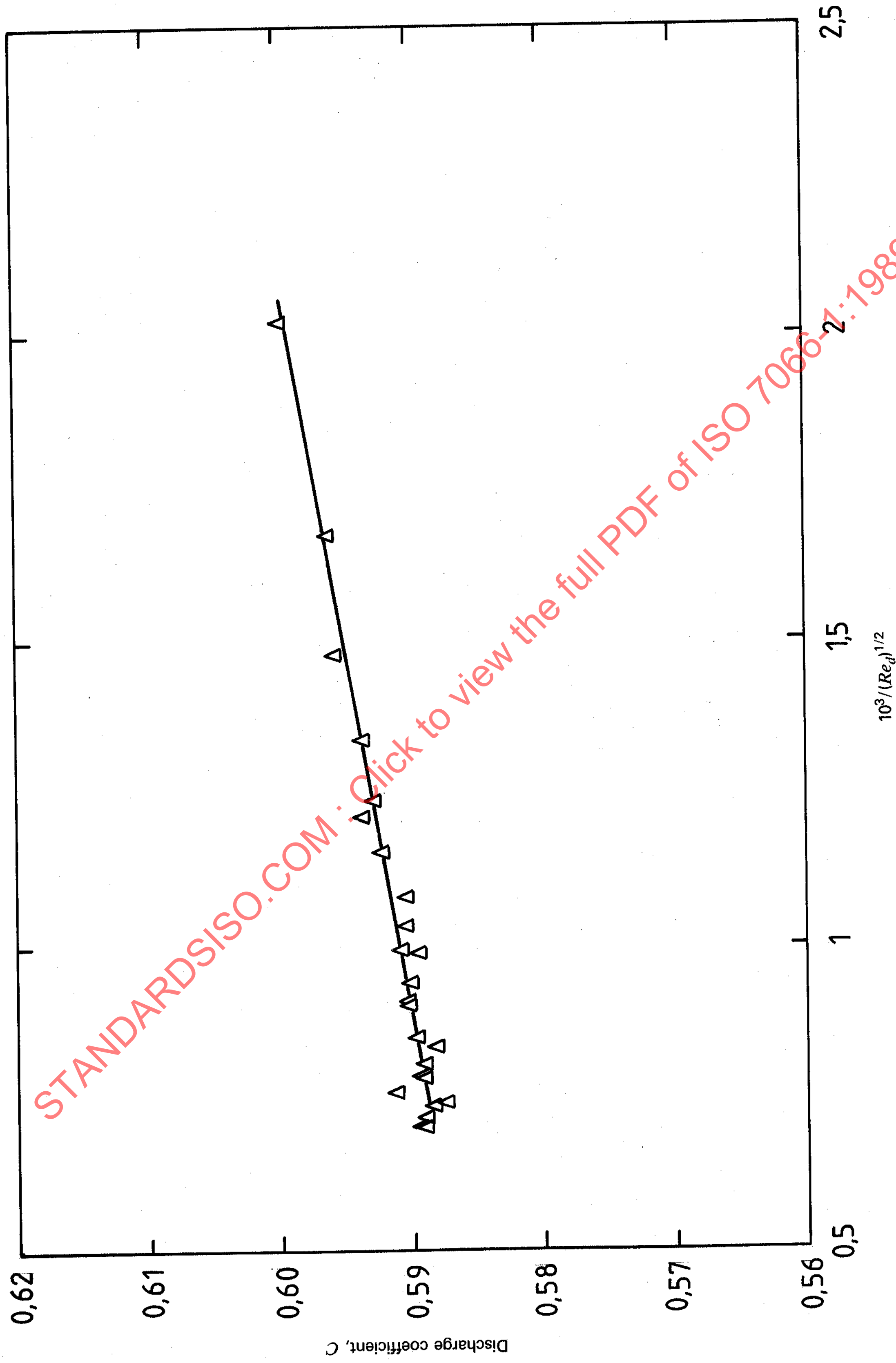

Figure A.2 — Discharge coefficient as a function of $10^3/(Re_d)^{1/2}$

Table A.2 — Component uncertainties

Variable	Percentage random uncertainty E_r	Percentage systematic uncertainty E_s
d	0	0,2
D	0	0,2
g	negligible	negligible
H	0,1	0,05
Q	0,15	0,15
v	0	0,5

Similarly,

$$E_s(C) = 0,75\ \%$$

From equation (A.3), Re_d is a function of Q, v and d. The random and systematic uncertainties in these three variables were calculated in accordance with ISO 5168, and the results are given in table A.2.

Let

$$X = \frac{1}{(Re_d)^{1/2}}$$

Then, since

$$X = \frac{1}{(Re_d)^{1/2}} = \left(\frac{\pi v d}{4Q}\right)^{1/2} \qquad \ldots \text{(A.6)}$$

the percentage random uncertainty in the variable on the abscissa of figure A.2 is calculated from

$$E_r^2(X) = \frac{1}{4}E_r^2(v) + \frac{1}{4}E_r^2(d) + \frac{1}{4}E_r^2(Q) \qquad \ldots \text{(A.7)}$$

Substituting the values from table A.2 into equation (A.7) gives

$$E_r(X) = 0,08\ \%$$

Similarly

$$E_s(X) = 0,28\ \%$$

A.5 Fitting the best straight line

For the purposes of this example, y is the discharge coefficient C and x is X [$= 1/(Re_d)^{1/2}$]. Values of $\bar{x}$, $\bar{y}$, $s^2(x)$, $s^2(y)$ and $s(x, y)$ calculated from the data in table A.1 are given in table A.3.

Table A.3 — Quantities required to calculate uncertainty in the calibration graph

Quantity	Value
$\bar{x}$	$1,014\ 17 \times 10^{-3}$
$\bar{y}$	0,591 064
$s^2(x)$	$1,087\ 864 \times 10^{-7}$
$s^2(y)$	$8,101\ 213 \times 10^{-6}$
$s(x, y)$	$8,985\ 401 \times 10^{-7}$

To check first whether the random uncertainty in x, $e_r(x)$, can be regarded as negligible, the results obtained from equations (A.5) and (A.7) are used:

$$e_r(x) = 8,1 \times 10^{-7} \text{ (at } \bar{x}\text{)}$$

$$e_r(y) = 9,5 \times 10^{-4} \text{ (at } \bar{y}\text{)}$$

Using equation (8) provisionally [on the assumption that $e_r(x)$ can be regarded as negligible], the following calculations can be made:

$$b = \frac{s(x, y)}{s^2(x)} = 8,26$$

and

$$\frac{b e_r(x)}{e_r(y)} = 7,0 \times 10^{-3}$$

Since this is less than one-fifth, $e_r(x)$ can be regarded as negligible, and the value of b obtained from equation (8) is the correct one to use.

From equation (7)

$$a = 0,582\ 7$$

and the equation of the best-fit line is therefore

$$C = 0,582\ 7 + \frac{8,26}{(Re_d)^{1/2}} \qquad \ldots \text{(A.8)}$$

Note that had equation (11) been used (uncertainties in x and y of similar magnitude), the value of b would have been 8,632, and the predicted value of C would have differed from that predicted by equation (A.8) by a maximum of 0,06 % over the calibration range.

A.6 Uncertainty of calibration

Orifice plate flow coefficients vary only slowly with the Reynolds number, and, in some cases, it may be appropriate to take the gradient, b, as zero, as explained in clause 9.

Using equations (18) and (19), with the values given in table A.3, to test whether the gradient differs significantly from zero gives the value

$$s(b) = 0,52$$

Since there are 25 data points, the value for t from table 2 is 2,1, and so the 95 % confidence limits for b are given as 7,74 and 8,78.

Since these limits do not include zero, the gradient shall have a non-zero value (as could have been seen by inspection of figure A.2). The random uncertainty in $1/(Re_d)^{1/2}$ is negligible, so the method given in 9.3 shall be used for the calculation of the uncertainty in the calibration graph.

The random component of the uncertainty in $\hat{y}$ (that is in C) is given by equation (25):

$$e_r(\hat{y}) = \pm t s_R \left[\frac{1}{n} + \frac{(x_k - \bar{x})^2}{(n-1)s^2(x)} \right]^{1/2}$$

Substituting the appropriate values from table A.3 into equation (24) to obtain a value for s_R gives the value

$$s_R = 0{,}842 \times 10^{-3}$$

Thus, substituting these values into equation (25)

$$e_r(\hat{y}) = \pm 2{,}1 \times 0{,}842 \times 10^{-3} \times$$

$$\times [0{,}04 + 3{,}83 \times 10^5 (x_k - \bar{x})^2]^{1/2} \qquad \ldots \text{(A.9)}$$

Note that at $x_k = \bar{x}$, $e_r(\hat{y}) = 3{,}5 \times 10^{-4}$ (0,06 %), whereas at the minimum and maximum values of x, the values of $e_r(\hat{y})$ are $4{,}8 \times 10^{-4}$ (0,08 %) and $11{,}4 \times 10^{-4}$ (0,20 %) respectively.

Equation (A.9) thus gives the random component in the uncertainty in C, the discharge coefficient, and this shall be added by the root-sum-square method to the systematic uncertainty in C, $E_s(C)$, given as 0,75 % (see clause A.4).

Thus the uncertainty in C arising from the calibration for any value of X [$= 1/(Re_d)^{1/2}$] is given by

$$e(C_c) = \{3{,}13[4 \times 10^{-8} + 0{,}383(X_k - \bar{X})^2] +$$

$$+ (0{,}007\,5C)^2\}^{1/2} \qquad \ldots \text{(A.10)}$$

At $X_k = \bar{X}$,

$$e(C_c) = 0{,}004\,4 \qquad (0{,}75\ \%)$$

At the minimum and maximum values of $1/(Re_d)^{1/2}$, $e(C_c)$ has the values 0,004 4 (0,75 %) and 0,004 6 (0,77 %) respectively.

It is therefore clear that for this particular set of data the random uncertainties are negligible in comparison with the systematic uncertainties, which results in the uncertainty in C being almost constant over the range of the calibration.

The above analysis has computed the uncertainty in the calibration, without regard to how it will be used. If, as is usually the case, the orifice plate is subsequently going to be used without remeasuring the diameter of the orifice bore, but simply using the value which was used to calculate A_o for equation (A.4), then uncertainties due to this source will not affect flow-rate measurements made subsequently with that orifice plate.

If the calibrated orifice plate in subsequent use registers a differential head, H', then the flow-rate Q' is calculated as

$$Q' = \frac{CA_o}{(1 - \beta^4)^{1/2}} (2gH')^{1/2} \qquad \ldots \text{(A.11)}$$

Using equation (A.4)

$$Q' = \frac{Q(1-\beta^4)^{1/2}}{A_o(2gH)^{1/2}} \frac{A_o(2gH')^{1/2}}{(1-\beta^4)^{1/2}} = Q\left(\frac{H'}{H}\right)^{1/2} \qquad \ldots \text{(A.12)}$$

Thus the quantity A_o cancels out, and so uncertainties in the value used for it during the original calibration will not affect the accuracy of subsequent measurements with the calibrated orifice plate.

When it is known that the same values will be used for constants when a flow-meter is used after a calibration, the uncertainties in these constants need not be considered in assessing the uncertainty in the calibration coefficient.

In this particular example, this means that the uncertainties in g, d and D may be ignored in equations (A.5) and (A.7). This gives

$$E_r(C) = 0{,}16\ \%$$

$$E_s(C) = 0{,}15\ \%$$

$$E_r(X) = 0{,}08\ \%$$

$$E_s(X) = 0{,}26\ \%$$

If the systematic uncertainty in C (0,15 %) is now combined by the root-sum-square method with the random uncertainty in the calibration graph, given by equation (A.10), the uncertainty in the value of C arising from the calibration becomes

$$e(C_c) = \pm \{3{,}13[4 \times 10^{-8} + 0{,}383(X_k - \bar{X})^2] +$$

$$+ (0{,}001\,5C)^2\}^{1/2} \qquad \ldots \text{(A.13)}$$

At $X_k = \bar{X}$,

$$e(C_c) = \pm 0{,}001\,0 \qquad (0{,}16\ \%)$$

At the minimum and maximum values of $1/(Re_d)^{1/2}$, $e(C_c)$ has the values 0,001 0 (0,16 %) and 0,001 5 (0,24 %).

When the data are considered in this way, the effects of the random uncertainties are comparable with those of the systematic uncertainties at the upper extreme of the calibration graph.

A.7 Uncertainty in flow-rate measurement using the calibrated orifice plate

In the following calculations, it will be assumed that the flow-rate measurement is carried out under the same conditions as existed during the calibration. After the orifice plate has been calibrated, it will be used to measure flow-rate by putting the appropriate value of the discharge coefficient, C, into equation (A.11).

To do this, an approximate value of C is assumed and, using the differential head, H', measured across the orifice plate, a value is obtained for Q'.

This value is substituted into equation (A.3) to give a value for Re_d and the value of C given by the calibration equation is thus calculated using equation (A.8):

$$C = 0{,}582\,7 + \frac{8{,}26}{(Re_d)^{1/2}}$$

This is substituted into equation (A.11) to give a more accurate value of Q' and the procedure is repeated until successive values of Q' differ by an insignificant amount. Note that this iteration procedure does not introduce an uncertainty into the value eventually used for C; the only additional uncertainty which is introduced is the uncertainty in H' and v, which would result in an uncertainty in the value for $1/(Re_d)^{1/2}$ at the point where the iteration converges.

The percentage random uncertainty in $1/(Re_d)^{1/2}$ $(= X)$ is given by equation (A.7):

$$E_r^2(X) = \frac{1}{4}E_r^2(v) + \frac{1}{4}E_r^2(d) + \frac{1}{4}E_r^2(Q)$$

The only contribution to the random uncertainty in Q (assuming that the uncertainty in g is negligible) is the random uncertainty in H', since C is fixed by the calibration graph and the uncertainties in d and D need not be considered, as explained in clause A.6. For the same reason, there is no contribution from the uncertainty in the value used for d to the uncertainty in X, and so equation (A.7) becomes

$$E_r^2(X) = \frac{1}{4}E_r^2(v) + \frac{1}{16}E_r^2(H) \quad \ldots \quad (A.14)$$

The random uncertainty $e_r(C_o)$ which this adds to the value used for C depends on the gradient of the calibration graph as indicated in figure C.1 in annex C, and is given by

$$e_r(C_o) = be_r(X) \quad \ldots \quad (A.15)$$

Similarly

$$e_s(C_o) = be_s(X) \quad \ldots \quad (A.16)$$

Note that equations (A.15) and (A.16) are valid only for absolute values of uncertainty, not percentage values.

The additional uncertainty in the value used for C is thus

$$e(C_o) = b\,[e_r^2(X) + e_s^2(X)]^{1/2} \quad \ldots \quad (A.17)$$

and the total uncertainty in the value used for C is obtained by combining this uncertainty with the uncertainty in the calibration graph [see equation (33)]. Thus

$$e(C) = \left\{ t^2 s^2(y, x) \left[\frac{1}{n} + \frac{(x_k - \bar{x})^2}{(n-1)s^2(x)} \right] + e_s^2(x) + e^2(C_o) \right\}^{1/2} \quad \ldots \quad (A.18)$$

The values for the random and systematic uncertainties in v and H shall be calculated for the conditions under which the orifice plate is being used and then substituted into equation (A.14) and the corresponding equation for the systematic uncertainties.

The percentage random uncertainty in the measurement of flow-rate is thus given, using equation (A.11), by

$$E_r^2(Q') = \frac{1}{4}E_r^2(v) + \frac{1}{16}E_r^2(H') \quad \ldots \quad (A.19)$$

The percentage systematic uncertainty is given by

$$E_s^2(Q') = e^2(C) + \frac{1}{4}E_s^2(v) + \frac{1}{16}E_s^2(H') \quad \ldots \quad (A.20)$$

where $e(C)$ is obtained using equation (A.18).

For the purposes of this example, let

$E_r(v) = 0$

$E_s(v) = 1\ \%$

$E_r(H') = 0{,}5\ \%$

$E_s(H') = 1\ \%$

Then from equation (A.14)

$$E_r(X) = \frac{0{,}5}{4} = 0{,}13\ \%$$

Similarly

$$E_s(X) = \left(\frac{1}{4} + \frac{1}{16}\right)^{1/2} = 0{,}6\ \%$$

If the value of Re_d at the point where the iteration converges is 8×10^5, then

$$X \left(= \frac{1}{(Re_d)^{1/2}} \right) = 0{,}001\,12$$

Thus

$e_r(X) = 1{,}45 \times 10^{-6}$

$e_s(X) = 6{,}7 \times 10^{-6}$

From equation (A.17), and letting $b = 8{,}26$,

$$e(C_o) = 8{,}26\,[(1{,}45 \times 10^{-6})^2 + (6{,}7 \times 10^{-6})^2]^{1/2}$$

$$= 5{,}7 \times 10^{-5}$$

Substituting this value into equation (A.18) gives

$$e(C) = \{3,13[4 \times 10^{-8} + 0,383(x_k - \overline{x})^2] + (0,001\,5)^2 + (5,7 \times 10^{-5})^2\}^{1/2}$$

$$= \{3,13[4 \times 10^{-8} + 0,383 \times (1,06 \times 10^{-4})^2] + (0,001\,5)^2 + (5,7 \times 10^{-5})^2\}^{1/2}$$

$$= 0,001\,5 \ (0,25\ \%)$$

The magnitude of $e(C_o)$ has in this case made negligible difference to the total uncertainty in C, since the gradient of the calibration graph is very slight. In other words, a very large uncertainty is required in $1/(Re_d)^{1/2}$ in order to introduce any significant uncertainty into the value used for C. In other cases, however, where the gradient of the calibration graph is appreciably steeper, this contribution to the uncertainty in C could be important.

Substituting the appropriate values into equations (A.19) and (A.20) gives

$$E_r(Q') = 0,25\ \%$$

$$E_s(Q') = 1,03\ \%$$

and these values represent the uncertainty in the flow-rate measurement.

Annex B
(normative)

Example for an open channel

B.1 Symbols

a	value of stage at zero flow
$e_r(g)$	random uncertainty in the recorded value of the stage, expressed in metres
$e_r(z)$	random uncertainty in the gauge zero, expressed in metres
h	depth of water at open channel gauging station
n	number of revolutions per second of current-meter rotor
N	number of observations
Q_c	discharge taken from stage-discharge relation corresponding to Q_i and $h + a$, where h is the stage and a is the value of the stage at zero flow
Q_{dm}	daily mean discharge
Q_i	current-meter observation
Q_{mm}	monthly mean discharge
s_e	standard error of estimate
s_{mr}	standard error of the mean
v	observations of carriage velocity, expressed in metres per second
v_c	velocity taken from current-meter rating equation corresponding to values of v and n
$X(Q)$	percentage uncertainty in Q (stage-discharge relation) (also known as standard error of the mean, s_{mr})
$X(Q_{am})$	percentage uncertainty in the annual mean discharge
$X(C)$	percentage uncertainty in the coefficient of discharge of a weir or flume
$X(Q_{dm})$	percentage uncertainty in the daily mean discharge
$X(h+a)$	percentage uncertainty in the head or stage
$X(Q_{mm})$	percentage uncertainty in the monthly mean discharge
$X(v)$	percentage uncertainty in v (current-meter rating, v-n relation)
β	exponent of the stage-discharge equation $Q = C(h + a)^{\beta}$, where C is the rating constant

NOTE — All uncertainties are expressed at the 95 % confidence level.

B.2 Uncertainty in the stage-discharge relation and in the daily mean discharge

B.2.1 General

The uncertainty in a single determination of discharge shall be evaluated according to ISO 5168. This clause deals with the uncertainty in the stage-discharge relation and in a continuous measurement of discharge; the principles of the method are given in ISO 1100-2.

B.2.2 Statistical analysis of the stage-discharge relation

The stage-discharge relation, being a best-fit line, should be more accurate than any of the individual gaugings. The equation of the relation may be computed as detailed in table B.1, which assumes that the relation plots as a straight line on logarithmic paper.

The standard error of estimate $s_e(\log_e Q)$ is first calculated for the logarithmic relation. The uncertainty in Q is subsequently calculated from $s_e(\log_e Q)$. Uncertainties are expressed as percentages at the 95 % confidence level.

The standard error of estimate s_e of $s_e(\log_e Q)$ may be calculated from one of the following equations:

$$s_e = \left\{ \left(\frac{N-1}{N-2} \right) \left[s^2(\log_e Q) - \beta^2 s^2(\log_e h) \right] \right\}^{1/2} \quad \ldots \text{(B.1)}$$

or

$$s_e = \left[\frac{\Sigma(\log_e Q_i - \log_e Q_c)^2}{N-2} \right]^{1/2} \quad \ldots \text{(B.2)}$$

where

N is the number of current-meter (discharge) observations;

$$s^2(\log_e Q) = \frac{\Sigma(\log_e Q_i - \overline{\log_e Q_i})^2}{N-1}$$

where

$\overline{\log_e Q_i}$ is the average value of $\log_e Q_i$;

$s(\log_e Q)$ is the standard deviation of the natural logarithms of the discharge observations;

Q_i is the current-meter observation;

$$s^2(\log_e h) = \frac{\Sigma[\log_e(h+a) - \overline{\log_e(h+a)}]^2}{N-1}$$

where

$s(\log_e h)$ is the standard deviation of the modified stage values $(h + a)$;

$(h + a)$ are the stage values corresponding to values of Q_i;

$\overline{\log_e(h+a)}$ is the average value of $\log_e(h + a)$;

β is the exponent of the stage-discharge equation;

Q_c is the discharge taken from the rating curve corresponding to Q_i and $(h + a)$ $[Q_c = C(h + a)^{\beta}]$.

The percentage uncertainty in Q is equal to $ts_e(Q)/N^{1/2}$ at the mean value of $(h + a)$. The uncertainty increases in moving away from the mean value of $(h + a)$ and an additional term is required within the square-root sign. Multiplication by t and by 100 is also necessary unless the percentage value of s_e is used; however, see note 1.

Then

$$X(Q) = ts_e \left\{ \frac{1}{N} + \frac{[\log_e(h+a) - \overline{\log_e(h+a)}]^2}{\Sigma[\log_e(h+a) - \overline{\log_e(h+a)}]^2} \right\}^{1/2} \times 100 \quad \ldots \text{(B.3)}$$

NOTES

1 Strictly equation (B.3) should be in the form 100 $[(e^z) - 1]$ where z equals the right-hand side of equation (B.3) with the term 100 transferred. This form gives asymmetrical uncertainty limits whereas equation (B.3) gives symmetrical limits.

2 In equation (B.3), t is Student's t distribution at the required probability level and with $N - 2$ degrees of freedom. t may be taken as 2 for 20 or more gaugings.

3 Equations (B.1) and (B.2) contain logarithms to the base e and give the standard error in absolute terms. The calculation may be carried out using logarithms to the base 10 and the answer multiplied by 2,3 and by 100 to give the standard error as a percentage.

4 Equations (B.1) and (B.2) assume that the uncertainty in stage is small compared to the uncertainty in discharge during the determination of the stage-discharge relation.

5 Equations (B.1) and (B.2) give a single value for the standard error of estimate of the logarithmic relation and provide two parallel straight lines, one on each side of the stage-discharge curve. In practice these lines are drawn at ts_e on each side of the stage-discharge curve where t is taken at the 95 % confidence level (see figure B.1).

6 Equation (B.1) needs to be calculated using a computer program in order to give the best result. This is because the bracketed term in the equation is small and therefore needs to be calculated to at least six decimal places. Equation (B.2) can be conveniently calculated using a pocket calculator.

7 The equation

$$s_e = \left\{ \frac{\left[\Sigma\left(\frac{Q_i - Q_c}{Q_c}\right) \times 100\right]^2}{N - 2} \right\}^{1/2} \quad \ldots \text{(B.4)}$$

also gives a good approximation of s_e provided that a logarithmic distribution is used to establish the rating equation. For the 95 % confidence level, the result should be multiplied by the appropriate value of t.

8 In equation (B.3), $X(Q)$ is also referred to as the 95 % confidence limits or as s_{mr} (standard error of the mean relation).

The value of s_{mr} shall be calculated for each observation of $(h + a)$ related to the corresponding gauging. The limits shall therefore be plotted on each side of the stage-discharge relation and shall be a minimum at the value of $\overline{\log_e(h + a)}$.

If the stage-discharge relation comprises one or more break points, s_e and s_{mr} shall be calculated for each range and $N - 2$ degrees of freedom allowed for each range, where N represents the number of observations in each segment.

At least 20 observations should be available in each range before a statistically acceptable estimate can be made of s_e and s_{mr}.

B.2.3 Uncertainty in the daily mean discharge

The value of discharge most commonly required for design and planning purposes is the daily mean discharge.

The daily mean discharge may be calculated by taking the average of the number of observations of discharge during the period of 24 h.

The percentage uncertainty in the daily mean discharge for a velocity area station may be calculated from the following

$$X(Q_{dm}) = \frac{\Sigma\{[X^2(Q) + \beta^2 X^2(h+a)]^{1/2} Q_c\}}{\Sigma Q_c} \quad \ldots \text{(B.5)}$$

where

$X(Q_{dm})$ is the percentage uncertainty in the daily mean discharge (95 % confidence level);

$X(Q)$ is the percentage uncertainty of the stage-discharge relation, (s_{mr}) (see B.2.2);

β is the exponent of the stage-discharge relation;

$X(h + a)$ is the percentage uncertainty in the recorded measurement of the stage;

Q_c are the values of discharge from the rating equation used to calculate the daily mean discharge.

The corresponding equation for a measuring structure is similar and may be expressed as follows:

$$X(Q_{dm}) = \frac{\Sigma[\{X^2(C) + \beta^2 X^2(h+a)\}^{1/2} Q_c]}{\Sigma Q_c} \quad \ldots \text{(B.6)}$$

where $X(C)$ is the percentage uncertainty in the coefficient of discharge.

NOTE — The percentage uncertainty in the length of crest (width of throat), $X(b)$, has been ignored.

The percentage uncertainty in the monthly mean and annual mean discharges may be estimated from the following equations

$$X(Q_{mm}) = \frac{\Sigma[X(Q_{dm})\, Q_{dm}]}{\Sigma Q_{dm}} \quad \ldots \text{(B.7)}$$

$$X(Q_{am}) = \frac{\Sigma[X(Q_{mm})\, Q_{mm}]}{\Sigma Q_{mm}} \quad \ldots \text{(B.8)}$$

where

$X(Q_{mm})$ is the percentage uncertainty in the monthly mean discharge (95 % confidence level);

$X(Q_{am})$ is the percentage uncertainty in the annual mean discharge;

Q_{dm} are the daily mean discharges;

Q_{mm} are the monthly mean discharges.

The percentage uncertainty in stage (or head) in the above equations may be found from the following equation

$$X(h + a) = \frac{100}{h + a}[e_r^2(g) + e_r^2(z)]^{1/2} \qquad \ldots \quad (B.9)$$

where

$(h + a)$ is the stage (or head), expressed in metres;

$e_r^2(g)$ is the random uncertainty in the recorded value of stage (or head) (recommended value: for punched tape recorder, 3 mm; for chart recorder, 5 mm);

$e_r^2(z)$ is the random uncertainty in the gauge zero (recommended value, 3 mm).

B.2.4 Example of calculation for s_e and s_{mr}

Using the stage-discharge curve tabulation given in table B.1 for the curve shown in figure B.1, the required tabulation for s_e and s_{mr} is given in table B.2 where the logarithms are to the base 10.

From equation (B.1), the value for s_e is calculated as follows:

$$s_e = \left[\frac{31}{30}\left(\frac{12{,}336\ 4}{31} - \frac{1{,}530\ 1^2 \times 5{,}266\ 5}{31}\right)\right]^{1/2}$$

$$= (0{,}000\ 2)^{1/2}$$

Then, converting to base e and multiplying by t (where $t = 2$ at the 95 % confidence level) and by 100 to obtain the percentage value

$$s_e = 2 \times 0{,}014\ 14 \times 2{,}3 \times 100$$

$$= 6{,}5\ \%$$

From equation (B.2), the value for s_e is calculated as follows:

$$s_e = \left(\frac{0{,}005\ 48}{30}\right)^{1/2}$$

Then, converting to base e and multiplying by t (where $t = 2$ at the 95 % confidence level) and by 100 to obtain the percentage value

$$s_e = 2 \times 0{,}013\ 5 \times 2{,}3 \times 100$$

$$= 6{,}2\ \%$$

From equation (B.4), the value for s_e is calculated as follows:

$$s_e = \left(\frac{291{,}18}{30}\right)^{1/2}$$

Then, multiplying by t (where $t = 2$ at the 95 % confidence level)

$$s_e = 2 \times 3{,}115$$

$$= 6{,}2\ \%$$

The calculation for s_{mr} for each of the 32 observations proceeds from equation (B.3), taking the previous value of $ts_e = 6{,}2$ %, as follows.

For observation No. 1

$$s_{mr} = ts_e\left(\frac{1}{32} + \frac{0{,}351\ 2}{5{,}266\ 5}\right)^{1/2}$$

$$= 6{,}2 \times 0{,}313$$

$$= 1{,}94\ \%\ \text{(at the 95 \% level)}$$

For observation No. 18

$$s_{mr} = ts_e\left(\frac{1}{32} + 0\right)^{1/2}$$

$$= 6{,}2 \times 0{,}177$$

$$= 1{,}1\ \%\ \text{(at the 95 \% level)}$$

For observation No. 32

$$s_{mr} = ts_e\left(\frac{1}{32} + \frac{0{,}518\ 3}{5{,}266\ 5}\right)^{1/2}$$

$$= 6{,}2 \times 0{,}360$$

$$= 2{,}23\ \%\ \text{(at the 95 \% level)}$$

NOTES

1 The s_{mr} values in table B.2 have been calculated by a computer program and therefore differ slightly from the above values.

2 The equation $s_{mr} = s_e/\sqrt{N}$ gives $s_{mr} = 6{,}2/\sqrt{32} = 1{,}1$ %, which is the value at $\overline{\log(h + a)} = -0{,}211\ 45$

B.2.5 Example of calculation for the uncertainty in the daily mean discharge $X(Q_{dm})$

The calculation proceeds as follows.

a) Calculate $X(h + a)$ for each of the N values of discharge, used to calculate the daily mean, from equation (B.9);

b) Calculate $X(Q_{dm})$ from equation (B.5) or (B.6), using the appropriate value for $X(Q)$ ($= s_{mr}$).

An example of the calculation for hourly values of discharge is given in table B.3.

Table B.1 — Example of the manual computation of the stage-discharge relation by the method of least squares

Observation No.	Flow-rate Q m³/s	Stage h m	$(h + a)$ where $a = -0,115$ m	$\log Q = Y$	$\log (h + a) = X$	XY	X^2
1	2,463	0,272	0,157	0,391 5	−0,804 1	−0,314 8	0,646 6
2	2,325	0,273	0,158	0,366 4	−0,801 3	−0,293 6	0,642 1
3	2,923	0,303	0,188	0,465 8	−0,725 8	−0,338 1	0,526 8
4	3,242	0,307	0,192	0,510 8	−0,716 7	−0,366 1	0,513 7
5	3,841	0,334	0,219	0,584 4	−0,659 6	−0,385 5	0,435 1
6	4,995	0,374	0,259	0,698 5	−0,586 7	−0,409 8	0,344 2
7	5,410	0,393	0,278	0,733 2	−0,556	−0,407 7	0,309 1
8	5,422	0,394	0,279	0,734 2	−0,554 4	−0,407	0,307 4
9	5,883	0,402	0,287	0,769 6	−0,542 1	−0,417 2	0,293 9
10	6,154	0,41	0,295	0,789 2	−0,530 2	−0,418 4	0,281 1
11	7,376	0,463	0,348	0,867 8	−0,458 4	−0,397 8	0,210 1
12	9,832	0,52	0,405	0,992 6	−0,392 5	−0,389 6	0,154 1
13	11,321	0,548	0,433	1,053 9	−0,363 5	−0,383 1	0,132 1
14	12,372	0,576	0,461	1,092 4	−0,336 3	−0,367 4	0,113 1
15	11,825	0,58	0,465	1,072 8	−0,332 5	−0,356 7	0,110 6
16	13,826	0,616	0,501	1,140 7	−0,300 2	−0,342 4	0,090 1
17	14,102	0,626	0,511	1,149 3	−0,291 6	−0,335 1	0,085
18	19,02	0,721	0,606	1,279 2	−0,217 5	−0,278 2	0,047 3
19	19,79	0,739	0,624	1,296 4	−0,204 8	−0,265 5	0,041 9
20	20,28	0,747	0,632	1,307 1	−0,199 3	−0,260 5	0,039 7
21	21,204	0,796	0,681	1,326 4	−0,166 9	−0,221 4	0,027 9
22	23,996	0,846	0,731	1,380 1	−0,136 1	−0,187 8	0,018 5
23	36,242	1,041	0,926	1,559 2	−0,033 4	−0,052 1	0,001 1
24	54,591	1,34	1,225	1,737 1	0,088 1	0,153	0,007 8
25	67,327	1,526	1,411	1,828 2	0,149 5	0,273 3	0,022 4
26	79,05	1,761	1,646	1,897 9	0,216 4	0,410 7	0,046 8
27	110,783	2,01	1,895	2,044 5	0,277 6	0,567 6	0,077 1
28	162,814	2,632	2,517	2,211 7	0,400 9	0,886 7	0,160 7
29	227,6	3,205	3,150	2,357 2	0,498 3	1,174 6	0,248 3
30	228,8	3,28	3,165	2,359 5	0,500 4	1,180 7	0,250 4
31	228,5	3,306	3,191	2,358 9	0,503 9	1,188 6	0,253 9
32	236,6	3,34	3,225	2,374	0,508 5	1,207 2	0,258 6
			Σ	40,730 5	−6,766 3	−0,553 4	6,697 5

For a least-squares regression where $Q = C(h + a)^\beta$

$$\Sigma(Y) - N(\log C) - \beta\Sigma(X) = 0$$

where N is the number of observations.

$$\Sigma(XY) - \Sigma(X)(\log C) - \beta\Sigma(X^2) = 0$$

Then

$$40,730\,5 - 32 \log C + 6,766\,3\beta = 0 \qquad \text{(i)}$$

$$-0,553\,4 + 6,766\,3 \log C - 6,697\,5\beta = 0 \qquad \text{(ii)}$$

Multiplying equation (i) by 6,766 3/32 and adding (i) and (ii) gives

$$\beta = 1,530\,1$$

$$C = 39,479$$

$$Q = 39,479\,(h - 0,115)^{1,530\,1}$$

NOTE — The stage-discharge curve is shown plotted in figure B.1.

Table B.2 — Tabulated values required to calculate s_e and s_{mr}

Observation No.	$(h + a)$ where $a = -0{,}115$ m	$\log(h + a)$	$[\log(h + a) - \overline{\log(h + a)}]^2$	Q_i (gauging) m^3/s	$\log Q_i$	$(\log Q_i - \overline{\log Q_i})^2$	Q_c (from rating equation) m^3/s	$\log Q_c$	$(\log Q_i - \log Q_c)^2$	$\left(\frac{Q_i - Q_c}{Q_c} \times 100\right)^2$ %	$\pm 2s_{mr}$ %
1	0,157	−0,804 1	0,351 2	2,463	0,391 46	0,776 789	2,323	0,366	0,000 65	36	2
2	0,158	−0,801 3	0,347 9	2,325	0,366 42	0,821 561	2,345	0,370 1	0,000 01	0,81	2
3	0,188	−0,725 8	0,264 6	2,923	0,465 83	0,651 233	3,06	0,485 7	0,000 4	20,25	1,8
4	0,192	−0,716 7	0,255 3	3,242	0,510 81	0,580 659	3,16	0,499 7	0,000 12	6,76	1,8
5	0,219	−0,659 6	0,200 8	3,841	0,584 44	0,473 592	3,865	0,587 1	0,000 01	0,36	1,7
6	0,259	−0,586 7	0,140 3	4,995	0,698 54	0,329 798	4,996	0,698 6	0	0	1,5
7	0,278	−0,556	0,118 7	5,41	0,733 2	0,291 189	5,568	0,746 7	0,000 16	7,84	1,5
8	0,279	−0,554 4	0,117 6	5,422	0,734 16	0,290 155	5,598	0,748	0,000 19	10,24	1,5
9	0,287	−0,542 1	0,109 3	5,883	0,769 6	0,253 230	5,846	0,766 9	0,000 01	0,36	1,4
10	0,295	−0,530 2	0,101 6	6,154	0,789 16	0,233 927	6,097	0,785 1	0,000 02	0,81	1,4
11	0,348	−0,458 4	0,061	7,376	0,867 82	0,164 025	7,851	0,894 9	0,000 74	36	1,3
12	0,405	−0,392 5	0,032 8	9,832	0,992 64	0,078 501	9,902	0,995 7	0,000 01	0,49	1,2
13	0,433	−0,363 5	0,023 1	11,321	1,053 88	0,047 935	10,968	1,040 1	0,000 19	10,24	1,2
14	0,461	−0,336 3	0,015 6	12,372	1,092 44	0,032 537	12,072	1,081 8	0,000 11	6,25	1,2
15	0,465	−0,332 5	0,014 7	11,825	1,072 8	0,040 008	12,233	1,087 5	0,000 22	10,89	1,2
16	0,501	−0,300 2	0,007 9	13,826	1,140 7	0,017 456	13,711	1,137 1	0,000 01	0,64	1,2
17	0,511	−0,291 6	0,006 4	14,102	1,149 28	0,015 262	14,132	1,150 2	0	0,04	1,1
18	0,606	−0,217 5	0	19,02	1,279 21	0,000 04	18,345	1,263 5	0,000 24	13,69	1,1
19	0,624	−0,204 8	0	19,79	1,296 44	0,000 558	19,185	1,283	0,000 18	10,24	1,1
20	0,632	−0,199 3	0,000 1	20,28	1,307 1	0,001 175	19,563	1,291 4	0,000 24	13,69	1,1
21	0,681	−0,166 9	0,002	21,204	1,326 41	0,002 872	21,931	1,341 1	0,000 21	10,89	1,1
22	0,731	−0,136 1	0,005 7	23,996	1,380 13	0,011 515	24,442	1,388 1	0,000 06	3,24	1,1
23	0,926	−0,033 4	0,031 7	36,242	1,559 21	0,082 019	35,098	1,545 3	0,000 19	10,89	1,2
24	1,225	−0,088 1	0,089 7	54 591	1,737 12	0,215 574	53,855	1,731 2	0,000 03	1,96	1,4
25	1,411	−0,149 5	0,130 3	67,327	1,828 19	0,308 436	66,859	1,825 2	0,000 01	0,49	1,5
26	1,646	−0,216 4	0,183 1	79,05	1,897 9	0,390 725	84,631	1,927 5	0,000 88	43,56	1,6
27	1,895	−0,277 6	0,239 2	110,783	2,044 47	0,595 444	104,989	2,021 1	0,000 54	30,25	1,8
28	2,517	−0,400 9	0,375 1	162,814	2,211 69	0,881 477	162,095	2,209 8	0	0,16	2
29	3,15	−0,498 3	0,503 7	227,6	2,357 17	1,175 814	228,478	2,358 8	0	0,16	2,2
30	3,165	−0,500 4	0,506 7	228,8	2,359 46	1,180 786	230,145	2,362	0,000 01	0,36	2,3
31	3,191	−0,503 9	0,511 7	228,5	2,358 48	1,179 548	233,044	2,367 4	0,000 07	3,61	2,3
32	3,225	−0,508 5	0,518 3	236,6	2,374 01	1,212 619	236,854	2,374 5	0	0,01	2,3
		$\overline{\log(h+a)} = -0{,}211\ 45$	Σ 5,266 5		Σ 40,730 38 $\overline{\log Q}$ = 1,272 82	Σ 12,336 459			Σ 0,005 48	Σ 291,18	

Table B.3 — Example of the computation for the uncertainty in the daily mean discharge using hourly values of discharge

Time	h m	$(h - 0{,}115)$ m	Q_c m³/s	$X(h + a)$ %	$2s_{mr}$ %	$[2s_{mr}^2 + \beta^2 X^2(h+a)]^{1/2} Q_c$
0900	1,225	1,11	46,314	0,4	1,3	64,84
1000	1,565	1,45	69,707	0,3	1,5	115,53
1100	1,971	1,856	101,699	0,2	1,8	183,06
1200	2,293	2,178	129,906	0,2	1,9	246,82
1300	2,52	2,405	151,186	0,2	2	302,37
1400	2,67	2,565	165,85	0,2	2	331,7
1500	2,789	2,674	177,814	0,2	2	355,63
1600	2,872	2,767	186,328	0,2	2,1	391,29
1700	2,929	2,814	192,255	0,2	2,1	403,74
1800	2,981	2,876	197,717	0,1	2,1	415,21
1900	3,034	2,929	203,339	0,1	2,2	447,34
2000	3,067	2,952	206,867	0,1	2,2	455,11
2100	3,082	2,967	208,478	0,1	2,2	458,65
2200	3,065	2,95	206,653	0,1	2,2	454,64
2300	3,026	2,911	202,487	0,1	2,2	445,47
2400	2,975	2,86	197,084	0,1	2,1	413,88
0100	2,915	2,8	190,793	0,2	2,1	400,66
0200	2,845	2,73	183,543	0,2	2,1	385,44
0300	2,747	2,632	173,558	0,2	2	347,11
0400	2,628	2,513	161,697	0,2	2	323,39
0500	2,495	2,38	148,788	0,2	2	297,57
0600	2,365	2,25	136,534	0,2	1,9	259,41
0700	2,257	2,142	126,635	0,2	1,9	240,61
0800	2,164	2,049	118,32	0,2	1,8	212,98
			Σ 3 883,56			
			$\bar{Q}_c = 161{,}815$			Σ 7 952,45

$$X(Q_{dm}) = \frac{\Sigma\left\{\left[(2s_{mr}^2 + \beta^2 X^2(h + a)\right]^{1/2} Q_c\right\}}{\Sigma\, Q_c}$$

$$= \frac{7\,952{,}45}{3\,883{,}56}$$

$$= 2\ \%$$

Then the daily mean discharge is equal to 161,8 m³/s ± 2 %.

B.2.6 Example of calculation for the standard error of estimate and the standard error of the mean of the current-meter rating equation

Values of v (in metres per second) and n (in revolutions per second) for the upper range of a current-meter rating are given in table B.4.

The equation of the line (rating equation) is

$$v = 0{,}006\,9 + 0{,}278\,5\,n$$

The standard error, s_e, of the experimental observations about the line is found from equation (17):

$$s_e = t\left[\frac{\Sigma(v - v_c)^2}{N - 2}\right]^{1/2} = 2\left(\frac{0{,}000\,165\,8}{18}\right)^{1/2} = 0{,}006\ \text{m/s}$$

By this method, as distinct from the method given in B.2.4, the percentage standard error varies along the line.

For example, for observation No. 1

$$s_e = \frac{0{,}006}{0{,}233\,5} \times 100 = 2{,}6\ \%$$

The values of s_e, in per cent, are given in table B.4, column 9.

The standard error of the mean of the line of the relation is found from equation (25):

$$X(v) = t s_e\left[\frac{1}{N} + \frac{(n - \bar{n})^2}{\Sigma(n - \bar{n})^2}\right]^{1/2}$$

For example, for observation No. 1

$$X(v) = 2{,}6\,(0{,}14)^{1/2} = 1\ \%$$

The values of $X(v)$, in per cent, are given in table B.4, column 10.

NOTES

1 v_c is the velocity taken from the rating equation corresponding to values of v.

2 n is the number of the revolutions per second for values of carriage velocity v.

3 N is the number of observations of carriage velocity v.

4 The limits for s_e and $X(v)$ are shown in figure B.2.

Table B.4 — Tabulated values required to calculate s_e and $X(v)$

Observation No.	v m/s	n rev/s	v_c m/s	$n - \overline{n}$	$(v - v_c)^2$ $\times 10^3$	$(n - \overline{n})^2$	$\frac{1}{N} + \frac{(n - \overline{n})^2}{\Sigma(n - \overline{n})^2}$	s_e [from equation (17)] %	$X(v)$ [from equation (25)] %
1	2	3	4	5	6	7	8	9	10
1	0,232 5	0,813 2	0,233 5	4,133 7	0,000 8	17,087 5	0,145	2,6	1
2	0,308 5	1,088 9	0,310 3	3,858	0,002 9	14,884 2	0,133	1,93	0,71
3	0,462 8	1,643 9	0,464 8	3,303	0,003 6	10,910	0,111	1,29	0,43
4	0,606	2,141 8	0,603 4	2,805 1	0,006 8	7,868 6	0,094	0,99	0,3
5	0,606 9	2,156 2	0,607 4	2,790 7	0,000 3	7,788	0,093	0,99	0,3
6	0,607 4	2,160 9	0,608 8	2,786	0,001 7	7,761 8	0,093	0,98	0,3
7	0,757 8	2,694 9	0,757 4	2,252	0,000 2	5,071 5	0,078	0,79	0,22
8	0,913 6	3,255 6	0,913 6	1,691 3	0	2,860 5	0,066	0,66	0,17
9	1,213 6	4,334 1	1,213 9	0,612 8	0,000 1	0,375 5	0,052	0,49	0,11
10	1,286 2	4,587 8	1,284 6	0,359 1	0,002 6	0,129	0,051	0,47	0,11
11	1,295 5	4,637 8	1,298 5	0,309 1	0,009	0,095 6	0,051	0,46	0,1
12	1,378 1	4,905 1	1,372 9	0,041 8	0,026 1	0,001 7	0,05	0,44	0,1
13	1,538 1	5,481 6	1,533 5	0,534 7	0,021 2	0,285 9	0,052	0,39	0,09
14	1,700 3	6,071 3	1,697 6	1,124 4	0,006 3	1,264 3	0,057	0,35	0,08
15	1,839 3	6,573 8	1,837 7	1,626 9	0,002 6	2,646 8	0,065	0,33	0,08
16	2,158 9	7,744 7	2,163 8	2,797 8	0,024	7,827 7	0,094	0,28	0,09
17	2,454 9	8,805 9	2,459 3	3,859	0,019 4	14,891 9	0,133	0,24	0,09
18	2,471	8,849 6	2,471 2	3,902 7	0,000 3	15,231 1	0,135	0,24	0,09
19	2,778	9,966 1	2,782 1	5,019 2	0,020 3	25,192 4	0,191	0,22	0,09
20	3,081 6	11,025 4	3,077 1	6,078 5	0,017 6	36,948 2	0,256	0,20	0,1
Σ	27,619	98,938 5			0,165 8	179,122 1			
Mean	1,384 6	4,946 9							

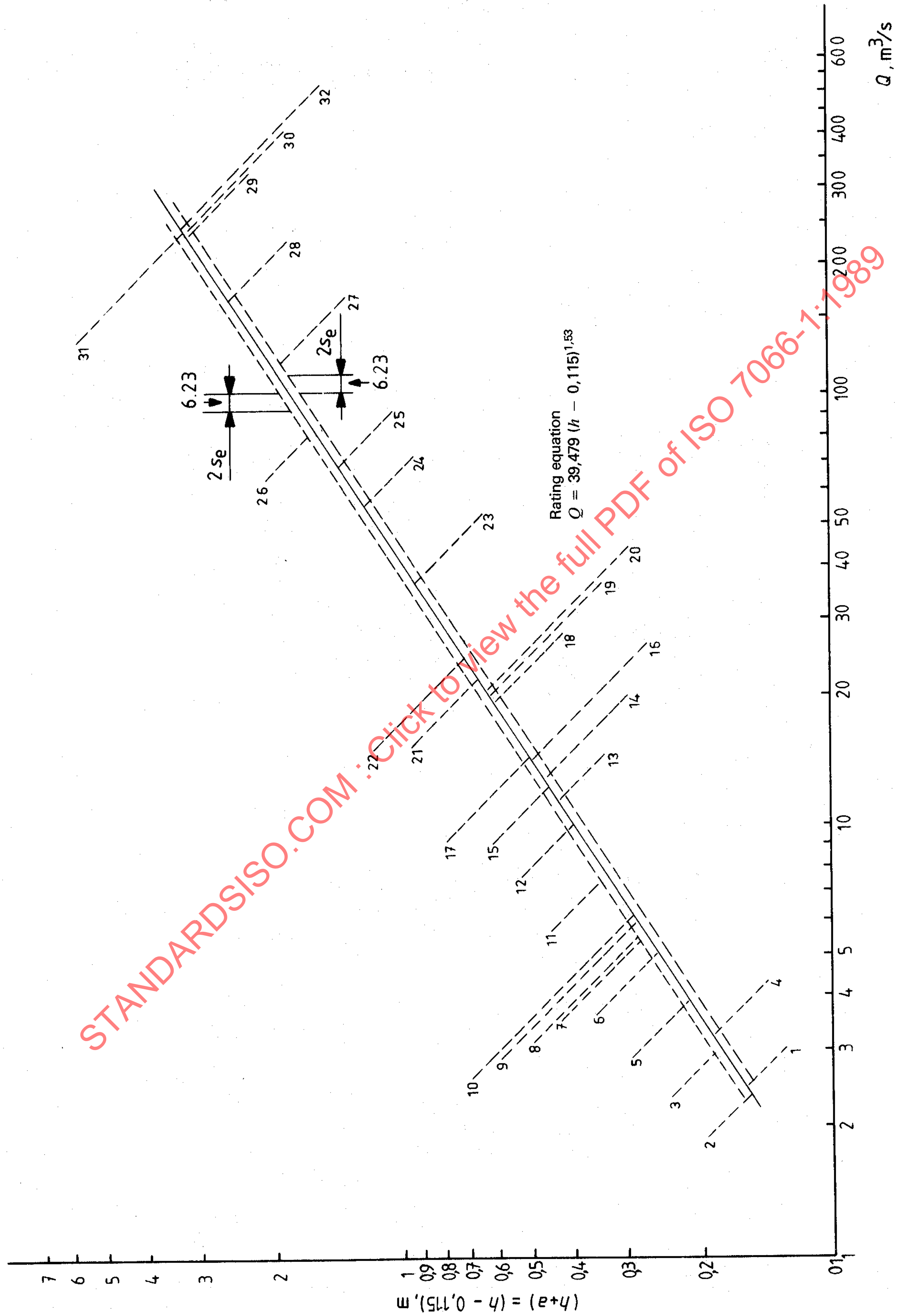

Figure B.1 — Stage-discharge curve

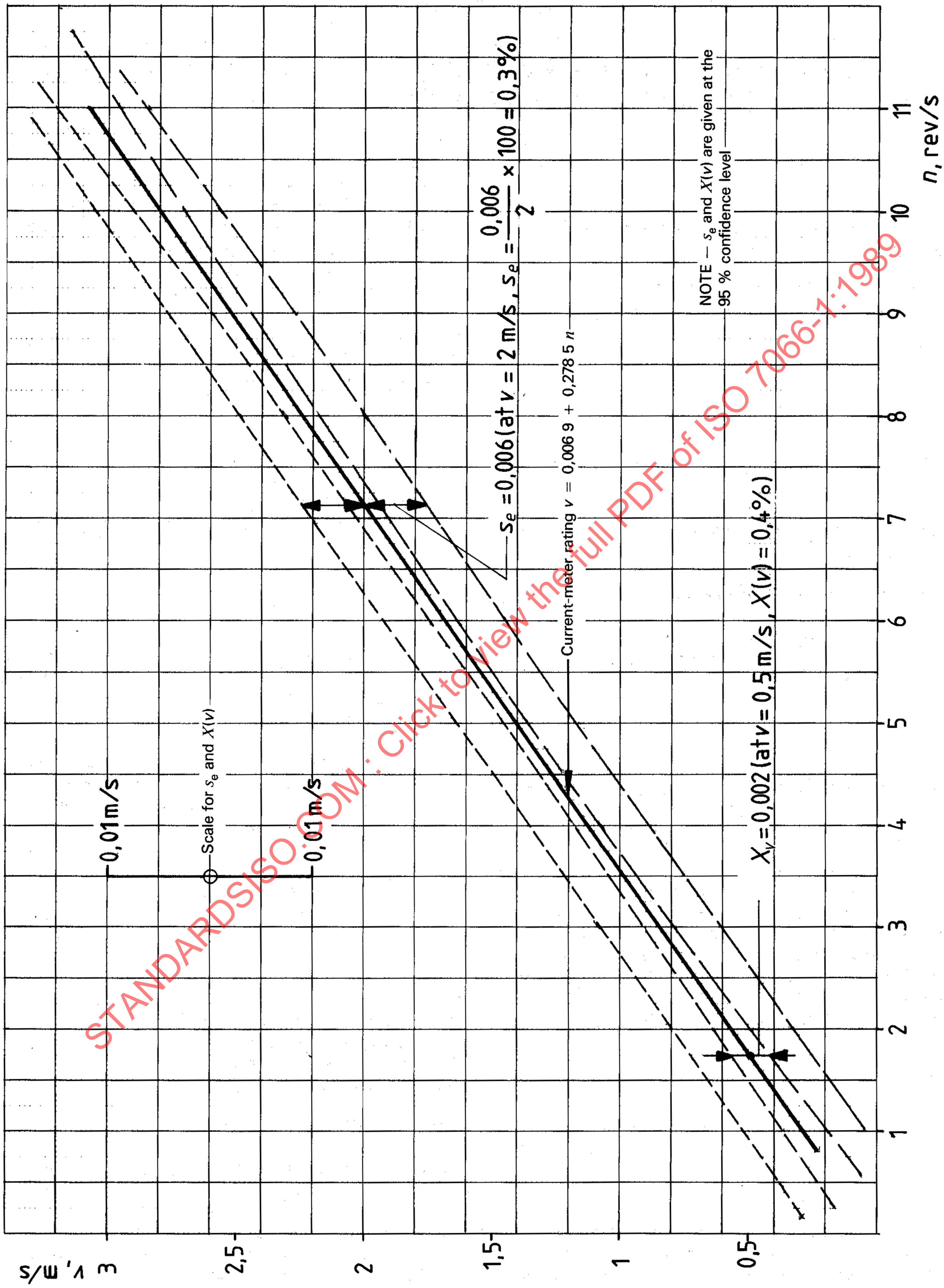

Figure B.2 — Current-meter rating

Annex C
(normative)

Uncertainty associated with the calibration coefficient when using a calibrated or standardized flow-meter

C.1 Symbols

β power index in calibration equations for stage-discharge relation

Δp differential pressure across an orifice plate

ε expansibility correction for an orifice plate

μ dynamic viscosity

ϱ density

C.2 Uncertainty associated with the calibration coefficient

The reason why, when a calibrated flow-meter is used to measure a flow-rate, the uncertainty in the value used for its calibration coefficient is greater than the uncertainty in the calibration graph is illustrated in figure C.1.

For example, if the calibration and use of a nozzle is considered, the calibration equation giving the flow coefficient, C, as a function of nozzle throat Reynolds number, Re_d, would take the form

$$C = a + b\,Re_d$$

This is the continuous, sloping line shown in figures C.1 a) and C.1 b), and the uncertainty in it is given by the chain-dot curves in figure C.1 a). This uncertainty arises from both random components [given by equation (25) or equation (29), depending on the relative magnitudes of the uncertainties in C and Re_d] and systematic components. The uncertainty in the calibration line is less than that in any one of the data points to which it is fitted, since the effect of the random uncertainty components is that a mean value (the calibration line in this case) can be determined more accurately than a single measurement (the individual data points). If equation (25) is considered, for example, the random component of the uncertainty in the line at $X_k = X$ is given by

$$ts(y, x)/\sqrt{N}$$

whereas the random component in a single data point can be shown to be

$$ts(y, x)$$

Although both random and systematic sources contribute to the uncertainty in the determination of the calibration line, when a calibrated flow-meter is used, the calibration equation adopted has a fixed value which will differ by a finite, unknown amount from the true value.

In particular, referring again to figure C.1 a) for the case of a nozzle, when the nozzle is used after calibration, the value used for C for any particular Reynolds number will be in error by a fixed amount which will not subsequently change, and so this will contribute a systematic uncertainty to the uncertainty in any flow-rate measurement using that nozzle.

In using the nozzle, a preliminary value for C will be assumed, and measurements or evaluations of Δp, ϱ and ε will be used to give a first estimate of q_V from

$$q_V = \frac{C\varepsilon}{(1-\beta^4)^{1/2}} \frac{\pi d^2}{4} \left(\frac{2\Delta p}{\varrho}\right)^{1/2}$$

This estimate of q_V will be combined with values of density, viscosity and throat diameter to give the Reynolds number from the formula

$$Re_d = \frac{4\varrho Q}{\pi \mu d}$$

This, in turn, will be used with the calibration graph [figure C.1 a)] to predict a more accurate value for C, and the procedure is repeated until there is no difference between successive values of C.

If it were possible to measure or know Δp, ϱ, ε and μ exactly, so that there was no uncertainty in any of these values, the uncertainty in the value used for C would simply be the uncertainty in the calibration graph, $e(C_c)$ [see figure C.1 a)].

Conversely, if the calibration equation were known exactly, with zero uncertainty, uncertainties in the measurement of Δp, ϱ, ε and μ would result in an uncertainty $e(Re_d)$ in the value used for Re_d and hence in the value used for C, as denoted in figure C.1 b) by $e(C_o)$.

In practice, both of these sources of uncertainty exist, and so $e(C_o)$ has to be combined by the root-sum-square method with $e(C_c)$. Hence the value used for C, when the calibrated nozzle is used, has a larger uncertainty than the uncertainty in the calibration graph itself, i.e.

$$e(C) = [e^2(C_c) + e^2(C_o)]^{1/2}$$

Although $e(C_c)$ has a systematic effect on the flow-rate measurement, there are both random and systematic components in $e(C_o)$. To be rigorous, therefore, these should be separated and only the systematic component combined with $e(C_c)$ at this stage, but as noted in clause 10, there is no need for this, in practice, for the purposes of assessing the uncertainty in the value of C adopted for the calculation of the flow-rate.

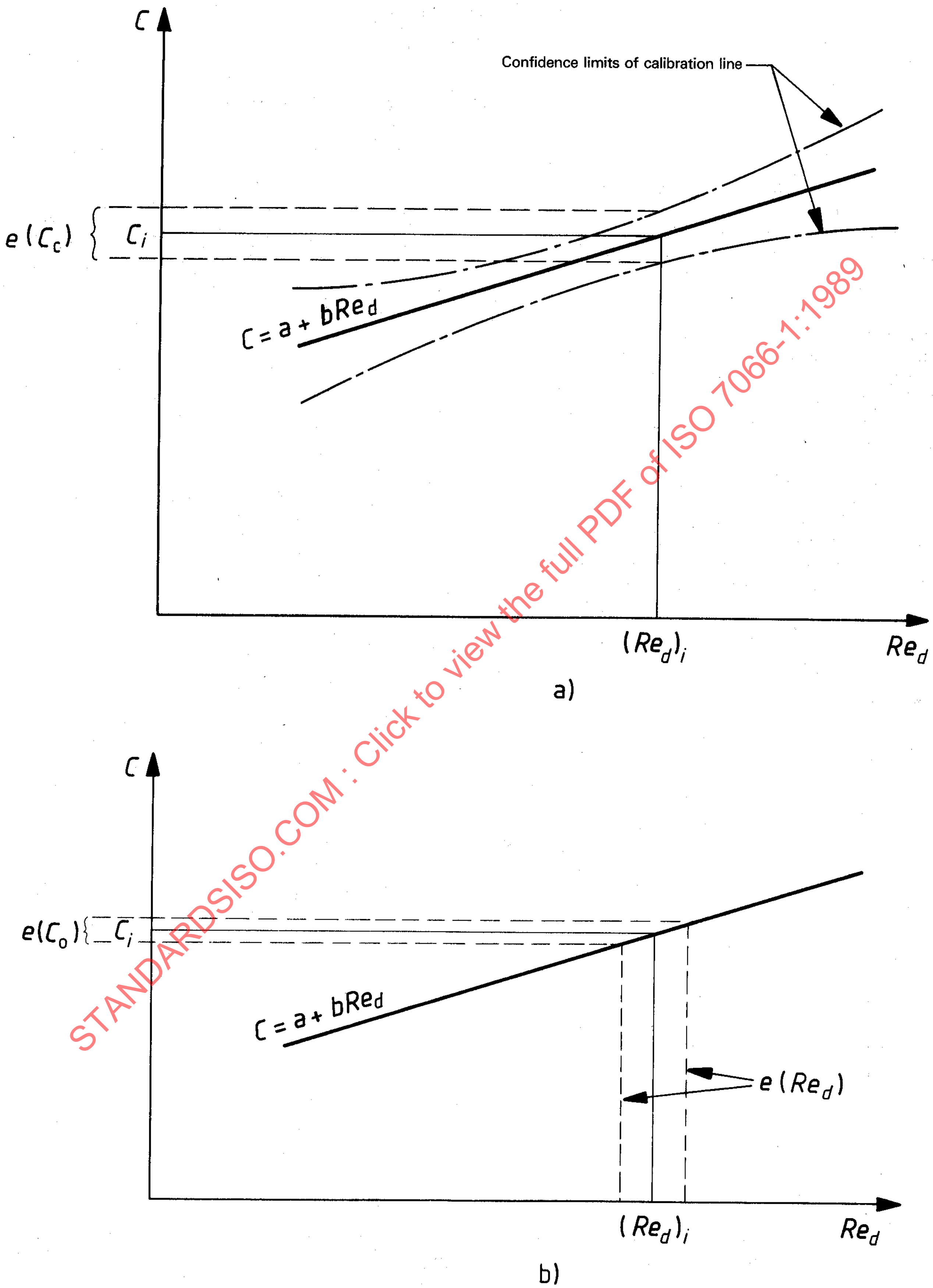

Figure C.1 — Contributions to uncertainty in the value used for C