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# International Standard



# 5406

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INTERNATIONAL ORGANIZATION FOR STANDARDIZATION • МЕЖДУНАРОДНАЯ ОРГАНИЗАЦИЯ ПО СТАНДАРТИЗАЦИИ • ORGANISATION INTERNATIONALE DE NORMALISATION

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## The mechanical balancing of flexible rotors

*Équilibrage mécanique des rotors flexibles*

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## Foreword

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Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council.

International Standard ISO 5406 was developed by Technical Committee ISO/TC 108, *Mechanical vibration and shock*, and was circulated to the member bodies in February 1979.

It has been approved by the member bodies of the following countries :

Australia	Germany, F. R.	South Africa, Rep. of
Austria	Italy	Spain
Belgium	Japan	Sweden
Brazil	Libyan Arab Jamahiriya	United Kingdom
Chile	Netherlands	USA
Czechoslovakia	New Zealand	
Finland	Poland	

The member body of the following country expressed disapproval of the document on technical grounds :

France

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# The mechanical balancing of flexible rotors

## 0 Introduction

This International Standard classifies rotors into groups in accordance with their balancing requirements, establishes methods of assessment of final unbalance, and gives initial guidance on the establishment of balance quality grades so that, ultimately, balance quality grades can be established for all types of flexible rotors.

As the next stage in the development of these balance quality grades, the criteria for evaluating the unbalance of flexible rotors will be further described in an addendum to this International Standard.

As this International Standard is complementary in many details to ISO 1940 it is recommended that, where applicable, the two should be considered together.

## 1 Scope and field of application

This International Standard classifies rotors into groups in accordance with their balancing requirements, specifies methods of assessment of final unbalance, and gives initial guidance on final balance quality criteria.

All rotors are therefore classified to indicate which can be balanced by normal or modified rigid rotor balancing techniques and which require some form of high speed balancing. Classification of rotors into different categories permits the use of simplified balancing methods for some rotors and ensures that for others, where necessary, an adequate balancing operation is performed by a suitable method.

As in the case of ISO 1940, this International Standard is not intended to serve as an acceptance specification for any rotor group, but rather to give indications of how to avoid gross deficiencies as well as exaggerated or unattainable requirements. Nevertheless, it may serve as a basis for more involved investigations, for example when a more exact determination of the required balance quality is necessary. If due regard is paid to the specified limits or methods of manufacture and balance, satisfactory running conditions can most probably be expected. However, there may be cases where deviations from this International Standard may be necessary.

## 2 References

ISO 1925, *Balancing — Vocabulary*.

ISO 1940, *Balance quality of rotating rigid bodies*.

ISO 2041, *Vibration and shock — Vocabulary*.

ISO 2953, *Balancing machines — Description and evaluation*.

## 3 Definitions

**3.1** The definitions relating to mechanical balancing in International Standard ISO 1925 and many of the definitions relating to vibration and shock in ISO 2041 are applicable to this International Standard.

**3.2** For the convenience of users of this International Standard, the following terms and definitions are repeated from ISO 1925 (in the case of 3.4 and 3.15 the entries are adapted from ISO 1925).

**3.3 rigid rotor** : A rotor is considered rigid when it can be corrected in any two (arbitrarily selected) planes and, after that correction, its unbalance does not significantly exceed the balancing tolerances (relative to the shaft axis) at any speed up to maximum service speed and when running under conditions which approximate closely to those of the final supporting system.

**3.4 flexible rotor** : A rotor not satisfying definition 3.3 due to elastic deflection.

**3.5 bearing support** : The part, or series of parts, that transmits the load from the bearing to the main body of the structure.

**3.6 foundation** : A structure that supports the mechanical system.

### NOTES

1 The foundation may be fixed in space or may undergo a motion that provides excitation for the supported system.

2 In the context of the balancing and vibration of rotating machines, the term "foundation" is usually applied to the heavy base structure on which the whole machine is mounted.

**3.7 controlled initial unbalance** : Initial unbalance which has been minimized by individual balancing of components and/or careful attention to design, manufacture and assembly of the rotor.

**3.8 (rotor) flexural critical speed** : A speed of a rotor at which there is maximum flexure of the rotor and where that flexure is significantly greater than the motion of the journals.

**3.9 (rotor) flexural principal mode** : For undamped rotor/bearing systems, that mode shape which the rotor takes up at one of the (rotor) flexural critical speeds.

**3.10 modal balancing** : A procedure for balancing flexible rotors in which balance corrections are made to reduce the amplitude of vibration in the separate significant principal flexural modes to within specified limits.

**3.11  $n^{\text{th}}$  modal unbalance** : That unbalance which affects only the  $n^{\text{th}}$  principal mode of the deflection configuration of a rotor/bearing system.

NOTE — This  $n^{\text{th}}$  modal unbalance is not a single unbalance but an unbalance distribution  $u_n(z)$  in the  $n^{\text{th}}$  principal mode. It can be mathematically represented with respect to its effect on the  $n^{\text{th}}$  principal mode by a single unbalance vector  $\vec{U}_n$  obtained from the formula :

$$\vec{U}_n = \int_0^1 \vec{u}_n(z) \phi_n(z) dz$$

where  $\phi_n(z)$  is the mode function.

**3.12 equivalent  $n^{\text{th}}$  modal unbalance** : The minimum single unbalance  $\vec{U}_{ne}$  equivalent to the  $n^{\text{th}}$  modal unbalance in its effect upon the  $n^{\text{th}}$  principal mode of the deflection configuration.

**NOTES**

1 There exists the relation  $\vec{U}_n = \vec{U}_{ne} \phi_n(z_e)$  where  $\phi_n(z_e)$  is the mode function value for  $z = z_e$ , the axial co-ordinate of the transverse plane where  $\vec{U}_{ne}$  is applied.

2 A set of balance masses distributed in an appropriate number of correction planes and so proportioned that the mode under consideration will be affected, may be called the equivalent  $n^{\text{th}}$  modal unbalance set.

3 An equivalent  $n^{\text{th}}$  modal unbalance will affect some modes other than the  $n^{\text{th}}$  mode.

**3.13 modal unbalance tolerance** : With respect to a mode, that amount of modal unbalance that is specified as the maximum below which the state of unbalance in that mode is considered acceptable.

**3.14 multiple-frequency vibration** : A vibration at a frequency corresponding to an integral multiple of the rotational speed.

NOTE — This vibration may be caused by anisotropy of the rotor, non-linear characteristics of the rotor/bearing system or other causes.

**3.15 thermally induced unbalance** : That change of condition exhibited by a rotor if its state of unbalance is significantly altered by its temperature changes.

NOTE — The change of condition may be permanent or temporary.

**3.16 low speed balancing** (relating to flexible rotors) : A procedure of balancing at a speed where the rotor to be balanced can be considered rigid.

**3.17 high speed balancing** (relating to flexible rotors) : A procedure of balancing at speeds where the rotor to be balanced cannot be considered rigid.

**4 Fundamentals of flexible rotor dynamics with respect to balancing**

**4.1 The motion of a flexible rotor**

Consider a thin slice of a shaft perpendicular to the shaft axis (see figure 1) where for simplicity of illustration the cross-section of the shaft is shown to be circular). Assume that, when the shaft is not rotating, the shaft axis intersects the slice at its geometric centre E (it is assumed throughout this International Standard that the deflection of the shaft due to gravity is ignored). The mass centre C of the slice is in general offset from E by a small distance  $e$  due to the small imperfections unavoidably produced in the shaft during manufacture (from errors in casting, machining tolerances and so on). The mass  $m$  of the slice and the offset distance  $e$  form a measure of unbalance in the slice, namely  $m \times e$ .

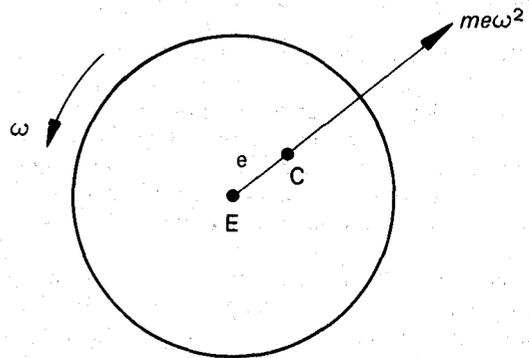


Figure 1 — Centrifugal force acting on an elementary slice of a shaft rotating about its mid-point

If the shaft starts to rotate about the shaft axis with an angular velocity  $\omega$ , the thin slice starts to rotate in its own plane with speed  $\omega$  about an axis through E. A centrifugal force  $me\omega^2$  is thus experienced by the slice. This force is transverse to the shaft axis and may be accompanied at other cross sections along the shaft by similar forces which are likely to vary in magnitude and direction along the shaft. These forces cause the shaft to bend and the deflection modifies the resultant forces experienced by the shaft.

Satisfactory operation of the shaft can be specified in terms of one of the following :

- a) vibration induced by the unbalance forces;
- b) limits on the resultant forces applied by the shaft to the bearings;
- c) residual unbalance.

In all cases in which it is necessary to reduce the unbalance forces, this is usually achieved by attaching a suitable axial distribution of correction masses along the shaft. It is not practical, and indeed not necessary, to balance the shaft exactly (that is to make  $e$  zero at all cross-sections along the shaft), so that there will invariably be some residual unbalance distributed along the shaft.

## 4.2 Unbalance distribution

Apart from any special design features, the axial distribution of unbalance along the rotor is likely to be random. The distribution may be significantly influenced by the presence of large local unbalances arising from shrink-fitted discs, couplings, etc.

The method of construction can significantly influence the magnitude and distribution of unbalance along the rotor.

Rotors may be machined from a single forging or they may be constructed by fitting several components together. For example, jet engine rotors are constructed by joining many shell and disc components, whereas alternator rotors are usually manufactured from a single piece of material, though they may still have additional components fitted.

Since the unbalance distribution along a rotor is likely to be random, the unbalance distribution along two nominally identical rotors may be similar, but they will rarely be identical. Indeed, significant differences in initial unbalance and residual unbalance are common in otherwise identical rotors. The distribution of unbalance is of greater significance in a flexible rotor than in a rigid rotor because it determines the degree to which any flexural mode of vibration is excited. Moreover, the magnitude of the unbalance force at any point along the rotor depends on the bending deflection of the rotor at that point.

The correction of unbalance in axial planes along the rotor other than those in which the unbalance occurs may induce vibrations at speeds other than that at which the rotor was originally corrected. In many circumstances the vibrations may exceed specified tolerances, particularly at critical speeds.

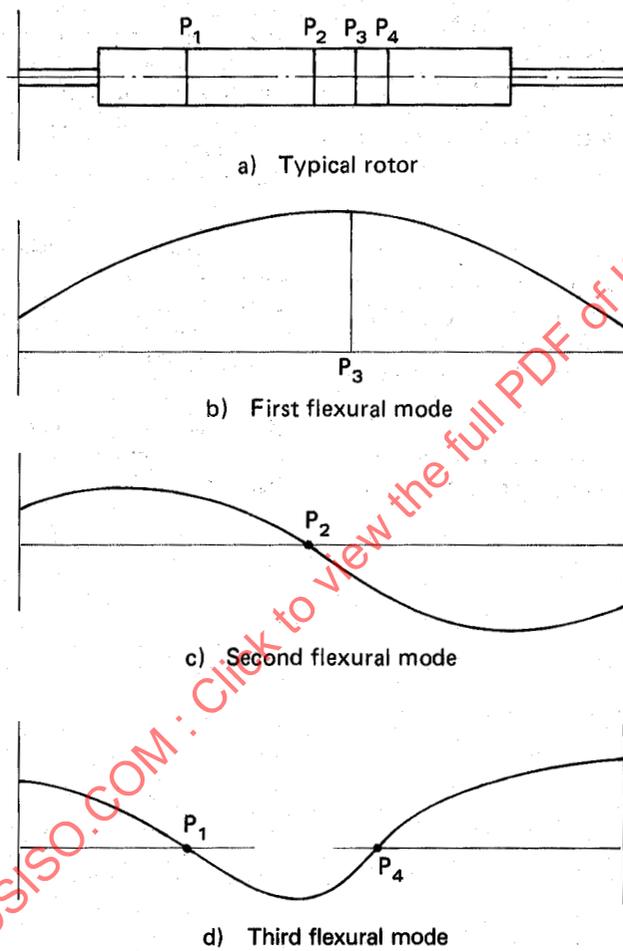
Rotors which become heated during operation are susceptible to thermal distortions which can lead to variations in the unbalance.

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**4.3 Flexible rotor mode shapes**

If damping is neglected, the modes of a rotor are the flexural principal modes and, for a rotor supported in "isotropic" bear-

ings, are plane curves rotating about the shaft axis. Typical curves for the three lowest principal modes for a simple rotor supported in flexible bearings near to its ends are illustrated in figure 2.



**Figure 2 – Typical mode shapes for flexible rotors on flexible supports**

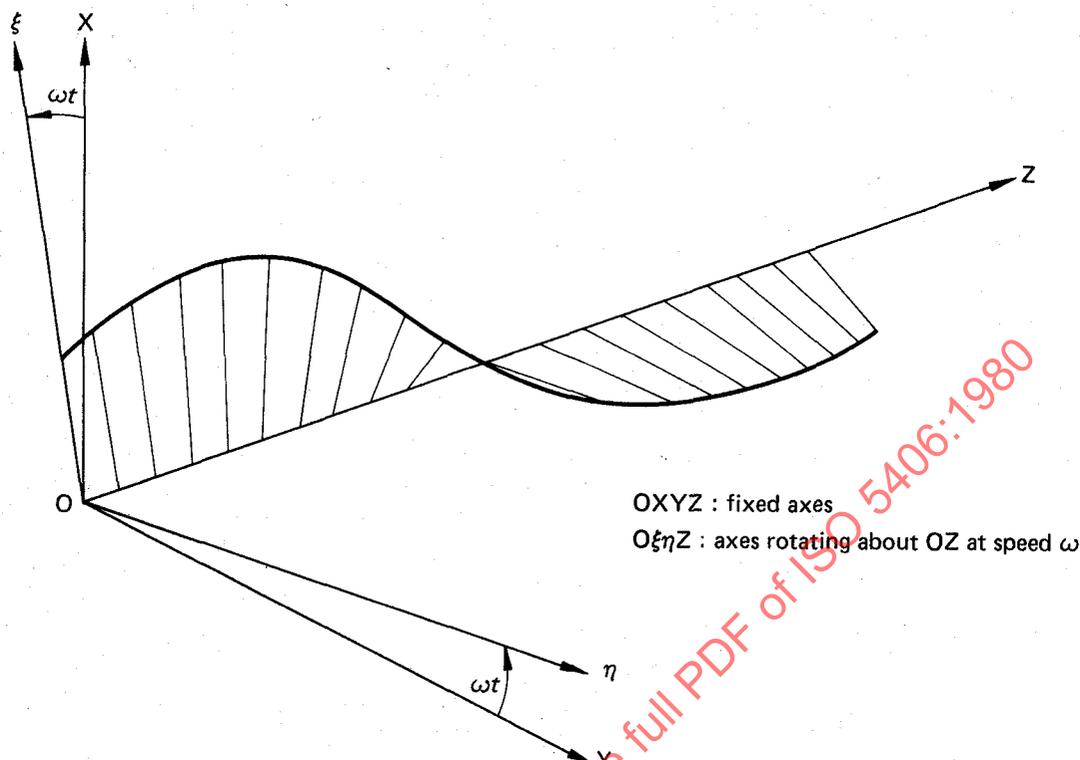


Figure 3 — Possible damped second mode shapes

For a damped rotor bearing system the flexural modes may be space curves rotating about the shaft axis, especially in the case of substantial damping, arising perhaps from fluid-film bearings. A possible substantially damped second mode is illustrated in figure 3. In many cases the damped modes can be treated approximately as principal modes and hence regarded as rotating plane curves. It must be stressed that the form of the mode shapes and the response of the rotor are strongly influenced by the dynamic properties and axial locations of the bearings and their supports.

#### 4.4 Response of a flexible rotor to unbalance

The unbalance distribution can be expressed in terms of modal components and the deflection in each mode is caused by the corresponding modal component of unbalance. Moreover, the response of the rotor in the vicinity of a critical speed is usually predominantly in the associated mode. The rotor modal response is a maximum at the rotor critical speed corresponding to that mode. Thus, when a rotor rotates at a speed near to a critical speed, it is disposed to adopt a deflection shape corresponding to the mode associated with this critical speed. The degree to which large amplitudes of rotor deflection occur in these circumstances is determined both by the component of unbalance in the mode in question and by the amount of damping experienced by the rotor system in this condition.

If the component of unbalance in a particular mode is reduced by a number of discrete masses, then the corresponding modal component of deflection is similarly reduced. The reduction of the modal components in this way forms the basis of two of the balancing procedures described in annex A.

If the rotor has a speed close to its first flexural critical value, then the deflection shape of the rotor tends to approximate to that shown in figure 2 b). Similarly, the deflection shapes of the rotor when rotating at speeds in the vicinity of its second or third critical speeds resemble those shown in figure 2 c) and 2 d). Similar comments apply to the higher modes.

Principal modes of the type shown in figures 2 b) to 2 d) determine the modal components of unbalance. Moreover the balancing effect produced by a given correction in a particular mode depends on the ordinate on the mode shape curve at the axial location of the correction. Thus a balancing mass attached to the rotor in figure 2 a) in the plane  $P_2$  will produce no change in the response in the second mode. Similarly a correction mass attached in either  $P_1$  or  $P_4$  will not affect the response in the third mode. Conversely, a balancing mass in plane  $P_3$  will produce the maximum effect on the first mode. If the rotor-bearing system has substantial damping, the rotor deflection will form space curves, which are related to the damped mode shapes mentioned above. A typical deflection curve under such circumstances for speeds near the second critical speed would resemble that shown in figure 3.

#### 4.5 Objectives of flexible rotor balancing

It has already been observed that it is not practical to balance a rotor exactly, that is, to ensure that the offset  $e$  is zero at all cross-sections along the rotor. Indeed, the aims of balancing are many and are primarily determined by the operational requirements of the machine. Before balancing any particular rotor it is desirable to decide what criteria of balance can be

regarded as adequate. In this way the balancing process can be made efficient and economic and still satisfy the needs of the user.

Balancing in general is usually a process whereby rotor vibration or bearing forces are reduced to within appropriate specified tolerances. For some applications it is only necessary to balance rotors at one speed, but in many cases the vibrations or oscillatory forces due to unbalance must be reduced to low levels over a range of speed, including several critical speeds.

It should also be remembered that the eventual aim of balancing is to ensure satisfactory running of the rotor in its operating environment and not only in the balancing facility. To this end it may be desirable to simulate service support conditions in specifying bearings for the balancing facility. Thus the bearings and pedestals used for balancing should reproduce to the necessary extent the mass and stiffness of the service bearings.

#### 4.6 Provision for correction planes

Correction masses are attached to a rotor to counteract the effect of an initial lack of balance. Although the unbalance invariably has a random distribution along the rotor, the correction masses are discrete in magnitude, in axial location along the rotor, and in angular location around the rotor. Rotors are often balanced in a modal sequence and in this process correction masses are situated along the rotor so that at each stage in the balancing procedure the new correction masses do not disturb modes already balanced (see annex A). The exact number of axial locations along the rotor that are needed for this process depends to some extent on the particular balancing procedure which is adopted. Generally, however, if the operating speed of the rotor exceeds its  $n^{\text{th}}$  critical speed, then at least  $(n + 2)$  correction planes (transverse to the rotor axis) are likely to be needed along the rotor.

In practice the number of axial locations that are available for use as correction planes is often limited by design considerations (and in field balancing by limitations on accessibility). An adequate number of correction planes should be included at the design stage. For turbine rotors, usually two end planes and a mid-span plane are available. For generator rotors, a minimum of two end planes and a mid-span plane have customarily been available in the balancing facility. For larger machines (with more flexible rotors and more critical speeds below the maximum operating speed), two additional planes or multiple planes have been used by some manufacturers. Centrifugal compressor rotors are usually assembly-balanced in the end planes only after each disc and the shaft have been separately balanced in a low speed balancing machine. With such restrictions, considerable ingenuity is often required from the balancing engineer.

#### 4.7 Rotors coupled together

When assessing coupled rotors, the nomenclature of the critical speeds requires some clarification for the following reason. Consider two rotors. Each rotor has a series of critical speeds and mode shapes that are usually different from those

of the other rotor. When the two rotors are coupled together, the complete unit will also have a series of critical speeds and mode shapes. However, these speeds are neither equal to nor simply related to the critical speeds of the uncoupled rotors. Moreover, the deflection shape of each part of the coupled unit, when vibrating in one of the coupled principal modes, need not be simply related to any mode shape of the corresponding uncoupled rotor. In theory, therefore, the unbalance distribution along two or more coupled rotors should be treated in terms of modal components with respect to the coupled system and not to the modes of the uncoupled rotors.

In practice, however, it is desirable for simplicity of production processes that each rotor should be balanced separately as an uncoupled shaft. Although no simple general indications can be formulated, it is often possible to make approximate comparisons between the coupled and uncoupled mode shapes and critical speeds, and in most cases such an approximate is adequate to ensure satisfactory operation of the coupled rotors. The degree to which this simple technique is practicable depends on the mode shapes and the critical speeds of the uncoupled and coupled rotors, the stiffness of the coupling and coupling shaft sections, the distribution of unbalance (which is not known) and the unbalance and especially the machining errors in the coupling assembly. The success of the technique is assisted if the coupling is flexible. It must, however be emphasized that, strictly speaking, each rotor may only be considered separately for balancing purposes provided that, when forming part of the coupled system, its modal deflection shapes do not differ significantly from its uncoupled mode shapes.

On the other hand, balancing a single-span rotor according to its mode shapes is not an aim in itself. If modal balancing techniques are used, the final goal is to gain information, as accurately as possible, about the unbalance and its distribution along the rotor, and as far as possible to correct the unbalance over the speed range. If this goal is reached it is not necessary that the modal shapes or natural frequencies should be the same when balancing and when the rotor is running in situ.

When two rotors, each separately supported in its own bearings, are coupled together, provided the coupling does not form a significant overhung mass on either rotor by comparison with the rotor mass, it is probable that each rotor may be balanced separately as an independent rotor.

### 5 Classification

**5.1** For the purposes of this International Standard, rotors are divided into five main classes as shown below and in the table. Each class requires different balancing techniques.

Class 1 — A rotor whose unbalance can be corrected in two (arbitrarily selected) planes so that, after the correction, its unbalance does not change significantly at any speed up to the maximum service speed. Rotors of this type can be corrected by rigid rotor balancing methods.<sup>1)</sup>

Class 2 — A rotor that cannot be considered rigid but that can be balanced using modified rigid rotor balancing techniques.

1) Recommendations for the balancing of rigid rotors are given in ISO 1940.

**Class 3** — A rotor that cannot be balanced using modified rigid rotor balancing techniques but instead requires the use of high speed balancing methods.

**Class 4** — A rotor that could fall into classes 1, 2 or 3 but has in addition one or more components that are themselves flexible or flexibly attached.

**Class 5** — A rotor that could fall into class 3 but for some reason, for example economy, is balanced for one speed of operation only.

**NOTE** — The number of modes that are considered in the balancing operation is not necessarily an indication of the number of critical speeds through which a rotor passes as it is run up to maximum service speed.

**5.2** Class 2 rotors are subdivided (see the table) into :

a) rotors in which the axial distribution of unbalance is known (classes 2a, 2b, 2c and 2d; also class 2e in which the axial distribution is partly known);

b) rotors in which the axial distribution of unbalance is not known (classes 2f, 2g and 2h).

The subdivision of class 2 rotors shows the many reasons why rotors can often be balanced satisfactorily at low speed as rigid rotors even though they are flexible. Some rotors will fit into more than one category of the subdivision.

**5.3** Class 3 is sub-divided (see the table) because the balancing techniques, criteria and bearing requirements may differ substantially for different rotors.

**5.4** A sub-division of class 4 rotors is indicated in 7.4.

## 6 Factors governing the classification of class 2 rotors

### 6.1 General

A low speed balancing machine considers only the static and couple unbalances in a rotor and does not evaluate the effect of deflection due to modal components of unbalance. Some rotors that are balanced in a low speed machine may therefore vibrate excessively both when running through critical speeds and at service speed. However, it is possible in some circumstances to balance a rotor in a low speed machine so that not only are the static and couple unbalances cancelled but also the remaining modal unbalances are sufficiently small to ensure satisfactory running when the rotor is installed in its final environment.

The amount of modal unbalance remaining in a rotor after the static and couple unbalances are corrected will depend partially on the modal shapes of the rotor and the axial positions of the unbalances relative to the correction planes used.

To assess to what extent it is likely that low speed balancing will be successful, it is necessary to consider the following factors.

### 6.2 Mass distribution of the rotor

No general rule can be laid down regarding the mass distribution of the rotor except that, if the axial positions of the unbalances are known, the balancing planes should be provided in the most suitable axial positions to cancel the effect of the unbalances.

### 6.3 Rotors made up of individual components

When a rotor is composed of separate components that are distributed axially and mounted concentrically on a shaft, it would considerably increase the probability of a satisfactory balance if one or both of the following methods of manufacture were adopted :

a) Each component and the shaft should be individually balanced as a rigid rotor to specified tolerances before assembly. In addition, the concentricities of the shaft diameters or other locating features that position the individual components on the shaft should be held to close tolerances relative to the shaft axis.

The concentricities of the mandrel diameters or other locating features that position each individual component on the mandrel should likewise be held within close tolerances relative to the shaft axis of the mandrel. Mandrel concentricity may be checked by turning the workpiece on the mandrel through 180°.

When balancing the components of the shaft individually, due allowance should be made for any unsymmetrical feature (such as keys) that form part of the complete rotor but may not be used in the individual balancing of the separate components.

It is advisable to check by calculation the unbalance produced by the eccentricities for the minimum practicable manufacturing tolerances.

When calculating the effect of the eccentricities of these location features on the mandrel and on the shaft, it is important to note that the effect of the eccentricities can be cumulative on the final assembly. If there are many effects to be taken into account, some statistical approach may be suitable. It may be found that the correction which may finally be necessary to compensate for these probable eccentricities in the mounting is large compared to the correction that is likely to be required in the component itself.

In such cases, the pre-balancing of the component on a separate mandrel is of relatively little value and it may then be considered preferable to proceed by the method described in b) below.

b) The shaft should first be balanced. The rotor should then be balanced as each component is mounted, correction being made only on the latest component added. This method avoids the necessity for such close control of the concentricities of the locating diameters or other features that position the individual components on the shaft.

It is important, if this method is adopted, to ensure that the balance of the parts of the rotor already treated is not changed by the addition of successive components.

Table — Classification of rotors

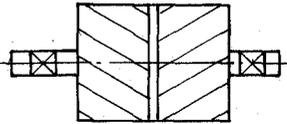
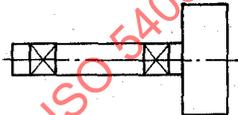
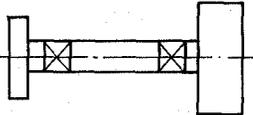
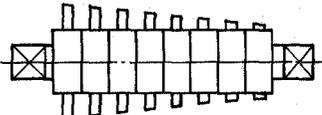
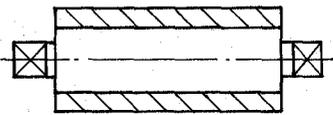
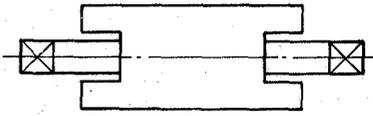
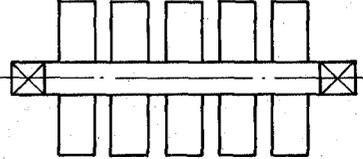
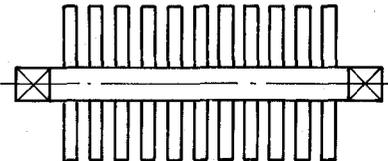
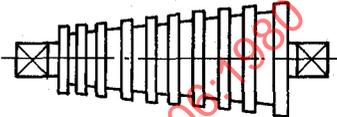
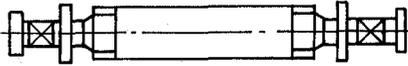
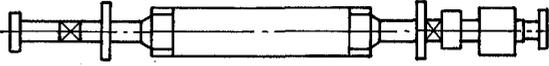
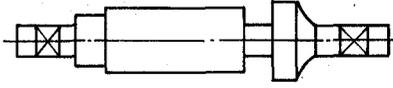
Class of rotor	Description	Example
<b>Class 1</b> (rigid rotors)	A rotor whose unbalance can be corrected in any two (arbitrarily selected) planes so that, after that correction, its unbalance does not change significantly at any speed up to maximum service speed.	 Gear wheel
<b>Class 2</b> (quasi-rigid rotor)	<i>A rotor that cannot be considered rigid but that can be balanced using modified rigid rotor balancing techniques.</i>	—
<b>Rotors in which the axial distribution of unbalance is known</b>		
<b>Class 2a</b>	A rotor with a single transverse plane of unbalance, for example a single mass on a light flexible shaft whose unbalance can be neglected.	 Grinding wheel
<b>Class 2b</b>	A rotor with two transverse planes of unbalance for example two masses on a light shaft whose unbalance can be neglected.	 Grinding wheel with pulley
<b>Class 2c</b>	A rotor with more than two transverse planes of unbalance.	 Compressor rotor
<b>Class 2d</b>	A rotor with uniformly or linearly varying unbalance.	 Printing press roller
<b>Class 2e</b>	A rotor consisting of a rigid mass of significant axial length supported by flexible shafts whose unbalance can be neglected.	 Computer memory drum
<b>Rotors in which the axial distribution of unbalance is not known</b>		
<b>Class 2f</b>	A symmetrical rotor with two end correction planes; whose maximum speed does not significantly approach second critical speed; whose service speed range does not contain first critical speed, and which has a controlled initial unbalance.	 Multi-stage centrifugal pump

Table — Classification of rotors (concluded)

Class of rotor	Description	Example
Class 2g	A symmetrical rotor with two end correction planes and a central correction plane; whose maximum speed does not significantly approach second critical speed and which has a controlled initial unbalance.	 <p data-bbox="1019 537 1286 559">High speed centrifugal pump</p>
Class 2h	An unsymmetrical rotor which has a controlled initial unbalance treated in a similar manner to a class 2f rotor.	 <p data-bbox="1025 754 1256 776">I.P. steam turbine rotor</p>
Class 3 (flexible rotors)	<i>A rotor that cannot be balanced using modified rigid rotor balancing techniques but instead requires the use of high speed balancing methods.</i>	
Class 3a	A rotor that, for any unbalance distribution, is significantly affected by only the first mode unbalance.	 <p data-bbox="1029 1075 1267 1097">Four pole generator rotor</p>
Class 3b	A rotor that, for any unbalance distribution, is significantly affected by only the first and second mode unbalance.	 <p data-bbox="1014 1225 1297 1247">Small two pole generator rotor</p>
Class 3c	A rotor significantly affected by more than the first and second mode unbalance.	 <p data-bbox="1010 1373 1297 1395">Large two pole generator rotor</p>
Class 4	A rotor that could fall into classes 1, 2 or 3 but has in addition one or more components that are themselves flexible or flexibly attached.	 <p data-bbox="1019 1550 1297 1572">Rotor with centrifugal switch</p>
Class 5	A rotor that could fall into class 3 but for some reason, for example economy, is balanced for one speed of operation only.	 <p data-bbox="1075 1727 1241 1749">High speed motor</p>

In some cases it may be possible to add more than one component at a time. Alternatively, in cases where several components form a short stiff unit or sub-assembly or core section of the complete rotor; it is permissible to treat this unit as a single component and to balance the rotor, after mounting the complete unit, by correction to the unit itself at or near its ends.

c) Where a rotor contains individual components that are mounted concentrically as a set (for example turbine blades, coupling bolts, pole pieces, etc.) it is desirable that these be carefully selected and assembled so that their resultant unbalance is within the specified tolerance.

NOTE — In the case of turbine blades, it may be necessary to consider the blade mass moment (mass  $\times$  radius) rather than its weight.

## 6.4 Service speed of the rotor

Preferably the speed or speeds at which the rotor is designed to operate should not be close to a flexural critical speed. How close such service speed can be to a critical speed will depend on the maximum acceptable level of vibration, the overall damping of the system and the accuracy with which the axial distribution of unbalance is known and controlled.

These factors can be assessed on the basis of experience with similar rotors or installations, during development testing of rotors, or by advanced statistical analysis methods.

If the service speed range includes a flexural critical speed, then class 2 balancing methods should only be used with caution.

It should be noted that a rotor that is balanced satisfactorily for a given service speed range may still experience excessive vibration if it has to run through a critical speed in order to reach this service speed.

## 6.5 Initial unbalance

As the process of balancing a flexible rotor in a low speed balancing machine is essentially a compromise, it follows that the initial unbalance is a major factor in predicting the degree of final modal unbalance that can be expected.

The maximum initial unbalance that can be tolerated will be dependent on the allowable bearing loads and the detailed characteristics of the rotor, and may be calculated using the methods described in annex C.

For example, if a rotor is manufactured from individual components that are treated by some method similar to that outlined in 6.3, then the axial distribution of unbalance of the assembled rotor can be kept under control and a higher initial unbalance can be tolerated. With the other rotors, the axial distribution of unbalance may be random and therefore not so well known.

In cases of random axial distribution that occur in rotors such as those for large steam turbines, it may happen that the sources of unbalance are close to the correction planes. It is then often found that considerable amounts of initial unbalance

can be corrected in a low speed balancing machine with satisfactory results at service speeds.

However, when the sources of unbalance are at some distance from the correction planes, it may be found that only very small amounts of initial unbalance can be corrected in this way.

It is outside the scope of this International Standard to compare the cost of high speed balancing with the cost of assembling to such close tolerances that the initial unbalance is sufficiently controlled to avoid all necessity for high speed balancing. Each case should be treated on its merits and it may be found expedient in some instances to assess the quality of the running of a rotor through its intended service speed range before deciding whether high speed balancing is necessary.

## 7 Balancing procedures

### 7.1 General

The ideal aim in balancing most flexible rotors is to correct the local unbalance occurring at each elemental length by means of balance corrections at the element itself. This will result in a rotor in which the centre of gravity of each elemental length lies on the shaft axis.

A rotor balanced in this ideal way will not only have no static and couple unbalance but will also have no modal components of unbalance. Such a perfectly balanced rotor will then run satisfactorily at all speeds insofar as unbalance is concerned.

Thus, in those cases where the axial locations of the unbalances in a rotor are known, any low speed balancing technique which ensures that each unbalance is corrected in its own transverse plane will be satisfactory. Such procedures are dealt with in 7.2.1 to 7.2.5.

However, where the axial locations of the unbalances are not known it is often possible to achieve an acceptable state of balance using a low speed balancing machine by controlling the initial unbalance of the rotor (i.e. before final balancing) during manufacture and assembly. These procedures are dealt with in 7.2.6, 7.2.7 and 7.2.8.

On the other hand, rotors that do not fulfil all the conditions for rigid or for class 2 flexible rotors as described in this International Standard may require methods of balancing other than those that are possible in a low speed balancing machine. These are dealt with in 7.3 to 7.5.

### 7.2 Balancing procedures for class 2 rotors

#### 7.2.1 Class 2a rotors with a single transverse plane of unbalance

If the initial unbalance is known to be wholly contained in one transverse plane and the balancing is also carried out in this plane, then the rotor will be balanced for all speeds.

In these circumstances, the unbalance can be corrected in a low speed balancing machine as effectively as at service speed, provided that the balancing machine has adequate sensitivity.

### 7.2.2 Class 2b rotors with two transverse planes of unbalance

If the initial unbalance is known to be contained in two transverse planes and the balancing is also carried out in these planes, then the shaft will be balanced for all speeds.

In these circumstances the unbalance can be measured and corrected in a low speed balancing machine as effectively as at service speed.

### 7.2.3 Class 2c rotors with more than two transverse planes of unbalance

When a rotor is composed of more than two separate components that are distributed axially, it is likely that there will be more than two transverse planes of unbalance. A satisfactory state of balance may however be achieved by balancing in a low speed balancing machine provided that the methods of manufacture and the precautions suggested in 6.3 b) are followed.

It is important to recognize that the assembly process may produce changes in the shaft run-out that may subsequently change during high speed service.

In some cases where a gross unbalance may occur in a single component it may be considered advantageous to balance this separately before mounting it on the rotor, in addition to carrying out the balancing procedure after it is mounted.

### 7.2.4 Class 2d rotors with uniformly distributed or linearly varying unbalance

If, because of design or method of construction, a rotor has unbalances that are distributed uniformly along the entire length of the rotor (for example a tube), it may be possible by a suitable axial disposition of two balancing planes to achieve satisfactory running over the entire speed range by balancing in a low speed balancing machine.

It is likely that the optimum disposition of the two balancing planes to give the best overall running conditions can only be determined by experimentation on a number of rotors of similar type. The method is based on the probability that rotors of such design will have a similar axial distribution of unbalance.

It is shown below and in annex E that for a simple rotor system (comprising a single span rotor with uniform or linear distribution of unbalance, no overhangs, uniform mass and flexibility, and operating speeds significantly below second critical speed) the optimum position for the two balancing planes is 22 % of the bearing span inboard of each bearing.

If rotors of this type have balancing planes in the middle and at their ends, it is possible to balance the rotors satisfactorily by three plane balancing which may be carried out in a low speed balancing machine.

To do this it is necessary to assess what proportion of the total initial unbalance is to be corrected at the centre plane. A

method of making this assessment is given in annex E for rotors that satisfy the following conditions :

- a) single span rotor with end bearings;
- b) uniform mass distribution with no overhangs;
- c) uniform bending flexibility of the shaft along its length;
- d) continuous service speeds below and not significantly approaching second critical speed.

For such rotors, annex E shows how the unbalance correction required at the centre plane can be found directly from the initial dynamic unbalance measured at the two end correction planes.

### 7.2.5 Class 2e rotors with a rigid core

If the unbalance in the rotor is known to be contained wholly within a substantially rigid section of the rotor and the balancing is carried out also within this section, then the unbalance will be zero in all modes.

Such a rotor that has a rigid core and flexibility derived solely from flexible shafts can be balanced on a low speed balancing machine provided the unbalance corrections are carried out at correction planes located within the rigid core section of the rotor.

### 7.2.6 Class 2f symmetrical rotors with controlled initial unbalance (two correction planes)

When a rotor is composed of separate components that are individually balanced before assembly as outlined in 6.3 a), a satisfactory state of balance may be achieved in a low speed balancing machine provided the initial unbalance of the completed rotor does not exceed specified tolerances.

For such rotors, the axial distribution and magnitude of the unbalance of the complete assembly will not be known. Since the maximum speed of this class of rotor does not significantly approach the second critical speed, the most unfavourable case that will occur with a given distribution of unbalance is when the individual contributions of the assembled components to the resultant unbalance have the same angular position. It is then possible to estimate the maximum initial unbalance of the assembly that may be corrected in two correction planes and that will result in satisfactory running conditions.

This type of analysis can therefore be carried out for any rotor whose initial unbalance consists of a distribution of small unbalances. The analysis does however require realistic data on shaft and bearing flexibilities.

Details of a possible approach are given in annex C.

### 7.2.7 Class 2g symmetrical rotors with controlled initial unbalance (three correction planes)

For rotors that conform to the requirements in 7.2.6 but that have in addition a third, central, correction plane, then, provided that it is possible to hold the initial unbalance of the com-

plete rotor to within twice the permissible initial unbalance arrived at in 7.2.6, the rotor may be balanced on a low speed balancing machine as a rigid rotor, correcting a portion of the initial unbalance at the central plane and the remainder at the two end planes.

NOTE — Experience has shown that between 30 and 60 % of the initial static unbalance should be located in the central plane.

### 7.2.8 Class 2h unsymmetrical rotors with controlled initial unbalance

For rotors that do not conform to the configuration defined in 7.2.6, for example as regards symmetry or overhangs, it may be possible to carry out a similar estimation to that given in annex C and hence to arrive at the maximum permissible unbalance that may be corrected satisfactorily at any given correction plane.

However, in extreme cases, the permissible initial unbalance arrived at in this way may be too small to make this method of balancing practicable and in these cases some other method of balancing the rotor will have to be adopted.

## 7.3 Balancing procedures for class 3 rotors

### 7.3.1 General

Various procedures can be applied to rotors in this class and three procedures are outlined for guidance. It is emphasized that only the principles of the techniques are described below and that they may require some refinement in practice. All three methods will balance a class 3 rotor but for a particular rotor one method may prove to be more advantageous than another.

Most of the possible methods of balancing class 3 rotors depend on the fact that the deflection of the rotor is the sum of the deflection components in its principal modes and that the distribution of the local centre of gravity displacement can be similarly expressed in terms of modal components. The vibration in each mode is caused by the corresponding modal component of unbalance. If the modal components of unbalance are corrected in the ways described it is possible to achieve smooth running of the rotor up to any desired service speed.

For balancing purposes, the rotor can be mounted on any suitable bearings. If, however, the bearings are chosen so that the support conditions simulate those on site, then it is possible that the degree of subsequent on-site balancing required will be significantly reduced.

Transducers are positioned to measure shaft, bearing or support vibration or bearing force, as appropriate to the application. The transducer should be capable of measuring the amplitude of the once-per-revolution component of the signal together with the phase angle relative to some fixed but arbitrarily-selected angular reference on the rotor. Alternatively, it is possible to use a measuring system which resolves the synchronous vibration into X and Y components.

It is desirable that the transducers should be of such a type that they do not undergo resonant vibration of their own or their suspension at any test speed.

If the rotor to be balanced has an overhang of significant mass that would normally be supported when installed on site, it may be desirable to provide an additional bearing to support it during the balancing operation.

### 7.3.2 Method 1 (modal balancing)

7.3.2.1 Mount the rotor in a suitable hard bearing balancing machine or similar high speed facility that reasonably imitates site foundations.

7.3.2.2 Run the rotor to some safe speed approaching first critical speed and note readings of bearing vibrations or forces.

7.3.2.3 Add a trial mass to the rotor. This can be added at any axial position along the rotor except at the nodes of the mode, but the maximum effect on first mode will usually be achieved if it is placed towards the middle of the rotor. Note readings of bearing vibrations or forces at exactly the same speed as in 7.3.2.2.

7.3.2.4 From the readings obtained in 7.3.2.2 and 7.3.2.3, compute and correct the unbalance in first mode using the construction given in annex D, adding the correction in the same axial position as the trial mass.

The rotor should now run at any speed up to and through first critical speed without any significant amplification of vibration.

7.3.2.5 Run the rotor to some safe speed approaching second critical speed, provided this is lower than the maximum balancing speed. Note readings of vibration or bearing forces.

NOTE — The maximum balancing speed is usually the maximum service speed of the rotor, but may lie in its overspeed range.

7.3.2.6 Add a pair of trial masses to the rotor to affect the unbalance in second mode. These should be positioned in radial planes 180° apart and proportioned so that the balance in first mode is not affected. Note readings of bearing vibrations or forces at exactly the same speed as in 7.3.2.5.

7.3.2.7 From readings obtained in 7.3.2.5 and 7.3.2.6, compute and correct the unbalance in second mode using a pair of correction masses which retain the proportionality and axial positions of the trial masses.

The rotor should now run at any speed up to and through first and second critical speeds without any significant amplification of vibration.

7.3.2.8 Continue the above operations for successive modes until the maximum balancing speed is attained. At each stage, choose correction masses so that they do not upset the balance already achieved in the lower modes.

7.3.2.9 If significant vibrations or bearing forces still occur at maximum balancing speed, they will be caused by unbalance in modes whose critical speeds are higher than that maximum speed. Correct these by the addition of suitable corrections which do not affect the lower modes. In this case, however, it

will not be possible to magnify the effect of unbalance in any of these higher modes by running close to their associated critical speeds. Some knowledge therefore of the shapes of the higher modes may be helpful in correcting these residual unbalances.

#### NOTES

1 At speeds near to a critical speed the effect of unbalance in the associated mode will be greatly magnified and so the vibrations or bearing forces will be largely caused by unbalance in the mode. If there is a significant interaction from modes other than the one being corrected, techniques for the separation of the modes are available.

2 Instead of bearing vibrations or forces, the shaft vibration or other suitable response data may be used.

### 7.3.3 Method 2 (combined rigid rotor and model balancing)

**7.3.3.1** Mount the rotor in a balancing machine, preferably of the hard-bearing type because these machines more closely simulate site conditions. This is particularly necessary for rotors requiring correction in more than two modes, for unsymmetrical rotors, and for rotors with heavy overhung masses.

**7.3.3.2** If only a soft bearing machine is available, it may be necessary to make trim balancing corrections in situ.

NOTE — The balancing speed should be lower than 50 % of the first flexural critical speed of the rotor.

**7.3.3.3** Balance the rotor at low speed, using the end correction planes for the couple unbalance, and, if possible, planes distributed over the rotor length for the static unbalance.

**7.3.3.4** Balance in first mode by the method indicated in 7.3.2.2, 7.3.2.3 and 7.3.2.4 but using a configuration of masses disposed in three balancing planes. The masses must be so chosen that the first modal component of unbalance is annulled and the rigid rotor balance is not upset. A suitable set of masses complying with these conditions can be computed from the rotor structure analysis data if such data is available for the complete rotor/bearing system, or it can be found experimentally by adding a known trial mass to the rotor as in 7.3.2.3. Then find the two unbalances in the end correction planes that compensate for its effect on the low speed balancing, as in 7.3.3.3. Such a set of three masses can therefore be used for annulling the first modal component of unbalance without upsetting the rigid rotor balance achieved in 7.3.3.3.

The rotor should now run at any speed up to and through the first critical speed without any significant amplification of vibration.

**7.3.3.5** Follow the steps specified in 7.3.2.5 to 7.3.2.9 but in each case the corrections must be chosen so that they correct the mode under consideration without affecting the lower modes and without affecting the rigid rotor balance.

Suitable sets of balance masses may again be found by computation or experimentally by extending the method described

in 7.3.3.4 so that the lower modes, already annulled, are not affected. Thus, after conclusion of balancing in the  $n^{\text{th}}$  mode, a single trial mass of known amount should be added to the rotor in a plane where it has maximum effect on the next higher mode, and then balancing up to the  $n^{\text{th}}$  mode should be repeated. This will result in the mass set needed for annulling the  $(n + 1)^{\text{th}}$  mode without upsetting either the rigid rotor balance or the modal balance up to the  $n^{\text{th}}$  mode.

The rotor should now run from low speed up to and through all  $(n + 1)$  critical speeds without any significant amplification of vibration.

**7.3.3.6** Final balancing at service speed may be added if necessary as described in 7.3.2.9 but in some cases the rigid rotor balancing at low speed may ensure sufficiently smooth running at service speed, provided all accessible modes of unbalance have been annulled.

### 7.3.4 Method 3 (influence coefficient matrix balancing)

**7.3.4.1** The influence coefficient matrix method is usually an automatic procedure under the control of a digital computer.

**7.3.4.2** The basic assumption of the method is that the rotor vibration at the measurement locations results from the rotor unbalance distribution which is considered to be represented as a number of discrete unbalances located in the selected correction planes. Thus the rotor response  $\{b\}$  is assumed to be related to this discrete unbalance distribution  $\{U\}$  by the influence coefficient matrix  $[A]$ , such that

$$\{b\} = [A] \{U\}$$

In this equation  $\{b\}$  is the vector of  $p$  rotor response measurements,  $[A]$  is the  $p \times q$  array of influence coefficients, and  $\{U\}$  is the vector of  $q$  rotor unbalance elements ( $q$  correction planes)<sup>1)</sup>. Each of the elements of  $\{b\}$ ,  $[A]$ , and  $\{U\}$  has associated with it a phase angle as well as a magnitude. In the case of  $\{b\}$  these are denoted by  $\lambda$  and  $b$  respectively, with appropriate suffixes. The rotor response data are measured directly; the influence coefficients are usually determined by placing a trial mass in each balancing plane in turn, and recording the change in rotor response due to the trial mass. Once the rotor response ( $b$ ,  $\lambda$ ) and influence coefficients ( $a_{ij}$ ) are known, the equation above may be solved for the rotor unbalance vector  $\{U\}$  or, equivalently, the balance correction  $\{C\} = -\{U\}$ . If the number of rotor response measurements  $n$  equals the number of correction planes  $q$ ,  $\{C\}$  can be chosen to make the resultant rotor response identically zero at the measuring locations and at the balancing speeds.

The procedure for determining the matrix  $[A]$  is described in 7.3.4.3 to 7.3.4.11 below and the underlying analysis is given in annex A. The matrix  $[A]$  may be influenced by the degree of randomness in the measurements taken. In more advanced influence coefficient methods, statistical procedures are used to overcome such problems.

1) Response data are usually taken at more than one rotor speed. Thus often  $p = m \times n$ , where  $m$  is the number of balancing speeds and  $n$  is the number of measuring locations.

**7.3.4.3** Install  $n$  transducers at suitable axial locations to measure shaft or bearing vibration ( $n$  transducers at  $m$  shaft speeds, so that  $p = m \times n$ ). Select also  $q$  prescribed correction planes. The minimum value of  $q$  depends on the objectives of the particular balance problem being handled.

**7.3.4.4** Select suitable balancing speeds  $\omega_1, \omega_2, \dots, \omega_m$ .

NOTE — It may be necessary to conduct a balancing operation at only the lower speeds before the higher speeds are attainable.

**7.3.4.5** Measure and record the vibration at each of the  $n$  transducers for each of the  $m$  speeds. Let the vibration at the  $i$ th transducer for the speed  $\omega_k$  be  $b_{io}^{(k)}$ .

**7.3.4.6** Attach a trial mass of known magnitude  $m_1$  at a known radius  $r_1$  in correction plane 1 at a known orientation angle  $\psi_1$  around the rotor from some angular reference location.

**7.3.4.7** Again measure and record the vibration at each of the  $n$  measuring planes for each of the  $m$  speeds. Let the vibration vector at the  $i$ th transducer for the speed  $\omega_k$  be  $b_{i1}^{(k)}$ .

**7.3.4.8** Remove the trial mass from the plane 1 and attach a trial mass  $m_j$  in correction plane  $j$  at a known radius  $r_j$  and a known angular position  $\psi_j$ .

**7.3.4.9** Measure and record the vibration at each of the  $n$  measuring planes for each of the  $m$  speeds. Let the vibration at the  $i$ th transducer for the speed  $\omega_k$  be  $b_{ij}^{(k)}$ .

**7.3.4.10** Perform the operations specified in 7.3.4.8 and 7.3.4.9 for  $j = 2, 3, \dots, q$ .

**7.3.4.11** Obtain the required balance corrections  $C_j$  and their angular locations  $\zeta_j$  using a procedure such as that described in annex A. This procedure usually involves the use of a digital computer.

**7.3.4.12** Usually, the number of response measurements does not equal the number of correction planes and  $\{C\}$  is chosen to minimize the rotor response after balancing with a least squares procedure.

**7.3.4.13** Because of the different amplification of the rotor modes at different speeds, it is often advisable to use more than one balancing speed. The number of transducers can sometimes be reduced if more balancing speeds are used.

**7.3.4.14** Care should be taken with the influence coefficient procedure if a trial run is necessary through a critical speed.

**7.3.4.15** The influence coefficient matrix may be nearly singular if the positions of the correction planes and transducers and the balancing speeds are not chosen sensibly.

**7.3.4.16** In each plane, the radius at which the correction mass is attached will normally be equal to the radius of the trial mass.

**7.3.4.17** To improve results, it is frequently advisable to use combinations of masses in suitable planes rather than a single trial mass, the mass combinations being similar to, but not necessarily equal to, the mass combinations recommended for the modal balancing techniques of methods 1 and 2 (see 7.3.2 and 7.3.3).

## 7.4 Balancing procedures for class 4 rotors

Rotors in this class may have a basic shaft and body construction that would fall into classes 1, 2, or 3. In addition they have one or more components that are either flexible or are flexibly mounted so that the balance of the whole system may change with change of speed.

Rotors in this class may fall into either of two categories :

- a) rotors whose balance changes continuously with change of speed, for example rubber bladed fans;
- b) rotors whose balance changes up to a certain speed and remains constant above that speed, for example rotors of single phase induction motors that carry a centrifugal starting switch.

It is sometimes possible to balance these rotors with counterbalances of similar characteristics. If not, the following procedures should be used.

Rotors that fall into category a) above should be balanced in a balancing machine at the speed at which it is specified that the rotor should be in balance.

Rotors that fall into category b) above may be balanced at any speed above that at which the balance ceases to change.

NOTE — It may be possible to minimize or counterbalance the effects of the flexible components by careful design and by attention to their locations, but it should be appreciated that rotors in this class are likely to be in balance at one speed only or within a limited range of speed.

## 7.5 Balancing procedures for class 5 rotors

Some rotors that are flexible and pass through one or more critical speeds on their way up to full speed may sometimes be required to be in balance for one speed only (usually service speed). In general, rotors that fall into this class fulfil one or more of the following conditions :

- a) The acceleration and deceleration up to and from full speed is so rapid that the amplitude of vibration at the critical speeds will not build up beyond acceptable limits.
- b) The damping of the system is sufficiently high to suppress vibrations at the critical speeds to acceptable limits.
- c) The rotor is supported in a sufficiently elastic environment to prevent serious vibrations being transmitted.
- d) A high level of vibration at the critical speeds is acceptable.
- e) A rotor runs at full speed for such long periods that otherwise unacceptable starting conditions can be accepted in this case.

A rotor that fulfils any of the conditions above should be balanced in a high speed balancing machine or equivalent facility at the speed at which it is determined that the rotor should be in balance.

If the rotor falls into category c) above, it is especially important that the stiffness of the balancing machine bearings should imitate site conditions sufficiently closely to ensure that at the balancing speed the predominant modes are the same as those that will be experienced on site.

Some consideration should be given to the axial balance weight distribution. Based on the probability that similar rotors will have similar unbalance distributions, it may be possible to choose optimum axial positions for the balancing planes and two planes may be sufficient. This may produce a minimum residual unbalance in the lower modes and thus minimize the vibrations when running through critical speeds.

## 8 Evaluation of final state of unbalance

The purpose of this clause is to specify the environmental conditions and methods of test for the evaluation of the final state of unbalance of a rotor.

### 8.1 General

Rotors fitted to machines or engines in quantity production where, as is usually the case, the product undergoes some development or proving tests before the production run, will in general have the methods and standard of balance checked and evaluated during the development stage.

Such rotors will have unbalance tolerances set initially on past experience. During the development stage it will become apparent whether these tolerances provide for a satisfactory level of vibration in the final product or whether some modifications are required, either in the method of balancing or in the unbalance tolerances, or indeed in the basic design.

After the development stage is completed, rotors manufactured on the production run would be expected to behave satisfactorily in service if they are balanced to the tolerances established during development.

It may be possible for the final state of unbalance of such rotors to be evaluated in a low speed balancing machine and for limits of unbalance to be specified in terms of unbalance remaining in the balancing planes. Where applicable, limits of initial unbalance may also be specified.

In other cases, for example, large generator rotors, prior development experience of the rotor in its final environment may not be possible. Thus a specification of unbalance remaining in the balancing planes can at present have little meaning as the factors that determine the effect of unbalance are not known.

In these cases, it is therefore necessary to specify limits of vibration resulting from residual unbalance and, at the same time, to specify the environmental conditions under which the evaluation is made. In special cases the environmental conditions will be agreed between the manufacturer and the purchaser.

However, many rotors, for example those which are mass produced, would normally be submitted to much simpler assessment procedure than that detailed below.

### 8.2 Evaluation of unbalance

Evaluation of the final state of unbalance may be carried out in one or more of the following situations, depending on the type and purpose of the rotor being assessed :

- a) in a low speed balancing machine;
- b) in a high speed balancing machine or high speed balancing facility;
- c) on a test bed as an assembled machine;
- d) at site in its final assembled condition.

### 8.3 Rotors whose final state of unbalance is evaluated in a low speed balancing machine

Class 2 rotors that are in quantity production have their balance quality assessed in a low speed balancing machine. In most cases a subsequent high speed check will be made on the test bed or on site. In specific cases, by agreement between the manufacturer and the customer, the high speed assessment may be dispensed with and the rotor accepted on the basis of the final low speed balance. This can apply particularly to class 2 rotors sold as spares where a final assessment at site may be delayed for a considerable time.

The rotor should be complete and all attachments such as half couplings, gear wheels, etc. should be fitted.

If the rotor has an overhung mass that if flexibly coupled and that would normally be supported on site, a steady bearing may at the discretion of the manufacturer be used to support it in the balancing machine. The object of the support is to ensure that the appropriate mass is being carried by the workpiece and it is important to ensure that the method of support does not introduce any questionable unbalance readings.

The balancing machine should be one that conforms to International Standard ISO 2953.

#### NOTES

1 If the journal supported by the steady bearing is offset due to a permanent bend in the shaft, unbalance simulating forces may be set up. This could lead to an error in the final balance of the rotor.

2 If required by the customer, the operation and sensitivity of the balancing machine may be demonstrated by adding test weights to the rotor equivalent to five times the claimed residual unbalance. Alternatively the full procedure described in ISO 2953 may be used if a more exhaustive test is felt to be needed.

Before the balance of the rotor is assessed, it should be run at some suitable speed to remove any temporary bend it may have taken.

When the above conditions have been satisfied, the rotor should be run at the balancing speed and readings taken of amount and angle of unbalance remaining in each measuring

plane, except in the case of class 2a rotors where only one measuring plane is used.

For class 2f and 2g rotors, the initial unbalance should also be stated.

In addition, for rotors that have been balanced in stages or that have been made up of balanced components, the degree of balance achieved at each stage should be stated.

#### 8.4 Rotors whose final state of unbalance is evaluated by means of vibration measurement in a high speed balancing and overspeed facility

The purpose of this test is to ensure that, as far as possible, the balance of the rotor is good enough to produce smooth running when it is finally installed on site.

##### 8.4.1 Installation

In cases where the modal shapes of the rotor depend significantly on the dynamic properties of the supports, the rotor should have supports whose dynamic properties as nearly as possible represent those of site conditions.

When special purpose balancing machine pedestals that have variable resonances are used, they must be locked in one particular state of adjustment throughout the test.

If the rotor has an overhung mass that would normally be supported when installed on site, a steady bearing may be used to support it during the test. In some cases it may be acceptable to measure the unbalance without having the overhung mass fitted and to measure the balance of this separately on a suitable mandrel.

If the rotor has an overhung mass that is not supported in any way when installed on site, it must be left unsupported during the test.

Two vibration transducers shall be installed on each bearing pedestal 90° apart on the same transverse plane to permit resolution of the vertical and horizontal transverse vibrations.

If isotropic bearings are used, the transducers may be placed to sense the transverse vibrations in any radial direction.

As an alternative or additionally, shaft-riding probes or non-contact proximity transducers may be used to measure journal vibrations. These must be positioned as close as possible to the bearing insert and placed to sense vertical and horizontal transverse vibrations. Their output should be read on equipment that is able to differentiate between ambient effects, slow-roll runout, and the vibrations due to unbalance.

In all cases, there should be no resonances of the transducer mountings within the speed range of the test.

The output from all transducers should be read on equipment that is able to differentiate between the synchronous component and other components of the vibration.

The drive for the rotor should be such as to impose negligible restraint on the vibration of the rotor and introduce negligible unbalance into the system.

NOTE — To establish that the coupling is introducing no unwanted unbalance forces, the coupling should be turned through 180° in relation to the rotor and the effect noted, as suggested in ISO 1940.

Before the balance of the rotor is assessed, it should be run at some convenient low speed or speeds to remove any temporary bend it may have taken.

##### 8.4.2 Procedure

When the above conditions have been satisfied, the rotor should be run up to speed at an acceleration that ensures that no peaks of vibration are suppressed. All significant peaks of vibration should be measured between 70 % of the observed first critical speed and maximum service speed.

The rotor should be held at maximum service speed for at least two minutes and vibration measurements should be taken of synchronous vibration.

At the conclusion of the test at maximum service speed the rotor should be run to a specified overspeed, if demanded by the specification.

##### NOTES

- 1 The overspeed to be attained will depend upon the overall test specification for the particular type of rotor. Where no specification exists, agreement should be reached between the manufacturer and the purchaser.
- 2 In some cases, final overspeeding is forbidden.

After the rotor has been held for a specified time at full overspeed, if overspeed is demanded by the specification, it should be decelerated to maximum service speed and the synchronous vibrations again measured.

NOTE — If significant changes of balance occur or are expected to occur due to overspeeding, the final balancing and evaluation of vibration should be carried out after overspeeding.

The rotor should then be decelerated and measurements of synchronous vibration should be taken at decrements of speed of not more than 5 % of full speed between the maximum service speed and 70 % of the first critical speed. In addition, all peaks of total vibration should be noted. During the test the rate of deceleration should not be greater than the rate of acceleration.

If desired, the above tests may be taken only to full speed and an initial run made for the full overspeed assessment. If this method is adopted then the first run should be as follows :

- a) the rotor should be accelerated up to full speed with any convenient acceleration;
- b) the rotor should be held at full speed until all readings have become reasonably steady and vibration readings have been recorded;

- c) at the end of the two minute run, speed should be increased to full overspeed for the specified period of time;
- d) at the conclusion of c), the rotor should be brought down in speed at any convenient deceleration.

### 8.5 Rotors whose final state of unbalance is evaluated on the test bed

Rotors whose final state of unbalance is evaluated on the test bed should have an instrumentation and test procedure as stated above but it should be appreciated that different procedures may be necessary in some cases, for example :

- a) the rotor may be assembled as a complete machine driven by its own power, and may therefore not be coupled to a prime mover;
- b) it may be advantageous to couple the rotor rigidly to a prime mover in order to simulate site conditions more closely;
- c) if the rotor is, for example, an induction motor, it may be possible to measure vibration over a speed range and only full speed readings can be obtained;
- d) a complete machine, it may not always be possible to place vibration transducers at the bearings. In these cases, the points where the vibrations should be measured should be agreed between the manufacturer and the customer;
- e) if the state of balance may be dependent on load, the range of load over which the balance is assessed should be agreed between the manufacturer and the customer.

### 8.6 Rotors whose final state of unbalance is evaluated at high speed as unbalance remaining in specified correction planes

In some cases, facilities that permit rotors to be run at high speeds will be used to assess balance in terms of vibration. However, in other cases, it may be desirable to specify unbalance remaining in specified correction planes. It should be appreciated that the specification of the levels of balance to be achieved must rely heavily on experience of the ultimate performance of similar rotors that have been balanced in a similar manner, using the same axial positions for the correction planes.

By agreement between the manufacturer and the customer, rotors in any class may be assessed in this manner.

The axial position of the correction planes and the balancing speed should be stated for each stage of the balance check, i.e. for each mode.

If the rotor is assessed in a balancing machine having its own instrumentation this should be used throughout the test.

If the rotor is assessed in an overspeed or similarly facility, the instrumentation and general installation of the rotor into the facility should be as stated in 8.4.

When the above conditions have been met, the rotor should be run at each balancing speed in turn and readings taken of the amplitude and angular position of the unbalance remaining in each correction plane relevant to each speed.

If requested by the customer the rotor may be checked by a similar method to that referred to in 8.3. The check should be carried out for each balancing speed.

### 8.7 Rotors whose final state of unbalance is evaluated at site

8.7.1 Machines that have their balance quality evaluated after final installation at site are subject to many disturbing factors that can produce vibration. Some of this vibration may be at fundamental frequency and can therefore be mistaken for unbalance. This is particularly true in installations where two or more flexible rotors are coupled together.

Some of the factors that can produce such vibrations, together with some of the precautions that should be taken, are mentioned in annex B.

It should also be appreciated that in the final installation at site, there may be many factors during commissioning that will conflict with the steady state conditions needed to assess the balance quality. It may be necessary, however, to combine the balance quality runs with those for other purposes. If the preliminary running of the installation shows the balance quality to be in doubt, special runs may be arranged specifically for the purpose of checking this point.

8.7.2 In many installations, for example where the prime mover is a "direct to line start" induction motor, it may be impossible to control the speed of rotation during run up and stable conditions can only be achieved at full speed. Agreement should therefore be reached between the manufacturer and the purchaser on the speed range over which the balance quality should be checked.

8.7.3 The balance check is normally made with the machine unloaded. If the machine is loaded, the load at which the balance quality is to be checked should be agreed between the manufacturer and the purchaser.

8.7.4 Vibration transducers should be mounted on the bearing supports at the following points :

- a) on the centreline of the top of the bearing bush placed at the axial centre of the bearing bush and positioned to sense the vertical component of the vibration;
- b) on the centreline at the side of the bearing bush and positioned to sense the horizontal transverse component of the vibration.

8.7.5 As an alternative, shaft riding probes or non-contact proximity transducers may be used to measure journal vibrations. These should be positioned as close as possible to the bearing bush and placed to sense vertical and horizontal transverse vibrations. Their output should be read on equipment that is able to differentiate between ambient effects, slow-roll runout, and the vibrations due to unbalance.

**8.7.6** In all cases there should be no resonance of the transducer mountings within the running range of the test.

**8.7.7** The output from the vibration transducers may be read on equipment that is able to differentiate between the fundamental component and the harmonics of the vibration.

**8.7.8** As an alternative to 8.6.4 or 8.6.5, vibrations may be read on portable apparatus using a hand-held vibrations transducer. The points chosen for the vibration readings should be the same as for the fixed transducers.

**8.7.9** As a further alternative where supervisory equipment is provided in the installation, this may be used instead, provided it is fully commissioned and steps have been taken to ensure that the vibration in the worst direction can be determined from the readings.

**8.7.10** If, as a result of the test using the supervisory equipment, the balance quality is in doubt, than a further test should be carried out using the equipment specified in 8.6.4 to 8.6.8.

## 9 Balance quality criteria

The balance quality for rigid rotors is frequently specified in terms of the displacement of the centre of gravity from the axis of rotation as a function of the speed. This method is adopted

in ISO 1940. Except in a few cases, the method is at present not directly applicable to flexible rotors. In general, therefore, the balance quality for flexible rotors is specified in terms of vibration of the journals or bearing pedestals. The balance quality for flexible rotors will most probably be specified as vibrations permissible at specified measuring points or unbalance remaining in specified balancing planes.

The maximum levels of vibration that are considered satisfactory for a particular rotor are normally stated in the product specification for the machine type, and these should be referred to, as applicable. If no such specification exists, agreement should be reached between the manufacturer and the customer on maximum permitted levels. In all cases, however, the methods and conditions under which the vibration readings are obtained should be in accordance with the appropriate part of clause 8.

It should be remembered that, in most product specifications where maximum levels of vibrations are stated, they refer to total vibration arising from all sources. The level quoted could therefore include the r.m.s. value of a multiplicity of frequencies and the manufacturer should consider what levels of vibration can be permitted from unbalance alone in order to keep within the limits of total vibration.

The balance quality criteria for flexible rotors that have so far been developed will be given in an addendum to this International Standard.

## Annex A

### Theory of the influence coefficient matrix method

The required  $p$  vibration amplitudes  $b_{ij}^{(k)}$  and vibration phase angles  $\lambda_{ij}^{(k)}$  are obtained by the procedure described in 7.3.4. These are stored in the  $\{b\}$  vector in accordance with the relations :

$$b_{ij} = r_{ij} + i S_{ij}$$

where

$$r_{ij} = b_{ij} \cos \lambda_{ij}^{(k)}$$

$$S_{ij} = b_{ij} \sin \lambda_{ij}^{(k)}$$

and

$$i = \sqrt{-1}$$

The influence coefficients are computed from the expression :

$$a_{ij}^{(k)} = \frac{b_{ij}^{(k)} - b_{i0}^{(k)}}{m_j r_j \exp(i \psi_j)}$$

The influence coefficient matrix  $[A]$  is formed from the above results for  $a_{ij}^{(k)}$  in accordance with the sequence described in 7.3.4. This has the form :

(1)	(1)	(1)
$a$	$a \dots \dots a$	
11	12	1 $q$
(1)	(1)	(1)
$a$	$a \dots \dots a$	
21	22	2 $q$
.	.	.
(1)	(1)	(1)
$a$	$a \dots \dots a$	
$n1$	$n2$	$nq$
(2)	(2)	(2)
$a$	$a \dots \dots a$	
11	12	1 $q$
.	.	.
(2)	(2)	(2)
$a$	$a \dots \dots a$	
$n1$	$n2$	$nq$
.	.	.
( $m$ )	( $m$ )	( $m$ )
$a$	$a \dots \dots a$	
$n1$	$n2$	$nq$

(matrix of order  $q \times q$  for inversion)

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The required balance corrections  $\{C\}$  are found by inverting the  $[A]$  matrix (which must be square, non-singular, and for good results preferably well-conditioned) as follows :

$$\{C\} = -[A]^{-1} \{b\}$$

i.e.

$$\begin{bmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ C_n \\ \cdot \\ \cdot \\ C_q \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = [A]^{-1} \begin{bmatrix} b_{10}^{(1)} \\ b_{20}^{(2)} \\ \cdot \\ \cdot \\ b_{n0}^{(1)} \\ b_{10}^{(2)} \\ b_{10}^{(2)} \\ b_{n0}^{(2)} \\ b_{10}^{(3)} \\ \cdot \\ \cdot \\ b_{n0}^{(n)} \end{bmatrix}$$

where  $C_j$  is a complex quantity of dimensions (mass  $\times$  length) which represents the correction factor in the  $j^{\text{th}}$  plane, its radius of attachment and its orientation around the rotor.

If more than the minimum data is acquired, then  $[A]$  will not be a square matrix. In these circumstances, some of the excess data may be eliminated to form a square matrix  $[A]$ . More commonly, the additional data is retained and at least squares technique is used to obtain an optimum balance. A number of such optimization procedures are available.

Details of the required balance corrections are found as follows :

$$\begin{aligned} \text{Generally, } C_j &= g_j + ih_j \\ &= C_j \exp i\xi_j \end{aligned}$$

where

$$C_j = \sqrt{g_j^2 + h_j^2}$$

$$\tan \xi_j = h_j/g_j$$

$C_j$  is the magnitude of the required balance correction in the  $j^{\text{th}}$  plane and

$\xi_j$  is the angular position of  $C_j$  from the angular reference location.

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## Annex B

### Cautionary notes concerning multi-span rotors on site

#### B.1 Introduction

The running quality of multi-span rotor systems, of which the most frequently encountered example is the large steam turbine generator unit, may sometimes be improved at site, as far as synchronous vibrations are concerned, by the application of correction masses at suitable points along the line of coupled rotors.

Before undertaking such operations, due consideration should be given to the possibility that factors other than unbalance are influencing the levels of synchronous vibrations of the machine. Such factors, which are not necessarily best corrected by the addition of balance masses, include those mentioned below.

Some other factors which may cause vibrations at other than synchronous frequency are mentioned in B.4 and B.5.

#### B.2 Bearing misalignment

Transverse or angular misalignment of the rotor bearings can produce effects which cannot be removed by balancing. Therefore, the alignment should be corrected as a preliminary to further assessment of the running quality of the machine (see also the last two paragraphs of B.3).

#### B.3 Coupling eccentricity and swash

There is no practical means of ensuring that large rotors, each with two or more journals of their own, can be coupled together without a small amount of eccentricity and/or swash between the mating halves of the coupling.

If any doubt exists concerning this point, or if the machine is not responding to balancing operations, the concentricity of the couplings should be checked, as well as the alignment.

Errors should be corrected to lie within whatever tolerances have been found to be satisfactory in practice for the size and

type of machine under consideration, before attempting further balancing operations.

If a rotor incorporating an auxiliary device and having only one bearing of its own is balanced in a balancing machine, a record may be made of the eccentricity of the rotor coupling after balancing. If the eccentricity is measured again after final assembly, the measured difference in eccentricity can be used to calculate the angular positions and axial distribution of the correction masses needed.

#### B.4 Bearing instability

Various forms of instability (for example fluid film whirl) may take place in the types of hydro-dynamically lubricated bearings which are normally used in multi-span flexible rotor systems.

The symptoms of these phenomena are well known, and it is necessary to ascertain whether such symptoms are present before attempting to improve the quality of running by carrying out balancing operations.

Discussions of such effects and possible remedial measures are outside the scope of this document.

#### B.5 Structural resonances

If any of the stationary parts of the machine or the supporting foundation structure are in resonance at the service speed, very high levels of vibration are sometimes produced even though the rotors are in a state of balance which is well within normally accepted tolerances.

In such circumstances, balancing within exceptionally fine limits may be required to effect an improvement in vibration level.

It may be necessary to eliminate the structural resonance in question before a satisfactory quality of running can be obtained.

## Annex C

### Derivation of maximum permissible initial unbalance

#### C.1 Introduction

**C.1.1** It may be possible to balance a flexible rotor satisfactorily by low speed balancing if the initial unbalance in the rotor due to manufacture and assembly is controlled. The effect of residual unbalance in a flexible rotor which has been balanced in this way varies with speed, due partially to the presence of initial unbalance not situated in the axial planes of the corrections applied at low speed. The amount of initial unbalance which can be corrected in this way will depend on the final unbalance limit.

**C.1.2** Preferably the maximum permissible bearing loads (due to unbalance) for any particular rotor should be specified and compared with predictions of the bearing loads which will arise at relevant speeds due to a) balance corrections being made at low speed and b) residual unbalance; i.e. the bearing loads which arise when the rotor operates a flexible rotor after it has only been balanced, to certain tolerances, as a rigid rotor. The calculation of the residual bearing loads will require the assumption of a typical initial unbalance distribution. On this basis the acceptability of low speed balancing could be truly assessed. If calculations show that the rotor is not sensitive to initial unbalance, then initial unbalance can be ignored.

It should be noted that effects other than bearing loads, for example bearing deflections, could be used.

**C.1.3** At present it is difficult to make accurate predictions of this nature so that simpler approaches must be adopted. One such method of estimating the maximum permissible initial unbalance measured in a low speed balancing machine is given in this annex for rotors which satisfy the following conditions :

- single span rotor with end bearings and two correction planes, one adjacent to each journal;
- fairly symmetrical mass distribution about midspan with no overhangs;
- fairly symmetrical flexibility distribution about midspan;
- continuous service speeds below and not significantly approaching second critical speed.

**C.1.4** It is further assumed that

- rotors which are built up by carefully controlled assembly of pre-balanced components will in general have an initial unbalance that is composed of small unbalances distributed along the rotor;
- the unbalance is distributed along the rotor in one radial direction. It should be noted that for any distribution this represents the most unfavourable condition for rotors

operating at speeds below and not significantly approaching second critical.

NOTE — The success of the method cannot be guaranteed for all rotors but it should in general give satisfactory guidance.

#### C.2 Calculation of bearing loads at speeds below first critical speed

##### C.2.1 Nomenclature

$M$	Rotor mass
$e_1$	Initial unbalance mass eccentricity
$e_2$	Maximum permissible residual unbalance mass eccentricity (i.e. residual unbalance at each end correction plane is $0,5 M e_2$ )
$k$	Rotor bending stiffness (at midspan)
$h$	Bearing stiffness (each bearing)
$\omega$	Rotor speed
$\omega_1$	Rotor first critical speed on its bearings
$\lambda$	$\frac{\omega}{\omega_1}$ i.e. rotor speed as a fraction of first critical speed
$\alpha$	$\frac{2h}{k}$ i.e. ratio of total bearing stiffness to rotor stiffness

##### C.2.2 Calculation of bearing loads

Consider a flexible shaft in flexible bearings with a central mass having an initial eccentricity  $e_1$  as in figure 4. This unbalance is corrected at end correction planes with a residual unbalance in each correction plane of  $0,5 M e_2$  as shown in figure 5.

The bearing load at each bearing arising from the unbalances shown in figure 5 at any speed  $\omega$  less than  $\omega_1$  is given by :

$$0,5 M \omega^2 \times \frac{\alpha \lambda^2 e_1 + (1 + \alpha - \alpha \lambda^2) e_2}{(1 + \alpha) (1 - \lambda^2)}$$

#### C.3 Estimation of permissible initial unbalance for speeds below first critical speed

##### C.3.1 Principle

The basis of this method is that the balancing recommendations of ISO 1940 are acceptable for rotors running up to 80 %

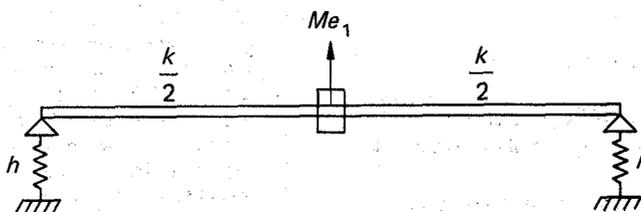


Figure 4 — Bearing deflections

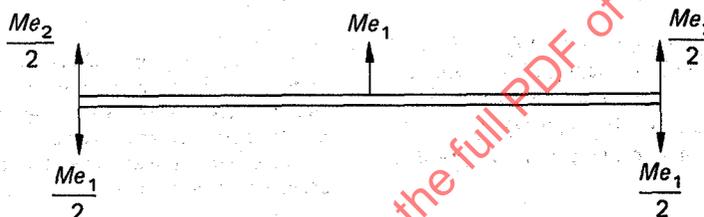


Figure 5 — Unbalance forces

of first critical speed. To determine the maximum permissible initial unbalance for a particular flexible rotor, consider an equivalent "rigid" rotor which is called the "datum" rotor and calculate the bearing loads for the "datum" rotor at 80 % of its first critical speed when balanced to the required quality grade in accordance with ISO 1940. The initial unbalance of the flexible rotor is then limited so that the bearing loads of the rotor after balancing are no greater than those calculated for the "datum" rotor.

**C.3.2 "Datum" rotor**

The "datum" rotor is a rotor of the same mass, general dimensions, running speed and quality grade as the flexible rotor but with its shaft and bearing flexibilities adjusted so that the rotor stiffness is twice the stiffness of each bearing and the first critical speed is 1,25 times the service speed. Further, it is assumed that an initial unbalance of ten times the maximum permissible residual unbalance has been corrected at end correction planes and that the maximum permissible residual unbalance remains in the correction planes. Such a rotor falls within the class of rigid rotors covered by ISO 1940 and should therefore run with safe bearing loads up to 80 % of first critical speed.

**C.3.3 Estimation**

For the datum rotor we have :

$$\alpha = 1$$

$$\lambda = 0,8$$

$$\frac{e_1}{e_2} = 10$$

and each bearing load =  $5,38 M\omega^2 e_2$

Thus to keep the bearing load (as given in C.2.2) of any flexible rotor within this limit we require :

$$\frac{e_1}{e_2} < \frac{9,76 (1 - \lambda^2) (1 + \alpha) - \lambda^2}{\alpha \lambda^2}$$

This relationship is plotted in figure 6.

**C.4 Estimation of permissible initial unbalance for speeds above first critical speed**

For speeds above first critical speed, the method of calculation is similar to C.2 and C.3 except that the phase of  $e_1$  relative to the shaft deflection is reversed and the phase of  $e_2$  must be selected to give the highest bearing load at the running speed concerned.

The resulting values of  $\frac{e_1}{e_2}$  are plotted in figure 6.

## C.5 Balancing procedure

The recommended balancing procedure is as follows :

- a) Estimate the ratio ( $\alpha$ ) of the total bearing stiffness to the stiffness of the rotor at midspan for a centrally applied load.
- b) Estimate the ratio ( $\lambda$ ) of maximum operating speed to first critical speed.
- c) Using these values, read off from figure 6 the appropriate value of  $\frac{e_1}{e_2}$ .
- d) From figure 4 in ISO 1940 find the value of residual specific unbalance ( $e_2$ ) for the appropriate operating speed and quality grade.
- e) Hence deduce the maximum permissible initial unbalance mass eccentricity ( $e_1$ ) and the maximum permissible initial unbalance  $Me_1$ .
- f) Provided that, by design, planned assembly, machine control and individual component balancing it is possible to hold the initial unbalance in each correction plane to within 50 % of the value of  $Me_1$  obtained in this way, then the rotor may be balanced at low speed using the two correction planes near the journals.

## NOTES

1 This analysis ignores damping and therefore produces a situation at the first critical speed where no initial unbalance can be tolerated. This is a theoretical condition and would not be met with in practice. It should be appreciated that the calculations are related to the condition at the running speed and in no way guarantee that the rotor will be capable of passing through the critical speed without excessive vibration.

2 This method has been simplified by putting the residual unbalance tolerance for the flexible rotor equal to that derived for the "datum" rigid rotor found from figure 4 in ISO 1940.

If it is practicable to balance the flexible rotor to smaller residual unbalance tolerances than are given in ISO 1940, then somewhat broader initial unbalance tolerances than are indicated by this method of calculation can be accepted. However, the advantages to be gained are relatively small and, in the average case, if the residual unbalance tolerance were to be reduced by 50 % the initial unbalance tolerance could be increased by some 5 to 10 %. The effect increases as  $\alpha$  diminishes.

NOTE — Although this analysis has been based on balancing planes adjacent to each bearing, the optimum position for these planes may be some distance from each bearing. In some cases, where it is practicable to position the balancing planes some distance away from the bearings, it may therefore be advantageous to investigate the optimum positions either experimentally or analytically.