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Sampling procedures for inspection by variables —

Part 2:
General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics

Règles d'échantillonnage pour les contrôles par mesures —

Partie 2: Spécification générale pour les plans d'échantillonnage simples, indexés d'après la limite d'acceptation de qualité (LAQ), pour les contrôles lot par lot des caractéristiques de qualité indépendantes

Reference number
ISO 3951-2:2006(E)

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 3951-2 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

This first edition of ISO 3951-2, together with ISO 3951, cancels and replaces ISO 3951:1989, of which it constitutes a technical revision.

The most significant differences between ISO 3951-2:2006 and ISO 3951:1989 are as follows:

— The acronym AQL now stands for Acceptance Quality Limit rather than Acceptable Quality Level, in order to reflect more accurately its function.

— ISO 3951-2:2006 not only covers the univariate procedures of ISO 3951:1989 but also includes separate and complex control of double specification limits and multivariate procedures for independent characteristics.

— The plans have been modified so that their operating characteristic curves match those of the plans in ISO 2859-1 more closely. The sample sizes for both the "s" method and the "σ" method are constant along rows of the master tables.

— The acceptability constants of Annexes B and C have been recalculated and tabulated to three decimal places for an extended range of AQLs corresponding to ISO 2859-1:1999. A new master table of Form p^* acceptability constants is provided as Annex G, tabulated to four significant figures.

— All tabulated values of operating characteristics have been recalculated and related directly to reduced inspection as well as to normal and tightened inspection.

— The annex containing statistical theory has been removed. It is planned ultimately to reintroduce this within a guidance document to sampling procedures for inspection by variables.

— Text and tables that are merely informative have been consigned to annexes wherever practicable.

— The annex dealing with the "R" method has been eliminated, now that the availability of calculators with a standard deviation function key is so widespread. Data for acceptance sampling by variables is often substantially more expensive to acquire than data for sampling by attributes, and the "s" method makes more efficient use of this data.

ISO 3951 currently consists of the following parts, under the general title *Sampling procedures for inspection by variables*:

⎯ *Part 1: Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL*

⎯ *Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

⎯ *Part 5: Sequential sampling plans indexed by acceptance quality limit (AQL) for inspection by variables (known standard deviation)*

The following part is under preparation:

⎯ *Part 3: Double sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

Introduction

This part of ISO 3951 specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the Acceptance Quality Limit (AQL), and is of a technical nature, aimed at users who are already familiar with sampling by variables or who have complicated requirements. (An introductory treatment is given in ISO 3951-1.)

The objectives of the methods laid down in this part of ISO 3951 are to ensure that lots of an acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide

a) automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected;

b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this part of ISO 3951, the acceptability of a lot is implicitly or explicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This part of ISO 3951 is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this part of ISO 3951 is applied to each one separately.

This part of ISO 3951 is complementary to ISO 2859-1. When specified by the responsible authority, both ISO 3951-2 and ISO 2859-1 may be referenced in a product specification, contract, inspection instructions, or other documents, and the provisions set forth therein shall govern. The "responsible authority" shall be designated in one of the above documents.

It is assumed in this part of ISO 3951 that measurement uncertainty is negligible (see ISO 10576-1:2003). For further information on allowing for measurement uncertainty in sampling by variables, see Reference [16] in the Bibliography.

CAUTION — The procedures in this part of ISO 3951 are not suitable for application to lots that have been screened previously for nonconforming items.

Inspection by variables for percent nonconforming items, as described in the present document, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

⎯ unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;

⎯ a single specification limit, or double specification limits with combined, separate or complex control;

⎯ univariate or multivariate cases;

⎯ normal inspection, tightened inspection or reduced inspection.

Fourteen annexes are provided.

— Annexes A to I provide the tables needed to support the procedures.

— Annex J indicates how the sample standard deviation, s, and the presumed known value of the process standard deviation, σ, should be determined.

— Annex K provides formulae for the estimation of the process fraction nonconforming, together with a highly accurate approximation for use when the process standard deviation is unknown.

— Annex L provides the statistical theory underlying the calculation of the consumer's risk qualities, together with tables showing these quality levels for normal, tightened and reduced inspection under the "s" and "σ" methods.

— Annex M provides similar information for the producer's risks.

— Annex N gives the general formula for the operating characteristic of the "σ" method.

Table 1 is intended to facilitate the use of this part of ISO 3951 by directing the user to the paragraphs and tables concerning any situation with which he may be confronted. Table 1 only deals with Clauses 15, 16, 17, 18, 19, 23, 24 and 25; in every case, it is necessary first of all to have read the other clauses.

Table 1 — Summary

Inspection type	Single specification limit				Double specification limits with combined control			
	"s" method		"σ" method		"s" method		"σ" method	
	Clauses or sub-clauses	Tables	Clauses or sub-clauses	Tables	Clauses or sub-clauses	Tables	Clauses or sub-clauses	Tables
Normal inspection	15.1, 15.2, 15.3, 16.2, 23.1	A.1, A.2, B.1	17.1, 17.2, 18, 19, 23.1	A.1, A.2, C.1	15.1, 15.4, 16.2, 23.1	A.1, A.2, D.1, F.1 (for $n = 3$), G.1	17.1, 17.3, 18, 19, 23.1	A.1, A.2, C.1, E.1
Switching between normal and tightened inspection	23.2, 23.3	B.1, B.2	23.2, 23.3	C.1, C.2	23.2, 23.3	D.1, D.2 / G.1, G.2	23.2, 23.3	E.1 / G.1, G.2
Switching between normal and reduced inspection	23.4, 23.5	B.1, B.3 / I.1	23.4, 23.5	C.1, C.3 / I.1	23.4, 23.5	D.1, D.3 / G.1, G.3 / I.1	23.4, 23.5	E.1 / G.1, G.3 / I.1
Switching between tightened and discontinued inspection	21, 24	B.2	24	C.2	21, 24	D.2 / G.2	24	E.1 / G.2
Switching between the "s" and "σ" methods	25 K.2.1 K.3, K.4, K.5	H.1	25 K.2.2	H.1	25 K.2.1 K.3, K.4, K.5	H.1	25 K.2.2	H.1

Table 1 (*continued*)

Inspection type	Double specification limits with separate control				Double specification limits with complex control			
	"s" method		"σ" method		"s" method		"σ" method	
	Clauses or sub-clauses	Tables	Clauses or sub-clauses	Tables	Clauses or sub-clauses	Tables	Clauses or sub-clauses	Tables
Normal inspection	15.1, 15.4, 16.2, 23.1	A.1, A.2, D.1, F.1 (for n = 3), G.1	17.1, 17.2, 17.3, 18, 19, 23.1	A.1, A.2, C.1, E.2	15.1, 15.5, 16.2, 23.1	A.1, A.2, D.1, F (for n = 3), G.1	17.1, 17.3, 18, 19, 23.1	A.1, A.2, C.1, E.3
Switching between normal and tightened inspection	23.2, 23.3	D.1, D.2, G.1, G.2	23.2, 23.3	E.1, G.1, G.2	23.2, 23.3	D.1, D.2, G.1, G.2	23.2, 23.3	E.1, G.1, G.2
Switching between normal and reduced inspection	23.4, 23.5	D.1, D.3, G.1, G.3, I.1	23.4, 23.5	E.2, G.1, G.3, I.1	23.4, 23.5	D.1, D.3, G.1, G.3, I.1	23.4, 23.5	E.3, G.1, G.3, I.1
Switching between tightened and discontinued inspection	21, 24	D.2, G.2	24	E.1, G.2	21, 24	D.2, G.2	24	E.1, G.2
Switching between the "s" and "σ" methods	25, K.2.1, K.3, K.4, K.5	H.1	25, K.2.2	H.1	25, K.2.1, K.3, K.4, K.5	H.1	25, K.2.2	H.1

Sampling procedures for inspection by variables —

Part 2:
General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics

1 Scope

This part of ISO 3951 specifies an acceptance sampling system of single sampling plans for inspection by variables, indexed in terms of the Acceptance Quality Limit (AQL).

This part of ISO 3951 is primarily designed for use under the following conditions:

a) where the inspection procedure is to be applied to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, apply this part of ISO 3951 to each one separately;

b) where the quality characteristics of the items of product are measurable on a continuous scale;

c) where the measurement error is negligible (i.e. with a standard deviation no more than 10 % of the corresponding process standard deviation);

d) where production is stable (under statistical control) and the quality characteristics are distributed, at least to a close approximation, according to normal distributions;

e) where, in the case of multiple quality characteristics, the characteristics are independent of one another, at least approximately;

f) where a contract or standard defines an upper specification limit U, a lower specification limit L, or both on each of the quality characteristics.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1:1999, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 9000, *Quality management systems — Fundamentals and vocabulary*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 3534-2, ISO 2859-1 and ISO 9000 apply, except where redefined below. References are given in square brackets for definitions that have been repeated here for convenience.

3.1
inspection by variables
inspection by measuring the magnitude(s) of a characteristic(s) of an item

[ISO 3534-2]

3.2
sampling inspection
inspection of selected items in the group under consideration

[ISO 3534-2]

3.3
acceptance sampling inspection
acceptance inspection where the acceptability is determined by means of **sampling inspection** (3.2)

[ISO 3534-2]

3.4
acceptance sampling inspection by variables
acceptance sampling inspection (3.3) in which the acceptability of the process is determined statistically from measurements on specified quality characteristics of each item in a sample from a lot

3.5
process fraction nonconforming
rate at which nonconforming items are generated by a process, expressed as a proportion

3.6
acceptance quality limit
AQL
worst tolerable **process fraction nonconforming** (3.5) when a continuing series of lots is submitted for acceptance sampling

NOTE   See Clause 5.

3.7
quality level
quality expressed as a rate of occurrence of nonconforming items

3.8
consumer's risk quality
CRQ
quality level (3.7) of a process which, in the acceptance sampling plan, corresponds to a specified consumer's risk

NOTE 1   In this part of ISO 3951, the quality level is the process fraction nonconforming.

NOTE 2   In this part of ISO 3951, the consumer's risk quality is such that the consumer's risk is 10 %.

3.9
producer's risk
PR
probability of non-acceptance when the **quality level** (3.7) has a value stated by the plan as acceptable

NOTE   Quality level relates to the **process fraction nonconforming** (3.5) and acceptable relates to the **AQL** (3.6).

3.10
nonconformity
non-fulfilment of a requirement

[ISO 9000]

NOTE   Nonconformity will generally be classified by its degree of seriousness, such as:

— **Class A.** Nonconformity of a type considered to be of the highest concern for the product or service. Such types of nonconformity will typically be assigned very small AQL values.

— **Class B.** Nonconformity of a type considered to have the next lower degree of concern; this is typically assigned a larger AQL value than that in Class A and smaller than that in Class C if a third class exists, and so on.

The number of classes and the assignment into a class should be appropriate to the quality requirements of the specific situation.

3.11
nonconforming unit
unit with one or more nonconformities

[ISO 3534-2]

3.12
"s" method acceptance sampling plan
s method
acceptance sampling plan by variables using the sample standard deviation

[ISO 3534-2]

NOTE   See Clause 15.

3.13
"σ" method acceptance sampling plan
sigma method
acceptance sampling plan by variables using the presumed value of the process standard deviation(s)

[ISO 3534-2]

NOTE   See Clause 16.

3.14
specification limit
limiting value stated for a characteristic

[ISO 3534-2]

3.15
lower specification limit
specification limit (3.14) that defines the lower limiting value

NOTE   The preferred symbol for the lower specification limit is L.

[ISO 3534-2]

3.16
upper specification limit
specification limit (3.14) that defines the upper limiting value

NOTE   The preferred symbol for the upper specification limit is U.

[ISO 3534-2]

3.17
combined control
requirement when **nonconformity** (3.10) beyond both the **upper** and the **lower specification limits** (3.16, 3.15) of a quality characteristic belongs to the same class, to which a single AQL is applied

NOTE 1   See 5.3, 15.3.2 and 17.3.

NOTE 2   The use of a combined **AQL** (3.6) requirement implies that nonconformities beyond either **specification limit** (3.14) are believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

3.18
separate control
requirement when nonconformity beyond the **upper** and the **lower specification limits** (3.16, 3.15) of a quality characteristic belongs to different classes, to which separate **AQL**s (3.6) are applied

NOTE   See 5.3, 15.3.3 and 17.2.

3.19
complex control
requirement when nonconformity beyond the **upper specification limit** (3.16) and the **lower specification limits** (3.15) of a quality characteristic belongs to one class, and nonconformity beyond either the upper specification limit or the lower specification limit belongs to a different class, with separate **AQL**s (3.6) being applied to the two classes

NOTE   See 5.3, 15.3.4 and 17.3.

3.20
acceptability constant
k, p^*
constant depending on the specified value of the **acceptance quality limit** (3.6) and the sample size, used in the criteria for accepting the lot in an acceptance sampling plan by variables

[ISO 3534-2]

NOTE   See 15.2 and 16.2.

3.21
quality statistic
Q
function of the **specification limit** (3.14), the sample mean, and the sample or process standard deviation, used in assessing the acceptability of a lot (see 4.1, symbols Q_L and Q_U)

[ISO 3534-2]

NOTE 1   For the case of a single specification limit, the lot may be sentenced on the result of comparing Q with the **acceptability constant** (3.20) k.

NOTE 2   See 15.2 and 16.2.

3.22
lower quality statistic
function of the **lower specification limit** (3.15), the sample mean, and the sample or process standard deviation

[ISO 3534-2]

NOTE 1   For a single, lower specification limit, the lot is sentenced on the result of comparing Q_L with the **acceptability constant** (3.20) k.

NOTE 2   See Clause 4, 15.2 and 16.2.

3.23
upper quality statistic
function of the **upper specification limit** (3.16), the sample mean, and the sample or process standard deviation

[ISO 3534-2]

NOTE 1   For a single, upper specification limit, the lot is sentenced on the result of comparing Q_U with the **acceptability constant** (3.20) k.

NOTE 2   See Clause 4, 15.2 and 16.2.

3.24
maximum sample standard deviation
MSSD
s_{max}
largest sample standard deviation for a given sample size code letter and **acceptance quality limit** (3.6) for which it is possible to satisfy the acceptance criterion for double **specification limits** (3.14) with a combined **AQL** (3.6) requirement and unknown process variability

NOTE   See 15.3.

3.25
maximum process standard deviation
MPSD
σ_{max}
largest process standard deviation for a given sample size code letter and **acceptance quality limit** (3.6) for which it is possible to satisfy the acceptance criterion for double **specification limits** with a combined **AQL** (3.6) requirement under tightened inspection with known process variability

NOTE   See 17.2 and 17.3.

3.26
switching rule
instruction within an acceptance sampling scheme for changing from one acceptance sampling plan to another of greater or lesser severity of sampling based on demonstrated quality history

[ISO 3534-2]

NOTE 1   Normal, tightened or reduced inspection, or discontinuation of inspection, are examples of "greater or lesser severity".

NOTE 2   See Clauses 6 and 23.

3.27
measurement
set of operations having the object of determining a value of a quantity

[ISO 3534-2]

3.28
responsible authority
generic term used to maintain the neutrality of this part of ISO 3951 (primarily for specification purposes), irrespective of whether it is being invoked or applied by the first, second or third party.

NOTE 1   The responsible authority may be

a)   the quality department within a supplier's organization (first party);

b)   the purchaser or procurement organization (second party);

c)   an independent verification or certification authority (third party);

d)   any of a), b) or c), differing according to function (see NOTE 2) as described in a written agreement between two of the parties, for example a document between supplier and purchaser.

NOTE 2   The duties and functions of a responsible authority are outlined in ISO 2859-1:1999 (5.2, 6.2, Clause 7, 9.1, 9.3.3, 9.4, 10.1 and 10.3).

4   Symbols

4.1   Univariate symbols

The symbols used when there is only one quality characteristic in the class are as follows.

f_s factor that relates the maximum sample standard deviation (MSSD) to the difference between U and L (see Annex D)

f_σ factor that relates the maximum process standard deviation (MPSD) to the difference between U and L (see Annex E)

k Form k acceptability constant, for use with a single specification limit and a single quality characteristic (see Annexes B and C)

L lower specification limit (as a suffix to a variable, denotes its value at L)

μ process mean

N lot size (number of items in a lot)

n sample size (number of items in a sample)

$\hat{p}$ estimate of the process fraction nonconforming (see Annex K)

p_L process fraction nonconforming below the lower specification limit

$\hat{p}_L$ estimate of the process fraction nonconforming below the lower specification limit

p_U process fraction nonconforming above the upper specification limit

$\hat{p}_U$ estimate of the process fraction nonconforming above the upper specification limit

p^* Form p^* acceptability constant, the maximum acceptable value of the estimate of the process fraction nonconforming (see Annex G)

P_a probability of acceptance

Q quality statistic

Q_L lower quality statistic

NOTE 1 Q_L is defined as $(\overline{x} - L)/s$ when the process standard deviation is unknown, and as $(\overline{x} - L)/\sigma$ when it is presumed to be known.

Q_U upper quality statistic

NOTE 2 Q_U is defined as $(U - \overline{x})/s$ when the process standard deviation is unknown, and as $(U - \overline{x})/\sigma$ when it is presumed to be known.

s Sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s = \sqrt{\dfrac{\sum\limits_{j=1}^{n} (x_j - \overline{x})^2}{n - 1}}.$$

(See also Annex J.)

s_{max} maximum sample standard deviation (MSSD)

σ standard deviation of a process that is under statistical control

NOTE 3 σ^2, the square of the process standard deviation, is known as the process variance.

σ_{max} maximum process standard deviation (MPSD)

U upper specification limit (as a suffix to a variable, denotes its value at U)

x_j measured value of the quality characteristic for the jth item of the sample

$\overline{x}$ arithmetic mean of the measured values of the quality characteristics in the sample, i.e.

$$\overline{x} = \dfrac{\sum\limits_{j=1}^{n} x_j}{n}$$

$\overline{x}_L$ lower acceptance value for $\overline{x}$

$\overline{x}_U$ upper acceptance value for $\overline{x}$

4.2 Multivariate symbols

Further symbols used when there are two or more quality characteristics in a class are as follows.

L_i lower specification limit for the ith quality characteristic

M number of quality characteristics in the class

$\hat{p}_i$ estimate of the process fraction nonconforming for the ith quality characteristic

$\hat{p}_{i,L}$ estimate of the process fraction nonconforming below the lower specification limit for the ith quality characteristic

$\hat{p}_{i,U}$ estimate of the process fraction nonconforming above the upper specification limit for the ith quality characteristic

s_i sample standard deviation for the ith quality characteristic, i.e.

$$s_i = \sqrt{\frac{\sum\limits_{j=1}^{n}(x_{ij} - \bar{x}_i)^2}{n-1}}$$

σ_i process standard deviation for the ith quality characteristic

U_i upper specification limit for the ith quality characteristic

x_{ij} measured value of the ith quality characteristic for the jth item in the sample

$\bar{x}_i$ sample mean value of the ith quality characteristic, i.e.

$$\bar{x}_i = \frac{\sum\limits_{j=1}^{n} x_{ij}}{n}$$

5 Acceptance quality limit (AQL)

5.1 Concept

The AQL is the quality level that is the worst tolerable process fraction nonconforming when a continuing series of lots is submitted for acceptance sampling. Although individual lots with quality as bad as the acceptance quality limit may be accepted with fairly high probability, the designation of an acceptance quality limit does not suggest that this is a desirable quality level. The sampling schemes found in this part of ISO 3951, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to keep process fractions nonconforming consistently better than the respective AQLs. Otherwise, there is a high risk that the inspection severity will be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection will be invoked pending such improvement.

5.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this part of ISO 3951.

5.3 Specifying AQLs

The AQL to be used will be designated in the product specification, contract or by the responsible authority. In all cases, one AQL shall be specified for each class of nonconformity (see 3.10).

Where both upper and lower specification limits are given for a quality characteristic, three cases may be identified:

a) *combined* control of double specification limits, where nonconformity beyond both limits belongs to the same class, to which a single AQL applies;

b) *separate* control, where nonconformity beyond both limits belongs to different classes, to which separate AQLs apply;

c) *complex* control, where nonconformity beyond the limit that is of greater seriousness belongs to one class to which one AQL applies, and nonconformity beyond both limits belongs to another class to which a larger AQL applies.

In other words, for a single quality characteristic that has a lower specification L, an upper specification limit U, an unknown process fraction nonconforming below L of p_L and an unknown process fraction nonconforming above U of p_U:

— *combined* control seeks simply to control the sum $p_L + p_U$ within one class of nonconformity, to which a single AQL applies;

— *separate* control seeks to control p_L within one class to which one AQL applies, and to separately control p_U within another class to which a second AQL applies;

— *complex* control seeks to control $p_L + p_U$ within one class, to which one AQL applies, and to separately control either p_L or p_U within another class to which a lower AQL applies.

Including the control of single upper and single lower specification limits, there are therefore five types of control. A class may contain nonconformities under any number of these types of control.

An acceptance test shall be carried out according to the provisions of this part of ISO 3951 for each class of nonconformity. The lot shall only be accepted if all classes of nonconformity satisfy their respective acceptance tests.

5.4 Preferred AQLs

The sixteen AQLs given in this part of ISO 3951, ranging in value from 0,01 % to 10 % nonconforming, are described as preferred AQLs. If, for any product or service, an AQL is designated other than a preferred AQL, then this part of ISO 3951 is not applicable (see 13.2).

5.5 Caution

From the above definition of the AQL, it follows that the desired protection can only be assured when a continuing series of lots is provided for inspection.

5.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming items of product.

6 Switching rules for normal, tightened and reduced inspection

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This part of ISO 3951 prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving his production process.

Tightened inspection and the discontinuation rule are integral, and therefore obligatory, procedures of this part of ISO 3951 if the protection implied by the AQL is to be maintained.

This part of ISO 3951 also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

9

When there is sufficient evidence from the control charts (see 22.1) that the variability is in statistical control, consideration should be given to switching to the "σ" method. If this appears advantageous, the consistent value of s (the sample standard deviation) shall be taken as σ (see Clause 25).

When it has been necessary to discontinue acceptance sampling inspection, inspection under this part of ISO 3951 shall not be resumed until action has been taken by the producer to improve the quality of the submitted product.

Details of the operation of the switching rules are given in Clauses 23, 24 and 25 and are shown diagrammatically in Figure 1.
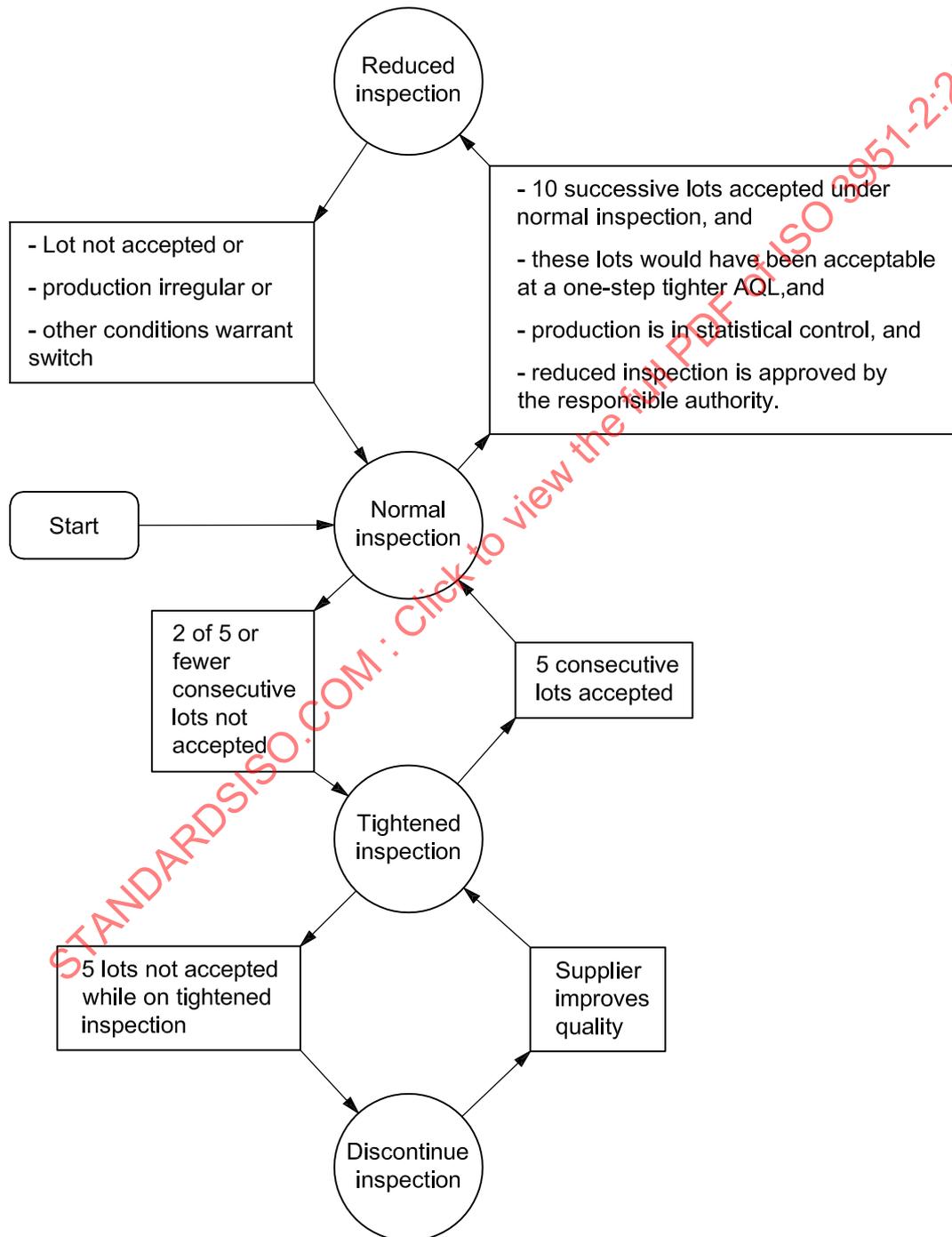
Figure 1 — Outline of the switching rules

7   Relationship to ISO 2859-1 and ISO 3951-1

7.1   Relationship to ISO 2859-1

7.1.1   Similarities to ISO 2859-1

a)   This part of ISO 3951 is complementary to ISO 2859-1; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.

b)   Both use the AQL to index the sampling plans, and the preferred values used in this document are identical with those given for percent nonconforming in ISO 2859-1 (i.e. from 0,01 % to 10 %).

c)   In both International Standards, lot size and inspection level (inspection level II in default of other instructions) determine a sample size code letter. Then general tables give the sample size to be taken and the acceptability criterion, indexed by the sample size code letter and the AQL. Separate tables are given for the "s" and "σ" methods, and for normal, tightened and reduced inspection.

d)   The switching rules are essentially equivalent.

e)   The classification of nonconformities by degree of seriousness into Class A, Class B, etc., remains unchanged.

7.1.2   Differences from ISO 2859-1

a)   **Determination of acceptability**: Acceptability for an ISO 2859-1 attributes sampling plan for percent nonconforming is determined by the number of nonconforming items found in the sample. Acceptability for a plan for inspection by variables is based on the distance of the estimated process mean from the specification limit(s) in terms of the estimated process standard deviation. In this part of ISO 3951, two methods are considered: the "s" method for use when the process standard deviation σ is unknown and the "σ" method for use when σ is presumed to be known. In the case of a class containing a single quality characteristic with a single specification limit, acceptability is determined most easily by comparing a quality statistic with a "Form k" acceptability constant (see 15.2 and 17.2). For more complicated classes with multiple quality characteristics and/or combined or complex control of double specification limits, acceptability is determined by comparing an estimate of the process fraction nonconforming for that class with a "Form p*" acceptability constant.

b)   **Normality**: In ISO 2859-1, there is no requirement relating to the distribution of the characteristics. However, in this part of ISO 3951, it is necessary for the efficient operation of the plans that the measurements on each quality characteristic should be distributed according to a normal distribution or at least a close approximation to a normal distribution.

c)   **Independence**: In ISO 2859-1, there is no requirement relating to independence of multiple quality characteristics. However, in this part of ISO 3951, for the efficient operation of a plan, it is necessary that the measurements for all quality characteristics in a class shall be independent, or at least approximately so.

d)   **Operating characteristic curves** (OC curves): The OC curves of the variables plans in this part of ISO 3951 are not identical to those of the corresponding attributes plans in ISO 2859-1. The curves have been matched as closely as possible subject to a number of pragmatic constraints, such as keeping the sample size the same for a given code letter, severity of inspection and whether the "s" or "σ" method is being used, regardless of the AQL.

e)   **Producer's risk**: For process quality precisely at the AQL, the producer's risk that a lot will not be accepted tends to decrease with one-step increases in sample size coupled with one-step decreases in AQL, i.e. down diagonals of the master tables running from top right to bottom left. The progressions of probabilities are similar, but not identical, to those in ISO 2859-1. (The producer's risks of the plans are given in Annex M.)

f)   **Sample sizes**: The variables sample sizes corresponding to given code letters are usually smaller than the attributes sample sizes for the same letters. This is particularly true for the "σ" method. (See Table A.2.)

g)   **Double sampling plans**: Double sampling plans by variables are presented separately, in ISO 3951-3[1]).

h)   **Multiple sampling plans**: No multiple sampling plans are given in this part of ISO 3951.

i)   **Average outgoing quality limit (AOQL)**: The AOQL concept applies when 100 percent inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans will generally be used under these circumstances, no tables of AOQL have been included in this part of ISO 3951.

7.2   Relationship to ISO 3951-1

7.2.1   Similarities

a)   This part of ISO 3951 is complementary to ISO 3951-1 and the two documents both present single sampling procedures for inspection by variables.

b)   The procedures in ISO 3951-1 are included in this part of ISO 3951 but referred to as of "Form k".

7.2.2   Differences

a)   ISO 3951-2 is more general than ISO 3951-1, as it includes multivariate procedures for independent quality characteristics and also includes procedures for separate or combined control of double specification limits.

b)   Because Form k procedures may only be used for a single quality characteristic with a single AQL, ISO 3951-2 also includes the more general Form p^* procedures.

NOTE   For users who are familiar with MIL-STD-414, Form k corresponds to Form 1 of the Military Standard, and Form p^* corresponds to Form 2. The new terminology is thought to be more helpful.

8   Consumer protection

8.1   Use of individual plans

This part of ISO 3951 is intended to be used as a system employing tightened, normal and reduced inspection on a continuing series of lots to provide consumer protection while assuring the producer that acceptance will be very likely to occur if quality is better than the AQL.

Sometimes specific individual plans are selected from this part of ISO 3951 and used without the switching rules. For example, a purchaser may be using the plans for verification purposes only. This is not the intended application of the system given in this part of ISO 3951 and its use in this way should not be referred to as "inspection in compliance with ISO 3951-2". When used in such a way, this part of ISO 3951 simply represents a collection of individual plans indexed by AQL. The operating characteristic curves and other measures of a plan so chosen shall be assessed individually from the tables provided.

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1)   To be published.

8.2 Consumer's risk quality tables

If the series of lots is not long enough to allow the switching rules to be applied, it may be desirable to limit the selection of sampling plans to those, associated with a designated AQL value, that give consumer's risk quality not more than the specified limiting quality protection. Sampling plans for this purpose can be selected by choosing a consumer's risk quality (CRQ) and a consumer's risk to be associated with it. Annex L gives values of consumer's risk quality levels for the "s" method and "σ" methods corresponding to a consumer's risk of 10 %.

However, application of this part of ISO 3951 to isolated lots is deprecated, as the theory of sampling by variables applies to a *process*. For isolated lots, it is appropriate and more efficient to use plans for sampling by attributes, such as from ISO 2859-2 [7]. (See also Reference [8] in the Bibliography.)

8.3 Producer's risk tables

Annex M gives the probability of non-acceptance under the "s" and "σ" methods for lots produced when the *process* fraction nonconforming equals the AQL. This probability is called the producer's risk.

8.4 Operating characteristic curves

The tables for consumer's risk quality and producer's risk provide information about only two points on the operating characteristic curves. The degree of consumer protection provided by an individual sampling plan at any *process* quality level may, however, be judged from its operating characteristic (OC) curve. OC curves for the normal inspection "s" method sampling plans of this part of ISO 3951 are given in Charts B to R of ISO 3951-1, which should be consulted when choosing a sampling plan. Also given in ISO 3951-1 are Tables B to R of process quality levels at nine standard probabilities of acceptance for all the "s" method sampling plans in this part of ISO 3951.

These OC curves and tables apply to a single specification limit under the "s" method. Most of them also provide a good approximation to the "σ" method, and to the case of combined control of double specification limits, particularly for the larger sample sizes. If more accurate OC values are required for the "σ" method, refer to Annex N.

9 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement and some knowledge both of statistics and the product to be inspected. Clauses 10 to 13 of this part of ISO 3951 are intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be borne in mind when deciding whether a variables plan would be suitable, and the choices to be made when selecting an appropriate standard plan.

10 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account.

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time-consuming and costly per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables, as the information obtained indicates more precisely how good the product is. Earlier warning will therefore be given if the quality is slipping.

c) An attributes scheme can be more readily understood and accepted. For example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See the examples in 15.3.2.2 and 15.3.2.4.)

d) A comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes (i.e. from ISO 2859-1) and the standard plans in this part of ISO 3951 is given in Table A.2. It will be seen that the smallest samples are required by the "σ" method (used when the process standard deviation is presumed to be known). The sample sizes for the "s" method (used when the process standard deviation is unknown) are also in general substantially smaller than for sampling by attributes.

e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.

f) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.

g) A variables scheme becomes relatively more complicated to operate as the number of quality characteristics and the number of measurements to be taken on each item increases.

h) The use of this part of ISO 3951 is only applicable when there is reason to believe that the distribution of measurements of each quality characteristic is normal and that the quality characteristics are independent. In case of doubt, the responsible authority should be consulted.

NOTE 1   ISO 5479 gives detailed procedures for tests for departure from normality.

NOTE 2   Departure from normality is also dealt with in ISO 2854:1976, Clause 2, which provides examples of graphical methods which can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

11 Choice between the "s" and "σ" methods

If it is desired to apply inspection by variables, the next question is whether to use the "s" method or the "σ" method. The "σ" method is the most economical in terms of sample size but, before this method may be employed, the value of σ has to be established.

Initially, it will be necessary to begin with the "s" method but, subject to the agreement of the responsible authority and provided the quality remains satisfactory, the standard switching rules will permit a switch to reduced inspection and the use of a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, will it be economical to change to the "σ" method? The size of the sample will generally be smaller and the acceptability criterion simpler under the "σ" method. (See 16.2.) On the other hand, it will still be necessary to calculate the sample standard deviation, s, for record purposes and to keep the control charts up to date. (See Clause 19.) The calculation of s can appear daunting, but the difficulty is more apparent than real; this is especially true if an electronic calculator is available. Methods of calculating s are given in Annex J.

12 Choice of inspection level and AQL

For a standard sampling plan, the inspection level in conjunction with the size of the lots and the AQL determines the size of the sample to be taken, and governs the severity of the inspection. The appropriate OC curve from Charts B to R or the appropriate table from Tables B to R of ISO 3951-1 show the extent of the risk that is involved in such a plan.

The choice of inspection level and AQL is governed by a number of factors, but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

13 Choice of sampling scheme

13.1 Standard plans

The standard procedure can be used only when the production of lots is continuing.

This standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the "s" method, has been found in practice to produce workable sampling schemes; but it rests on the assumption that the order of priority is first the AQL, second the sample size and last, the limiting quality.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see Clauses 23, 24 and 25), which quickly increase the severity of inspection and finally terminate inspection altogether if the quality of the process remains worse than the AQL.

NOTE   It is pointed out that the limiting quality is the quality which, if offered for inspection, would have a 10 % probability of acceptance. The actual risk taken by the consumer varies according to the probability that goods of such a low quality are offered for inspection.

If, in certain circumstances, the limiting quality has a higher priority than the sample size (for example, when only a limited number of lots are being produced), a suitable plan from this part of ISO 3951 may be selected by using Chart A, (see Figure 2). Construct a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 % probability of acceptance (i.e. approximately equal to the AQL). The point of intersection of these two lines will lie on, or under, a line indexed with the sample size code letter of a standard normal inspection plan, which meets the specified requirements. (This should be verified by inspecting the OC curve from among Charts B to R of ISO 3951-1 relating to this code letter and AQL.)

The methods of this part of ISO 3951 are not applicable to short series of lots or isolated lots. Use attributes plans for fewer than 10 lots.

EXAMPLE   Suppose that an acceptable value for the limiting quality is 1,5 % nonconforming and that the desired quality with a 95 % probability of acceptance is 0,15 % nonconforming. A vertical line on Chart A at 1,5 % nonconforming and a horizontal line at 0,15 % nonconforming intersect just below the sloping line indexed by the letter L. (Examination of Chart L of ISO 3951-1 confirms that a plan with sample size code letter L and AQL 0,15 % meets the requirements.)

If the lines intersect at a point above the line marked R in Chart A, this implies that, for example, a sample of over 250 would be necessary for the "s" method and the specification cannot be met by any of the plans in this part of ISO 3951.

13.2 Special plans

If standard plans are not acceptable, it will be necessary to devise a special plan. It then has to be decided which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent for, when any two have been chosen, the third follows.

This choice is not completely unfettered; the fact that the size of the sample is necessarily a whole number imposes some limitations. If a special plan is necessary, it should be devised only with the assistance of a statistician experienced in quality control.
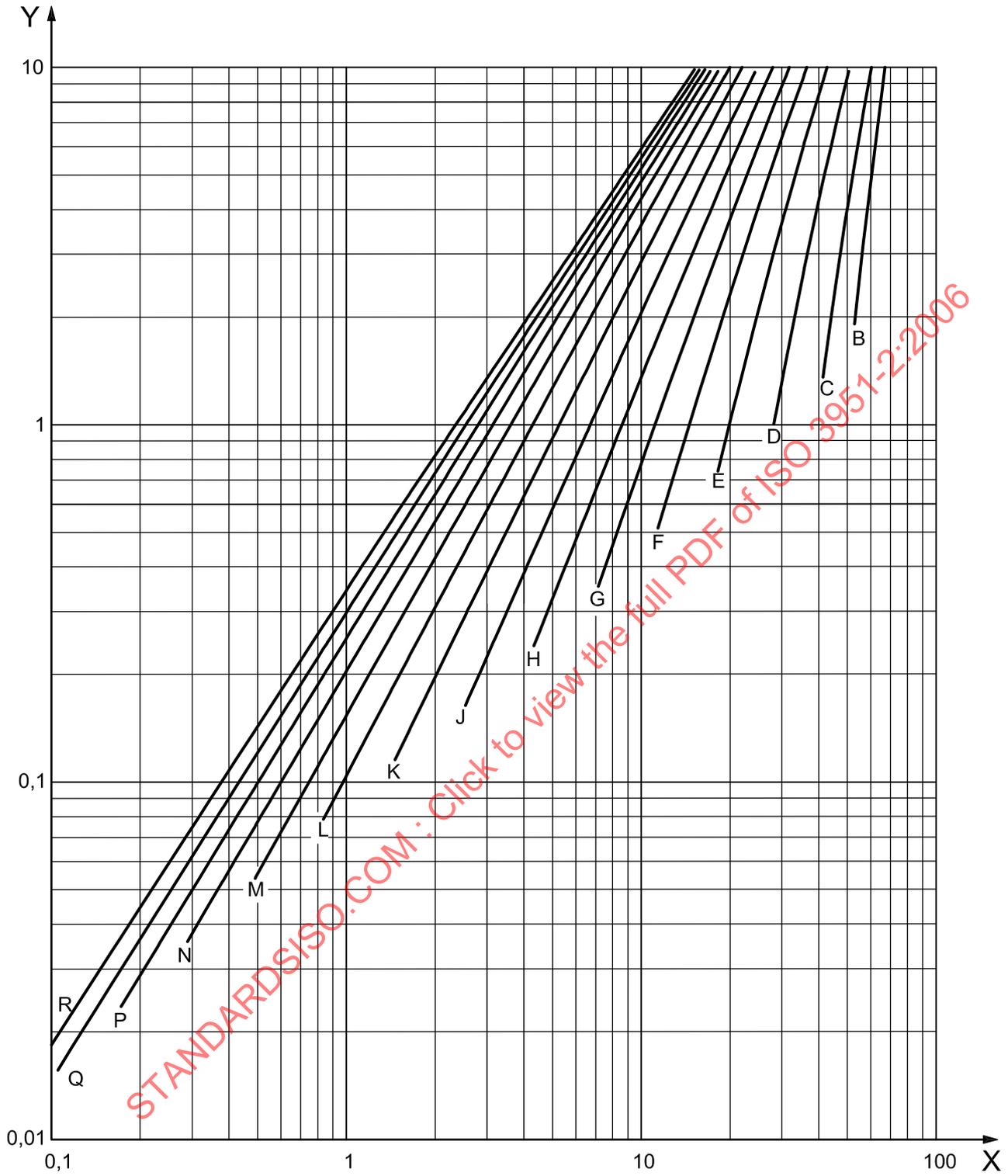
Key

X   limiting quality, i.e. quality level at 10 % probability of acceptance (in percent nonconforming)

Y   quality level at 95 % probability of acceptance (in percent nonconforming)

Sample size code letters are shown in boldface type.

Figure 2 — Chart A — Sample size code letters of standard single sampling plans for specified quality levels at 95 % and 10 % probabilities of acceptance

14 Preliminary operations

Before starting inspection by variables,

a) check that production is considered to be continuing and that the distribution of the quality characteristics can be considered to be normal and independent;

NOTE 1   For tests for departure from normality, see ISO 5479.

NOTE 2   If lots have been screened for nonconforming items prior to acceptance sampling, then the distribution will have been truncated and this part of ISO 3951 will not be applicable.

b) check for each quality characteristic separately whether the "s" method is to be used initially or whether the process standard deviation is stable and known, in which case the "σ" method should be used;

c) check that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;

d) check, for every quality characteristic with double specification limits, whether the limits are under combined, separate or complex control, and to which class of nonconformity each limit has been assigned. For combined control, check that nonconformity beyond each limit is of equal importance;

e) check that an AQL has been designated for each class of nonconformity, and that it is one of the preferred AQLs for use with this part of ISO 3951. If it is not, then the tables are not applicable.

15 Standard univariate "s" method procedures

15.1 Obtaining a plan, sampling and preliminary calculations

The procedure for obtaining and implementing a plan is as follows.

a) With the inspection level given (normally this will be II) and with the lot size, obtain the sample size code letter using Table A.1.

b) For a single specification limit, enter Table B.1, B.2 or B.3 as appropriate with this code letter and the AQL, and obtain the sample size, n, and the Form k acceptability constant, k. For separate control of double specification limits, do this for both limits. For combined control of double specification limits, enter Table G.1, G.2 or G.3 as appropriate and obtain the sample size, n, and the Form p^* acceptability constant. For complex control of double specification limits, enter Table G.1, G.2 or G.3 as appropriate twice, once with the combined control part of the specification and once with the smaller AQL applying to the specification limit of greater concern.

c) Take a random sample of size n, measure the characteristic x in each item and then calculate $\overline{x}$, the sample mean and s, the estimate of the process standard deviation (see Annex J). If $\overline{x}$ is outside the specification limit(s), the lot can be judged unacceptable without even calculating s. It is, however, necessary to calculate s for record purposes.

15.2 Form k acceptability criterion for the "s" method

If single specification limits are given, or separate control of double specification limits is required, the most straightforward procedure is as follows. Calculate the quality statistic

$$Q_U = \frac{U - \overline{x}}{s}$$

and/or

$$Q_L = \frac{\overline{x} - L}{s}$$

as appropriate, then compare the quality statistic (Q_U or Q_L) with the Form k acceptability constant obtained from either Table B.1, B.2 or B.3 for normal, tightened or reduced inspection, respectively. If the quality statistic is greater than or equal to the acceptability constant, the lot is acceptable; if less, it is not acceptable.

Thus, if only the upper specification limit U is given, the lot is

acceptable if $Q_U \geqslant k$,

not acceptable if $Q_U < k$,

or, if only the lower specification limit L is given, the lot is

acceptable if $Q_L \geqslant k$,

not acceptable if $Q_L < k$.

Under separate control of double specification limits, the Form k acceptability constants at L and U may be different. Denote them by k_L and k_U respectively. In this case, the lot is

acceptable if $Q_U \geqslant k_U$ and $Q_L \geqslant k_L$,

not acceptable if $Q_U < k_U$ and/or $Q_L < k_L$.

EXAMPLE 1   **Single, upper specification limit**

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items. Inspection level II, normal inspection with AQL = 2,5 % is to be used. From Table A.1, the sample size code letter is F; from Table B.1, it is seen that a sample size of 13 is required and that the acceptability constant k is 1,405. Suppose that the measurements are as follows: 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 50 °C; 50 °C; 55 °C; 54 °C; 57 °C. Compliance with the acceptability criterion is to be determined.

Information needed	Values obtained
Sample size: n	13
Sample mean: $\overline{x} = \dfrac{\sum\limits_{j=1}^{n} x_j}{n}$	54,615 °C
Sample standard deviation: $s = \sqrt{\dfrac{\sum\limits_{j=1}^{n} (x_j - \overline{x})^2}{n-1}}$	3,330 °C
(See J.1.2, Annex J.)	
Upper specification limit: U	60 °C
Upper quality statistic: $Q_U = (U - \overline{x}) / s$	1,617

Form k acceptability constant: k (see Table B.1)	1,405
Acceptability criterion: Is $Q_U \geqslant k$?	Yes (1,617 > 1,405)

The lot meets the acceptability criterion, and is therefore acceptable.

EXAMPLE 2 **Single, lower specification limit**, requiring the use of an arrow in the master table

A certain pyrotechnic delay mechanism has a specified minimum delay time of 4,0 s. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,10 % applied to the lower limit. From Table A.1, it is seen that the sample size code letter is J, and from Table A.2, it is seen that the sample size is 35 for the "s" method. However, on entering Table B.1 with sample size code letter J and AQL 0,10 %, it is found that there is an arrow pointing to the cell below. This means that an entirely suitable plan is unavailable, and the next best plan is given by sample size code letter k, i.e. sample size 50 and acceptability constant $k = 2,569$. A random sample of size 50 is drawn. Suppose the sample delay times, in seconds, are as follows:

6,95	6,04	6,68	6,63	6,65	6,52	6,59	6,86	6,57	6,91
6,40	6,44	6,34	6,04	6,15	6,29	6,63	6,70	6,67	6,67
6,44	7,15	6,70	6,59	6,51	6,80	5,94	5,92	6,56	6,53
6,35	7,17	6,83	6,25	6,96	7,00	6,38	6,83	6,29	6,39
6,80	5,84	6,16	6,25	6,57	6,71	6,77	6,55	6,87	6,25

Compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Sample size: n	50
Sample mean: $\bar{x} = \dfrac{\sum\limits_{j=1}^{n} x_j}{n}$	6,542 s
Sample standard deviation: $s = \sqrt{\dfrac{\sum\limits_{j=1}^{n} (x_j - \bar{x})^2}{n-1}}$ (See J.1.2, Annex J.)	0, 312 0 s
Lower specification limit: L	4, 0 s
Lower quality statistic: $Q_L = (\bar{x} - L)/s$	8,147
Form k acceptability constant: k (see Table B.1)	2,569
Acceptability criterion: Is $Q_L \geqslant k$?	Yes (8,147 > 2,569)

The lot meets the acceptability criteria, so it is acceptable.

15.3 Form p^* acceptability criterion for the "s" method

15.3.1 Introduction

This part of ISO 3951 also provides a Form p^* method for determining lot acceptability. Whereas Form k applies only to a single quality characteristic with either a single specification limit or with double specification limits that are to be controlled separately, Form p^* applies much more generally to single or multiple quality characteristics with any combination of single or double specification limits with combined, separate or complex control.

15.3.2 Combined control for the "s" method

15.3.2.1 General

If, for the univariate "s" method, combined or complex control of both the upper and lower specification limits is required, i.e. there is an overall AQL for the percentage of the process outside the two specification limits, the first step is to check that the sample standard deviation s is not so large that lot acceptability is impossible. If the value of s exceeds the value of the maximum sample standard deviation (MSSD) determined from Table D.1, D.2 or D.3, no further calculation or reference to graphs is required and the lot shall be immediately judged unacceptable.

If the value of s does not exceed the value of the MSSD, the estimate $\hat{p}$ of the process fraction nonconforming shall be calculated and compared with the Form p^* acceptability constant. The lot is determined to be

acceptable if $\hat{p} \leqslant p^*$,

not acceptable if $\hat{p} > p^*$,

where

$$\hat{p} = \hat{p}_L + \hat{p}_U$$

with

$$\hat{p}_L = B_{(n-2)/2}\left[\frac{1}{2}\left(1 - \frac{\overline{x} - L}{s}\frac{\sqrt{n}}{n-1}\right)\right],$$

$$\hat{p}_U = B_{(n-2)/2}\left[\frac{1}{2}\left(1 - \frac{U - \overline{x}}{s}\frac{\sqrt{n}}{n-1}\right)\right],$$

in which $B_{(n-2)/2}$ represents the distribution function of the symmetric beta distribution with both parameters equal to $(n-2)/2$. (See Annex K for details.)

Form p^* may also be applied to a single specification limit, although in that case, Form k is easier to apply provided an estimate of the process fraction nonconforming is not required.

In the absence of tables of the beta distribution or corresponding computer software, one of the following three procedures shall be used, depending on the sample size.

15.3.2.2 Combined control for the "s" method with $n = 3$

It may be seen from Tables B.1, B.2 and B.3 that the required sample size is 3 for the "s" method for sample size code letter B under normal and tightened inspection and for sample size code letters B to D under reduced inspection.

If combined control of double specification limits is required then, after calculating the sample mean $\bar{x}$ and the sample standard deviation s, the applicable value of the coefficient f_s shall be found from the first row of Table D.1, D.2 or D.3. Determine the maximum sample standard deviation (i.e. the maximum allowable) from the formula

$$\text{MSSD} = s_{\max} = (U - L)f_{s.}$$

Then compare s with $s_{\max}$. If s is greater than $s_{\max}$, then the lot may be rejected without further calculation.

Otherwise, determine the values of $Q_U = (U - \bar{x})/s$ and/or $Q_L = (\bar{x} - L)/s$. Multiply Q_U and/or Q_L by $\sqrt{n}/(n-1) = \sqrt{3}/2$ (i.e. approximately 0,866) and use Table F.1 to determine the estimates $\hat{p}_U$ and/or $\hat{p}_L$ of the fraction of items in the process that are nonconforming beyond the upper and/or lower limits respectively.

NOTE 1   Negative values of Q correspond to estimates of the process fraction nonconforming in excess of 0,5 at that specification limit and will consequently always result in lot non-acceptance under the provisions of this part of ISO 3951. However, in order to obtain a numerical value for record-keeping purposes, the estimate of the process fraction nonconforming may be obtained by entering Table F.1 with the absolute value of $\sqrt{3}Q/2$ and subtracting the result from 1,0. For example, if $Q_U = -0,156$ then $\sqrt{3}Q_U/2 = -0,135$; entering Table F.1 with 0,135 gives an estimate of 0,456 9; subtracting this from 1,0 gives $\hat{p}_U = 0,543\ 1$.

NOTE 2   The basis of Table F.1 is given in Annex K. Instead of using Table F.1, the estimate of the process fraction nonconforming beyond each specification limit when $n = 3$ may be calculated directly as

$$\hat{p} = \begin{cases} 0 & \text{if } Q > 2/\sqrt{3}, \\ \dfrac{2}{\pi}\arcsin\left\{\sqrt{\left(1 - Q\sqrt{3}/2\right)/2}\right\} & \text{if } -2/\sqrt{3} \leqslant Q \leqslant 2/\sqrt{3}, \\ 1 & \text{if } Q < -2/\sqrt{3}. \end{cases}$$

These two estimates must be added to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^*, given in Table G (i.e. Table G.1 for normal inspection, Table G.2 for tightened inspection or Table G.3 for reduced inspection), the lot is considered to be acceptable; otherwise, the lot is considered to be not acceptable.

EXAMPLE   Determination of acceptability for combined control of double specification limits when the sample size is 3.

Torpedoes supplied in batches of 100 are to be inspected for accuracy in the horizontal plane. Positive or negative angular errors are equally unacceptable, so a combined AQL requirement for double specification limits is appropriate. The specification limits are set at 10 metres from the point of aim at a distance of 1 km, with an AQL of 4 %. Because testing is destructive and very costly, it has been agreed between the producer and the responsible authority that special inspection level S-2 is to be used. From Table A.1, the sample size code letter is found to be B. From Table A.2, it is seen that a sample of size 3 is required. Three torpedoes are tested, yielding errors –5,0 m, 6,7 m and 8,8 m. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed	Value obtained
Sample size: n	3
Sample mean: $\bar{x} = \dfrac{\sum\limits_{j=1}^{n} x_j}{n}$	3,5 m
Sample standard deviation: $s = \sqrt{\dfrac{\sum\limits_{j=1}^{n}(x_j - \bar{x})^2}{n-1}}$	7,436 m

(See J.1.2, Annex J.)

Value of f_s for MSSD (Table D.1)	0,474
MSSD = s_{max} = $(U - L)f_s$ = [10 − (−10)] × 0,474	9,48

As $s = 7,436 < s_{max} = 9,48$, the lot *may* be acceptable, so continue with the calculations.

$Q_U = (U - \bar{x})/s$ = (10 − 3,5)/7,436	0,874 1
$Q_L = (\bar{x} - L)/s$ = (3,5 + 10)/7,436	1,815
$\sqrt{3}Q_U/2$	0,757
$\sqrt{3}Q_L/2$	1,572
$\hat{p}_U$ (from Table F.1)	0,226 7
$\hat{p}_L$ (from Table F.1)	0,000 0
$\hat{p} = \hat{p}_U + \hat{p}_L$	0,226 7
p^* (from Table G.1 as it is normal inspection)	0,190 5

As $\hat{p} > p^*$, the lot is not acceptable.

NOTE   This lot is not acceptable even though all inspected items **in the sample** are within the specification limits.

15.3.2.3   Combined control for the "*s*" method with $n = 4$

It may be seen from Tables B.1, B.2 and B.3 that the required sample size is 4 for the "*s*" method for sample size code letter C under normal and tightened inspection and for sample size code letter E under reduced inspection.

After calculating the sample mean $\bar{x}$ and the sample standard deviation s, find the applicable value of the coefficient f_s from the second row of Table D.1, D.2 or D.3. Determine the maximum sample standard deviation (i.e. the maximum allowable) from the formula

$$MSSD = s_{max} = (U - L)f_s.$$

Then compare s with the MSSD. If s is greater than the MSSD, then the lot may be rejected without further calculation.

Otherwise determine the values of $Q_U = (U - \bar{x})/s$ and $Q_L = (\bar{x} - L)/s$. Calculate

$$\hat{p}_U = \begin{cases} 1 & \text{if } Q_U \leqslant \text{-1,5} \\ 0,5 - Q_U/3 & \text{if -1,5} < Q_U < 1,5 \\ 0 & \text{if } Q_U \geqslant 1,5 \end{cases} \tag{1}$$

and

$$\hat{p}_L = \begin{cases} 1 & \text{if } Q_L \leqslant \text{-1,5} \\ 0,5 - Q_L/3 & \text{if -1,5} < Q_L < 1,5 \\ 0 & \text{if } Q_L \geqslant 1,5. \end{cases} \tag{2}$$

Add these two estimates to obtain the estimate $\hat{p} = \hat{p}_U + \hat{p}_L$ of the overall process fraction nonconforming. If $\hat{p}$ does not exceed the applicable maximum allowable value, p^*, given in Table G (i.e. Table G.1 for normal inspection, Table G.2 for tightened inspection or Table G.3 for reduced inspection), the lot is considered to be acceptable; otherwise, the lot is considered to be not acceptable.

NOTE   The basis of Equations (1) and (2) is given in Annex K.

EXAMPLE   Determination of acceptability for combined control of double specification limits when the sample size is 4.

Items are being manufactured in lots of size 25. The lower and upper specification limits on their diameters are 82 mm to 84 mm. Items with diameters that are too large are equally unsatisfactory as those with diameters that are too small, and it has been decided to control the total fraction nonconforming beyond either limit using an AQL of 2,5 % at inspection level II. Normal inspection is to be instituted at the beginning of inspection operations. From Table A.1, the sample size code letter is found to be C. From Table A.2, it is seen that a sample of size 4 is required. The diameters of four items from the first lot are measured, yielding diameters 82,4 mm, 82,2 mm, 83,1 mm and 82,3 mm. Compliance with the acceptability criterion under normal inspection is to be determined.

Information needed	**Value obtained**
Sample size: n	4
Sample mean: $\bar{x} = \dfrac{\sum\limits_{j=1}^{n} x_j}{n}$	82,50 mm
Sample standard deviation: $s = \sqrt{\dfrac{\sum\limits_{j=1}^{n}(x_j - \bar{x})^2}{n-1}}$	0,408 2 mm
(See J.1.2, Annex J.)	
Upper specification limit: U	84,0 mm
Lower specification limit: L	82,0 mm
Value of f_s for MSSD (Table D.1)	0,376
MSSD = s_{max} = $(U - L)f_s$ = $(84 - 82)$ 0,376	0,752 mm

As $s = 0{,}4082 < s_{max} = 0{,}752$, the lot *may* be acceptable, so continue with the calculations.

$Q_U = (U - \bar{x})/s$ = $(84 - 82{,}5)/0{,}408\ 2$	3,674 7
$Q_L = (\bar{x} - L)/s$ = $(82{,}5 - 82)/0{,}408\ 2$	1,224 9
$\hat{p}_U$ [from Equation (1) above]	0,000 0
$\hat{p}_L$ [from Equation (2) above]	0,091 7
$\hat{p} = \hat{p}_U + \hat{p}_L$	0,091 7
p^* (from Table G.1, as it is normal inspection)	0,112 3

As $\hat{p} < p^*$, the lot is acceptable.

15.3.2.4   Combined control for the "s" method with $n \geqslant 5$

After calculating the sample mean $\bar{x}$ and the sample standard deviation s, find the applicable value of the coefficient f_s from Table D.1, D.2 or D.3. Determine the maximum sample standard deviation (i.e. the maximum allowable) from the equation

$$\text{MSSD} = s_{\text{max}} = (U - L)f_s.$$

Then compare s with s_{max}. If s is greater than s_{max}, then the lot may be rejected without further ado.

Otherwise, compute the upper and lower quality statistics $Q_U = (U - \bar{x})/s$ and $Q_L = (\bar{x} - L)/s$. If tables of the beta distribution function or corresponding software are available, determine estimates $\hat{p}_U$ and $\hat{p}_L$ of the process fractions nonconforming in accordance with K.2.1. Otherwise use the method given in K.3.

EXAMPLE   Determination of acceptability for combined control of double specification limits when the sample size is 5 or more.

The minimum temperature of operation for a certain device is specified as 60 °C and the maximum temperature as 70 °C. Production is in inspection lots of 96 items. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. From Table A.1, the sample size code letter is F; from Table A.2 it is seen that a sample of 13 is required, and from Table D.1 that the value of f_s for the MSSD under normal inspection is 0,274. Suppose the measurements obtained are as follows: 63,5 °C; 62,0 °C; 65,2 °C; 61,7 °C; 69,0 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C; 63,4 °C; 60,7 °C; 65,8 °C. Compliance with the acceptability criterion is to be determined.

Information needed	Value obtained
Sample size: n	13
Sample mean: $\bar{x} = \dfrac{\sum\limits_{j=1}^{n} x_j}{n}$	64,276 9 °C
Sample standard deviation: $s = \sqrt{\dfrac{\sum\limits_{j=1}^{n} (x_j - \bar{x})^2}{n-1}}$ (See J.1.2, Annex J)	2,861 9 °C
Upper specification limit: U	70,0 °C
Lower specification limit: L	60,0 °C
Value of f_s for MSSD (Table D.1 for normal inspection)	0,274
MSSD $= s_{\text{max}} = (U - L)f_s = (70 - 60) \times 0,274$	2,74 °C

As the value of s exceeds s_{max}, the lot may immediately be adjudged unacceptable.

NOTE   This lot is not acceptable even though all inspected items **in the sample** are within the specification limits.

Suppose that the AQL had been 4,0 % instead of 1,5 %. In that case, the value of f_s would be 0,328, so s_{max} is equal to 3,28 °C. As s is now less than s_{max}, it is not possible to determine at this stage whether or not the lot is acceptable and further calculations are required.

Two methods of completing the necessary calculations are described. The first applies when tables or software are available for the beta distribution function (see K.2.1). Note that five significant figures are retained throughout the intermediate calculations.

Information needed	Value obtained
$Q_U = (U - \bar{x})/s$	1,999 8
$x_U = \frac{1}{2}[1 - Q_U \sqrt{n}/(n-1)]$	0,199 57
$\hat{p}_U = G_{(n-2)/2}(x_U)$	0,0149 37
$Q_L = (\bar{x} - L)/s$	1,494 4
$x_L = \frac{1}{2}[1 - Q_L \sqrt{n}/(n-1)]$	0,275 49
$\hat{p}_L = G_{(n-2)/2}(x_L)$	0,061 881
p^* (from Table G.1, as it is normal inspection)	0,115 4

The overall process fraction nonconforming is estimated as

$$\hat{p} = \hat{p}_L + \hat{p}_U = 0,061\,881 + 0,014\,937 = 0,076\,82$$

which is less than the acceptability constant p^*. The lot is therefore accepted.

15.3.2.5   Combined control for the "s" method with $n \geqslant 5$ – Approximative method

When beta distribution tables or software are not available, the highly accurate approximative method described in K.3 is recommended. It is demonstrated by applying it below to the foregoing example.

Information needed	Value obtained
$Q_U = (U - \bar{x})/s$	1,999 8
$x_U = \frac{1}{2}[1 - Q_U \sqrt{n}/(n-1)]$	0,199 57
a_n (from Table K.1)	1,583 745
$y_U = a_n \ln[x_U/(1 - x_U)]$	–2,199 8
$w_U = y_U^2 - 3$	1,839 1
As $w_U \geqslant 0$, $t_U = \dfrac{12(n-1)y_U}{12(n-1) + w_U}$	–2,172 1
$\hat{p}_U = \Phi(t_U)$	0,014 924
$Q_L = (\bar{x} - L)/s$	1,494 4
$x_L = \frac{1}{2}[1 - Q_L \sqrt{n}/(n-1)]$	0,275 49
$y_L = a_n \ln[x_L/(1 - x_L)]$	–1,531 4
$w_L = y_L^2 - 3$	–0,654 81

$$w_L < 0, \ t_L = \frac{12(n-2)y_L}{12(n-2)+w_L} \qquad\qquad -1{,}539\ 0$$

$$\hat{p}_L = \Phi(t_L) \qquad\qquad 0{,}061\ 902$$

p^* (from Table G.1 as it is normal inspection) $\qquad\qquad 0{,}115\ 4$

The overall process fraction nonconforming is estimated as

$$\hat{p} = \hat{p}_L + \hat{p}_U = 0{,}061\ 902 + 0{,}014\ 924 = 0{,}076\ 83$$

which is less than the acceptability constant p^*. The lot is therefore accepted.

NOTE   The result of using the approximative method differs by only one unit in the fourth significant figure from the correct value. The approximative method is typically very accurate.

15.3.3 Separate control for the "s" method

When separate AQLs apply to both specification limits, Table G.1, G.2 or G.3 is entered with the sample size code letter and the AQLs at the upper and lower limits to obtain p_U^* and p_L^*. The acceptance criterion is then $\hat{p}_U \leqslant p_U^*$ and $\hat{p}_L \leqslant p_L^*$.

15.3.4 Complex control for the "s" method

Complex control consists of combined control of both specification limits and simultaneous separate control of one of the limits using a separate and smaller AQL. The lot is therefore accepted if $\hat{p} \leqslant p^*$ and *either* $\hat{p}_U \leqslant p_U^*$ *or* $\hat{p}_L \leqslant p_L^*$, whichever is relevant.

16 Standard multivariate "s" method procedures for independent quality characteristics

16.1 General methodology

The general methodology for dealing with a class containing m independent quality characteristics is as follows. Denoting the estimated process fraction nonconforming for the ith quality characteristic by $\hat{p}_i$, the estimated process fraction nonconforming for the class is given by

$$\hat{p} = 1 - (1 - \hat{p}_1)(1 - \hat{p}_2)\ldots(1 - \hat{p}_m),$$

i.e. one minus the product of the estimated process fractions *conforming*.

NOTE   If $\hat{p}_1, \hat{p}_2, \ldots, \hat{p}_m$ are all small, say no greater than 0,01, then $\hat{p}$ is approximately equal to the sum of the individual estimates, i.e. $\hat{p} \approx \hat{p}_1 + \hat{p}_2 + \ldots + \hat{p}_m$.

If there is only one class, say Class A, then the estimated process fraction nonconforming for the class may be denoted by $\hat{p}_A$. The lot is accepted if

$$\hat{p}_A \leqslant p^*$$

and not accepted otherwise, where p^* is the Form p^* acceptability constant given in Table G (i.e. Table G.1 for normal inspection, Table G.2 for tightened inspection or Table G.3 for reduced inspection) for the applicable sample size code letter and the AQL applying to the class.

If there are two or more classes, say Class A, Class B, … with acceptability constants p_A^*, p_B^*, … the lot is accepted if $\hat{p}_A \leqslant p_A^*$ and $\hat{p}_B \leqslant p_B^*$ and … but not accepted if one or more of the inequalities is violated.

If there is more than one class of nonconformity, Class A will contain nonconformities of the greatest level of seriousness and generally have the lowest AQL and therefore the lowest Form $p*$ acceptability constant, Class B will contain nonconformities of the next lower level of seriousness, and have a higher AQL and value of $p*$; and so on. It is possible that different classes of nonconformity will be under inspection at different levels of severity at any one time.

16.2 Example

Consider a product that has five independent quality characteristics x_1, x_2, x_3, x_4 and x_5, none of whose process standard deviations are known. The sample size code letter is H and the sample size is 25 under normal inspection for all five characteristics. Suppose that the requirements and results are as summarized in Table 2.

Table 2 — Example of requirements and results for five quality characteristics with unknown process standard deviations

Variable	Limits	Type of control	Class	Sample mean	Sample standard deviation	Quality statistic Q	$\dfrac{1-Q\sqrt{n}\,/(n-1)}{2}$	$\hat{p}$
x_1	$U_1 = 70,0$	Single	A	$\bar{x}_1 = 68,5$	$s_1 = 0,50$	3,0000	0,1875	0,000418
x_2	$L_2 = 10,0$	Single	B	$\bar{x}_2 = 10,4$	$s_2 = 0,20$	2,0000	0,2917	0,019134
x_3	$U_3 = 4,050$ $L_3 = 3,950$	Combined	A	$\bar{x}_3 = 4,005$	$s_3 = 0,015$	3,0000 3,6667	0,1875 0,1181	0,000418 0,000004 0,000422
x_4	$U_4 = 1,950$ $L_4 = 1,750$	Separate	B A	$\bar{x}_4 = 1,862$	$s_4 = 0,032$	2,7500 3,5000	0,2135 0,1354	0,001380 0,000018
x_5	$U_5 = 214$ $L_5 = 206$	Complex, i.e. Separate + Combined	A B	$\bar{x}_5 = 210$	$s_5 = 1,25$	3,2000 3,2000	0,1667 0,1667	0,000140 0,000140 0,000280

Suppose also that the AQL for Class A nonconformities is 0,25 % and the AQL for Class B is 1,0 %. From Table G.1, it is found that the corresponding Form $p*$ acceptability constants are $p_A^* = 0,010\,12$ and $p_B^* = 0,030\,10$.

The fraction nonconforming for Class A is estimated as

$$\hat{p}_A = 1 - (1 - \hat{p}_1)(1 - \hat{p}_3)(1 - \hat{p}_{4,L})(1 - \hat{p}_{5,U})$$
$$= 1 - (1 - 0,000\,418)(1 - 0,000\,422)(1 - 0,000\,018)(1 - 0,000\,140)$$
$$= 1 - 0,999\,582 \times 0,999\,578 \times 0,999\,982 \times 0,999\,860$$
$$= 1 - 0,999\,002$$
$$= 0,000\,998.$$

The fraction nonconforming for Class B is estimated as

$$\hat{p}_B = 1 - (1 - \hat{p}_2)(1 - \hat{p}_{4,U})(1 - \hat{p}_5)$$
$$= 1 - (1 - 0,019\,134)(1 - 0,001\,380)(1 - 0,000\,280)$$
$$= 1 - 0,980\,866 \times 0,998\,620 \times 0,999\,720$$
$$= 1 - 0,979\,238$$
$$= 0,020\,762.$$

As $\hat{p}_A < p_A^*$ and $\hat{p}_B < p_B^*$, the lot is accepted.

NOTE   The corresponding approximate estimates of the process fractions nonconforming in each class obtained by simply adding the component estimates are

$$\hat{p}_A \cong \hat{p}_1 + \hat{p}_3 + \hat{p}_{4,L} + \hat{p}_{5,U}$$
$$= 0,000\,418 + 0,000\,422 + 0,000\,018 + 0,000\,140$$
$$= 0,000\,998$$

and

$$\hat{p}_B \cong \hat{p}_2 + \hat{p}_{4,U} + \hat{p}_5$$
$$= 0,019\,134 + 0,001\,380 + 0,000\,280$$
$$= 0,020\,794.$$

17 Standard univariate "σ" method procedures

17.1 Obtaining a plan, sampling and preliminary calculations

The "σ" method is only to be used when there is valid evidence that the standard deviation of the process can be considered constant and taken to be σ.

From Table A.1, obtain the sample size code letter. Then, depending on the severity of inspection, enter Table C.1, C.2 or C.3 with the sample size code letter and the specified AQL to obtain the sample size n and acceptability constant k.

Take a random sample of this size, measure the characteristic under inspection, x, for all items of the sample and calculate the sample mean $\bar{x}$. (The sample standard deviation s should also be calculated, but only for the purpose of checking the continued stability of the process standard deviation. See Clause 19.)

17.2 Acceptability criterion for a single specification limit or for double specification limits with separate control

The acceptability criterion can be found by following the procedure given for the "s" method. First replace the s derived from the individual samples by σ, the presumed known value of the standard deviation of the process, and then compare the calculated value of Q with the value of the acceptability constant k obtained from one of the Tables C.1, C.2 and C.3.

Note, for example, that the acceptability criterion $Q_U \; [= (U - \bar{x})/\sigma] \geqslant k$ for an upper specification may be written as $\bar{x} \leqslant U - k\sigma$. As U, k and σ are all known in advance, the acceptance value $\bar{x}_U \; [= U - ks]$ should therefore be determined before inspection begins.

For an upper specification limit, a lot will be

acceptable if $\bar{x} \leqslant \bar{x}_U \; [= U - k\sigma]$; not acceptable if $\bar{x} > \bar{x}_U \; [= U - k\sigma]$.

Similarly, for a lower specification limit, a lot will be

acceptable if $\bar{x} \geqslant \bar{x}_L \; [= L + k\sigma]$; not acceptable if $\bar{x} < \bar{x}_L \; [= L + k\sigma]$.

EXAMPLE   Determination of acceptability for a single specification limit using the "σ" method.

The specified minimum yield point for certain steel castings is 400 N/mm^2. A lot of 500 items is submitted for inspection. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. The value of σ is considered to be 21 N/mm^2. From Table A.1 it is seen that the sample size code letter is H. Then, from Table C.1, it is seen that for an AQL of 1,5 % the sample size n is 12 and the acceptability constant k is 1,613. Suppose the yield points of the sample specimens are 431; 417; 469; 407; 450; 452; 427; 411; 429; 420; 400; 445. Compliance with the acceptability criterion is to be determined.

Information needed	Value obtained
Acceptability constant: k	1,613
Known σ	21 N/mm^2
Product: $k\sigma$	33,9 N/mm^2
Lower specification limit: L	400 N/mm^2
Lower acceptance value: $\bar{x}_L = L + k\sigma$	433,9 N/mm^2
Sum of measurement results: $\sum x$	5 184 N/mm^2
Sample size: n	12
Sample mean: $\bar{x} = \dfrac{\sum_{j=1}^{n} x_j}{n}$	429,8 N/mm^2
Acceptability criterion: Is $\bar{x} \geqslant \bar{x}_L$?	No

The sample mean of the lot does not meet the acceptability criterion, so the lot is not acceptable.

For double specification limits with separate control, the lot may at once be declared to be not acceptable if σ is greater than the MPSD derived from Table E.2. If $\sigma \leqslant$ MPSD, determine the acceptability constants for the upper and lower limits, say k_U and k_L. The lot will be acceptable if

$$\bar{x} \leqslant \bar{x}_U \ [= U - k_U\sigma] \text{ and } \bar{x} \geqslant \bar{x}_L \ [= L + k_L\sigma],$$

and not acceptable if

$$\bar{x} > \bar{x}_U \ [= U - k_U\sigma] \text{ and/or } \bar{x} < \bar{x}_L \ [= L + k_L\sigma].$$

17.3 Acceptability criterion for double specification limits with combined or complex control

If there is a combined AQL requirement for the upper and the lower specification limits, i.e. an overall AQL for the percentage of the process outside both specification limits, the following procedure is recommended.

a) Before sampling, determine the value of the factor f_σ by entering Table E.1 (for combined control) with the single AQL or by entering Table E.3 (for complex control) with both AQLs.

b) Calculate the maximum allowable value of the process standard deviation, using the formula $\sigma_{max} = (U - L)f_\sigma$ for the MPSD.

c) Compare the value of the process standard deviation σ with σ_{max}. If σ exceeds σ_{max}, the process is unacceptable and sampling inspection is pointless until it is demonstrated that the process variability has been adequately reduced.

d) If $\sigma \leqslant \sigma_{max}$, then use the lot size and given inspection level to determine the sample size code letter from Table A.1.

e) From the sample size code letter and inspection severity (i.e. whether inspection is normal, tightened or reduced) determine the sample size, n, and acceptability constant, p^*, from Table G.1, G.2 or G.3.

f) Select a random sample of size n from the lot and calculate the sample mean $\bar{x}$.

g) Using the method given in K.2.2, calculate $\hat{p}_U$, $\hat{p}_L$ and $\hat{p} = \hat{p}_U + \hat{p}_L$.

h) If $\hat{p} > p^*$, the lot is not acceptable for either combined or complex control and no other calculations or comparisons are required.

i) For combined control the lot is acceptable if $\hat{p} \leqslant p^*$.

j) For complex control, determine from Table G.1, G.2 or G.3 the acceptability constant for the single specification limit, i.e. p_U^* for an upper specification limit or p_L^* for a lower specification limit. For complex control that includes a separate AQL for the upper specification limit, the lot is acceptable if $\hat{p} \leqslant p^*$ and $\hat{p}_U \leqslant p_U^*$. For complex control that includes a separate AQL for the lower specification limit, the lot is acceptable if $\hat{p} \leqslant p^*$ and $\hat{p}_L \leqslant p_L^*$.

EXAMPLE   Determination of acceptability for combined control under the "σ" method.

The specification for electrical resistance of a certain electrical component is 520 ± 50 Ω. Production is at a rate of 2 500 items per inspection lot. Inspection level II, normal inspection, with a single AQL of 4 %, is to be used for the two specification limits (470 Ω and 570 Ω). σ is known to be 21,0 Ω. Entering Table A.1 with the lot size and inspection level, it is found that the sample size code letter is K; from Table A.2, it is seen that a sample size of 18 is required under normal inspection. Suppose the values of the sample resistance in Ω are as follows: 515; 491; 479; 507; 543; 521; 536; 483; 509; 514; 507; 484; 526; 552; 499; 530; 512; 492.

Information needed	Value obtained
Factor from Table E.1: f_σ	0,223
Upper specification limit: U	570 Ω
Lower specification limit: L	470 Ω
Maximum process standard deviation, $\sigma_{max} = (U - L)f_\sigma$	22,3 Ω
Known σ	21,0 Ω

(As σ is less than σ_{max}, the sample is analysed further for lot acceptability.)

Sample size: n	18
Acceptability constant k (from Table C.1)	1,340
Upper bound for $\bar{x}$: $\bar{x}_U = U - k\sigma$	541,86 Ω
Lower bound for $\bar{x}$: $\bar{x}_L = L + k\sigma$	498,14 Ω
Sum of measurement results: $\sum x$	9 200 Ω
Sample mean: $\bar{x} = \dfrac{\sum_{j=1}^{n} x_j}{n}$	511,11 Ω

As $\bar{x}$ at 511, 11 Ω lies between the acceptance limits for $\bar{x}$ of 498, 14 Ω and 541,86 Ω, the lot is acceptable.

All the calculations other than the last two lines should be completed before sampling begins.

If, for example, σ had been known to be 25, then σ exceeds the MPSD and therefore sampling inspection should not even have taken place.

18 Standard multivariate "σ" method procedures for independent quality characteristics

18.1 General methodology

The general methodology for dealing with a class containing m independent quality characteristics x_1, x_2, ..., x_m under the "σ" method is similar to that for the multivariate "s" method. Thus, denoting the estimated process fraction nonconforming for the ith quality characteristic by $\hat{p}_i$, the estimated process fraction nonconforming for the class is given by

$$\hat{p} = 1 - (1 - \hat{p}_1)(1 - \hat{p}_2)...(1 - \hat{p}_m),$$

i.e. one minus the product of the estimated process fractions *conforming*.

If there is only one class, say Class A, then the estimated process fraction nonconforming for the class may be denoted by $\hat{p}_A$. The lot is accepted if $\hat{p}_A \leqslant p^*$ and not accepted otherwise, where p^* is the Form p^* acceptability constant given in Table G (i.e. Table G.1 for normal inspection, Table G.2 for tightened inspection or Table G.3 for reduced inspection) for the applicable sample size code letter and the AQL applying to the class.

If there are two or more classes, say Class A, Class B, ... with acceptability constants p_A^*, p_B^*, ... the lot is accepted if $\hat{p}_A \leqslant p_A^*$ and $\hat{p}_B \leqslant p_B^*$ and ... but not accepted if one or more of the inequalities is violated.

If there is more than one class of nonconformity, Class A will contain nonconformities of the greatest level of seriousness and generally have the lowest AQL and therefore the lowest Form p^* acceptability constant, Class B will contain nonconformities of the next lower level of seriousness, and have a higher AQL and value of p^*; and so on. It is possible that different classes of nonconformity will be under inspection at different levels of severity at any one time.

The only difference from the multivariate "σ" method is that the process fraction nonconforming for each characteristic is estimated in accordance with K.2.2 instead of K.2.1.

18.2 Example

The example given in 16.2 is repeated with the sample standard deviations re-designated as process standard deviations.

Consider a product that has five independent quality characteristics x_1, x_2, x_3, x_4 and x_5, for all of which the process standard deviations are known. The sample size code letter is H and the sample size is 12 under normal inspection for all five characteristics. Suppose that the requirements and results are as summarized in Table 3.

Suppose also that the AQL for Class A nonconformities is 0,25 % and the AQL for Class B is 1,0 %. From Table G.1, it is found that the corresponding Form p^* acceptability constants are $p_A^* = 0{,}010\ 12$ and $p_B^* = 0{,}030\ 10$.

Table 3 — Example of requirements and results for five quality characteristics with known process standard deviations

Variable	Limits	Sample mean	Process standard deviation	Type of control	Quality statistic Q	$Q\sqrt{\dfrac{n}{n-1}}$ (n = 12)	$\hat{p}$	Class
x_1	$U_1 = 70,0$	$\overline{x}_1 = 68,5$	$\sigma_1 = 0,50$	Single	3,0000	3,1334	0,000864	A
x_2	$L_2 = 10,0$	$\overline{x}_2 = 10,4$	$\sigma_2 = 0,20$	Single	2,0000	2,0889	0,018357	B
x_3	$U_3 = 4,050$ $L_3 = 3,950$	$\overline{x}_3 = 4,005$	$\sigma_3 = 0,015$	Combined	3,0000 3,6667	3,1334 3,8297	0,000864 0,000064 0,000928	A
x_4	$U_4 = 1,950$ $L_4 = 1,750$	$\overline{x}_4 = 1,862$	$\sigma_4 = 0,032$	Separate	2,7500 3,5000	2,8723 3,6556	0,002038 0,000128	B A
x_5	$U_5 = 214$ $L_5 = 206$	$\overline{x}_5 = 210$	$\sigma_5 = 1,25$	Complex, i.e. Separate + Combined	3,2000 3,2000	3,3423 3,3423	0,000415 0,000415 0,000830	A A B

The fraction nonconforming for Class A is estimated as

$$\hat{p}_{A} = 1 - (1 - \hat{p}_1)(1 - \hat{p}_3)(1 - \hat{p}_{4,L})(1 - \hat{p}_{5,U})$$
$$= 1 - (1 - 0,000\,864)(1 - 0,000\,928)(1 - 0,000\,128)(1 - 0,000\,415)$$
$$= 1 - 0,999\,136 \times 0,999\,072 \times 0,999\,872 \times 0,999\,585$$
$$= 1 - 0,997\,667$$
$$= 0,002\,333.$$

The fraction nonconforming for Class B is estimated as

$$\hat{p}_{B} = 1 - (1 - \hat{p}_2)(1 - \hat{p}_{4,U})(1 - \hat{p}_5)$$
$$= 1 - (1 - 0,018\,357)(1 - 0,002\,038)(1 - 0,000\,830)$$
$$= 1 - 0,981\,643 \times 0,997\,962 \times 0,999\,170$$
$$= 1 - 0,978\,829$$
$$= 0,021\,171.$$

As $\hat{p}_{A} < p_{A}^{*}$ and $\hat{p}_{B} < p_{B}^{*}$, the lot is accepted.

19 Standard multivariate combined "s" and "σ" method procedures for independent quality characteristics

19.1 General methodology

Cases may arise in which the process standard deviations of some of the quality characteristics in a class are known and some are unknown. The general methodology for dealing with such a class containing m independent quality characteristics is, as before, to estimate the process fraction nonconforming for the class by

$$\hat{p} = 1 - (1 - \hat{p}_1)(1 - \hat{p}_2)...(1 - \hat{p}_m).$$

If there is only one class, say Class A, then the estimated process fraction nonconforming for the class may be denoted by $\hat{p}_A$. The lot is accepted if

$$\hat{p}_A \leqslant p^*$$

and not accepted otherwise, where p^* is the Form p^* acceptability constant given in Table G (i.e. Table G.1 for normal inspection, Table G.2 for tightened inspection or Table G.3 for reduced inspection) for the applicable sample size code letter and the AQL applying to the class.

If there are two or more classes, say Class A, Class B, etc. with acceptability constants p_A^*, p_B^*, etc., the lot is accepted if $\hat{p}_A \leqslant p_A^*$, $\hat{p}_B \leqslant p_B^*$, etc., but not accepted if one or more of the inequalities is violated.

The difference between this procedure and the procedures given in Clauses 16 and 18 is that the sub-class of quality characteristics with unknown process standard deviations require one sample size while the sub-class of characteristics with known process standard deviations require a smaller sample size; estimates of the process fractions nonconforming for each characteristic in the former sub-class are obtained in accordance with K.2.1 while estimates for the latter sub-class are obtained in accordance with K.2.2.

19.2 Example

Consider, as before, a product that has five independent quality characteristics x_1, x_2, x_3, x_4 and x_5. However, in this case only characteristics x_1 and x_4 have process standard deviations whose values are known. The sample size code letter is H and the sample size is 12 or 25 under normal inspection for all five characteristics, depending on whether the process standard deviation is known or unknown respectively. Suppose that the requirements and results are as given in Table 4.

Again, suppose that the AQL for Class A nonconformities is 0,25 % and the AQL for Class B is 1,0 %, so from Table G.1 it is found that the corresponding Form p^* acceptability constants are $p_A^* = 0,01012$ and $p_B^* = 0,03010$.

The fraction nonconforming for Class A is estimated as

$$\hat{p}_A = 1 - (1 - \hat{p}_1)(1 - \hat{p}_3)(1 - \hat{p}_{4,L})(1 - \hat{p}_{5,U})$$
$$= 1 - (1 - 0,000\,864)(1 - 0,000\,422)(1 - 0,000\,018)(1 - 0,000\,140)$$
$$= 1 - 0,999\,136 \times 0,999\,578 \times 0,999\,982 \times 0,999\,860$$
$$= 1 - 0,998\,557$$
$$= 0,001\,443.$$

Table 4 — Example of requirements and results for five quality characteristics, some with known and some with unknown process standard deviations

Variable	Limits	Sample size, n	Sample mean	Standard deviation	Type of control	Quality statistic Q	$\dfrac{1 - Q\sqrt{n}/(n-1)}{2}$ (n = 25)	$Q\sqrt{\dfrac{n}{n-1}}$ (n = 12)	$\hat{p}$	Class
x_1	$U_1 = 70,0$	12	$\bar{x}_1 = 68,5$	$\sigma_1 = 0,500$	Single	3,0000		3,1334	0,000864	A
x_2	$L_2 = 10,0$	25	$\bar{x}_2 = 10,4$	$s_2 = 0,200$	Single	2,0000	0,2917		0,019134	B
x_3	$U_3 = 4,050$ $L_3 = 3,950$	25	$\bar{x}_3 = 4,005$	$s_3 = 0,015$	Combined	3,0000 3,6667	0,1875 0,1181		0,000418 0,000004 0,000422	A
x_4	$U_4 = 1,950$ $L_4 = 1,750$	12	$\bar{x}_4 = 1,862$	$\sigma_4 = 0,032$	Separate	2,7500 3,5000		2,8723 3,6556	0,002038 0,000018	B A
x_5	$U_5 = 214$ $L_5 = 206$	25	$\bar{x}_5 = 210$	$s_2 = 1,250$	Complex: i.e. Separate + Combined	3,2000 3,2000	0,1667 0,1667		0,000140 0,000140 0,000280	A A B

The fraction nonconforming for Class B is estimated as

$$\hat{p}_B = 1 - (1 - \hat{p}_2)(1 - \hat{p}_{4,U})(1 - \hat{p}_5)$$
$$= 1 - (1 - 0{,}019\,134)(1 - 0{,}002\,038)(1 - 0{,}000\,280)$$
$$= 1 - 0{,}980\,866 \times 0{,}997\,962 \times 0{,}999\,720$$
$$= 1 - 0{,}978\,593$$

As $\hat{p}_A < p_A^*$ and $\hat{p}_B < p_B^*$, the lot is accepted.

20 Procedure during continuing inspection

As a variables sampling inspection plan can only operate efficiently if

a) the characteristic being inspected is normally distributed,

b) records are kept, and

c) the switching rules are obeyed,

it is necessary to ensure that these requirements are being met.

21 Normality and outliers

21.1 Normality

The responsible authority should have checked for normality before sampling began. In case of doubt, a statistician should advise whether the distribution appears suitable for sampling by variables, or whether use should be made of the tests for departure from normality such as those given in ISO 5479. Normality should be reconfirmed periodically, particularly if there is a significant change of any kind in production, e.g. in personnel, design, materials or production method.

21.2 Outliers

An outlier (or an outlying observation) is one that appears to deviate markedly from other observations in the sample in which it occurs. A single outlier, even when it lies within specification limits, will produce an increase in variability and change the mean and may consequently lead to non-acceptance of the lot. (See, for example, ISO 5725-2.) When outliers are detected, the disposition of the lot should be a matter for negotiation between the vendor and vendee.

22 Records

22.1 Control charts

One of the advantages of inspection by variables is that trends in the quality level of the product can be detected and a warning given before an unacceptable standard is reached, but this is only possible if adequate records are kept.

Whatever the method used, "s" or "σ", records should be kept of the values of $\bar{x}$ and s, preferably in the form of control charts. (See ISO 7870 and ISO 8258.)

This procedure should be applied especially with the "σ" method in order to verify that the values of s obtained from the samples fall within the limits of the prescribed value of σ.

For double specification limits with a combined AQL requirement, the value of the MSSD, given in Table D.1, D.2 or D.3, should be plotted on the s control chart, as an indication of an unacceptable value.

NOTE   Control charts are used to detect trends. The ultimate decision as to the acceptability of an individual lot is governed by the procedures given in Clauses 15 to 19.

22.2 Lots that are not accepted

Particular care shall be taken to record all lots that are not accepted and to see that switching rules are implemented. Any lot not accepted by the sampling plan shall not be resubmitted either in whole or in part without the permission of the responsible authority.

23 Operation of switching rules

The standard switching rules are as follows.

a) **Normal inspection** is used at the start of inspection (unless otherwise designated) and shall continue to be used during the course of inspection until tightened inspection becomes necessary or reduced inspection is allowed.

b) **Tightened inspection** shall be **instituted** when two lots on original normal inspection are not accepted within any five or fewer successive lots.

   Tightened inspection is generally achieved by increasing the values of the acceptability constant. The values are tabulated in Table B.2 for the "s" method and Table C.2 for the "σ" method. For neither method is there a change in the size of the sample in switching from normal to tightened inspection, unless the AQL is so small that the tables indicate, with a downward arrow, that an increase in sample size is necessary.

c) **Tightened inspection** shall be **relaxed** when five successive lots on original inspection have been accepted on tightened inspection; then normal inspection shall be reinstated.

d) **Reduced inspection** may be instituted after ten successive lots have been accepted under normal inspection, provided that

   1) these lots would have been acceptable if the AQL had been one step tighter;

   NOTE   If a value of k for this tighter AQL is not given in Table B.1 ("s" method) or Table C.1 ("σ" method), or a value of p^* is not given in Table G.1, refer to Table I.

   2) production is in statistical control;

   3) reduced inspection is considered desirable by the responsible authority.

   Reduced inspection is conducted on a much smaller sample than normal inspection and the value of the acceptability constant is also decreased. The values of n and k for reduced inspection are given in Table B.3 for the "s" method and Table C.3 for the "σ" method.

e) **Reduced inspection** shall **cease** and normal inspection be reinstated if any of the following occur on original inspection:

   1) a lot is not accepted;

   2) production becomes irregular or delayed;

   3) reduced inspection is no longer considered desirable by the responsible authority.

24 Discontinuation and resumption of inspection

If the cumulative number of lots not accepted in a sequence of consecutive lots on original tightened inspection reaches 5, the acceptance procedures of this part of ISO 3951 shall be discontinued.

Inspection under the provisions of this part of ISO 3951 shall not be resumed until action has been taken by the supplier to improve the quality of the submitted product or service. Tightened inspection shall then be used as if 23 b) had been invoked.

25 Switching between the "s" and "σ" methods

25.1 Estimating the process standard deviation

While this part of ISO 3951 is being used, the weighted root mean square of the values of s shall be calculated periodically as estimates of the process standard deviation σ, for both the "s" and the "σ" methods. (See Clause J.2 in Annex J.) The value of σ shall be estimated at five-lot intervals, unless the responsible authority specifies another interval. The estimate shall be based on the preceding 10 lots, unless the responsible authority specifies another number of lots.

25.2 State of statistical control

Calculate the upper control limit for each of the 10 lots (or other number of lots specified by the responsible authority) from the expression $c_U\sigma$, where c_U is a factor which depends on the sample size n and is given in Table H. If none of the sample standard deviations, s_i, exceeds the corresponding control limit, then the process may be considered to be in a state of statistical control; otherwise the process shall be considered to be out of statistical control.

NOTE 1   If the sample sizes from the lots are all equal, then the value of $c_U\sigma$ is common to all the lots.

NOTE 2   If the sample sizes from each lot vary, it is not necessary to calculate $c_U\sigma$ for those lots for which the sample standard deviation, s_i, is less than or equal to σ.

NOTE 3   The values of c_U are such that, in a stable process with constant standard deviation σ, in ten consecutive lots the probability of one or more sample standard deviations exceeding its $c_U\sigma$ is 5 %. Thus the probability of a false alarm is constrained to 5 %.

25.3 Switching from the "s" method to the "σ" method

If the process is considered to be in a state of statistical control under the "s" method, then the "σ" method may be instituted using the latest value of σ.

NOTE   This switch is made at the discretion of the responsible authority.

25.4 Switching from the "σ" method to the "s" method

It is recommended that a control chart for s be kept even under the "σ" method. As soon as there is any doubt that the process remains in statistical control, inspection shall be switched to the "s" method.

Annex A
(normative)

Tables for determining the appropriate sample size

Table A.1 — Sample size code letters and inspection levels

Lot or batch size	Special inspection levels				General inspection levels		
	S-1	S-2	S-3	S-4	I	II	III
2 to 8	B	B	B	B	B	B	B
9 to 15	B	B	B	B	B	B	C
16 to 25	B	B	B	B	B	C	D
26 to 50	B	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1 200	C	C	E	F	G	J	K
1 201 to 3 200	C	D	E	G	H	K	L
3 201 to 10 000	C	D	F	G	J	L	M
10 001 to 35 000	C	D	F	H	K	M	N
35 001 to 150 000	D	E	G	J	L	N	P
150 001 to 500 000	D	E	G	J	M	P	Q
500 000 and over	D	E	H	K	N	Q	R

NOTE   The sample size code letters and inspection levels in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

Table A.2 — Sample sizes for sample size code letters and inspection method

Sample size code letter	"*s*" method		"*σ*" method		Equivalent attributes sample size in ISO 2859-1	
	Normal and tightened inspection	Reduced inspection	Normal and tightened inspection	Reduced inspection	Normal and tightened inspection	Reduced inspection
B	3	3	2	2	3	2
C	4	3	3	2	5	2
D	6	3	4	2	8	3
E	9	4	6	3	13	5
F	13	6	8	4	20	8
G	18	9	10	6	32	13
H	25	13	12	8	50	20
J	35	18	15	10	80	32
K	50	25	18	12	125	50
L	70	35	21	15	200	80
M	95	50	25	18	315	125
N	125	70	32	21	500	200
P	160	95	40	25	800	315
Q	200	125	50	32	1250	500
R	250	160	65	40	2000	800

NOTE   The sample size code letters and inspection levels in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

Annex B
(normative)

Form k single sampling plans for the "s" method

Table B.1 — Form k single sampling plans for normal inspection (master table): "s" method

Code letter	Sample size	— Acceptance quality limit (in percent nonconforming)

Code letter	Sample size	0,01 k	0,015 k	0,025 k	0,04 k	0,065 k	0,10 k	0,15 k	0,25 k	0,40 k	0,65 k	1,0 k	1,5 k	2,5 k	4,0 k	6,5 k	10,0 k
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,954	0,818	0,526
C	4	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,163	1,046	0,853	0,580
D	6	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,395	1,275	1,108	0,902	0,587
E	9	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,615	1,494	1,338	1,159	0,907	0,597
F	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,830	1,712	1,565	1,405	1,189	0,938	0,614
G	18	↓	↓	↓	↓	↓	↓	↓	↓	2,025	1,910	1,770	1,622	1,429	1,212	0,944	0,718
H	25	↓	↓	↓	↓	↓	↓	↓	2,215	2,102	1,969	1,829	1,652	1,457	1,225	1,035	0,809
J	35	↓	↓	↓	↓	↓	↓	2,399	2,289	2,160	2,028	1,862	1,684	1,476	1,311	1,118	0,912
K	50	↓	↓	↓	↓	↓	2,569	2,461	2,336	2,209	2,052	1,885	1,693	1,543	1,372	1,193	0,947
L	70	↓	↓	↓	↓	2,736	2,631	2,510	2,389	2,239	2,082	1,904	1,766	1,611	1,451	1,238	↑
M	95	↓	↓	↓	2,889	2,787	2,670	2,553	2,410	2,261	2,093	1,965	1,822	1,676	1,484	↑	↑
N	125	↓	↓	3,037	2,937	2,824	2,711	2,574	2,432	2,274	2,154	2,021	1,886	1,710	↑	↑	↑
P	160	↓	3,179	3,082	2,973	2,865	2,733	2,597	2,447	2,334	2,209	2,083	1,921	↑	↑	↑	↑
Q	200	3,310	3,215	3,109	3,004	2,877	2,747	2,603	2,495	2,377	2,258	2,106	↑	↑	↑	↑	↑
R	250	3,350	3,247	3,146	3,023	2,898	2,760	2,657	2,545	2,432	2,289	↑	↑	↑	↑	↑	↑

NOTE 1   The sample size code letters in this area correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2   Symbols:

↓ (down)   There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ (up)   There is no suitable plan in this area; use the first sampling plan above the arrow.

Table B.2 — Form k single sampling plans for tightened inspection (master table): "s" method

Symbols: ↓ = use the first sampling plan below the arrow; ↑ = use the first sampling plan above the arrow.

Code letter	Sample size	Acceptance quality limit (in percent nonconforming) — k															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,954	0,818
C	4	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,163	1,046	0,853
D	6	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,395	1,275	1,108	0,902
E	9	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,615	1,494	1,338	1,159	0,907
F	13	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,830	1,712	1,565	1,405	1,189	0,938
G	18	↓	↓	↓	↓	↓	↓	↓	↓	↓	2,025	1,910	1,770	1,622	1,429	1,212	0,944
H	25	↓	↓	↓	↓	↓	↓	↓	↓	2,215	2,102	1,969	1,829	1,652	1,457	1,225	0,995
J	35	↓	↓	↓	↓	↓	↓	↓	2,399	2,289	2,160	2,028	1,862	1,684	1,476	1,242	1,010
K	50	↓	↓	↓	↓	↓	↓	2,569	2,461	2,336	2,209	2,052	1,885	1,693	1,481	1,277	1,044
L	70	↓	↓	↓	↓	↓	2,736	2,631	2,510	2,389	2,239	2,082	1,904	1,710	1,526	1,322	↑
M	95	↓	↓	↓	↓	2,889	2,787	2,670	2,553	2,410	2,261	2,093	1,913	1,745	1,559	↑	↑
N	125	↓	↓	↓	3,037	2,937	2,824	2,711	2,574	2,432	2,274	2,105	1,949	1,779	↑	↑	↑
P	160	↓	↓	3,179	3,082	2,973	2,865	2,733	2,597	2,447	2,288	2,141	1,984	↑	↑	↑	↑
Q	200	↓	3,310	3,215	3,109	3,004	2,877	2,747	2,603	2,452	2,313	2,165	↑	↑	↑	↑	↑
R	250	3,442	3,350	3,247	3,146	3,023	2,898	2,760	2,616	2,485	2,345	↑	↑	↑	↑	↑	↑

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols: ↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table B.3 — Form k single sampling plans for reduced inspection (master table): "s" method

Acceptance quality limit (in percent nonconforming)

Code letter	Sample size	0,01 k	0,015 k	0,025 k	0,04 k	0,065 k	0,10 k	0,15 k	0,25 k	0,40 k	0,65 k	1,0 k	1,5 k	2,5 k	4,0 k	6,5 k	10,0 k
B - D	3												0,954	0,907	0,818	0,526	0,023
E	4											1,163	1,119	1,046	0,853	0,580	0,099
F	6										1,395	1,348	1,275	1,108	0,902	0,587	0,161
G	9									1,615	1,566	1,494	1,338	1,159	0,907	0,597	0,368
H	13								1,830	1,782	1,712	1,565	1,405	1,189	0,938	0,763	0,461
J	18							2,025	1,978	1,910	1,770	1,622	1,429	1,212	1,065	0,823	0,619
K	25						2,215	2,168	2,102	1,969	1,829	1,652	1,457	1,329	1,123	0,955	0,809
L	35					2,399	2,353	2,289	2,160	2,028	1,862	1,684	1,569	1,387	1,242	1,118	
M	50				2,569	2,524	2,461	2,336	2,209	2,052	1,885	1,778	1,612	1,481	1,372		
N	70			2,736	2,692	2,631	2,510	2,389	2,239	2,082	1,982	1,829	1,710	1,611			
P	95		2,889	2,846	2,787	2,670	2,553	2,410	2,261	2,067	2,023	1,913	1,822				
Q	125	3,037	2,995	2,937	2,824	2,711	2,574	2,432	2,344	2,208	2,105	2,021					
R	160	3,139	3,082	2,973	2,865	2,733	2,597	2,513	2,385	2,288	2,209						

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

NOTE 2

Symbols: → There is no suitable plan in this area; use the first sampling plan below the arrow.

← There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex C
(normative)

Form k single sampling plans for the "σ" method

Table C.1 — Form *k* single sampling plans for normal inspection (master table): "σ" method

Code letter	Sample size	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
		k	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*
B	2														0,620	0,478	0,273
C	3													0,991	0,841	0,643	0,412
D	4												1,296	1,148	0,964	0,760	0,478
E	6											1,578	1,432	1,256	1,068	0,818	0,528
F	8										1,821	1,682	1,517	1,344	1,121	0,872	0,564
G	10									2,030	1,897	1,742	1,581	1,378	1,157	0,893	0,675
H	12								2,223	2,096	1,949	1,800	1,613	1,412	1,179	0,991	0,771
J	15							2,410	2,289	2,150	2,009	1,835	1,650	1,439	1,273	1,082	0,879
K	18						2,576	2,459	2,327	2,193	2,029	1,857	1,662	1,511	1,340	1,162	0,919
L	21					2,738	2,627	2,500	2,374	2,218	2,057	1,876	1,737	1,582	1,422	1,210	
M	25				2,890	2,783	2,661	2,540	2,393	2,240	2,070	1,941	1,797	1,650	1,459		
N	32			3,041	2,937	2,820	2,704	2,563	2,419	2,258	2,136	2,001	1,866	1,690			
P	40		3,186	3,086	2,974	2,862	2,727	2,589	2,436	2,321	2,194	2,068	1,905				
Q	50	3,319	3,222	3,113	3,005	2,875	2,742	2,596	2,487	2,367	2,247	2,094					
R	65	3,359	3,254	3,150	3,025	2,897	2,758	2,653	2,539	2,426	2,281						

NOTE 1   The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2   Symbols:

⬇ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

⬆ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table C.2 — Form k single sampling plans for tightened inspection (master table): "σ" method

Code letter	Sample size	Acceptance quality limit (in percent nonconforming)															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
		k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
B	2															0,620	0,478
C	3														0,991	0,841	0,643
D	4													1,296	1,148	0,964	0,760
E	6												1,578	1,432	1,256	1,068	0,818
F	8											1,821	1,682	1,517	1,344	1,121	0,872
G	10										2,030	1,897	1,742	1,581	1,378	1,157	0,893
H	12									2,223	2,096	1,949	1,800	1,613	1,412	1,179	0,913
J	15								2,410	2,289	2,150	2,009	1,835	1,650	1,439	1,204	0,976
K	18							2,576	2,459	2,327	2,193	2,029	1,857	1,662	1,449	1,245	1,015
L	21						2,738	2,627	2,500	2,374	2,218	2,057	1,876	1,681	1,497	1,293	
M	25					2,890	2,783	2,661	2,540	2,393	2,240	2,070	1,888	1,719	1,534		
N	32				3,041	2,937	2,820	2,704	2,563	2,419	2,258	2,087	1,929	1,758			
P	40			3,186	3,086	2,974	2,862	2,727	2,589	2,436	2,274	2,127	1,968				
Q	50		3,319	3,222	3,113	3,005	2,875	2,742	2,596	2,443	2,303	2,154					
R	65	3,454	3,359	3,254	3,150	3,025	2,897	2,758	2,611	2,478	2,337						

NOTE 1   The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2   Symbols: ⬇   There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

⬆   There is no suitable plan in this area; use the first sampling plan above the arrow.

Table C.3 — Form k single sampling plans for reduced inspection (master table): "σ" method

Code letter	Sample size	Acceptance quality limit (in percent nonconforming)															
		0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
		k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
B - D	2	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,620	0,565	0,478	0,273	0,011
E	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	0,991	0,931	0,841	0,643	0,412	0,067
F	4	↓	↓	↓	↓	↓	↓	↓	↓	↓	1,296	1,236	1,148	0,964	0,760	0,478	0,129
G	6	↓	↓	↓	↓	↓	↓	↓	↓	1,578	1,518	1,432	1,256	1,068	0,818	0,528	0,323
H	8	↓	↓	↓	↓	↓	↓	↓	1,821	1,764	1,682	1,517	1,344	1,121	0,872	0,705	0,422
J	10	↓	↓	↓	↓	↓	↓	2,030	1,975	1,897	1,742	1,581	1,378	1,157	1,012	0,776	0,581
K	12	↓	↓	↓	↓	↓	2,223	2,170	2,096	1,949	1,800	1,613	1,412	1,283	1,078	0,913	0,771
L	15	↓	↓	↓	↓	2,410	2,360	2,289	2,150	2,009	1,835	1,650	1,533	1,349	1,204	1,082	↑
M	18	↓	↓	↓	2,576	2,527	2,459	2,327	2,193	2,029	1,857	1,748	1,580	1,449	1,340	↑	↑
N	21	↓	↓	2,738	2,691	2,627	2,500	2,374	2,218	2,057	1,956	1,801	1,681	1,582	↑	↑	↑
P	25	↓	2,890	2,845	2,783	2,661	2,540	2,393	2,240	2,145	1,999	1,888	1,797	↑	↑	↑	↑
Q	32	3,041	2,998	2,937	2,820	2,704	2,563	2,419	2,328	2,191	2,087	2,001	↑	↑	↑	↑	↑
R	40	3,144	3,086	2,974	2,862	2,727	2,589	2,503	2,373	2,274	2,194	↑	↑	↑	↑	↑	↑

NOTE 1   The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2   Symbols: ↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex D
(normative)

Values of f_s for maximum sample standard deviation (MSSD)

Table D.1 — Values of f_s for maximum sample standard deviation (MSSD) for combined control of double specification limits: normal inspection, "s" method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B	→	→	→	→	→	→	→	→	→	→	→	→	→	0,474	0,507	0,595
C	→	→	→	→	→	→	→	→	→	→	→	→	0,376	0,393	0,425	0,481
D	→	→	→	→	→	→	→	→	→	→	→	0,314	0,331	0,357	0,396	0,471
E	→	→	→	→	→	→	→	→	→	→	0,274	0,289	0,310	0,338	0,386	0,464
F	→	→	→	→	→	→	→	→	→	0,245	0,257	0,274	0,295	0,328	0,375	0,457
G	→	→	→	→	→	→	→	→	0,224	0,234	0,248	0,264	0,289	0,321	0,372	0,426
H	→	→	→	→	→	→	→	0,206	0,215	0,227	0,240	0,259	0,283	0,317	0,351	0,401
J	→	→	→	→	→	→	0,192	0,200	0,209	0,220	0,235	0,254	0,279	0,302	0,335	0,376
K	→	→	→	→	→	0,180	0,187	0,195	0,205	0,217	0,232	0,252	0,269	0,292	0,320	0,368
L	→	→	→	→	0,170	0,176	0,183	0,191	0,202	0,214	0,230	0,243	0,261	0,281	0,312	←
M	→	→	→	0,162	0,167	0,174	0,180	0,189	0,200	0,213	0,224	0,237	0,253	0,276	←	←
N	→	→	0,155	0,160	0,165	0,171	0,179	0,188	0,199	0,208	0,219	0,231	0,249	←	←	←
P	→	0,149	0,153	0,158	0,163	0,170	0,177	0,187	0,194	0,203	0,213	0,227	←	←	←	←
Q	0,143	0,147	0,152	0,156	0,162	0,169	0,177	0,183	0,191	0,199	0,211	←	←	←	←	←
R	0,142	0,146	0,150	0,155	0,161	0,168	0,174	0,180	0,187	0,197	←	←	←	←	←	←

NOTE   The MSSD is obtained by multiplying the standardized MSSD f_s by the difference between the upper specification limit U and the lower specification limit L, i.e. MSSD = s_{max} = $(U - L)f_s$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under normal inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD, then there is a possibility, but not a certainty, that the lot will be accepted.

Table D.2 — Values of f_s for maximum sample standard deviation (MSSD) for combined control of double specification limits: tightened inspection, "s" method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B															0,474	0,507
C														0,376	0,393	0,425
D													0,314	0,331	0,357	0,396
E												0,274	0,289	0,310	0,338	0,386
F											0,245	0,257	0,274	0,295	0,328	0,375
G										0,224	0,234	0,248	0,264	0,289	0,321	0,372
H									0,206	0,215	0,227	0,240	0,259	0,283	0,317	0,368
J								0,192	0,200	0,209	0,220	0,235	0,254	0,279	0,313	0,355
K							0,180	0,187	0,195	0,205	0,217	0,232	0,252	0,277	0,307	0,348
L						0,170	0,176	0,183	0,191	0,202	0,214	0,230	0,249	0,271	0,299	
M					0,162	0,167	0,174	0,180	0,189	0,200	0,213	0,228	0,245	0,266		
N				0,155	0,160	0,165	0,171	0,179	0,188	0,199	0,212	0,225	0,241			
P			0,149	0,153	0,158	0,163	0,170	0,177	0,187	0,197	0,208	0,222				
Q		0,143	0,147	0,152	0,156	0,162	0,169	0,177	0,186	0,196	0,206					
R	0,138	0,142	0,146	0,150	0,155	0,161	0,168	0,176	0,184	0,193						

NOTE   The MSSD is obtained by multiplying the standardized MSSD f_s by the difference between the upper specification limit U and the lower specification limit L, i.e. MSSD = s_{max} = $(U - L)f_s$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under tightened inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD, then there is a possibility, but not a certainty, that the lot will be accepted.

Table D.3 — Values of f_s for maximum sample standard deviation (MSSD) for combined control of double specification limits: reduced inspection, "s" method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s	f_s
B-D												0,474	0,485	0,507	0,595	0,849
E											0,376	0,382	0,393	0,425	0,481	0,625
F										0,314	0,320	0,331	0,357	0,396	0,471	0,623
G									0,274	0,280	0,289	0,310	0,338	0,386	0,464	0,542
H								0,245	0,250	0,257	0,274	0,295	0,328	0,375	0,416	0,507
J							0,224	0,228	0,234	0,248	0,264	0,289	0,321	0,347	0,399	0,455
K						0,206	0,210	0,215	0,227	0,240	0,259	0,283	0,301	0,335	0,368	0,401
L					0,192	0,195	0,200	0,209	0,220	0,235	0,254	0,267	0,291	0,313	0,335	
M				0,180	0,183	0,187	0,195	0,205	0,217	0,232	0,243	0,261	0,277	0,292		
N			0,170	0,173	0,176	0,183	0,191	0,202	0,214	0,223	0,237	0,249	0,261			
P		0,162	0,164	0,167	0,174	0,180	0,189	0,200	0,207	0,219	0,228	0,237				
Q	0,155	0,157	0,160	0,165	0,171	0,179	0,188	0,194	0,203	0,212	0,219					
R	0,150	0,153	0,158	0,163	0,170	0,177	0,183	0,191	0,197	0,203						

NOTE   The MSSD is obtained by multiplying the standardized MSSD f_s by the difference between the upper specification limit U and the lower specification limit L, i.e. MSSD = $s_{max} = (U - L)f_s$.

The above MSSDs indicate the greatest allowable magnitudes of the sample standard deviation under reduced inspection when using plans for combined control of double specification when the process variability is unknown. If the sample standard deviation is less than the MSSD, there is a possibility, but not a certainty, that the lot will be accepted.

Annex E
(normative)

Values of f_σ process standard deviation

Table E.1 — Values of f_σ for maximum process standard deviation for combined control of double specification limits: "σ" method

Acceptance quality limit (in percent nonconforming)	f_σ
0,010	0,125
0,015	0,129
0,025	0,132
0,040	0,137
0,065	0,141
0,10	0,147
0,15	0,152
0,25	0,157
0,40	0,165
0,65	0,174
1,0	0,184
1,5	0,194
2,5	0,206
4,0	0,223
6,5	0,243
10,0	0,271

NOTE   The MPSD is obtained by multiplying the standardized MPSD f_σ by the difference between the upper specification limit U and the lower specification limit L, i.e. MPSD = σ_{max} = $(U - L)f_\sigma$

The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for combined control of double specification limits when the process variability is known. If the process standard deviation is less than the MPSD, there is a possibility but not a certainty that the lot will be accepted.

Table E.2 — Values of f_σ for maximum process standard deviation (MPSD) for separate control of double specification limits: "σ" method

AQL % (lower limit)	Acceptance quality limit in percent nonconforming (upper limit)															
	0,010	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ
0,010	0,131	0,133	0,134	0,137	0,139	0,142	0,145	0,147	0,151	0,154	0,158	0,163	0,167	0,173	0,179	0,187
0,015	0,133	0,134	0,136	0,139	0,141	0,144	0,147	0,150	0,153	0,157	0,161	0,165	0,170	0,176	0,183	0,191
0,025	0,134	0,136	0,138	0,141	0,144	0,146	0,149	0,152	0,156	0,160	0,164	0,168	0,173	0,179	0,186	0,195
0,040	0,137	0,139	0,141	0,144	0,146	0,149	0,152	0,155	0,159	0,163	0,168	0,172	0,177	0,184	0,191	0,200
0,065	0,139	0,141	0,144	0,146	0,149	0,152	0,155	0,158	0,162	0,167	0,171	0,176	0,181	0,188	0,196	0,205
0,10	0,142	0,144	0,146	0,149	0,152	0,155	0,159	0,162	0,166	0,170	0,175	0,180	0,186	0,193	0,201	0,211
0,15	0,145	0,147	0,149	0,152	0,155	0,159	0,162	0,165	0,170	0,174	0,179	0,185	0,190	0,198	0,207	0,217
0,25	0,147	0,150	0,152	0,155	0,158	0,162	0,165	0,168	0,173	0,178	0,183	0,189	0,195	0,203	0,212	0,223
0,40	0,151	0,153	0,156	0,159	0,162	0,166	0,170	0,173	0,178	0,183	0,189	0,195	0,201	0,210	0,219	0,231
0,65	0,154	0,157	0,160	0,163	0,167	0,170	0,174	0,178	0,183	0,189	0,195	0,201	0,207	0,217	0,227	0,240
1,0	0,158	0,161	0,164	0,168	0,171	0,175	0,179	0,183	0,189	0,195	0,201	0,208	0,215	0,225	0,236	0,250
1,5	0,163	0,165	0,168	0,172	0,176	0,180	0,185	0,189	0,195	0,201	0,208	0,215	0,222	0,233	0,245	0,260
2,5	0,167	0,170	0,173	0,177	0,181	0,186	0,190	0,195	0,201	0,207	0,215	0,222	0,230	0,242	0,255	0,271
4,0	0,173	0,176	0,179	0,184	0,188	0,193	0,198	0,203	0,210	0,217	0,225	0,233	0,242	0,255	0,269	0,288
6,5	0,179	0,183	0,186	0,191	0,196	0,201	0,207	0,212	0,219	0,227	0,236	0,245	0,255	0,269	0,286	0,306
10,0	0,187	0,191	0,195	0,200	0,205	0,211	0,217	0,223	0,231	0,240	0,250	0,260	0,271	0,288	0,306	0,330

NOTE   The MPSD is obtained by multiplying the standardized MPSD f_σ by the difference between the upper specification limit U and the lower specification limit L, i.e. MPSD $= \sigma_{max} = (U - L)f_\sigma$.

The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for separate control of double specification limits when the process standard deviation is known. If the process standard deviation is less than the MPSD, there is a possibility but not a certainty that the lot will be accepted.

Table E.3 — Values of f_σ for maximum process standard deviation (MPSD) for complex control of double specification limits: "σ" method

AQL % (single limit)	Acceptance quality limit in percent nonconforming (both limits combined)														
	0,015	0,025	0,040	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ	f_σ
0,010	0,129	0,132	0,135	0,138	0,141	0,144	0,147	0,151	0,154	0,158	0,162	0,167	0,173	0,179	0,187
0,015		0,132	0,136	0,140	0,143	0,146	0,149	0,153	0,157	0,161	0,165	0,170	0,176	0,183	0,191
0,025			0,137	0,141	0,145	0,148	0,151	0,155	0,159	0,164	0,168	0,173	0,179	0,186	0,195
0,040				0,141	0,146	0,150	0,154	0,158	0,162	0,167	0,172	0,177	0,184	0,191	0,200
0,065					0,147	0,152	0,156	0,161	0,166	0,171	0,176	0,181	0,188	0,196	0,205
0,10						0,152	0,157	0,163	0,169	0,174	0,180	0,185	0,193	0,201	0,211
0,15							0,157	0,165	0,171	0,178	0,183	0,189	0,197	0,206	0,217
0,25								0,165	0,173	0,180	0,187	0,193	0,202	0,211	0,223
0,40									0,174	0,183	0,191	0,198	0,208	0,218	0,230
0,65										0,184	0,194	0,202	0,213	0,225	0,238
1,0											0,194	0,205	0,219	0,232	0,247
1,5												0,206	0,222	0,238	0,255
2,5													0,223	0,242	0,262
4,0														0,243	0,269
6,5															0,271

NOTE   The MPSD is obtained by multiplying the standardized MPSD f_σ by the difference between the upper specification limit U and the lower specification limit L, i.e. MPSD $= \sigma_{max} = (U - L)f_\sigma$

The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for complex control of double specification limits when the process standard deviation is known. If the process standard deviation is less than the MPSD, there is a possibility but not a certainty that the lot will be accepted.

Annex F
(normative)

Estimating the process fraction nonconforming
for sample size 3: "s" method

Table F.1 — Estimated process fraction nonconforming, $\hat{p}$, as a function of the quality statistic Q

First two decimal places of $Q\sqrt{3}/2$	Third decimal place of $Q\sqrt{3}/2$									
	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
0,00	0,5000	0,4997	0,4994	0,4990	0,4987	0,4984	0,4981	0,4978	0,4975	0,4971
0,01	0,4968	0,4965	0,4962	0,4959	0,4955	0,4952	0,4949	0,4946	0,4943	0,4940
0,02	0,4936	0,4933	0,4930	0,4927	0,4924	0,4920	0,4917	0,4914	0,4911	0,4908
0,03	0,4904	0,4901	0,4898	0,4895	0,4892	0,4889	0,4885	0,4882	0,4879	0,4876
0,04	0,4873	0,4869	0,4866	0,4863	0,4860	0,4857	0,4854	0,4850	0,4847	0,4844
0,05	0,4841	0,4838	0,4834	0,4831	0,4828	0,4825	0,4822	0,4818	0,4815	0,4812
0,06	0,4809	0,4806	0,4803	0,4799	0,4796	0,4793	0,4790	0,4787	0,4783	0,4780
0,07	0,4777	0,4774	0,4771	0,4767	0,4764	0,4761	0,4758	0,4755	0,4751	0,4748
0,08	0,4745	0,4742	0,4739	0,4735	0,4732	0,4729	0,4726	0,4723	0,4720	0,4716
0,09	0,4713	0,4710	0,4707	0,4704	0,4700	0,4697	0,4694	0,4691	0,4688	0,4684
0,10	0,4681	0,4678	0,4675	0,4672	0,4668	0,4665	0,4662	0,4659	0,4656	0,4652
0,11	0,4649	0,4646	0,4643	0,4640	0,4636	0,4633	0,4630	0,4627	0,4624	0,4620
0,12	0,4617	0,4614	0,4611	0,4607	0,4604	0,4601	0,4598	0,4595	0,4591	0,4588
0,13	0,4585	0,4582	0,4579	0,4575	0,4572	0,4569	0,4566	0,4563	0,4559	0,4556
0,14	0,4553	0,4550	0,4546	0,4543	0,4540	0,4537	0,4534	0,4530	0,4527	0,4524
0,15	0,4521	0,4518	0,4514	0,4511	0,4508	0,4505	0,4501	0,4498	0,4495	0,4492
0,16	0,4489	0,4485	0,4482	0,4479	0,4476	0,4472	0,4469	0,4466	0,4463	0,4459
0,17	0,4456	0,4453	0,4450	0,4447	0,4443	0,4440	0,4437	0,4434	0,4430	0,4427
0,18	0,4424	0,4421	0,4417	0,4414	0,4411	0,4408	0,4404	0,4401	0,4398	0,4395
0,19	0,4392	0,4388	0,4385	0,4382	0,4379	0,4375	0,4372	0,4369	0,4366	0,4362
0,20	0,4359	0,4356	0,4353	0,4349	0,4346	0,4343	0,4340	0,4336	0,4333	0,4330
0,21	0,4327	0,4323	0,4320	0,4317	0,4314	0,4310	0,4307	0,4304	0,4300	0,4297
0,22	0,4294	0,4291	0,4287	0,4284	0,4281	0,4278	0,4274	0,4271	0,4268	0,4265
0,23	0,4261	0,4258	0,4255	0,4251	0,4248	0,4245	0,4242	0,4238	0,4235	0,4232
0,24	0,4229	0,4225	0,4222	0,4219	0,4215	0,4212	0,4209	0,4206	0,4202	0,4199
0,25	0,4196	0,4192	0,4189	0,4186	0,4183	0,4179	0,4176	0,4173	0,4169	0,4166
0,26	0,4163	0,4159	0,4156	0,4153	0,4150	0,4146	0,4143	0,4140	0,4136	0,4133
0,27	0,4130	0,4126	0,4123	0,4120	0,4117	0,4113	0,4110	0,4107	0,4103	0,4100
0,28	0,4097	0,4093	0,4090	0,4087	0,4083	0,4080	0,4077	0,4073	0,4070	0,4067
0,29	0,4063	0,4060	0,4057	0,4053	0,4050	0,4047	0,4043	0,4040	0,4037	0,4033
0,30	0,4030	0,4027	0,4023	0,4020	0,4017	0,4013	0,4010	0,4007	0,4003	0,4000

Table F.1 (*continued*)

First two decimal places of $Q\sqrt{3}/2$	Third decimal place of $Q\sqrt{3}/2$									
	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
0,31	0,3997	0,3993	0,3990	0,3987	0,3983	0,3980	0,3977	0,3973	0,3970	0,3967
0,32	0,3963	0,3960	0,3956	0,3953	0,3950	0,3946	0,3943	0,3940	0,3936	0,3933
0,33	0,3930	0,3926	0,3923	0,3919	0,3916	0,3913	0,3909	0,3906	0,3902	0,3899
0,34	0,3896	0,3892	0,3889	0,3886	0,3882	0,3879	0,3875	0,3872	0,3869	0,3865
0,35	0,3862	0,3858	0,3855	0,3852	0,3848	0,3845	0,3841	0,3838	0,3835	0,3831
0,36	0,3828	0,3824	0,3821	0,3818	0,3814	0,3811	0,3807	0,3804	0,3800	0,3797
0,37	0,3794	0,3790	0,3787	0,3783	0,3780	0,3776	0,3773	0,3770	0,3766	0,3763
0,38	0,3759	0,3756	0,3752	0,3749	0,3745	0,3742	0,3739	0,3735	0,3732	0,3728
0,39	0,3725	0,3721	0,3718	0,3714	0,3711	0,3707	0,3704	0,3701	0,3697	0,3694
0,40	0,3690	0,3687	0,3683	0,368	0,3676	0,3673	0,3669	0,3666	0,3662	0,3659
0,41	0,3655	0,3652	0,3648	0,3645	0,3641	0,3638	0,3634	0,3631	0,3627	0,3624
0,42	0,3620	0,3617	0,3613	0,3610	0,3606	0,3603	0,3599	0,3596	0,3592	0,3589
0,43	0,3585	0,3582	0,3578	0,3575	0,3571	0,3567	0,3564	0,356	0,3557	0,3553
0,44	0,3550	0,3546	0,3543	0,3539	0,3536	0,3532	0,3528	0,3525	0,3521	0,3518
0,45	0,3514	0,3511	0,3507	0,3504	0,3500	0,3496	0,3493	0,3489	0,3486	0,3482
0,46	0,3478	0,3475	0,3471	0,3468	0,3464	0,3461	0,3457	0,3453	0,3450	0,3446
0,47	0,3443	0,3439	0,3435	0,3432	0,3428	0,3424	0,3421	0,3417	0,3414	0,3410
0,48	0,3406	0,3403	0,3399	0,3395	0,3392	0,3388	0,3385	0,3381	0,3377	0,3374
0,49	0,3370	0,3366	0,3363	0,3359	0,3355	0,3352	0,3348	0,3344	0,3341	0,3337
0,50	0,3333	0,3330	0,3326	0,3322	0,3319	0,3315	0,3311	0,3308	0,3304	0,3300
0,51	0,3296	0,3293	0,3289	0,3285	0,3282	0,3278	0,3274	0,3270	0,3267	0,3263
0,52	0,3259	0,3256	0,3252	0,3248	0,3244	0,3241	0,3237	0,3233	0,3229	0,3226
0,53	0,3222	0,3218	0,3214	0,3211	0,3207	0,3203	0,3199	0,3196	0,3192	0,3188
0,54	0,3184	0,3180	0,3177	0,3173	0,3169	0,3165	0,3161	0,3158	0,3154	0,3150
0,55	0,3146	0,3142	0,3139	0,3135	0,3131	0,3127	0,3123	0,3120	0,3116	0,3112
0,56	0,3108	0,3104	0,3100	0,3096	0,3093	0,3089	0,3085	0,3081	0,3077	0,3073
0,57	0,3069	0,3066	0,3062	0,3058	0,3054	0,3050	0,3046	0,3042	0,3038	0,3034
0,58	0,3031	0,3027	0,3023	0,3019	0,3015	0,3011	0,3007	0,3003	0,2999	0,2995
0,59	0,2991	0,2987	0,2983	0,2979	0,2975	0,2972	0,2968	0,2964	0,2960	0,2956
0,60	0,2952	0,2948	0,2944	0,2940	0,2936	0,2932	0,2928	0,2924	0,2920	0,2916
0,61	0,2912	0,2908	0,2904	0,2900	0,2896	0,2892	0,2888	0,2883	0,2879	0,2875
0,62	0,2871	0,2867	0,2863	0,2859	0,2855	0,2851	0,2847	0,2843	0,2839	0,2835
0,63	0,2831	0,2826	0,2822	0,2818	0,2814	0,2810	0,2806	0,2802	0,2798	0,2793
0,64	0,2789	0,2785	0,2781	0,2777	0,2773	0,2769	0,2764	0,2760	0,2756	0,2752
0,65	0,2748	0,2743	0,2739	0,2735	0,2731	0,2727	0,2722	0,2718	0,2714	0,2710

Table F.1 (*continued*)

First two decimal places of $Q\sqrt{3}/2$	Third decimal place of $Q\sqrt{3}/2$									
	0,000	0,001	0,002	0,003	0,004	0,005	0,006	0,007	0,008	0,009
	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$	$\hat{p}$
0,66	0,2706	0,2701	0,2697	0,2693	0,2689	0,2684	0,2680	0,2676	0,2672	0,2667
0,67	0,2663	0,2659	0,2654	0,2650	0,2646	0,2641	0,2637	0,2633	0,2628	0,2624
0,68	0,2620	0,2615	0,2611	0,2607	0,2602	0,2598	0,2594	0,2589	0,2585	0,2580
0,69	0,2576	0,2572	0,2567	0,2563	0,2558	0,2554	0,2550	0,2545	0,2541	0,2536
0,70	0,2532	0,2527	0,2523	0,2518	0,2514	0,2509	0,2505	0,2500	0,2496	0,2491
0,71	0,2487	0,2482	0,2478	0,2473	0,2469	0,2464	0,2460	0,2455	0,2451	0,2446
0,72	0,2441	0,2437	0,2432	0,2428	0,2423	0,2418	0,2414	0,2409	0,2405	0,2400
0,73	0,2395	0,2391	0,2386	0,2381	0,2377	0,2372	0,2367	0,2362	0,2358	0,2353
0,74	0,2348	0,2344	0,2339	0,2334	0,2329	0,2324	0,2320	0,2315	0,2310	0,2305
0,75	0,2301	0,2296	0,2291	0,2286	0,2281	0,2276	0,2272	0,2267	0,2262	0,2257
0,76	0,2252	0,2247	0,2242	0,2237	0,2232	0,2227	0,2222	0,2217	0,2213	0,2208
0,77	0,2203	0,2198	0,2193	0,2188	0,2183	0,2177	0,2172	0,2167	0,2162	0,2157
0,78	0,2152	0,2147	0,2142	0,2137	0,2132	0,2127	0,2121	0,2116	0,2111	0,2106
0,79	0,2101	0,2096	0,2090	0,2085	0,2080	0,2075	0,2069	0,2064	0,2059	0,2054
0,80	0,2048	0,2043	0,2038	0,2032	0,2027	0,2022	0,2016	0,2011	0,2006	0,2000
0,81	0,1995	0,1989	0,1984	0,1978	0,1973	0,1967	0,1962	0,1956	0,1951	0,1945
0,82	0,1940	0,1934	0,1929	0,1923	0,1917	0,1912	0,1906	0,1900	0,1895	0,1889
0,83	0,1883	0,1878	0,1872	0,1866	0,1860	0,1855	0,1849	0,1843	0,1837	0,1831
0,84	0,1826	0,1820	0,1814	0,1808	0,1802	0,1796	0,1790	0,1784	0,1778	0,1772
0,85	0,1766	0,1760	0,1754	0,1748	0,1742	0,1736	0,1729	0,1723	0,1717	0,1711
0,86	0,1705	0,1698	0,1692	0,1686	0,1680	0,1673	0,1667	0,1660	0,1654	0,1648
0,87	0,1641	0,1635	0,1628	0,1622	0,1615	0,1609	0,1602	0,1595	0,1589	0,1582
0,88	0,1575	0,1569	0,1562	0,1555	0,1548	0,1542	0,1535	0,1528	0,1521	0,1514
0,89	0,1507	0,1500	0,1493	0,1486	0,1479	0,1472	0,1465	0,1457	0,1450	0,1443
0,90	0,1436	0,1428	0,1421	0,1414	0,1406	0,1399	0,1391	0,1384	0,1376	0,1368
0,91	0,1361	0,1353	0,1345	0,1338	0,1330	0,1322	0,1314	0,1306	0,1298	0,1290
0,92	0,1282	0,1274	0,1266	0,1257	0,1249	0,1241	0,1232	0,1224	0,1215	0,1207
0,93	0,1198	0,1189	0,1181	0,1172	0,1163	0,1154	0,1145	0,1136	0,1127	0,1118
0,94	0,1108	0,1099	0,1089	0,1080	0,1070	0,1061	0,1051	0,1041	0,1031	0,1021
0,95	0,1011	0,1001	0,0990	0,0980	0,0969	0,0959	0,0948	0,0937	0,0926	0,0915
0,96	0,0903	0,0892	0,0880	0,0869	0,0857	0,0845	0,0832	0,0820	0,0807	0,0795
0,97	0,0782	0,0768	0,0755	0,0741	0,0727	0,0713	0,0699	0,0684	0,0669	0,0653
0,98	0,0638	0,0621	0,0605	0,0588	0,0570	0,0552	0,0533	0,0514	0,0494	0,0473
0,99	0,0451	0,0427	0,0403	0,0377	0,0349	0,0318	0,0285	0,0247	0,0201	0,0142
1,00	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

NOTE   For negative values of Q, enter the table with the absolute value of $Q\sqrt{3}/2$ and subtract the result from 1,0.

Annex G
(normative)

Form p^* single sample plans

Table G.1 — Form p* single sampling plans for normal inspection (master table): "s" and "σ" methods

Code letter	n_σ	n_s	Acceptance quality limit (in percent nonconforming) 0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
			100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*	100 p*
B	2	3													↓	19,05	24,94	34,95
C	3	4												↓	11,23	15,13	21,57	30,67
D	4	6											↓	6,724	9,246	13,29	19,02	29,03
E	6	9										↓	4,196	5,833	8,437	12,12	18,52	28,13
F	8	13									↓	2,578	3,605	5,245	7,537	11,54	17,54	27,31
G	10	18								↓	1,620	2,275	3,323	4,782	7,315	11,12	17,33	23,84
H	12	25							↓	1,012	1,428	2,084	3,010	4,603	7,010	10,91	15,03	21,03
J	15	35						↓	0,6299	0,8900	1,304	1,880	2,880	4,379	6,820	9,375	13,14	18,13
K	18	50					↓	0,4021	0,5690	0,8335	1,203	1,840	2,800	4,364	6,006	8,406	11,59	17,20
L	21	70				↓	0,2511	0,3553	0,5209	0,7500	1,150	1,750	2,725	3,753	5,255	7,257	10,74	↑
M	25	95			↓	0,1593	0,2254	0,3304	0,4765	0,7298	1,110	1,732	2,383	3,336	4,607	6,827	↑	
N	32	125		↓	0,1001	0,1421	0,2081	0,3005	0,4602	0,7006	1,090	1,499	2,098	2,898	4,301	↑		
P	40	160	↓	0,06265	0,08893	0,1302	0,1874	0,2875	0,4381	0,6818	0,9368	1,311	1,812	2,685	↑			
Q	50	200	0,03998	0,05689	0,08333	0,1202	0,1842	0,2801	0,4366	0,6007	0,8397	1,161	1,720	↑				
R	65	250	0,03553	0,05209	0,07495	0,1151	0,1750	0,2729	0,3753	0,5245	0,7258	1,076	↑					

NOTE 1   The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2   Symbols:
↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.
↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Table G.2 — Form p^* single sampling plans for tightened inspection (master table): "s" and "σ" methods

Code letter	Sample size n_s	Sample size n_σ	0,01 $100\,p^*$	0,015 $100\,p^*$	0,025 $100\,p^*$	0,04 $100\,p^*$	0,065 $100\,p^*$	0,10 $100\,p^*$	0,15 $100\,p^*$	0,25 $100\,p^*$	0,40 $100\,p^*$	0,65 $100\,p^*$	1,0 $100\,p^*$	1,5 $100\,p^*$	2,5 $100\,p^*$	4,0 $100\,p^*$	6,5 $100\,p^*$	10,0 $100\,p^*$
B	3	2														↓	19,05	24,94
C	4	3													↓	11,23	15,13	21,57
D	6	4												↓	6,724	9,246	13,29	19,02
E	9	6											↓	4,196	5,833	8,437	12,12	18,52
F	13	8										↓	2,578	3,605	5,245	7,537	11,54	17,54
G	18	10									↓	1,620	2,275	3,323	4,782	7,315	11,12	17,33
H	25	12								↓	1,012	1,428	2,084	3,010	4,603	7,010	10,91	17,02
J	35	15							↓	0,6299	0,8900	1,304	1,880	2,880	4,379	6,820	10,62	15,63
K	50	18						↓	0,4021	0,5690	0,8335	1,203	1,840	2,800	4,364	6,808	10,01	14,82
L	70	21					↓	0,2511	0,3553	0,5209	0,7500	1,150	1,750	2,725	4,251	6,257	9,249	↑
M	95	25				↓	0,1593	0,2254	0,3304	0,4765	0,7298	1,110	1,732	2,699	3,965	5,879	↑	
N	125	32			↓	0,1001	0,1421	0,2081	0,3005	0,4602	0,7006	1,090	1,700	2,498	3,698	↑		
P	160	40		↓	0,06265	0,08893	0,1302	0,1874	0,2875	0,4381	0,6818	1,062	1,565	2,311	↑			
Q	200	50	↓	0,03998	0,05689	0,08333	0,1202	0,1842	0,2801	0,4366	0,6798	1,001	1,481	↑				
R	250	65	0,02501	0,03553	0,05209	0,07495	0,1151	0,1750	0,2729	0,4249	0,6242	0,9242	↑					

NOTE 1   The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2   Symbols: ↓   There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.

↑   There is no suitable plan in this area; use the first sampling plan above the arrow.

Table G.3 — Form p^* single sampling plans for reduced inspection (master table): "s" and "σ" methods

Code letter	Sample size n_σ	Sample size n_s	Acceptance quality limit (in percent nonconforming) 0,01 100 p^*	0,015 100 p^*	0,025 100 p^*	0,04 100 p^*	0,065 100 p^*	0,10 100 p^*	0,15 100 p^*	0,25 100 p^*	0,40 100 p^*	0,65 100 p^*	1,0 100 p^*	1,5 100 p^*	2,5 100 p^*	4,0 100 p^*	6,5 100 p^*	10,0 100 p^*
B - D	2	3											↓	19,05	21,24	24,94	34,95	49,37
E	3	4										↓	11,23	12,70	15,13	21,57	30,67	46,70
F	4	6									↓	6,724	7,671	9,246	13,29	19,02	29,03	44,10
G	6	9								↓	4,196	4,819	5,833	8,437	12,12	18,52	28,13	36,16
H	8	13							↓	2,578	2,967	3,605	5,245	7,537	11,54	17,54	22,56	32,59
J	10	18						↓	1,620	1,867	2,275	3,323	4,782	7,315	11,12	14,32	20,68	27,03
K	12	25					↓	1,012	1,171	1,428	2,084	3,010	4,603	7,010	9,014	13,02	17,02	21,03
L	15	35				↓	0,6299	0,7296	0,8900	1,304	1,880	2,880	4,379	5,628	8,125	10,62	13,14	↑
M	18	50			↓	0,4021	0,4656	0,5690	0,8335	1,203	1,840	2,800	3,604	5,201	6,808	8,406	↑	
N	21	70		↓	0,2511	0,2909	0,3553	0,5209	0,7500	1,150	1,750	2,254	3,251	4,251	5,255	↑		
P	25	95	↓	0,1593	0,1847	0,2254	0,3304	0,4765	0,7298	1,110	1,429	2,066	2,699	3,336	↑			
Q	32	125	0,1001	0,1161	0,1421	0,2081	0,3005	0,4602	0,7006	0,8992	1,301	1,700	2,098	↑				
R	40	160	0,07248	0,08893	0,1302	0,1874	0,2875	0,4381	0,5628	0,8129	1,062	1,311	↑					

NOTE 1 The sample size code letters in this part of ISO 3951 correspond to those given in ISO 2859-1 and ISO 3951-1.

NOTE 2 Symbols:
↓ There is no suitable plan in this area; use the first sampling plan below the arrow. If the sample size equals or exceeds the lot size, carry out 100 % inspection.
↑ There is no suitable plan in this area; use the first sampling plan above the arrow.

Annex H
(normative)

Values of c_U for upper control limit on the sample standard deviation

Table H.1 — Values of c_U for upper control limit on the sample standard deviation

Sample size n	Factor c_U	Sample size n	Factor c_U	Sample size n	Factor c_U	Sample size n	Factor c_U
2	2,800	10	1,617	25	1,377	70	1,221
3	2,297	12	1,558	32	1,331	95	1,189
4	2,065	13	1,534	35	1,316	125	1,165
6	1,827	15	1,494	40	1,295	160	1,145
8	1,700	18	1,448	50	1,263	200	1,130
9	1,654	21	1,413	65	1,230	250	1,116

Annex I
(normative)

Supplementary acceptability constants for qualifying towards reduced inspection

Table I.1 — Supplementary acceptability constants for qualifying towards reduced inspection

Sample size code letter	AQL (%)	Form k acceptability constant for AQL that is one step tighter		Form p^* acceptability constant for AQL that is one step tighter
		"s" method	"σ" method	"s" and "σ" methods
		k	k	$100\,p^*$
B	4,0	1,118	0,991	8,047
C	2,5	1,325	1,281	5,833
D	1,5	1,516	1,465	4,540
E	1,0	1,740	1,739	2,840
F	0,65	1,967	1,990	1,671
G	0,40	2,153	2,182	1,074
H	0,25	2,350	2,378	0,6495
J	0,15	2,503	2,526	0,4461
K	0,10	2,678	2,694	0,2784
L	0,065	2,856	2,866	0,1659
M	0,040	3,002	3,008	0,1069
N	0,025	3,157	3,167	0,06470
P	0,015	3,272	3,282	0,04433
Q	0,01	3,407	3,419	0,02760
R	0,01	3,448	3,460	0,02443

Annex J
(normative)

Procedures for obtaining s or σ

J.1 Procedure for obtaining s

J.1.1 The estimate from a sample of the standard deviation of a population is generally denoted by the symbol s. Its value may be obtained from the mathematical formula

$$s = \sqrt{\dfrac{\sum\limits_{j=1}^{n}(x_j - \bar{x})^2}{n-1}} \tag{J.1}$$

where

x is the value of the quality characteristic of the jth item in a sample of size n; and

$\bar{x}$ is the mean value of the x_j, i.e.

$$\bar{x} = \sum_{j=1}^{n} x_j \tag{J.2}$$

J.1.2 The above formula for s is not recommended for the purpose of computation, as it tends to introduce an unnecessary amount of rounding error. An equivalent but computationally better formula is

$$s = \sqrt{\dfrac{n\sum\limits_{j=1}^{n} x_j^2 - (\sum\limits_{j=1}^{n} x_j)^2}{n(n-1)}} \tag{J.3}$$

J.1.3 This formula can be improved upon still further by subtracting a suitable arbitrary constant a from all the values of x_j before computing s, i.e.

$$s = \sqrt{\dfrac{n\sum\limits_{j=1}^{n}(x_j - a)^2 - \left(\sum\limits_{j=1}^{n}(x_j - a)\right)^2}{n(n-1)}} \tag{J.4}$$

This form tends to provide a more accurate result when the variability is very small relative to the mean, i.e. s is very small in comparison with $\bar{x}$.