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Sampling procedures and charts for inspection by variables for percent nonconforming

Règles et tables d'échantillonnage pour les contrôles par mesures des pourcentages de non conformes

Reference number
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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for approval before their acceptance as International Standards by the ISO Council. They are approved in accordance with ISO procedures requiring at least 75 % approval by the member bodies voting.

International Standard ISO 3951 was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods.*

This second edition cancels and replaces the first edition (ISO 3951 : 1981), of which it constitutes a technical revision.

The principal changes to the first edition are as follows:

a) a distinction has been drawn between maximum process standard deviation (MPSD) under the "σ" method and maximum sample standard deviation (MSSD) under the "*s*" method;

b) all the "σ" method acceptance curves have been truncated at the appropriate maximum process standard deviations;

c) the terminology has been aligned on that of ISO 2859 and ISO 3534.

Users should note that all International Standards undergo revision from time to time and that any reference made herein to any other International Standard implies its latest edition, unless otherwise stated.

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Sampling procedures and charts for inspection by variables for percent nonconforming

Section one : General

1 Scope and field of application

1.1 Scope

1.1.1 This International Standard establishes sampling plans and procedures for inspection by variables. It is complementary to ISO 2859. When specified by the responsible authority, both this International Standard and ISO 2859 may be referenced in a product or process specification, contract, inspection instructions, or other documents, and the provisions set forth therein shall govern. The "responsible authority" shall be designated in one of the above documents.

1.1.2 The object of the methods laid down in this International Standard is to ensure that lots of an acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as possible.

1.1.3 In common with ISO 2859, the percentage of nonconforming products in the lots is used to define the quality of these lots and of the production process in question.

1.2 Field of application

This International Standard is primarily designed for use under the following conditions:

a) where the inspection procedure is to be applied to a **continuous series of lots** of discrete products all supplied by one producer using one production process. If there are different producers, this International Standard shall be applied to each one separately;

b) where only a **single quality characteristic** x of these products is taken into consideration, which must be **measurable on a continuous scale.** If several such characteristics are of importance, this International Standard shall be applied to each separately;

c) where production is stable (under statistical control) and the quality characteristic x is distributed according to a normal distribution or a close approximation to the normal distribution;

d) where a contract or standard defines an **upper specification limit** U, a **lower specification limit** L, or both; a product is qualified as nonconforming when its measured quality characteristic x satisfies one of the following inequalities:

$$x > U \quad \ldots(1)$$

$$x < L \quad \ldots(2)$$

$$\text{either } x > U \text{ or } x < L \quad \ldots(3)$$

Inequalities (1) and (2) are called cases with a **single specification limit**, and (3) a case with **double specification limits.** In this last situation a further distinction is made between separate or combined double limits according to whether the AQL is applied to each limit separately or to both limits combined (see clause 4).

2 References

ISO 2854, *Statistical interpretation of data — Techniques of estimation and tests relating to means and variances.*

ISO 2859, *Sampling procedures and tables for inspection by attributes.*

ISO 3534, *Statistics — Vocabulary and symbols.*

ISO 5479, *Normality tests.* [1)]

ISO 5725, *Precision of test methods — Determination of repeatability and reproducibility for a standard test method by inter-laboratory tests.*

3 Definitions and symbols

3.1 Definitions

For the purposes of this International Standard, the definitions given in ISO 3534 and ISO 2859, together with the following, apply.

1) At present at the stage of draft.

3.1.1 inspection by variables (contrôle par mesures) : A method which consists in measuring a quantitative characteristic for each item of a population or a sample taken from this population.

3.1.2 acceptance sampling by variables (échantillonnage par mesures en vue d'acceptation) : An acceptance procedure wherein a specified characteristic is measured to establish statistically the acceptability of a lot from the result obtained from the items in a sample.

3.1.3 acceptable quality level (AQL) [niveau de qualité acceptable (NQA)] : When a continuous series of lots is considered, a quality level which for the purposes of sampling inspection is the limit of a satisfactory process average percent nonconforming. (See clause 4.)

3.1.4 limiting quality (qualité limite) : When a lot is considered in isolation, a quality level which for the purposes of sampling inspection is limited to a low probability of acceptance (in this International Standard : 10 %). (See 12.1.)

3.1.5 nonconformity (non-conformité) : Failure to fulfil a specified requirement by a quality characteristic of an item or service, the assessment of which does not depend essentially on the passage of time.

Nonconformities will generally be classified by their degree of seriousness, such as:

Class A. Those nonconformities of a type considered to be of the highest concern for the product or service. In acceptance sampling, such types of nonconformity will be assigned very small AQL values.

Class B. Those nonconformities of a type considered to have the next lower degree of concern; therefore these can be assigned a larger AQL value than those in class A and smaller than in class C, if a third class exists, and so on.

The number of classes and the assignment into a class should be appropriate to the quality requirements of the specific situation.

3.1.6 nonconforming unit (unité non conforme): A unit of product or service containing at least one nonconformity.

3.1.7 *"s"* method (méthode «*s*»): A method of assessing the acceptability of a lot by using the sample standard deviation. (See clause 14.)

3.1.8 *"σ"* method (méthode «*σ*»): A method of assessing the acceptability of a lot by using knowledge of the process standard deviation. (See clause 15.)

3.1.9 *"R"* method (méthode «*R*»): A method of assessing the acceptability of a lot by indirectly using an estimate of the standard deviation of the process based on the average range of the measurements of the items in sub-groups of the sample. (See annex C.)

3.1.10 specification limit (limite de spécification): The limiting value (lower or upper) specified for a quantitative characteristic.

3.1.11 lower specification limit (*L*) [limite inférieure de spécification (L_i)]: A specification limit that defines the lower conformance boundary for an individual unit of a manufacturing or service operation.

3.1.12 upper specification limit (*U*) [limite supérieure de spécification (L_s)]: A specification limit that defines the upper conformance boundary for an individual unit of a manufacturing or service operation.

3.1.13 single specification limit (limite unique de spécification) : The term used when one limit only is specified.

3.1.14 separate double specification limits (limites de spécifications doubles séparées) : The term used when both upper and lower limits are specified and separate AQLs are applied to each limit individually. (See 4.3.)

3.1.15 combined double specification limit (limite de spécification double combinée) : The term used when both upper and lower limits are specified and an AQL is given which applies to the combined percent nonconforming at the two limits. (See 4.3.)

3.1.16 acceptability constant (*k*) [constante d'acceptabilité (*k*)] : A constant dependent on the specified value of the acceptable quality level and the sample size. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.17 quality statistic (*Q*) [statistique de qualité (*Q*)] : A function of the specification limit, the sample mean, and the standard deviation. The lot is sentenced on the result of comparing *Q* with the acceptability constant *k*. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.18 lower quality statistic (Q_L) [statistique de qualité correspondant à la limite inférieure (Q_i)] : A function of the lower specification limit, the sample mean, and the standard deviation. The lot is sentenced on the result of comparing Q_L with the acceptability constant *k*. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.19 upper quality statistic (Q_U) [statistique de qualité correspondant à la limite supérieure (Q_s)] : A function of the upper specification limit, the sample mean, and the standard deviation. The lot is sentenced on the result of comparing Q_U with the acceptability constant *k*. (See 14.2 and 15.2, or clause C.5 in annex C.)

3.1.20 maximum sample standard deviation (MSSD) [écart-type maximal de l'échantillon (ETME)]: Under given conditions, the largest acceptable sample standard deviation. (See 14.6 and B.8.3 in annex B.)

3.1.21 maximum process standard deviation (MPSD) [écart-type maximal du procédé (ETMP)]: Under given conditions, the largest acceptable process standard deviation. (See 15.3 and B.5.2 in annex B).

3.1.22 switching rules (règles de modification du contrôle): Instructions within a sampling scheme for shifting from one sampling plan to another based on demonstrated quality history. (See clause 19.)

3.2 Symbols

The symbols used are as follows:

f_s A factor, given in table IV-s, that relates the maximum sample standard deviation to the difference between U and L.

f_σ A factor, given in table IV-σ, that relates the maximum process standard deviation to the difference between U and L.

k The acceptability constant when using the "s" method, "σ" method or "R" method.

K The acceptability constant when both μ and σ are known.

L Lower specification limit. (As a suffix to a variable, denotes its value at L.)

U Upper specification limit. (As a suffix to a variable, denotes its value at U.)

n Sample size (number of units in a sample).

N Lot size (number of units in a lot).

P_a The probability of acceptance.

Q The quality statistic.

Q_L Lower quality statistic.

Q_U Upper quality statistic.

s Standard deviation of a sample (estimate of the standard deviation of the process).

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}$$

(See also annex A.)

x Measured value of a characteristic in the sample.

$\bar{x}$ Mean value of x for the sample of n items.

$\bar{x}_L$ Lower acceptance value.

$\bar{x}_U$ Upper acceptance value.

μ Mean of the process.

σ Standard deviation of the process. (σ^2, the square of the standard deviation, is known as the variance.)

Σ "The sum of" (for example, $\Sigma\, x$ = the sum of the x values).

$\sum_{i=1}^{n} x_i$ The sum of all the x values when i takes integral values from 1 to n.

$>$ "Greater than" (for example, $a > b$ means a is greater than b).

$\geqslant$ "Greater than or equal to" (for example, $a \geqslant b$ means a is greater than or equal to b).

$<$ "Less than" (for example, $a < b$ means a is less than b).

$\leqslant$ "Less than or equal to" (for example, $a \leqslant b$ means a is less than or equal to b).

3.3 Bibliography

A bibliography of documents used in the development of this International Standard is given in annex D.

4 Acceptable Quality Level (AQL)

4.1 Definition

When a continuous series of lots is considered, a quality level which for the purposes of sampling inspection is the limit of a satisfactory process average percent nonconforming.

4.2 Use

The AQL, together with the sample size code letter, is used to index the sampling plans in this International Standard.

4.3 Specifying AQLs

The AQL to be used will be designated in the product specification contract or by the responsible authority. Where both upper and lower specifications limits are given, separate AQLs may be given to the individual limits, which are then known as "separate double specification limits". Alternatively, an overall AQL may be given which applies to the combined percent nonconforming at both the upper and lower limits; this is then known as a "combined double specification limit".

4.4 Preferred AQLs

Eleven AQLs given in this International Standard, ranging in value from 0,10 % to 10 % nonconforming, are described as preferred AQLs. If, for any product or service, an AQL is designated other than a preferred AQL, then this International Standard is not applicable. (See 12.2.)

Two further AQLs, 0,065 % and 15 %, are given to complete the range of plans needed for the operation of the switching rules. (See clauses 19 and 21.) Plans and curves designated by an AQL of 0,065 % or 15 % will only be used when the AQL under normal inspection is 0,10 % or 10 % respectively and the switching rules are invoked.

4.5 Caution

From the above definition of the AQL, it follows that desired protection can only be obtained when a continuous series of lots is provided for inspection.

4.6 Limitation

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming unit of product.

5 Switching rules for normal, tightened and reduced inspection

5.1 In order to discourage the producer from operating at a process average percent nonconforming that exceeds the AQL, this International Standard prescribes a switch to tightened inspection when inspection results indicate that the process average does exceed the AQL, and discontinuation of sampling inspection altogether when tightened inspection does not in time stimulate the producer to improve his production process.

5.2 Hence, tightened inspection and the discontinuation rule are integral, and therefore obligatory, procedures of this International Standard if the protection implied by the AQL is to be maintained.

5.3 This International Standard also provides the possibility of switching to reduced inspection when inspection results indicate that the process average percent nonconforming is stable and reliable at a level below the AQL. This practice is, however, optional (at the discretion of the responsible authority).

5.4 When there is sufficient evidence from the control charts (see 18.1) that the variability is in statistical control, consideration should be given to switching to the "σ" method. If this appears advantageous, the consistent value of s shall be taken as σ.

5.5 When it has been necessary to discontinue sampling inspection, inspection may not be resumed until action is taken by the producer to improve the quality of the submitted product.

5.6 Details of the operation of the switching rules are given in clauses 19 and 20.

6 Relation to ISO 2859

6.1 Similarities

a) This International Standard is a complement to ISO 2859; the two documents share a common philosophy and, as far as possible, their procedures and vocabulary are the same.

b) Both use the AQL to index the sampling plans and the preferred values used in this document are identical with those given in ISO 2859 for the same range of values (i.e. from 0,1 % to 10 %).

c) In the two International Standards, lot size and inspection level (inspection level II being preferred in default of other instructions) determine a code letter. Then general tables give the sample size to be taken and the acceptability criterion in terms of the code letter and the AQL according to the method chosen ("s", "σ" or, contingently, "R"). Separate tables are given for normal, tightened and reduced inspection.

d) The switching rules are essentially the same.

e) The classification of nonconformities by degree of seriousness into class A, class B, etc. remains unchanged.

6.2 Differences

a) **Determination of acceptability.** The acceptability of an attributes sampling plan, taken from ISO 2859, is determined by the number of nonconforming units found in the sample; the acceptability criterion in inspection by variables is based on estimates of the location and variability of the distributed measurements of the lot, in relation to the specification limits, that is in terms of the mean and standard deviation. In this International Standard two methods are considered: the "s" method for use when the process standard deviation σ is unknown and the "σ" method for use when σ is considered to be known. A third method, called the "R" method, is given in annex C. In the case of a single specification limit or of two separate limits, the acceptability may be calculated from a formula (see 14.2 and 15.2), but is more easily established by a graphical method (see 14.3). In the case of a combined double limit, this International Standard provides for a graphical method (see 14.6 and 15.3).

b) **Normality.** In ISO 2859 there is no requirement relating to the distribution of the characteristics, but in this International Standard it is necessary to the efficient operation of a plan that the measurements should be distributed according to a normal distribution or a close approximation to the normal distribution.

c) **Operating characteristic curves** (OC curves). While an individual variables plan may be devised the OC curve of which corresponds closely to that of a given attributes plan, it would not be possible to make all the OC curves in this International Standard identical with the corresponding OC curves in ISO 2859 (which are indexed with the same code letter and AQL), without the sample size increasing with the AQL for a given sample size code letter. For the "s" method the sample size has been kept fixed for a given lot size across the full range of AQLs; for the "σ" method this constraint has been removed in order to match the "s" and "σ" method OC curves as closely as possible, both at the AQL and at the limiting quality.

d) **Probability of acceptance at the AQL.** The probability that a lot, whose quality is precisely at the AQL, will be accepted increases with the sample size and follows a similar, but not identical, scale to that used in ISO 2859.

e) **Sample sizes.** The variables sample sizes corresponding to given code letters are usually smaller than the attributes sample sizes for the same letters.

f) **Double sampling plans.** No double sampling plans are given in this International Standard.

g) **Average Outgoing Quality Limit (AOQL).** Under destructive or expensive testing, where 100 % inspection and rectification of non-accepted lots is not feasible, the AOQL concept cannot be applied. As variables plans will generally be used under these circumstances, no AOQL tables have been included in this International Standard.

7 Non-continuous production and operating characteristic curves

7.1 Non-continuous production

a) The sampling schemes contained in this International Standard were not designed to be applied under circumstances different from those specified in 1.2, for example to an isolated lot or limited number of lots, where tightened inspection and the discontinuation rules cannot be applied.

b) Under such conditions, the concept of an AQL becomes irrelevant, as the consumer's concern narrows to the quality of the limited number of submitted lots, and he is no longer involved with exerting controls on the quality of the production process. The AQL will still indicate a submitted quality which has a high probability of acceptance, and can therefore still be used as an index to a sampling plan. (See clause 12.)

7.2 Operating characteristic curves

a) The degree of protection of the consumer provided by the individual sampling plans of this International Standard can, however, be judged from their operating characteristic (OC) curves as given in charts V-B to V-P and tables V-B-1 to V-P-1, and these should be consulted when choosing a sampling plan.

b) These curves are for normal inspection using the "s" method with a single specification limit, but they provide a good approximation to the case of a combined double specification limit.

c) These curves also provide a good approximation to the OC curves for the "σ" method (and the "R" method) sampling plans indexed by the same code letters and AQL, unless the sample size is small.

d) Separate OC curves are not given for tightened or reduced inspection, but the curves may be found from among those given for normal inspection (see tables VI-A and VI-B).

Section two: Choice of sampling plan

8 Planning

The choice of the most suitable variables plan, if one exists, requires experience, judgement and some knowledge of both statistics and the product to be inspected. This section of this International Standard is intended to suggest to those responsible for specifying sampling plans the considerations that should be borne in mind when deciding whether a variables plan would be suitable and the choices to be made when selecting an appropriate standard plan.

9 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect by variables rather than by attributes. The following points should be taken into account :

a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more expensive in time and money per item.

b) In terms of the knowledge gained, the advantage lies with inspection by variables as the more precise information obtained indicates how good the product is; earlier warning will be given if the quality is slipping.

c) An attributes scheme can be more readily understood and accepted; for example, it may at first be difficult to accept that, when inspecting by variables, a lot can be rejected on measurements taken of a sample that does not contain any nonconforming items. (See the example in 14.6.)

d) A comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes (i.e. from ISO 2859) and the standard plans in this International Standard is given in table I-B. It will be seen that the smallest samples are required by the "σ" method, used when the standard deviation of the process is known.

e) Inspection by variables is appropriate particularly in conjunction with the use of control charts for variables.

f) Variables sampling has a substantial advantage when the inspection process is expensive, for example in the case of destructive testing.

g) A variables scheme becomes less suitable as the number of measurements to be taken on one item increases, as each characteristic has to be considered separately. It may be advantageous to apply "attributes" to the majority of the characteristics and "variables" to one or two of the more important requirements, for example proof load tests, safety and reliability requirements.

h) The use of this International Standard is only applicable when there is reason to believe that the distribution of measurements is normal. In case of any doubt, the responsible authority should be consulted.

NOTES

1 Tests for departure from normality are dealt with in section two of ISO 2854, which provides examples of graphical methods which can be used to verify that the distribution of the data is sufficiently normal to justify the use of sampling by variables.

2 ISO 5479 gives a more detailed study on the subject of normality tests.

10 Choice of method

If it is desired to apply inspection by variables, the next question is which method should be used, the "s" method or the "σ" method (or the "R" method).

The "σ" method is the most economical in sample size, but before this method may be employed, the value of σ has to be established.

In terms of sample size, the "s" method has a slight advantage over the "R" method, but the calculation of s involves more computation; the extent and difficulty of this is more apparent than real, especially if an electronic calculator is available. Methods of calculating s are given in annex A.

The "R" method (given in annex C) is simple to calculate, but requires a somewhat larger sample size for the same AQL. Also it has the undesirable property that for samples of size 10 or more the acceptability of a lot can depend upon how the sample is divided into sub-groups.

Initially, it will be necessary to begin with the "s" (or the "R" method), but if the quality is satisfactory, the standard switching rules will permit the responsible authority to commence reduced inspection and use a smaller sample size.

The question then is, if the variability is under control and lots continue to be accepted, will it be economical to change to the "σ" method?

The size of the sample will generally be smaller and the acceptability criteria become simpler. (See clauses 15.2 and 15.3.) On the other hand, it will still be necessary to calculate s for record purposes and to keep the control charts up to date. (See clause 18.)

11 Choice of inspection level and AQL

In standard sampling plans, the inspection level in conjunction with the size of the lots and the AQL determines the size of the sample to be taken, and governs the severity of the inspection. The appropriate OC curve given in one of the tables V-B to V-P shows the extent of the risk that is involved in such a plan.

The choice of the inspection level and AQL is governed by a number of factors, but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

The normal practice is to use inspection level II, unless special circumstances indicate that another level is more appropriate.

12 Choice of sampling plan

12.1 Standard plans

The standard procedure can be used only when the production of lots is continuous.

The standard procedure, with its semi-automatic steps from lot size to sample size, using inspection level II and beginning with the "*s*" method, has been found in practice to produce workable sampling plans; but it assumes that the order of priority is first the AQL, second the sample size and last, the limiting quality.

The acceptability of this system is due to the fact that the consumer is protected by the switching rules (see clause 19), which quickly increase the severity of inspection and finally terminate it, if the quality of the process is worse than the AQL.

NOTE — It should also be remembered that the limiting quality is the quality which if offered for inspection would have a 10 % probability of acceptance. The actual risk taken by the consumer therefore also depends on the probability of goods of this low quality being offered for inspection.

However, if, in certain circumstances, the limiting quality has a higher priority than the sample size (for example, when only a limited number of lots are being produced), a suitable plan in this International Standard may be selected by using diagram A. The intersection of a vertical line through the acceptable value for the limiting quality and a horizontal line through the desired quality with a 95 % probability of acceptance (approximately equal to AQL) will lie on,or under, a sloping line indexed with the sample size code letter of a standard plan which meets the specified requirements. This should be verified by inspecting the OC curve given in table V for this code letter and AQL.

If the lines intersect at a point above the line marked P (see diagram A), this implies that, for example, a sample of over 200 would be necessary for the *s* method and the specification cannot be met by the plans in this International Standard.

12.2 Special plans

If standard plans are not acceptable, it will be necessary to devise a special plan. The choice is then to decide which combination of AQL, limiting quality, and sample size is most suitable, remembering that these are not independent, for, when any two have been chosen, the third follows.

NOTE — This choice is not completely unfettered; the fact that the size of the sample is necessarily a whole number imposes some constraints. If a special scheme is necessary it should be devised only with the assistance of a statistician experienced in quality control.

Section three: Operation of a variables sampling plan

13 Preliminary operations

Before starting inspection by variables, check

a) that the distribution can be considered to be normal and that production is considered to be continuous;

b) whether the "*s*" (or "*R*") method is to be used initially or whether the standard deviation is stable and known, in which case the "σ" method should be used;

c) that the inspection level to be used has been designated. If none has been given, inspection level II shall be used;

d) that the AQL has been designated and that it is one of the preferred AQLs for use with this International Standard. If it is not, the tables are not applicable;

e) if a double specification limit has to be met, whether the limits are separate or combined and, if the limits are separate, whether AQLs are determined for each limit.

14 Standard procedure for "*s*" method

14.1 Obtaining a plan

The procedure for obtaining a plan is as follows :

a) With the inspection level given (normally this will be II) and with the lot size, obtain the sample size code letter using table I-A.

b) With this code letter and the AQL, enter table II-A and obtain the sample size n and acceptability constant k.

c) Taking a random sample of this size, measure the characteristic x in each item and then calculate $\bar{x}$, the sample mean, and s, the estimated standard deviation (see annex A). If $\bar{x}$ is outside the specification limit, the lot can be judged unacceptable without calculating s. It may, however, be necessary to calculate s for record purposes.

14.2 Acceptability criteria for single or separate double specification limits

If single or separate specification limits are given, calculate the quality statistic

$$Q_U = \frac{U - \bar{x}}{s}$$

and/or

$$Q_L = \frac{\bar{x} - L}{s}$$

as appropriate,

then compare the quality statistic (Q_U and/or Q_L) with the acceptability constant k obtained from table II-A for normal inspection. If the appropriate quality statistic is greater than or equal to the acceptability constant, the lot is acceptable; if less, it is not acceptable.

Thus, if only the upper specification limit U is given, the lot is

acceptable if $Q_U \geqslant k$

not acceptable if $Q_U < k$

Or, if only the lower specification limit L is given, the lot is

acceptable if $Q_L \geqslant k$

not acceptable if $Q_L < k$

When both U and L are given (k values are different if the AQLs are different for the upper limit and the lower limit), the lot is

acceptable if both $Q_L \geqslant k_L$ and $Q_U \geqslant k_U$

not acceptable if either $Q_L < k_L$ or $Q_U < k_U$

Example

The maximum temperature of operation for a certain device is specified as 60 °C. Production is inspected in lots of 100 items. Inspection level II, normal inspection with AQL = 2,5 % is to be used. From table I-A, the sample size code letter is F; from table II-A it is seen that a sample size of 10 is required and that the acceptability constant k is 1,41. Suppose the measurements are as follows: 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 55 °C; 50 °C. Compliance with the acceptability criterion is to be determined.

Information needed	**Values obtained**
Sample size : n	10
Sample mean $\bar{x}$: $\Sigma\, x/n$	54,9
Standard deviation s : $\sqrt{\sum_i (x_i - \bar{x})^2/(n-1)}$ (See A.1.2, annex A.)	3,414
Specification limit (upper) : U	60
$Q_U = (U - \bar{x})/s$	1,494
Acceptability constant : k (see table II-A)	1,41
Acceptability criterion : compare Q_U with k	1,494 > 1,41

The lot meets the acceptability criterion, and is therefore acceptable.

14.3 Graphical method for a single specification limit

When a graphical criterion is desired, draw the line

$\bar{x} = U - k\,s$ (for an upper limit) or

$\bar{x} = L + k\,s$ (for a lower limit), as appropriate, on graph paper with $\bar{x}$ as the vertical axis and s as the horizontal axis. When the inspection concerns an upper specification limit the accept zone is the zone below the line. When a lower specification limit is considered, the accept zone is the zone above the line. Using the values of s and $\bar{x}$ calculated from the measurements obtained from a sample (see annex A for the calculation of s), plot the point (s, $\bar{x}$) on the graph. If this point lies in the accept zone, the lot is acceptable; if outside, it is not acceptable.

Example

Using the data given in the example in 14.2, mark the point $U = 60$ on the $\bar{x}$ (vertical) axis and draw a line through this point with a slope $- k$ [as $k = 1{,}41$, this means the line passes through points ($s = 1$, $\bar{x} = 58{,}59$), ($s = 2$, $\bar{x} = 57{,}18$), ($s = 3$, $\bar{x} = 55{,}77$), etc.]. Select a suitable point and draw a straight line through it and ($s = 0$, $\bar{x} = 60$), i.e. U. The accept zone is then the area under this line. The calculated values of s and $\bar{x}$ are 3,414 and 54,9. Plotting the point (s, $\bar{x}$), it will be seen from figure 1 that it lies just inside the accept zone; the lot is acceptable.

The graph can be prepared before beginning the inspection of a series of lots. Then, for each lot plot the point (s, $\bar{x}$) and decide whether or not the lot is acceptable.

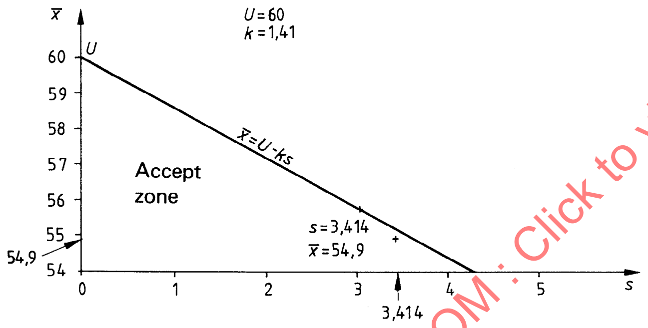


Figure 1 — Example of the use of an acceptance chart for a single specification limit: "s" method

14.4 Numerical method for single or separate double specification limits

Example

A certain pyrotechnic delay has a specified minimum delay time of 4,0 s and a maximum time of 9,0 s. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,1 % applied to the lower limit and an AQL of 2,5 % applied to the upper limit. From table I-A it is seen that the sample size code letter is J; from table I-B it is seen that the sample size is 35 for the "s" method and from table II-A it is found that the upper and lower acceptability constants are $k_U = 1{,}57$ and $k_L = 2{,}54$ respectively. Suppose the sample delay times are as follows:

6,95	6,04	6,68	6,63	6,65
6,40	6,44	6,34	6,04	6,15
6,44	7,15	6,70	6,59	6,51
6,35	7,17	6,83	6,25	6,96
6,80	5,84	6,15	6,25	6,57
6,52	6,59	6,86	6,57	6,91
6,29	6,63	6,70	6,67	6,67

Compliance with the acceptability criteria is to be determined.

Information needed	Value obtained
Sample size: n	35
Sample mean $\bar{x}$: $\Sigma\, x/n$	6,55 s
Sample standard deviation s: $\sqrt{\sum_i (x_i - \bar{x})^2/(n - 1)}$ (See A.1.2, annex A.)	0,31 s
Upper specification limit: U	9,0 s
$Q_U = (U - \bar{x})/s$	7,90
Acceptability constant: k_U (see table II-A)	1,57
Lower specification limit: L	4,0 s
$Q_L = (\bar{x} - L)/s$	8,23
Acceptability constant: k_L (see table II-A)	2,54
Acceptability criterion: is $Q_U \geqslant k_U$ and $Q_L \geqslant k_L$?	$7{,}90 > 1{,}57$ and $8{,}23 > 2{,}54$

The lot meets the acceptability criteria, and is acceptable.

14.5 Graphical method for separate double specification limits

When a graphical criterion is desired for separate double specification limits, draw the lines

$\bar{x} = U - k_U\, s$ (for the upper limit) and

$\bar{x} = L + k_L\, s$ (for the lower limit)

on graph paper with $\bar{x}$ as the vertical axis and s as the horizontal axis. Using the values of s and x calculated from the measurements obtained from a sample, plot the point (s, $\bar{x}$) on the graph. If this point lies in the accept zone, the lot is acceptable; if outside, it is not acceptable.

Example

Using the data given in the example in 14.4 mark the point $U = 9{,}0$ on the $\bar{x}$ (vertical) axis and draw a line through this point with a slope $- k_U$ [as $k_U = 1{,}57$, this means that the line will pass through the point ($s = 1$, $\bar{x} = 7{,}43$)]. Also mark the point $L = 4{,}0$ on the $\bar{x}$ axis and draw a line through this point with slope $+ k_L$ [as $k_L = 2{,}54$, this means that the line will pass through the point ($s = 1$, $\bar{x} = 6{,}54$)]. The accept zone is then the area bounded by these lines and the $\bar{x}$ axis. The calculated values of s and $\bar{x}$ are 0,31 and 6,55. Plotting the point ($s = 0{,}31$, $\bar{x} = 6{,}55$) as shown in figure 2 it will be seen that it lies well within the accept zone and that the lot is acceptable.

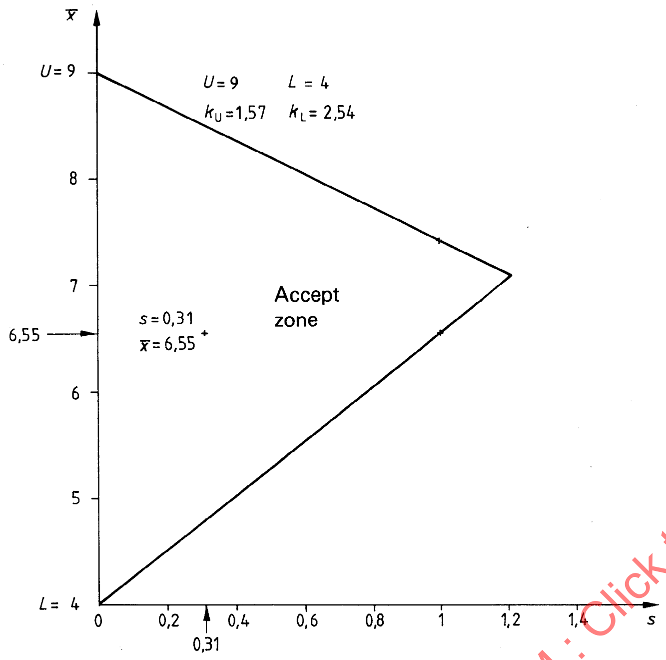


Figure 2 — Example of the use of an acceptance chart for separate double specification limits: "s" method

14.6 Acceptability criterion for a combined double specification limit

If a combined upper and lower specification limit has been given, it will be necessary to use a graphical method, unless s is greater than the value of the MSSD obtained from table IV-s, in which case the lot shall be immediately judged unacceptable.

Consult the chart in the "s" series which is labelled with the appropriate code letter and select the acceptance curve with the AQL specified for the two limits.

Then calculate the values of

$$\frac{s}{U - L} \quad \text{and} \quad \frac{\bar{x} - L}{U - L}$$

and plot a point representing these values on the graph (or a copy of it). If the point lies inside the accept zone, the lot is acceptable; if outside, it is not acceptable.

For greater convenience it is recommended that, before the inspection operations begin, the acceptance curves for normal and tightened inspection be copied (or traced) onto graph paper. (A sheet of the special graph paper needed for this is given in annex D.) The scales should be adjusted so that s and $\bar{x}$ can be plotted directly (e.g. the upper limit is given instead of 1,0 and the lower limit instead of 0 on the $\bar{x}$ scale).

Then plot on the chart the values of s and $\bar{x}$ found from the sample and, if the point lies inside the accept zone, the lot is acceptable; if outside, it is not acceptable.

NOTE — For sample size code letters B and C (i.e. sample sizes 3 and 4), the accept zone is bounded by four straight lines: the $\bar{x}$ axis, the line $\bar{x} = U - k\,s$, a line parallel to the $\bar{x}$ axis through the MSSD (see table IV-s) and the line $\bar{x} = L + ks$. The value of k is obtained from the table II-A, II-B or II-C.

Example

The minimum temperature of operation for a certain device is specified as 60,0 °C and the maximum temperature as 70,0 °C. Production is in inspection lots of 96 items. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. From table I-A, the sample size code letter is F; from table I-B it is seen that a sample size of 10 is required and from table IV-s that the value of f_s for the MSSD is 0,276. Suppose the measurements obtained are as follows : 63,5 °C; 62,0 °C; 65,2 °C; 61,7 °C; 69,0 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C. Compliance with the acceptability criterion is to be determined.

Information needed	**Value obtained**
Sample size : n	10
Sample mean : $\bar{x}$: $\Sigma\, x/n$	64,57
Sample standard deviation s : $\sqrt{\sum_i (x_i - \bar{x})^2/(n-1)}$ (See clause A.1.2, annex A.)	3,01
Value of f_s for MSSD (table IV-s)	0,276
MSSD = $f_s\,(U - L)$	2,76

The appropriate acceptance curve is taken from diagram s-F.

If, as on figure 3, the scales have been adjusted to the real measurements, plot the point ($s = 3{,}01$, $\bar{x} = 64{,}57$). This lies outside the acceptance curve and the lot is not acceptable. It could have been judged not acceptable as soon as it was seen that s was greater than the MSSD.

If the scale of the chart has not been adjusted to the values of s and $\bar{x}$, the following additional calculations are required :

Standardized mean :
$(\bar{x} - L)/(U - L) = (64{,}57 - 60)/(70 - 60) = 0{,}457$

Standardized s : $s/(U - L) = 3{,}01/(70 - 60) = 0{,}301$

The point (0,301, 0,457) is plotted on figure 3.

As it lies outside the acceptance curve for AQL = 1,5 %, the lot is not acceptable.

NOTE — This lot is not acceptable even though all tested items **in the sample** are within specification limits.

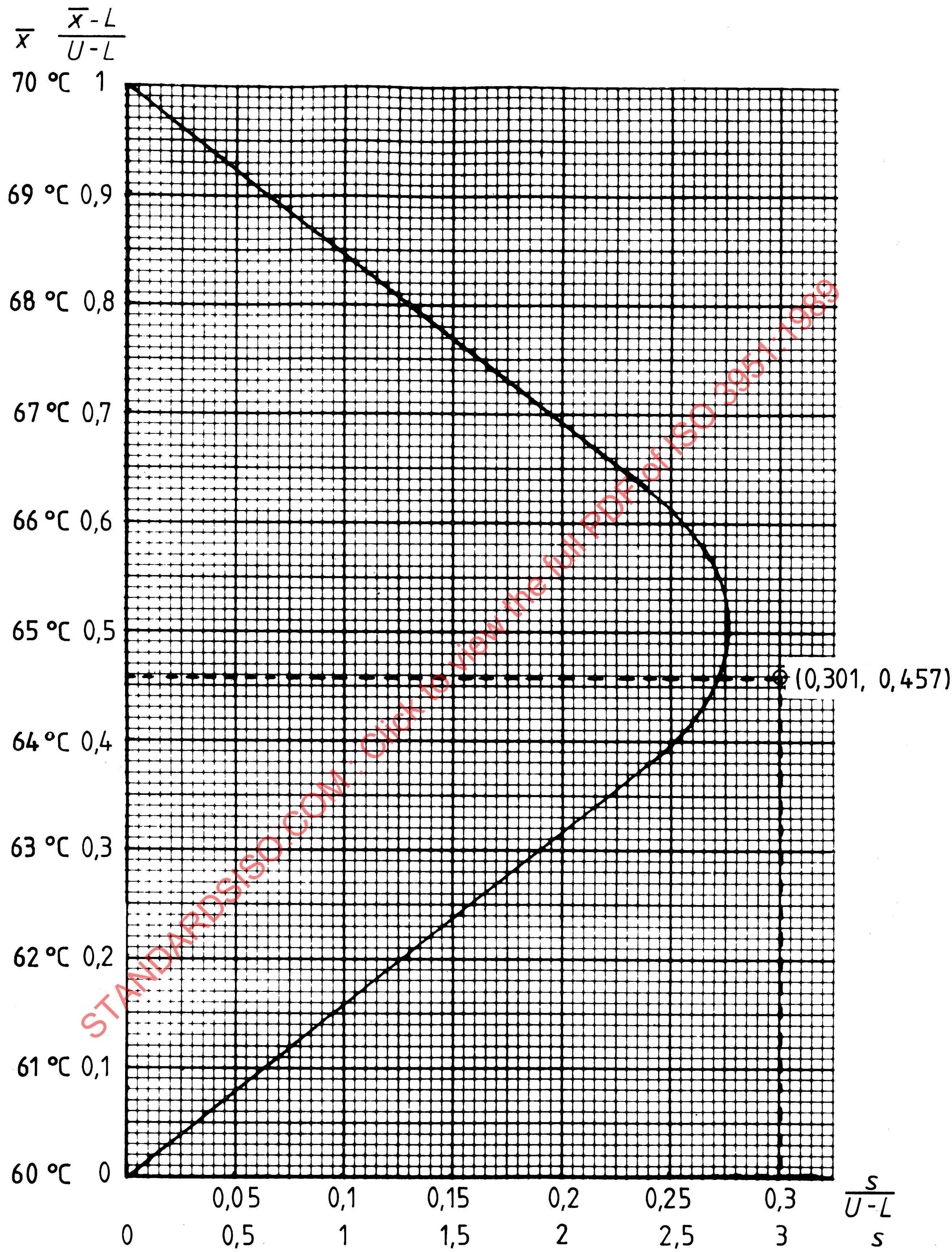


Figure 3 — Acceptance curve for sample size code letter F, AQL = 1,5 %, MSSD = 2,76 : "s" method

15 Standard procedure for "σ" method

15.1 Obtaining a plan

This method is only to be used when there is valid evidence that the standard deviation of the process can be considered constant and taken to be "σ".

From table I-A obtain the code letter; then, for normal inspection, from table III-A with the sample size code letter and the specified AQL, obtain the sample size n and acceptability constant k.

Take a random sample of this size, measure the characteristic under inspection x for all items of the sample and calculate the mean $\bar{x}$.

15.2 Acceptability criteria for single or separate specification limits

It is possible to obtain an acceptability criterion by following the procedure given for the "s" method, by substituting σ, the given value of the standard deviation of the process, for the s derived for the individual samples and comparing the calculated value of Q with the value of the acceptability constant k obtained from table III-A.

However, if $Q_U [= (U - \bar{x})/\sigma] \geqslant k_U$, the acceptability criterion for an upper specification is written as $\bar{x} \leqslant U - k_U \sigma$.

Since U, k_U and σ are all known in advance, the acceptance value $\bar{x}_U [= U - k_U \sigma]$ should be determined before inspection begins. The acceptability criterion then becomes, for an upper specification limit,

acceptable if $\bar{x} \leqslant \bar{x}_U [= U - k_U \sigma]$

not acceptable if $\bar{x} > \bar{x}_U [= U - k_U \sigma]$

similarly, for a lower specification limit,

acceptable if $\bar{x} \geqslant \bar{x}_L [= L + k_L \sigma]$

not acceptable if $\bar{x} < \bar{x}_L [= L + k_L \sigma]$

When both separate limits are given,

acceptable if both $\bar{x} \leqslant \bar{x}_U$ and $\bar{x} \geqslant \bar{x}_L$

not acceptable if either $\bar{x} > \bar{x}_U$ or $\bar{x} < \bar{x}_L$

Example

The specified minimum yield point for certain steel castings is 400 N/mm^2. A lot of 500 items is submitted for inspection. Inspection level II, normal inspection, with AQL = 1,5 %, is to be used. σ is known to be 21 N/mm^2. From table I-A it is seen that the sample size code letter is I and then, from table III-A, that for AQL of 1,5 % the sample size n is 10 and the acceptability constant k is 1,70. Suppose the yield points of the sample specimens are

431; 417; 469; 407; 452; 427; 421; 476; 400; 445;

and compliance with the acceptability criterion is to be determined.

Information needed	Value obtained
Acceptability constant : k	1,70
Known σ	21
Product : $k\sigma$	35,70
Specification limit : L	400
Acceptance value : $L + k\sigma = \bar{x}_L$	435,70
Sum of measurements : Σx	4 345
Sample size : n	10
Sample mean : $\bar{x}$	434,5
Acceptability criterion : Is $\bar{x} \geqslant \bar{x}_L$?	No

The sample mean of the lot does not meet the acceptability criterion so the lot is not accepted.

15.3 Acceptability criterion for a combined double specification limit

If a combined upper and lower specification limit has been given, it will be necessary to use a graphical method (unless σ is greater than the value of the MPSD obtained from table IV-σ, in which case the lot has to be immediately judged not acceptable without a sample being drawn). The following procedure is recommended:

a) Using the lot size and given inspection level, find the code letter from table I-A and the sample size from table I-B using the sample size code letter and AQL.

b) From the chart in the sigma (σ) series indexed by the sample size code letter, select the acceptance curve for the AQL given for this plan.

c) Calculate the value $\sigma/(U - L)$ and draw a vertical line through this point.

d) If this line intercepts the acceptance curve (see note 1), measure the values of $(\bar{x} - L)/(U - L)$ at the intersection points. These give the upper and lower "standardized" acceptable values of the mean. (See figure 4.) The acceptance values of the mean, $\bar{x}_U$ and $\bar{x}_L$, are then deduced as shown in the example.

NOTES

1 If the line does not intercept the acceptance curve, the process is not acceptable and sampling inspection is pointless.

2 If the line intercepts the straight line portions of the acceptance curve, this is equivalent to two separate specification limits and the acceptance criteria are those given in 15.2.

e) In accordance with 15.3 a), take a sample of the appropriate size and obtain the mean of the characteristic under inspection.

f) The acceptability criterion is: if the mean lies between the upper and lower acceptance values for $\bar{x}$ (i.e. if $\bar{x}_L \leqslant \bar{x} \leqslant \bar{x}_U$) the lot is acceptable, otherwise it is not acceptable.

Example

The specification for electrical resistance of a certain electrical component is 520 ± 50 Ω. Production is at a rate of 2 500 items per inspection lot. Inspection level II, normal inspection, with a single AQL of 4 %, is to be used for the two specification limits (470 and 570). σ is known to be 21,0. From table I-A the sample size code letter is K; from table I-B it is seen that a sample size of 25 is required. Suppose the values of the sample resistance are as follows :

515; 491; 479; 507; 543; 521; 536; 483; 509; 514; 507; 484; 526; 552; 499; 530; 512; 492; 521; 467; 489; 513; 535; 501; 529.

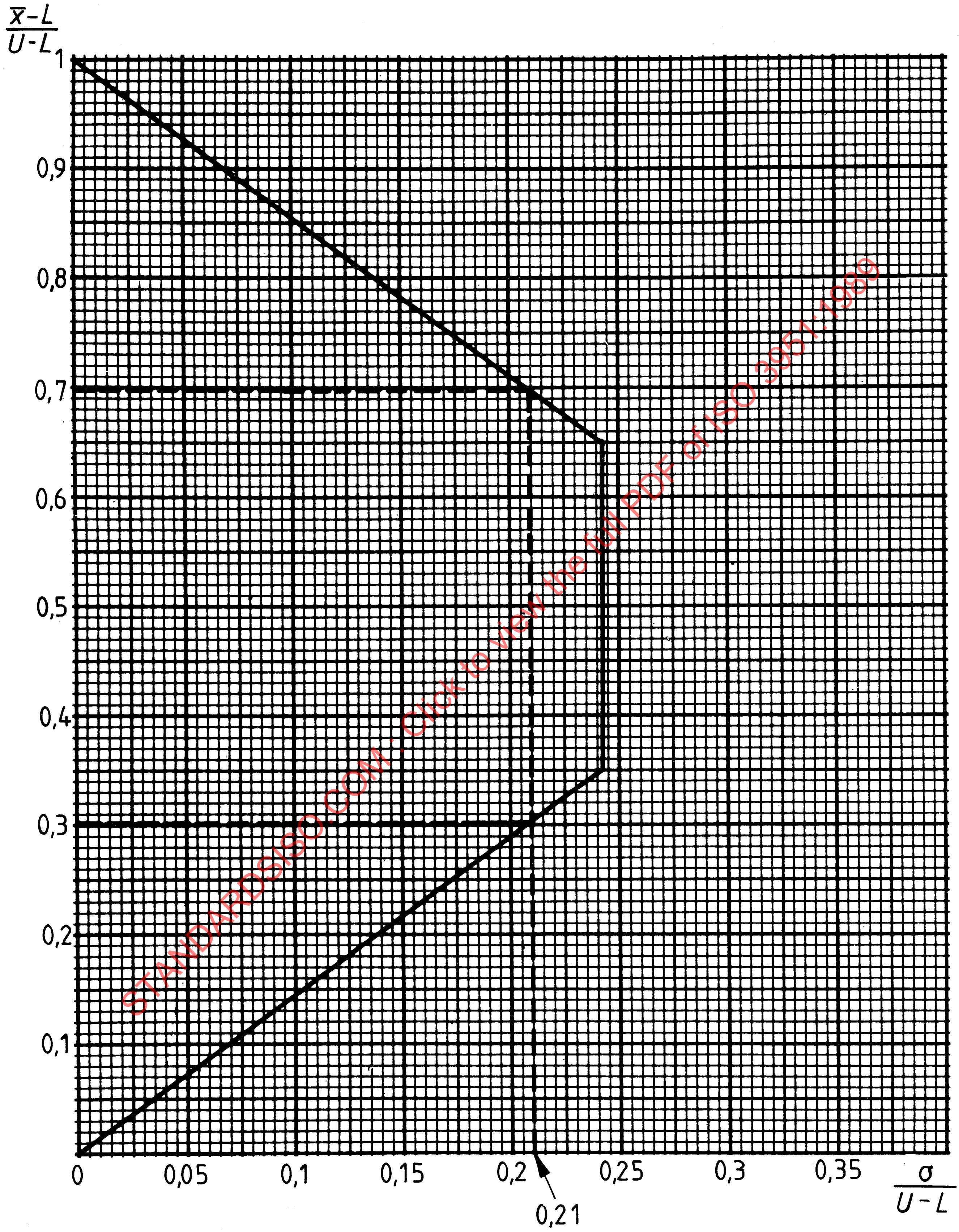


Figure 4 — Example of the use of an acceptance chart for a combined double specification limit: "σ" method

Information needed	Value obtained
Sample size : n	25
Upper specification limit : U	570
Lower specification limit : L	470
Known σ :	21,0
"Standardized" σ : $\dfrac{\sigma}{U - L} = \dfrac{21,0}{100}$	0,21

"Standardized" limiting values from acceptance curve (see figure 4)

Maximum	Minimum
$\dfrac{\bar{x}_U - L}{U - L} = 0,700$	$\dfrac{\bar{x}_L - L}{U - L} = 0,300$
$\bar{x}_U = L + 0,700\,(U - L)$	$\bar{x}_L = L + 0,300\,(U - L)$
$= 470 + 70,0$	$= 470 + 30,0$

Acceptance values for $\bar{x}$

$\bar{x}_U = 540,0$	$\bar{x}_L = 500,0$
Sum of measurements :	12 755
Mean of measurements :	510,2

Since $\bar{x}$ at 510,2 lies between the acceptance limits for $\bar{x}$ of 540,0 and 500,0 the lot is acceptable.

NOTES

1 All the calculations other than the last two lines should be completed before sampling begins.

2 In this example, since the intercepts lie for all practical purposes on the straight line portions of the acceptance curve, the acceptance values for $\bar{x}$ could alternatively have been obtained without difficulty by direct calculation.

From table IV-σ

$f_\sigma = 0,244$

MPSD $= 0,244\ (570 - 470) = 24,4$

As $\sigma <$ MPSD, the next step is to calculate $\bar{x}_U$ and $\bar{x}_L$.

From table III-A

$k = 1,42$

$k\sigma = 29,8$

$\bar{x}_U = U - k\sigma = 540,2$

$\bar{x}_L = L + k\sigma = 499,8$

and again the lot, whose mean is 510,2, would be acceptable.

NOTE — If σ had been known to be 25, then $\sigma >$ MPSD and therefore sampling inspection should not have taken place.

16 Procedure during continuing inspection

As a variables sampling inspection plan can only operate efficiently if

a) the characteristic being inspected is normally distributed,

b) records are kept,

c) the switching rules are obeyed,

it is necessary to ensure that these requirements are being met.

17 Normality and outliers

17.1 Normality

Normality should have been checked by the responsible authority before sampling begins. In case of doubt, a statistician should advise whether the distribution appears suitable for sampling by variables or whether use should be made of the graphical tests for normality given in ISO 2854, section two.

17.2 Outliers

An outlier (or an outlying observation) is one that appears to deviate markedly from other observations in the sample in which it occurs. A single outlier, even when it lies within specification limits, may produce an increase in variability, change the mean and consequently lead to rejection of the lot. When an outlying observation is suspected, a test for outliers should precede, and be made independently of, a decision on the acceptability of a lot. (See, for example, ISO 5725.) When outliers are detected, the disposition of the lot should be a matter for negotiation between the vendor and vendee.

18 Records

18.1 Control charts

One of the advantages of inspection by variables is that trends in the quality level of the product can be detected and warning given before an unacceptable standard is reached, but this is only possible if adequate records are kept.

Whatever the method used, s, σ, (or R), records should be kept of the values of $\bar{x}$ and s (or R), preferably in the form of control charts.

This procedure should be applied especially with the "σ" method in order to verify that the values of s obtained from the samples fall within the limits of the prescribed value of σ.

When a combined double specification limit has been laid down, the value of the MSSD, given in table IV-s (or MAR given in table RIV), should be plotted on the s (or R) control chart, as an indication of an unacceptable value.

NOTE — Control charts are used to detect trends. The ultimate decision as to the acceptability of an individual lot is governed by the procedure given in clauses 14 and 15.

18.2 Lots which are not accepted

Particular care shall be taken to record all lots which are not accepted and see that switching rules are implemented. Any lot not accepted by the sampling plan shall not be resubmitted either in whole or in part without the permission of the responsible authority.

19 Operation of switching rules

The standard switching rules are as follows.

19.1 Normal inspection is used at the start of inspection (unless otherwise designated) and shall continue to be used during the course of inspection until tightened inspection becomes necessary or reduced inspection is allowed.

19.2 Tightened inspection shall be instituted when two lots on original normal inspection are not accepted within any five or fewer successive lots.

Tightened inspection is achieved by increasing the values of the acceptability constant. The values are tabulated in table II-B for the *"s"* method (table *R*II-B for the *"R"* method) and table III-B for the *"σ"* method. In the *"s"* method (and also in the *"R"* method), there is no change in the size of the sample, unless the sample size is so small that the tables indicate, with a downward arrow, that an increase in sample size is necessary.

19.3 Tightened inspection shall be relaxed when five successive lots on original inspection have been accepted on tightened inspection; then reinstate normal inspection.

19.4 Reduced inspection may be instituted after ten successive lots have been accepted under normal inspection, provided that

a) these lots would have been acceptable if the AQL had been one step tighter;

NOTE — If a value of *k* for this tighter AQL is not given in table II-A (*"s"* method), a sufficiently accurate acceptance curve may be extrapolated by eye in comparison with the other curves of the appropriate diagram in the *"s"* series. The same procedure applies to the *"σ"* method (and the *"R"* method).

b) production is in statistical control;

c) reduced inspection is considered desirable by the responsible authority.

Reduced inspection is conducted on a much smaller sample than normal inspection and the value of the acceptability constant is also decreased. The values of *n* and *k* for reduced inspection are given in table II-C for the *"s"* method, table III-C for the *"σ"* method (table *R*II-C for the *"R"* method).

19.5 Reduced inspection shall cease and normal inspection be reinstated if any of the following occur on original inspection :

a) a lot is not accepted;

b) production becomes irregular or delayed;

c) other conditions warrant that normal inspection shall be instituted.

20 Discontinuation and resumption of inspection

If the cumulative number of lots not accepted in a sequence of consecutive lots on original tightened inspection reaches 5, the acceptance procedures of this International Standard shall be discontinued.

Inspection under the provisions of this International Standard shall not be resumed until action has been taken by the supplier to improve the quality of the submitted product or service. Tightened inspection shall then be used as if 19.2 had been invoked.

21 Acceptance curves for tightened and reduced inspection

a) Separate acceptance curves have not been produced for tightened or reduced inspection, so when combined double specification limits have to be met under tightened or reduced inspection, it is necessary to find the curve among those given for use in normal inspection which has the required sample size and acceptability constant.

b) When switching to tightened inspection, the required acceptance curve can be found using table VI-A. Usually it will turn out to be the curve just inside that being used in normal inspection (i.e. corresponding to the same sample size, but the next lower AQL) but in some cases there is an accompanying increase in sample size.

If the *"σ"* method is being used and *"σ"* exceeds the MPSD corresponding to the next lower AQL, reversion to tightened inspection under the *"s"* method is required.

c) When switching to reduced inspection, the required acceptance curve is found from table VI-B. This curve will correspond to an AQL under normal inspection that is one step larger and to a sample size that is up to three steps smaller.

Example

Suppose that the *"s"* method is being used with sample size code letter I and AQL 0,25. Table VI-A shows that the acceptance curve for tightened inspection is the one for code letter I and AQL 0,15 under normal inspection. Table VI-B shows that the acceptance curve for reduced inspection is the one for code letter F and AQL 0,40 under normal inspection.

22 Switching to *"σ"* method

If it appears that the value of *s* (or $\bar{R}$) is in control, the (weighted) root mean square value of *s* (or $\bar{R}/c$) may be presumed to be *σ*, the "known" standard deviation of the process, and the *"σ"* method may be adopted. In order to verify that the variability remains under control, the value of *s* should still be calculated and plotted on a control chart. (See clause A.2 in annex A.)

NOTE — This switch should not be undertaken without the permission of the responsible authority.

Section four: Tables and diagrams

Tables

I-A	Sample size code letters and inspection levels
I-B	Sample size code letters and sample sizes for normal inspection
II-A	Single sampling plans for normal inspection (master table) : "*s*" method
II-B	Single sampling plans for tightened inspection (master table) : "*s*" method
II-C	Single sampling plans for reduced inspection (master table) : "*s*" method
III-A	Single sampling plans for normal inspection (master table) : "σ" method
III-B	Single sampling plans for tightened inspection (master table) : "σ" method
III-C	Single sampling plans for reduced inspection (master table) : "σ" method
IV-*s*	Values of f_s for maximum sample standard deviation (MSSD) : "*s*" method
IV-σ	Values of f_σ for maximum process standard deviation (MPSD): "σ" method
V (V-B to V-P)	Tables and operating characteristic curves for sampling plans (sample size code letters B to P)
VI-A	Correspondence between tightened and normal inspection plans
VI-B	Correspondence between reduced and normal inspection plans

Diagrams

A	Sample size code letters of standard sampling plans for specified qualities at 95 % and 10 % probabilities of acceptance
s-D to *s*-P	Acceptance curves for combined double specification limits : "*s*" method
σ-C to σ-P	Acceptance curves for combined double specification limits : "σ" method

Table I-A — Sample size code letters and inspection levels

Lot or batch size	Special inspection levels		General inspection levels		
	S-3	S-4	I	II	III
2 to 8	↓	↓	↓		C
9 to 15	↓	↓	↓	B	D
16 to 25	↓	↓	B	C	E
26 to 50	↓	↓	C	D	F
51 to 90	↓	B	D	E	G
91 to 150	↓	C	E	F	H
151 to 280	B	D	F	G	I
281 to 500	C	E	G	H/I*	J
501 to 1 200	D	F	H	J	K
1 201 to 3 200	E	G	I	K	L
3 201 to 10 000	F	H	J	L	M
10 001 to 35 000	G	I	K	M	N
35 001 to 150 000	H	J	L	N	P
150 001 to 500 000	I	K	M	P	↑
500 001 and over	J	L	N	↑	↑

* Use H for lot size 281 to 400 and I for lot size 401 to 500.

NOTES

1 The sample size code letters and inspection levels in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414[8].

2 Symbol

↓ There is no suitable sampling plan in this area; use the first sampling plan below or above the arrow. This refers to both the sample size and acceptability constant *k*.

Table I-B — Sample size code letters and sample sizes for normal inspection

Method	"s"	"σ"											Equivalent attributes sample (ISO 2859)	
Acceptable quality level	All AQL	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	Code letter	Sample size
Sample size code letter B	3	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	B	3
C	4	↓	↓	↓	↓	↓	2	2	2	2	3	3	C	5
D	5	↓	↓	↓	↓	2	2	2	3	3	3	4	D	8
E	7	↓	↓	2	2	3	3	3	4	4	5	5	E	13
F	10	↓	3	3	3	4	4	4	5	5	6	7	F	20
G	15	4	4	4	5	5	6	6	7	8	9	11	G	32
H	20	5	5	6	6	7	7	8	9	10	12	14	H	50
I	25	6	6	7	8	8	9	10	11	13	15	17	—	—
J	35	8	9	9	10	11	12	14	15	18	20	24	J	80
K	50	11	12	13	14	16	17	19	22	25	29	33	K	125
L	75	16	17	19	21	23	25	28	32	36	42	49	L	200
M	100	22	23	25	27	30	33	36	42	48	55	64	M	315
N	150	31	34	37	40	44	49	54	61	70	82	95	N	500
P	200	42	45	49	54	59	65	71	81	93	109	127	P	800

NOTES

1 All AQL values are in percent nonconforming.

2 The sample size code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.

3 Symbol

↓ There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.

"*s*" method

Table II-A — Single sampling plans for normal inspection (master table) : "*s*" method

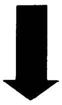
Sample size code letter	Sample size	Acceptable quality level (normal inspection)										
		0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
		k	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*
B	3								1,12	0,958	0,765	0,566
C	4						1,45	1,34	1,17	1,01	0,814	0,617
D	5					1,65	1,53	1,40	1,24	1,07	0,874	0,675
E	7			2,00	1,88	1,75	1,62	1,50	1,33	1,15	0,955	0,755
F	10		2,24	2,11	1,98	1,84	1,72	1,58	1,41	1,23	1,03	0,828
G	15	2,42	2,32	2,20	2,06	1,91	1,79	1,65	1,47	1,30	1,09	0,886
H	20	2,47	2,36	2,24	2,11	1,96	1,82	1,69	1,51	1,33	1,12	0,917
I	25	2,50	2,40	2,26	2,14	1,98	1,85	1,72	1,53	1,35	1,14	0,936
J	35	2,54	2,45	2,31	2,18	2,03	1,89	1,76	1,57	1,39	1,18	0,969
K	50	2,60	2,50	2,35	2,22	2,08	1,93	1,80	1,61	1,42	1,21	1,00
L	75	2,66	2,55	2,41	2,27	2,12	1,98	1,84	1,65	1,46	1,24	1,03
M	100	2,69	2,58	2,43	2,29	2,14	2,00	1,86	1,67	1,48	1,26	1,05
N	150	2,73	2,61	2,47	2,33	2,18	2,03	1,89	1,70	1,51	1,29	1,07
P	200	2,73	2,62	2,47	2,33	2,18	2,04	1,89	1,70	1,51	1,29	1,07

NOTES

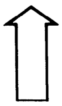
1 All AQL values are in percent nonconforming.

2 The sample size code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.

3 Symbols



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.



The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.

The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"s" method

Table II-B — Single sampling plans for tightened inspection (master table) : "s" method

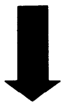
Sample size code letter	Sample size	Acceptable quality levels (tightened inspection)										
		0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
		k	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*
B	3	↓	↓	↓	↓	↓	↓	↓	↓	1,12	0,958	0,765
C	4	↓	↓	↓	↓	↓	↓	1,45	1,34	1,17	1,01	0,814
D	5	↓	↓	↓	↓	↓	1,65	1,53	1,40	1,24	1,07	0,874
E	7	↓	↓	↓	2,00	1,88	1,75	1,62	1,50	1,33	1,15	0,955
F	10	↓	↓	2,24	2,11	1,98	1,84	1,72	1,58	1,41	1,23	1,03
G	15	2,53	2,42	2,32	2,20	2,06	1,91	1,79	1,65	1,47	1,30	1,09
H	20	2,58	2,47	2,36	2,24	2,11	1,96	1,82	1,69	1,51	1,33	1,12
I	25	2,61	2,50	2,40	2,26	2,14	1,98	1,85	1,72	1,53	1,35	1,14
J	35	2,65	2,54	2,45	2,31	2,18	2,03	1,89	1,76	1,57	1,39	1,18
K	50	2,71	2,60	2,50	2,35	2,22	2,08	1,93	1,80	1,61	1,42	1,21
L	75	2,77	2,66	2,55	2,41	2,27	2,12	1,98	1,84	1,65	1,46	↑ 1,24
M	100	2,80	2,69	2,58	2,43	2,29	2,14	2,00	1,86	1,67	↑ 1,48	1,26
N	150	2,84	2,73	2,61	2,47	2,33	2,18	2,03	1,89	↑ 1,70	1,51	1,29
P	200	2,85	2,73	2,62	2,47	2,33	2,18	2,04	↑ 1,89	1,70	1,51	1,29

NOTES

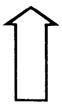
1 All AQL values are in percent nonconforming.

2 The sample size code letters in this International Standard correspond to those given in ISO 2859; they are not identical with those given in MIL STD 414.

3 Symbols



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.



The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.

The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"*s*" method

Table II-C — Single sampling plans for reduced inspection (master table) : "*s*" method

Sample size code letter	Sample size	Acceptable quality levels (reduced inspection)										
		0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
		k	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*	*k*
B	3	↓	↓	↓	↓	↓	↓	1,12	0,958	0,765	0,566	0,341
C	3							1,12	0,958	0,765	0,566	0,341
D	3							1,12	0,958	0,765	0,566	0,341
E	3							1,12	0,958	0,765	0,566	0,341
F	4					1,45	1,34	1,17	1,01	0,814	0,617	0,393
G	5				1,65	1,53	1,40	1,24	1,07	0,874	0,675	0,455
H	7		2,00	1,88	1,75	1,62	1,50	1,33	1,15	0,955	0,755	0,536
I	10	2,24	2,11	1,98	1,84	1,72	1,58	1,41	1,23	1,03	0,828	0,611
J	15	2,32	2,20	2,06	1,91	1,79	1,65	1,47	1,30	1,09	0,886	0,664
K	20	2,36	2,24	2,11	1,96	1,82	1,69	1,51	1,33	1,12	0,917	0,695
L	25	2,40	2,26	2,14	1,98	1,85	1,72	1,53	1,35	1,14	0,936	↑ 0,712
M	35	2,45	2,31	2,18	2,03	1,89	1,76	1,57	1,39	1,18	↑ 0,969	0,745
N	50	2,50	2,35	2,22	2,08	1,93	1,80	1,61	1,42	↑ 1,21	1,00	0,774
P	75	2,55	2,41	2,27	2,12	1,98	1,84	1,65	↑ 1,46	1,24	1,03	0,804

NOTES

1 All AQL values are in percent nonconforming.

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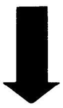
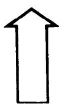
3 Symbols



There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.



The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.

The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"σ" method

Table III-A — Single sampling plans for normal inspection (master table) : "σ" method

Sample size code letter	Acceptable quality levels (normal inspection)																				
	0,10		0,15		0,25		0,40		0,65		1,00		1,50		2,50		4,00		6,50		10,00	
	n	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*
B	↓		↓		↓		↓		↓		↓		↓		↓		↓		↓		↓	
C	↓		↓		↓		↓		↓		2	1,36	2	1,25	2	1,09	2	0,936	3	0,755	3	0,573
D	↓		↓		↓		↓		2	1,58	2	1,42	2	1,33	3	1,17	3	1,01	3	0,825	4	0,641
E	↓		↓		2	1,94	2	1,81	3	1,69	3	1,56	3	1,44	4	1,28	4	1,11	5	0,919	5	0,728
F	↓		3	2,19	3	2,07	3	1,91	4	1,80	4	1,69	4	1,53	5	1,39	5	1,20	6	0,991	7	0,797
G	4	2,39	4	2,30	4	2,14	5	2,05	5	1,88	6	1,78	6	1,62	7	1,45	8	1,28	9	1,07	11	0,877
H	5	2,46	5	2,34	6	2,23	6	2,08	7	1,95	7	1,80	8	1,68	9	1,49	10	1,31	12	1,11	14	0,906
I	6	2,49	6	2,37	7	2,25	8	2,13	8	1,96	9	1,83	10	1,70	11	1,51	13	1,34	15	1,13	17	0,924
J	8	2,54	9	2,45	9	2,29	10	2,16	11	2,01	12	1,88	14	1,75	15	1,56	18	1,38	20	1,17	24	0,964
K	11	2,59	12	2,49	13	2,35	14	2,21	16	2,07	17	1,93	19	1,79	22	1,61	25	1,42	29	1,21	33	0,995
L	16	2,65	17	2,54	19	2,41	21	2,27	23	2,12	25	1,97	28	1,84	32	1,65	36	1,46	42	1,24	49	1,03
M	22	2,69	23	2,57	25	2,43	27	2,29	30	2,14	33	2,00	36	1,86	42	1,67	48	1,48	55	1,26	64	1,05
N	31	2,72	34	2,62	37	2,47	40	2,33	44	2,17	49	2,03	54	1,89	61	1,69	70	1,51	82	1,29	95	1,07
P	42	2,73	45	2,62	49	2,48	54	2,34	59	2,18	65	2,04	71	1,89	81	1,70	93	1,51	109	1,29	127	1,07

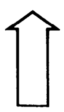
NOTES

1 All AQL values are in percent nonconforming.

2 The sample size code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.

3 Symbols

There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.



The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.

The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"σ" method

Table III-B — Single sampling plans for tightened inspection (master table) : "σ" method

Sample size code letter	Acceptable quality levels (tightened inspection)																					
	0,10		0,15		0,25		0,40		0,65		1,00		1,50		2,50		4,00		6,50		10,00	
	n	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*
B	↓		↓		↓		↓		↓		↓		↓		↓		↓		↓		↓	
C	↓		↓		↓		↓		↓		↓		2	1,36	2	1,25	2	1,09	2	0,936	3	0,755
D	↓		↓		↓		↓		↓		2	1,58	2	1,42	2	1,33	3	1,17	3	1,01	3	0,825
E	↓		↓		↓		2	1,94	2	1,81	3	1,69	3	1,56	3	1,44	4	1,23	4	1,11	5	0,919
F	↓		↓		3	2,19	3	2,07	3	1,91	4	1,80	4	1,69	4	1,53	5	1,39	5	1,20	6	0,991
G	3	2,49	4	2,39	4	2,30	4	2,14	5	2,05	5	1,88	6	1,78	6	1,62	7	1,45	8	1,28	9	1,07
H	4	2,55	5	2,46	5	2,34	6	2,23	6	2,08	7	1,95	7	1,80	8	1,68	9	1,49	10	1,31	12	1,11
I	6	2,59	6	2,49	6	2,37	7	2,25	8	2,13	8	1,96	9	1,83	10	1,70	11	1,51	13	1,34	15	1,13
J	7	2,63	8	2,54	9	2,45	9	2,29	10	2,16	11	2,01	12	1,86	14	1,75	15	1,56	18	1,38	20	1,17
K	11	2,72	11	2,59	12	2,49	13	2,35	14	2,21	16	2,07	17	1,93	19	1,79	22	1,61	25	1,42	29	1,21
L	15	2,77	16	2,65	17	2,54	19	2,41	21	2,27	23	2,12	25	1,97	28	1,84	32	1,65	36	1,46	42 ⇧	1,24
M	20	2,80	22	2,69	23	2,57	25	2,43	27	2,29	30	2,14	33	2,00	36	1,86	42	1,67	48 ⇧	1,48	55 ⇧	1,26
N	30	2,84	31	2,72	34	2,62	37	2,47	40	2,33	44	2,17	49	2,03	54	1,89	61 ⇧	1,69	70 ⇧	1,51	82 ⇧	1,29
P	40	2,85	42	2,73	45	2,62	49	2,48	54	2,34	59	2,18	65	2,04	71 ⇧	1,89	81 ⇧	1,70	93 ⇧	1,51	109 ⇧	1,29

NOTES

1 All AQL values are in percent nonconforming.

2 The sample size code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.

3 Symbols

↓ There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.

⇧ The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.

The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

"σ" method

Table III-C — Single sampling plans for reduced inspection (master table) : "σ" method

Sample size code letter	Acceptable quality levels (reduced inspection)																					
	0,10		0,15		0,25		0,40		0,65		1,00		1,50		2,50		4,00		6,50		10,00	
	n	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*	*n*	*k*
B	↓		↓		↓		↓		↓		↓		↓		↓		↓		↓		↓	
C																						
D																						
E																						
F									2	1,36	2	1,25	2	1,09	2	0,936	3	0,755	3	0,573	4	0,344
G							2	1,58	2	1,42	2	1,33	3	1,17	3	1,01	3	0,825	4	0,641	4	0,429
H			2	1,94	2	1,81	3	1,69	3	1,56	3	1,44	4	1,28	4	1,11	5	0,919	5	0,728	6	0,515
I	3	2,19	3	2,07	3	1,91	4	1,80	4	1,69	4	1,53	5	1,39	5	1,20	6	0,991	7	0,797	8	0,584
J	4	2,30	4	2,14	5	2,05	5	1,88	6	1,78	6	1,62	7	1,45	8	1,28	9	1,07	11	0,877	12	0,649
K	5	2,34	6	2,23	6	2,08	7	1,95	7	1,80	8	1,68	9	1,49	10	1,31	12	1,11	14	0,906	16	0,685
L	6	2,37	7	2,25	8	2,13	8	1,96	9	1,83	10	1,70	11	1,51	13	1,34	15	1,13	17	0,924	20 ↑	0,706
M	9	2,45	9	2,29	10	2,16	11	2,01	12	1,88	14	1,75	15	1,56	18	1,38	20	1,17	24 ↑	0,964	27	0,737
N	12	2,49	13	2,35	14	2,21	16	2,07	17	1,93	19	1,79	22	1,61	25	1,42	29 ↑	1,21	33	0,995	38	0,770
P	17	2,54	19	2,41	21	2,27	23	2,12	25	1,97	28	1,84	32	1,65	36 ↑	1,46	42	1,24	49	1,03	56	0,803

NOTES

1 All AQL values are in percent nonconforming.

2 The sample size code letters in this International Standard correspond to those given in ISO 2859; they are *not* identical with those given in MIL STD 414.

3 Symbols

↓ There is no suitable sampling plan in this area; use the first sampling plan below the arrow. This refers to both the sample size and acceptability constant *k*.

↑ The plan given in this area gives a high degree of security but at the expense of a large sample. At the discretion of the responsible authority the next plan above the arrow may be used.

The heavy lines indicate the boundary of the equivalent attribute sampling plans in ISO 2859.

Table IV-*s* — Values of f_s for maximum sample standard deviation (MSSD): "*s*" method

Sample size	Acceptable quality levels (normal inspection)												
		0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00	
3									0,436	0,453	0,475	0,502	0,538
4							0,339	0,353	0,374	0,399	0,432	0,472	0,528
5						0,294	0,308	0,323	0,346	0,372	0,408	0,452	0,511
7				0,242	0,253	0,266	0,280	0,295	0,318	0,345	0,381	0,425	0,485
10			0,214	0,224	0,235	0,248	0,261	0,276	0,298	0,324	0,359	0,403	0,460
15	0,188	0,195	0,202	0,211	0,222	0,235	0,248	0,262	0,284	0,309	0,344	0,386	0,442
20	0,183	0,190	0,197	0,206	0,216	0,229	0,242	0,255	0,277	0,302	0,336	0,377	0,432
25	0,180	0,187	0,193	0,203	0,212	0,225	0,238	0,251	0,273	0,297	0,331	0,372	0,426
35	0,176	0,183	0,189	0,198	0,208	0,220	0,232	0,245	0,266	0,291	0,323	0,364	0,416
50	0,172	0,178	0,184	0,194	0,203	0,214	0,227	0,241	0,261	0,284	0,317	0,356	0,408
75	0,168	0,174	0,181	0,189	0,199	0,211	0,223	0,235	0,255	0,279	0,310	0,348	0,399
100	0,166	0,172	0,179	0,187	0,197	0,208	0,220	0,233	0,253	0,276	0,307	0,345	0,395
150	0,163	0,170	0,175	0,185	0,193	0,206	0,216	0,230	0,249	0,271	0,302	0,341	0,388
200	0,163	0,168	0,175	0,183	0,193	0,203	0,215	0,228	0,248	0,269	0,302	0,338	0,386
	0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00		
	Acceptable quality levels (tightened inspection)												
			0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
			Acceptable quality levels (reduced inspection)										

NOTE — The MSSD is obtained by multiplying the standardized MSSD f_s by the difference between the upper specification limit U and lower specification limit L, i.e. MSSD = $f_s (U - L)$.

The MSSD indicates the greatest allowable magnitude of the sample standard deviation when using plans for the double specification limit case with unknown variability. If the sample standard deviation is less than the MSSD there is a possibility but not a certainty that the lot will be accepted.

Table IV-*σ* — Values of f_σ for maximum process standard deviation (MPSD): "*σ*" method

Acceptable quality levels (normal inspection)												
	0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00	
0,147	0,152	0,157	0,165	0,174	0,184	0,194	0,206	0,223	0,243	0,271	0,304	0,347
0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00		
Acceptable quality levels (reduced inspection)												
		0,10	0,15	0,25	0,40	0,65	1,00	1,50	2,50	4,00	6,50	10,00
		Acceptable quality levels (reduced inspection)										

NOTE — THE MPSD is obtained by multiplying the standardized MPSD f_σ by the difference between the upper specification limit U and lower specification limit L, i.e. MPSD = $f_\sigma (U - L)$.

The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for the double specification limit case with known variability. If the process standard deviation is less than the MPSD there is a possibility but not a certainty that the lot will be accepted.

The responsible authority may specify the use of the f_σ value for tightened inspection for both normal and reduced inspection, in which case the selection between "*σ*" and "*s*" method is kept independent of the switching rules.

B

Table V-B — Sample size code letter B

Chart V-B — Operating characteristic curves for single sampling plans

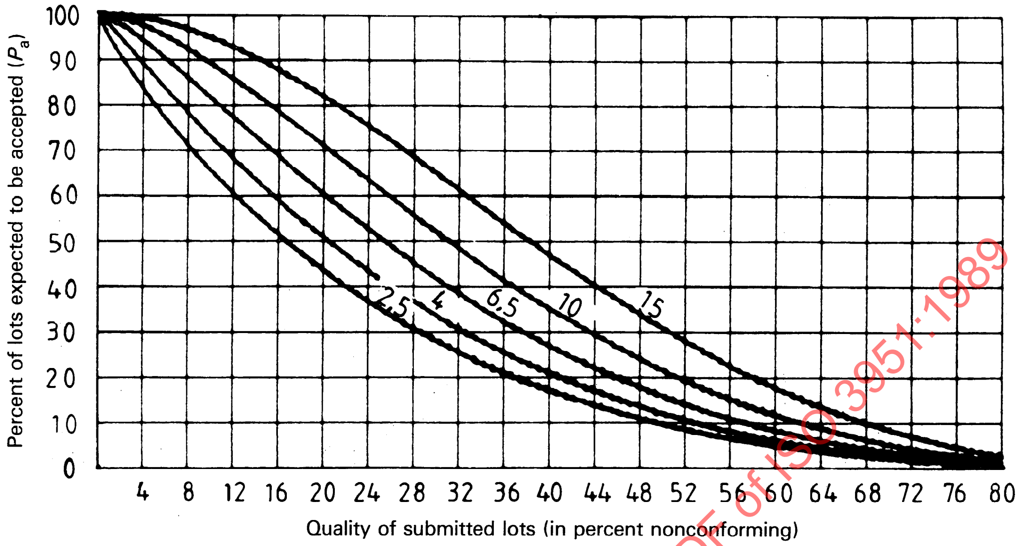


Table V-B-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0									0,18	0,44	1,07	2,28	99,0
95,0									1,04	1,89	3,52	6,02	95,0
90,0									2,26	3,65	6,03	9,39	90,0
75,0									6,66	9,18	12,95	17,71	75,0
50,0									16,68	20,30	25,22	30,97	50,0
25,0									32,40	36,35	41,45	47,14	25,0
10,0									49,34	52,83	57,24	62,08	10,0
5,0									59,45	62,44	66,20	70,30	5,0
1,0									75,99	77,93	80,34	82,98	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

C

Table V-C — Sample size code letter C

Chart V-C — Operating characteristic curves for single sampling plans

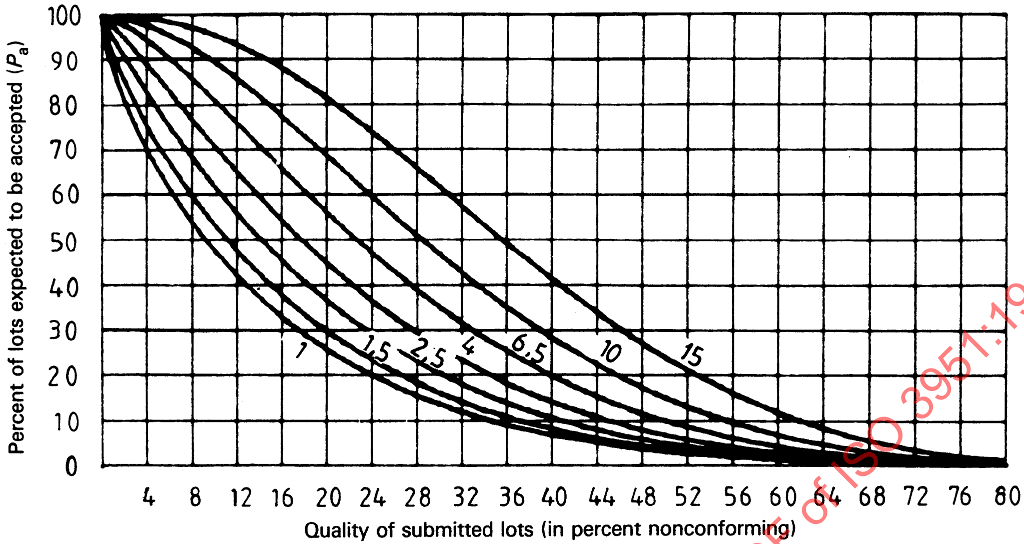


Table V-C-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0							0,07	0,13	0,32	0,69	1,55	3,05	99,0
95,0							0,44	0,69	1,32	2,29	4,13	6,85	95,0
90,0							1,02	1,48	2,53	3,98	6,51	10,00	90,0
75,0							3,36	4,37	6,37	8,81	12,61	17,35	75,0
50,0							9,52	11,28	14,44	17,93	22,89	28,61	50,0
25,0							20,81	23,11	27,00	31,02	36,43	42,37	25,0
10,0							34,88	37,26	41,15	45,05	50,13	55,55	10,0
5,0							44,29	46,53	50,14	53,72	58,33	68,20	5,0
1,0							61,76	63,48	66,23	68,95	72,37	75,98	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

D

Table V-D — Sample size code letter D

Chart V-D — Operating characteristic curves for single sampling plans

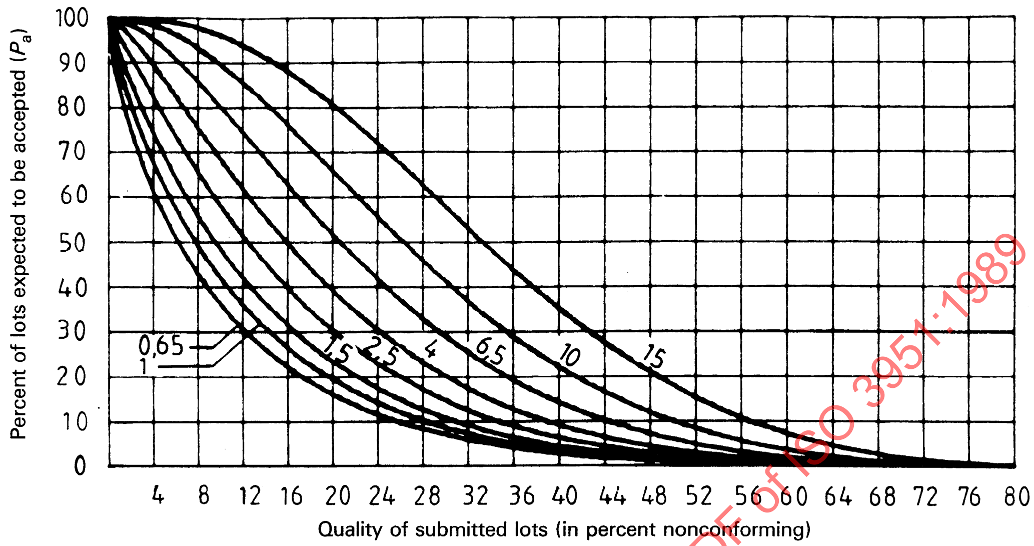


Table V-D-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0						0,04	0,09	0,18	0,40	0,84	1,79	3,44	99,0
95,0						0,28	0,46	0,77	1,38	2,43	4,30	7,11	95,0
90,0						0,64	0,98	1,51	2,48	3,99	6,49	10,00	90,0
75,0						2,15	2,93	4,02	5,78	8,23	11,89	16,55	75,0
50,0						6,34	7,82	9,71	12,47	15,97	20,75	26,40	50,0
25,0						14,64	16,79	19,36	22,88	27,06	32,43	38,46	25,0
10,0						25,94	28,40	31,24	34,98	39,25	44,55	50,32	10,0
5,0						34,06	36,52	39,33	42,97	47,06	52,06	57,42	5,0
1,0						50,47	52,63	55,04	58,11	61,51	65,57	69,89	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

E

Table V-E — Sample size code letter E

Chart V-E — Operating characteristic curves for single sampling plans

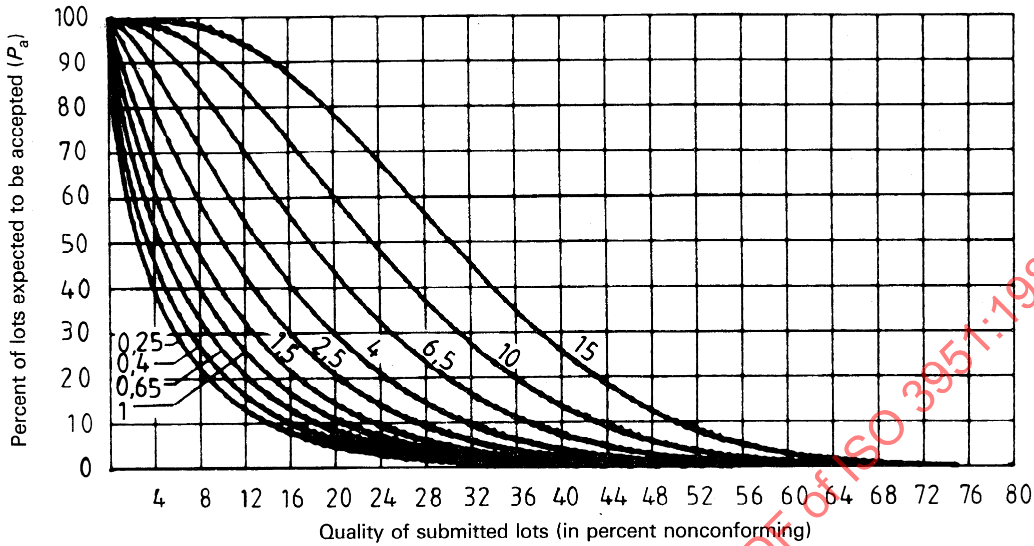


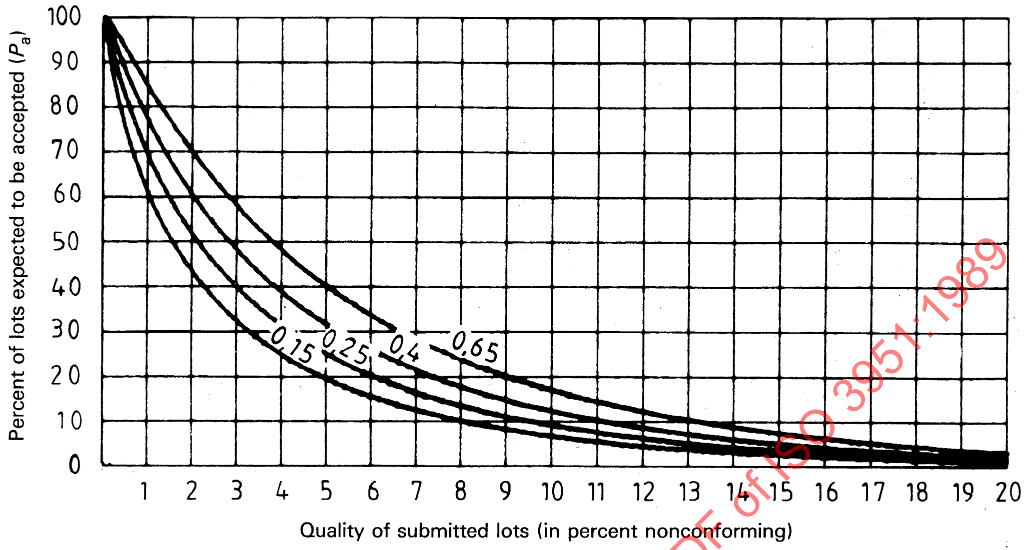
Table V-E-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0				0,02	0,03	0,07	0,14	0,25	0,53	1,09	2,19	4,04	99,0
95,0				0,11	0,18	0,32	0,53	0,83	1,50	2,65	4,57	7,46	95,0
90,0				0,25	0,40	0,64	1,01	1,48	2,47	4,04	6,50	9,99	90,0
75,0				0,90	1,27	1,83	2,58	3,47	5,15	7,56	11,00	15,49	75,0
50,0				2,89	3,72	4,83	6,18	7,69	10,28	13,66	18,11	23,53	50,0
25,0				7,38	8,80	10,57	12,60	14,71	18,11	22,27	27,41	33,35	25,0
10,0				14,42	16,33	18,60	21,09	23,58	27,43	31,93	37,28	43,25	10,0
5,0				20,09	22,20	24,65	27,29	29,88	33,82	38,33	43,60	49,38	5,0
1,0				33,10	35,32	37,83	40,45	42,95	46,72	50,89	55,64	60,73	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Table V-F — Sample size code letter F

F

Chart V-F — Operating characteristic curves for single sampling plans



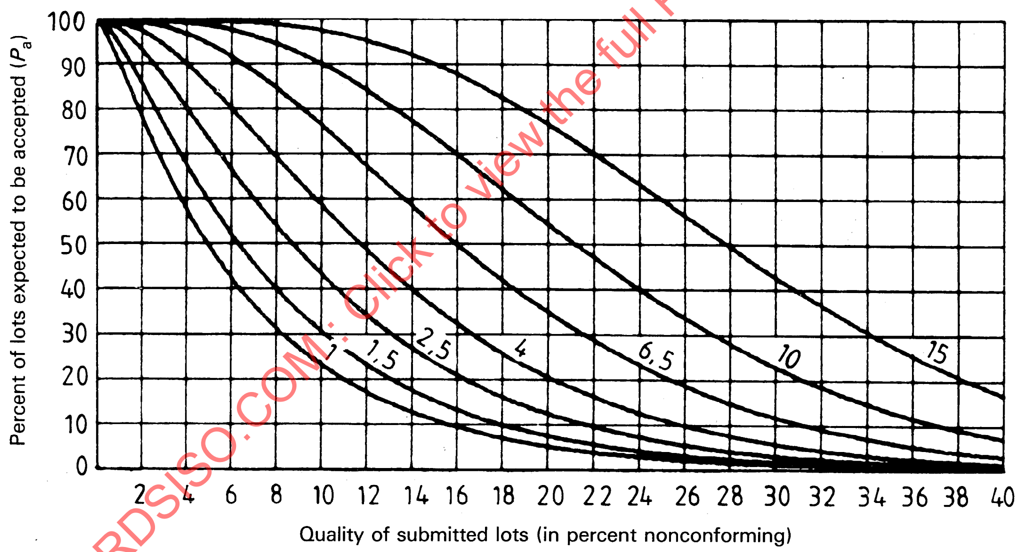


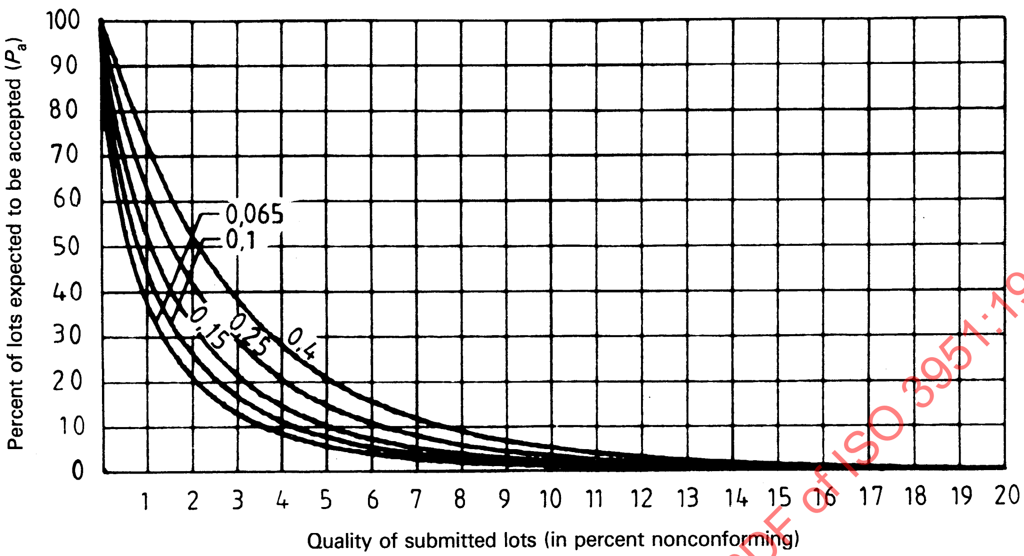
Table V-F-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0			0,01	0,03	0,05	0,11	0,19	0,35	0,69	1,35	2,62	4,69	99,0
95,0			0,07	0,12	0,21	0,36	0,57	0,94	1,65	2,83	4,84	7,81	95,0
90,0			0,15	0,25	0,40	0,66	0,98	1,52	2,50	4,04	6,52	10,01	90,0
75,0			0,49	0,74	1,08	1,61	2,21	3,15	4,70	6,93	10,25	14,60	75,0
50,0			1,53	2,08	2,79	3,77	4,82	6,33	8,62	11,69	15,91	21,09	50,0
25,0			3,93	4,95	6,16	7,72	9,29	11,41	14,45	18,25	23,20	28,96	25,0
10,0			7,95	9,44	11,15	13,23	15,23	17,84	21,40	25,66	30,99	36,98	10,0
5,0			11,40	13,17	15,13	17,47	19,68	22,49	26,27	30,68	36,09	42,06	5,0
1,0			20,10	22,24	24,53	27,19	29,58	32,59	36,50	40,92	46,18	51,82	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

G

Table V-G — Sample size code letter G

Table V-G — Operating characteristic curves for single samping plans



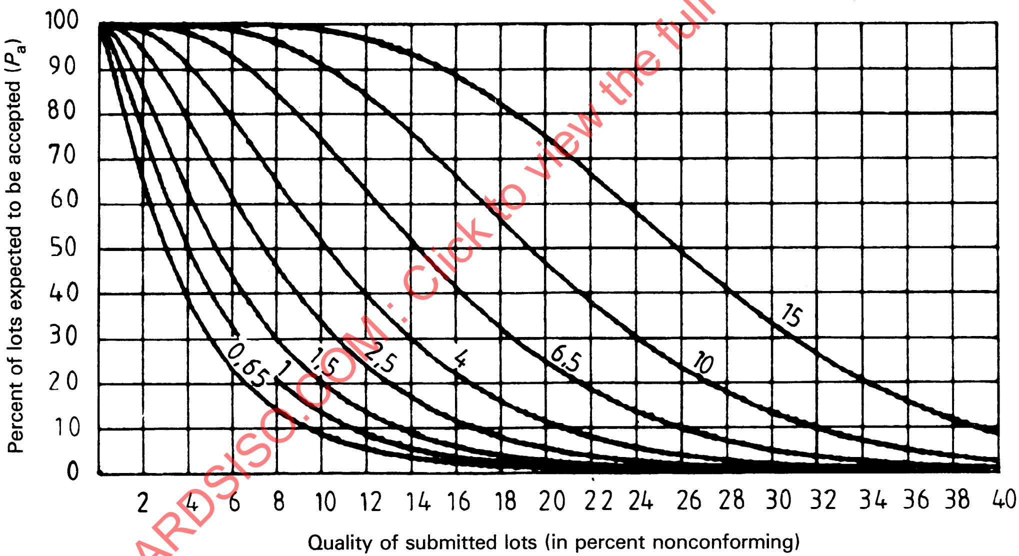
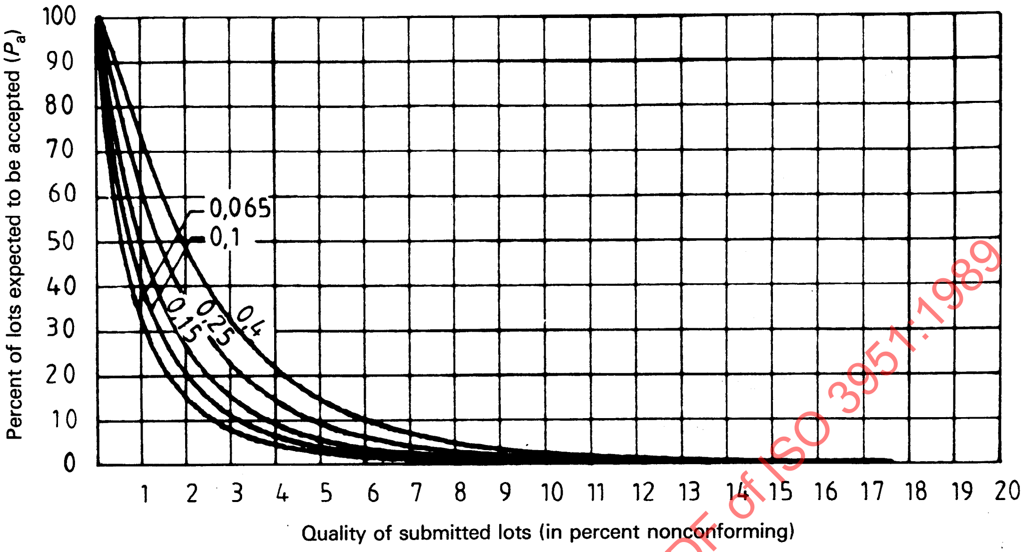


Table V-G-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,01	0,01	0,02	0,04	0,08	0,17	0,28	0,49	0,96	1,71	3,23	5,58	99,0
95,0	0,03	0,06	0,09	0,15	0,25	0,45	0,68	1,09	1,91	3,09	5,30	8,41	95,0
90,0	0,07	0,11	0,17	0,26	0,43	0,72	1,06	1,61	2,67	4,14	6,76	10,30	90,0
75,0	0,22	0,32	0,45	0,65	0,98	1,50	2,07	2,94	4,49	6,50	9,83	14,09	75,0
50,0	0,67	0,90	1,17	1,57	2,20	3,09	3,99	5,32	7,51	10,15	14,27	19,25	50,0
25,0	1,73	2,18	2,67	3,38	4,41	5,77	7,09	8,92	11,77	15,02	19,84	25,38	25,0
10,0	3,58	4,31	5,07	6,13	7,58	9,41	11,12	13,38	16,77	20,48	25,76	31,63	10,0
5,0	5,27	6,19	7,13	8,40	10,11	12,22	14,13	16,63	20,28	24,20	29,67	35,63	5,0
1,0	9,91	11,18	12,45	14,11	16,24	18,76	21,00	23,83	27,82	31,97	37,57	43,50	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Table V-H — Sample size code letter H

Chart V-H — Operating characteristic curves for single sampling plans



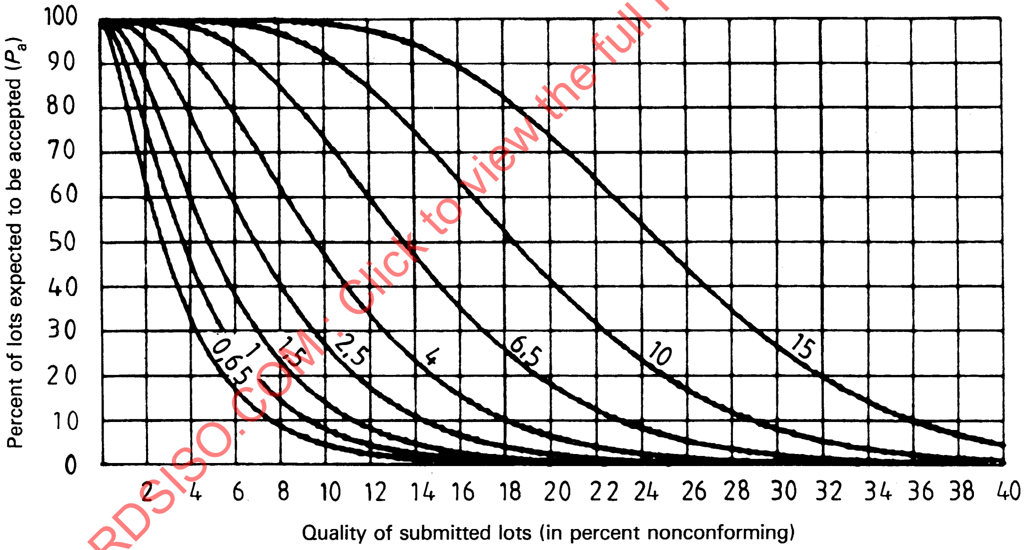
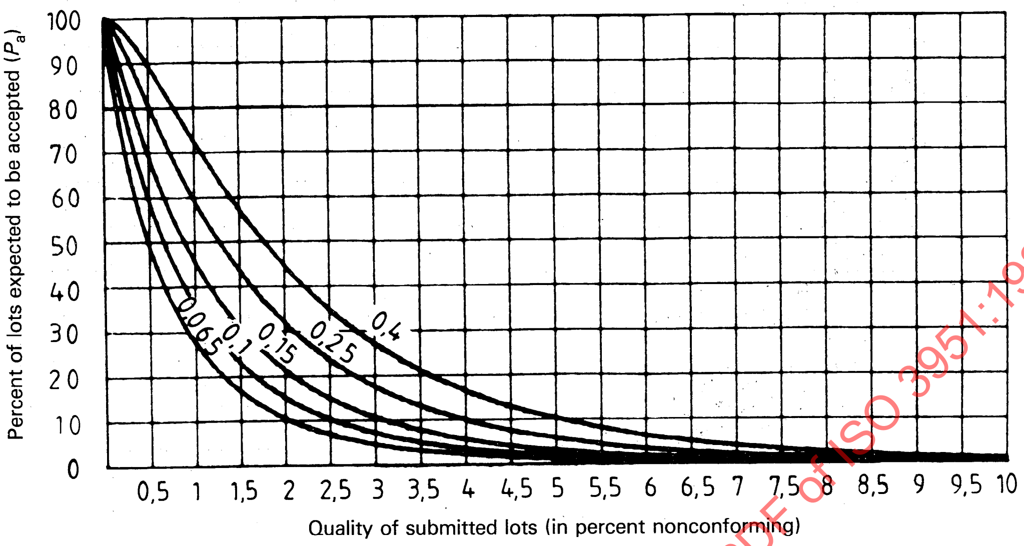


Table V-H-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0		0,02	0,04	0,06	0,11	0,21	0,37	0,61	1,15	2,04	3,73	6,25	99,0
95,0	0,04	0,07	0,11	0,17	0,29	0,49	0,79	1,21	2,07	3,39	5,69	8,88	95,0
90,0	0,08	0,12	0,19	0,29	0,45	0,75	1,15	1,69	2,77	4,35	7,01	10,58	90,0
75,0	0,21	0,31	0,44	0,63	0,93	1,42	2,06	2,85	4,35	6,43	9,71	13,89	75,0
50,0	0,56	0,76	1,01	1,38	1,90	2,69	3,66	4,81	6,85	9,51	13,49	18,31	50,0
25,0	1,31	1,68	2,13	2,74	3,56	4,75	6,12	7,67	10,29	13,52	18,13	23,47	25,0
10,0	2,58	3,16	3,85	4,73	5,88	7,46	9,23	11,14	14,25	17,94	23,01	28,70	10,0
5,0	3,71	4,44	5,29	6,36	7,71	9,54	11,53	13,65	17,03	20,95	26,24	32,05	5,0
1,0	6,81	7,85	9,22	10,44	17,17	14,42	16,79	19,24	23,02	27,26	32,79	38,70	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Table V-I — Sample size code letter I

Chart V-I — Operating characteristic curves for single sampling plans



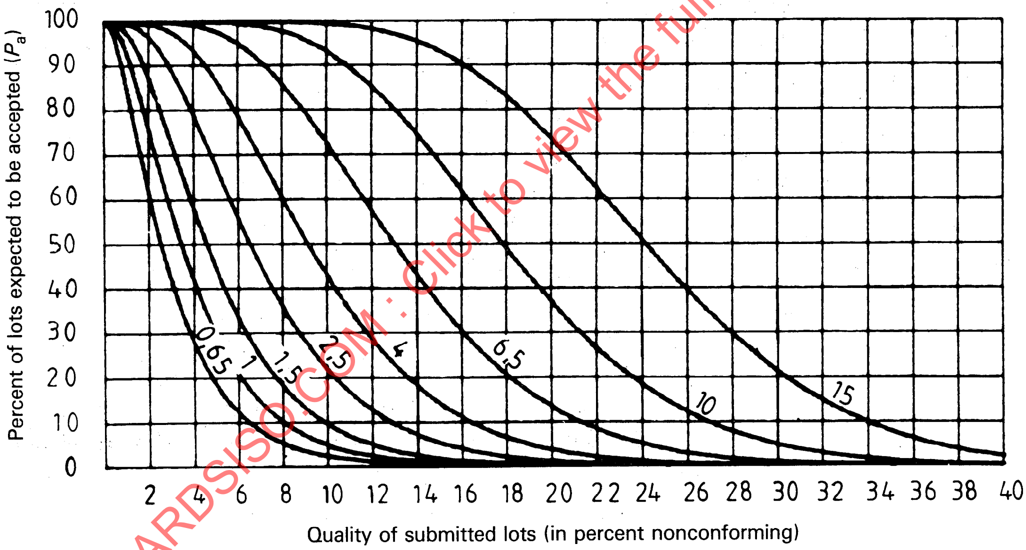
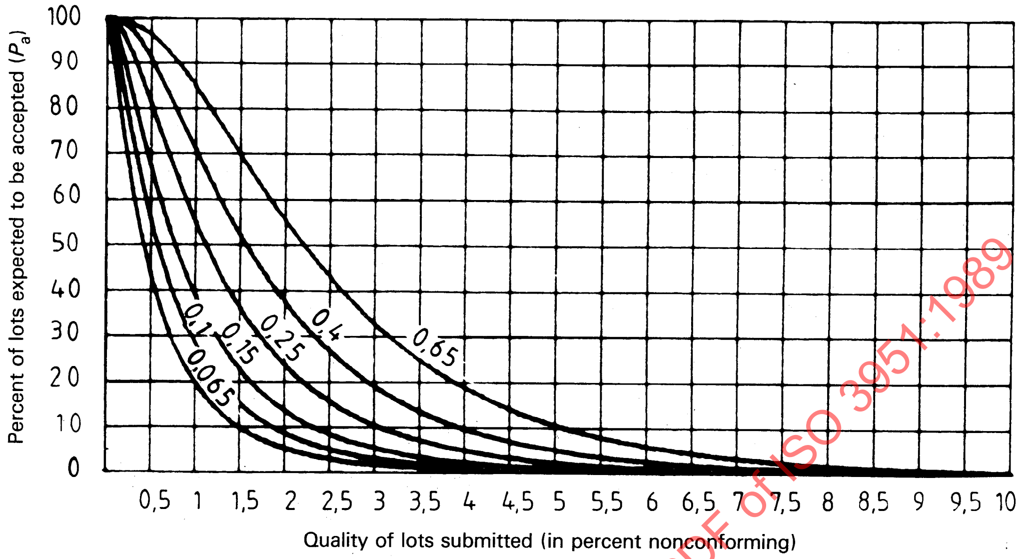


Table V-I-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,02	0,03	0,04	0,08	0,14	0,26	0,44	0,70	1,33	2,31	4,12	6,79	99,0
95,0	0,05	0,08	0,12	0,20	0,32	0,56	0,85	1,28	2,23	3,61	5,98	9,27	95,0
90,0	0,09	0,13	0,19	0,32	0,48	0,80	1,19	1,73	2,89	4,51	7,21	10,83	90,0
75,0	0,21	0,30	0,41	0,64	0,91	1,42	2,00	2,78	4,34	6,39	9,65	13,82	75,0
50,0	0,50	0,68	0,89	1,28	1,73	2,53	3,39	4,47	6,54	9,12	13,00	17,74	50,0
25,0	1,09	1,42	1,77	2,41	3,09	4,25	5,43	6,87	9,47	12,57	17,03	22,27	25,0
10,0	2,05	2,55	3,08	3,99	4,93	6,46	7,97	9,73	12,81	16,34	21,24	26,82	10,0
5,0	2,89	3,52	4,17	5,26	6,37	8,14	9,83	11,78	15,14	18,89	24,01	29,75	5,0
1,0	5,17	6,06	6,97	8,43	9,85	12,04	14,08	16,36	20,14	24,24	29,66	35,56	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Table V-J — Sample size code letter J

Chart V-J — Operating characteristic curves for single sampling plans



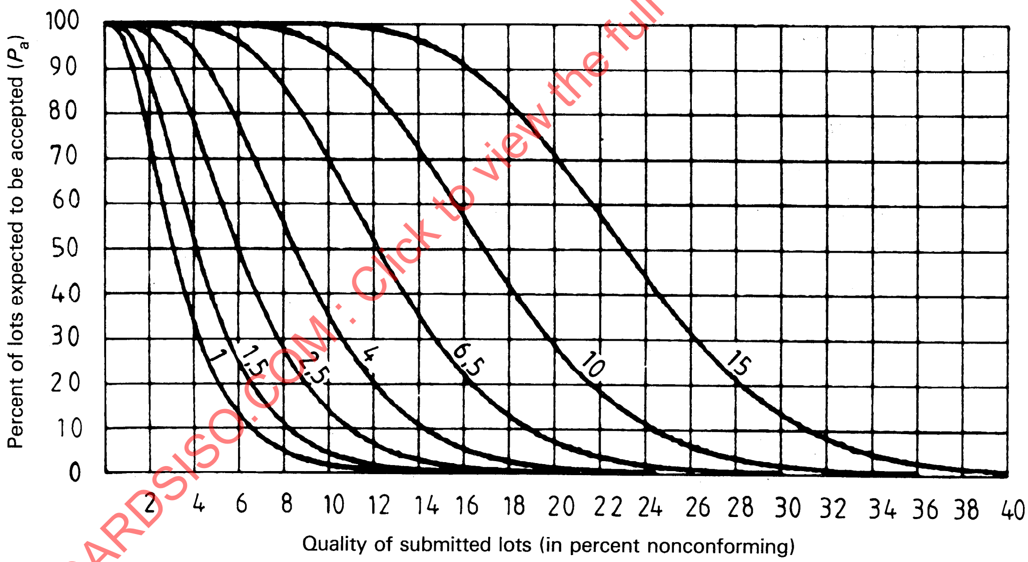


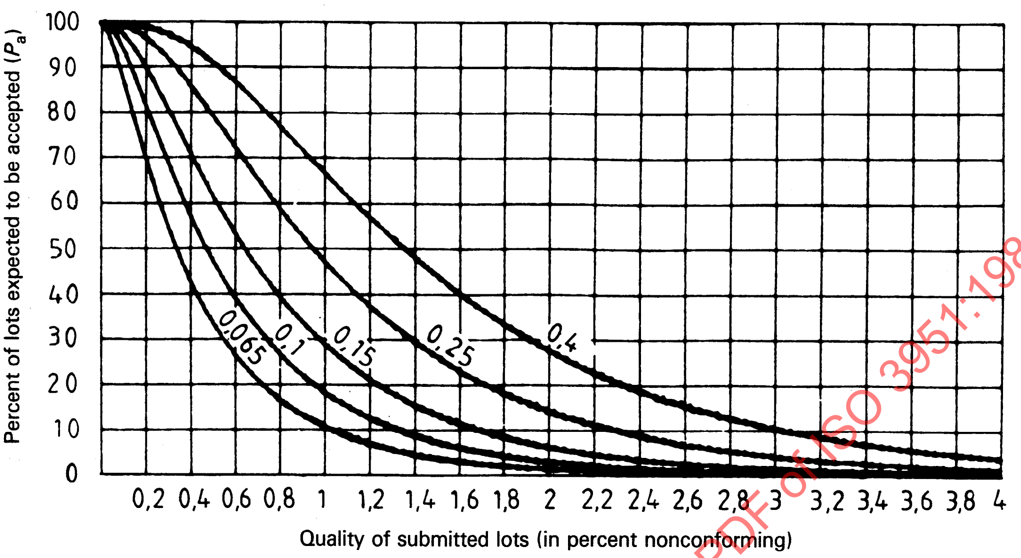
Table V-J-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,02	0,04	0,06	0,11	0,18	0,32	0,54	0,84	1,54	2,61	4,54	7,44	99,0
95,0	0,06	0,09	0,13	0,23	0,36	0,60	0,94	1,40	2,38	3,80	6,21	9,65	95,0
90,0	0,10	0,15	0,20	0,33	0,51	0,82	1,25	1,80	2,97	4,59	7,28	11,01	90,0
75,0	0,20	0,29	0,39	0,60	0,88	1,34	1,94	2,69	4,19	6,18	9,34	13,56	75,0
50,0	0,43	0,59	0,76	1,10	1,54	2,21	3,05	4,05	5,98	8,41	12,10	16,82	50,0
25,0	0,86	1,12	1,39	1,92	2,55	3,50	4,62	5,91	8,29	11,16	15,35	20,53	25,0
10,0	1,50	1,90	2,29	3,02	3,87	5,10	6,50	8,07	10,85	14,11	18,71	24,23	10,0
5,0	2,05	2,54	3,01	3,89	4,89	6,29	7,87	9,60	12,62	16,09	20,90	26,60	5,0
1,0	3,50	4,12	4,84	6,02	7,30	9,04	10,95	12,98	16,42	20,24	25,39	31,32	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

K

Table V-K — Sample size code letter K

Chart V-K — Operating characteristic curves for single sampling plans



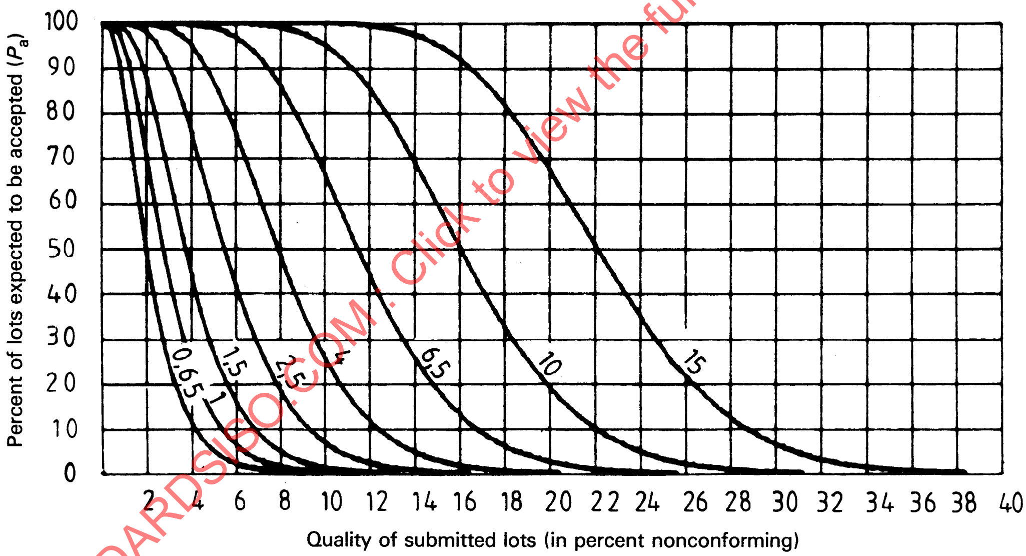
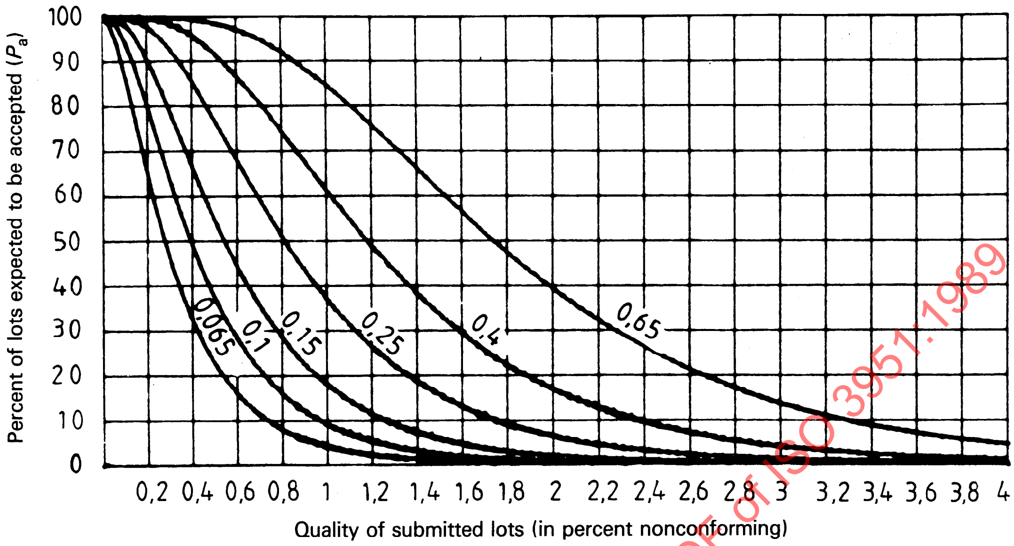


Table V-K-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (nominal inspection)											P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,03	0,05	0,08	0,14	0,23	0,38	0,64	0,98	1,74	2,96	5,03	8,05	99,0
95,0	0,07	0,10	0,15	0,26	0,40	0,64	1,02	1,49	2,51	4,04	6,52	10,00	95,0
90,0	0,10	0,15	0,21	0,35	0,54	0,83	1,29	1,84	3,01	4,73	7,44	11,16	90,0
75,0	0,19	0,27	0,37	0,58	0,85	1,26	1,87	2,59	4,04	6,08	9,19	13,31	75,0
50,0	0,35	0,49	0,65	0,98	1,37	1,94	2,76	3,68	5,48	7,90	11,45	16,00	50,0
25,0	0,64	0,86	1,10	1,58	2,12	2,89	3,95	5,11	7,27	10,09	14,06	19,01	25,0
10,0	1,06	1,36	1,70	2,35	3,07	4,03	5,33	6,72	9,23	12,39	16,72	21,98	10,0
5,0	1,40	1,77	2,18	2,94	3,77	4,87	6,32	7,84	10,55	13,92	18,45	23,88	5,0
1,0	2,29	2,81	3,36	4,36	5,42	6,78	8,52	10,30	13,39	17,10	21,97	27,65	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Table V-L — Sample size code letter L

Chart V-L — Operating characteristic curves for single sampling plans



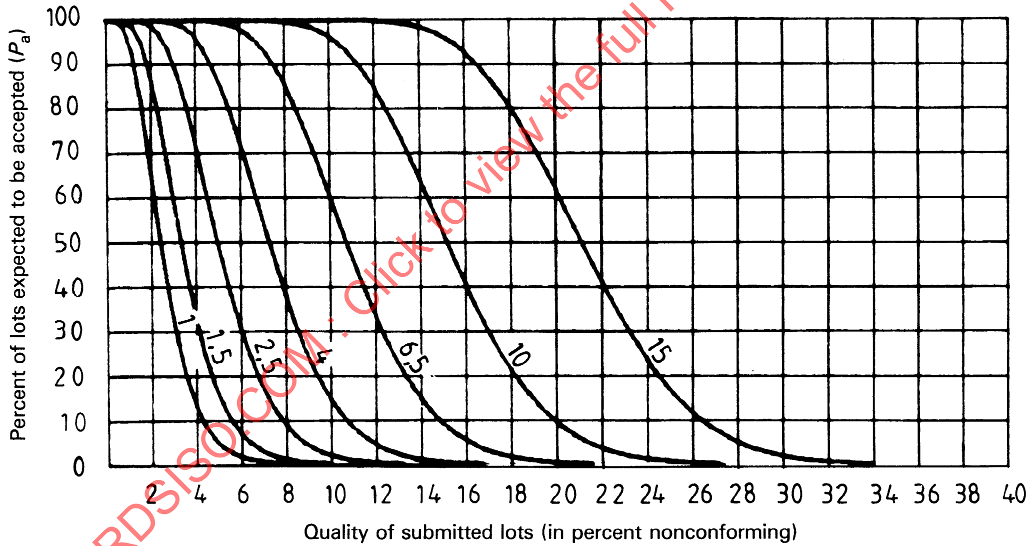


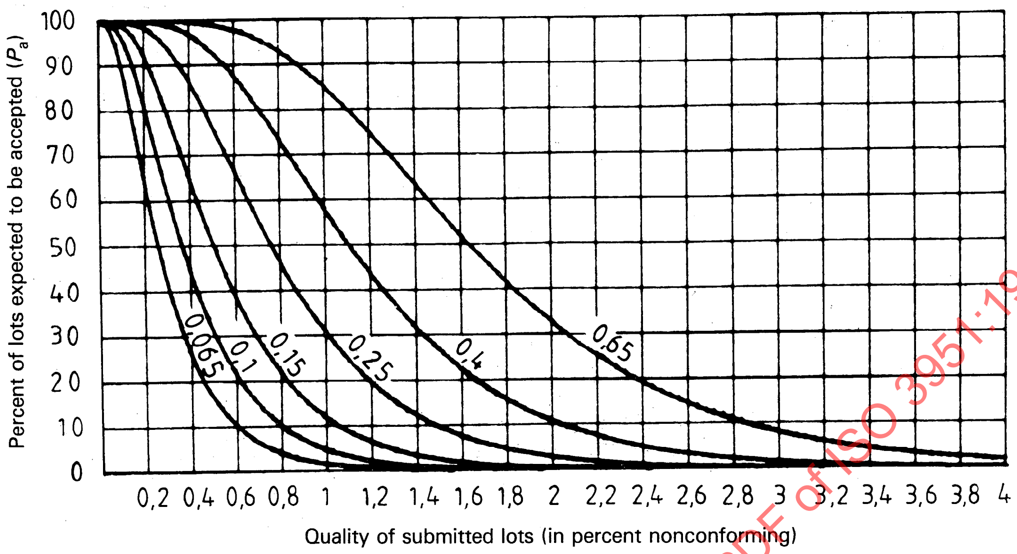
Table V-L-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,04	0,06	0,09	0,16	0,27	0,46	0,73	1,12	1,95	3,24	5,52	8,67	99,0
95,0	0,07	0,11	0,17	0,27	0,43	0,70	1,06	1,58	2,62	4,18	6,81	10,34	95,0
90,0	0,10	0,15	0,22	0,35	0,55	0,86	1,29	1,88	3,05	4,76	7,59	11,31	90,0
75,0	0,17	0,24	0,35	0,53	0,80	1,21	1,76	2,49	3,89	5,86	9,02	13,08	75,0
50,0	0,29	0,40	0,56	0,82	1,19	1,74	2,43	3,34	5,02	7,29	10,84	15,24	50,0
25,0	0,48	0,65	0,87	1,23	1,73	2,44	3,30	4,41	6,38	8,97	12,89	17,62	25,0
10,0	0,74	0,97	1,27	1,74	2,37	3,24	4,28	5,58	7,82	10,70	14,94	19,95	10,0
5,0	0,95	1,23	1,57	2,12	2,84	3,82	4,97	6,38	8,79	11,84	16,26	21,42	5,0
1,0	1,47	1,84	2,30	3,01	3,91	5,10	6,47	8,11	10,84	14,19	18,93	24,34	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

M

Table V-M — Sample size code letter M

Chart V-M — Operating characteristic curves for single sampling plans



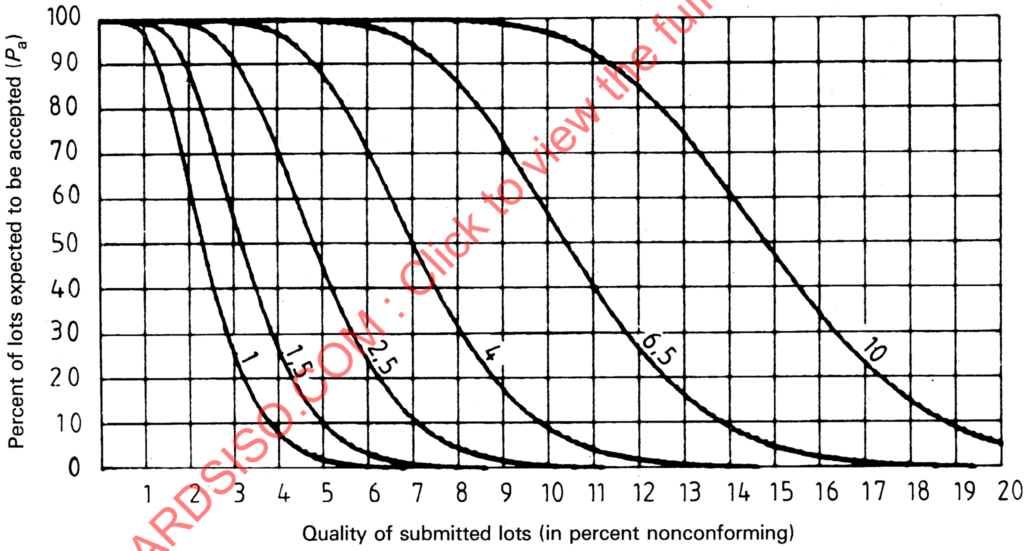
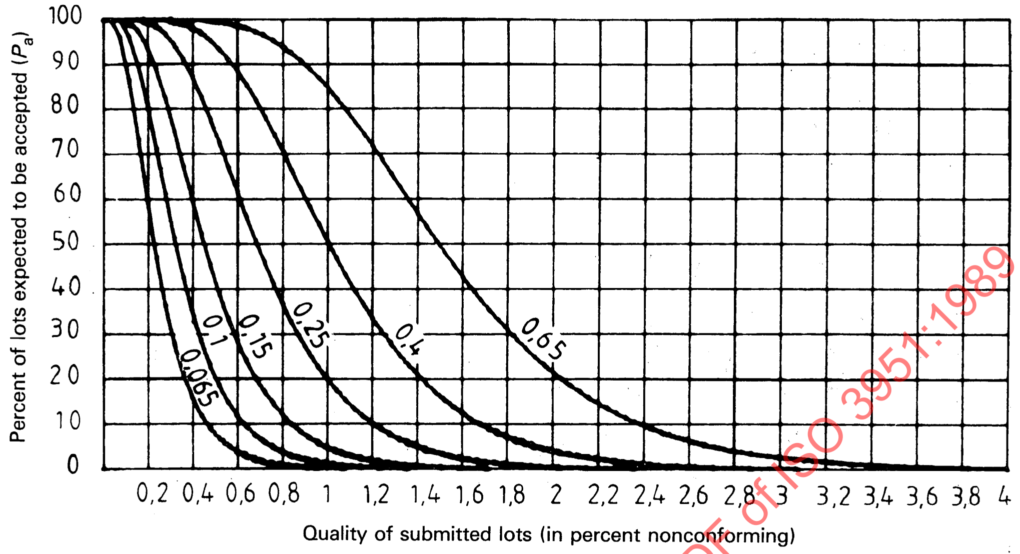


Table V-M-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)												P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,05	0,07	0,11	0,19	0,31	0,52	0,81	1,24	2,12	3,46	5,81	9,03	99,0
95,0	0,08	0,12	0,18	0,29	0,47	0,74	1,12	1,66	2,73	4,31	6,97	10,51	95,0
90,0	0,10	0,15	0,22	0,37	0,57	0,89	1,33	1,93	3,11	4,82	7,65	11,36	90,0
75,0	0,16	0,23	0,33	0,53	0,79	1,20	1,74	2,46	3,84	5,78	8,90	12,90	75,0
50,0	0,26	0,37	0,51	0,77	1,12	1,64	2,31	3,18	4,80	7,00	10,45	14,75	50,0
25,0	0,41	0,56	0,75	1,11	1,56	2,22	3,02	4,07	5,93	8,41	12,18	16,78	25,0
10,0	0,61	0,80	1,05	1,50	2,06	2,86	3,81	5,01	7,11	9,84	13,89	18,73	10,0
5,0	0,75	0,99	1,28	1,79	2,43	3,31	4,35	5,65	7,89	10,77	14,99	19,97	5,0
1,0	1,12	1,43	1,81	2,46	3,24	4,30	5,52	7,02	9,53	12,68	17,19	22,41	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Table V-N — Sample size code letter N

Chart V-N — Operating characteristic curves for single sampling plans



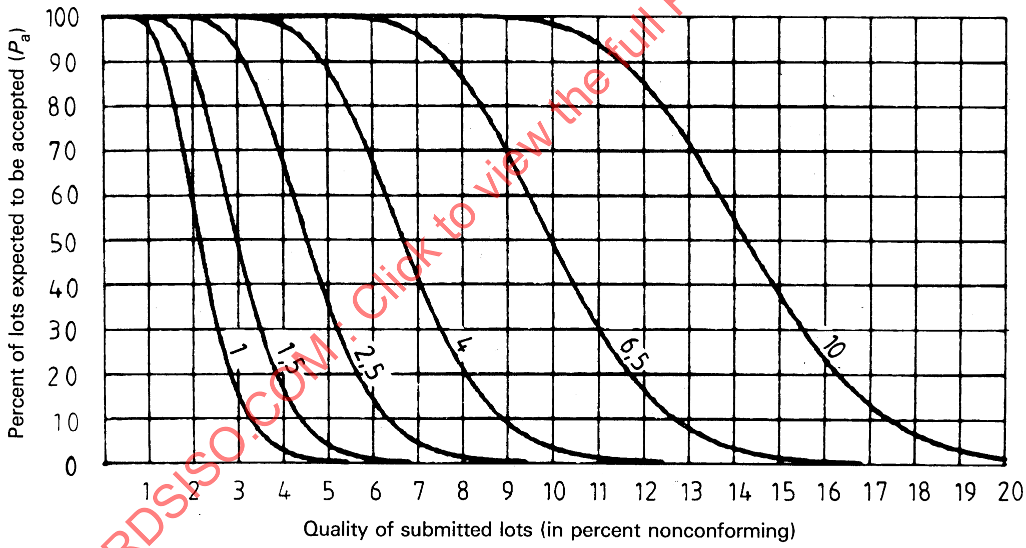


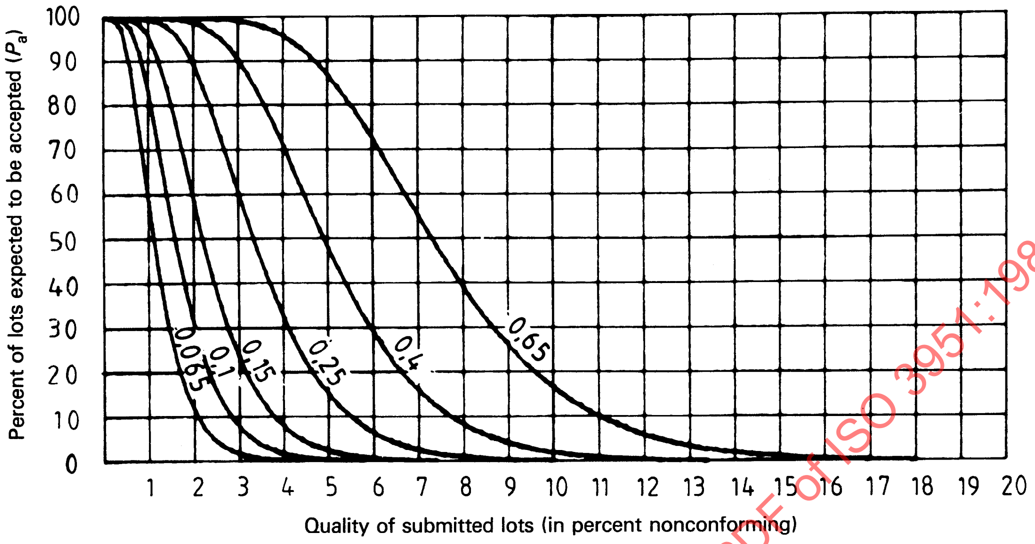
Table V-N-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,05	0,08	0,13	0,22	0,35	0,57	0,91	1,36	2,29	3,69	6,10	9,55	99,0
95,0	0,08	0,13	0,19	0,31	0,48	0,77	1,18	1,73	2,82	4,41	7,07	10,80	95,0
90,0	0,11	0,16	0,24	0,37	0,57	0,89	1,35	1,96	3,13	4,84	7,64	11,51	90,0
75,0	0,15	0,22	0,33	0,50	0,75	1,14	1,69	2,39	3,73	5,62	8,66	12,77	75,0
50,0	0,23	0,32	0,46	0,69	1,00	1,48	2,14	2,96	4,49	6,59	9,90	14,28	50,0
25,0	0,34	0,46	0,64	0,93	1,32	1,90	2,68	3,64	5,36	7,69	11,26	15,89	25,0
10,0	0,46	0,62	0,85	1,21	1,68	2,36	3,26	4,34	6,26	8,78	12,58	17,44	10,0
5,0	0,56	0,74	1,00	1,40	1,93	2,68	3,65	4,81	6,84	9,48	13,43	18,40	5,0
1,0	0,79	1,03	1,35	1,84	2,48	3,36	4,48	5,79	8,04	10,91	15,11	20,31	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

P

Table V-P — Sample size code letter P

Chart V-P — Operating characteristic values for single sampling plans



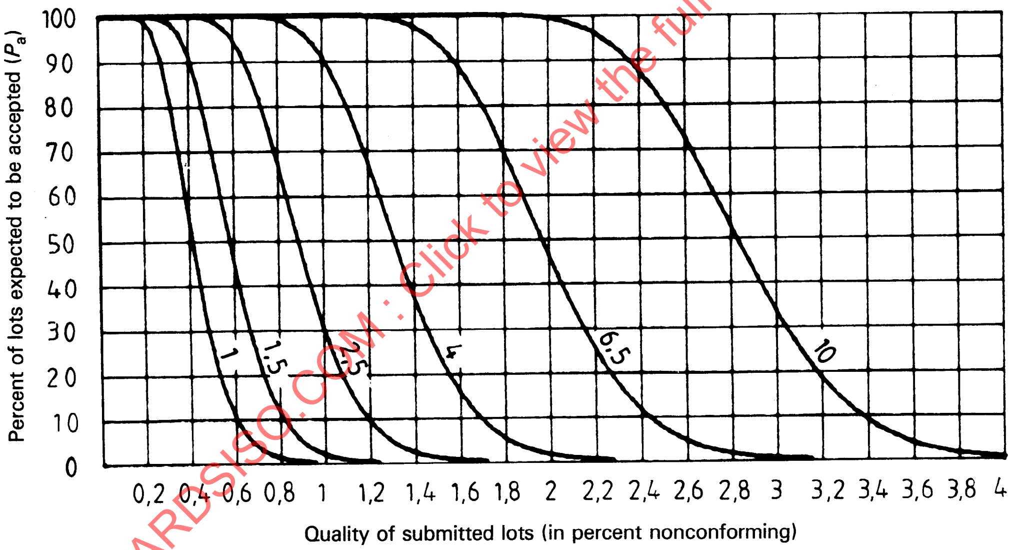


Table V-P-1 — Tabulated values for operating characteristic curves for single sampling plans

P_a	Acceptable quality level (normal inspection)											P_a
		0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
99,0	0,064	0,101	0,150	0,254	0,405	0,65	0,99	1,52	2,52	4,00	6,52	10,10	99,0
95,0	0,094	0,143	0,210	0,344	0,534	0,84	1,25	1,86	3,00	4,66	7,40	11,22	95,0
90,0	0,114	0,173	0,249	0,402	0,617	0,95	1,40	2,07	3,29	5,04	7,91	11,85	90,0
75,0	0,157	0,233	0,330	0,519	0,779	1,18	1,70	2,46	3,82	5,73	8,80	12,96	75,0
50,0	0,222	0,321	0,445	0,683	1,000	1,48	2,08	2,96	4,48	6,58	9,88	14,27	50,0
25,0	0,309	0,437	0,594	0,889	1,272	1,83	2,54	3,53	5,23	7,52	11,05	15,66	25,0
10,0	0,411	0,571	0,763	1,116	1,567	2,22	3,02	4,12	5,98	8,45	12,19	16,98	10,0
5,0	0,485	0,666	0,882	1,275	1,770	2,47	3,33	4,51	5,47	9,04	12,90	17,80	5,0
1,0	0,657	0,884	1,150	1,621	2,206	3,02	4,00	5,32	7,46	10,23	14,31	19,41	1,0
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0		
	Acceptable quality level (tightened inspection)												

Table VI-A — Correspondence between tightened and normal inspection plans

Sample size code letter \ AQL	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B[1]								C-1,5	B-2,5	B-4,0	B-6,5
C						D-0,65	C-1,0	C-1,5	C-2,5	C-4,0	C-6,5
D					E-0,40	D-0,65	D-1,0	D-1,5	D-2,5	D-4,0	D-6,5
E			F-0,15	E-0,25	E-0,40	E-0,65	E-1,0	E-1,5	E-2,5	E-4,0	E-6,5
F		G-0,10	F-0,15	F-0,25	F-0,40	F-0,65	F-1,0	F-1,5	F-2,5	F-4,0	F-6,5
G	G-0,065	G-0,10	G-0,15	G-0,25	G-0,40	G-0,65	G-1,0	G-1,5	G-2,5	G-4,0	G-6,5
H	H-0,065	H-0,10	H-0,15	H-0,25	H-0,40	H-0,65	H-1,0	H-1,5	H-2,5	H-4,0	H-6,5
I	I-0,065	I-0,10	I-0,15	I-0,25	I-0,40	I-0,65	I-1,0	I-1,5	I-2,5	I-4,0	I-6,5
J	J-0,065	J-0,10	J-0,15	J-0,25	J-0,40	J-0,65	J-1,0	J-1,5	J-2,5	J-4,0	J-6,5
K	K-0,065	K-0,10	K-0,15	K-0,25	K-0,40	K-0,65	K-1,0	K-1,5	K-2,5	K-4,0	K-6,5
L	L-0,065	L-0,10	L-0,15	L-0,25	L-0,40	L-0,65	L-1,0	L-1,5	L-2,5	L-4,0	L-6,5
M	M-0,065	M-0,10	M-0,15	M-0,25	M-0,40	M-0,65	M-1,0	M-1,5	M-2,5	M-4,0	M-6,5
N	N-0,065	N-0,10	N-0,15	N-0,25	N-0,40	N-0,65	N-1,0	N-1,5	N-2,5	N-4,0	N-6,5
P	P-0,065	P-0,10	P-0,15	P-0,25	P-0,40	P-0,65	P-1,0	P-1,5	P-2,5	P-4,0	P-6,5

1) *"s"* method only.

NOTE — The table is entered with the sample size code letter (at the left) and the AQL (at the top) for the plan under normal inspection. The table gives the sample size code letter and the AQL of the normal inspection plan to use for tightened inspection.

Table VI-B — Correspondence between reduced and normal inspection plans

Sample size code letter \ AQL	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B[1]								B-4,0	B-6,5	B-10,0	B-15,0
C						C-1,5	B-2,5	B-4,0	B-6,5	B-10,0	B-15,0
D					C-1,0	C-1,5	B-2,5	B-4,0	B-6,5	B-10,0	B-15,0
E			E-0,40	D-0,65	C-1,0	C-1,5	B-2,5	B-4,0	B-6,5	B-10,0	B-15,0
F		E-0,25	E-0,40	D-0,65	C-1,0	C-1,5	C-2,5	C-4,0	C-6,5	C-10,0	C-15,0
G	F-0,15	E-0,25	E-0,40	D-0,65	D-1,0	D-1,5	D-2,5	D-4,0	D-6,5	D-10,0	D-15,0
H	F-0,15	E-0,25	E-0,40	E-0,65	E-1,0	E-1,5	E-2,5	E-4,0	E-6,5	E-10,0	E-15,0
I	F-0,15	F-0,25	F-0,40	F-0,65	F-1,0	F-1,5	F-2,5	F-4,0	F-6,5	F-10,0	F-15,0
J	G-0,15	G-0,25	G-0,40	G-0,65	G-1,0	G-1,5	G-2,5	G-4,0	G-6,5	G-10,0	G-15,0
K	H-0,15	H-0,25	H-0,40	H-0,65	H-1,0	H-1,5	H-2,5	H-4,0	H-6,5	H-10,0	H-15,0
L	I-0,15	I-0,25	I-0,40	I-0,65	I-1,0	I-1,5	I-2,5	I-4,0	I-6,5	I-10,0	I-15,0
M	J-0,15	J-0,25	J-0,40	J-0,65	J-1,0	J-1,5	J-2,5	J-4,0	J-6,5	J-10,0	J-15,0
N	K-0,15	K-0,25	K-0,40	K-0,65	K-1,0	K-1,5	K-2,5	K-4,0	K-6,5	K-10,0	K-15,0
P	L-0,15	L-0,25	L-0,40	L-0,65	L-1,0	L-1,5	L-2,5	L-4,0	L-6,5	L-10,0	L-15,0

1) *"s"* method only.

NOTE — The table is entered with the sample size code letter (at the left) and the AQL (at the top) for the plan under normal inspection. The table gives the sample size code letter and the AQL of the normal inspection plan to use for reduced inspection.

Diagram A — Sample size code letters of standard sampling plans for specified qualities at 95 % and 10 % probabilities of acceptance

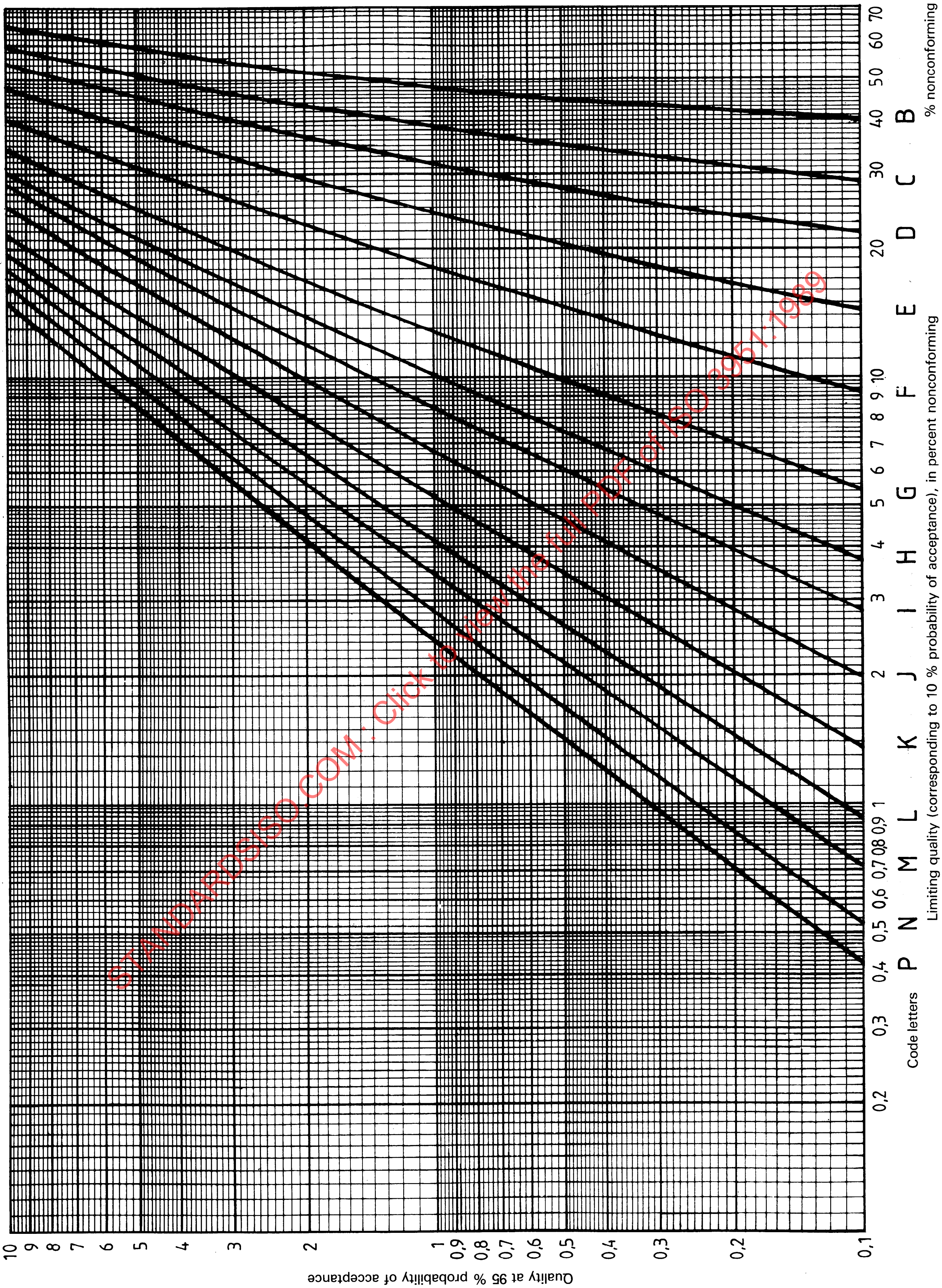
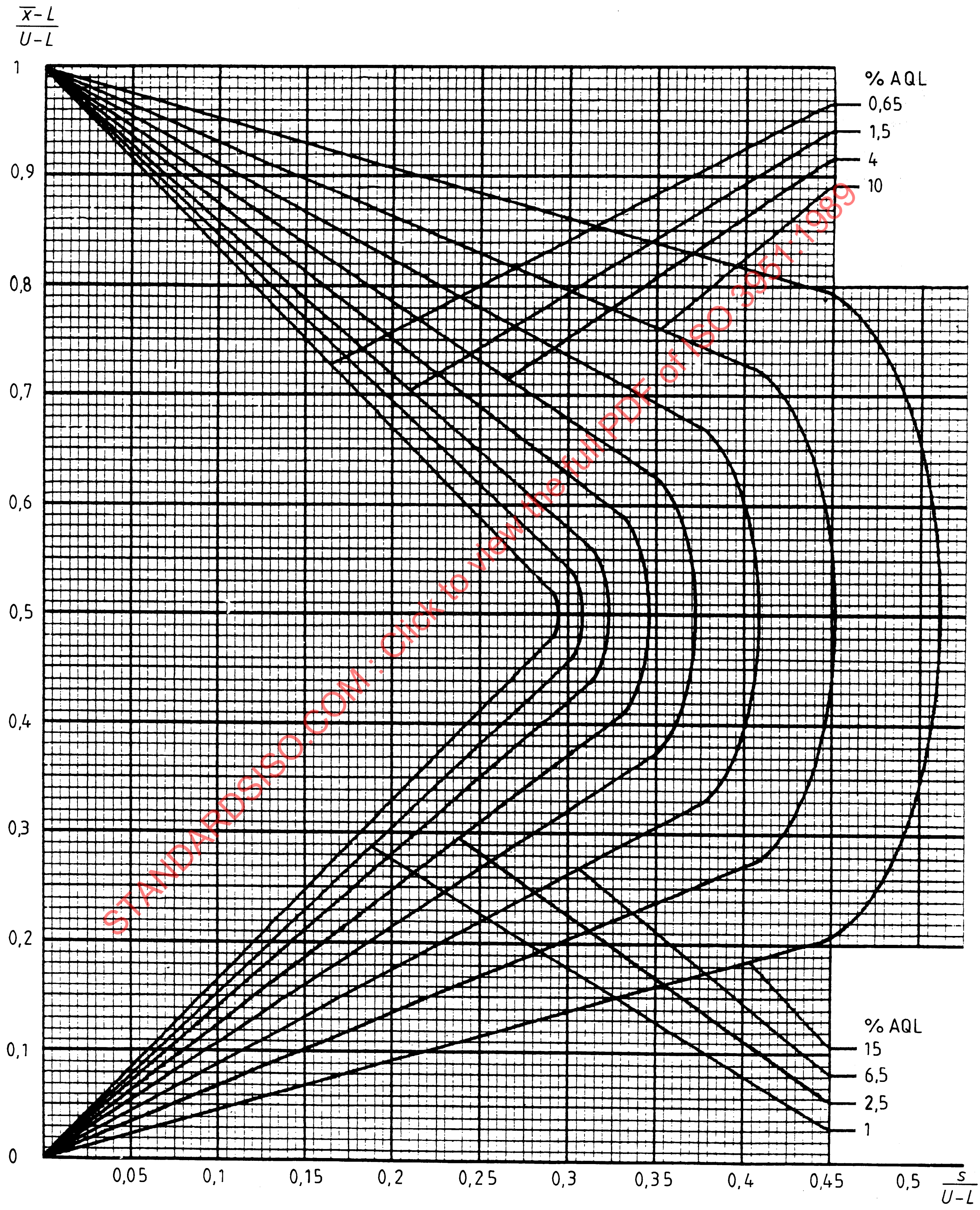


Diagram *s*-D — Acceptance curves for combined double specification limits: "*s*" method — Sample size code letter D (sample size 5)

D



E

Diagram *s*-E — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter E (sample size 7)

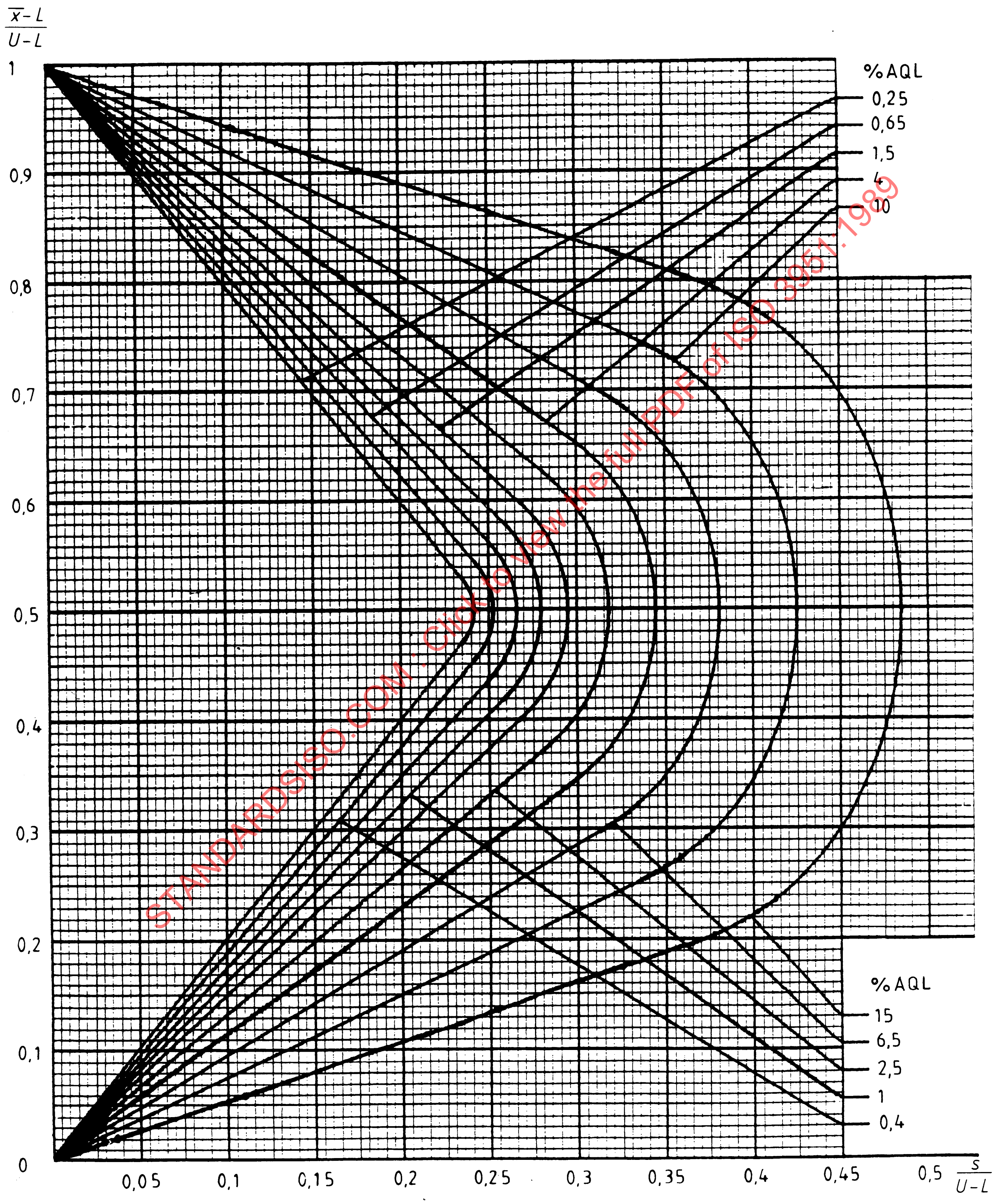
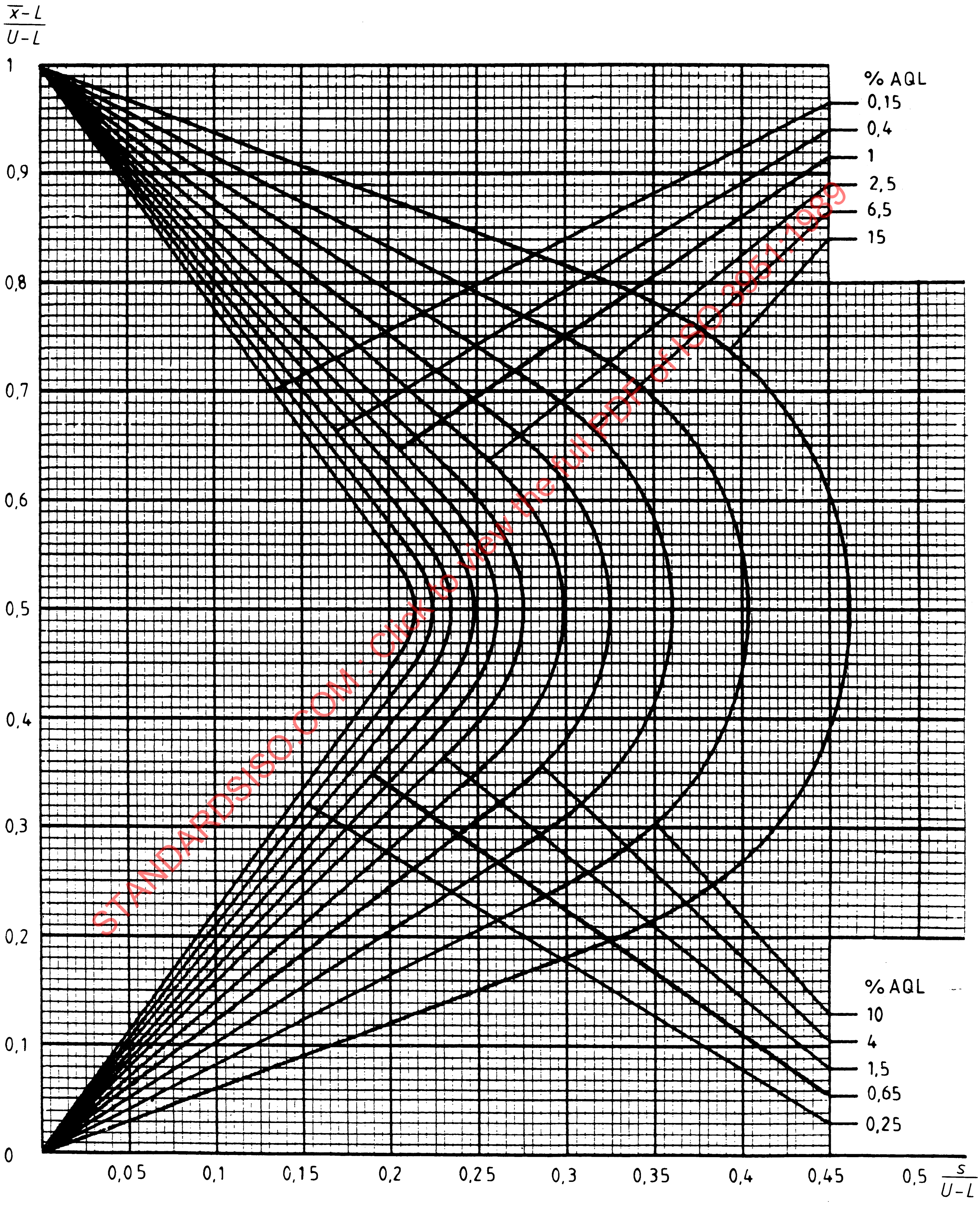


Diagram *s*-F — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter F (sample size 10)

F



G

Diagram *s*-G — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter G (sample size 15)

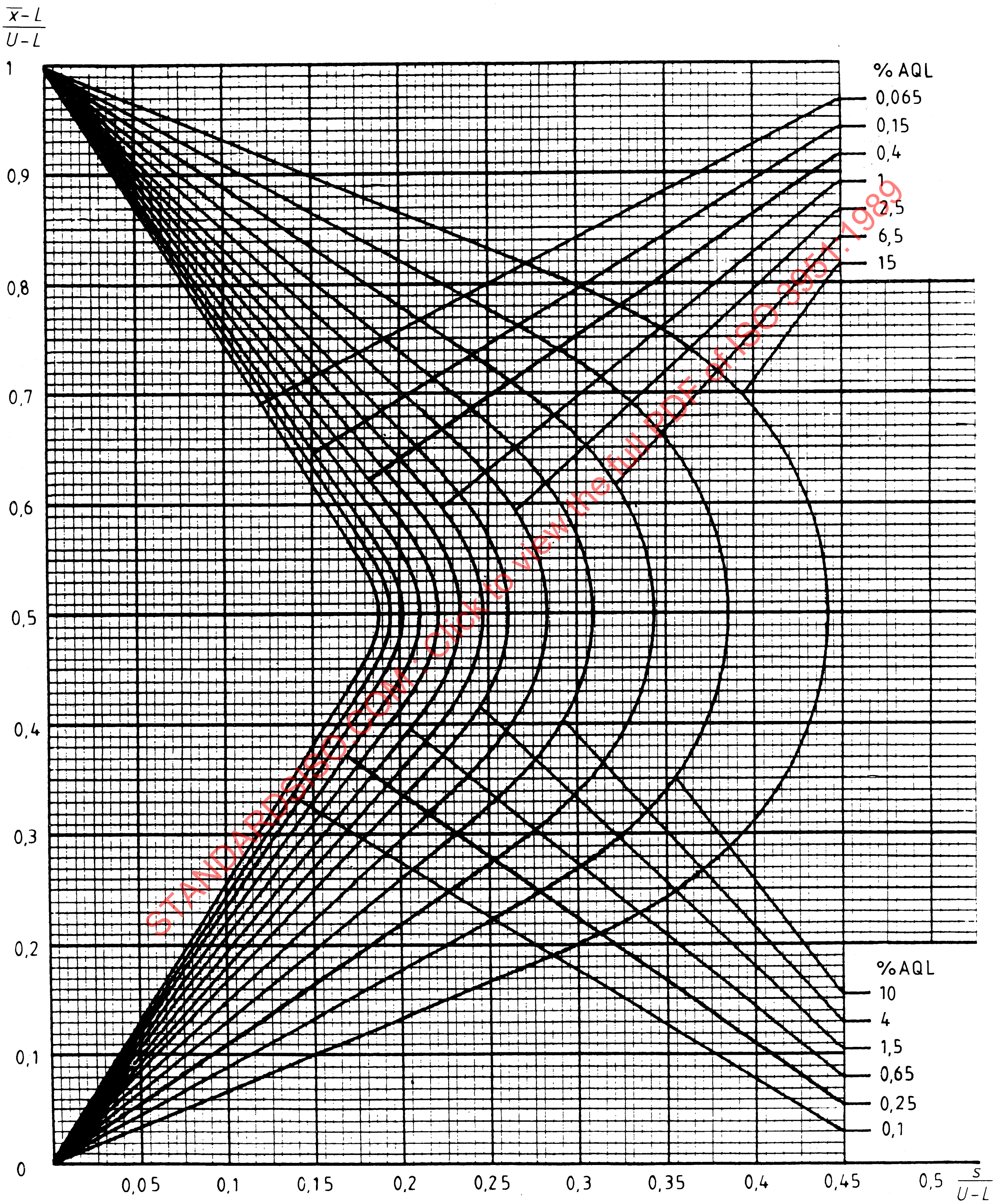
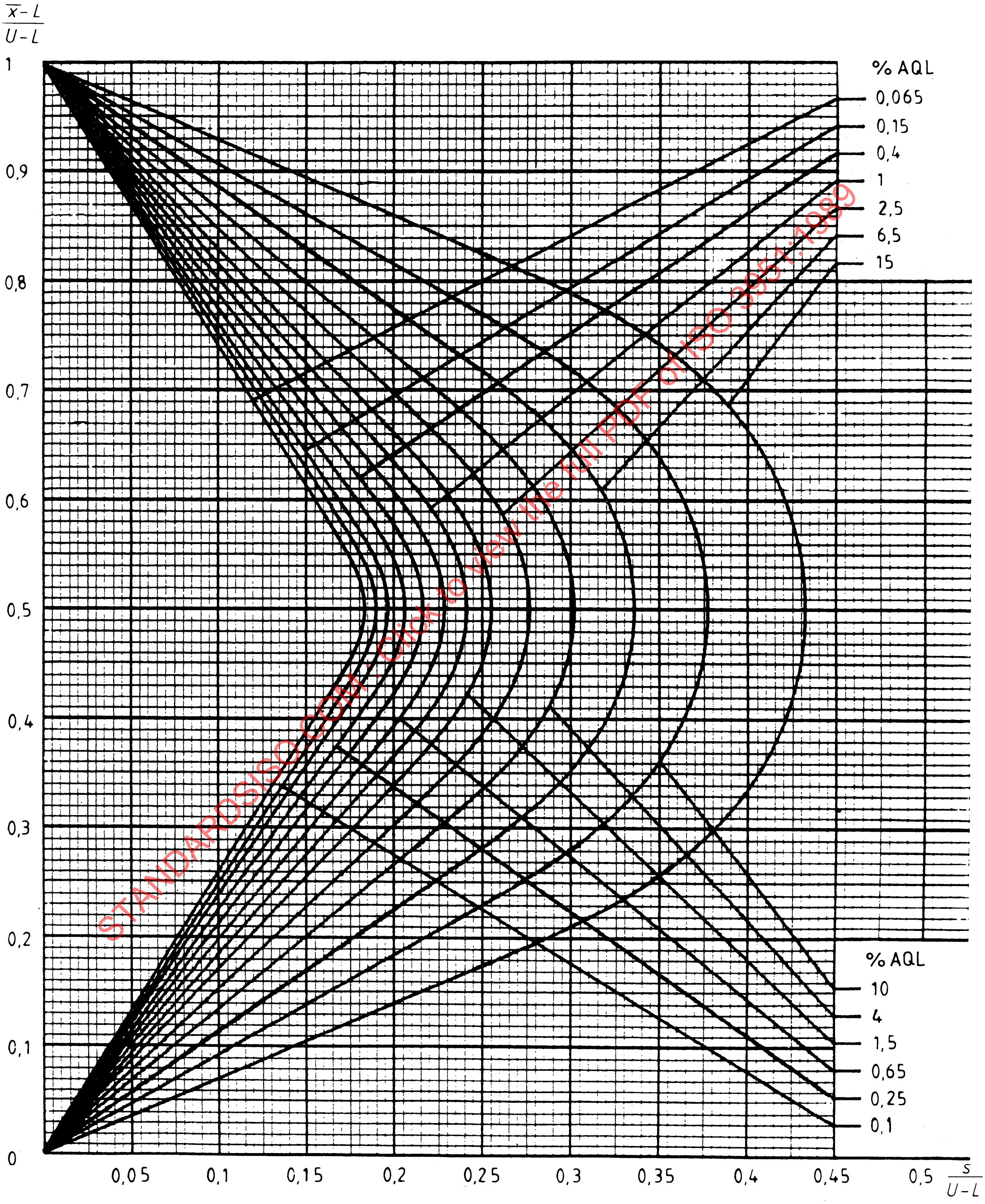


Diagram *s*-H — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter H (sample size 20)

H



I

Diagram s-I — Acceptance curves for combined double specification limits : "s" method — Sample size code letter I (sample size 25)

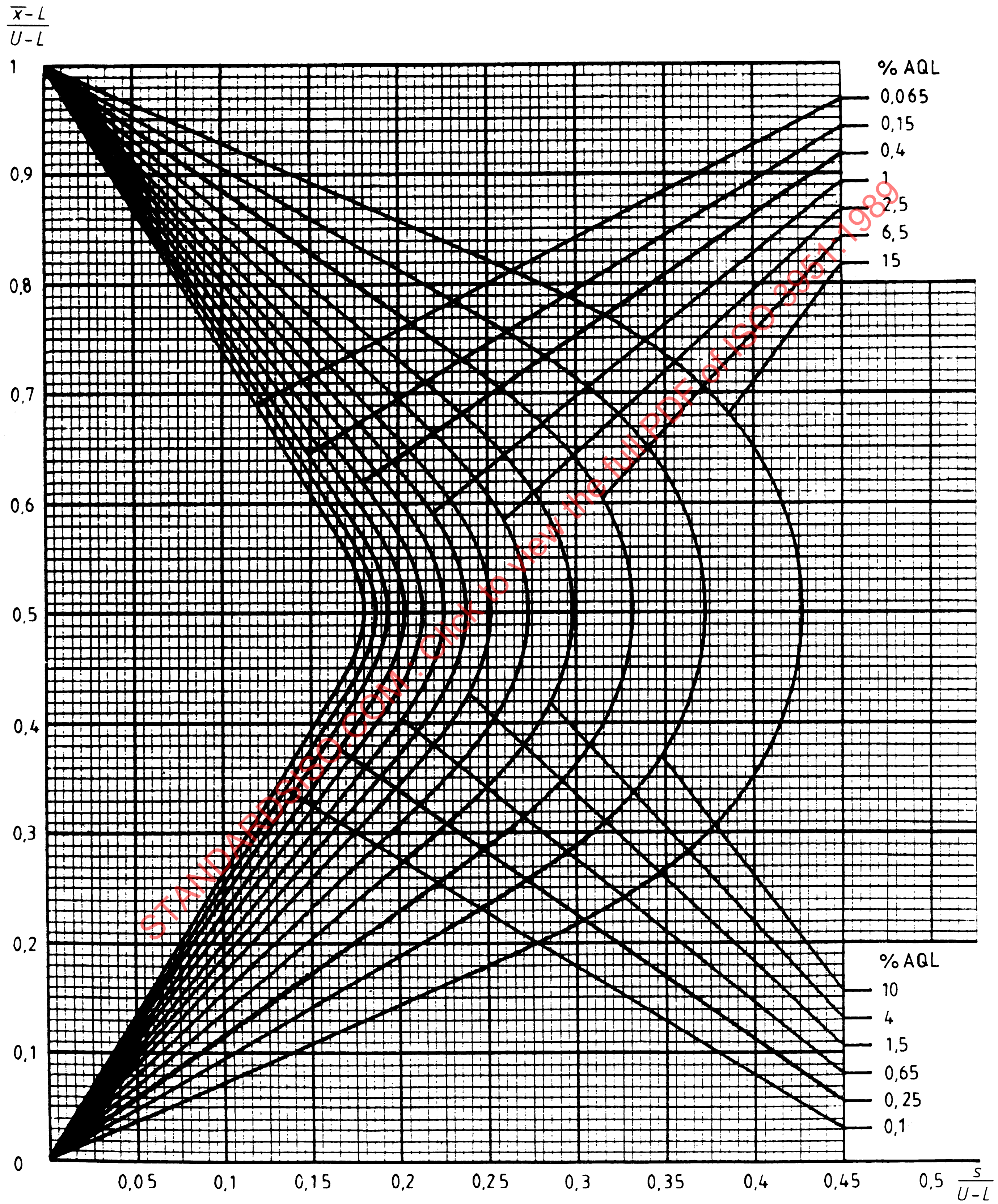
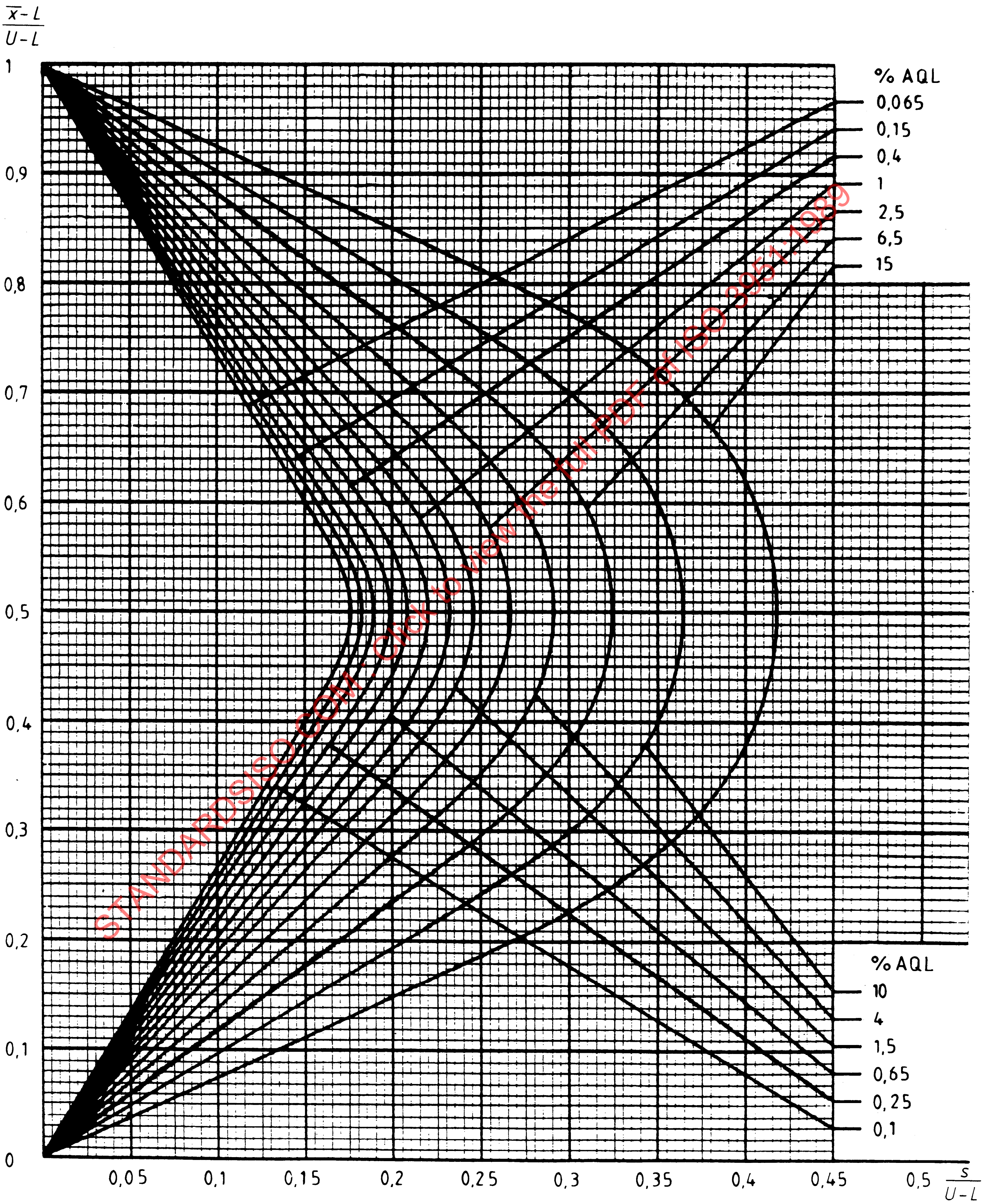


Diagram *s*-J — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter J (sample size 35)

J



K

Diagram *s*-K — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter K (sample size 50)

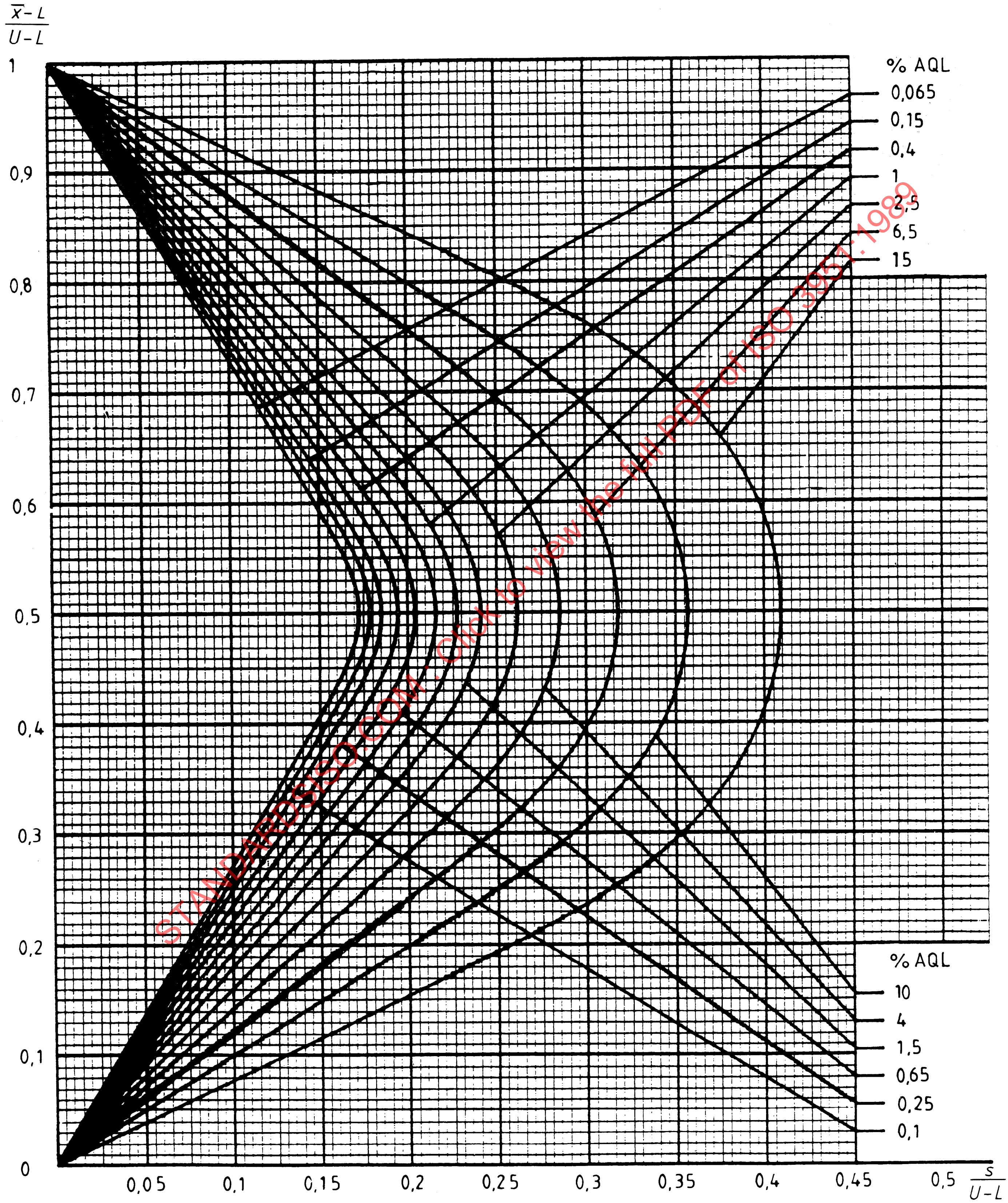
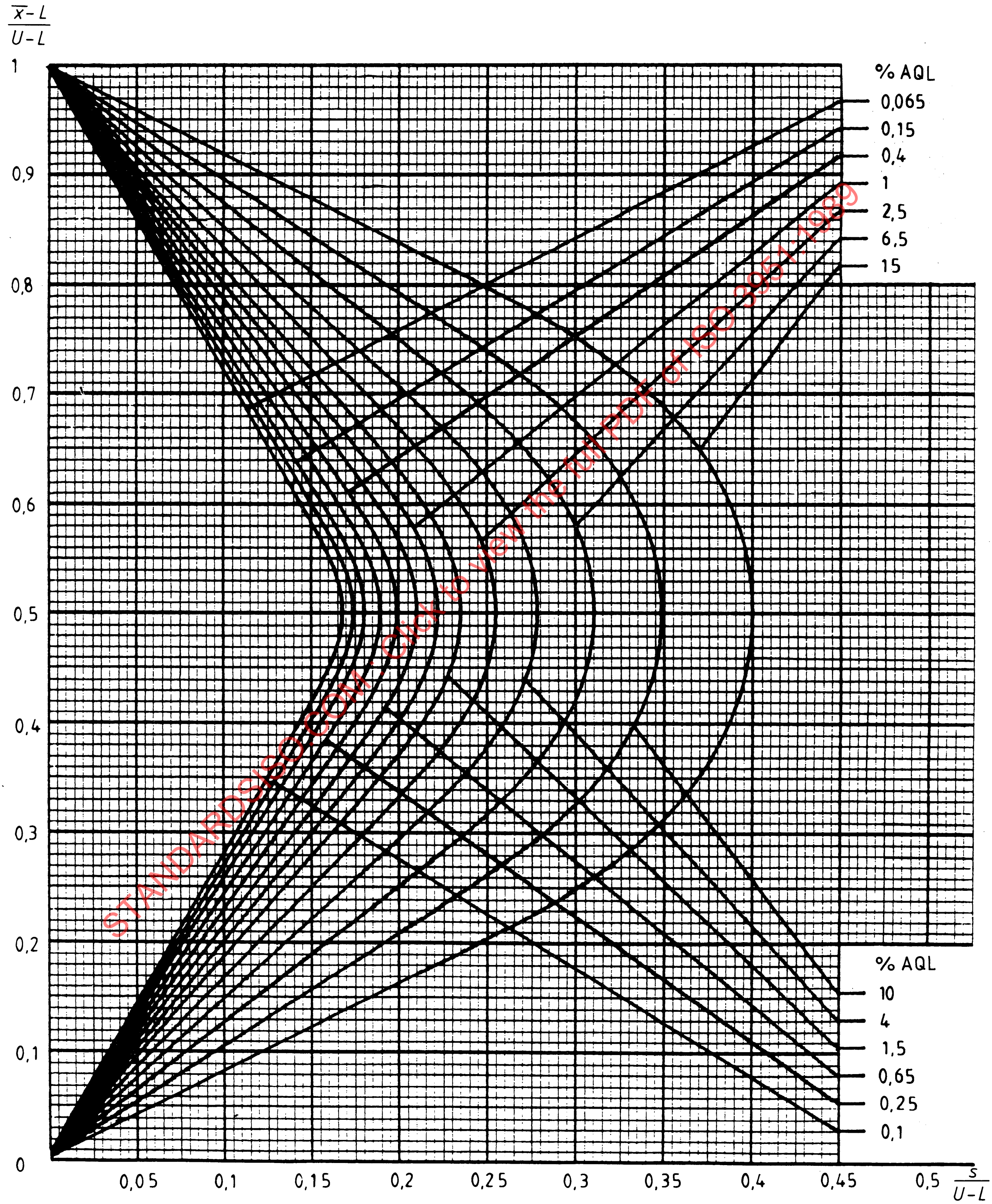


Diagram *s*-L — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter L (sample size 75)

L



M

Diagram *s*-M — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter M (sample size 100)

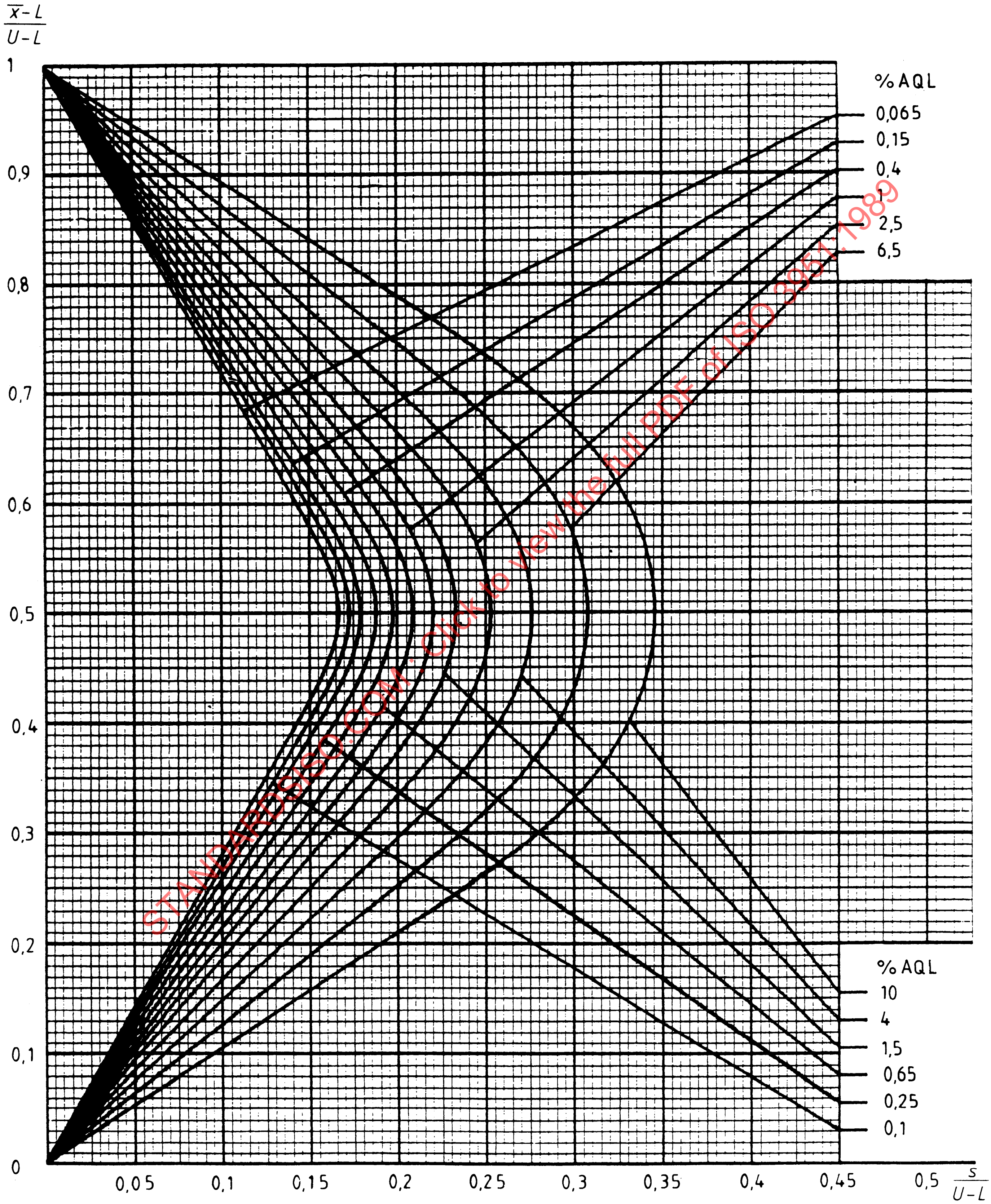
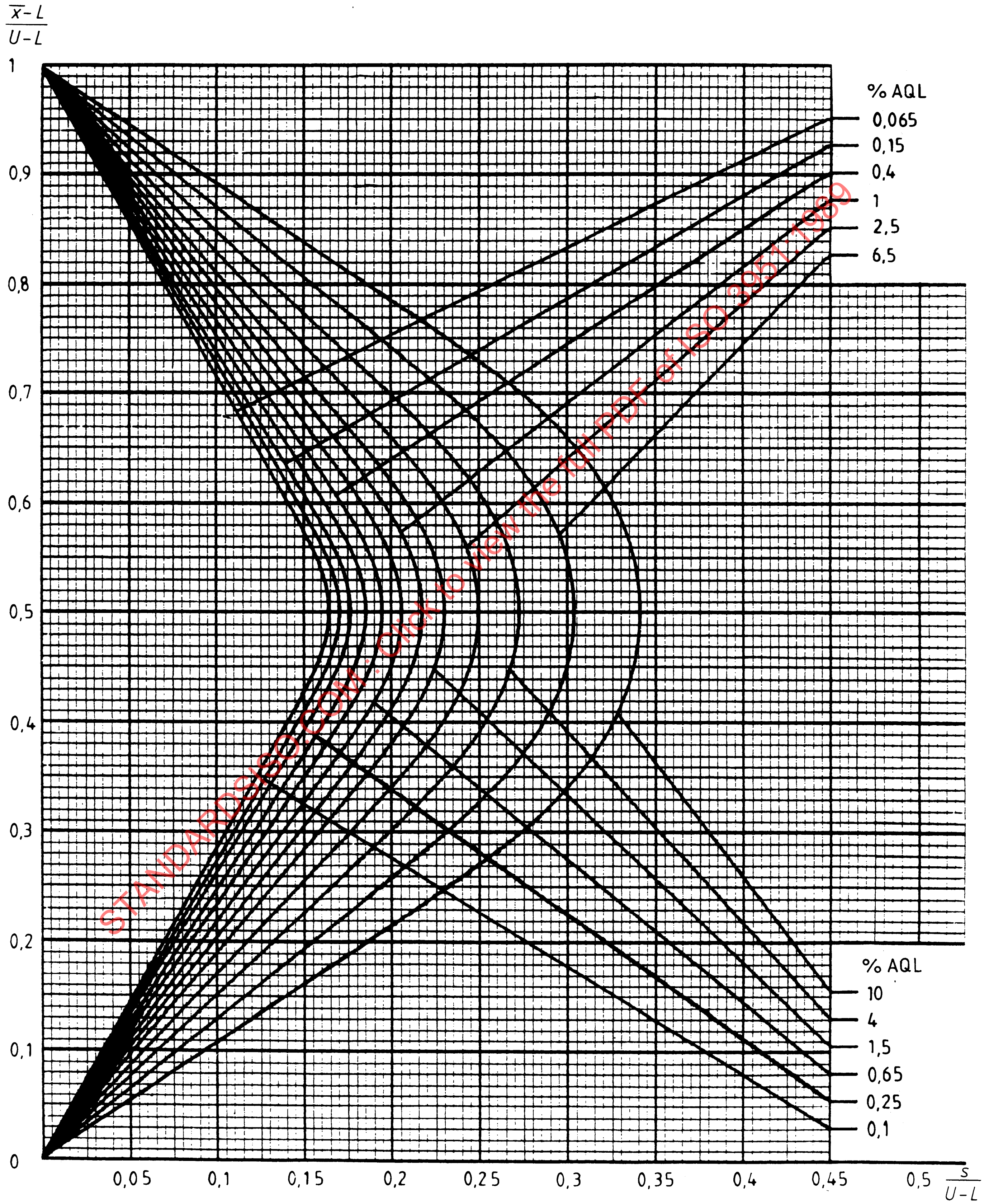


Diagram *s*-N — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter N (sample size 150)

N



P

Diagram *s*-P — Acceptance curves for combined double specification limits : "*s*" method — Sample size code letter P (sample size 200)

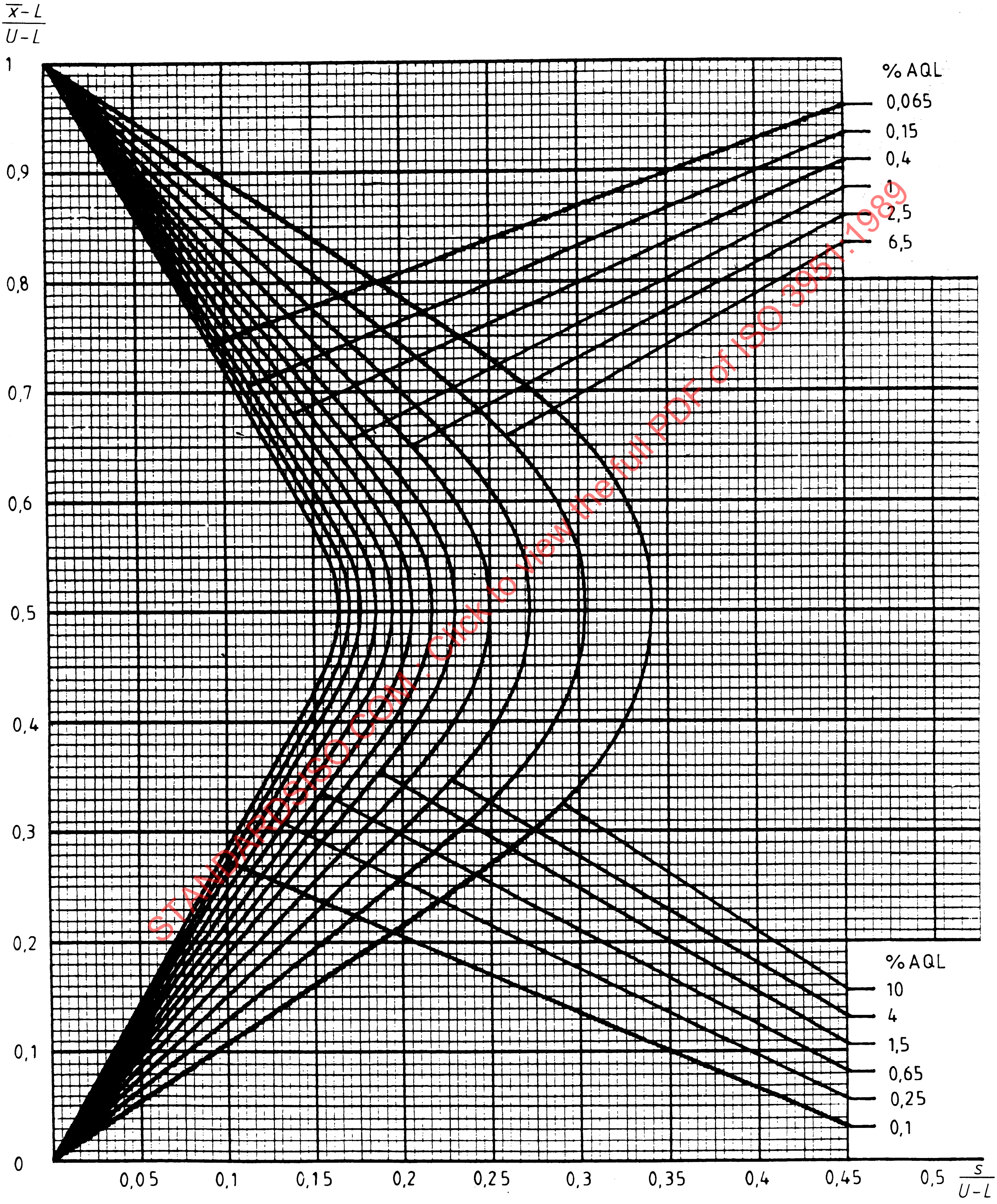
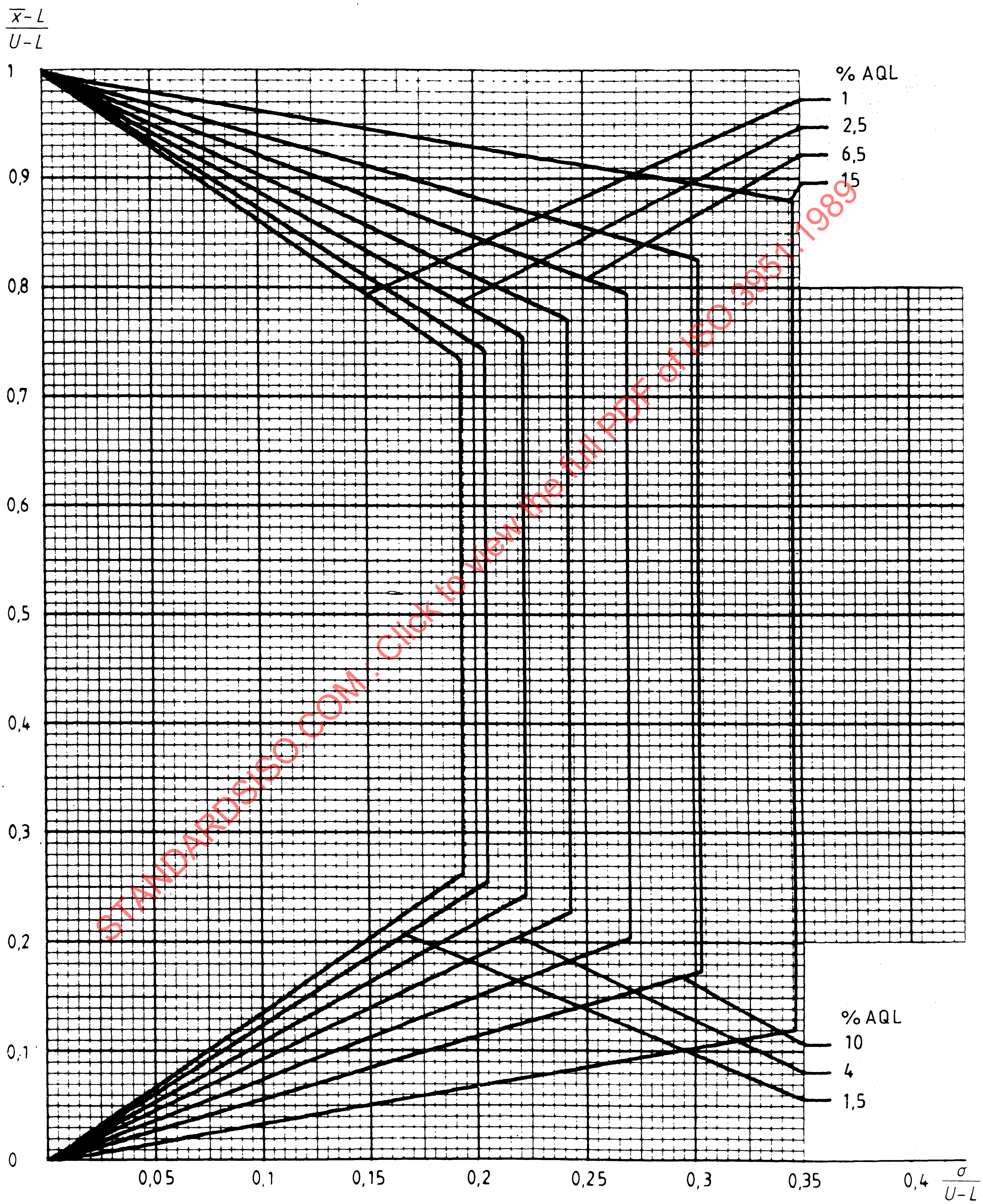


Diagram σ-C — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter C

C



D

Diagram σ-D — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter D

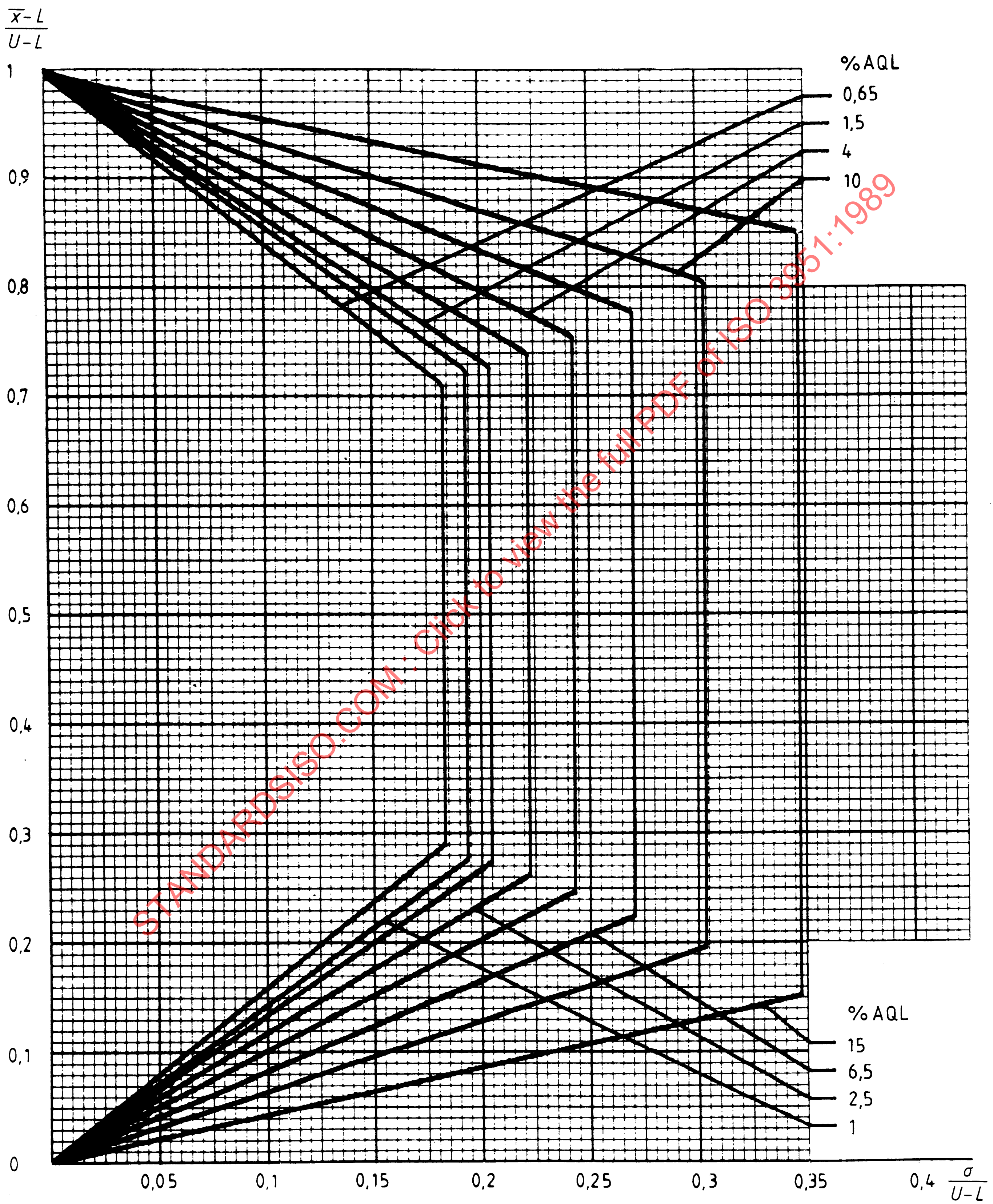
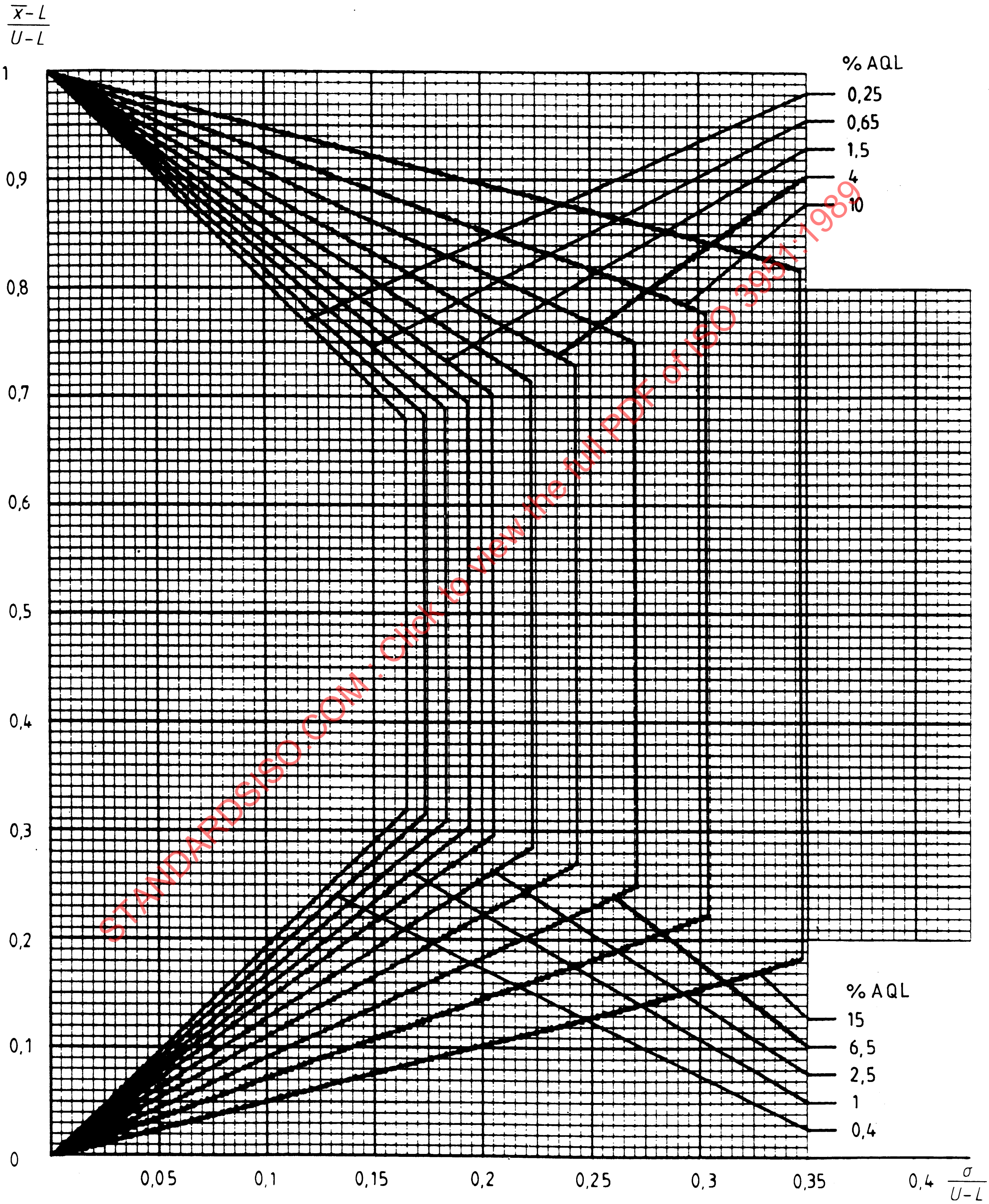


Diagram σ-E — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter E

E



F

Diagram σ-F — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter F

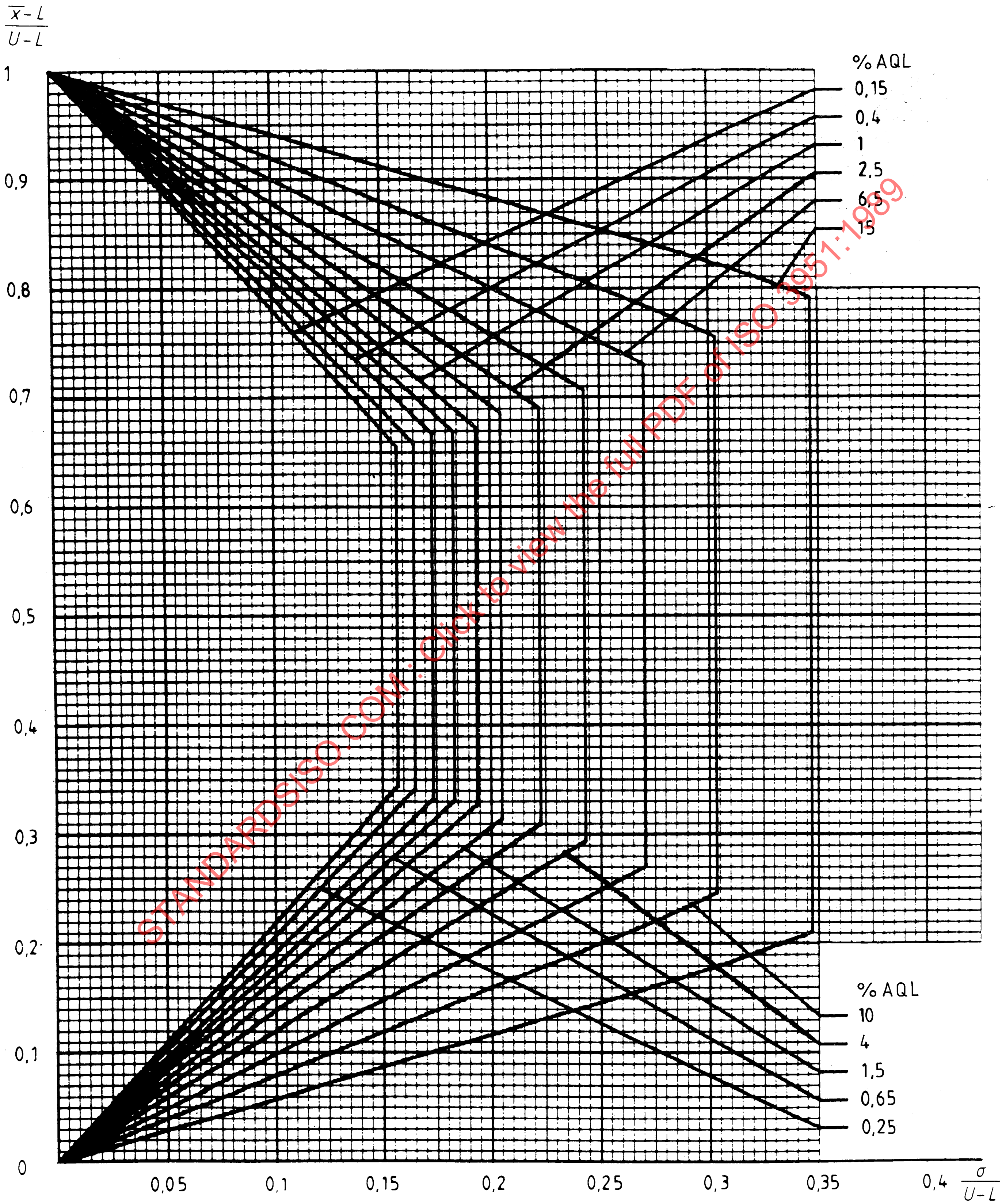
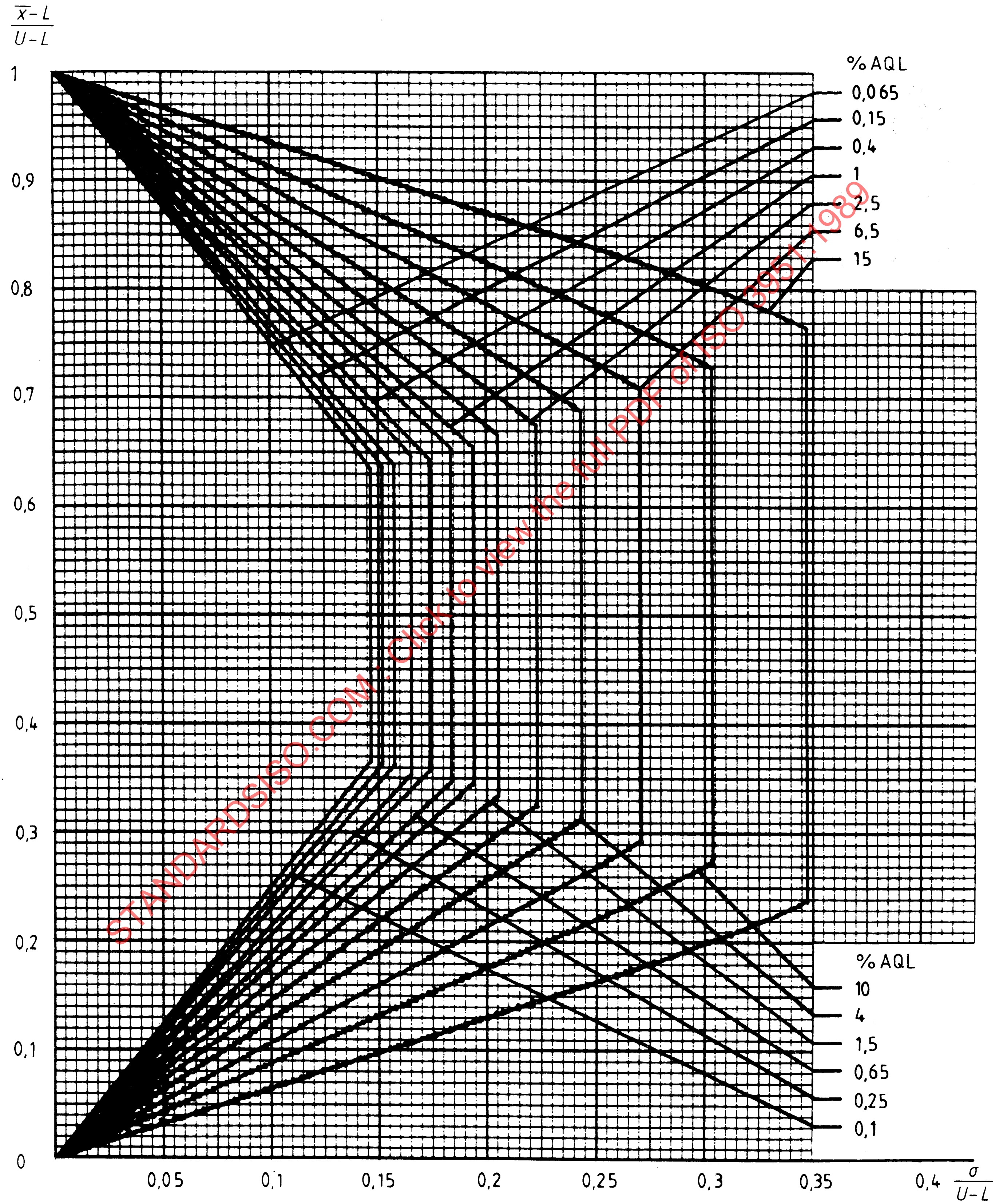


Diagram σ-G — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter G

G



H

Diagram σ-H — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter H

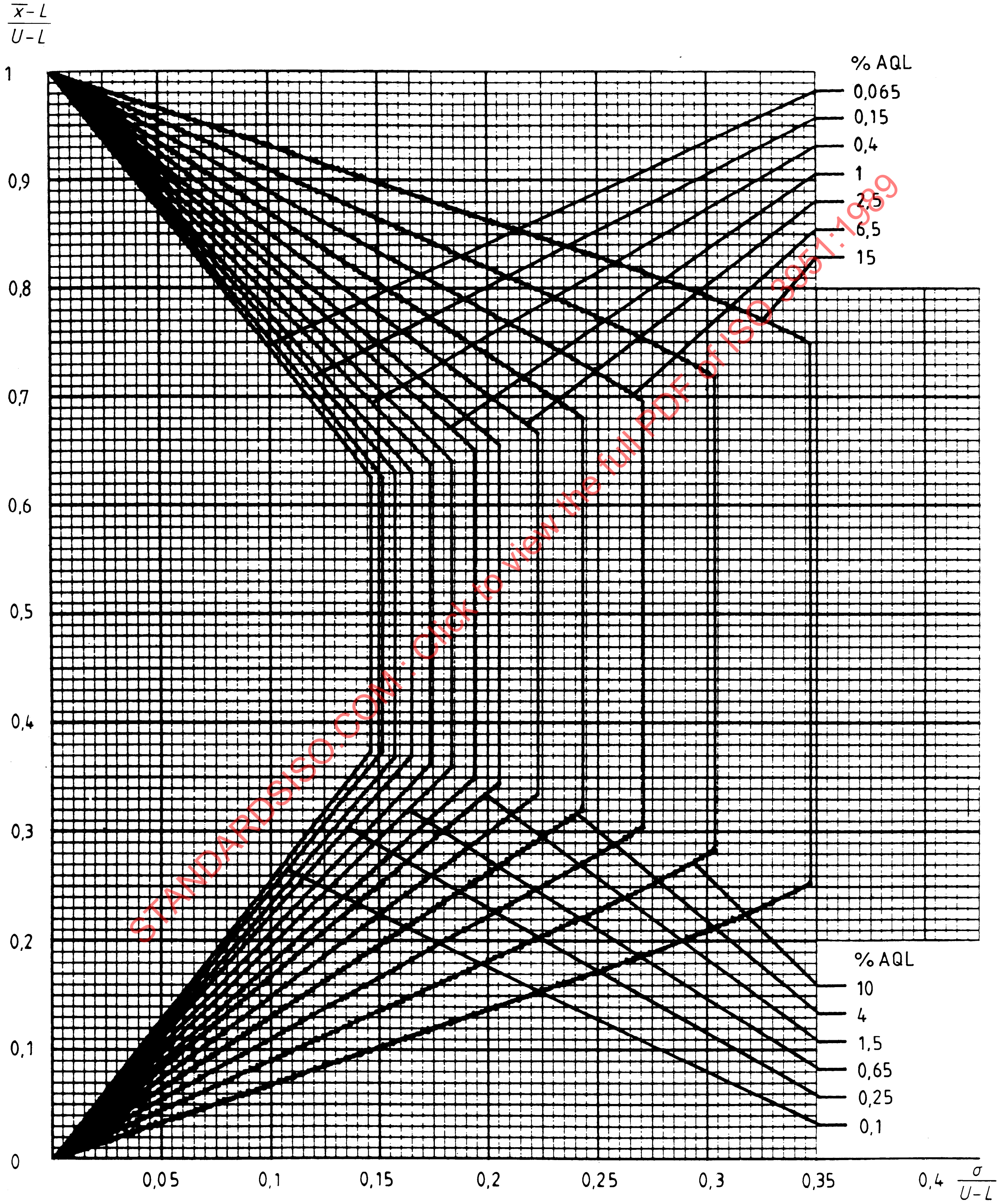
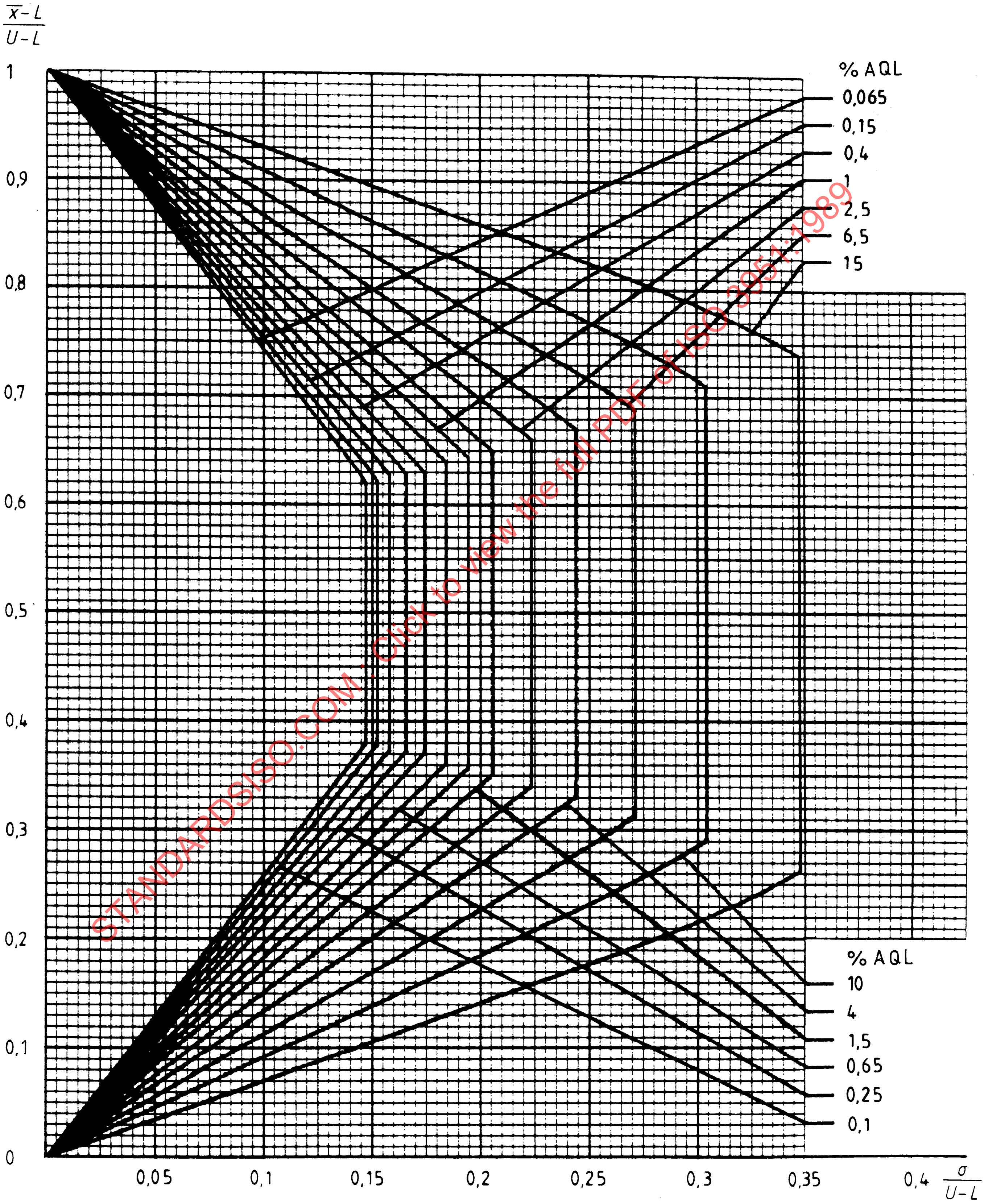


Diagram σ-I — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter I



J

Diagram σ-J — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter J

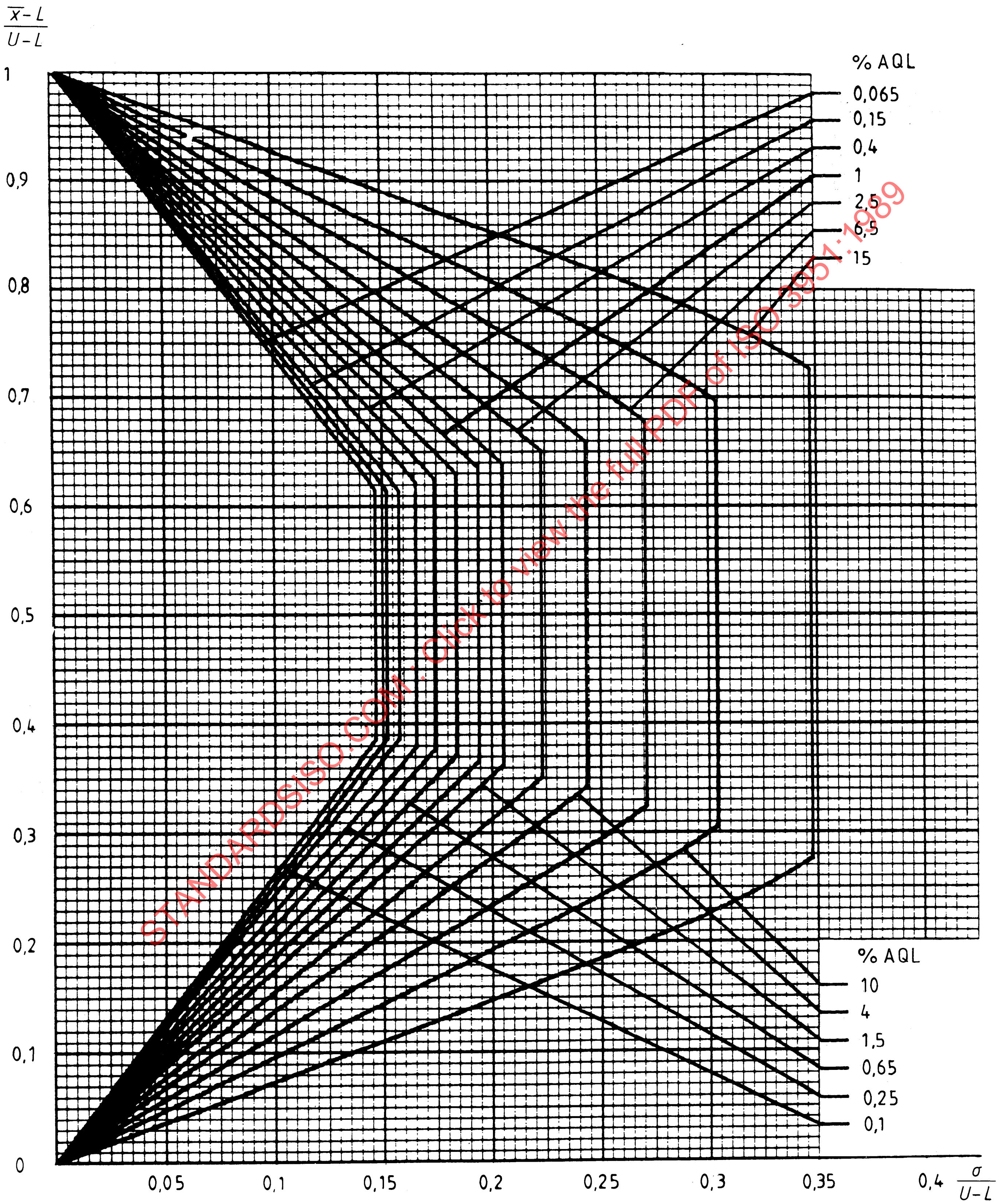
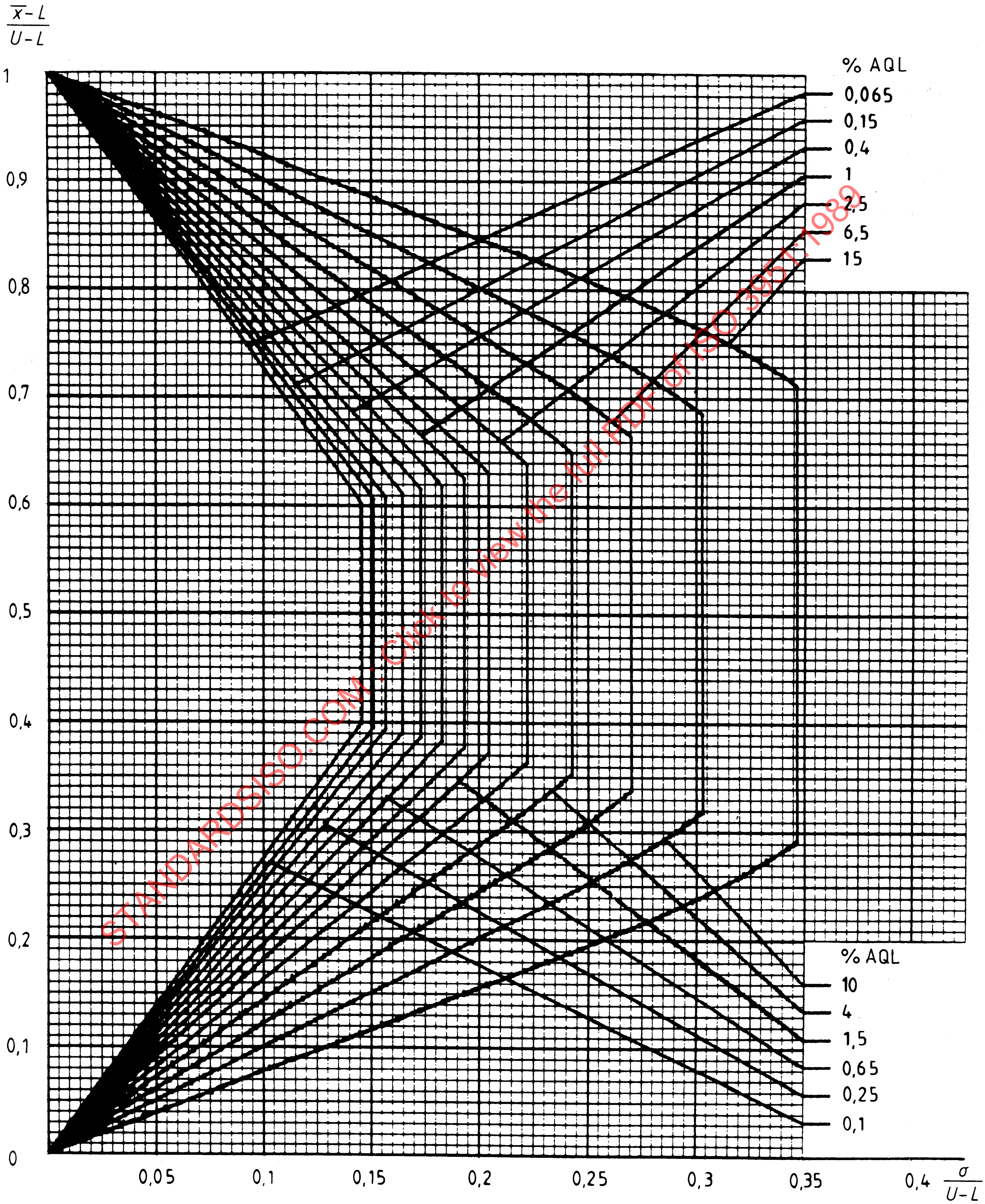


Diagram σ-K — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter K

K



L

Diagram σ-L — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter L

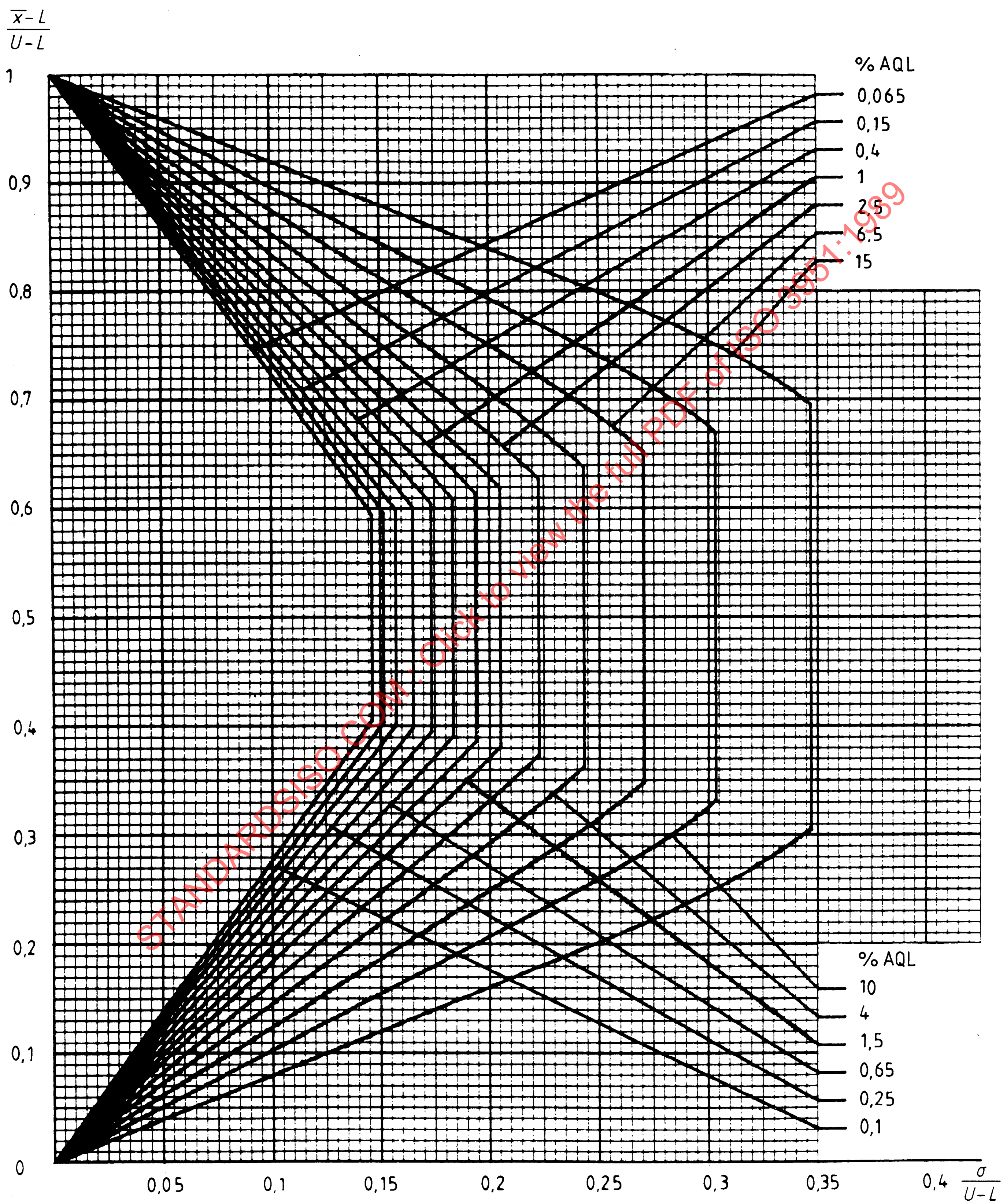
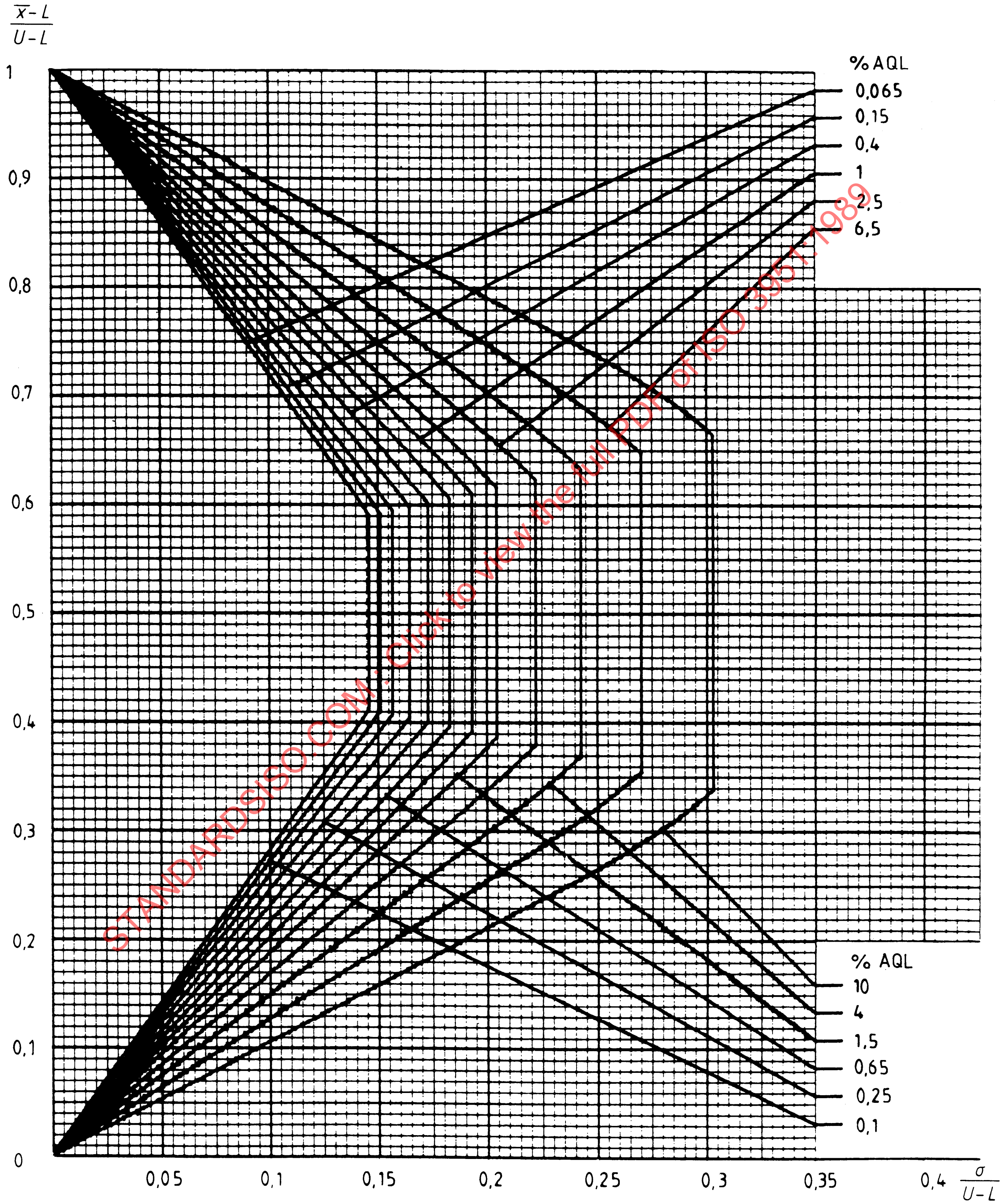


Diagram σ-M — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter M

M



N

Diagram σ-N — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter N

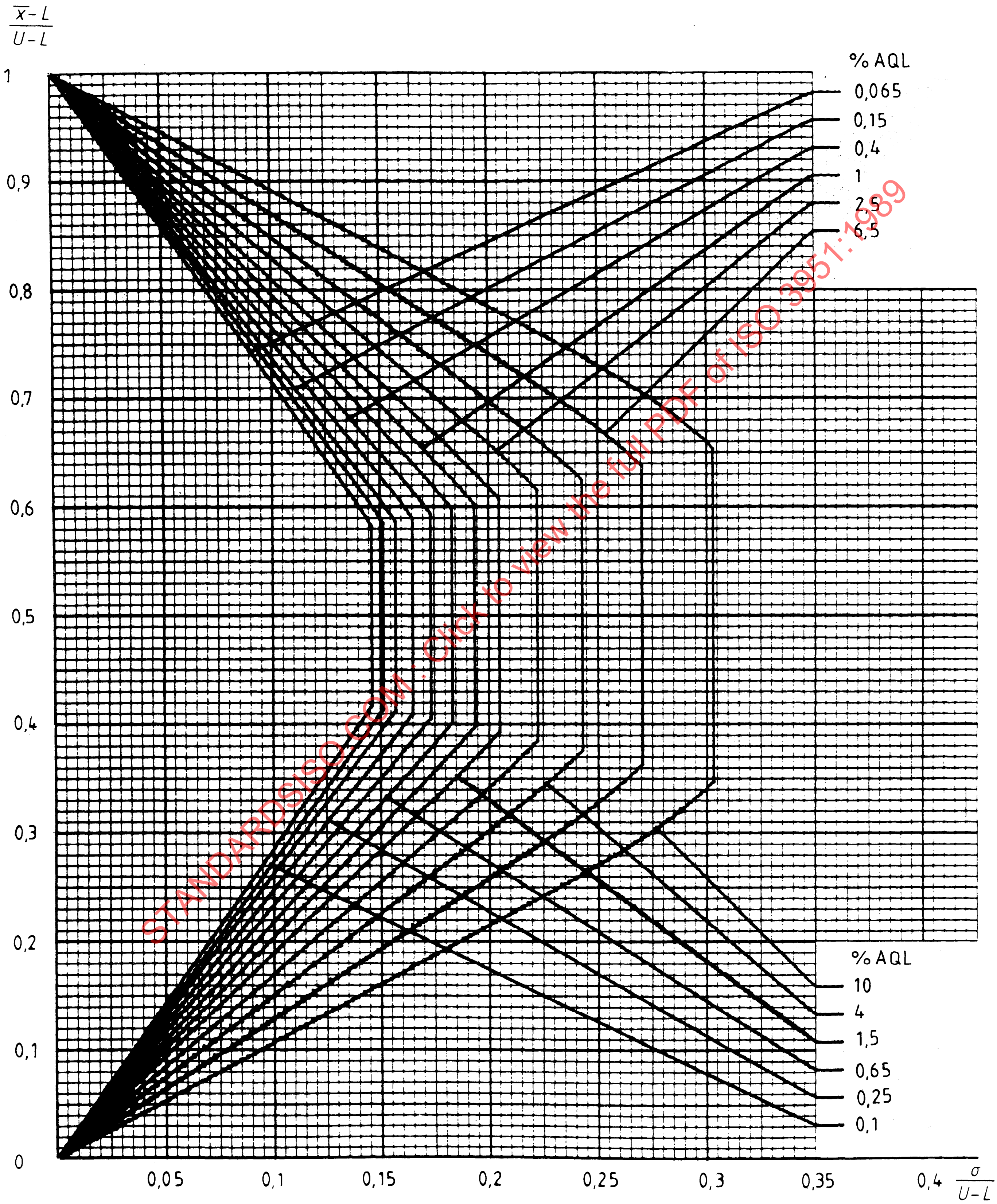
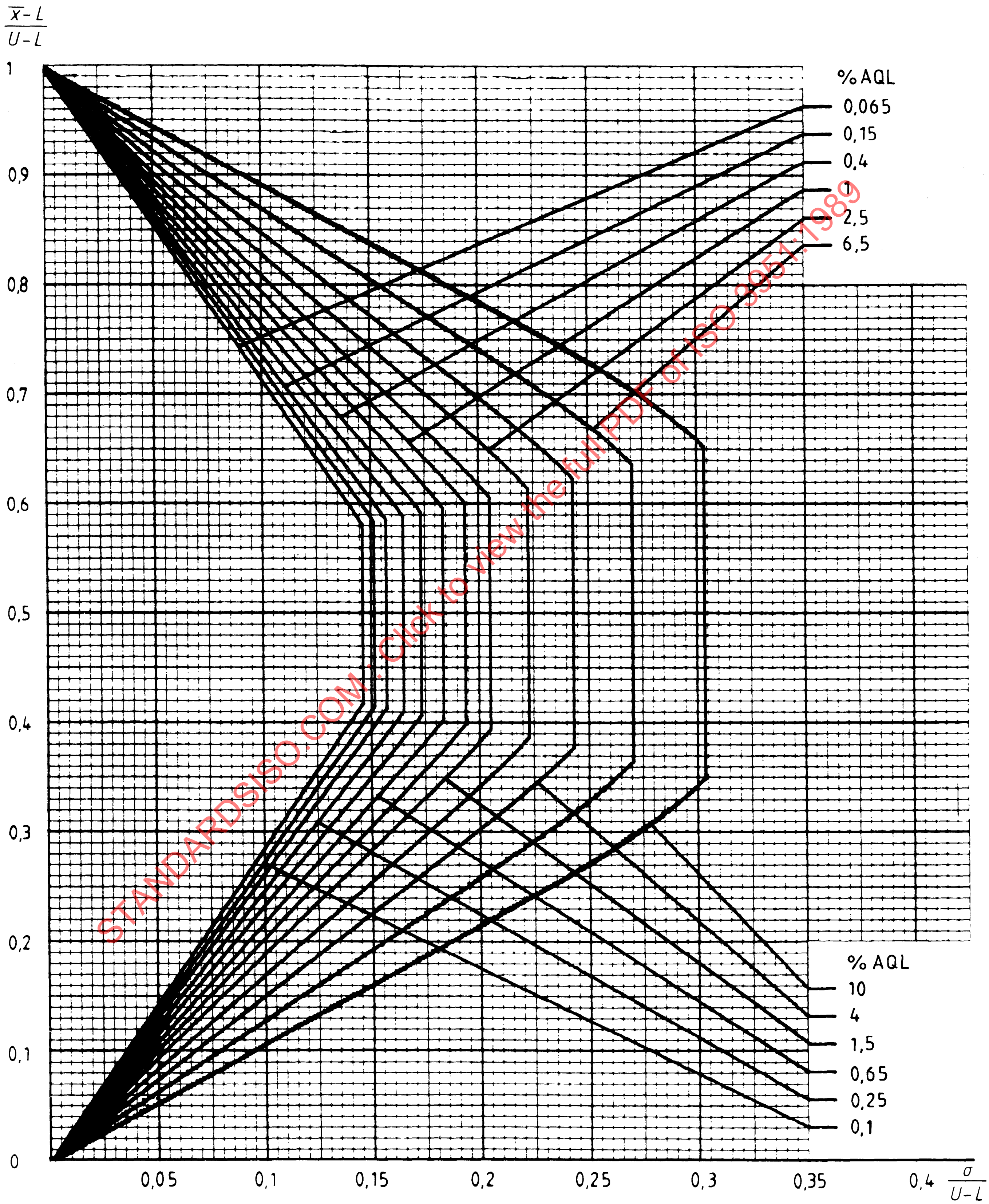


Diagram σ-P — Acceptance curves for combined double specification limits : "σ" method — Sample size code letter P

P



Annex A

Procedures for obtaining s and σ

(This annex forms an integral part of the Standard.)

A.1 Procedure for obtaining s

A.1.1 The estimate from a sample of the standard deviation of a population is given the symbol s and its value may be obtained from the mathematical formula

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1}}$$

where x_i is the measured characteristic of the ith item in a sample of n items and $\overline{x}$ is the mean value of the x_i, i.e.:

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

A.1.2 The above formula for s is not recommended for the purpose of computation, since the calculations can be simplified by subtracting an arbitrary whole number a from the x_i and using the alternative equivalent formula for s:

$$s^2 = \left\{ \sum_{i=1}^{n} (x_i - a)^2 - n(a - \overline{x})^2 \right\} \Big/ (n - 1)$$

For example, consider the measurements given in the example to 14.6. Take a = 65 and set out the calculations in tabular forms as follows:

x_i	a	$(x_i - a)$	$(x_i - a)^2$
63,5	65	− 1,5	2,25
62	65	− 3,0	9,00
65,2	65	0,2	0,04
61,7	65	− 3,3	10,89
69	65	4,0	16,00
67,1	65	2,1	4,41
60	65	− 5,0	25,00
66,4	65	1,4	1,96
62,8	65	− 2,2	4,84
68	65	3,0	9,00

Σx_i = 645,7
n = 10
$\overline{x}$ = 64,57

a = 65
$(a - \overline{x})$ = 0,43
$(a - \overline{x})^2$ = 0,185
$n(a - \overline{x})^2$ = 1,85

$\Sigma(x_i - a)^2$ = 83,39
$n(a - \overline{x})^2$ = 1,85

Difference = 81,54 = $(n - 1)s^2$
$(n - 1)$ = 9
s^2 = 9,06
s = 3,01

A.1.3 When a desk calculator is available, the value of a in clause A.1.2 may be taken as zero and the formula becomes

$$s^2 = \left\{ \sum_{i=1}^{n} (x_i)^2 - n(\overline{x})^2 \right\} \Big/ (n-1)$$

With a modern machine, it is possible to obtain the cumulative sum of both x_i and $(x_i)^2$ simultaneously and it is unnecessary to write down the individual values of x_i and $(x_i)^2$. But to illustrate the process, these values are given in the following example. Note how large the figures in the fourth column can become and when using an electronic calculator ensure that they are not truncated. The figures are taken from the second example given in C.9.2.5 in annex C, but in this case s will be calculated.

x_i	Cumulative sum of x_i	x_i^2	Cumulative sum of x_i^2
515	515	265 225	265 225
491	1 006	241 081	506 306
479	1 485	229 441	735 747
507	1 992	257 049	992 796
543	2 535	294 849	1 287 645
521	3 056	271 441	1 559 086
536	3 592	287 296	1 846 382
483	4 075	233 289	2 079 671
509	4 584	259 081	2 338 752
514	5 098	264 196	2 602 948
507	5 605	257 049	2 859 997
484	6 089	234 256	3 094 253
526	6 615	276 676	3 370 929
552	7 167	304 704	3 675 633
499	7 666	249 001	3 924 634
530	8 196	280 900	4 205 534
512	8 708	262 144	4 467 678
492	9 200	242 064	4 709 742
521	9 721	271 441	4 981 183
467	10 188	218 089	5 199 272
489	10 677	239 121	5 438 393
513	11 190	263 169	5 701 562
535	11 725	286 225	5 987 787
501	12 226	251 001	6 238 788
529	12 755	279 841	6 518 629

so $\sum_{i=1}^{n} x_i = 12\,755$

$n = 25$

$\overline{x} = \sum_{i=1}^{n} x_i \Big/ n = 510{,}2$

$(\overline{x})^2 = 260\,304{,}04$

$n(\overline{x}) = 6\,507\,601$

$\sum_{i=1}^{n} x_i^2 = 6\,518\,629$

$n(\overline{x})^2 = 6\,507\,601$

$\sum_{i=1}^{n} x_i^2 - n(\overline{x})^2 = 11\,028 = (n-1)s^2$

$n - 1 = 24$

$s^2 = 459{,}5$

$s = 21{,}43$

Thus the mean $\overline{x} = 510{,}2$ and the estimate of the lot standard deviation $s = 21{,}4$.

A.1.4 If the variation between readings is small, it is recommended that only that portion of the measurement which changes from item to item be considered; e.g. if the values of x_i had been 27,515; 27,491; 27,479; etc., they should be considered as if the values had been 515; 491; 479; etc.

These are then the figures in the previous example; thus $\bar{x}$ = 27,510 and s = 0,021.

The above method not only reduces the arithmetic labour but also reduces or helps to prevent the accumulation of rounding errors.

It should be noted that a computer with single precision can give less accuracy than a pocket calculator unless it uses a program which minimizes rounding errors. A possible way of doing this is to substract a constant from every value as described above. It is convenient to choose the first observation as this constant — in the example above 27,515 would be subtracted from each observation.

A.1.5 Other forms of the equation for s can be derived. Depending on the mechanism of the calculator, either of the following may be useful:

$$s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2 / n}{n - 1}}$$

or

$$s = \sqrt{\frac{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n - 1)}}$$

A.2 Procedure for obtaining σ

A.2.1 If it appears that the value of s (or $\bar{R}$) is in control, σ may be presumed to be the weighted root mean square value of s (or $\bar{R}/c$) and the weighted root mean square value may be obtained from the following formula:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{l} (n_i - 1) s_i^2}{\sum_{i=1}^{l} (n_i - 1)}}$$

where

l is the number of lots;

n_i is the sample size from the ith lot;

s_i is the sample size standard deviation of the ith lot.

A.2.2 If the sample sizes from each of the lots are equal, then the above formula reduces to:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{l} s_i^2}{l}}$$

A.2.3 In the case of the "R" method, then s_i^2 in the above equations should be replaced by $(\bar{R}_i/c_i)^2$, where $\bar{R}_i$ is the average range and c_i the scale factor for the sample from the ith lot.

Annex B

Statistical theory

(This annex forms an integral part of the Standard.)

B.0 Symbols

Further symbols used in this annex are as follows:

p The total process percent or fraction nonconforming.

$p = p_U + p_L$

p_L The process percent or fraction nonconforming below L.

p_U The process percent or fraction nonconforming above U.

P The cumulative probability function (distribution function).

q The quality parameter: equal to z_U or $-z_L$

z Value of the standardized variate.

$$z = \frac{x - \mu}{\sigma}$$

z_L Value of z at the lower specification limit.

$$z_L = \frac{L - \mu}{\sigma}$$

z_U Value of z at the upper specification limit.

$$z_U = \frac{U - \mu}{\sigma}$$

B.1 The normal distribution

B.1.1 The theory governing the calculation of risks in inspection by variables depends on the properties of the normal distribution and, therefore, this International Standard is only accurately applicable when there is reason to believe that the frequency distribution of the measured characteristic is normal or nearly normal.

B.1.2 A normal distribution can be defined completely in terms of the mean μ and standard deviation σ of the population; when these two parameters are known, it is possible to calculate the probability with which any measured value x will fall between two given values and, in particular, the probability of any measured value occurring outside given upper or lower limits. (See figure 5.)

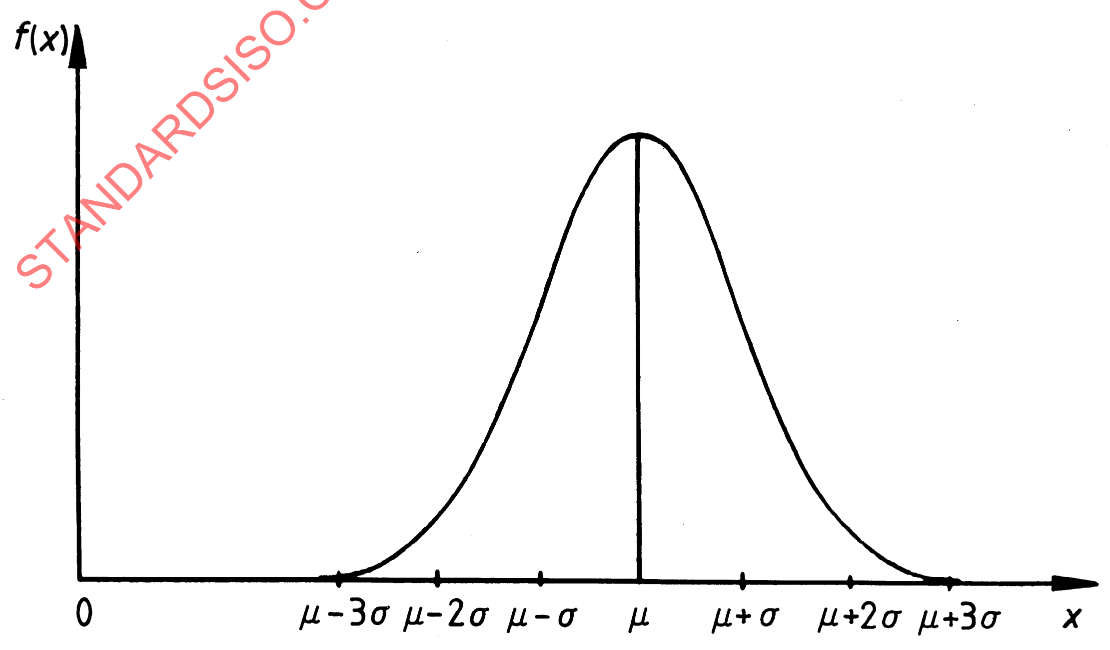


Figure 5 — Normal distribution

B.1.3 In order to facilitate the tabulation of these probabilities, the measured value x is transformed into a normal standardized variate z such that

$$z = \frac{x - \mu}{\sigma}$$

This device scales all normal distribution curves to a common shape (see figure 6), simplifies the equation of the curve and, as the total area under the curve is equal to 1, permits the calculation of a single table which, for a given value of z, gives the value of P, the cumulative probability function (the unshaded area under the curve).

B.2 The fraction nonconforming

B.2.1 The fraction nonconforming is represented by the area (or areas) under the distribution curve beyond the specification limit line (or lines if both upper and lower limits have been given).

NOTE — The fraction nonconforming is, in practice, often expressed as a percentage nonconforming; accordingly, in sections one to four it is given in this form.

B.2.2 When the measurements x are normally distributed and their mean μ and standard deviation σ are known, the fraction nonconforming p relative to given specification limits L and/or U can be obtained from tables of the normal distribution, as $p_U = 1 - P_U$ and $p_L = P_L$. (See figure 7.)

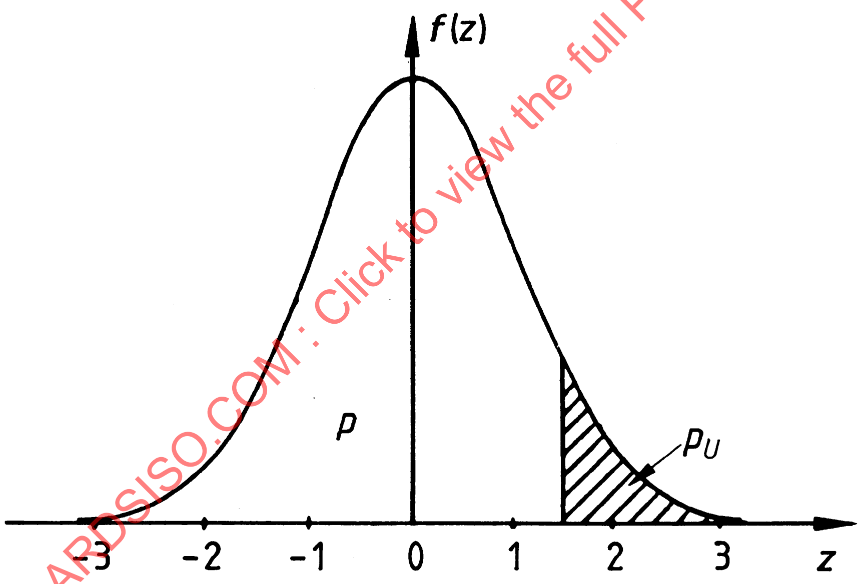


Figure 6 — Standardized normal distribution

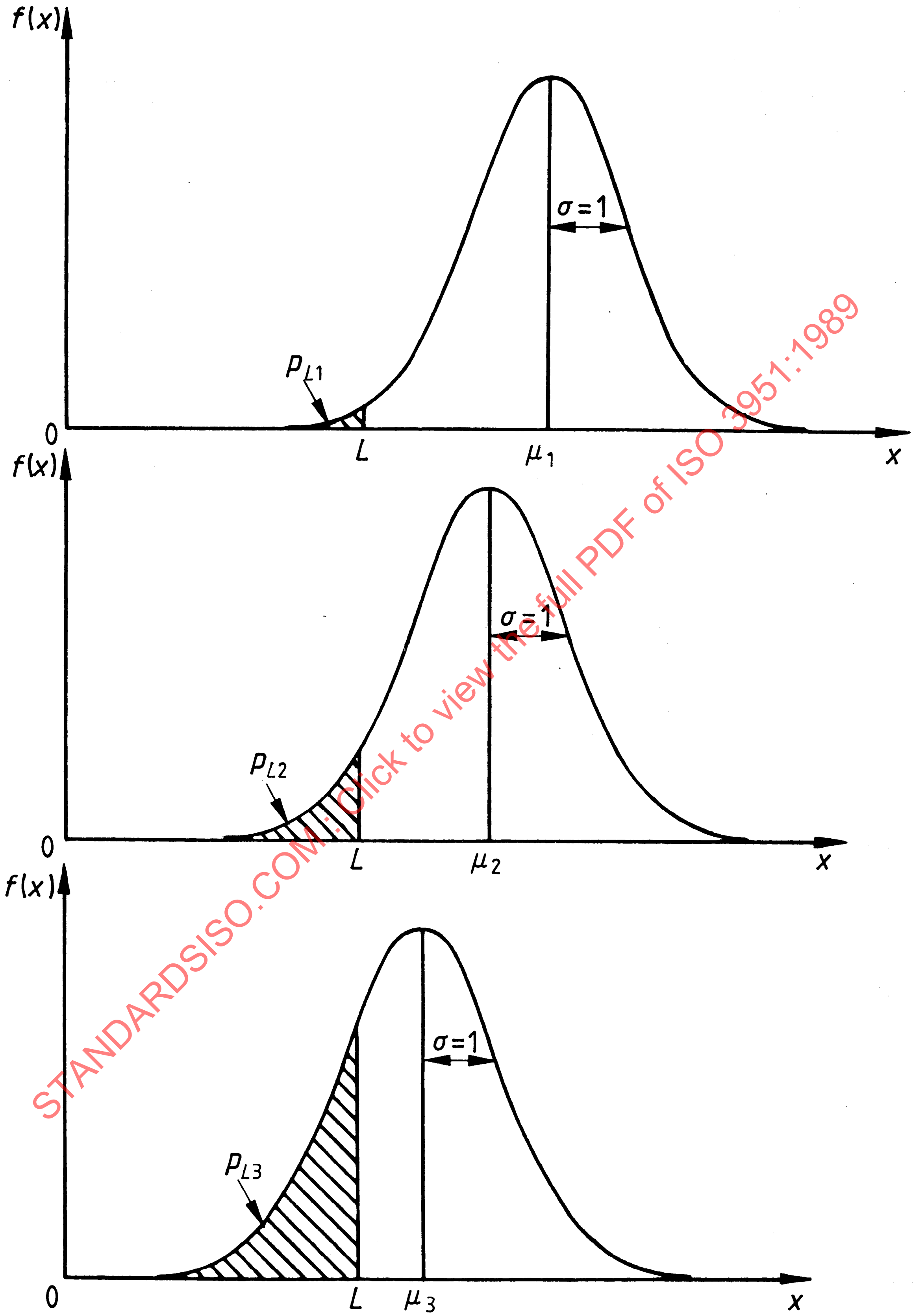


Figure 7 — Fraction nonconforming for constant σ = 1 and different values of the mean (μ_1, μ_2, μ_3) in relation to a fixed lower specification limit

B.3 The quality parameter

B.3.1 When the normal distribution curve has been "standardized" by measuring the deviation from the mean in units of the standard deviation (i.e. to a scale $\sigma = 1$), the fraction nonconforming will be uniquely related to the standardized deviate

$$-z_L = \frac{\mu - L}{\sigma}$$

B.3.2 Accordingly, a quality parameter has been devised in terms of this difference and the standard deviation which, in relation to one specification limit, may be used in acceptance inspection instead of the fraction nonconforming. When U, the upper specification limit, is given, this is defined as

$$q_U = \frac{U - \mu}{\sigma} = z_U$$

and when L, the lower specification limit, is given, as

$$q_L = \frac{\mu - L}{\sigma} = -z_L$$

B.4 Acceptable parameter region for single or separate double specification limits

The AQL is defined as the quality level which for the purposes of sampling inspection is the limit of a satisfactory process average percent nonconforming for a continuous series of lots.

When considering a single specification limit or both limits separately, the AQL can be related readily to the quality parameter q. Since there is one-to-one correspondence between the fraction beyond the given specification limit and the quality parameter, we can define a K which would be described as the smallest value of q that would be deemed acceptable as a process characteristic for purposes of acceptance sampling, i.e. for acceptability $q \geqslant K$.

This would then give

— in attributes : the AQL

— in variables : K

Since for a given set of specification limits, p_L or p_U depend on μ and σ, there will be a region in the μ, σ plane where p_L (or p_U) will be less than or equal to the AQL and will thus mark off processes that will be acceptable. There will conversely be another region in which p_L (or p_U) will be greater than the AQL and will thus mark off processes that are considered unacceptable. Fortunately, these regions can be separated by a single straight line.

For a single lower specification limit, the line of AQL in the μ, σ plane is given by the relationship

$$q_L = \frac{\mu - L}{\sigma} = K$$

or

$$\mu = L + K\sigma$$

For all points above this line, the process average percent nonconforming would be less than or equal to the AQL and hence be acceptable. For points below the line, it would not be acceptable. (See figure 8.)

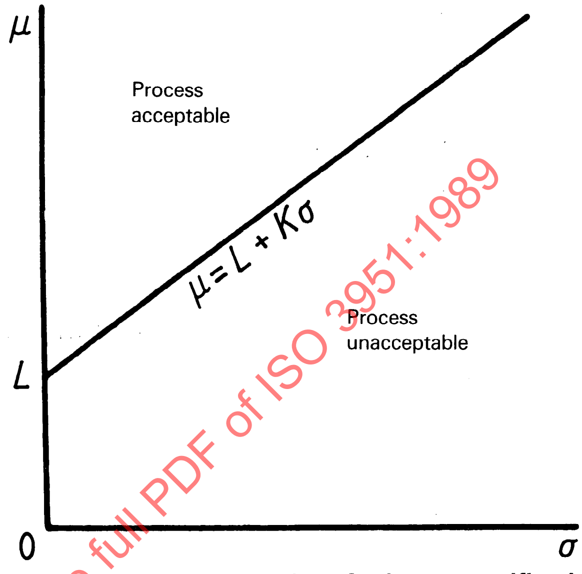


Figure 8 — Acceptance chart for lower specification limit only, σ and μ known

For a single upper specification limit, a similar relationship would be

$$q_U = \frac{U - \mu}{\sigma} = K$$

or

$$\mu = U - K\sigma$$

For points below this line, the process average percent nonconforming would be less than the AQL and hence acceptable; above, they would not. (See figure 9.)

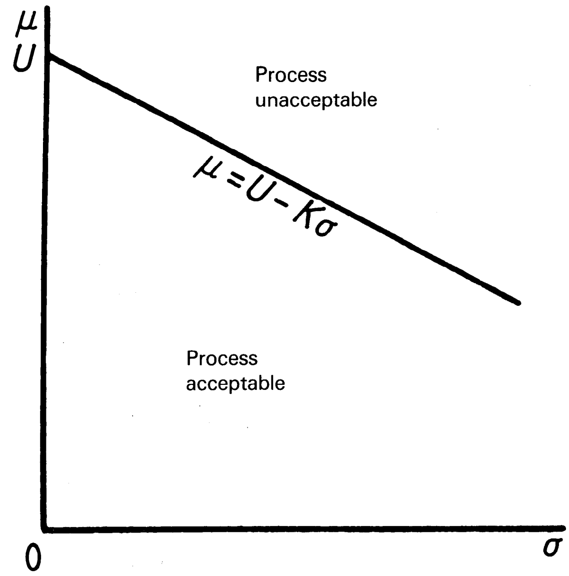


Figure 9 — Acceptance chart for upper specification limit only, σ and μ known

With two separate specification limits, the situation would be as depicted in figure 10.

B.5 Acceptable parameter region for a combined double specification limit

B.5.1 Combined double specification limit

When upper and lower limits are specified and a single, overall AQL has to be met, this is known as a combined double specification limit.

Specification of an AQL for a combined double specification limit does not separately specify the fractions p_U and p_L nonconforming beyond the upper and lower limits but specifies the value that their sum p should not exceed, There is then no longer a way of obtaining a single variables figure such as K corresponding to the AQL. Instead, the mathematical relationship between the process mean μ, the process standard deviation σ and the total process fraction nonconforming must be used to find pairs of values (σ, μ) for which this fraction nonconforming is equal to the AQL.

For a given value of σ, not only do the relative values of p_U and p_L depend on the value of μ but their total $p_U + p_L = p$ is also dependent on the value of μ in relation to U and L. (See figure 11.) The value of p is a minimum when μ is midway between U and L and increases as μ moves away from this value.

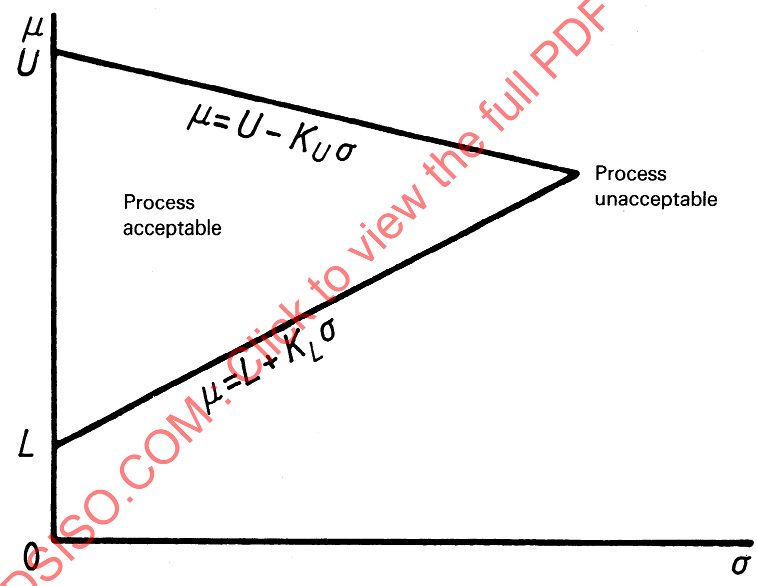


Figure 10 — Acceptance chart for separate double specification limits, σ and μ known

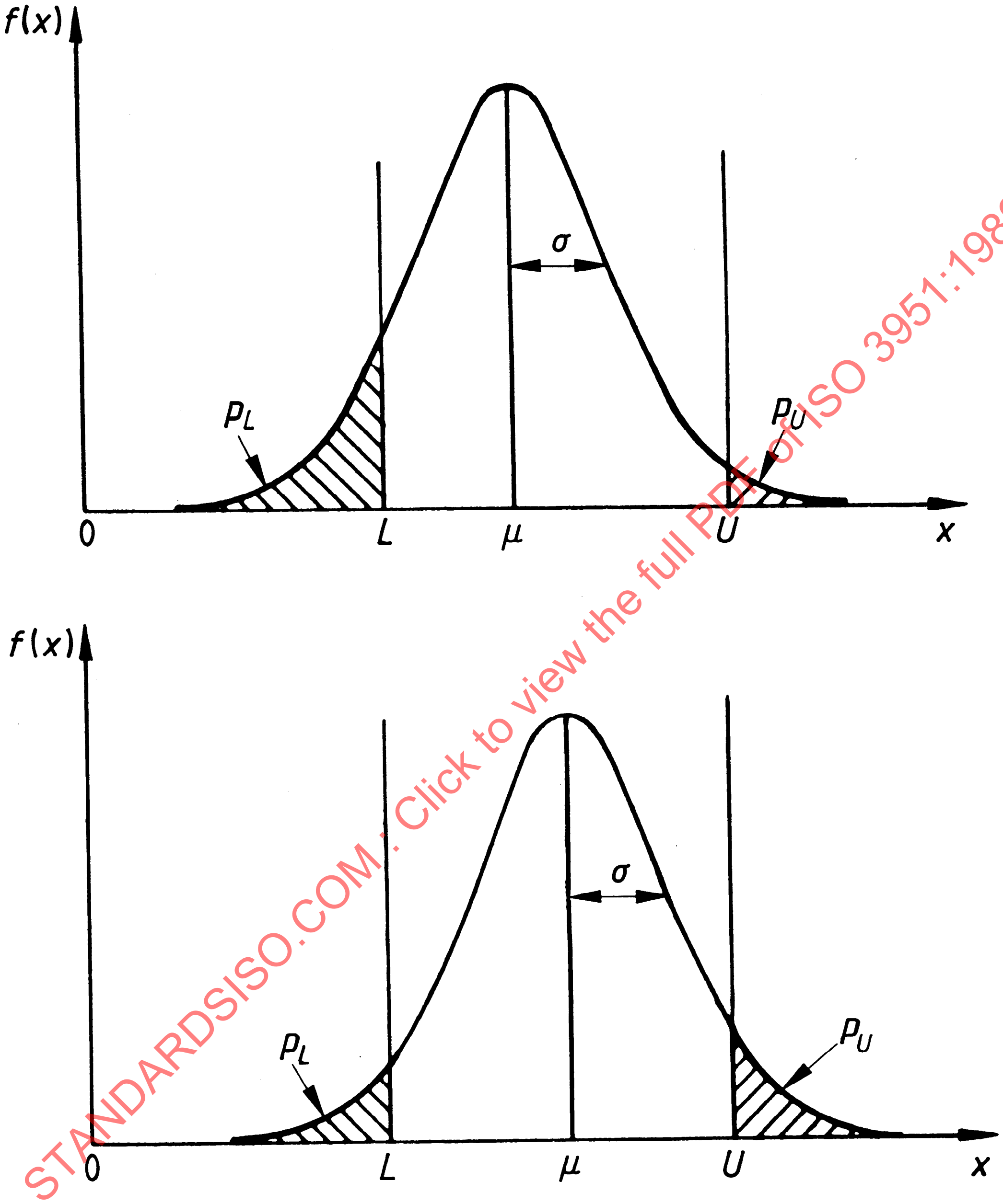


Figure 11 — The effect on process fraction nonconforming of a change in μ, with σ held constant

The extent to which the distribution curve overlaps the two limits, for a given value of μ, is governed by the value of σ. (See figure 12.)

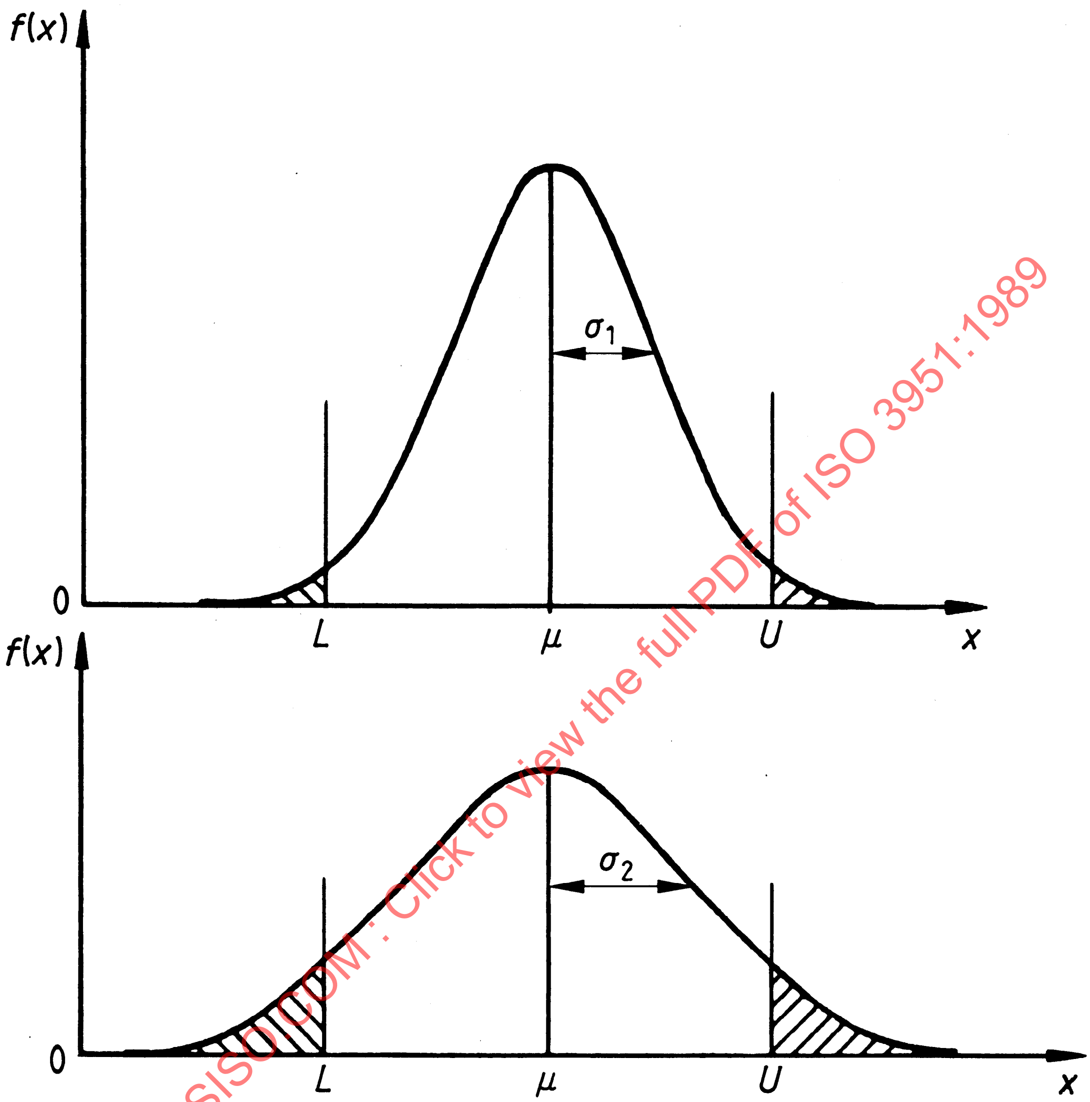


Figure 12 — The effect on process fraction nonconforming of a change in σ, with μ held constant

B.5.2 Maximum process standard deviation (MPSD)

The largest value of σ that will just give the AQL when μ is midway between U and L may be viewed as a maximum process standard deviation (MPSD) under normal inspection. If a process has a σ greater than this, its fraction nonconforming exceeds the AQL. The converse, that if the standard deviation is less than this maximum a process fraction nonconforming is less than the AQL, is not necessarily true.

Under tightened or reduced inspection, the MPSD is the largest process standard deviation for which the next lower or higher preferred AQL may be achieved, respectively.

At the discretion of the responsible authority, the value of f_σ for tightened inspection may be invoked for normal and reduced inspection.

Under this alternative procedure, the choice between the "σ" and "s" methods is unrelated to the switching rules.

B.5.3 Acceptance curve

When a combined double specification has been given, a series of values of μ and σ can be calculated that give values of p_L and p_U that in total equal the AQL. These values can be plotted in the (μ, σ) plane and a curve drawn through them (see figure 13). This curve lies within the triangle that would be appropriate for separate upper and lower specification limits each with the given AQL (see figure 10). In the vicinity of U and L, the curve is indistinguishable from a straight line with slope $-K$ and K respectively and then becomes a smooth curve passing through the point σ = maximum standard deviation and $\mu = (U + L)/2$. (See figure 13.)

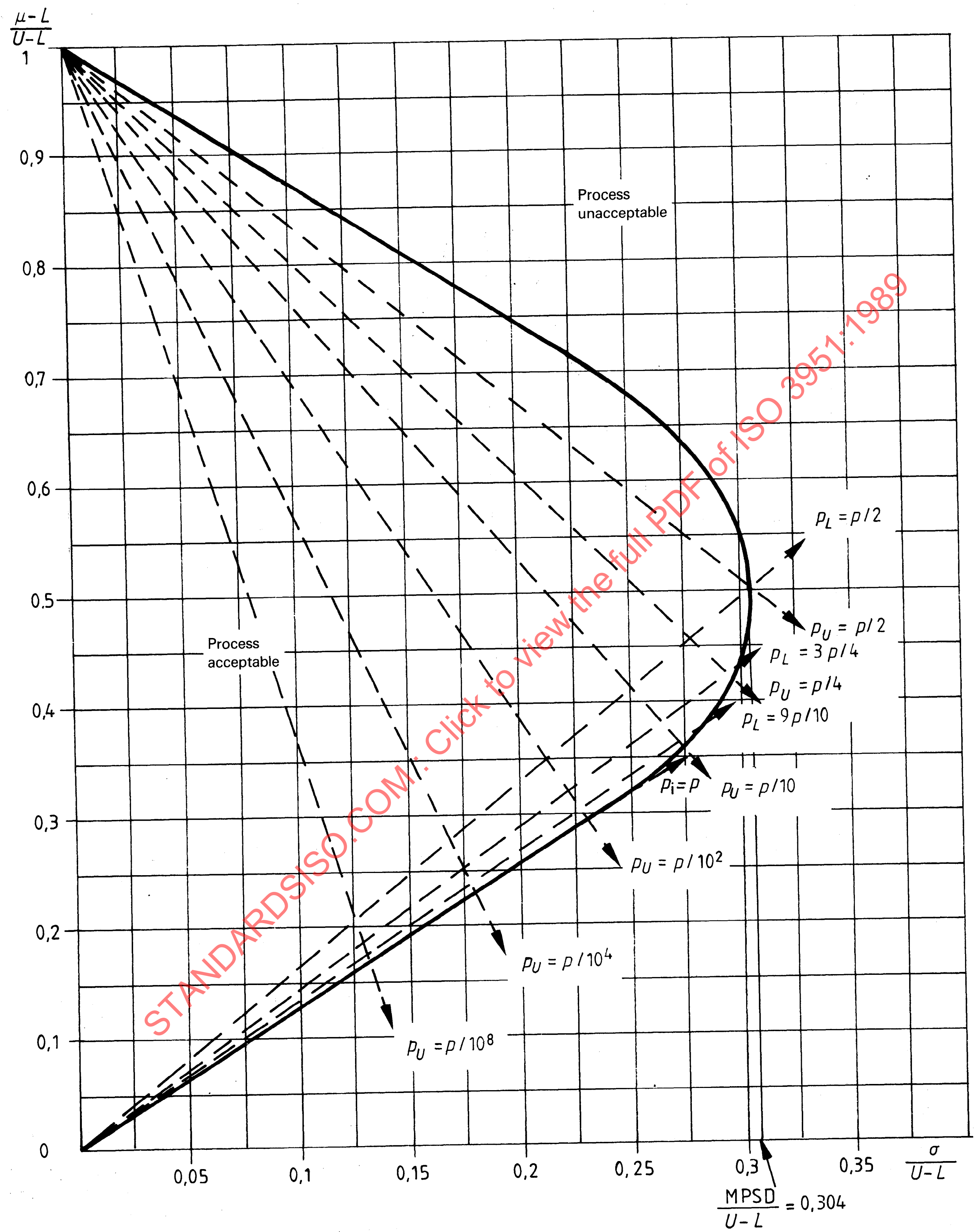


Figure 13 — Contour in (σ, μ) plane for p = AQL = 10 % nonconforming for a combined double specification limit

B.6 Sampling inspection

In practice, μ is not known and has to be estimated from a sample; usually, σ has to be estimated also.

The best estimate of μ is given by $\bar{x}$, the mean of the sample. Depending on the circumstances, σ is either assumed to be known or estimated from the sample by one of two methods. These three treatments of σ lead to what are called in this International Standard the "σ" method, the "s" method (and the "R" method given in annex C), for sentencing a lot.

Using these estimates of the values of μ and σ, an estimate of the quality parameter q is obtained. This is known as the quality statistic Q.

It is possible to calculate values of K which, for a given sample size and method of assessing the process standard deviation, will ensure that, with a given probability, the lot is acceptable when the quality statistic Q is greater than or equal to this K which is then known as the acceptability constant.

B.7 The "σ" method

B.7.1 Derivation of the acceptability constant

Under certain circumstances, e.g. when the process has been running for some time and is under statistical quality control, σ is assumed to be known and constant.

The value of the means for successive lots is not assumed to be constant or known, so each has to be estimated from the mean of a random sample of items taken from the population.

The mean of the sample $\bar{x}$ is not necessarily equal to the mean of the population μ, but $(\bar{x} - \mu)\sqrt{n}/\sigma$ is normally distributed with a zero mean and a standard deviation of 1, so it is possible to show that, for a given probability, the difference between the mean of the sample and the mean of the lot will be not more than a definite figure. For example, for a 95 % probability, this figure is $\pm$ 1,96 $\sigma/\sqrt{n}$.

The equation for the upper quality statistic $Q_U = (U - \bar{x})/\sigma$ may be rewritten

$$Q_U\sqrt{n} = (U - \mu)\sqrt{n}/\sigma - (\bar{x} - \mu)\sqrt{n}/\sigma$$

The first term on the right-hand side is a constant and the second is the standardized normal deviate of the mean discussed above. So $Q_U\sqrt{n}$ (and similarly $Q_L\sqrt{n}$) is also normally distributed with a standard deviation of 1. Hence, it is possible to compute a value k such that, for a given sample size and AQL, the sample's quality statistic is, with a stated probability, greater than or equal to k when the lot is acceptable.

B.7.2 Acceptability criteria for single or separate double specification limits

For a value of the acceptability constant k computed as indicated in B.7.1 and which, for normal, tightened and reduced inspection is given in tables III-A, III-B and III-C respectively, the acceptability criterion becomes, for a single upper specification limit,

$$\text{acceptable if } Q_U = \frac{U - \bar{x}}{\sigma} \geqslant k$$

and, for a single lower specification limit,

$$\text{acceptable if } Q_L = \frac{\bar{x} - L}{\sigma} \geqslant k$$

otherwise, not acceptable.

Where separate double specification limits are given, the acceptability criterion is

acceptable if $Q_U \geqslant k_U$ and $Q_L \geqslant k_L$

not acceptable if either $Q_U < k_U$ or $Q_L < k_L$

Since σ is a known constant, it will, in practice, be more convenient to re-arrange the acceptability criteria in the form $\bar{x} \leqslant U - k\sigma$ for an upper specification limit and $\bar{x} \geqslant L + k\sigma$ for a lower limit, as the right-hand side of these criteria can be determined before inspection begins.

B.7.3 Acceptability criterion for a combined double specification limit

When a combined double specification limit is given, in order to establish whether the lot is acceptable, the point

$$\left(\frac{\sigma}{U - L}, \frac{\bar{x} - L}{U - L}\right)$$

may be plotted on the appropriate diagram taken from section four, diagrams σ-C to σ-P (compare with B.5.3); however, since σ is a known constant the point will lie on a vertical line corresponding to this value of σ. It is therefore sufficient in practice to obtain $\bar{x}_U$ and $\bar{x}_L$, the upper and lower acceptance values of $\bar{x}$. (See figure 14.)

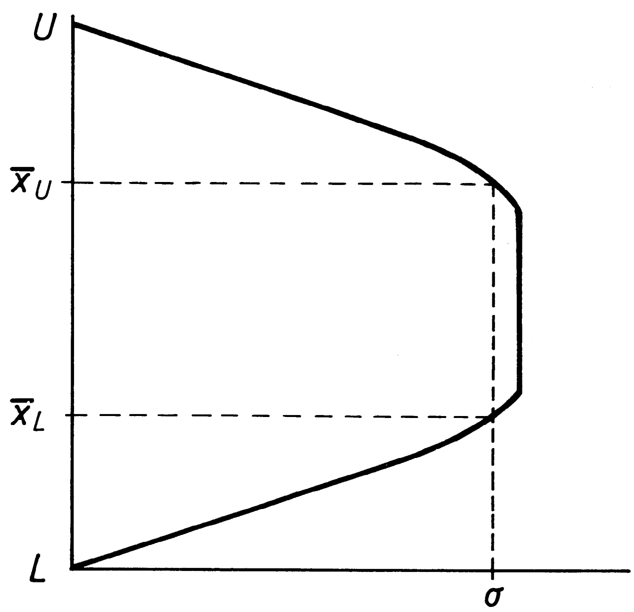


Figure 14 — Acceptance chart for a combined double specification limit: "σ" method

The acceptability criteria then becomes: the lot is

acceptable if $\bar{x} \leqslant \bar{x}_U$ and $\bar{x} \geqslant \bar{x}_L$,

not acceptable if either $\bar{x} > \bar{x}_U$ or $\bar{x} < \bar{x}_L$.

B.8 The "s" method

B.8.1 Derivation of the acceptability constant

When neither the mean nor the standard deviation of a population can be presumed to be known, they both have to be estimated from a sample taken from that population. In the "s" method, the mean of the sample, $\bar{x}$, is taken instead of μ, the mean of the population, and

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}$$

from the sample is taken instead of σ, so

$$Q_U = \frac{U - \bar{x}}{s} \quad \text{and} \quad Q_L = \frac{\bar{x} - L}{s}$$

Compared with the "σ" method, this increases the possibility of error, for when s is substituted for σ in the expression $(\bar{x} - \mu)\sqrt{n}/\sigma$ (see B.7.1), it is found that $(\bar{x} - \mu)\sqrt{n}/s$, which may be written

$$\frac{\sigma}{s} \; \frac{(\bar{x} - \mu)\sqrt{n}}{\sigma}$$

is no longer normally distributed, but has a "t" distribution with $(n - 1)$ degrees of freedom. For instance, if $n = 10$, there is now a 95 % probability that the difference between the mean of the sample and the mean of the lot will be not more than $\pm$ 2,26 $s/\sqrt{10}$, compared with $\pm$ 1,96 $\sigma/\sqrt{10}$ obtained by putting $n = 10$.

Further,

$$Q_U\sqrt{n} = \frac{(U - \bar{x})\sqrt{n}}{s}$$

which may be written as

$$Q_U\sqrt{n} = \frac{\sigma}{s} \left[\frac{(U - \mu)\sqrt{n}}{\sigma} - \frac{(\bar{x} - \mu)\sqrt{n}}{\sigma} \right]$$

and

$$Q_L\sqrt{n} = \frac{(\bar{x} - L)\sqrt{n}}{s}$$

which may be written as

$$Q_L\sqrt{n} = \frac{\sigma}{s} \left[\frac{(\mu - L)\sqrt{n}}{\sigma} + \frac{(\bar{x} - \mu)\sqrt{n}}{\sigma} \right]$$

are also not normally distributed, but they do follow a "non-central t" distribution and this distribution has been tabulated. The Resnikoff and Lieberman tables[10] were produced for sampling inspection by variables in particular. The detailed theory of this application of the non-central t distribution is covered in the introduction to the tables and the arguments used fit our standard sample sizes and preferred AQLs.

The acceptability constant k has been calculated, using the non-central t tables, for a standard range of sample size and AQLs, and is given in tables II-A, II-B and II-C for normal, tightened and reduced inspection respectively.

B.8.2 Acceptability criteria for single or separate double specification limits

The acceptability criteria are, as stated in clause B.6, similar to those given in clause B.5, but substituting the estimated values of the mean, standard deviation and quality statistic obtained from a sample and comparing the latter with the acceptability constant k calculated for this method.

The acceptability criterion for a single specification limit is: the lot is

acceptable if $Q_U \geqslant k$ or $Q_L \geqslant k$

not acceptable if $Q_U < k$ or $Q_L < k$

When separate double specification limits are given, the acceptability criterion is: the lot is

acceptable if both $Q_U \geqslant k_U$ and $Q_L \geqslant k_L$

not acceptable if either $Q_U < k_U$ or $Q_L < k_L$

The graphical method illustrated in figures 8, 9 and 10 is also applicable to both the above criteria. An example is given in 14.3 and illustrated in figure 2.

The upper boundary for the accept zone of an upper limit is given by the straight line $\bar{x} = U - ks$ and the lower boundary for the accept zone of a lower limit is given by the straight line $\bar{x} = L + ks$. When separate upper and lower specification limits are given, the accept zone is the area between these two lines and the $\bar{x}$ axis, as in figure 2.

B.8.3 Acceptability criteria for a combined double specification limit

When a combined double specification limit is given, it was shown in B.5.3 that the acceptability of a process depended on whether the point (σ,μ) lies within the appropriate acceptance curve on the acceptance diagram. (See figure 13.)

The acceptability of a lot may be found when the values of s and $\bar{x}$, obtained from a sample, are plotted on a similar acceptance diagram, indexed in terms of the sample size and AQL.

Acceptances curves have been calculated for the standard plans given in this International Standard, making allowance for the uncertainty introduced by the need to use $\bar{x}$ as an estimate for μ and s as an estimate for σ in the "s" method.

These curves are shown in diagrams s-D to s-P in section four. They are to a common scale, i.e. instead of plotting $\bar{x}$ against s,

$$\frac{\bar{x} - L}{U - L} \text{ is plotted against } \frac{s}{U - L}$$

when using the "s" method.

For sample size code letters B and C (i.e. sample sizes 3 and 4), the accept zone is bounded by four straight lines, the $\bar{x}$ axis, the line $\bar{x} = U - k\,s$, a line parallel to the $\bar{x}$ axis through the MSSD, (see table IV-*s*) and the line $\bar{x} = L + k\,s$, the value of k being obtained from table II-A, II-B or II-C.

The acceptability criterion is that the lot is acceptable if the point $[s/(U - L), (\bar{x} - L)/(U - L)]$ lies inside the accept zone and that the lot is not acceptable otherwise.

In practice, when a series of lots is being inspected, the Inspector's task is made easier if special acceptance curves in terms of coordinates s and $\bar{x}$ (see figure 3) are produced for both the normal and the tightened inspection plan.

If s is larger than the MSSD (which may be found in table IV-*s*), the lot has to be immediately judged unacceptable. (Compare with B.5.2.)

B.8.4 Complex specification limits

When combinations of the above specifications are given, e.g. a separate AQL (corresponding to an acceptability constant k_1) for the upper limit, combined with an overall AQL, the corresponding limit lines or curves are drawn and the accept zone is the area common to both requirements. (See figure 15.)

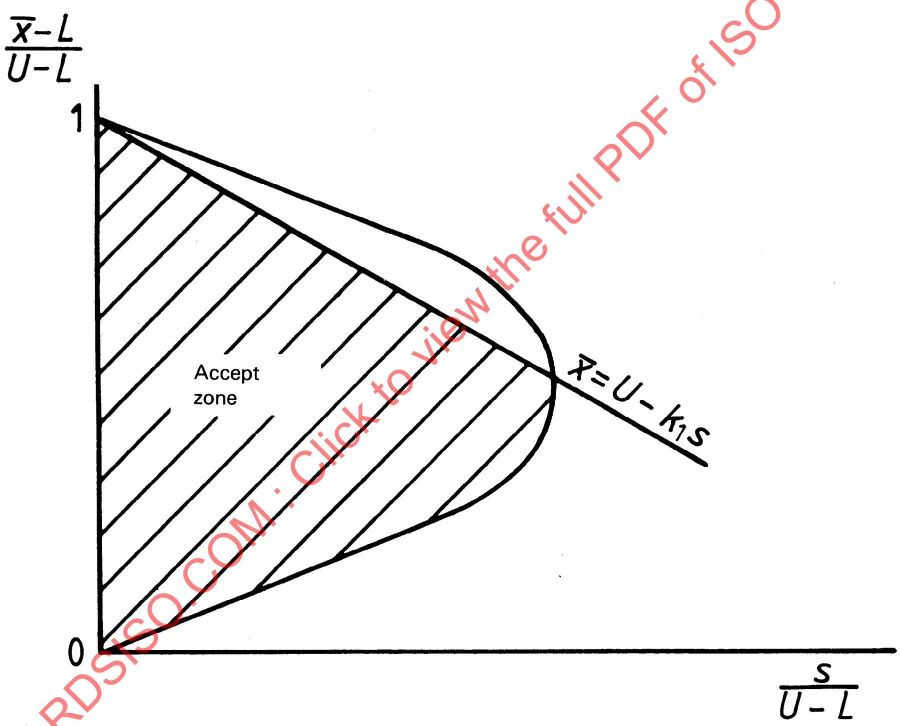


Figure 15 — Acceptance chart for complex specification limits: "*s*" method

Annex C

Sampling plans for "R" method

(This annex forms an integral part of the standard.)

C.1 Object

This method provides an alternative to the "s" method given in clause 14.

C.2 Definitions

The additional terms for use with this method are as follows :

C.2.1 "R" method : A method of assessing the acceptability of a lot by using an estimate of the standard deviation based on the average range of the measurements of the items in sub-groups of the sample.

C.2.2 sub-group : Five units taken in the order in which they were measured.

C.2.3 range (R) : Restricted sense, when using the "R" method : the range of a sub-group.

NOTE — The range, in its ordinary meaning, is the difference between the greatest and the smallest observed values.

C.2.4 average range ($\overline{R}$) : The mean of the ranges of the sub-groups. For samples of less than ten items, the range of the whole sample is taken as $\overline{R}$.

C.2.5 maximum average range (MAR) : Under given conditions, the largest acceptable average range.

C.3 Symbols

Additional symbols for use with this method are as follows :

c Scale factor given in table RI for assessing σ from $\overline{R}$.

F A factor, given in table RIV, which relates the maximum average range to the difference between U and L.

k The acceptability constant.

R Range of a sub-group.

$\overline{R}$ Average range, based on several sub-groups.

C.4 Theory

The "R" method provides an alternative way of estimating the standard deviation.

The theoretical basis of this method is that for small samples the range R and s, where both are calculated from the same normal sample, are highly correlated, which enables R to be used as an estimator instead of s. This relationship is used when dealing with samples of less than ten items. However, as the sample size increases, R becomes a less precise estimator and may no longer be used, but if the sample is split randomly into small sub-groups of equal size, the average of their ranges $\overline{R}$ may be used to estimate the standard deviation.

In the standard plans for the "R" method the sample sizes are, with the exception of three plans having sample sizes 3, 4, and 7, all multiples of 5. A sample is split into sub-groups of five items and the range R of each sub-group is obtained and hence the average range $\overline{R}$. When the sample consists of less than 10 items, it is not split into sub-groups and the range is treated as though it were $\overline{R}$.

A **scale factor** c has been calculated in terms of the sub-group sizes and the number of sub-groups, such that s may be replaced by $\overline{R}/c$. This scale factor c is given for the various sample sizes, on the basis of the sub-group sizes used in this International Standard, in table RI. This table is based on table 30 of the Biometrika Tables, Volume 1[11].

However, the use of the scale factor c given in table RI is not usually required as, in order to reduce the numerical work to be carried out, a special set of tables has been provided for the "R" method. Then the upper and lower quality statistics are directly defined as

$$Q_U = \frac{U - \overline{x}}{\overline{R}}$$

and

$$Q_L = \frac{\overline{x} - L}{\overline{R}}$$

Table RI provides also a comparison by a code letter of the sample sizes for the "R" method, the "s" method and the attributes method given in ISO 2859. The sample sizes are somewhat larger in the "R" method than in the "s" method, but they are both smaller than for the attributes method.

C.5 The acceptability criteria

For each plan an acceptability constant k is calculated such that the acceptability criteria are, for a single upper specification limit,

$$\text{accept if } Q_U = \frac{U - \overline{x}}{\overline{R}} \geqslant k$$

and, for a single lower specification limit,

$$\text{accept if } Q_L = \frac{\overline{x} - L}{\overline{R}} \geqslant k$$

otherwise, do not accept.

When separate upper and lower specification limits are given, the acceptability criterion is that the lot is

acceptable if both $Q_U \geqslant k_U$ and $Q_L \geqslant k_L$

not acceptable if either $Q_U < k_U$ or $Q_L < k_L$

The graphical method described in 14.3 is applicable to this method putting $\bar{R}$ in the place of s.

When a combined double specification limit is given, the point

$$\left(\frac{\bar{R}}{U - L}, \frac{\bar{x} - L}{U - L} \right)$$

is plotted on the appropriate diagram R-D to R-P in order to establish whether the lot is acceptable. Note that in the "R" method the equivalent of the maximum sample standard deviation (MSSD) is the maximum average range (MAR). Its value can be obtained by entering table RIV with the sample size and AQL. The point $[\bar{R} = \text{MAR}, \bar{x} = 1/2\ (U + L)]$ gives the apex of the acceptance curve. If the value of $\bar{R}$ is greater than the MAR, the lot is not to be accepted.

C.6 The operating characteristic curve

The operating characteristic (OC) curves calculated for the "s" method are applicable to this method with little loss of accuracy. They are identified by code letters and AQL values.

C.7 Control charts

The technique of maintaining records in the form of control charts recommended in 18.1 applies equally to the "R" method.

The MAR should be plotted on the R chart when a combined double specification limit has been specified.

If it appears that $\bar{R}$ is generally in control, the weighted root mean square values of $\bar{R}/c$ may be presumed to be σ, the factor c being given in table RI. The "σ" method may be then adopted when permission to switch to this method has been received from the responsible authority concerned. (See clause A.2 in annex A.)

C.8 Choice of method

If a modern calculator is available, the use of the "s" method is normally recommended, but, if the calculation of s poses an impediment to this method, the "R" method may be used as an alternative to the "s" method.

The "R" method is simple to calculate, but requires a somewhat larger sample size.

C.9 Operation of an "R" method sampling plan

C.9.1 Obtaining a plan

Check at first that the instructions given in clause 13 have been applied.

The standard procedure for an "R" method plan is as follows :

a) With the inspection level given (normally this will be II) and the lot size, obtain the sample size code letter from table I-A.

b) Using the sample size code letter and the AQL specified, enter table RII-A and obtain the sample size n and acceptability constant k.

C.9.2 Operating a plan

C.9.2.1 Take the individual items of the sample in the random order in which they were selected and measure them for the characteristic being inspected. Record the measurements in this order.

C.9.2.2 Find the sum Σx of all these measurements and, dividing by n, the number of items in the sample, **obtain the sample mean**

$$\bar{x} = \frac{\Sigma x}{n}$$

C.9.2.3 Obtain the value of $\bar{R}$.

a) Where there are ten or more items, split the record of the measurements, as they were taken, into sub-groups of five. (Since the larger sample sizes for standard plans are in multiples of five, this will always be possible.) Obtain the range for each sub-group by subtracting the smallest from the largest measurement and then calculate the average range $\bar{R}$.

b) Samples of less than ten items are not subdivided, the difference between the largest and smallest measurement gives the range and this is then treated as though it were the average range $\bar{R}$.

C.9.2.4 Apply the acceptability criterion.

If a single specification limit or separate double specification limits are given, calculate the quality statistic

$$Q_U = (U - \bar{x})/\bar{R}$$

and/or

$$Q_L = (\bar{x} - L)/\bar{R}$$

as appropriate;

then compare the quality statistic (Q_U and/or Q_L) with the acceptability constant (k_U and/or k_L) obtained from table RII-A (for normal inspection). If the quality statistic is greater than or equal to this acceptability constant, the lot is acceptable; if less, it is not acceptable. Thus, if an upper specification limit U is given, the lot is

acceptable if $Q_U \geqslant k$

not acceptable if $Q_U < k$

Or, if a lower specification limit L only is given, the lot is

acceptable if $Q_L \geqslant k$

not acceptable if $Q_L < k$

When both U and L are given : (k values are different when AQLs are different for the upper and the lower limits), the lot is

acceptable if both $Q_L \geqslant k_L$ and $Q_U \geqslant k_U$

not acceptable if either $Q_L < k_L$ or $Q_U < k_U$

Example

The lower specification limit for electrical resistance of a certain electrical component is 580 Ω. A lot of 100 items is submitted for inspection. Inspection level II, normal inspection, with AQL = 1 % is to be used. From table I-A the sample size code letter is F; from *R*II-A it is seen that a sample of size 10 is required and that the acceptability constant k is 0,703. Suppose that values of the sample resistances are obtained in the following order :

First sub-group 610; 615; 629; 593; 617
(R = 629 − 593 = 36).

Second sub-group 623; 589; 608; 591; 611
(R = 623 − 589 = 34).

Compliance with the acceptability criterion is to be determined.

Information needed	Value obtained
Sample size : n	10
Sample mean $\bar{x}$: $\Sigma\, x/n$	608,6
Average range $\bar{R}$: $\Sigma\, R$/No. of sub-groups $= \dfrac{36 + 34}{2}$	35
Specification limit (lower) : L	580
$Q_L = (\bar{x} - L)/\bar{R}$	0,817
Acceptability constant : k (see table **R**II-A)	0,703
Acceptability criterion : compare Q_L with k	0,817 > 0,703

The lot meets the acceptability criterion, so the lot is acceptable.

C.9.2.5 When a graphical criterion is desired for a single or separate double specification limit, draw the line

$\bar{x} = U - k\,\bar{R}$ (for an upper limit), straight line through the point ($\bar{R}$ = 0, $\bar{x}$ = U) with a slope $-k$,

and/or

$\bar{x} = L + k\,\bar{R}$ (for a lower limit), straight line through the point ($\bar{R}$ = 0, $\bar{x}$ = L) with a slope k,

as appropriate, on graph paper with $\bar{x}$ as the vertical axis and $\bar{R}$ as the horizontal axis. So the graph can be prepared before beginning the inspection of a series of lots (following the procedure given in 14.3 for the "s" method). Then use the values of $\bar{R}$ and $\bar{x}$ from the measurements obtained from each sample and plot the point ($\bar{R}$, $\bar{x}$) on the graph. If this point lies in the accept zone, the lot is acceptable; if outside, it is not acceptable.

Example

Using the figures given in the example above, mark the point L = 580 on the $\bar{x}$ (vertical) axis, draw a line through this point with a slope k [as k = 0,703, this means the line will pass through points ($\bar{R}$ = 10, $\bar{x}$ = 587), ($\bar{R}$ = 20, $\bar{x}$ = 594,1), ($\bar{R}$ = 40, $\bar{x}$ = 608,1), etc.]. Select a suitable point and draw a straight line through it and ($\bar{R}$ = 0, $\bar{x}$ = 580), i.e. L. The accept zone is then the area above this line. The calculated values of $\bar{R}$ and $\bar{x}$ are 35 and 608,6. Plotting the point ($\bar{R}$, $\bar{x}$), it will be seen from figure 16 that it lies inside the accept zone; the lot is accepted.

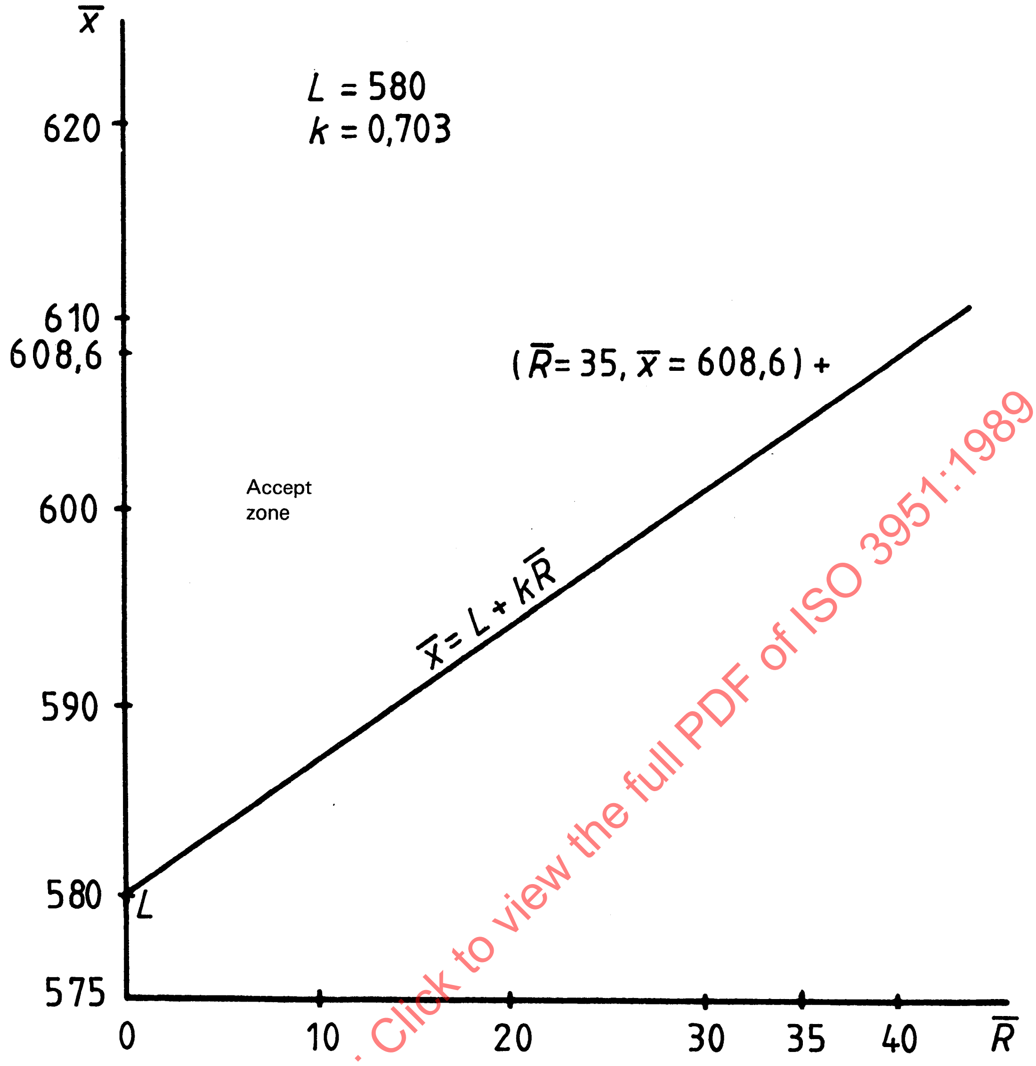


Figure 16 — Graphical method for single or separate specification limits

If a combined upper and lower specification limit has been given, it will be necessary to use a graphical method, unless $\bar{R}$ is greater than the MAR obtained from table *R*IV, in which case the lot has to be immediately judged not acceptable.

Using the sample size code letter, select the appropriate chart in the *R* series. From this chart, choose the acceptance curve marked with the selected AQL.

Take a tracing of this acceptance curve and the one immediately inside it. (The inner curve will be required if it is necessary to switch to tightened inspection and the area between the two provides a warning zone.)

Calculate the values of

$$\frac{\bar{R}}{U - L} \quad \text{and} \quad \frac{\bar{x} - L}{U - L}$$

and plot them on the tracing just produced.

If the point lies inside the accept zone, the lot is acceptable; if outside, it is not acceptable.

NOTE — It may be more convenient to adjust the scale of the working version of this curve so that $\bar{R}$ and $\bar{x}$ can be plotted directly. (See figure 17.)

Example

The specification for electrical resistance of a certain electrical component is 520 ± 50 Ω. A lot of 350 items is submitted for inspection. Inspection level II, normal inspection, with AQL = 4 % is to be used. From table I-A, the sample size code letter is H; from table *R*II-A it is seen that a sample size of 25 is required and from table *R*IV that the *F* value of MAR is 0,707. Suppose the values of the sample resistance are obtained in the following order.

First sub-group 515; 491; 479; 507; 543
(R = 543 − 479 = 64).

Second sub-group 521; 536; 483; 509; 514
(R = 536 − 483 = 53).

Third sub-group 507; 484; 526; 552; 499
(R = 552 − 484 = 68).

Fourth sub-group 530; 512; 492; 521; 467
(R = 530 − 467 = 63).

Fifth sub-group 489; 513; 535; 501; 529
(R = 535 − 489 = 46).

Information needed	Value obtained
Sample size : n	25
Sample mean $\overline{x}$: $\Sigma\, x/n$	510,2
Average range $\overline{R}$: $\Sigma\, R$/No. of sub-groups = 294/5	58,8
Upper specification limit : U	570
Lower specification limit : L	470
"Standardized" mean : $(\overline{x} - L)/(U - L)$	0,402
"Standardized" average range : $\overline{R}/(U - L)$	0,588
F value (see table RIV)	0,707
MAR = $F\,(U - L)$	70,7

The "standardized" mean and average range are then plotted on the graph (R-H, AQL 4 %). (See figure 17.) If this plotted point lies outside the acceptance curve, the lot is not to be accepted.

The point (0,588, 0,402) lies inside the acceptance curve and the lot is accepted.

C.10 Procedure during continuing inspection

The procedures given in clauses 16 to 22 for the "s" method are to be applied when using the "R" method.

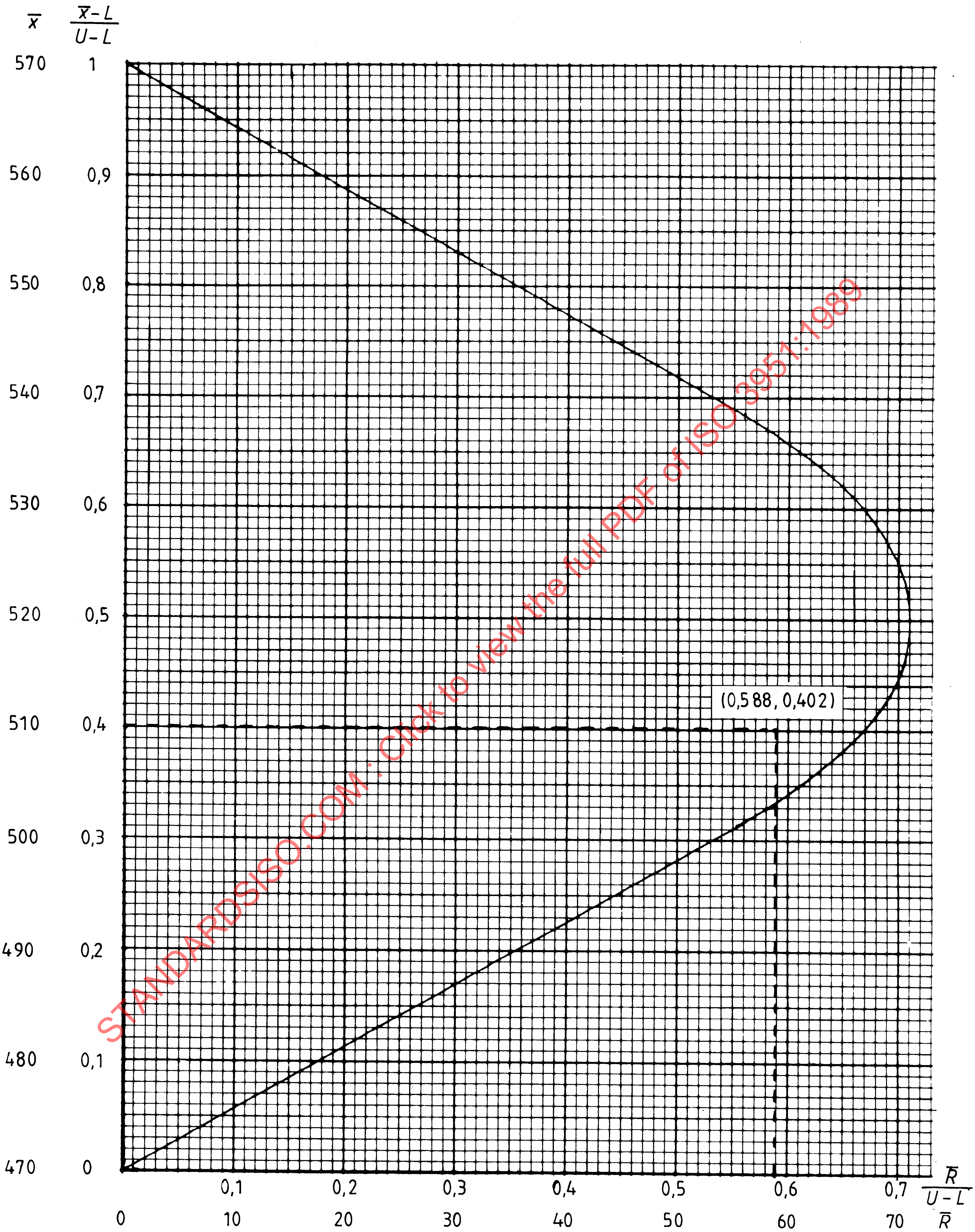


Figure 17 — Acceptance curve for code letter H, *n* = 25, AQL = 4 %, MAR = 70,7 : "*R*" method