# INTERNATIONAL STANDARD

**ISO**

**39511**

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# Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)

*Plans d'échantillonnage progressif pour le contrôle par mesures des pourcentages de non-conformes (écart-type connu)*

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# Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

This first edition of ISO 39511 cancels and replaces ISO 8423:2008, of which it constitutes a minor revision to change the reference number from 8423 to 39511.

# Introduction

In contemporary production processes, quality is often expected to reach such high levels that the number of nonconforming items is reported in parts per million. Under such circumstances, popular acceptance sampling plans by attributes, such as those presented in ISO 2859-1, require prohibitively large sample sizes. When it is possible to apply acceptance sampling plans by variables, such as those presented in ISO 3951-1, the sample sizes are much smaller. However, especially in the case of acceptance of a product of extremely high quality, those sample sizes are still too large. Therefore, there is a need to apply standardized statistical procedures that require the smallest possible sample sizes; sequential sampling plans are the only statistical procedures that satisfy that need. It has been mathematically proved that among all possible sampling plans having similar statistical properties the sequential sampling plan has the smallest average sample size.

The principal advantage of sequential sampling plans is the reduction in the average sample size. The *average sample size* is the average of all the sample sizes that may occur under a sampling plan for a given lot or process quality level. The use of sequential sampling plans leads to a smaller average sample size than single sampling plans having the equivalent operating characteristic.

Other factors that should be taken into account are as follows.

a)   Complexity

The rules of a sequential sampling plan are more easily misunderstood by inspectors than the simple rules for a single sampling plan.

b)   Variability in the amount of inspection

As the actual number of items inspected for a particular lot is not known in advance, the use of sequential sampling plans brings about various organizational difficulties. For example, scheduling of inspection operations may be difficult.

c)   Difficulty of drawing sample items

If drawing sample items is rather difficult, the reduction in the average sample size by sequential sampling plans may be cancelled out by the increased sampling cost.

d)   Duration of test

If the test of a single item is of long duration and a number of items can be tested simultaneously, sequential sampling plans are much more time-consuming than the corresponding single sampling plan.

e)   Variability of quality within the lot

If the lot consists of two or more sublots from different sources and if there is likely to be any substantial difference between the qualities of the sublots, drawing of a representative sample under a sequential sampling plan is far more difficult than under the corresponding single sampling plan.

The balance between the advantage of a smaller average sample size of the sequential sampling plan and the above disadvantages leads to the conclusion that sequential sampling plans are suitable only when inspection of individual items is costly in comparison with inspection overheads.

The choice between single and sequential sampling plans should be made before the inspection of a lot is started. During inspection of a lot, it is not permitted to switch from one type to another, because the operating characteristic of the plan may be drastically changed if the actual inspection results influence the choice of acceptability criteria.

Although a sequential sampling plan is on average much more economical than the corresponding single sampling plan, it may occur, during inspection of a particular lot, that acceptance or non-acceptance comes at a very late stage because the cumulative leeway (the statistic used for the determination of lot acceptability) remains between the acceptance value and the rejection value for a long time. With the

graphical method, this corresponds to the random progress of the step-wise linear curve remaining in the indecision zone.

In order to alleviate this disadvantage, the curtailment values are set before the inspection of a lot (or a process) is started, and inspection terminates if the cumulative sample size reaches the curtailment value, $n_t$, without determination of lot acceptability. The acceptance and non-acceptance of the lot (or the process) is then determined using the curtailment acceptance and rejection values.

For sequential sampling plans in common use, curtailment usually represents a deviation from their intended usage, leading to a distortion of their operating characteristics. In this International Standard, however, the operating characteristics of the sequential sampling plans have been determined with curtailment taken into account, so curtailment is an integral component of the provided plan.

Sequential sampling plans for inspection by variables are also provided in ISO 3951-5. However, the design principle of those plans is fundamentally different from that of this International Standard. The sampling plans in ISO 3951-5 are designed to supplement the ISO 3951-1 acceptance sampling system for inspection by variables, which is a counterpart of the popular ISO 2859-1 acceptance sampling system for inspection by attributes. Thus, they should be used for the inspection of a continuing series of lots, that is, a series long enough to permit the switching rules of the ISO 3951 system to take effect. The application of the switching rules is the only means of providing enhanced protection to the consumer (by means of tightened sampling inspection criteria or discontinuation of sampling inspection) when the sequential sampling plans from ISO 3951-5 are used. However, in certain circumstances, there is a strong need to have both producer's and consumer's risks under strict control. Such circumstances occur, for example, when sampling is performed for regulatory reasons, for the demonstration of quality of production processes or for hypothesis testing. In such cases, individual sampling plans selected from the ISO 3951-5 sampling scheme may be inappropriate. The sampling plans from this International Standard have been designed in order to meet these specific requirements.

# Sequential sampling plans for inspection by variables for percent nonconforming (known standard deviation)

## 1 Scope

This International Standard specifies sequential sampling plans and procedures for inspection by variables of discrete items.

The plans are indexed in terms of producer's risk point and the consumer's risk point. Therefore, they are suitable not only for the purposes of acceptance sampling, but for the more general purpose of the testing of simple statistical hypotheses for proportions.

The purpose of this International Standard is to provide procedures for the sequential assessment of inspection results that may be used to induce the supplier to supply lots of a quality having a high probability of acceptance. At the same time, the consumer is protected by a prescribed upper limit to the probability of accepting a lot (or process) of poor quality.

This International Standard is primarily designed for use under the following conditions:

a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process. In such a case, sampling of particular lots is equivalent to the sampling of the process. If there are different producers or production processes, this International Standard shall be applied to each one separately;

b) where only a single quality characteristic $x$ of these products is taken into consideration, which must be measurable on a continuous scale;

c) where the measurement error is negligible (i.e. with a standard deviation no more than 10 % of the process standard deviation);

d) where production is stable (under statistical control) and the quality characteristic $x$ has a known standard deviation, and is distributed according to a normal distribution or a close approximation to the normal distribution;

**CAUTION — The procedures in this International Standard are not suitable for application to lots that have been screened previously for nonconforming items.**

e) where a contract or standard defines an upper specification limit $U$, a lower specification limit $L$, or both; an item is qualified as conforming if and only if its measured quality characteristic, $x$, satisfies the appropriate one of the following inequalities:

   1) $x \leq U$ (i.e. the upper specification limit is not violated);

   2) $x \geq L$ (i.e. the lower specification limit is not violated);

   3) $x \leq U$ and $x \geq L$ (i.e. neither the upper nor the lower specification limit is violated.)

Inequalities 1) and 2) are called cases with a "single specification limit", and 3) is the case with "double specification limits".

In this International Standard, it is assumed that, where double specification limits apply, conformance to both specification limits is either equally important to the integrity of the product or is considered separately for both specification limits. In the first case, it is appropriate to control the combined percentage of product outside the two specification limits. This is referred to as combined control. In the second case, nonconformity beyond each of the limits is controlled separately, and this is referred to as separate control.

## 2   Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2:2006, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3951-1, *Sampling procedures for inspection by variables — Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

## 3   Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 3534-2 and ISO 3951-1 and the following apply.

**3.1**
**inspection by variables**
inspection by measuring the magnitude(s) of the characteristic(s) of an item

[SOURCE: ISO 3534-2:2006, definition 4.1.4]

**3.2**
**sampling inspection**
inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2:2006, definition 4.1.6]

**3.3**
**acceptance sampling**
sampling after which decisions are made to accept or not to accept a lot, or other grouping of products, materials or services, based on sample results

[SOURCE: ISO 3534-2:2006, definition 1.3.17]

**3.4**
**acceptance sampling inspection**
acceptance inspection where the acceptability is determined by means of sampling inspection

[SOURCE: ISO 3534-2:2006, definition 4.1.8]

**3.5**
**acceptance sampling inspection by variables**
**acceptance sampling inspection** in which the acceptability of a process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

[SOURCE: ISO 3534-2:2006, definition 4.2.11]

**3.6**
**quality level**
quality expressed as a rate of occurrence of nonconforming units

**3.7**
**nonconformity**
non-fulfilment of a requirement

[SOURCE: ISO 9000:2015, definition 3.6.9, and ISO 3534-2:2006, definition 3.1.11]

**3.8**
**nonconforming unit**
unit with one or more nonconformities

[SOURCE: ISO 3534-2:2006, definition 1.2.15]

**3.9**
**specification limit**
limiting value stated for a characteristic

[SOURCE: ISO 3534-2:2006, definition 3.1.3]

**3.10**
**lower specification limit**
$L$
specification limit that defines the lower limiting value

[SOURCE: ISO 3534-2:2006, definition 3.1.5]

**3.11**
**upper specification limit**
$U$
specification limit that defines the upper limiting value

[SOURCE: ISO 3534-2:2006, definition 3.1.4]

**3.12**
**combined control**
requirement when both upper and lower limits are specified for the quality characteristic and specified risks apply to the combined percent nonconforming beyond the two limits

Note 1 to entry: The use of combined control implies that nonconformities beyond either specification limit are believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

**3.13**
**separate control**
requirement when both upper and lower limits are specified for the quality characteristic and separate risks are given which apply to each limit

Note 1 to entry: The use of separate control implies that nonconformities beyond either specification limit are believed to be of different importance to the lack of integrity of the product.

**3.14**
**maximum process standard deviation**
$\sigma_{max}$
largest process standard deviation for a given sampling plan for which it is possible to satisfy the acceptance criteria for a combined double specification limit when the process variability is known

Note 1 to entry: Maximum process standard deviation $\sigma_{max}$ was denoted by its acronym MPSD in older standards.

Note 2 to entry: This definition is different from the similar definition given in ISO 3534-2 in which the concept of AQL is used.

**3.15**
**measurement**
set of operations having the object of determining a value of a quantity

[SOURCE: ISO 3534-2:2006, definition 3.2.1]

**3.16**
**leeway**
quantity derived from a measured value of an item

Note 1 to entry: In the case of a single lower specification limit and in the case of double specification limits, the leeway is obtained by subtracting the numerical value of the lower specification limit from the measured value. In the case of an upper specification limit, the leeway is obtained by subtracting the measured value from the numerical value of the upper specification limit.

**3.17**
**cumulative leeway**
value calculated by summing the leeways obtained from the start of the inspection up to, and including, that of the item last inspected

**3.18**
**cumulative sample size**
total number of inspected items, counting from the start of the inspection up to, and including, the item last inspected

**3.19**
**acceptance value for sequential sampling**
value derived from the specified parameters of the sampling plan and the cumulative sample size

Note 1 to entry: Whether the lot may yet be accepted is determined by comparing the cumulative leeway with the acceptance value.

**3.20**
**rejection value for sequential sampling**
value derived from the specified parameters of the sampling plan and the cumulative sample size

Note 1 to entry: Whether the lot may yet be considered unacceptable is determined by comparing the cumulative leeway with the rejection value.

**3.21**
**consumer's risk quality**
**CRQ**
$Q_{CR}$
⟨acceptance sampling⟩ quality level of a lot or process which, in the acceptance sampling plan, corresponds to a specified consumer's risk

Note 1 to entry: The specified consumer's risk is usually 10 %.

[SOURCE: ISO 3534-2:2006, definition 4.6.9]

**3.22**
**producer's risk quality**
**PRQ**
$Q_{PR}$
⟨acceptance sampling⟩ quality level of a lot or process which, in the acceptance sampling plan, corresponds to a specified producer's risk

[SOURCE: ISO 3534-2:2006, definition 4.6.10]

Note 1 to entry: The specified producer's risk is usually 5 %.

**3.23**
**average sample size**
**ASSI**
⟨acceptance sampling⟩ average number of units in a sample inspected per lot in reaching decisions to accept or not to accept when using a given acceptance sampling plan

[SOURCE: ISO 3534-2:2006, definition 4.7.3]

**3.24**
**sequential acceptance sampling inspection**
acceptance sampling inspection in which, after each item has been inspected, the decision to accept the lot, not accept the lot, or to inspect another item is taken based on the cumulative sampling evidence to date

[SOURCE: ISO 3534-2:2006, definition 4.2.7]

**3.25**
**sequential sampling plan**
plan which states acceptance criteria in sequential acceptance sampling inspection

**3.26**
**operating characteristic curve**
curve showing the relationship between probability of acceptance of product and the incoming quality level for a given acceptance sampling plan

[SOURCE: ISO 3534-2:2006, definition 4.5.1]

**3.27**
**producer's risk point**
**PRP**
⟨acceptance sampling⟩ point on the operating characteristic curve corresponding to a predetermined high probability of acceptance

[SOURCE: ISO 3534-2:2006, definition 4.6.7]

**3.28**
**consumer's risk point**
**CRP**
⟨acceptance sampling⟩ point on the operating characteristic curve corresponding to a predetermined low probability of acceptance

[SOURCE: ISO 3534-2:2006, definition 4.6.5]

# 4   Symbols

The symbols used are as follows.

| | |
|---|---|
| $A$ | acceptance value for sequential sampling |
| $A_t$ | acceptance value corresponding to the curtailed value of the cumulative sample size |
| $f$ | a factor given in Tables 5 and 6, that relates the maximum process standard deviation to the difference between $U$ and $L$ |
| $g$ | multiplier of the cumulative sample size that is used to determine the acceptance values and the rejection values (slope of the acceptance and rejection lines) |
| $h_A$ | constant that is used to determine the acceptance values (intercept of the acceptance line) |
| $h_R$ | constant that is used to determine the rejection values (intercept of the rejection line) |
| $L$ | lower specification limit (as a suffix to a variable, denotes its value at $L$) |
| $N$ | lot size (number of items in a lot) |
| $n$ | sample size (number of items in a sample) |
| $n_{cum}$ | cumulative sample size |

$n_t$      curtailment value of the cumulative sample size

$P_a$      probability of acceptance

$Q_{CR}$      consumer's risk quality

$Q_{PR}$      producer's risk quality

$R$      rejection value for sequential sampling

$U$      upper specification limit (as a suffix to a variable, denotes its value at $U$)

$x$      measured value of the quality characteristic for the item of the sample

$y$      leeway, defined as

     $y = U - x$      for a single upper specification limit

     $y = x - L$      for a single lower specification limit

     $y = x - L$      for double specification limits

$Y$      cumulative leeway obtained by adding the leeways up to, and including, the item last inspected

$\alpha$      producer's risk

$\beta$      consumer's risk

$\sigma$      standard deviation of a process that is under statistical control

     NOTE    $\sigma^2$, the square of the process standard deviation, is known as the process variance.

$\sigma_{max}$      maximum process standard deviation

## 5   Principles of sequential sampling plans for inspection by variables

Under a sequential sampling plan by variables, sample items are drawn at random and inspected one by one, and the cumulative leeway (which measures a "distance" between the process level and specification limits) is obtained. After the inspection of each item, the cumulative leeway is compared with the acceptability criteria in order to assess whether there is sufficient information to determine lot or process acceptability at that stage of the inspection.

If, at a given stage, the cumulative leeway is such that the risk of accepting a lot of unsatisfactory quality level is sufficiently low, the lot is considered acceptable and the inspection is terminated.

If, on the other hand, the cumulative leeway is such that the risk of non-acceptance of a lot of satisfactory quality level is sufficiently low, the lot is considered not acceptable and the inspection is terminated.

If the cumulative leeway does not allow either of the above decisions to be taken, then an additional item is sampled and inspected. The process is continued until sufficient sample information has been accumulated to warrant a decision that the lot is acceptable or not acceptable.

## 6   Selection of a sampling plan

### 6.1   Producer's risk point and consumer's risk point

The general method described in 6.1 and 6.2 is used when the requirements of the sequential sampling plan are specified in terms of two points on the operating characteristic curve of the plan. The point

corresponding to the higher probability of acceptance shall be designated the "producer's risk point"; the other shall be designated the "consumer's risk point".

The first step when designing a sequential sampling plan is to choose these two points, if they have not already been dictated by circumstances. For this purpose, the following combination is often used:

— a producer's risk of $\alpha \leq 0{,}05$ and the corresponding producer's risk quality ($Q_{PR}$);

— a consumer's risk of $\beta \leq 0{,}10$ and the corresponding consumer's risk quality ($Q_{CR}$).

This combination of requirements is used in this International Standard for the design of the sampling plans.

When the desired sequential sampling plan is required to have approximately the same operating characteristic curve as an existing single sampling plan, the producer's risk point and the consumer's risk point may be read off from a graph or a table of the operating characteristic of that plan. When no such plan exists, the producer's and the consumer's risk points have to be determined from direct consideration of the conditions under which the sampling plan operates.

## 6.2 Preferred values of $Q_{PR}$ and $Q_{CR}$

Table 4 gives 21 preferred values of $Q_{PR}$ (producer's risk quality) ranging from 0,1 % to 10,0 %, and 17 preferred values of $Q_{CR}$ (consumer's risk quality) ranging from 0,8 % to 31,5 %. This International Standard is only applicable to a combination of the preferred values of $Q_{PR}$ and $Q_{CR}$.

## 6.3 Pre-operation preparations

### 6.3.1 Obtaining the parameters $h_A$, $h_R$ and $g$

The criteria for acceptance and non-acceptance of a lot are determined from the parameters $h_A$, $h_R$ and $g$.

Table 4 gives the values of these parameters corresponding to each combination of preferred values of $Q_{PR}$ and $Q_{CR}$ together with the producer's risk $\alpha$ approximately equal to 0,05 and the consumer's risk $\beta$ approximately equal to 0,1.

### 6.3.2 Obtaining the curtailment values

The curtailment value, $n_t$, of the cumulative sample size of the sequential sampling plan is given in Table 4 together with the other parameters.

# 7 Operation of a sequential sampling plan

## 7.1 Specification of the plan

Before operating a sequential sampling plan, the inspector shall record on the sampling document the specified values of the parameters, $h_A$, $h_R$, $g$ and $n_t$.

## 7.2 Drawing a sample item

As a rule, the individual sample items shall be drawn at random from the lot and inspected one by one in the order in which they were drawn. If, for convenience, successive items are drawn at the same time, the order in which each sample item is inspected shall be at random.

## 7.3 Leeway and cumulative leeway

Following the inspection of each item, record the inspection result $x$ against the current value, $n_{cum}$, of the cumulative sample size.

Calculate the leeway $y$ for that item as:

— $y = x - L$ in the case of combined control of double specification limits or a single lower specification limit;

— $y = U - x$ in the case of a single upper specification limit.

Record the cumulative leeway $Y$ as the sum of the leeways found so far in the sample from the lot.

## 7.4  Choice between numerical and graphical methods

This International Standard provides two methods of operating a sequential sampling plan: a numerical method and a graphical method, either one of which may be chosen.

The numerical method uses an acceptability table for operating, and has the advantage of being accurate, thereby avoiding disputes about acceptance or non-acceptance. An acceptability table can also be used as an inspection record sheet, after inscribing the inspection results.

The graphical method uses an acceptability chart for its operation, and has the advantage of displaying the increase in the information on the lot quality as additional items are inspected, information being represented by the piecewise linear curve within the indecision zone, until the line reaches, or crosses, one of the boundaries of that zone. On the other hand, the method is less accurate, due to the inaccuracy inherent in plotting points and in drawing lines.

The numerical method is the standard method so far as acceptance or non-acceptance is concerned. When the numerical method is applied, it is recommended that the calculation and preparation of an acceptability table be done using appropriate software.

The following provisions are based on the assumption that either the acceptability table or the acceptability chart is prepared on a sheet of paper. However, if a computer program is used, the acceptability table can be displayed on a computer screen so that entering a minimum of data may be sufficient to determine the acceptability of the lot. Furthermore, it is possible to incorporate additional features, such as

— displaying both the acceptability table and the acceptability chart on different windows of the same computer screen,

— printing out the inspection record sheet after determination of the lot acceptability, or

— compressing the inspection record to the necessary minimum.

## 7.5  Numerical method for a single specification limit

### 7.5.1  Acceptance and rejection values

When the numerical method is used, the following calculations shall be carried out and an acceptability table shall be prepared.

For each value, $n_{cum}$, of the cumulative sample size, that is less than the curtailment value of the sample size, the acceptance value, $A$, is given by the following equation:

$$A = g\sigma n_{cum} + h_A \sigma \tag{1}$$

For each value of $n_{cum}$, the rejection value, $R$, is given by the following equation:

$$R = g\sigma n_{cum} - h_R \sigma \tag{2}$$

The acceptance value $A_t$ corresponding to the curtailment sample size $n_t$ is determined as:

$$A_t = g\sigma n_t \tag{3}$$

The values, $A$ and $R$, given by Equations (1) and (2), shall be recorded to one decimal place more than the inspection results.

### 7.5.2 Determination of acceptability

Inscribe the leeway and the cumulative leeway into the acceptability table prepared in accordance with 7.5.1, after the inspection of each item.

Compare the cumulative leeway, $Y$, with the corresponding acceptance value, $A$, and rejection value, $R$.

a)  If the cumulative leeway, $Y$, is greater than or equal to the acceptance value, $A$, for the cumulative sample size, $n_{cum}$, the lot shall be considered acceptable and the inspection shall be terminated.

b)  If the cumulative leeway, $Y$, is less than or equal to the rejection value, $R$, for the cumulative sample size, $n_{cum}$, the lot shall be considered not acceptable and the inspection shall be terminated.

c)  If neither a) nor b) is satisfied, another item shall be sampled and inspected.

When the cumulative sample size reaches the curtailment value $n_t$, the lot shall be considered acceptable if $Y \geq A_t$, otherwise the lot shall be considered not acceptable.

## 7.6 Graphical method for a single specification limit

### 7.6.1 Acceptance chart

When the graphical method is used, an acceptability chart shall be prepared in accordance with the following procedures. Prepare a graph with the cumulative sample $n_{cum}$ as the horizontal axis, and the cumulative leeway, $Y$, as the vertical axis. Draw two straight lines with the same slope, $g\sigma$, corresponding to the acceptance and rejection values, $A$ and $R$, given by Equations (1) and (2). The lower line with the intercept of $-h_R\sigma$ is designated the "rejection line", and the upper line with the intercept of $h_A\sigma$ is designated the "acceptance line". Add a vertical line, the "curtailment line", at $n_{cum} = n_t$.

The lines define three zones on the chart.

—  The "acceptance zone" is the zone above (and including) the acceptance line together with that part of the curtailment line that is above and includes the point $(n_t, A_t)$.

—  The "rejection zone" is the zone below (and including) the rejection line together with that part of the curtailment line that is below the point $(n_t, A_t)$.

—  The "indecision zone" is the strip between acceptance and rejection lines that is to the left of the curtailment line.

An example of the prepared graph is given as Figure 1.
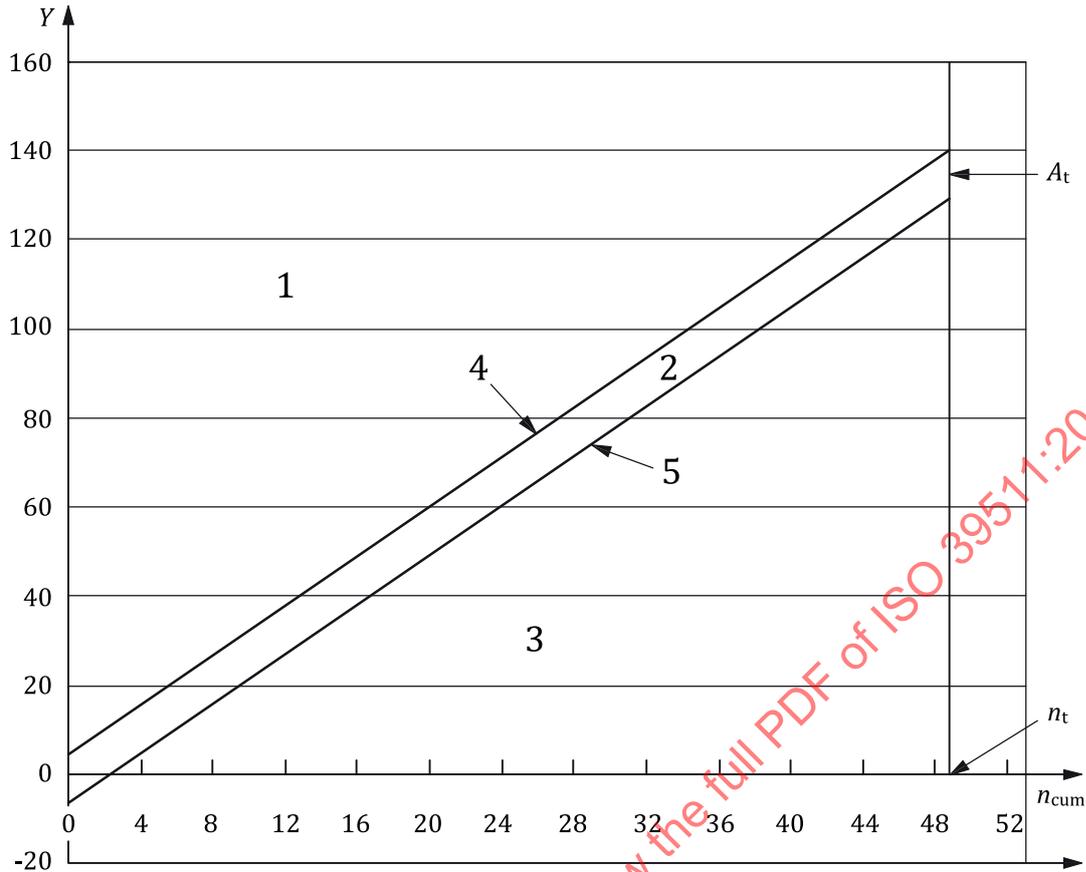
**Key**

1 acceptance zone
2 indecision zone
3 rejection zone
4 acceptance line
5 rejection line

**Figure 1 — Example of acceptance chart for the sequential sampling plan for a single specification limit**

### 7.6.2 Determination of acceptability

When the graphical method is used, the following procedures shall be followed.

Plot the point ($n_{cum}$, $Y$) on the acceptability chart prepared in accordance with 7.6.1, after the inspection of each item.

a) If the point lies in the acceptance zone, the lot shall be considered acceptable and the inspection shall be terminated.

b) If the point lies in the rejection zone, the lot shall be considered not acceptable and the inspection shall be terminated.

c) If the point lies in the indecision zone, another item shall be sampled and inspected.

The successive points on the acceptability chart shall be connected by a step curve to show up any trend in the inspection results.

**CAUTION — If the point is close to the acceptance or rejection lines, the numerical method shall be used to make the decision.**

## 7.7 Numerical method for combined control of double specification limits

### 7.7.1 Maximum values of process standard deviation

In the case of the combined control of double specification limits, sequential sampling is only applicable if the process standard deviation, $\sigma$, is sufficiently small in relation to the specification interval $(U - L)$. The limiting value of the process standard deviation is given by:

$$\sigma_{\text{max}} = (U - L)f$$

where $f$ depends only on the value of $Q_{\text{PR}}$, and can be found in Table 5.

If, in the case of the combined control of double specification limits, $\sigma$ exceeds $\sigma_{\text{max}}$, the lot shall immediately be judged not acceptable without a sample being drawn.

### 7.7.2 Acceptance and rejection values

When the numerical method is used, the following calculations shall be carried out and an acceptability table shall be prepared.

For each value, $n_{\text{cum}}$, of the cumulative sample size that is less than the curtailment value of the sample size, a pair of acceptance values and a pair of rejection values are determined.

The upper acceptance value, $A_U$, is found as:

$$A_U = (U - L - g\sigma)n_{\text{cum}} - h_A\sigma \tag{4}$$

The lower acceptance value, $A_L$, is found as:

$$A_L = g\sigma\, n_{\text{cum}} + h_A\sigma \tag{5}$$

The upper rejection value, $R_U$, is found as:

$$R_U = (U - L - g\sigma)n_{\text{cum}} + h_R\sigma \tag{6}$$

The lower rejection value, $R_L$, is found as:

$$R_L = g\sigma\, n_{\text{cum}} - h_R\sigma \tag{7}$$

Whenever the value of $A_U$ is less than the corresponding value of $A_L$, the cumulative sample size is too small to allow acceptance of the lot.

The acceptance values $A_{\text{t},U}$ and $A_{\text{t},L}$ that correspond to the curtailment sample size are determined as:

$$A_{\text{t},U} = (U - L - g\sigma)n_{\text{t}} \tag{8}$$

and

$$A_{\text{t},L} = g\sigma\, n_{\text{t}} \tag{9}$$

The acceptance and rejection values shall be recorded to one decimal place more than the inspection results.

### 7.7.3 Determination of acceptability

Inscribe the leeway and the cumulative leeway into the acceptability table prepared in accordance with 7.7.2, after the inspection of each item.

Compare the cumulative leeway, $Y$, with the corresponding upper and lower acceptance values, $A_U$ and $A_L$, and the corresponding upper and lower rejection values, $R_U$ and $R_L$.

a) If, for the cumulative sample size, $n_{cum}$, the cumulative leeway, $Y$, is greater than or equal to the lower acceptance value, $A_L$, and less than or equal to the upper acceptance value, $A_U$, the lot shall be considered acceptable and the inspection shall be terminated.

b) If, for the cumulative sample size, $n_{cum}$, the cumulative leeway, $Y$, is either less than or equal to the lower rejection value, $R_L$, or greater than or equal to the upper rejection value, $R_U$, the lot shall be considered not acceptable and the inspection shall be terminated.

c) If neither a) nor b) is satisfied, another item shall be sampled and inspected.

When the cumulative sample size reaches the curtailment value $n_t$, the lot shall be considered acceptable if $A_{t,L} \leq Y \leq A_{t,U}$, otherwise the lot shall be considered not acceptable.

## 7.8 Graphical method for combined control of double specification limits

### 7.8.1 Acceptance chart

When the graphical method is used, an acceptability chart shall be prepared in accordance with the following procedures. Prepare a graph with the cumulative sample size $n_{cum}$ as the horizontal axis, and the cumulative leeway, $Y$, as the vertical axis. Draw two straight lines with the same slope, $U - L - g\sigma$, corresponding to the upper acceptance and rejection values, $A_U$ and $R_U$, given by Equations (4) and (6), and two straight lines with the same slope, $g\sigma$, corresponding to the lower acceptance and rejection values, $A_L$ and $R_L$, given by Equations (5) and (7). Add a vertical line, the "curtailment line", at $n_{cum} = n_t$.

The uppermost line with the slope $U - L - g\sigma$ and intercept $h_R\sigma$ is called the "upper rejection line". The upper acceptance line has the slope $U - L - g\sigma$ and intercept $-h_A\sigma$. The lowermost line with the slope $g\sigma$ and intercept $-h_R\sigma$ is called the "lower rejection line". The lower acceptance line has the slope $g\sigma$ and intercept $h_A\sigma$.

The lines define the following zones on the chart.

— The "acceptance zone" is the triangular sector on the chart which is bounded by the upper acceptance line, the lower acceptance line and the curtailment line. The acceptance zone includes the two acceptance lines; moreover, that part of the curtailment line which is between (and including) the points $(n_t, A_{t,L})$ and $(n_t, A_{t,U})$ belongs to the acceptance zone.

— The "upper rejection zone" is the zone above (and including) the upper rejection line together with that part of the curtailment line that is above the point $(n_t, A_{t,U})$.

— The "lower rejection zone" is the zone below (and including) the lower rejection line together with that part of the curtailment line that is below the point $(n_t, A_{t,L})$.

— The "indecision zone" is the V-shaped strip between the acceptance and rejection zones that is to the left of the curtailment line.

An example of the prepared graph is given as Figure 2.
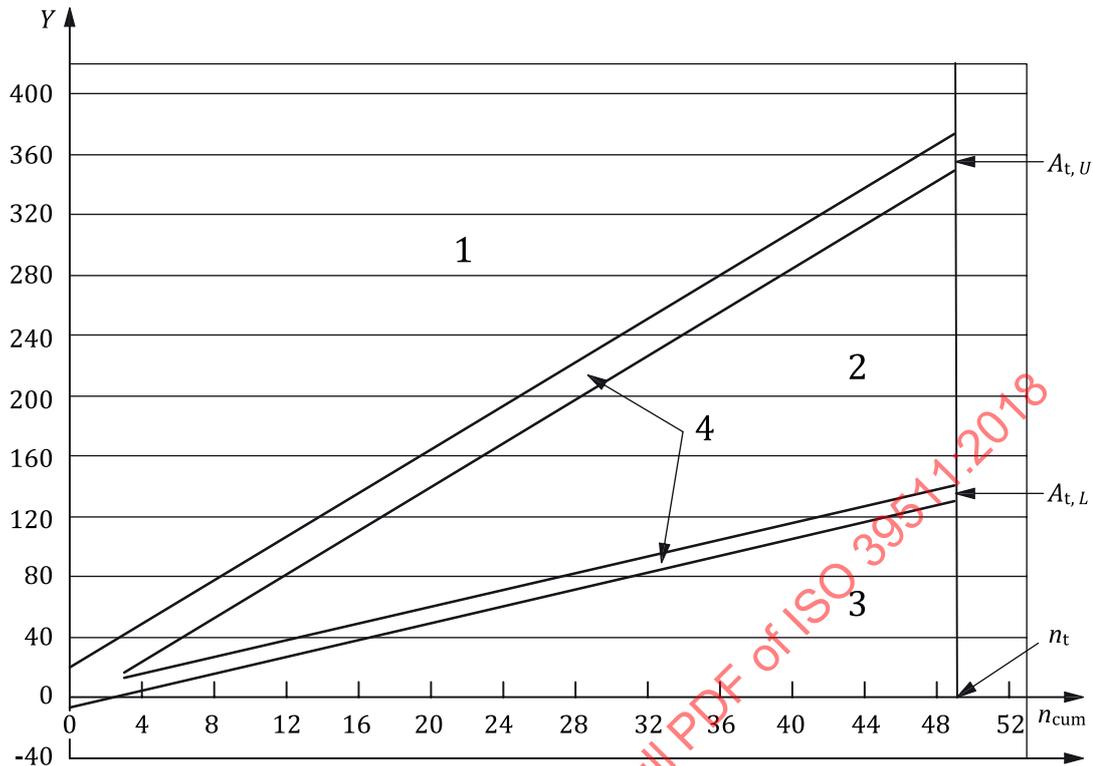
**Key**

1 upper rejection zone
2 acceptance zone
3 lower rejection zone
4 indecision zone(s)

**Figure 2 — Example of acceptance chart for the sequential sampling plan for combined control of double specification limits**

### 7.8.2 Determination of acceptability

When the graphical method is used, the following procedures shall be followed.

Plot the point ($n_{cum}$, $Y$) on the acceptability chart prepared in accordance with 7.8.1, after the inspection of each item.

a) If the point lies in the acceptance zone, the lot shall be considered acceptable and the inspection shall be terminated.

b) If the point lies in one of the rejection zones, the lot shall be considered not acceptable and the inspection shall be terminated.

c) If the point lies in the indecision zone, another item shall be sampled and inspected.

The successive points on the acceptability chart shall be connected by a step curve to show up any trend in the inspection results.

**CAUTION — If the point is close to the acceptance or rejection lines, the numerical method shall be used to make the decision.**

## 7.9 Numerical method for separate control of double specification limits

### 7.9.1 Maximum values of process standard deviation

In the case of the separate control of double specification limits, sequential sampling is only applicable if the process standard deviation $\sigma$ is sufficiently small in relation to the specification interval $(U - L)$. The limiting value of the process standard deviation is given by:

$$\sigma_{\max} = (U - L) f$$

where $f$ depends only on the values of $Q_{PR}$ specified for the upper and lower limit, and can be found in Table 6.

If, in the case of the separate control of double specification limits, $\sigma$ exceeds $\sigma_{\max}$, the lot shall immediately be judged not acceptable without a sample being drawn.

### 7.9.2 Acceptance and rejection values

When the numerical method is used, the following calculations shall be carried out and an acceptability table shall be prepared.

For each value, $n_{\text{cum}}$, of the cumulative sample size, that is less than the curtailment value of the sample size, a pair of acceptance values and a pair of rejection values are determined.

The acceptance value $A_U$ for the upper specification limit is found as:

$$A_U = (U - L - g_U \sigma) n_{\text{cum}} - h_{A,U} \sigma \qquad (10)$$

The acceptance value $A_L$ for the lower specification limit is found as:

$$A_L = g_L \sigma n_{\text{cum}} + h_{A,L} \sigma \qquad (11)$$

The rejection value $R_U$ for the upper specification limit is found as:

$$R_U = (U - L - g_U \sigma) n_{\text{cum}} + h_{R,U} \sigma \qquad (12)$$

The rejection value $R_L$ for the lower specification limit is found as:

$$R_L = g_L \sigma n_{\text{cum}} - h_{R,L} \sigma \qquad (13)$$

The acceptance values $A_{t,U}$ and $A_{t,L}$ corresponding to the curtailment sample size are determined as:

$$A_{t,U} = (U - L - g_U \sigma) n_t \qquad (14)$$

and

$$A_{t,L} = g_L n_t \sigma \qquad (15)$$

The acceptance and rejection values shall be recorded to one decimal place more than the inspection results.

### 7.9.3 Determination of acceptability

#### 7.9.3.1 General

Inscribe the leeway and the cumulative leeway into the acceptability table prepared in accordance with 7.9.2, after the inspection of each item.

The acceptability criteria in 7.9.3.2 and 7.9.3.3 shall be applied to determine the acceptability for each specification limit separately. The lot shall be considered acceptable and inspection shall terminate if the lot has been considered acceptable with respect to both limits according to 7.9.3.2 a) and 7.9.3.3 a).

#### 7.9.3.2 Determination of acceptability for the upper specification limit

Compare the cumulative leeway $Y$ with the corresponding acceptance value $A_U$ and rejection value $R_U$.

a) If the cumulative leeway, $Y$, is less than or equal to the acceptance value, $A_U$, for the cumulative sample size, $n_{cum}$, the lot shall be considered acceptable with respect to the upper specification limit and the inspection with respect to that limit shall be terminated.

b) If the cumulative leeway, $Y$, is greater than or equal to the rejection value, $R_U$, for the cumulative sample size, $n_{cum}$, the lot shall be considered not acceptable and the inspection with respect to both limits shall be terminated.

c) If neither a) nor b) is satisfied, another item shall be sampled and inspected with respect to the upper specification limit.

When the cumulative sample size reaches the curtailment value $n_t$, the lot shall be considered not acceptable if $Y > A_{t,U}$ and inspection shall terminate.

When the cumulative sample size reaches the curtailment value $n_t$ and $Y \leq A_{t,U}$, the lot shall be considered acceptable with respect to the upper limit. If the lot has already been considered acceptable with respect to the lower limit, or if $Y \geq A_{t,L}$, the lot shall be considered acceptable and inspection shall terminate, otherwise the lot shall be considered not acceptable, and inspection shall terminate.

#### 7.9.3.3 Determination of acceptability for the lower specification limit

Compare the cumulative leeway $Y$ with the corresponding acceptance value $A_L$ and rejection value $R_L$.

a) If the cumulative leeway, $Y$, is greater than or equal to the acceptance value, $A_L$, for the cumulative sample size, $n_{cum}$, the lot shall be considered acceptable with respect to the lower specification limit and the inspection with respect to that limit shall be terminated.

b) If the cumulative leeway, $Y$, is less than or equal to the rejection value, $R_L$, for the cumulative sample size, $n_{cum}$, the lot shall be considered not acceptable and the inspection with respect to both limits shall be terminated.

c) If neither a) nor b) is satisfied, another item shall be sampled and inspected with respect to the lower specification limit.

When the cumulative sample size reaches the curtailment value $n_t$, the lot shall be considered not acceptable if $Y < A_{t,L}$ and inspection shall terminate.

When the cumulative sample size reaches the curtailment value $n_t$ and $Y \geq A_{t,L}$, the lot shall be considered acceptable with respect to the lower limit. If the lot has already been considered acceptable with respect to the upper limit, or if $Y \leq A_{t,U}$, the lot shall be considered acceptable and inspection shall terminate, otherwise the lot shall be considered not acceptable, and inspection shall terminate.

## 7.10 Graphical method for separate control of double specification limits

### 7.10.1 Acceptance chart

When the graphical method is used, an acceptability chart shall be prepared in accordance with the following procedures. Prepare a graph with the cumulative sample $n_{cum}$ as the horizontal axis, and the cumulative leeway, $Y$, as the vertical axis. Draw two straight lines with the same slope, $U - L - g_U\sigma$, corresponding to the upper acceptance and rejection values, $A_U$ and $R_U$, given by Equations (10) and (12), and two straight lines with the same slope, $g_L\sigma$, corresponding to the lower acceptance and rejection values, $A_L$ and $R_L$, given by Equations (11) and (13). Add a vertical line, the "curtailment line", at $n_{cum} = n_t$.

The uppermost line with the slope $U - L - g_U\sigma$ and intercept $h_{R,U}\sigma$ is called the "upper rejection line". The upper acceptance line has the slope $U - L - g_U\sigma$ and intercept $-h_{A,U}\sigma$. The lowermost line with the slope $g_L\sigma$ and intercept $-h_{R,L}\sigma$ is called the "lower rejection line". The lower acceptance line has the slope $g_L\sigma$ and intercept $h_{A,L}\sigma$.

The lines define the following zones on the chart.

— The "acceptance zone for the upper specification limit" is the zone below (and including) the acceptance line for the upper specification limit together with that part of the curtailment line that is below and includes the point $(n_t, A_{t,U})$.

— The "rejection zone for the upper specification limit" is the zone above (and including) the rejection line for the upper specification limit together with that part of the curtailment line that is above the point $(n_t, A_{t,U})$.

— The "indecision zone for the upper specification limit" is the strip between acceptance and rejection lines for the upper specification limit that is to the left of the curtailment line.

— The "acceptance zone for the lower specification limit" is the zone above (and including) the acceptance line for the lower specification limit together with that part of the curtailment line that is above and includes the point $(n_t, A_{t,L})$.

— The "rejection zone for the lower specification limit" is the zone below (and including) the rejection line for the lower specification limit together with that part of the curtailment line that is below the point $(n_t, A_{t,L})$.

— The "indecision zone for the lower specification limit" is the strip between acceptance and rejection lines for the lower specification limit that is to the left of the curtailment line.

An example of the prepared graph is given as Figure 3.
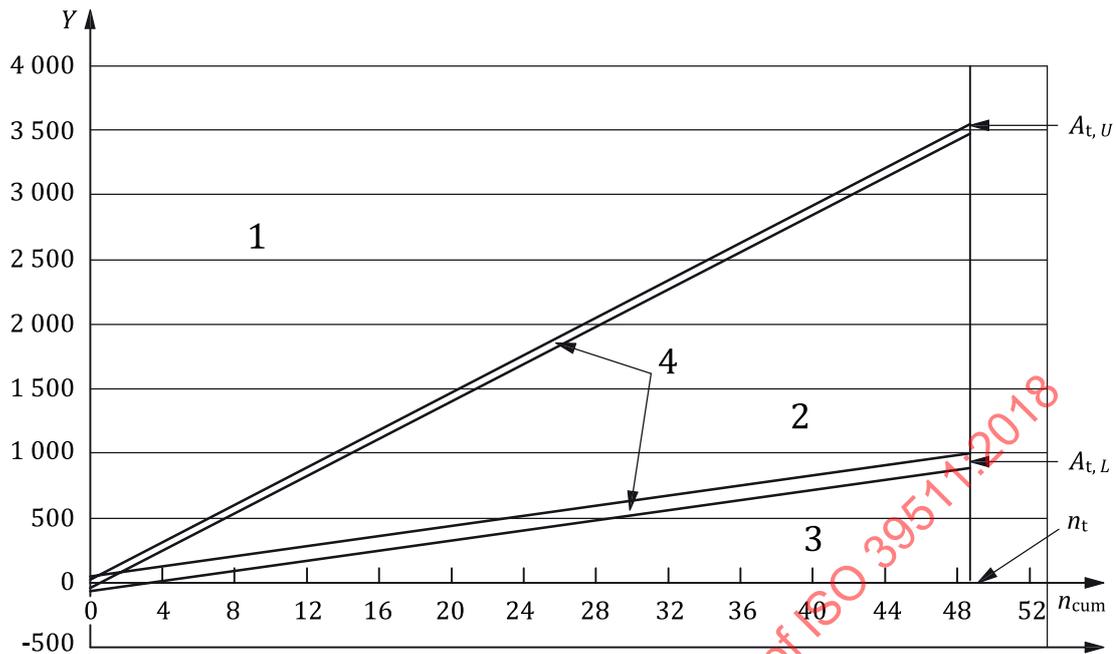
**Key**

1   upper rejection zone
2   acceptance zone
3   lower rejection zone
4   indecision zone(s)

**Figure 3 — Acceptance chart for the sequential sampling plan for separate control of double specification limits**

### 7.10.2 Determination of acceptability

#### 7.10.2.1 General

When the graphical method is used, the following procedures shall be followed.

Plot the point ($n_{cum}$, $Y$) on the acceptability chart prepared in accordance with 7.10.1, after the inspection of each item.

The acceptability criteria in 7.10.2.2 and 7.10.2.3 shall be applied to determine the acceptability for each specification limit separately. The lot shall be considered acceptable and inspection shall terminate if the lot has been considered acceptable with respect to both limits according to 7.10.2.2 a) and 7.10.2.3 a).

The successive points on the acceptability chart shall be connected by a step curve to show up any trend in the inspection results.

**CAUTION — If the point is close to the acceptance or rejection lines, the numerical method shall be used to make the decision.**

#### 7.10.2.2 Determination of acceptability for the upper specification limit

Apply the following criteria.

a)   If the point lies in the acceptance zone for the upper specification limit, the lot shall be considered acceptable with respect to the upper specification limit and inspection with respect to that limit shall terminate.

b) If the point lies in the rejection zone for the upper specification limit, the lot shall be considered not acceptable with respect to the upper specification limit and inspection with respect to both limits shall terminate.

c) If the point lies in the indecision zone for the upper specification limit, another item shall be sampled and inspected with respect to the upper specification limit.

### 7.10.2.3 Determination of acceptability for the lower specification limit

Apply the following criteria.

a) If the point lies in the acceptance zone for the lower specification limit, the lot shall be considered acceptable with respect to the lower specification limit and inspection with respect to that limit shall terminate.

b) If the point lies in the rejection zone for the lower specification limit, the lot shall be considered not acceptable with respect to the lower specification limit and inspection with respect to both limits shall terminate.

c) If the point lies in the indecision zone for the lower specification limit, another item shall be sampled and inspected with respect to the lower specification limit.

## 8 Examples

### 8.1 Example 1

The specified minimum withstand voltage for certain insulators is 200 kV. Lots from a steady production are submitted for inspection. Production is stable and it has been verified that the withstand voltages vary within a lot in accordance with a normal distribution. It has further been documented that the standard deviation within a lot is stable and can be taken to be $\sigma$ = 1,2 kV.

It has been decided to use a sequential sampling plan with the following properties.

a) If submitted quality is 0,5 % nonconforming, then the probability of acceptance shall be 0,95.

b) If submitted quality is 2,0 % nonconforming, then the probability of acceptance shall be 0,10.

These requirements are achieved by fixing the producer's risk quality at $Q_{PR}$ = 0,5 %, and the consumer's risk quality at $Q_{CR}$ = 2,0 %.

The specification refers to a single, lower limit. From Table 4, it is seen that the required sequential sampling plan has the parameters:

— $h_A$ = 3,826;

— $h_R$ = 5,258;

— $g$ = 2,315;

— $n_t$ = 49.

The equation for the acceptance value $A$ becomes:

$$A = 2,778 n_{cum} + 4,591$$

and the equation for the rejection value $R$ becomes:

$$R = 2,778 n_{cum} - 6,310$$

The acceptance and rejection values corresponding to the cumulative sample sizes $n_{cum}$ = 1,2, …, 48 are determined by successively inserting the values of $n_{cum}$ in these equations. The acceptance value $A_t$ that corresponds to the curtailment sample size is determined from:

$$A_t = 2,778 n_t$$

with the curtailment sample size $n_t$ = 49.

Since the withstand voltage of the insulators is determined to one decimal place, the acceptance and rejection values are rounded to two decimal places.

| | Information needed | Value obtained |
|---|---|---|
| $g$ | slope of the acceptance and rejection lines | 2,315 |
| $h_A$ | intercept of the acceptance line | 3,826 |
| $h_R$ | intercept of the rejection line | 5,258 |
| $n_t$ | curtailment value | 49 |
| $\sigma$ | known standard deviation | 1,2 kV |
| $L$ | lower specification limit | 200 kV |

**Table 1 — Example of the operation of the sequential sampling plan in the case of a single specification limit**

| Cumulative sample size $n_{cum}$ | Inspection result $x$ kV | Leeway $y$ | Rejection value $R$ | Cumulative leeway $Y$ | Acceptance value $A$ |
|---|---|---|---|---|---|
| 1 | 202,5 | 2,5 | −3,53 | 2,5 | 7,37 |
| 2 | 203,8 | 3,8 | −0,75 | 6,3 | 10,15 |
| 3 | 201,9 | 1,9 | 2,02 | 8,2 | 12,93 |
| 4 | 205,6 | 5,6 | 4,80 | 13,8 | 15,70 |
| 5 | 199,9 | −0,1 | 7,58 | 13,7 | 18,48 |
| 6 | 202,7 | 2,7 | 10,36 | 16,4 | 21,26 |
| 7 | 203,2 | 3,2 | 13,14 | 19,6 | 24,04 |
| 8 | 203,6 | 3,6 | 15,91 | 23,2 | 26,82 |
| 9 | 204,0 | 4,0 | 18,69 | 27,2 | 29,59 |
| 10 | 203,6 | 3,6 | 21,47 | 30,8 | 32,37 |
| 11 | 203,3 | 3,3 | 24,25 | 34,1 | 35,15 |
| 12 | 204,7 | 4,7 | 27,03 | 38,8[a] | 37,93 |
| [a] The lot is accepted. | | | | | |

The sample mean of the lot meets the acceptability criterion so the lot is accepted.

## 8.2   Example 2

The specification for the dimension of an industrially manufactured mechanical part is 205 mm ± 5 mm. Production is stable and it has been verified that the dimension varies within a lot in accordance with a normal distribution. It has further been documented that the standard deviation within a lot is stable and can be taken to be $\sigma = 1,2$ mm.

It has been decided to use a sequential sampling plan with the producer's risk quality $Q_{PR} = 0,5$ %, and the consumer's risk quality $Q_{CR} = 2$ % for both limits combined.

The parameters of the sampling plan are found from Table 4 to be $h_A = 3,826$, $h_R = 5,258$, $g = 2,315$ and $n_t = 49$.

The equations for the upper and lower acceptance values $A_U$ and $A_L$ become:

$$A_U = 7,222 n_{cum} - 4,591$$

and

$$A_L = 2,778 n_{cum} + 4,591$$

Similarly, the equations for the upper and lower rejection values $R_U$ and $R_L$ become:

$$R_U = 7,222 n_{cum} + 6,310$$

and

$$R_L = 2,778 n_{cum} - 6,310$$

The acceptance and rejection values corresponding to the cumulative sample sizes $n_{cum} = 1, 2, ..., 48$ are determined by successively inserting the values of $n_{cum}$ in these equations. The acceptance values $A_{t,U}$ and $A_{t,L}$ that correspond to the curtailment sample size are determined from:

$$A_{t,U} = 7,222 n_t$$

and

$$A_{t,L} = 2,778 n_t$$

with the curtailment sample size $n_t = 49$.

Since the dimension of the mechanical part is determined to one decimal place, the acceptance and rejection values are rounded to two decimal places.

| | **Information needed** | **Value obtained** |
|---|---|---|
| $f$ | factor from Table 5 | 0,165 |
| $g$ | slope of the acceptance and rejection lines | 2,315 |
| $h_A$ | intercept of the acceptance line | 3,826 |
| $h_R$ | intercept of the rejection line | 5,258 |
| $n_t$ | curtailment value | 49 |
| $\sigma$ | known standard deviation | 1,2 mm |
| $L$ | lower specification limit | 200 mm |
| $U$ | upper specification limit | 210 mm |
| Maximum process standard deviation, $\sigma_{max}$: $(U - L)f$ | | 1,65 mm |

As $\sigma$ is less than $\sigma_{max}$, the sample is analysed further for lot acceptability.

**Table 2 — Example of the operation of the sequential sampling plan in the case of combined control of double specification limits**

| Cumulative sample size $n_{cum}$ | Inspection result $x$ mm | Leeway $y$ | Lower rejection value $R_L$ | Lower acceptance value $A_L$ | Cumulative leeway $Y$ | Upper acceptance value $A_U$ | Upper rejection value $R_U$ |
|---|---|---|---|---|---|---|---|
| 1 | 202,5 | 2,5 | −3,53 | 7,37[a] | 2,5 | 2,63[a] | 13,53 |
| 2 | 203,8 | 3,8 | −0,75 | 10,15[a] | 6,3 | 9,85[a] | 20,75 |
| 3 | 201,9 | 1,9 | 2,02 | 12,93 | 8,2 | 17,08 | 27,98 |
| 4 | 205,6 | 5,6 | 4,80 | 15,70 | 13,8 | 24,30 | 35,20 |
| 5 | 199,9 | −0,1 | 7,58 | 18,48 | 13,7 | 31,52 | 42,42 |
| 6 | 202,7 | 2,7 | 10,36 | 21,26 | 16,4 | 38,74 | 49,64 |
| 7 | 203,2 | 3,2 | 13,14 | 24,04 | 19,6 | 45,96 | 56,86 |
| 8 | 203,6 | 3,6 | 15,91 | 26,82 | 23,2 | 53,19 | 64,09 |
| 9 | 204,0 | 4,0 | 18,69 | 29,59 | 27,2 | 60,41 | 71,31 |
| 10 | 203,6 | 3,6 | 21,47 | 32,37 | 30,8 | 67,63 | 78,53 |
| 11 | 203,3 | 3,3 | 24,25 | 35,15 | 34,1 | 74,85 | 85,75 |
| 12 | 204,7 | 4,7 | 27,03 | 37,93 | 38,8[b] | 82,07 | 92,97 |

[a] Acceptance is not permitted for this cumulative sample size since the lower acceptance value would have exceeded the upper acceptance value.

[b] The lot is accepted.

The sample mean of the lot meets the acceptability criterion so the lot is acceptable.

NOTE 1    For a single sampling plan from ISO 2859-1, the required sample size is $n = 32$.

NOTE 2    If, for example, $\sigma$ had been known to be 2,0 mm, then $\sigma$ exceeds the $\sigma_{max}$ and therefore sampling inspection should not even have taken place.

## 8.3 Example 3

The specification for the output voltage of an electronic component is 5 950 mV ± 50 mV. Production is stable and it has been verified that the output voltage varies within a lot in accordance with a normal distribution. It has further been documented that the standard deviation within a lot is stable and can be taken to be $\sigma = 12$ mV.

It has been decided to use a sequential sampling plan with the producer's risk quality $Q_{\mathrm{PR},U} = 0{,}5$ %, and the consumer's risk quality $Q_{\mathrm{CR},U} = 2$ % for the upper specification limit $U = 6\,000$ mV and the producer's risk quality $Q_{\mathrm{PR},L} = 2{,}5$ %, and the consumer's risk quality $Q_{\mathrm{CR},L} = 10$ % for the lower specification limit $L = 5\,900$ mV.

Since the quality levels have been specified for each of the limits separately, two sets of parameters are determined for the sequential sampling plan

The parameters of the sampling plan referring to the upper specification limit are found from Table 4 to be $h_{\mathrm{A},U} = 3{,}826$, $h_{\mathrm{R},U} = 5{,}258$, $g_U = 2{,}315$ and $n_{\mathrm{t},U} = 49$.

Similarly, the parameters of the sampling plan referring to the lower specification limit are found from Table 4 to be $h_{\mathrm{A},L} = 2{,}812$, $h_{\mathrm{R},L} = 3{,}914$, $g_L = 1{,}621$ and $n_{\mathrm{t},L} = 29$.

Since the larger of the two curtailment values is $n_{\mathrm{t},U} = 49$, the curtailment value that shall be used for the sequential sampling plan is $n_{\mathrm{t}} = 49$.

The equations for the acceptance value $A_U$ and the rejection value $R_U$ corresponding to the upper specification limit become:

$$A_U = 72{,}22 n_{\mathrm{cum}} - 45{,}91$$

and

$$R_U = 72{,}22 n_{\mathrm{cum}} + 63{,}10$$

Similarly, the equations for the acceptance value $A_L$ and the rejection value $R_L$ corresponding to the lower specification limit become:

$$A_L = 19{,}45 n_{\mathrm{cum}} + 33{,}74$$

and

$$R_L = 19{,}45 n_{\mathrm{cum}} - 46{,}97$$

The acceptance and rejection values corresponding to the cumulative sample sizes $n_{\mathrm{cum}} = 1, 2, \ldots, 48$ are determined by successively inserting the values of $n_{\mathrm{cum}}$ in these equations. The acceptance values $A_{\mathrm{t},U}$ and $A_{\mathrm{t},L}$ corresponding to the curtailment sample size are determined from:

$$A_{\mathrm{t},U} = 72{,}22 n_{\mathrm{t}}$$

and

$$A_{\mathrm{t},L} = 19{,}45 n_{\mathrm{t}}$$

with the curtailment sample size $n_{\mathrm{t}} = 49$.

Since the output voltage of the insulators is determined in millivolts without decimals, the acceptance and rejection values are rounded to one decimal place. The results are shown in Figure 3.

| | **Information needed** | **Value obtained** |
|---|---|---|
| $f$ | factor from Table 6 | 0,220 |
| $g_U$ | slope of the acceptance and rejection lines for upper limit | 2,315 |
| $h_{A,U}$ | intercept of the acceptance line for upper limit | 3,826 |
| $h_{R,U}$ | intercept of the rejection line for upper limit | 5,258 |
| $n_t$ | curtailment value | 49 |
| $g_L$ | slope of the acceptance and rejection lines for lower limit | 1,621 |
| $h_{A,L}$ | intercept of the acceptance line for lower limit | 2,812 |
| $h_{R,L}$ | intercept of the rejection line for lower limit | 3,914 |
| $\sigma$ | known standard deviation | 12 mV |
| $L$ | lower specification limit | 5 900 |
| $U$ | upper specification limit | 6 000 |
| Maximum process standard deviation, $\sigma_{max}$: $(U - L)f$ | | 22 mV |

As $\sigma$ is less than $\sigma_{max}$, the sample is analysed further for lot acceptability.

**Table 3 — Example of the operation of the sequential sampling plan in the case of separate control of double specification limits**

| Cumulative sample size | Inspection result | Leeway | Rejection value for lower limit | Acceptance value for lower limit | Cumulative leeway | Acceptance value for upper limit | Rejection value for upper limit |
|---|---|---|---|---|---|---|---|
| $n_{cum}$ | $x$ mV | $y$ | $R_L$ | $A_L$ | $Y$ | $A_U$ | $R_U$ |
| 1 | 5 930 | 30 | −27,5 | 53,2 | 30 | 26,3 | 135,3 |
| 2 | 5 909 | 9 | −8,1 | 72,6 | 39 | 98,5 | 207,5 |
| 3 | 5 921 | 21 | 11,4 | 92,1 | 60 | 170,7 | 279,8 |
| 4 | 5 924 | 24 | 30,8 | 111,6 | 84 | 243,0 | 352,0 |
| 5 | 5 927 | 27 | 50,3 | 131,0 | 111 | 315,2 | 424,2 |
| 6 | 5939 | 39 | 69,7 | 150,5 | 150 | 387,4 | 496,4 |
| 7 | 5 914 | 14 | 89,2 | 169,9 | 164 | 459,6 | 568,6 |
| 8 | 5 916 | 16 | 108,6 | 189,4 | 180 | 531,8 | 640,9 |
| 9 | 5 932 | 32 | 128,1 | 208,8 | 212[a] | 604,1 | 713,1 |

| |
|---|
| [a]  The lot is accepted. |

## 9   Tables

Tables 4 to 6 are given on the following pages.

Table 4 — Parameters for sequential sampling plans for percent nonconforming (Master table for $\alpha \approx 0,05$ and $\beta \approx 0,1$)

Table 5 — Values of $f$ for maximum process standard deviation (combined control of double specification limits)

Table 6 — Values of $f$ for maximum process standard deviation (separate control of double specification limits)

**Table 4 — Parameters for sequential sampling plans for percent nonconforming (Master table for $\alpha \sim 0{,}05$ and $\beta \sim 0{,}1$)**

| $Q_{PR}$ (in %) | Parameters | $Q_{CR}$ (in %) | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0,800 | 1,00 | 1,25 | 1,60 | 2,00 | 2,50 | 3,15 | 4,00 | 5,00 | 6,30 | 8,00 | 10,0 | 12,5 | 16,0 | 20,0 | 25,0 | 31,5 |
| 0,100 | $h_A$ | 2,794 | 2,431 | 2,126 | 1,842 | 1,636 | 1,452 | 1,273 | 1,125 | 0,976 | 0,846 | 0,715 | 0,609 | 0,492 | 0,371 | 0,254 | 0,138 | 0,012 |
| | $h_R$ | 3,882 | 3,403 | 2,987 | 2,593 | 2,331 | 2,092 | 1,840 | 1,667 | 1,460 | 1,304 | 1,142 | 1,035 | 0,894 | 0,764 | 0,634 | 0,508 | 0,377 |
| | $g$ | 2,750 | 2,708 | 2,666 | 2,617 | 2,572 | 2,525 | 2,475 | 2,420 | 2,368 | 2,310 | 2,248 | 2,186 | 2,120 | 2,042 | 1,966 | 1,882 | 1,786 |
| | $n_t$ | 29 | 23 | 19 | 16 | 13 | 11 | 10 | 8 | 8 | 7 | 7 | 5 | 5 | 4 | 4 | 4 | 4 |
| 0,125 | $h_A$ | 3,168 | 2,715 | 2,349 | 2,019 | 1,774 | 1,572 | 1,384 | 1,205 | 1,067 | 0,926 | 0,783 | 0,675 | 0,549 | 0,418 | 0,304 | 0,184 | 0,055 |
| | $h_R$ | 4,396 | 3,773 | 3,271 | 2,816 | 2,487 | 2,229 | 1,984 | 1,742 | 1,583 | 1,409 | 1,225 | 1,120 | 0,962 | 0,810 | 0,688 | 0,557 | 0,422 |
| | $g$ | 2,716 | 2,675 | 2,632 | 2,584 | 2,539 | 2,492 | 2,441 | 2,387 | 2,334 | 2,277 | 2,214 | 2,152 | 2,087 | 2,009 | 1,932 | 1,849 | 1,753 |
| | $n_t$ | 35 | 28 | 23 | 19 | 16 | 13 | 11 | 10 | 8 | 7 | 7 | 5 | 5 | 5 | 4 | 4 | 4 |
| 0,160 | $h_A$ | 3,688 | 3,119 | 2,663 | 2,269 | 1,992 | 1,749 | 1,516 | 1,337 | 1,158 | 1,012 | 0,866 | 0,734 | 0,619 | 0,480 | 0,362 | 0,236 | 0,104 |
| | $h_R$ | 5,075 | 4,309 | 3,684 | 3,157 | 2,814 | 2,488 | 2,145 | 1,933 | 1,678 | 1,510 | 1,330 | 1,164 | 1,048 | 0,880 | 0,755 | 0,614 | 0,472 |
| | $g$ | 2,678 | 2,637 | 2,595 | 2,546 | 2,501 | 2,454 | 2,404 | 2,349 | 2,296 | 2,239 | 2,176 | 2,115 | 2,049 | 1,971 | 1,895 | 1,811 | 1,715 |
| | $n_t$ | 46 | 35 | 28 | 22 | 17 | 14 | 13 | 10 | 8 | 8 | 7 | 7 | 5 | 5 | 4 | 4 | 4 |
| 0,200 | $h_A$ | 4,337 | 3,588 | 3,022 | 2,554 | 2,208 | 1,914 | 1,666 | 1,458 | 1,269 | 1,111 | 0,952 | 0,806 | 0,689 | 0,540 | 0,412 | 0,287 | 0,151 |
| | $h_R$ | 5,970 | 4,938 | 4,169 | 3,567 | 3,101 | 2,685 | 2,356 | 2,097 | 1,835 | 1,647 | 1,445 | 1,255 | 1,139 | 0,951 | 0,804 | 0,670 | 0,522 |
| | $g$ | 2,644 | 2,602 | 2,560 | 2,511 | 2,466 | 2,419 | 2,369 | 2,314 | 2,262 | 2,204 | 2,142 | 2,080 | 2,014 | 1,936 | 1,860 | 1,776 | 1,680 |
| | $n_t$ | 59 | 44 | 34 | 25 | 20 | 17 | 14 | 11 | 10 | 8 | 7 | 7 | 5 | 5 | 5 | 4 | 4 |
| 0,250 | $h_A$ | 5,208 | 4,204 | 3,495 | 2,887 | 2,457 | 2,133 | 1,837 | 1,588 | 1,387 | 1,197 | 1,033 | 0,887 | 0,743 | 0,605 | 0,470 | 0,341 | 0,200 |
| | $h_R$ | 7,109 | 5,756 | 4,836 | 4,001 | 3,410 | 3,001 | 2,584 | 2,255 | 1,989 | 1,733 | 1,537 | 1,356 | 1,176 | 1,030 | 0,868 | 0,731 | 0,574 |
| | $g$ | 2,608 | 2,567 | 2,524 | 2,476 | 2,430 | 2,383 | 2,333 | 2,279 | 2,226 | 2,169 | 2,106 | 2,044 | 1,979 | 1,901 | 1,824 | 1,741 | 1,644 |
| | $n_t$ | 83 | 58 | 41 | 31 | 25 | 19 | 16 | 13 | 11 | 10 | 8 | 7 | 7 | 5 | 5 | 4 | 4 |
| 0,315 | $h_A$ | 6,564 | 5,104 | 4,117 | 3,345 | 2,815 | 2,395 | 2,041 | 1,769 | 1,519 | 1,326 | 1,145 | 0,911 | 0,823 | 0,680 | 0,534 | 0,396 | 0,253 |
| | $h_R$ | 8,929 | 6,971 | 5,653 | 4,636 | 3,918 | 3,344 | 2,852 | 2,522 | 2,151 | 1,918 | 1,699 | 1,452 | 1,274 | 1,127 | 0,946 | 0,785 | 0,632 |
| | $g$ | 2,570 | 2,529 | 2,487 | 2,438 | 2,393 | 2,346 | 2,295 | 2,241 | 2,188 | 2,131 | 2,068 | 2,007 | 1,941 | 1,863 | 1,787 | 1,703 | 1,607 |
| | $n_t$ | 125 | 80 | 55 | 38 | 29 | 23 | 19 | 14 | 13 | 10 | 8 | 8 | 7 | 5 | 5 | 5 | 4 |

**Table 4** *(continued)*

| $Q_{PR}$ (in %) | Parameters | $Q_{CR}$ (in %) | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0,800 | 1,00 | 1,25 | 1,60 | 2,00 | 2,50 | 3,15 | 4,00 | 5,00 | 6,30 | 8,00 | 10,0 | 12,5 | 16,0 | 20,0 | 25,0 | 31,5 |
| 0,400 | $h_A$ | 8,919 | 6,512 | 5,039 | 3,952 | 3,269 | 2,743 | 2,313 | 1,967 | 1,697 | 1,470 | 1,246 | 1,082 | 0,915 | 0,744 | 0,607 | 0,460 | 0,313 |
| | $h_R$ | 12,090 | 8,868 | 6,908 | 5,416 | 4,527 | 3,820 | 3,231 | 2,775 | 2,404 | 2,117 | 1,801 | 1,600 | 1,394 | 1,175 | 1,032 | 0,857 | 0,698 |
| | $g$ | 2,530 | 2,489 | 2,447 | 2,398 | 2,353 | 2,306 | 2,256 | 2,201 | 2,148 | 2,091 | 2,029 | 1,967 | 1,901 | 1,823 | 1,747 | 1,663 | 1,567 |
| | $n_t$ | 218 | 122 | 77 | 52 | 37 | 28 | 22 | 17 | 14 | 11 | 10 | 8 | 7 | 7 | 5 | 5 | 4 |
| 0,500 | $h_A$ | 13,263 | 8,674 | 6,323 | 4,751 | 3,826 | 3,158 | 2,631 | 2,205 | 1,886 | 1,614 | 1,396 | 1,183 | 1,002 | 0,823 | 0,683 | 0,525 | 0,374 |
| | $h_R$ | 17,874 | 11,758 | 8,610 | 6,506 | 5,258 | 4,377 | 3,675 | 3,097 | 2,666 | 2,296 | 1,970 | 1,698 | 1,494 | 1,274 | 1,130 | 0,932 | 0,770 |
| | $g$ | 2,492 | 2,451 | 2,409 | 2,360 | 2,315 | 2,268 | 2,218 | 2,163 | 2,110 | 2,053 | 1,990 | 1,929 | 1,863 | 1,785 | 1,709 | 1,625 | 1,529 |
| | $n_t$ | 463 | 208 | 116 | 71 | 49 | 35 | 26 | 20 | 16 | 13 | 11 | 10 | 8 | 7 | 5 | 5 | 4 |
| 0,630 | $h_A$ | 26,286 | 13,137 | 8,522 | 6,002 | 4,641 | 3,727 | 3,029 | 2,501 | 2,121 | 1,787 | 1,531 | 1,307 | 1,117 | 0,917 | 0,749 | 0,598 | 0,431 |
| | $h_R$ | 35,313 | 17,693 | 11,551 | 8,185 | 6,349 | 5,142 | 4,179 | 3,479 | 2,983 | 2,509 | 2,145 | 1,889 | 1,656 | 1,397 | 1,200 | 1,021 | 0,826 |
| | $g$ | 2,452 | 2,411 | 2,368 | 2,320 | 2,274 | 2,227 | 2,177 | 2,123 | 2,070 | 2,012 | 1,950 | 1,888 | 1,823 | 1,745 | 1,668 | 1,585 | 1,488 |
| | $n_t$ | 1 739 | 454 | 202 | 106 | 68 | 46 | 34 | 25 | 19 | 16 | 13 | 10 | 8 | 7 | 7 | 5 | 5 |
| 0,800 | $h_A$ | | 27,416 | 13,215 | 8,149 | 5,918 | 4,556 | 3,607 | 2,913 | 2,430 | 2,019 | 1,706 | 1,458 | 1,227 | 1,017 | 0,841 | 0,682 | 0,504 |
| | $h_R$ | | 36,720 | 17,806 | 11,049 | 8,072 | 6,248 | 4,973 | 4,040 | 3,404 | 2,818 | 2,421 | 2,098 | 1,775 | 1,514 | 1,304 | 1,130 | 0,920 |
| | $g$ | | 2,368 | 2,325 | 2,277 | 2,231 | 2,184 | 2,134 | 2,080 | 2,027 | 1,969 | 1,907 | 1,845 | 1,780 | 1,702 | 1,625 | 1,542 | 1,445 |
| | $n_t$ | | 1886 | 460 | 185 | 103 | 65 | 44 | 31 | 23 | 19 | 14 | 11 | 10 | 8 | 7 | 5 | 5 |
| 1,00 | $h_A$ | | | 26,619 | 12,114 | 7,890 | 5,718 | 4,347 | 3,420 | 2,793 | 2,299 | 1,904 | 1,615 | 1,377 | 1,136 | 0,949 | 0,748 | 0,587 |
| | $h_R$ | | | 35,722 | 16,370 | 10,691 | 7,804 | 5,953 | 4,727 | 3,883 | 3,209 | 2,674 | 2,300 | 1,953 | 1,687 | 1,426 | 1,182 | 1,006 |
| | $g$ | | | 2,284 | 2,235 | 2,190 | 2,143 | 2,093 | 2,039 | 1,986 | 1,928 | 1,866 | 1,804 | 1,738 | 1,660 | 1,584 | 1,500 | 1,404 |
| | $n_t$ | | | 1 781 | 389 | 175 | 97 | 61 | 40 | 29 | 22 | 17 | 13 | 11 | 8 | 7 | 7 | 5 |
| 1,25 | $h_A$ | | | | 23,253 | 11,729 | 7,621 | 5,459 | 4,112 | 3,271 | 2,661 | 2,162 | 1,801 | 1,511 | 1,246 | 1,036 | 0,839 | 0,658 |
| | $h_R$ | | | | 31,226 | 15,833 | 10,339 | 7,458 | 5,646 | 4,511 | 3,726 | 3,024 | 2,531 | 2,141 | 1,801 | 1,541 | 1,294 | 1,099 |
| | $g$ | | | | 2,193 | 2,148 | 2,101 | 2,050 | 1,996 | 1,943 | 1,886 | 1,823 | 1,761 | 1,696 | 1,618 | 1,542 | 1,458 | 1,362 |
| | $n_t$ | | | | 1 367 | 367 | 164 | 89 | 55 | 38 | 26 | 20 | 16 | 13 | 10 | 8 | 7 | 5 |

**Table 4** *(continued)*

| $Q_{PR}$ (in %) | Parameters | $Q_{CR}$ (in %) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0,800 | 1,00 | 1,25 | 1,60 | 2,00 | 2,50 | 3,15 | 4,00 | 5,00 | 6,30 | 8,00 | 10,0 | 12,5 | 16,0 | 20,0 | 25,0 | 31,5 |
| 1,60 | $h_A$ | | | | | 24,899 | 11,941 | 7,511 | 5,273 | 4,030 | 3,169 | 2,526 | 2,075 | 1,732 | 1,412 | 1,158 | 0,968 | 0,739 |
| | $h_R$ | | | | | 33,511 | 16,117 | 10,191 | 7,188 | 5,540 | 4,398 | 3,521 | 2,906 | 2,462 | 2,028 | 1,679 | 1,452 | 1,182 |
| | $g$ | | | | | 2,099 | 2,052 | 2,002 | 1,948 | 1,895 | 1,837 | 1,775 | 1,713 | 1,647 | 1,569 | 1,493 | 1,409 | 1,313 |
| | $n_t$ | | | | | 1 564 | 379 | 160 | 85 | 53 | 35 | 25 | 19 | 14 | 11 | 10 | 7 | 7 |
| 2,00 | $h_A$ | | | | | | 24,055 | 11,309 | 7,032 | 5,054 | 3,812 | 2,965 | 2,393 | 1,961 | 1,581 | 1,306 | 1,065 | 0,835 |
| | $h_R$ | | | | | | 32,298 | 15,249 | 9,540 | 6,895 | 5,235 | 4,109 | 3,342 | 2,764 | 2,247 | 1,893 | 1,581 | 1,298 |
| | $g$ | | | | | | 2,007 | 1,956 | 1,902 | 1,849 | 1,792 | 1,729 | 1,668 | 1,602 | 1,524 | 1,448 | 1,364 | 1,268 |
| | $n_t$ | | | | | | 1 462 | 341 | 142 | 79 | 49 | 32 | 23 | 17 | 13 | 10 | 8 | 7 |
| 2,50 | $h_A$ | | | | | | | 22,347 | 10,459 | 6,742 | 4,781 | 3,571 | 2,812 | 2,246 | 1,785 | 1,477 | 1,184 | 0,945 |
| | $h_R$ | | | | | | | 30,067 | 14,137 | 9,175 | 6,546 | 4,934 | 3,914 | 3,121 | 2,506 | 2,132 | 1,716 | 1,435 |
| | $g$ | | | | | | | 1,910 | 1,855 | 1,802 | 1,745 | 1,683 | 1,621 | 1,555 | 1,477 | 1,401 | 1,317 | 1,221 |
| | $n_t$ | | | | | | | 1 267 | 295 | 131 | 71 | 43 | 29 | 22 | 16 | 11 | 10 | 7 |
| 3,15 | $h_A$ | | | | | | | | 20,714 | 10,196 | 6,425 | 4,493 | 3,404 | 2,650 | 2,068 | 1,670 | 1,345 | 1,067 |
| | $h_R$ | | | | | | | | 27,850 | 13,794 | 8,739 | 6,153 | 4,699 | 3,667 | 2,896 | 2,365 | 1,929 | 1,587 |
| | $g$ | | | | | | | | 1,805 | 1,752 | 1,695 | 1,632 | 1,570 | 1,505 | 1,427 | 1,350 | 1,267 | 1,170 |
| | $n_t$ | | | | | | | | 1 093 | 281 | 120 | 64 | 40 | 28 | 19 | 14 | 11 | 8 |
| 4,00 | $h_A$ | | | | | | | | | 21,268 | 9,893 | 6,094 | 4,339 | 3,253 | 2,468 | 1,944 | 1,543 | 1,210 |
| | $h_R$ | | | | | | | | | 28,531 | 13,378 | 8,305 | 5,971 | 4,502 | 3,470 | 2,735 | 2,189 | 1,752 |
| | $g$ | | | | | | | | | 1,698 | 1,640 | 1,578 | 1,516 | 1,451 | 1,373 | 1,296 | 1,213 | 1,116 |
| | $n_t$ | | | | | | | | | 1 148 | 265 | 109 | 59 | 37 | 23 | 17 | 13 | 10 |
| 5,00 | $h_A$ | | | | | | | | | | 19,542 | 9,053 | 5,775 | 4,069 | 2,955 | 2,269 | 1,773 | 1,385 |
| | $h_R$ | | | | | | | | | | 26,306 | 12,271 | 7,894 | 5,571 | 4,097 | 3,162 | 2,486 | 1,988 |
| | $g$ | | | | | | | | | | 1,587 | 1,525 | 1,463 | 1,398 | 1,320 | 1,243 | 1,160 | 1,063 |
| | $n_t$ | | | | | | | | | | 976 | 224 | 98 | 55 | 32 | 22 | 16 | 11 |