INTERNATIONAL STANDARD

ISO 4351

First edition
2023-12

Geometrical product specifications (GPS) — Association

Spécification géométrique des produits (GPS) — Association

Reference number
ISO 4351:2023(E)

ISO copyright office
CP 401 • Ch. de Blandonnet 8
CH-1214 Vernier, Geneva
Phone: +41 22 749 01 11
Email: copyright@iso.org
Website: www.iso.org

Published in Switzerland

Contents

Page

Foreword ..... **iv**
Introduction ..... **v**
1 Scope **1**
2 Normative references **1**
3 Terms and definitions **1**
4 Association specification elements description of association process **3**
5 Association criterion **4**
5.1 General 4
5.2 Objective function description 5
5.3 Association constraint 6
5.4 Material offset 7
6 Specification elements **7**
Annex A (informative) **Non-enclosing feature/enclosing feature — Definitions** **11**
Annex B (informative) **Illustration of results of association** **13**
Annex C (informative) **Relationship to the ISO GPS matrix model** **15**
Bibliography **16**

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

ISO draws attention to the possibility that the implementation of this document may involve the use of (a) patent(s). ISO takes no position concerning the evidence, validity or applicability of any claimed patent rights in respect thereof. As of the date of publication of this document, ISO had not received notice of (a) patent(s) which may be required to implement this document. However, implementers are cautioned that this may not represent the latest information, which may be obtained from the patent database available at www.iso.org/patents. ISO shall not be held responsible for identifying any or all such patent rights.

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 213, *Dimensional and geometrical product specifications and verification*, in collaboration with the European Committee for Standardization (CEN) Technical Committee CEN/TC 290, *Dimensional and geometrical product specification and verification,* in accordance with the Agreement on technical cooperation between ISO and CEN (Vienna Agreement).

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

This document is a geometrical product specifications (GPS) standard and is to be regarded as a general ISO GPS standard (see ISO 14638). It influences the chain links A, B, C, E, F and G in all chains of standards in the general ISO GPS matrix.

The ISO GPS matrix plan given in ISO 14638 gives an overview of the ISO GPS system of which this document is a part. The fundamental rules of ISO GPS given in ISO 8015 apply to this document and the default decision rules given in ISO 14253-1 apply to the specifications made in accordance with this document, unless otherwise indicated.

For more detailed information on the relationship of this document to other International Standards and to the ISO GPS matrix model, see Annex C.

Geometrical product specifications (GPS) — Association

1 Scope

This document gives the terminology and basic concepts of association, including objective functions and association constraints and material offset.

This document is not intended to specify association defaults and GPS syntax which are introduced in other (ISO GPS) International Standards.

NOTE The association can be used to establish for example:

— a datum;

— a reference feature for a geometrical specification or for a surface texture specification;

— an associated toleranced feature;

— any dimensional characteristic;

— an intersection plane, an orientation plane, a collection plane or a direction feature.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 17450-1, *Geometrical product specifications (GPS) — General concepts — Part 1: Model for geometrical specification and verification*

ISO 17450-4, *Geometrical product specifications (GPS) — Basic concepts — Part 4: Geometrical characteristics for quantifying GPS deviations*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 17450-1 and ISO 17450-4 and the following apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
association
feature operation used to fit one or more ideal features to one or more *input features* (3.2) according to an association criterion

Note 1 to entry: The definition has been adapted from the definition of ISO 17450-1, to consider the term input feature.

3.2
input feature
<association> portion of an *associated feature* (3.3) or non-ideal feature, which is an extracted feature or a real feature, which can be filtered or not

3.3
associated feature
ideal feature which is fitted to an *input feature* (3.2) with a specific *association criterion* (3.5)

Note 1 to entry: An ideal feature is defined by a mathematical description using a finite set of real numbers called description parameters (see EXAMPLE below).

Note 2 to entry: An associated feature can be a feature of size. In this case, it is possible to use the term associated feature of size.

EXAMPLE An extruded surface having as directrix an ellipse, can be described by the formula

"$a \cdot (x-c)^2 + b \cdot (y-d)^2 = 1, \forall z$" (with description parameter *a, b, c* and *d* with *a* non-equal to *b*). It describes the points (having as coordinate *x*, *y* and *z*, in a local Cartesian coordinate system) of an ideal feature which belongs to the prismatic invariance class (prismatic surface with an elliptic base).

3.4
restricted associated feature
set of points of an *associated feature* (3.3), where local geometrical deviations with the *input feature* (3.2) exist

3.5
association criterion
objective function (3.6) with or without *association constraints* (3.9), and with or without material offset, defined for an association

Note 1 to entry: Several association constraints may be used for one association.

Note 2 to entry: Association results (associated features) can differ, depending upon the choice of association criterion.

3.6
objective function
optimization function
<association> formula that describes the goal of association from the *input feature* (3.2) and the ideal feature [*associated feature* (3.3)]

3.7
L-function
objective function (3.6) defined from the set of the signed local geometrical deviations between an *input feature* (3.2) and the *associated feature* (3.3)

Note 1 to entry: The local geometrical deviations are defined in ISO 17450-4, with the convention sign based on the material boundary defined from the ideal geometry.

3.8
S-function
objective function (3.6) based on the size of the associated feature of size

Note 1 to entry: The maximum inscribed feature and the minimum circumscribed feature are associated feature obtained with S-function.

3.9
association constraint
set of restrictions on variability of the mathematical parameters describing an *associated feature* (3.3) in the optimization process.

EXAMPLE Orientation constraint, location constraint, material constraint or size constraint are the different types of association constraint.

3.10
orientation constraint
association constraint (3.9) related to one or more rotational degrees of freedom of *associated feature* (3.3)

3.11
location constraint
association constraint (3.9) related to one or more translational degrees of freedom of *associated feature* (3.3)

3.12
material constraint
association constraint (3.9) on the *associated feature* (3.3), in relation to the material boundary of the *input feature* (3.2)

EXAMPLE The outside material constraint implies that all distances between the associated feature and the input feature are negative or equal to zero.

3.13
size constraint
association constraint (3.9) on the size of *associated feature* (3.3)

Note 1 to entry: Without size constraint, the size of associated feature is variable for the association.

4 Association specification elements description of association process

The association operation is an optimization process intended to fit an ideal feature with a predefined geometry to an input feature or to fit a collection of ideal features to predefined geometries to a collection of input features. The input feature for this optimization process is a geometrical feature (portion of an associated feature, an extracted feature or a real feature, which can be filtered or not).

To perform this optimization process, an association objective function shall be specified without or with association constraints. After this step of optimization, an offset step can be optionally performed (see Figure 1).

An associated feature or a collection of associated features is thus defined from which it is possible to specify the set of one or more situation features (allowing the situation in the space of this associated feature or this collection of associated features).

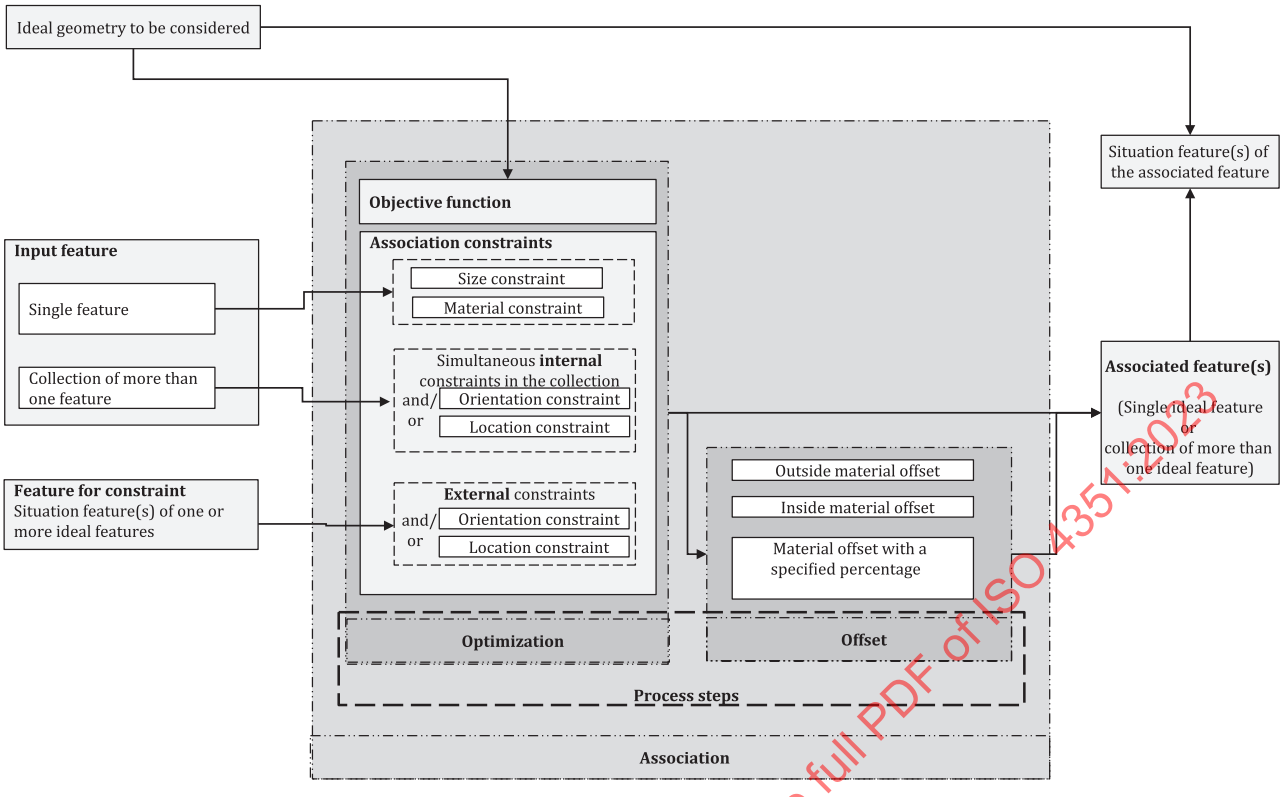


Figure 1 — Concept diagram illustrating the association process

If associations are performed on more than one non-ideal single features (input features), then the associations can be made:

— Independently.

— Simultaneously, i.e. the collection of associated features is established simultaneously with location constraints (if applicable) and orientation constraints (if applicable) between them. Those are internal constraints in the collection of the ideal features (to be associated).

— In a specific order, i.e. the associated feature of the rank n+1 is established with location constraints (if specified) and orientation constraints (if specified) related to the situation feature(s) of the associated feature(s) of rank 1 to n. Those are external constraints of ideal feature(s) of rank n+1 from associated feature(s) of rank n.

5 Association criterion

5.1 General

An association criterion specifies an optimization process combining an objective function and association constraints optionally followed by a material offset.

The objective function shall be specified from an S-function or an L-function:

— the first one, by maximizing or minimizing the size of the associated feature of size, by considering no association constraint or one or more association constraints but without size constraint;

— the second one, through an optimization of a function of the set of signed local geometrical deviations with or without association constraint.

The local geometrical deviations are established between the points on the input feature and the corresponding points on the associated feature. This set of the points defines a restricted associated

feature. The restricted associated feature can be an enclosing feature or non-enclosing feature (see Annex A).

5.2 Objective function description

An (mathematical) objective function is described from the size of associated feature, S, and/or from the (signed) local geometrical deviations, D_F, between an input feature and an ideal feature. The optimization of this objective function defines the final result (final ideal feature) of the association, which is the associated feature.

NOTE When the input feature is a derived feature, the local geometrical deviations, D_F, are not signed: no material constraint is applied.

The result of this optimization is the associated feature (see Table 1).

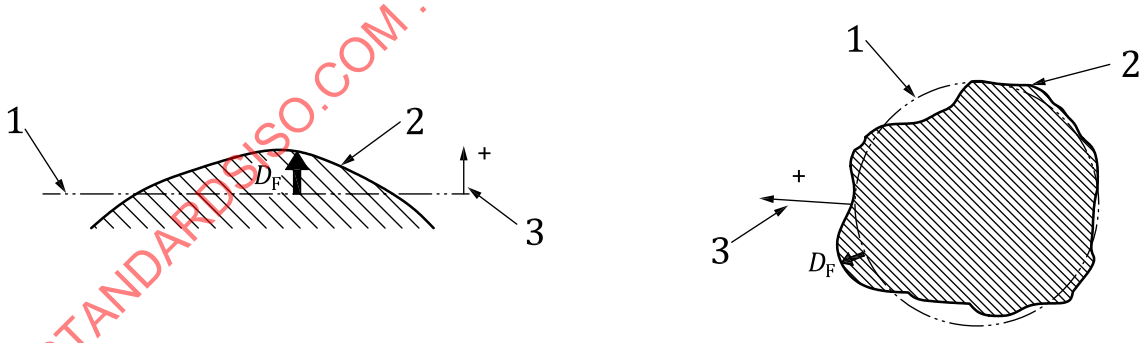
To establish a material constraint, the sign convention is established in relation with the material, see Figure 2. The local geometrical deviations, between a point of the non-ideal single feature and the associated feature, is positive when the deviation direction from a point on the associated feature to its homologous point on the input feature goes to the direction outside of material from the associated feature.

The implications of this sign convention for S-function (i.e. maximum inscribed, or minimum circumscribed) are illustrated in Figure 3.

When the associated feature is considered as a feature of size and the restricted associated feature is a non-enclosing feature, the maximum inscribed or minimum circumscribed association criteria should not be applied considering the risk to create an association result, which does not represent the intent.

When the extent of the restricted associated feature is smaller or close to its size, the minimum circumscribed objective function should be performed with orientation constraint, from a datum considering the risk to enlarge uncertainty contributor related to the association.

For one input feature, the application of a same set of association constraints with different objective function generally results in different associated features.

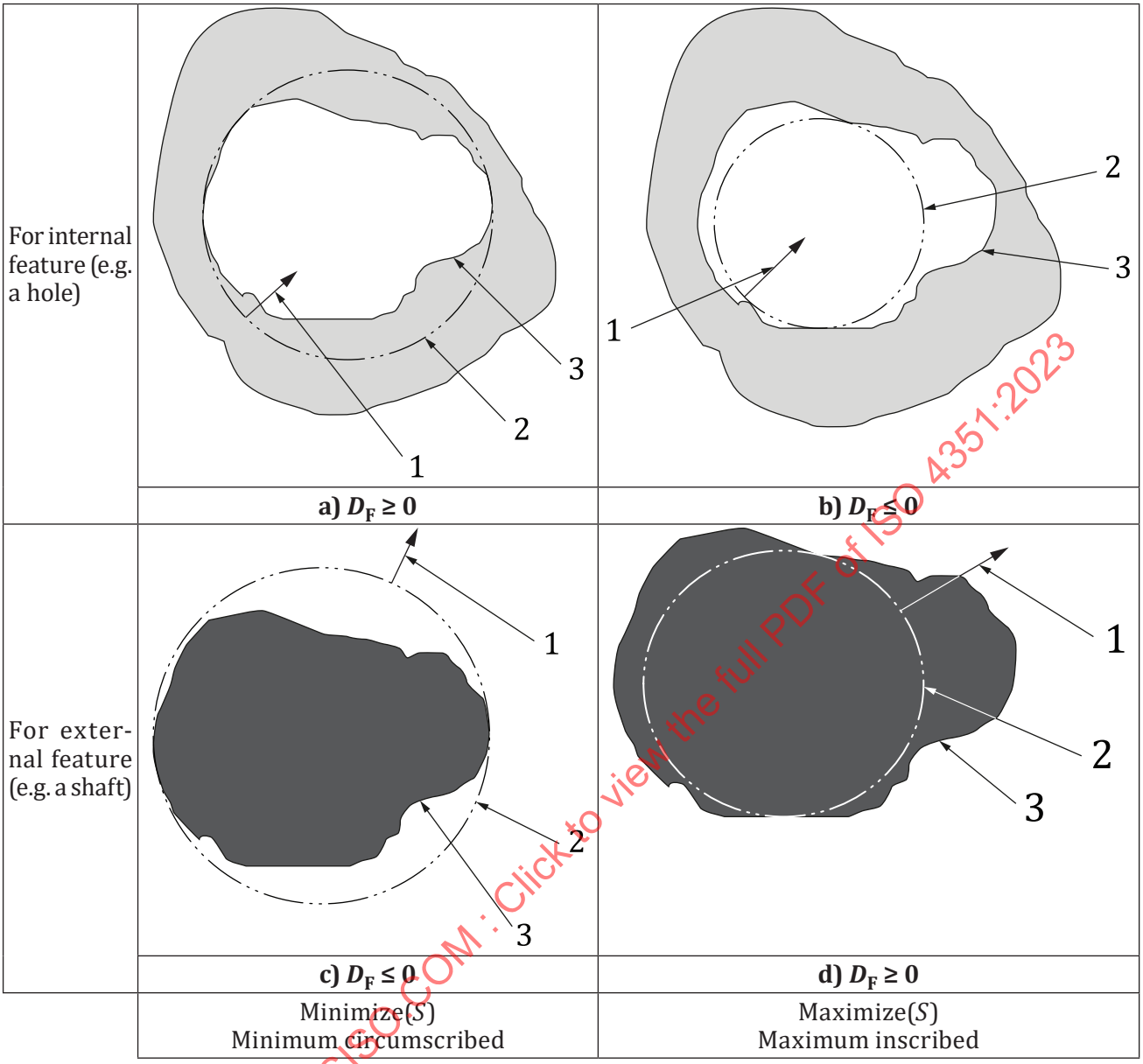


a) On a non-enclosing feature **b) On an enclosing feature**

Key

1 associated feature
2 input feature
3 conventional positive direction for D_F
D_F signed local geometrical deviation between a point of 2 and 1

Figure 2 — Conventional positive direction for D_F



Key

1 material surface normal, pointing away from the material (positive direction for D_F)

2 associated feature

3 input feature

D_F signed local geometrical deviation between a point of 3 and 2

Figure 3 — Illustration of the optimization result for S-function (minimum circumscribed and maximum inscribed) with its material constraints based on the signed local geometrical deviations

5.3 Association constraint

Within the optimization process, when an objective function is applied, an association constraint restricts the variability of the parameters describing the associated feature.

— The size constraint fixes the size of the associated feature. It shall not be used with S-function objective function.

— The material constraint fixes the situation of the associated feature with respect to the material boundary of the input feature.

— The orientation constraint fixes the rotational degree of freedom related to the situation features of the associated feature.

— The location constraint fixes the translational degree of freedom related to the situation features of the associated feature.

The orientation constraint and location constraint can be defined internally to a collection of associated feature or externally coming from one or more situation features of one or more other associated features.

For one input feature, the application of a same objective function with a different set of association constraints generally results in different associated features.

5.4 Material offset

After the optimization process, defining the first associated feature, it is possible to apply an offset linked with the range of local geometrical deviation and the material boundaries, offsetting the first associated feature, to:

— the outside material boundary of the input feature; or

— the inside material boundary of the input feature; or

— a specified level between the outside and inside material boundaries.

When a material offset is specified:

— on the outside material boundary, then the final associated feature is the offset feature obtained by a dilation considering an offset of the maximum value of the local geometric deviations;

— on the inside material boundary, then the final associated feature is the offset feature obtained by an erosion considering an offset of the minimum value of the local geometrical deviations;

— at a level (specified in percentage, *x* %) of material boundary, then the final associated feature is the offset feature obtained by a dilation of a percentage, *x*, of the range of the local geometrical deviations from a primary erosion corresponding to the inside boundary.

6 Specification elements

In cases where the association used is not the default association for a specific purpose (e.g. to establish datum, section feature, evaluation feature), it is recommended to describe the association specification elements by indicating

— the association objective function symbol, in case of non-default objective function (see Table 1), followed without space, by

— the material constraint symbol, in case of non-default material constraint (see Table 2) followed without space, if necessary, by

— the level of the material offset expressed by a sign + or – or the value of percentage followed by % (see Table 3).

The first associated feature is determined by using the specified objective function and the specified material constraint. The material constraint is indicated by the letter “E” for the outside material constraint (external of material) and by the letter “I” for the inside material constraint (internal of material). When no material constraint is applied, the letter “M” is indicated.

If a material offset applies, it is defined from a boundary of the input feature offsetting a first associated feature (applying objective function and association constraint):

— on outer point of the input feature when the modifier + is indicated;

— on inner point of the input feature when the modifier - is indicated;

— at a level of boundary material when a percentage (x %) is indicated followed by a space and the character % (0 % corresponding to the inner point and 100 % to the outer point).

This sequence should be enclosed in square brackets, e.g. [CM100 %], [C–], [CM], [CM50 %], [CM86 %].

NOTE 1 The indications for size constraint, orientation constraint and location constraint are given as follows:

— the size constraint implies taking into account the size defined as a theoretical exact dimension (TED);

— the orientation constraint implies taking into account one or more angular TEDs;

— the location constraint implies to take into account one or more linear TEDs.

The symbols for the objective functions and material constraints for association and material offset are given in Table 1.

Annex B provides some illustrations of results of association with different association criteria applied to a same feature.

Table 1 — Symbols for the objective functions

Objective function		
Symbol	**Name**	**Description**
G	L_2-function Gaussian (total least squares)	Minimize $\left[\left(\int_F \lvert D_F\rvert^2 \times dF\right)^{\frac{1}{2}}\right]$
C	L_∞-function Minimax (Chebyshev)	Minimize[a] $\left[\lim_{p\to\infty}\left(\int_F \lvert D_F\rvert^p \times dF\right)^{\frac{1}{p}}\right]$
K	L_1-function Minimum volume	Minimize $\left[\int_F \lvert D_F\rvert \times dF\right]$
L_p (e.g. L_4)	Lp-function with p different from 1, 2 and ∞	Minimize $\left[\left(\int_F \lvert D_F\rvert^p \times dF\right)^{\frac{1}{p}}\right]$
X	S_{max}-function Maximum inscribed	Maximize S, with $D_F \geq 0$ for external feature (shaft) or $D_F \leq 0$ for internal feature (hole)
N	S_{min}-function Minimum circumscribed	Minimize S, with $D_F \leq 0$ for external feature (shaft) or $D_F \geq 0$ for internal feature (hole)
D_F Signed local geometrical deviation F Input feature S Size of associated feature, which is a feature of size [a] Equivalent to minimize (Max($\lvert D_F\rvert$)).		

Table 2 — Symbols for the material constraints

Symbol	Name	Description
E	Outside material constraint	$\forall P \in F, D_F \leq 0$
I	Inside material constraint	$\forall P \in F, D_F \geq 0$
M	Without material constraint	$\forall P \in F$ no sign constraint on D_F
D_F Signed local geometrical deviation F Input feature P Any point of the input feature		

Table 3 — Symbols for the material offset

Symbol	Name	Description
+	Shifted tangent outside material	Shift to D_{Fmax}
–	Shifted tangent inside material	Shift to D_{Fmin}
x %	Material ratio percentage	Shift to $D_{Fmin} + x/100 \cdot (D_{Fmax} - D_{Fmin})$
D_F Signed local geometrical deviation x Percentage of ratio		

NOTE 2 Other ISO GPS standards give rules to indicate association criteria on TPD, e.g. ISO 5459 and ISO 1101.

Annex A
(informative)

Non-enclosing feature/enclosing feature — Definitions

A.1 Non-enclosing feature

This is an integral feature, for which the normal vectors are never opposed over the extent of the restricted associated feature.

NOTE 1 At least one plane (or straight line in two dimensions) exists for which all the normal vectors are pointing to the same side of this plane (or this straight line). See Figure A.1 a) and Figure A.1 b).

NOTE 2 A cylindrical surface with a central angle of less than 180° is an example of a non-enclosing feature.

NOTE 3 This definition is intended to be included in ISO 17450-1 during its next revision.

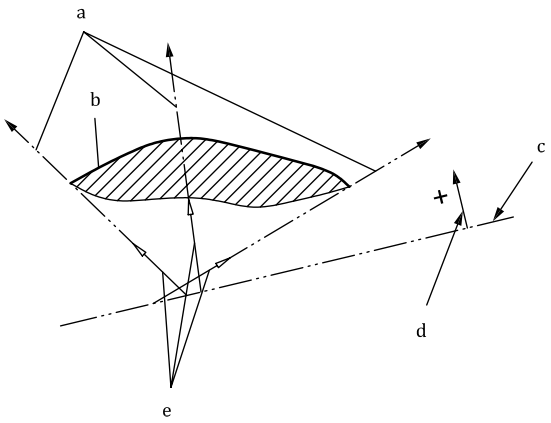
A.2 Enclosing feature

This is an integral feature, for which the normal vectors are opposed in at least one instance over the extent of the restricted associated feature.

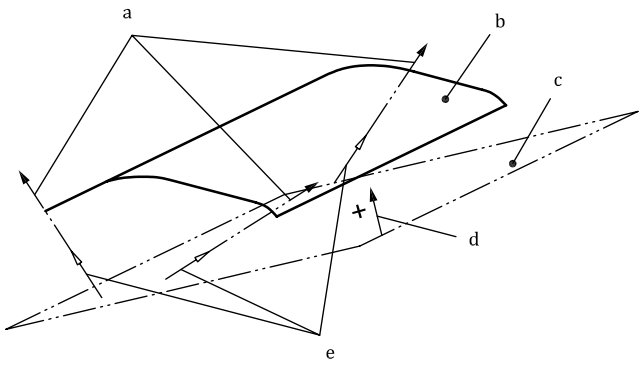
NOTE 1 No plane (or straight line in two dimensions) exists for which all the normal vectors are pointing to the same side of this plane (or this straight line). See Figure A.1 c) and Figure A.1 d).

NOTE 2 A cylindrical surface with a central angle of 180° or greater is an example of an enclosing feature.

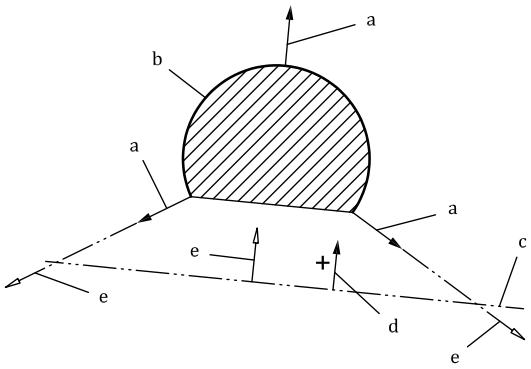
NOTE 3 This definition is intended to be included in ISO 17450-1 during its next revision.



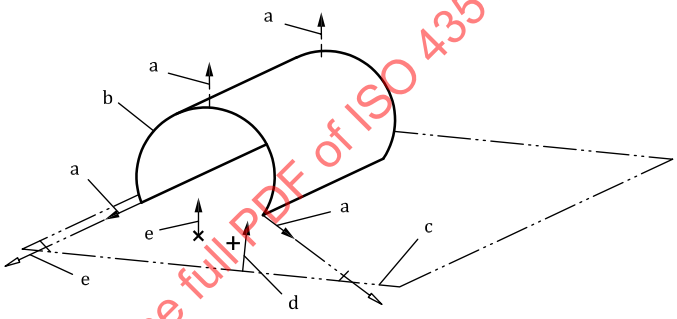
a) Non-enclosing feature — 2D illustration



b) Non-enclosing feature — 3D illustration



c) Enclosing feature — 2D illustration



d) Enclosing feature — 3D illustration

Key

a Normal vectors to b.

b Nominal feature.

c Plane in 3D and straight line in 2D, of verifying of the assumption.

d Positive side of c.

e Directions observed of a related to d.

NOTE In the case of an enclosing feature, no plane or no straight line exists for which all normal vectors are pointing to the same side.

Figure A.1 — Illustration of non-enclosing features and enclosing features

Annex B
(informative)

Illustration of results of association

Table B.1 shows the resulting associated features for different associations of one particular input feature (all the possibilities of association constraint and objective function are not presented).

Table B.1 — Illustration of associated features (result of an association)

Nominal feature	Input feature	Association criteria		Illustration of the result of association
		Objective function	Association constraints	
(considered as a shaft)		Minimum circumscribed	/	
		Maximum inscribed	/	
		Gaussian (total least squares)	Without constraint	