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**Sampling procedures for inspection by variables —**

Part 1:

**Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL**

*Règles d'échantillonnage pour les contrôles par mesures —*

*Partie 1: Spécification pour les plans d'échantillonnage simples indexés d'après un niveau de qualité acceptable (NQA) pour un contrôle lot par lot pour une caractéristique qualité unique et un NQA unique*



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## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see [www.iso.org/iso/foreword.html](http://www.iso.org/iso/foreword.html).

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 5, *Acceptance sampling*.

This third edition cancels and replaces the second edition (ISO 3951-1:2013), which has been technically revised.

The main changes are as follows:

- procedures have been introduced to accommodate measurement uncertainty;
- many of the sampling plans have been adjusted to improve the match between their operating characteristic curves and the operating characteristic curves of the corresponding plans for single sampling by attributes in ISO 2859-1.

A list of all parts in the ISO 3951 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at [www.iso.org/members.html](http://www.iso.org/members.html).

## Introduction

This document specifies an acceptance sampling system of single sampling plans for inspection by variables. It is indexed in terms of the acceptance quality limit (AQL). A more comprehensive and technical treatment of the AQL scheme is given in ISO 3951-2. This document is complementary to ISO 2859-1.

The objectives of the methods laid down in this document are to ensure that lots of acceptable quality have a high probability of acceptance and that the probability of not accepting inferior lots is as high as practicable. This is achieved by means of the switching rules, which provide the following:

- a) an automatic protection to the consumer (by means of a switch to tightened inspection or discontinuation of sampling inspection) should a deterioration in quality be detected; and
- b) an incentive (at the discretion of the responsible authority) to reduce inspection costs (by means of a switch to a smaller sample size) should consistently good quality be achieved.

In this document, the acceptance of a lot is implicitly determined from an estimate of the percentage of nonconforming items in the process, based on a random sample of items from the lot.

This document is intended for application to a continuing series of lots of discrete products all supplied by one producer using one production process. If there are different producers or production processes, this document is applied to each one separately.

This document is intended for application to a single quality characteristic that is measurable on a continuous scale and is normally distributed. For two or more such quality characteristics, see ISO 3951-2. For information on normality and data transformations, see [Clause 12](#).

It is assumed in the body of this document that measurement error is negligible (see ISO 10576-1:2003). For information on allowing for measurement error, see [Annex B](#).

For double specification limits, this document covers combined control. For other types of control, refer to ISO 3951-2.

**CAUTION — The procedures in this document are not suitable for application to lots that have been screened for nonconforming items.**

Inspection by variables for nonconforming items, as described in this document, includes several possible modes, the combination of which leads to a presentation that may appear quite complex to the user:

- unknown standard deviation, or originally unknown then estimated with fair precision, or known since the start of inspection;
- a single specification limit, or combined control of double specification limits;
- normal inspection, tightened inspection, or reduced inspection.

The choice of the most suitable variables plan, if one exists, requires experience, judgement, and some knowledge of both statistics and the product to be inspected. [Clause 5](#) of this document is intended to help those responsible for specifying sampling plans in making this choice. They suggest the considerations that should be kept in mind when deciding whether a variables plan is suitable and the choices to be made when selecting an appropriate standard plan.

The basic definitions and notations are provided in [Clauses 3](#) and [4](#). The basic operational rules are contained in [Clauses 5](#) through [9](#). [Clause 10](#) informs about the relations between this document and the attributes sampling standard ISO 2859-1. [Clauses 11](#), [12](#) and [13](#) provide background on accounting for measurement uncertainty, the normality assumption, and monitoring of inspection results and the underlying process. All tables needed for the sampling procedure can be found in [Clause 14](#) and examples for the  $s$ -method and the  $\sigma$ -method for both one and two specification limits can be found in [Clause 15](#).

Nine annexes are provided. [Annex A](#) indicates how the sample standard deviation,  $s$ , and the presumed known value of the process standard deviation,  $\sigma$ , should be determined. [Annex B](#) provides procedures for accommodating measurement uncertainty. [Annex C](#) shows five different sampling strategies. [Annex D](#) gives the general formula for the operating characteristic of the  $\sigma$ -method and provides tables with values of the operating characteristics of single sampling plans with known  $\sigma$ . [Annex E](#) gives the general formula for the operating characteristic of the  $s$ -method and provides tables with values of the operating characteristics of single sampling plans with unknown  $\sigma$ . [Annex F](#) provides the statistical theory underlying the calculation of the consumer's risk qualities, together with tables showing these quality levels for normal, tightened, and reduced inspection, as well as for the  $s$ -method and  $\sigma$ -method. [Annex G](#) provides similar information for the producer's risks. [Annex H](#) give details of how Acceptance diagrams for double specification limits are constructed, [Annex I](#) shows the use of the underlying software (R package to support implementation of this document).

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# Sampling procedures for inspection by variables —

## Part 1:

# Specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection for a single quality characteristic and a single AQL

## 1 Scope

This document specifies single sampling plans for lot-by-lot inspection under the following conditions:

- a) where the inspection procedure is applied to a continuing series of lots of discrete products, all supplied by one producer using one production process;
- b) where only a single quality characteristic,  $x$ , of these products is taken into consideration, which is measurable on a continuous scale;
- c) where production is under statistical control and the quality characteristic,  $x$ , is distributed according to a normal distribution or a close approximation to the normal distribution;
- d) where a contract or standard defines a lower specification limit,  $L$ , an upper specification limit,  $U$ , or both. An item is qualified as conforming if its measured quality characteristic,  $x$ , satisfies as appropriate one of the following inequalities:
  - 1)  $x \geq L$  (i.e. the lower specification limit is not violated);
  - 2)  $x \leq U$  (i.e. the upper specification limit is not violated);
  - 3)  $x \geq L$  and  $x \leq U$  (i.e. neither the lower nor the upper specification limit is violated).

Inequalities 1) and 2) are cases with a single specification limit, and 3) is a case with double specification limits.

Where double specification limits apply, it is assumed in this document that conformity to both specification limits is equally important to the integrity of the product. In such cases, it is appropriate to apply a single AQL to the combined percentage of a product outside the two specification limits. This is referred to as combined control.

## 2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2859-1, *Sampling procedures for inspection by attributes — Part 1: Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection*

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 3951-2, *Sampling procedures for inspection by variables — Part 2: General specification for single sampling plans indexed by acceptance quality limit (AQL) for lot-by-lot inspection of independent quality characteristics*

### 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2859-1, ISO 3534-1 and ISO 3534-2 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at <https://www.iso.org/obp>

— IEC Electropedia: available at <https://www.electropedia.org/>

#### 3.1 inspection by variables

inspection by measuring the magnitude(s) of a characteristic(s) of an item

[SOURCE: ISO 3534-2:2006, 4.1.4]

#### 3.2 sampling inspection

inspection of selected items in the group under consideration

[SOURCE: ISO 3534-2:2006, 4.1.6]

#### 3.3 acceptance sampling inspection

acceptance inspection where the acceptability is determined by means of *sampling inspection* (3.2)

[SOURCE: ISO 3534-2:2006, 4.1.8]

#### 3.4 acceptance sampling inspection by variables

*acceptance sampling inspection* (3.3) in which the acceptance of the process is determined statistically from measurements from *inspection by variables* (3.1)

[SOURCE: ISO 3534-2:2006, 4.2.11 modified — “on specified quality characteristics of each item in a sample from a lot” has been replaced by “from inspection by variables”]

#### 3.5 process fraction nonconforming

rate at which nonconforming items are generated by a process

Note 1 to entry: It is expressed as a proportion.

#### 3.6 acceptance quality limit

**AQL**  
(acceptance sampling) worst tolerable *quality level* (3.7)

Note 1 to entry: This concept only applies when an acceptance sampling scheme with rules for switching and for discontinuation, such as ISO 2859-1 and ISO 3951, is used.

Note 2 to entry: See 5.4.

[SOURCE: ISO 3534-2:2006, 4.6.15 modified — Notes 2 to 4 to entry have been deleted and Note 1 to entry has been added.]

**3.7****quality level**

quality expressed as the fraction nonconforming

[SOURCE: ISO 3534-2:2006, 4.6.16 modified — “or rate of number of nonconformities” has been removed”.]

**3.8****limiting quality****LQ**

*quality level* (3.7), when a lot is considered in isolation, which, for the purposes of *acceptance sampling inspection* (3.3), is limited to a low probability of acceptance

Note 1 to entry: In this document: 10 %.

[SOURCE: ISO 3534-2:2006, 4.6.13, modified — Note 1 to entry has been added.]

**3.9****nonconformity**

non-fulfilment of a requirement

[SOURCE: ISO 3534-2:2006, 3.1.11]

**3.10****nonconforming unit**

unit with one or more nonconformities

[SOURCE: ISO 3534-2:2006, 1.2.15]

**3.11*****p*\*-method acceptance sampling plan**

*acceptance sampling* (3.3) plan by variables where the the estimated fraction nonconforming  $\hat{p}$  is compared to the maximum acceptable value  $p^*$ .

Note 1 to entry: The method is applicable to both the *s-method* (3.12) and the *σ-method* (3.13) and gives equivalent results. It has the advantage that it deals directly with the fraction nonconforming.

**3.12*****s*-method acceptance sampling plan**

*acceptance sampling* (3.3) plan by variables using the sample standard deviation

Note 1 to entry: See [Clause 6](#).

[SOURCE: ISO 3534-2:2006, 4.3.10 modified — “s method” has been replaced by “s-method” and “acceptance sampling plan” has been added]

**3.13*****σ*-method acceptance sampling plan**

*acceptance sampling* (3.3) plan by variables using the presumed value of the process standard deviation

Note 1 to entry: See [Clause 7](#).

[SOURCE: ISO 3534-2:2006, 4.3.9 modified — “sigma method” has been replaced by “σ-method” and “acceptance sampling plan” has been added]

**3.14****specification limit**

conformance boundary specified for a characteristic

[SOURCE: ISO 3534-2:2006, 3.1.3, modified — “limiting value stated” has been replaced with “conformance boundary specified”.]

**3.15**  
**lower specification limit**

*L*

*specification limit* (3.14) that defines the lower conformance boundary

[SOURCE: ISO 3534 2:2006, 3.1.5, modified — "limiting value" has been replaced with "conformance boundary".]

**3.16**  
**upper specification limit**

*U*

*specification limit* (3.14) that defines the upper conformance boundary

[SOURCE: ISO 3534-2:2006, 3.1.4, modified — "limiting value" has been replaced with "conformance boundary".]

**3.17**  
**combined control**

requirement when both upper and lower limits are specified for the quality characteristic and an *AQL* (3.6) that applies to the combined percent nonconforming beyond the two limits is given

Note 1 to entry: See 5.4.

Note 2 to entry: The use of combined control implies that nonconformity beyond either *specification limit* (3.14) is believed to be of equal, or at least roughly equal, importance to the lack of integrity of the product.

**3.18**  
**form *k* acceptance constant**

*k*

constant depending on the specified value of the *acceptance quality limit* (3.6) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

Note 1 to entry: See [Clause 6](#) and [Clause 7](#).

[SOURCE: ISO 3534-2:2006, 4.4.4, modified — "acceptability constant" has been replaced with "acceptance constant". The original Note 1 to entry has been deleted, a new Note 1 to entry has been added.]

**3.19**  
**form *p*\* acceptance constant**

*p*\*

constant depending on the specified value of the *acceptance quality limit* (3.6) and the sample size, used in the criteria for accepting the lot in an *acceptance sampling* (3.3) plan by variables

[SOURCE: ISO 3534-2:2006, 4.4.4, modified — "acceptability constant" has been replaced with "acceptance constant". The original Note 1 to entry has been deleted, a new Note 1 to entry has been added.]

**3.20**  
**quality statistic**

*Q*

function of the *specification limit* (3.14), the sample mean and the sample or process standard deviation, used in assessing the acceptance of a lot

Note 1 to entry: For the case of a single specification limit, the lot can be accepted or rejected on the result of comparing *Q* with the *form k acceptance constant* (3.18).

Note 2 to entry: See [Clause 6](#) and [Clause 7](#).

[SOURCE: ISO 3534-2:2006, 4.4.9, modified — In the Note 1 to entry, "may be sentenced" has been replaced with "can be accepted or rejected". Note 2 to entry has been added.]

### 3.21 lower quality statistic

$Q_L$

function of the *lower specification limit* (3.15), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single lower specification limit, the lot can be accepted or rejected on the result of comparing  $Q_L$  with the *form k acceptance constant* (3.18).

Note 2 to entry: See [Clause 4](#), [Clause 6](#), and [Clause 7](#).

[SOURCE: ISO 3534-2:2006, 4.4.11, modified — Note 2 to entry has been added.]

### 3.22 upper quality statistic

$Q_U$

function of the *upper specification limit* (3.16), the sample mean, and the sample or process standard deviation

Note 1 to entry: For a single upper specification limit, the lot can be accepted or rejected on the result of comparing  $Q_U$  with the *form k acceptance constant* (3.18).

Note 2 to entry: See [Clause 4](#), [Clause 6](#), and [Clause 7](#).

[SOURCE: ISO 3534-2:2006, 4.4.10, modified — Note 2 to entry has been added.]

### 3.23 maximum process standard deviation MPSD

$\sigma_{\max}$

largest process standard deviation for a given sample size code letter and *AQL* (3.6) for which it is possible to satisfy the acceptance criterion for a double specification limit under all inspection severities (i.e. normal, tightened and reduced) when the process variability is known

Note 1 to entry: The MPSD depends on whether the double specification limits are combined, separate or complex, but does not depend on the inspection severity.

Note 2 to entry: See [5.2](#).

[SOURCE: ISO 3534-2:2006, 4.4.8, modified — Note 2 to entry has been added.]

### 3.24 switching rule

instruction within an *acceptance sampling* (3.3) scheme for changing from one acceptance sampling plan to another of greater or lesser severity based on demonstrated quality history

Note 1 to entry: Normal, tightened, or reduced inspection or discontinuation of inspection are examples of 'severity of sampling'.

Note 2 to entry: See [Clause 9](#).

[SOURCE: ISO 3534-2:2006, 4.3.4, modified — Note 2 to entry has been added.]

### 3.25 measurement

set of operations to determine the value of some quantity

[SOURCE: ISO 3534-2:2006, 3.2.1, modified — "having the object of determining a value of a quantity" has been replaced with "to determine the value of some quantity". Notes 1 and 2 to entry have been deleted.]

## 4 Symbols

$f_\sigma$	factor that relates the maximum process standard deviation to the difference between $U$ and $L$ (see <a href="#">14.5</a> )
$F_{BETA(\alpha,\beta)}(x)$	the distribution of the standard beta distribution with parameters $\alpha$ and $\beta$ . In this document $\alpha = \beta = n/2 - 1$ throughout.
$F_{t(\nu,\delta)}(x)$	the distribution function of the non-central $t$ -distribution with $\nu$ degrees of freedom and non-centrality parameter $\delta$
$K_p$	the upper $p$ -quantile of the standardized normal distribution i.e. $x$ such that $1 - \Phi(x) = p$ , which corresponds to the process fraction nonconforming $p$
$k$	form $k$ acceptance constant for use with a single quality characteristic and a single specification limit (see <a href="#">14.1</a> for the $s$ -method acceptance sampling plan or <a href="#">14.2</a> for the $\sigma$ -method acceptance sampling plan)
$L$	lower specification limit (as a subscript to a variable, it denotes its value at $L$ )
$M$	unknown process mean
$N$	lot size (number of items in a lot)
$n$	sample size (number of items in a sample)
$\hat{p}$	estimate of the process fraction nonconforming
$\hat{p}_L$	estimate of the process fraction nonconforming below the lower specification limit
$\hat{p}_U$	estimate of the process fraction nonconforming above the upper specification limit
$p^*$	form $p^*$ acceptance constant i.e. the maximum acceptable value for the estimate of the process fraction nonconforming
$P_a$	probability of acceptance
$\Phi(x)$	the distribution function of the standardized normal distribution
$Q$	quality statistic
$Q_L$	lower quality statistic  NOTE $Q_L$ is defined as $(\bar{x} - L)/s$ when the process standard deviation is unknown, and as $(\bar{x} - L)/\sigma$ when it is presumed to be known.
$Q_U$	upper quality statistic  NOTE $Q_U$ is defined as $(U - \bar{x})/s$ when the process standard deviation is unknown, and as $(U - \bar{x})/\sigma$ when it is presumed to be known.
$S$	sample standard deviation of the measured values of the quality characteristic (also an estimate of the standard deviation of the process), i.e.

$$s = \sqrt{\frac{\sum_{j=1}^n (x_j - \bar{x})^2}{n-1}}$$

(See [Annex A](#).)

$\sigma$	process standard deviation NOTE $\sigma^2$ , the square of the process standard deviation, is known as the process variance.
$\sigma_{\max}$	maximum process standard deviation (MPSD)
$\sigma_{\text{root}}$	weighted root mean square of $s$
$U$	upper specification limit (as a suffix to a variable, it denotes its value at $U$ )
$x_j$	measured value of the quality characteristic for the $j^{\text{th}}$ item of the sample
$\bar{x}$	the arithmetic mean of the measured values of the quality characteristic in the sample, i.e.

$$\bar{x} = \frac{\sum_{j=1}^n x_j}{n}$$

## 5 Choice of a sampling plan

### 5.1 Choice between variables and attributes

The first question to consider is whether it is desirable to inspect variables rather than attributes. The following points should be taken into account.

- a) In terms of economics, it is necessary to compare the total cost of the relatively simple inspection of a larger number of items by means of an attributes scheme with the generally more elaborate procedure required by a variables scheme, which is usually more time-consuming and costly per item.
- b) In terms of the knowledge gained, the advantage lies with inspection by variables as the information obtained indicates more precisely the quality of the product. Therefore, earlier warning can be given if the quality is slipping.
- c) An attributes scheme can be more readily understood and accepted. For example, two phenomena of variables sampling are difficult to understand:
  - 1) lots containing 100 % conforming items can be rejected, occasionally even with a high probability (see example 3 in [15.1](#));
  - 2) there are cases of larger AQL and smaller lot size where lots containing 100 % nonconforming items are accepted.
- d) From a comparison of the size of the samples required for the same AQL from standard plans for inspection by attributes, such as from ISO 2859-1, and the standard plans in this document, the smallest samples are generally required by the  $\sigma$ -method (used when the process standard deviation is presumed to be known). The sample sizes for the  $s$ -method (used when the process standard deviation is presumed to be unknown) are larger than for the  $\sigma$ -method but are, in general, substantially smaller than for sampling by attributes.
- e) Inspection by variables is particularly appropriate in conjunction with the use of control charts for variables.
- f) Variables sampling has a substantial advantage when the inspection process is expensive, for example, in the case of destructive testing.
- g) A variables scheme becomes relatively more complicated to operate as the number of measurements to be taken on each item increases.

NOTE For two or more quality characteristics, that need to be treated together using a single AQL, this document does not apply. See ISO 3951-2 for details.

## 5.2 General

The sampling schemes found in this document, with their rules for switching and for discontinuation of sampling inspection, are designed to encourage suppliers to keep the process fraction nonconforming consistently better than the respective AQLs.

Otherwise, there is a high risk that the inspection severity be switched to tightened inspection, under which the criteria for lot acceptance become more demanding. Once on tightened inspection, unless action is taken to improve the process, it is very likely that the rule requiring discontinuation of sampling inspection be invoked pending such improvement.

The following procedures shall be followed before starting an inspection by variables:

- a) check that an AQL has been designated, and that it is one of the preferred AQLs for use with this document (see [Clause 5](#)). If it is not, then the sampling tables of this document are not applicable;

NOTE If an AQL other than a preferred AQL is designated, then this document is only applicable under the advice of a statistical expert.

- b) check that the inspection level to be used has been designated. If none has been given, the inspection level II shall be used (see [5.4](#));

- c) determine the quality characteristic  $x$  and an upper limit  $U$  and/or a lower limit  $L$  for  $x$ ;

- d) for a quality characteristic with double specification limits, check that nonconformities beyond each limit are of equal importance. If this is not the case, refer to ISO 3951-2;

NOTE If the lot size, the inspection level,  $U$ ,  $L$  or the AQL change during the process, the user might need to stop and begin again with [5.1](#).

- e) check whether the  $s$ -method ([Clause 6](#)) is to be used initially or whether the standard deviation is stable and known, in which case the  $\sigma$ -method ([Clause 7](#)) should be used (see [5.3](#));

- f) for the  $\sigma$ -method and a quality characteristic with double specification limits, a process capability study in the following sense should be done:

- 1) enter [Table 11](#) in [14.4](#) with the AQL to determine the value of the factor  $f_\sigma$ ;
- 2) calculate the maximum allowable value of the process standard deviation using the formula  $\sigma_{\max} = (U - L) f_\sigma$ ;
- 3) If  $\sigma$  exceeds  $\sigma_{\max}$ , the process is not capable and sampling inspection is pointless until it is demonstrated that the process variability has been adequately reduced.

- g) obtain the sample size code letter with the inspection limit (normally level II) and the lot size from [Table 1](#).

## 5.3 Choice between the $s$ -method and $\sigma$ -method

If the inspection by variables method as proposed in this document is chosen, the decision shall be made whether to use the  $s$ -method or the  $\sigma$ -method. The  $\sigma$ -method is the more economical in terms of sample size, but before this method can be applied, it is necessary to have a reliable value of  $\sigma$ , usually obtained from previous process analyses.

In the case that no reliable assumptions on the value of  $\sigma$  can be made, it shall be necessary to begin with the  $s$ -method, but subject to the agreement of the responsible authority and provided the quality remains satisfactory, the operational rules of this document permit a change to the  $\sigma$ -method and consequently the use of a smaller sample size.

If the variability is under control, a reliable estimation for the process standard deviation can be made from the observed data, in which case it may be economical to change to the  $\sigma$ -method. The size of the sample is generally smaller and the acceptance criterion is simpler using the  $\sigma$ -method (see [Clause 7](#)).

On the other hand, it shall still be necessary to calculate the sample standard deviation,  $s$ , for record and continued monitoring purposes of the product quality and to keep the pattern or control charts up to date (see [Clause 13](#)). Methods of determining  $s$  and  $\sigma$  are given in [Annex A](#).

In case of using the  $\sigma$ -method, it is recommended to perform, periodically and systematically, comparisons of observed data, i.e. observed process standard deviations with the assumption made on  $\sigma$ . In the framework of continued monitoring, this can be done in several ways:

- use of descriptive, graphical methods, e.g. pattern charts;
- use of statistical control charts;
- descriptive numerical analysis; or
- statistical tests, e.g. Bartlett's test for the comparison of variances.

If based on these analyses there is doubt in the reliability of the assumptions made on  $\sigma$ , it can be necessary to switch to the  $s$ -method.

#### 5.4 Choice of inspection level and AQL

The AQL is the quality level that is the worst tolerable nonconforming process fraction when a continuing series of lots is submitted for acceptance sampling. Although individual lots determined to be of "bad" quality may be accepted by the AQL with fairly high probability, the designation of an AQL does not suggest that this is a desirable quality level.

The AQL, together with the sample size code letter, is used to index the sampling plans in this document. For a standard sampling plan, the inspection level, in conjunction with the size of the lots and the AQL, determines the size of the sample to be taken and governs the severity of the inspection. The appropriate operating characteristic from [Annex E](#) shows the extent of the risk that is involved in such a plan.

The 16 AQLs given in this document, ranging in value from 0,01 % to 10 % nonconforming, are described as "preferred AQLs". They are only preferred in the sense that they are the AQL values used in the tabulations and charts. It follows that, if for any product or service, an AQL other than a preferred AQL is designated, then this document is only applicable under the advice of a statistical expert.

The AQL to be used shall be designated in the product specification or in the contract, or by the responsible authority. Where both upper and lower specification limits are given, this document addresses only the case of an overall AQL applying to the combined percent nonconforming beyond the two limits; this is known as "combined control." (See ISO 3951-2 for "separate" and "complex" control of double specification limits.)

From the definition of the AQL, it follows that the desired protection can only be ensured when a continuing series of lots is provided for inspection.

The designation of an AQL shall not imply that the supplier has the right to supply knowingly any nonconforming product.

The choice of inspection level and AQL is governed by a number of factors but is mainly a balance between the total cost of inspection and the consequences of nonconforming items passing into service.

Three inspection levels, I, II, and III, are given in [Table 1](#) for general use. Inspection level II shall be used unless special circumstances indicate that another level is more appropriate. Level I may be used when less discrimination is needed, and level III when greater discrimination is required. Four additional special levels, S-1, S-2, S-3, and S-4, are also given in [Table 1](#) and may be used when relatively small sample sizes are necessary and larger sampling risks can be tolerated, or when very high process capability has been demonstrated.

**Table 1 — Sample size code letters and inspection levels**

Lot or batch size	Special inspection levels				General inspection levels		
	S-1	S-2	S-3	S-4	I	II	III
2 to 8	B	B	B	B	B	B	B
9 to 15	B	B	B	B	B	B	C
16 to 25	B	B	B	B	B	C	D
26 to 50	B	B	B	C	C	D	E
51 to 90	B	B	C	C	C	E	F
91 to 150	B	B	C	D	D	F	G
151 to 280	B	C	D	E	E	G	H
281 to 500	B	C	D	E	F	H	J
501 to 1 200	C	C	E	F	G	J	K
1 201 to 3 200	C	D	E	G	H	K	L
3 201 to 10 000	C	D	F	G	J	L	M
10 001 to 35 000	C	D	F	H	K	M	N
35 001 to 150 000	D	E	G	J	L	N	P
150 000 to 500 000	D	E	G	J	M	P	Q
500 000 and over	D	E	H	K	N	Q	R

NOTE The sample size code letters and inspection levels in this document correspond to those given in ISO 2859-1 and ISO 3951-2.

## 6 Standard procedures for the s-method

### 6.1 General

The s-method shall be used if information about the process standard deviation is missing or unreliable. Using the s-method, the standard deviation is estimated directly from each sample. As soon as the conditions for the use of the  $\sigma$ -method are warranted, one may switch from the s-method to the  $\sigma$ -method (see 5.3).

### 6.2 Single specification limits

Before starting the inspection by variables, see Clause 5.

The procedure for a single specification limit is as follows.

- a) Enter Table 2, Table 3, or Table 4, as appropriate, with the code letter and the AQL to obtain the sample size,  $n$ , and the acceptance constant,  $k$ .
- b) Take a random sample of size  $n$ , measure the characteristic  $x$  in each item, and then calculate  $\bar{x}$ , the sample mean, and  $s$ , the sample standard deviation (see Annex A).

NOTE Some sampling strategies are provided in Annex C.

- c) Calculate the quality statistic  $Q_U = (U - \bar{x}) / s$  or  $Q_L = (\bar{x} - L) / s$  and compare it with the acceptance constant  $k$ . The lot is accepted if  $Q_U \geq k$  or  $Q_L \geq k$ ; or rejected if  $Q_U < k$  or  $Q_L < k$ .
- d) Proceed with Clause 9.

For examples of single lower and upper specification limits using the s-method, see Example 1 and Example 2 in 15.1.

### 6.3 Double specification limits

Before starting the inspection by variables, see [Clause 5](#).

The procedure for double specification limits for the  $s$ -method is as follows.

- a) Enter [Table 2](#), [Table 3](#), or [Table 4](#), as appropriate, with the code letter and the AQL to obtain the sample size,  $n$ , and the acceptance constant,  $k$ .
- b) Take a random sample of size,  $n$ , measure the characteristic  $x$  in each item, and then calculate  $\bar{x}$ , the sample mean, and  $s$ , the sample standard deviation (see [Annex A](#)).

NOTE Some sampling strategies are provided in [Annex C](#).

- c) Determination of acceptance.
  - 1) If  $\bar{x} < L$  or  $\bar{x} > U$ , reject the lot; or continue with the next step.
  - 2) Plot  $(s, \bar{x})$  on the acceptance diagram, which can be obtained using the accCurve function in the underlying software (see [L.5](#)).

NOTE The standardized values  $(s_S, \bar{x}_S)$  where  $s_S = \frac{s}{U-L}$  and  $\bar{x}_S = \frac{\bar{x}-L}{U-L}$  may be used in which case  $(s_S, \bar{x}_S)$  is plotted on the standardized acceptance diagram, which can be obtained using the accCurve function in the underlying software without providing the standardized lower and upper specification limits  $L = 0, U = 1$  (see [L.5](#)).

- 3). If the plotted point is outside the acceptance region the lot is rejected; otherwise the lot is accepted.
- d) Proceed with [Clause 9](#).

For examples of combined control of double specification limits using the  $s$ -method, see Example 3 to Example 6 in [15.2](#).

## 7 Standard procedures for the $\sigma$ -method

### 7.1 General

The  $\sigma$ -method shall only be used when there is valid evidence that the standard deviation  $\sigma$  of the process can be considered constant with a known value.

### 7.2 Single specification limits

Before starting the inspection by variables, see [Clause 5](#).

The procedure for a single specification limit is as follows.

- a) Enter [Table 5](#), [Table 6](#), or [Table 7](#), as appropriate, with the code letter and the AQL and obtain the sample size,  $n$ , and the acceptance constant,  $k$ .
- b) Take a random sample of size  $n$ , measure the characteristic  $x$  in each item, and then calculate  $\bar{x}$ , the sample mean, and  $s$ , the sample standard deviation (see [Annex A](#)).

NOTE 1 Some sampling strategies are provided in [Annex C](#).

NOTE 2 The sample standard deviation is needed for the records (see [Clause 13](#)).

- c) Calculate the quality statistic  $Q_U = (U - \bar{x})/\sigma$  or  $Q_L = (\bar{x} - L)/\sigma$  and compare it with the acceptance constant  $k$ . The lot is accepted if  $Q_U \geq k$  or  $Q_L \geq k$ ; or rejected if  $Q_U < k$  or  $Q_L < k$ .

For examples of single lower and upper specification limits using the  $\sigma$ -method, see Example 1 and Example 2 in [15.2](#).

### 7.3 Double specification limits

Before starting the inspection by variables, see [Clause 5](#).

The procedure for double specification limits is as follows.

- a) Enter [Table 2](#), [Table 3](#), or [Table 4](#), as appropriate, with the code letter and the AQL and obtain the sample size,  $n$ , and the acceptance constant,  $k$ .
- b) Take a random sample of size  $n$ , measure the characteristic  $x$  in each item, and then calculate  $\bar{x}$ , the sample mean, and  $s$ , the sample standard deviation (see [Annex A](#)).

NOTE 1 Some sampling strategies are provided in [Annex C](#).

NOTE 2 The sample standard deviation is needed for the records (see [Clause 13](#)).

- c) Determination of acceptance.
  - 1) If  $\bar{x} < L$  or  $\bar{x} > U$ , reject the lot; or continue with the next step.
  - 2) Calculate the quality statistics  $Q_L = (\bar{x} - L) / \sigma$  and  $Q_U = (U - \bar{x}) / \sigma$ . If  $Q_L < k$  or  $Q_U < k$ , reject the lot; or continue with the next step.
  - 3) If  $\sigma \leq 0,75 \sigma_{\max}$  accept the lot; otherwise continue with the next step.
  - 4) If neither  $Q_L$  or  $Q_U$  are close to  $k$  accept the lot; otherwise use the  $p^*$ -method in [Clause 8](#).
- d) Proceed with [Clause 9](#).

For an example of combined control of double specification limits using the  $\sigma$ -method, see Example 3 in [15.2](#).

## 8 The $p^*$ -method

The  $p^*$ -method acceptance sampling plan is an alternative to the standard procedures in [Clause 6](#) and [Clause 7](#). For the application of this method, distribution functions are calculated.

Before starting the inspection by variables, see [Clause 5](#).

The procedure for the  $p^*$ -method for a single limit and double limits is as follows.

- a) Enter [Tables 8, 9](#) or [10](#) in [14.3](#) with the code letter and the AQL, to obtain the sample size, where  $n = n_s$  is the sample size for the  $s$ -method, and  $n = n_\sigma$  is the sample size for the  $\sigma$ -method, and to obtain the acceptance constant, i.e. the maximum fraction nonconforming,  $p^*$ .
- b) Take a random sample of size  $n$ , measure the characteristic  $x$  in each item, and then calculate  $\bar{x}$ , the sample mean, and  $s$ , the sample standard deviation (see [Annex A](#)).

NOTE 1 Some sampling strategies are provided in [Annex C](#).

NOTE 2 In the case of known standard deviation,  $\sigma$ , the sample standard deviation is needed for the records (see [Clause 13](#)).

- c) Estimated fraction nonconforming:
  - 1) **unknown standard deviation:** For a lower specification limit, calculate  $\hat{p}_L$ , for an upper limit, calculate  $\hat{p}_U$ , and for two specification limits, calculate both,

$$\hat{p}_L = F_{BETA\left(\frac{n-1}{2}, \frac{n-1}{2}\right)}\left(\max\left\{0, \frac{1}{2} - \frac{1}{2} \frac{\bar{x} - L}{s} \frac{\sqrt{n}}{n-1}\right\}\right) = F_{BETA\left(\frac{n-1}{2}, \frac{n-1}{2}\right)}\left(\max\left\{0, \frac{1}{2} \left[1 - Q_L \frac{\sqrt{n}}{n-1}\right]\right\}\right)$$

$$\hat{p}_U = F_{BETA\left(\frac{n-1}{2}, \frac{n-1}{2}\right)}\left(\max\left\{0, \frac{1}{2} - \frac{1}{2} \frac{U - \bar{x}}{s} \frac{\sqrt{n}}{n-1}\right\}\right) = F_{BETA\left(\frac{n-1}{2}, \frac{n-1}{2}\right)}\left(\max\left\{0, \frac{1}{2} \left[1 - Q_U \frac{\sqrt{n}}{n-1}\right]\right\}\right)$$

- 2) **known standard deviation:** For a lower specification limit, calculate  $\hat{p}_L$ , for an upper limit, calculate  $\hat{p}_U$ , and for two specification limits, calculate both,

$$\hat{p}_L = \Phi\left(\frac{L - \bar{x}}{\sigma} \sqrt{\frac{n}{n-1}}\right) = \Phi\left(-Q_L \sqrt{\frac{n}{n-1}}\right)$$

$$\hat{p}_U = \Phi\left(\frac{\bar{x} - U}{\sigma} \sqrt{\frac{n}{n-1}}\right) = \Phi\left(-Q_U \sqrt{\frac{n}{n-1}}\right)$$

NOTE The fraction nonconforming can be calculated with the R package, see [Annex I](#).

- d) Acceptance criteria:

- 1) **single limit:** A lot shall be accepted if  $\hat{p}_L \leq p^*$  for a lower limit, and if  $\hat{p}_U \leq p^*$  for an upper limit; if  $\hat{p}_L > p^*$  or  $\hat{p}_U > p^*$  respectively, the lot is rejected;
- 2) **double limits:** A lot shall be accepted if  $\hat{p}_L + \hat{p}_U \leq p^*$ ; if  $\hat{p}_L + \hat{p}_U > p^*$ , the lot is rejected;

- e) Proceed with [Clause 9](#).

For examples of single specification limits see Example 1 and Example 3 in [15.3](#). For examples of combined control of double specification limits see Example 2 and Example 4 in [15.3](#).

## 9 Switching between inspection severities

Switching rules discourage the producer from operating at a quality level that is worse than the AQL. This document prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded. It further prescribes a discontinuation of sampling inspection altogether if tightened inspection fails to stimulate the producer into rapidly improving this production process.

Tightened inspection and the discontinuation rule are integral and, therefore, obligatory procedures of this document if the protection implied by the AQL is to be maintained.

This document also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. This practice is, however, optional (at the discretion of the responsible authority).

When it has been necessary to discontinue acceptance sampling inspection, inspection as outlined by this document shall not be resumed until action has been taken by the producer to improve the quality of the submitted product.

The sampling system specified by this document can only work efficiently if the switching rules in [9.1](#) are obeyed (see also [Figure 1](#)), and therefore records of some values of the inspection plan need to be kept (see [9.2](#)).

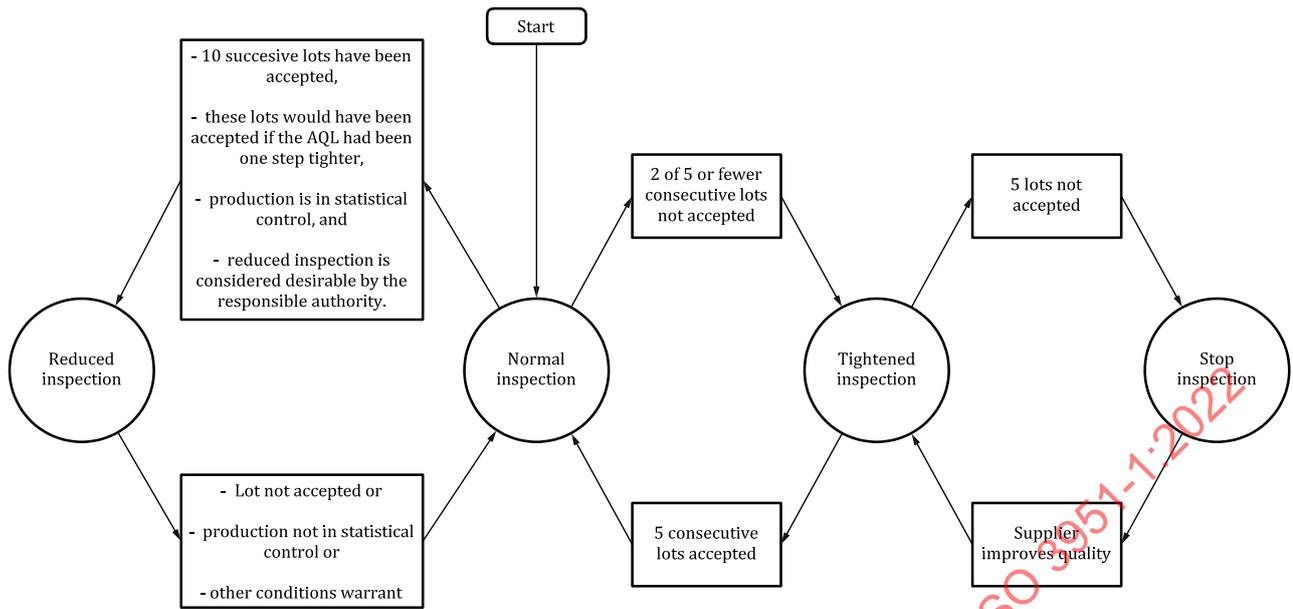


Figure 1 — Outline of the switching rules

9.1 Rules for switching between inspection severities

Normal inspection is used at the start of inspection (unless otherwise designated). It shall be used during inspection until tightened inspection becomes necessary or reduced inspection is allowed. For further information about the three inspection severities see 5.4.

This document prescribes a switch to tightened inspection when inspection results indicate that the AQL is being exceeded, and also provides the possibility of switching to reduced inspection when inspection results indicate that the quality level is stable and reliable at a level better than the AQL. The switch to reduced inspection is, however, optional (at the discretion of the responsible authority).

The standard switching rules are as follows.

a) Within normal inspection:

- 1) when two lots are rejected within any five or fewer successive lots, tightened inspection shall be instituted;

NOTE Tightened inspection is generally achieved by increasing the value of the acceptance constant *k*. The values are tabulated in Table 3 for the *s*-method and in Table 6 for the *σ*-method.

- 2) after 10 successive lots have been accepted, reduced inspection may be instituted, provided that

- i) these lots would have been accepted if the AQL had been one step tighter (e.g. 0,65 % instead of 1,0 %),

NOTE 1 To determine this, it is useful to keep a record of the values of *Q<sub>U</sub>* or *Q<sub>L</sub>*, respectively. Next, follow the procedures in Clause 6 or Clause 7 respectively, with the sample size code letter, the new AQL, the values of *Q<sub>U</sub>* or *Q<sub>L</sub>* and the last 10 values of  $\bar{x}$  and *s*.

NOTE 2 If a value of *k* for the tighter AQL is not given in Table 2 (*s*-method) or Table 6 (*σ*-method), refer to the supplementary acceptance constants provided in Table 13.

- ii) production is in statistical control (see 5.3), and
- iii) reduced inspection is considered desirable by the responsible authority.

NOTE 1 When the previous 10 lots have been accepted under original inspection, reduced inspection can be instituted without the condition that these lots would have been accepted if the AQL had been one step tighter, subject to the approval of the responsible authority.

NOTE 2 Reduced inspection is conducted on a much smaller sample than normal inspection and the value of the acceptance constant is also decreased. The values of  $n$  and  $k$  for reduced inspection are given in [Table 4](#) for the  $s$ -method and [Table 7](#) for the  $\sigma$ -method.

- b) Within reduced inspection: Normal inspection shall be reinstated if at least one of the following occur:
  - 1) a lot is rejected;
  - 2) production is no longer in statistical control or is delayed; or
  - 3) reduced inspection is no longer considered desirable by the responsible authority.
- c) Within tightened inspection:
  - 1) when five successive lots have been accepted, normal inspection shall be reinstated;
  - 2) if the cumulative number of lots rejected in a sequence of consecutive lots on original tightened inspection reaches five, the acceptance procedures of this document shall be discontinued.

Inspection under the provisions of this document shall not be resumed until action has been taken by the supplier to improve the quality of the submitted product or service and the responsible authority has agreed that this action is likely to be effective. Tightened inspection shall then be used again.

## 9.2 Records for switching between inspection severities

As a variables sampling inspection plan can only operate efficiently if records are kept, this is an important part of the inspection. For further information about the advantages of keeping records, and control charts especially, see [Clause 13](#). In particular, the following values should be kept:

- a) the values of  $\bar{x}$  and  $s$ , the lot size, the inspection level, the inspection severity, the code letter,  $U$ ,  $L$  and the AQL (especially when they changed since the last inspection);

When one of the prescribed values changes, apart from the inspection severity, one needs to start again with [Clause 5](#).

- b) whether the lot was accepted;

Any lot rejected by the sampling plan shall not be resubmitted either in whole or in part without the permission of the responsible authority.

- c) if the lot has been accepted; the values of  $Q_U$  and  $Q_L$ , if they have been evaluated.

NOTE These values are important for switching to reduced inspection.

## 10 Relation to ISO 2859-1

### 10.1 Similarities

The similarities are as follows.

- a) This document is complementary to ISO 2859-1; the two documents share a common principle and, as far as possible, their procedures and vocabulary are the same.
- b) Both use the AQL to index the sampling plans, and the preferred values used in this document are identical to those given for percent nonconforming in ISO 2859-1 (i.e. from 0,01 % to 10 %).

- c) In both this document and in ISO 2859-1, lot size and inspection level (inspection level II) determine a sample size code letter. General tables give the sample size to be taken and the acceptance criterion, indexed by the sample size code letter and the AQL. Separate tables are given for the  $s$ -method and  $\sigma$ -method, and for normal, tightened, and reduced inspection.
- d) The switching rules are essentially equivalent.

## 10.2 Differences

- a) **Determination of acceptance.** In ISO 2859-1, acceptance for an attributes sampling plan for percent nonconforming is determined by the number of nonconforming items found in the sample. Acceptance for a plan for inspection by variables is based on the distance of the estimated value of the process mean from the specification limit(s) in terms of the estimated or presumed value of the process standard deviation. In this document, two methods are considered: the  $s$ -method, for use when the process standard deviation,  $\sigma$ , is unknown, and the  $\sigma$ -method, for use when  $\sigma$  is presumed to be known.
- b) **Normality.** In ISO 2859-1, there is no requirement relating to the distribution of the characteristics. However, in this document, it is necessary for the efficient operation of the plans that the measurements be distributed according to a normal distribution (see [12.1](#)), or a close approximation to a normal distribution, or that the data is appropriately transformed to a normal distribution (see [12.2](#)).
- c) **Operating characteristic curves (OC curves).** The OC curves of the variables plans in this document are not identical to those of the corresponding attributes plans in ISO 2859-1. The curves for  $s$ -method plans have been matched by minimizing the area between the curves representing the squares of the OC values, which gives greater emphasis to the match at the top of the OC curves. In most cases, the resulting match between the OC curves is so close that, for most practical purposes, the attributes and variables of OC curves may be considered to be identical.

NOTE The  $\sigma$ -method plans are derived by minimizing the area between the squared OC functions subject to keeping the same Form  $p^*$  acceptance constant as for the corresponding  $s$ -method plan, i.e. only the sample size is open to choice, so the match is, in general, less perfect.

- d) **Producer's risk.** For process quality precisely at the AQL, the producer's risk that a lot not be accepted tends to decrease with increments in sample size coupled with decrements in AQL, as visible down the 'right' diagonals of the master tables, i.e., the tables in [Clause 14](#), running from top right to bottom left. The progressions of probabilities are similar, but not identical, to those in ISO 2859-1.

NOTE The producer's risks of the plans are given in [Annex G](#).

- e) **Sample sizes.** The variables sample sizes for given combinations of sample size code letter and AQL are usually smaller than the corresponding attributes sample sizes. This is particularly true for the  $\sigma$ -method. Moreover, due to the method by which the variables plans were derived, their sample sizes vary over AQL for a given sample size code letter.
- f) **Double sampling plans.** Double sampling plans by variables are presented separately in ISO 3951-3.
- g) **Multiple sampling plans.** There are no multiple sampling plans by variables in any part of ISO 3951 series.
- h) **Average outgoing quality limit (AOQL).** The AOQL concept is mainly of value when 100 % inspection and rectification is feasible for non-accepted lots. It follows that the AOQL concept cannot be applied under destructive or expensive testing. As variables plans shall generally be used under these circumstances, no tables of AOQL have been included in this document.

## 11 Allowing for measurement uncertainty

The master tables, i.e. the tables in [Clause 14](#), are based on the assumption that the quality characteristic,  $x$ , of the items in the lots is normally distributed with unknown process mean,  $\mu$ , and either known or unknown process standard deviation,  $\sigma$ . The assumption is also made that  $x$  can be measured without measurement error, i.e. that measurement of an item with the true value,  $x_i$ , results in the value  $x_i$ . However, the tables in [Clause 14](#) can also be used, with appropriate adjustments, in the result of measurement error.

Ideally, as a prerequisite for the application of the proposed sampling schemes, the measurement uncertainty, i.e. the measurement standard deviation, is determined by a structured measurement analysis followed by a measurement system capability assessment. If the measurement standard deviation is not higher than 10 % of the process standard deviation, the measurement system can be considered as capable and no further adjustments on sample sizes are required.

For measurement standard deviation higher than 10 % of the process standard deviation, the sample size needs to be increased, although the acceptance constant remains the same. Moreover, if neither the measurement standard deviation nor the process standard deviation is known, more than one measurement needs to be made on each sampled item and the total variability of the measurements needs to be separated into the components.

Details are provided in [Annex B](#).

## 12 Normality, data transformations and outliers

### 12.1 Normality

The assumption of a normal distribution for the inspected quality characteristic  $x$  is the fundamental requirement for the application of the proposed variables sampling plans. Therefore, it is essential to assess the data normality before applying the sampling plans, which can be done by several approaches:

- a) assessment based on practical, engineering experience and expertise;
- b) assessment based on graphical, descriptive methods, such as histograms, quantile/quantile or normal probability plots (see ISO 5479); or
- c) statistical tests for departure from normality, such as those given in ISO 5479 or in ISO 5725-2:2019, Clause 2.

### 12.2 Data transformations

For non-normal distributed data, the first approach is usually to try to find a suitable transformation  $f$  of the observed data  $x$ , so that  $Z = f(x)$  now follows an (approximate) normal distribution. The most commonly used data transformations are (see ISO 16269):

- power transformations;
- Box-Cox transformations;
- Johnson transformations;
- transformations using Pearson functions.

For the transformed data, the goodness of fit to a normal distribution can be assessed using normal probability plots and/or statistical tests for normality. For subsequent capability analyses and assessments, the targets and specification limits of the quality characteristic  $x$  have to be transformed in the same way as the observed process data.

### 12.3 Outliers

An outlier (or an outlying observation) is one that appears to deviate markedly from other observations in the sample in which it occurs. A single outlier, even when it lies within specification limits, increase variability, changes the mean, and may consequently lead to non-acceptance of the lot (see ISO 16269-4).

Generally, outliers can be a result of

- a) (unexpectedly high) natural process variability, and/or
- b) presence of special causes not representing the controlled and capable process status.

For case b), it might be considered to exclude the outlying observations for the quality assessment, whereas for case a), these values have to be considered in the analysis. Nevertheless, a sensitivity analysis including and excluding the outliers can help to assess the impact of these values on the decision on lot acceptance.

When outliers are detected, the disposition of the lot should be a matter for the responsible authority and may involve negotiation between the vendor and the vendee.

## 13 Monitoring and recording of inspection results

### 13.1 Monitoring of inspection results

The results of the quality assessment using the inspection by variables sampling plans proposed in the corresponding tables should be monitored continuously. The monitoring results should be analysed to detect changes or trends in the quality level of the product, so that warning can be given before an unacceptable quality level is reached. The adequate data recording of quality assessment results is a necessary prerequisite for monitoring.

Independent of the performed inspection, continuous monitoring should serve to assess the inherent product and process quality level of the production. Monitoring allows the user of this document to obtain a long-term view of the production quality in addition to the information on lot-by-lot decisions.

### 13.2 Process capability and performance assessment

Before implementing this document, it is recommended to assess the process capability, i.e. the ability of the production process to fulfil predefined quality requirements. These assessments shall be done in a scientifically planned manner or performance study. Several methods for estimating process capability or performance depending on the assumed process data distribution model are provided in ISO 22514-2.

### 13.3 Monitoring of process parameters

For variable inspection procedures, monitoring of product and process quality mainly focuses on process mean and process standard deviation. Whatever method used,  $s$ -method or  $\sigma$ -method, records should be kept of the values of  $\bar{x}$  and  $s$ , preferably in the form of pattern or control charts.

An introduction to the use of control charts is given in ISO 7870, and special control chart applications for assessing process means and standard deviations can be found for standard Shewhart control charts in ISO 7870-2 or ISO 8258 and, for more complex CUSUM (cumulated sum) charts, in ISO 7870-4.

The process standard deviation should be monitored particularly when using the  $\sigma$ -method in order to verify that the values of  $s$  obtained from the samples fall within the limits of the prescribed value of  $\sigma$ . With the  $s$ -method, the standard deviation shall be monitored to indicate whether a switch to the  $\sigma$ -method is possible. In particular, the values of  $s$  or  $\sigma$ , respectively, and whether they exceed their control limit should be kept.

## 14 Tables

### 14.1 Form $k$ for single sampling plans: $s$ -method

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Table 2 — Single sampling plans of form k for normal inspection: s-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
B														3	4	4
														0,950	0,735	0,586
C													4	6	6	5
													1,242	1,061	0,939	0,550
D											6		9	9	6	7
											1,476		1,323	1,218	0,887	0,507
E											9	13	13	9	9	9
											1,696	1,569	1,475	1,190	0,869	0,618
F										11	17	18	13	14	14	14
										1,889	1,769	1,682	1,426	1,147	0,935	0,601
G									15	22	23	18	20	21	21	21
									2,079	1,972	1,893	1,659	1,411	1,227	0,945	0,724
H								18	28	30	24	27	30	32	33	33
								2,254	2,153	2,079	1,862	1,636	1,471	1,225	1,036	0,806
J								36	38	31	37	41	46	49	52	53
								2,331	2,263	2,061	1,853	1,702	1,482	1,316	1,120	0,911
K								47	40	48	54	63	69	75	79	82
								2,425	2,493	2,428	2,043	1,702	1,552	1,377	1,195	0,946
L								58	61	70	84	94	105	115	124	
								2,592	2,230	2,101	1,914	1,777	1,619	1,456	1,239	
M								60	89	108	124	143	159	178		
								2,573	2,279	2,104	1,970	1,832	1,683	1,488		
N								93	137	159	186	213	247			
								2,564	2,285	2,166	2,031	1,894	1,716			
P								134	202	239	277	332				
								2,614	2,347	2,220	2,092	1,928				
Q	63	101	110	102	132	159	207	244	293	348	424					
	3,288	3,219	3,167	3,016	2,867	2,762	2,615	2,508	2,388	2,268	2,114					

NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.

Table 2 (continued)

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
R	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
3,351	116	127	120	155	189	247	298	362	438	541						
	3,301	3,156	3,012	3,012	2,912	2,771	2,670	2,556	2,443	2,298						
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection.															
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.															
NOTE	The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.															

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Table 3 — Single sampling plans of form k for tightened inspection: s-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
B															3	4
															0,950	0,735
C														4	6	6
														1,242	1,061	0,939
D													6	9	9	6
													1,476	1,323	1,218	0,887
E											9		13	13	9	9
											1,696		1,569	1,475	1,190	0,869
F											11	17	18	13	14	14
											1,889	1,769	1,682	1,426	1,147	0,935
G										15	22	23	18	20	21	21
										2,079	1,972	1,893	1,659	1,411	1,227	0,945
H							18			28	30	24	27	30	32	33
							2,254			2,153	2,079	1,862	1,636	1,471	1,225	0,954
J							36			38	31	37	41	46	50	53
							2,425			2,263	2,061	1,853	1,702	1,482	1,245	1,010
K							44			47	48	54	63	71	78	82
							2,493			2,493	2,043	1,904	1,702	1,489	1,281	1,045
L							54			64	71	84	99	111	122	
							2,580			2,237	2,101	1,914	1,720	1,533	1,325	
M							69			89	108	131	150	170		
							2,744			2,279	2,104	1,924	1,752	1,564		
N							73			137	169	201	233			
							3,023			2,285	2,117	1,958	1,785			
P			55				88			214	260	333				
			3,161				2,723			2,300	2,152	1,992				
Q		63	101				159	207	262	323	395					
		3,288	3,219				2,762	2,615	2,464	2,324	2,174					

NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.

Table 3 (continued)

Code letter	Acceptance quality limit (in percent nonconforming)																
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
R	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k	n k
	90	116	127	155	189	247	320	398	498								
	3,408	3,351	3,301	3,156	2,912	2,771	2,628	2,495	2,354								
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection.																
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.																
NOTE	The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.																

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Table 4 — Single sampling plans of form k for reduced inspection: s-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B-D	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
E	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
F	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
G	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
H	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
J	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
K	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
L	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
M	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
N	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
P	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
Q	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
R	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
Key	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k

There is no suitable plan in this area; use the first sampling plan in the appropriate column.  
If the sample size equals or exceeds the lot size, carry out 100 % inspection.

NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.

**Table 4 (continued)**

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>	<i>n</i>
<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>	<i>k</i>
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.															
NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.																

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**14.2 Form  $k$  for single sampling plans:  $\sigma$ -method**

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Table 5 — Single sampling plans of form k for normal inspection:  $\sigma$ -method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
B														3	4	3
														0,709	0,571	0,417
C													3	5	5	4
													1,115	0,945	0,821	0,436
D												4	6	6	5	5
												1,406	1,240	1,128	0,770	0,431
E											4	7	8	7	7	7
											1,595	1,506	1,419	1,115	0,792	0,555
F										5	8	9	8	10	9	11
										1,845	1,720	1,635	1,366	1,094	0,877	0,564
G									5	9	10	9	12	13	13	15
									2,006	1,934	1,856	1,610	1,370	1,186	0,906	0,694
H										11	10	13	16	16	19	23
								6	2,218	2,122	2,046	1,599	1,439	1,191	1,009	0,786
J										11	15	19	21	24	29	34
								7	2,401	2,302	2,234	1,823	1,456	1,293	1,102	0,897
K										12	17	27	29	35	42	53
								7	2,541	2,468	2,210	1,882	1,533	1,361	1,182	0,937
L										13	19	34	42	52	66	
								8	2,710	2,629	2,209	1,906	1,606	1,446	1,231	
M										14	24	36	61	79		
								8	2,844	2,726	2,264	1,963	1,674	1,481		
N										15	36	50	94			
								9	2,996	2,709	2,274	1,887	1,710			
P										17	45	72				
								2,996	2,709	2,274	2,022	1,887				
								10	3,141	2,603	2,212	1,923				
								18	3,207	2,450	2,212	1,923				
Q										20	92					
								3,275	3,155	2,500	2,262	2,110				

NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.

Table 5 (continued)

Code letter	Acceptance quality limit (in per cent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
R	19	21	22	30	38	54	64	81	105	142						
	3,339	3,289	3,145	3,002	2,903	2,764	2,663	2,550	2,438	2,294						
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection.															
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.															
NOTE	The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.															

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Table 6 — Single sampling plans of form k for tightened inspection:  $\sigma$ -method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
B															3	4
															0,709	0,571
C														3	5	5
														1,115	0,945	0,821
D													4	6	6	5
													1,406	1,240	1,128	0,770
E												4	7	8	7	7
												1,595	1,506	1,419	1,115	0,792
F											5	8	9	8	10	9
											1,845	1,720	1,635	1,366	1,094	0,877
G										5	9	10	9	12	13	13
										2,006	1,934	1,856	1,610	1,370	1,186	0,906
H										10	11	10	13	16	16	20
										2,218	2,046	1,820	1,599	1,439	1,191	0,929
J								7	11	12	11	15	19	21	25	32
								2,401	2,302	2,234	2,025	1,823	1,677	1,456	1,223	0,994
K								7	13	13	17	21	27	31	39	49
								2,541	2,401	2,210	2,018	1,882	1,683	1,471	1,267	1,035
L								8	14	19	24	32	37	47	61	
								2,710	2,387	2,209	2,088	1,900	1,705	1,521	1,316	
M								8	21	27	36	43	55	72		
								2,844	2,382	2,264	2,094	1,912	1,742	1,556		
N								9	30	40	49	65	85			
								2,996	2,437	2,274	2,106	1,950	1,779			
P								10	45	55	74	99				
								3,142	2,450	2,291	2,145	1,987				
Q								11	61	83	112					
								3,275	2,456	2,318	2,169					

NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.

Table 6 (continued)

Code letter	Acceptance quality limit (in per cent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
R	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
Key	14	19	21	22	30	38	54	68	92	126						
	3,391	3,391	3,289	3,145	3,002	2,903	2,764	2,621	2,490	2,350						
	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection.															
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.															
NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.																

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Table 7 — Single sampling plans of form k for reduced inspection:  $\sigma$ -method

Code letter	Acceptance quality limit (in percent nonconforming)																																					
	0,01		0,015		0,025		0,04		0,065		0,10		0,15		0,25		0,40		0,65		1,0		1,5		2,5		4,0		6,5		10,0							
	n	k	n	k	n	k	n	k	n	k	n	k	n	k	n	k	n	k	n	k	n	k	n	k	n	k	n	k	n	k	n	k						
B-D	3		3		3		3		3		3		3		3		3		3		3		3		3		3		3		3		3					
E	3		3		3		3		3		3		3		3		3		3		3		3		3		3		3		3		3					
F	4		4		4		4		4		4		4		4		4		4		4		4		4		4		4		4		4					
G	4		4		4		4		4		4		4		4		4		4		4		4		4		4		4		4		4		4			
H	5		5		5		5		5		5		5		5		5		5		5		5		5		5		5		5		5		5			
J	5		5		5		5		5		5		5		5		5		5		5		5		5		5		5		5		5		5			
K	6		6		6		6		6		6		6		6		6		6		6		6		6		6		6		6		6		6			
L	6		6		6		6		6		6		6		6		6		6		6		6		6		6		6		6		6		6		6	
M	7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7	
N	7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7	
P	7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7	
Q	7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7	
R	7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7		7	
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection.																																					

NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.

Table 7 (continued)

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n	n
k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k	k
There is no suitable plan in this area; use the last sampling plan in the appropriate column.																
NOTE: The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.																

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### 14.3 Form $p^*$ single sampling plans

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Table 8 — Single sampling plans of form  $p^*$  for normal inspection ( $n_s$  sample size for s-method,  $n_\sigma$  sample size for  $\sigma$ -method)

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$	$n_s$ $p^*$
B														3 3	4 4	4 3
C													4 3	6 5	6 5	5 4
D												6 4	9 6	9 6	6 5	7 5
E											9 4	13 7	13 8	9 7	9 7	9 7
F										11 5	17 8	18 9	13 8	14 10	14 9	14 11
G										0,019 6	0,033 0	0,041 4	0,072 0	0,124 5	0,176 1	0,277 1
H									15 5	22 9	23 10	18 9	20 12	21 13	21 13	21 15
I									0,012 4	0,020 1	0,025 2	0,043 8	0,076 3	0,108 5	0,172 9	0,236 2
J									28 10	30 11	24 10	27 13	30 16	32 16	33 19	33 23
K									0,007 5	0,015 9	0,027 5	0,048 0	0,068 6	0,109 4	0,150 0	0,210 9
L									38 12	31 11	37 15	41 19	46 21	49 24	52 29	53 34
M									0,004 8	0,007 9	0,009 8	0,016 8	0,026 4	0,042 4	0,067 8	0,101 4
N									44 12	47 13	40 13	63 27	69 29	75 35	79 42	82 53
P									0,003 0	0,005 0	0,006 2	0,018 8	0,026 9	0,043 1	0,059 4	0,083 6
Q	63 11	101 18	110 20	102 20	132 28	159 35	207 49	244 57	293 72	348 92	424 125	494 145	594 175	714 215	864 265	1014 315
	0,000 3	0,000 5	0,000 6	0,001 0	0,001 8	0,002 6	0,005 8	0,008 2	0,008 2	0,011 5	0,017 1	0,026 7	0,042 8	0,068 0	0,107 4	0,172 2

NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.

Table 8 (continued)

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$
R	116 19 0,000 3	127 21 0,000 4	120 22 0,000 6	155 30 0,001 1	189 38 0,001 6	247 54 0,002 6	298 64 0,003 6	362 81 0,005 1	438 105 0,007 1	541 142 0,010 7						
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection.															
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.															
NOTE	The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.															

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Table 9 — Single sampling plans of form  $p^*$  for tightened inspection ( $n_s$  sample size for s-method,  $n_\sigma$  sample size for  $\sigma$ -method)

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$
B															3 3	4 4
C														4 3	6 5	0,255 0
D													6 4	9 6	0,145 3	0,179 3
E												9 4	13 7	0,087 2	0,108 2	0,194 6
F											11 5	17 8	0,032 8	0,064 7	0,114 3	0,196 1
G										15 5	22 9	23 10	18 9	20 12	21 13	21 13
H									18 6	28 10	30 11	24 10	27 13	30 16	32 16	33 20
J								23 7	36 11	38 12	31 11	37 15	41 19	46 21	50 25	53 32
K								28 7	47 13	40 13	48 17	54 21	63 27	71 31	78 39	82 49
L								34 8	54 13	58 15	61 19	84 32	99 37	111 47	122 61	
M								40 8	69 16	60 15	89 27	131 43	150 55	170 72		
N								47 9	73 17	60 15	108 36	201 65	233 85			
P								55 10	86 19	134 33	169 49	312 99				
Q								63 11	101 18	112 26	260 74	0,022 9				
								0,000 3	0,000 5	0,004 1	0,015 4	0,022 9				
								0,001 0	0,001 8	0,002 6	0,009 8	0,014 7				

NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.

Table 9 (continued)

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$
R	90 14 0,000 2	116 19 0,000 3	127 21 0,000 4	155 30 0,001 1	189 38 0,001 6	247 54 0,002 6	320 68 0,004 1	398 92 0,006 2	498 126 0,009 2							
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection.															
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.															
NOTE	The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.															

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Table 10 — Single sampling plans of form  $p^*$  for reduced inspection ( $n_s$  sample size for s-method,  $n_\sigma$  sample size for  $\sigma$ -method)

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$	$n_s$ $n_\sigma$ $p^*$
B-D												3 3	4 4	4 4	4 3	7 6
E											4 3	6 5	6 5	6 5	5 4	9 8
F										6 4	8 5	9 6	9 6	6 5	7 5	8 7
G									9 4	0,052 2	0,070 9	0,087 2	0,108 2	0,194 6	0,314 9	0,413 0
H							11 5	11 5	0,032 8	0,041 6	0,051 9	0,064 7	0,114 3	0,196 1	0,274 3	0,408 8
J							11 5	11 5	0,026 7	0,033 0	0,041 4	0,072 0	0,124 5	0,176 1	0,277 1	0,328 4
K							15 5	15 5	0,019 6	0,025 2	0,043 8	0,076 3	0,108 5	0,172 9	0,204 5	0,267 5
L							18 6	18 6	0,007 5	0,012 4	0,020 1	0,048 0	0,109 4	0,129 6	0,170 3	0,210 9
M							23 7	23 7	0,006 2	0,007 9	0,015 9	0,042 4	0,105 9	0,131 1		
N							28 7	28 7	0,004 8	0,006 2	0,016 6	0,043 1	0,067 4	0,083 6		
P							34 8	34 8	0,003 0	0,004 0	0,011 6	0,051 3	0,067 4			
Q							40 8	40 8	0,003 9	0,005 0	0,016 6	0,041 9	0,052 0			
R							44 10	44 10	0,002 5	0,003 1	0,011 6	0,041 9	0,052 0			
							47 9	47 9	0,001 9	0,002 6	0,011 6	0,041 9	0,052 0			
							52 11	52 11	0,001 5	0,002 4	0,010 5	0,032 9				
							58 15	58 15	0,004 1	0,006 6	0,016 9	0,041 9				
							60 15	60 15	0,004 6	0,006 6	0,016 9	0,041 9				
							64 14	64 14	0,001 5	0,002 6	0,010 5	0,032 9				
							69 16	69 16	0,001 5	0,002 6	0,010 5	0,032 9				
							73 17	73 17	0,001 5	0,002 6	0,010 5	0,032 9				
							75 15	75 15	0,001 2	0,001 2	0,010 5	0,032 9				
							76 21	76 21	0,001 2	0,001 2	0,010 5	0,032 9				
							82 17	82 17	0,001 5	0,002 6	0,010 5	0,032 9				
							86 19	86 19	0,001 5	0,002 6	0,010 5	0,032 9				
							88 17	88 17	0,001 5	0,002 6	0,010 5	0,032 9				
							96 19	96 19	0,001 5	0,002 6	0,010 5	0,032 9				
							112 26	112 26	0,001 5	0,002 6	0,010 5	0,032 9				
							134 33	134 33	0,001 5	0,002 6	0,010 5	0,032 9				
							171 45	171 45	0,001 5	0,002 6	0,010 5	0,032 9				
							187 50	187 50	0,001 5	0,002 6	0,010 5	0,032 9				
							214 55	214 55	0,001 5	0,002 6	0,010 5	0,032 9				
							239 65	239 65	0,001 5	0,002 6	0,010 5	0,032 9				
							299 79	299 79	0,001 5	0,002 6	0,010 5	0,032 9				
							317 43	317 43	0,001 5	0,002 6	0,010 5	0,032 9				
							326 9	326 9	0,001 5	0,002 6	0,010 5	0,032 9				
							331 43	331 43	0,001 5	0,002 6	0,010 5	0,032 9				
							343 50	343 50	0,001 5	0,002 6	0,010 5	0,032 9				
							357 57	357 57	0,001 5	0,002 6	0,010 5	0,032 9				
							365 65	365 65	0,001 5	0,002 6	0,010 5	0,032 9				
							389 69	389 69	0,001 5	0,002 6	0,010 5	0,032 9				
							404 1	404 1	0,001 5	0,002 6	0,010 5	0,032 9				
							419 9	419 9	0,001 5	0,002 6	0,010 5	0,032 9				
							432 21	432 21	0,001 5	0,002 6	0,010 5	0,032 9				
							442 21	442 21	0,001 5	0,002 6	0,010 5	0,032 9				
							454 21	454 21	0,001 5	0,002 6	0,010 5	0,032 9				
							468 26	468 26	0,001 5	0,002 6	0,010 5	0,032 9				
							481 31	481 31	0,001 5	0,002 6	0,010 5	0,032 9				
							498 31	498 31	0,001 5	0,002 6	0,010 5	0,032 9				
							513 37	513 37	0,001 5	0,002 6	0,010 5	0,032 9				
							526 42	526 42	0,001 5	0,002 6	0,010 5	0,032 9				
							542 42	542 42	0,001 5	0,002 6	0,010 5	0,032 9				
							557 42	557 42	0,001 5	0,002 6	0,010 5	0,032 9				
							571 49	571 49	0,001 5	0,002 6	0,010 5	0,032 9				
							586 49	586 49	0,001 5	0,002 6	0,010 5	0,032 9				
							607 57	607 57	0,001 5	0,002 6	0,010 5	0,032 9				
							626 65	626 65	0,001 5	0,002 6	0,010 5	0,032 9				
							647 65	647 65	0,001 5	0,002 6	0,010 5	0,032 9				
							671 65	671 65	0,001 5	0,002 6	0,010 5	0,032 9				
							699 65	699 65	0,001 5	0,002 6	0,010 5	0,032 9				
							729 65	729 65	0,001 5	0,002 6	0,010 5	0,032 9				
							761 65	761 65	0,001 5	0,002 6	0,010 5	0,032 9				
							797 65	797 65	0,001 5	0,002 6	0,010 5	0,032 9				
							837 65	837 65	0,001 5	0,002 6	0,010 5	0,032 9				
							881 65	881 65	0,001 5	0,002 6	0,010 5	0,032 9				
							929 65	929 65	0,001 5	0,002 6	0,010 5	0,032 9				
							981 65	981 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1037 65	1037 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1097 65	1097 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1161 65	1161 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1229 65	1229 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1301 65	1301 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1377 65	1377 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1457 65	1457 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1541 65	1541 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1629 65	1629 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1721 65	1721 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1817 65	1817 65	0,001 5	0,002 6	0,010 5	0,032 9				
							1917 65	1917 65	0,001 5	0,002 6	0,010 5	0,032 9				
							2021 65	2021 65	0,001 5	0,002 6	0,010 5	0,032 9				
							2129 65	2129 65	0,001 5	0,002 6	0,010 5	0,032 9				
							2241 65	2241 65	0,001 5	0,002 6	0,010 5	0,032 9				
							2357 65	2357 65	0,001 5	0,002 6	0,010 5	0,032 9				
							2477 65	2477 65	0,001 5	0,002 6	0,010 5	0,032 9				

Table 10 (continued)

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$	$n_s$
	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$	$n_\sigma$
	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$	$p^*$

There is no suitable plan in this area; use the last sampling plan in the appropriate column.

NOTE The sample size code letters in this document correspond to those given in ISO 2859-1 and ISO 3951-2 in all cases where a correspondence can reasonably be considered.

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14.4 Values of  $f_\sigma$  for maximum process standard deviation (MPSD)

Table 11 — Values of  $f_\sigma$  for maximum process standard deviation for combined control of double specification limits:  $\sigma$ -method

Acceptance quality limit (in percent nonconforming)	$f_\sigma$
0,010	0,128 5
0,015	0,131 9
0,025	0,136 5
0,040	0,141 2
0,065	0,146 6
0,10	0,152 0
0,15	0,157 5
0,25	0,165 4
0,40	0,173 7
0,65	0,183 7
1,0	0,194 1
1,5	0,205 6
2,5	0,223 1
4,0	0,243 5
6,5	0,271 0
10	0,304 0

NOTE The MPSD is obtained by multiplying the standardized MPSD,  $f_\sigma$ , by the difference between the upper specification limit,  $U$ , and the lower specification limit,  $L$ , i.e.  $MPSD = (U - L)f_\sigma$ .

The MPSD indicates the greatest allowable magnitude of the process standard deviation when using plans for combined control of double specification limits when the process variability is known. If the process standard deviation is less than the MPSD, then there is a possibility, but not a certainty, that the lot shall be accepted.

14.5 Supplementary acceptance constants for qualifying towards reduced inspection

Table 12 — Supplementary acceptance constants for qualifying towards reduced inspection

Sample size code letter	AQL %	Acceptance constant for AQL that is one step tighter		
		s-method		$\sigma$ -method
		$k$	$p^*$ %	$k$
B	4,0	1,114	8,502	0,918
C	2,5	1,409	3,041	1,325
D	1,5	1,601	n/a	1,562
E	1,0	1,825	n/a	1,752
F	0,65	2,029	n/a	2,013
G	0,40	2,209	n/a	2,161
H	0,25	2,390	n/a	2,379
J	0,15	2,530	n/a	2,523
K	0,10	2,689	n/a	2,667

NOTE For this document, n/a is "not applicable."

Table 12 (continued)

Sample size code letter	AQL %	Acceptance constant for AQL that is one step tighter		
		s-method		$\sigma$ -method
		$k$	$p^*$ %	$k$
L	0,065	2,857	n/a	2,847
M	0,040	2,995	n/a	2,972
N	0,025	3,143	n/a	3,131
P	0,015	3,254	n/a	3,246
Q	0,010	3,385	n/a	3,382
R	0,010	3,449	n/a	3,446

NOTE For this document, n/a is "not applicable."

NOTE These constants were calculated so that the probability of acceptance at the next lower AQL is the same as the probability of acceptance at the given AQL. For example, the normal inspection s-method sampling plan for sample size code letter B and AQL of 4,0 % is found from Table 2 to be  $n = 3$  and  $k = 0,950$ . This can be shown to have a probability of acceptance of 96,440 4 % at process quality level 4,0 %. The next smaller AQL is 2,5 %. The s-method form  $k$  acceptance constant that provides the same probability of acceptance 96,440 4 % with the same sample size  $n = 3$  can be shown to be  $k = 1,114$ . Thus, in order to be acceptable at a one level tighter AQL, the sample mean needs to be at least 1,114 times the sample standard deviation within specification, rather than the 0,950 times the sample standard deviation needed to be considered merely acceptable.

## 15 Examples

### 15.1 General

Examples are included for the s-method (15.2), the  $\sigma$ -method (15.3) and the  $p^*$ -method (15.4) with details of the calculations and acceptance decisions given. The examples used for the  $p^*$ -method are a subset of those used for the s-method and the  $\sigma$ -method and this illustrates the equivalence of the methods. The software that supports the use of this standard, described in Annex I, can carry out all the calculations and acceptance decisions and produce the information given here.

### 15.2 Examples for the s-method

EXAMPLE 1 Single upper specification limit using the s-method.

The maximum temperature of operation for a certain device is specified as 60 °C and the operating temperature is known from previous experience to be normally distributed. Production is inspected in lots of 100 items and the process standard deviation is unknown. Inspection level II, normal inspection with AQL of 2,5 % is to be used. From Table 1, the sample size code letter is found to be F; from Table 2, it is seen that the sample size,  $n$ , is 13 and the form  $k$  acceptance constant is 1,426. Suppose that the measurements are as follows: 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 50 °C; 50 °C; 55 °C; 54 °C; 57 °C. Conformity with the acceptance criterion is to be determined.

#### Information needed

Sample size:  $n$

Form  $k$  acceptance constant:  $k$

Sample mean:  $\bar{x} = \sum x / n$

#### Values obtained

13

1,426

54,62 °C

Sample standard deviation:  $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$  3,330 °C

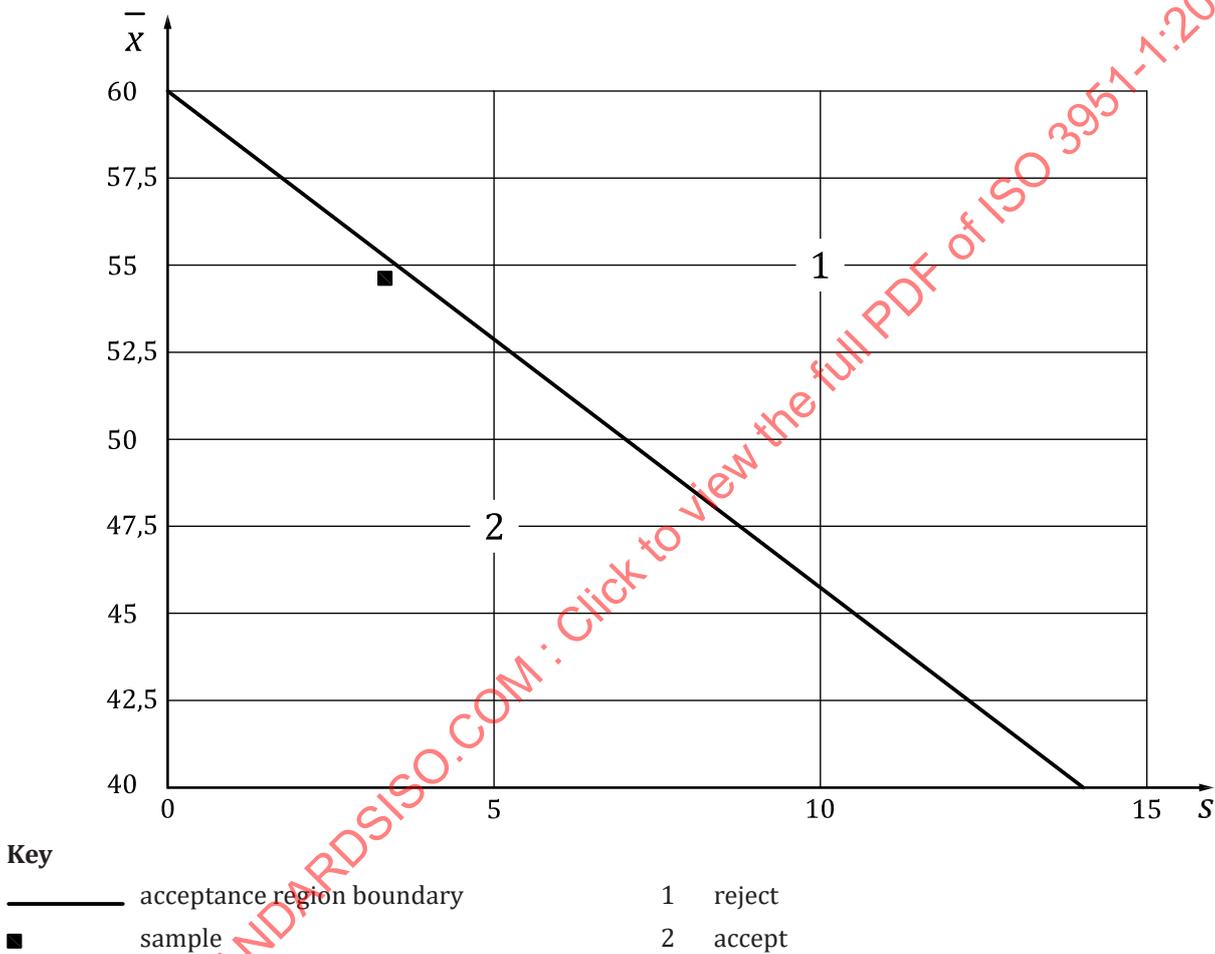
Upper specification limit:  $U$  60 °C

Upper quality statistic:  $Q_U = (U - \bar{x}) / s$  1,617

Acceptance criterion: Is  $Q_U \geq k$ ? Yes (1,617  $\geq$  1,426)

The quality statistic is greater than or equal to the acceptance constant; therefore, the lot is accepted.

Note that the acceptance test,  $Q_U \geq k$ , is equivalent to checking that  $(s, \bar{x})$  is below the acceptance region boundary. This is shown in the following [Figure 2](#).



**Figure 2 — Example of the use of an acceptance diagram, s-method**

**EXAMPLE 2** Single, lower specification limit using the s-method.

A certain pyrotechnic delay mechanism has a specified minimum delay time of 4,0 s. The process standard deviation is unknown. Production is inspected in lots of 1 000 items and inspection level II, normal inspection, is to be used with an AQL of 0,1 % applied to the lower limit. From Table 1, it is seen that the sample size code letter is J. However, on entering Table 2 with sample size code letter J and AQL of 0,1 %, it is found that there is an arrow pointing to the cell below. This means that an entirely suitable plan is unavailable, and the next best plan is given by sample size code letter K, with sample size,  $n = 28$  and form k acceptance constant 2,580. A random sample of size 28 is drawn. Suppose the sample delay times, in seconds, are as follows:

6,95 6,04 6,68 6,63 6,65 6,52 6,59 6,40 6,44 6,34 6,04 6,15 6,29 6,63  
 6,44 7,15 6,70 6,59 6,51 6,80 5,94 6,35 7,17 6,83 6,25 6,96 7,00 6,38

Conformity with the acceptance criterion is to be determined.

**Information needed**

**Values obtained**

Sample size:  $n$

28

Form  $k$  acceptance constant:  $k$

2,580

Sample mean:  $\bar{x} = \sum x / n$

6,551 s

Sample standard deviation:  $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$

0,325 1 s

Lower specification limit:  $L$

4,0 s

Lower quality statistic:  $Q_L = (\bar{x} - L) / s$

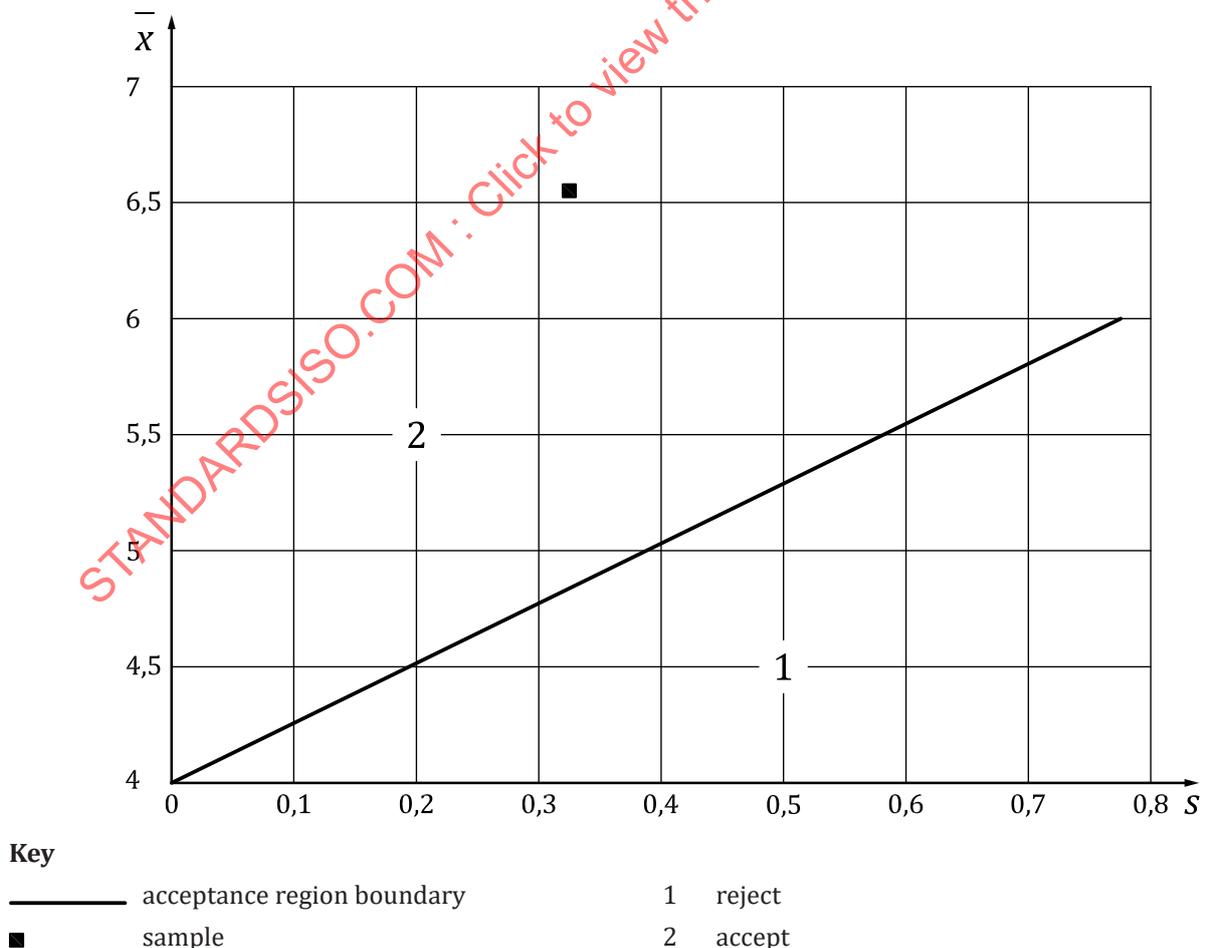
7,847

Acceptance criterion: Is  $Q_L \geq k$  ?

Yes ( $7,847 \geq 2,580$ )

The quality statistic is greater than or equal to the acceptance constant; therefore, the lot is accepted.

Note that the acceptance test,  $Q_L \geq k$ , is equivalent to checking that  $(s, \bar{x})$  is above the acceptance region boundary. This is shown in the following [Figure 3](#).



**Figure 3 — Example of the use of an acceptance diagram, s-method**

EXAMPLE 3 Combined control of double specification limits using the s-method.

Torpedoes supplied in batches of 100 are to be inspected for accuracy in the horizontal plane. Positive or negative angular errors are equally unacceptable, so combined control of the double specification limits is appropriate. The specification limits are set at 10 m either side of the point of aim at a distance of 1 km, with an AQL of 4 %. Because testing is destructive and very costly, it has been agreed between the producer and the responsible authority that special inspection level S-2 is to be used. From Table 1, the sample size code letter is found to be B; from Table 2, it is seen that the sample size,  $n$ , is 3 and the form  $k$  acceptance constant is 0,950. Three torpedoes are tested, yielding errors -5,0 m, 6,7 m, and 8,8 m. Conformity with the acceptance criterion is to be determined.

Information needed	Value obtained
Sample size: $n$	3
Acceptance constant: $k$	0,950
Sample mean: $\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j$	3,5 m
Sample standard deviation: $s = \sqrt{\sum_{j=1}^n (x_j - \bar{x})^2 / (n-1)}$	7,436 m
Upper specification limit: $U$	10,0 m
Lower specification limit: $L$	-10,0 m

Determination of acceptance. Neither  $\bar{x} < L$  or  $\bar{x} > U$  so  $(s, \bar{x})$  can be plotted on the acceptance diagram, as shown in [Figure 4](#). The point is outside the acceptance region; therefore the lot is rejected.

NOTE This lot is rejected even though all inspected items in the sample are within the specification limits.

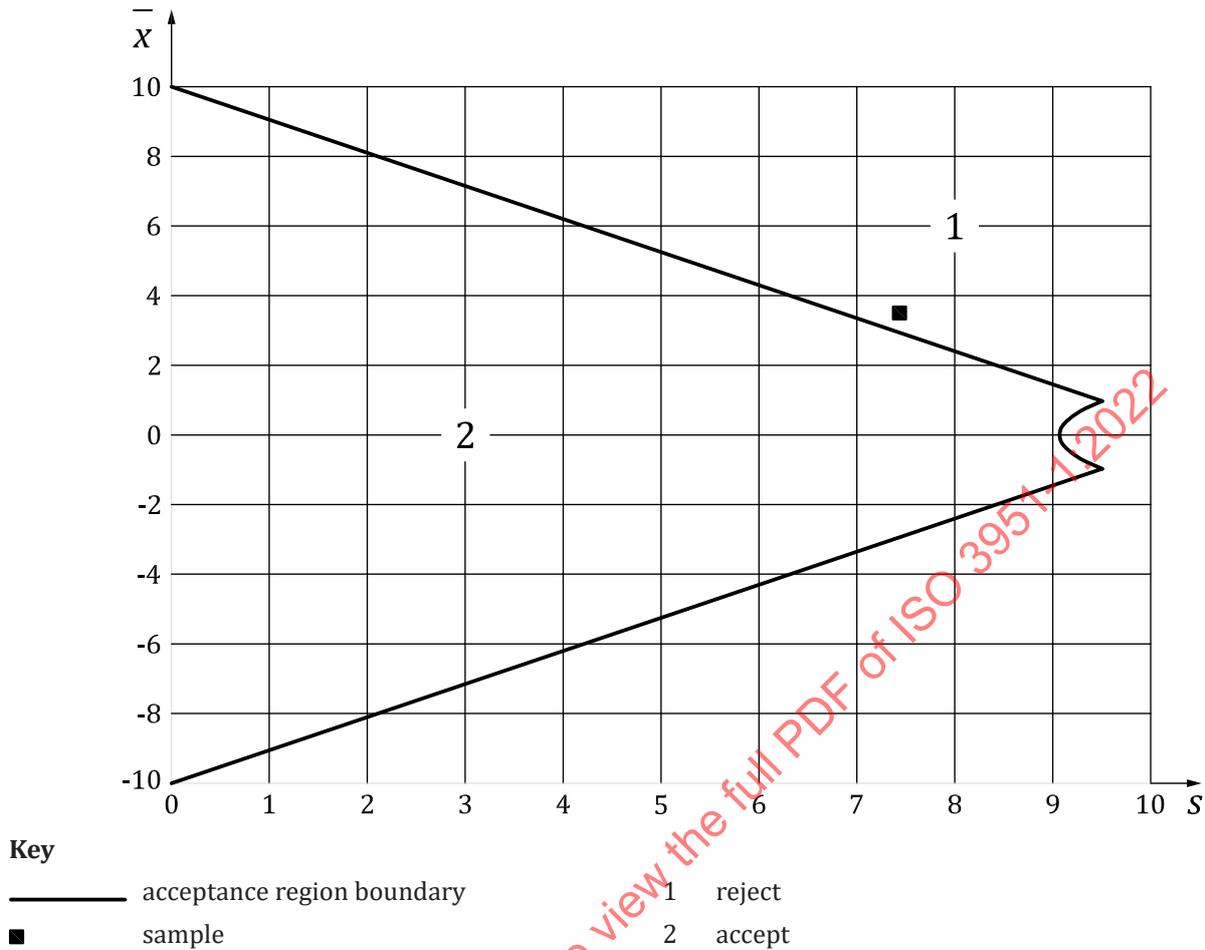


Figure 4 — Example of the use of an acceptance diagram, s-method

EXAMPLE 4 Combined control of double specification limits using the s-method.

Items are being manufactured in lots of size 25. The lower and upper specification limits on their diameters are 82 mm to 84 mm. Items with diameters that are too large are equally unsatisfactory as those with diameters that are too small, and it has been decided to control the total fraction nonconforming using an AQL of 10 % at inspection level II. Normal inspection is to be instituted at the beginning of inspection operations. From Table 1, the sample size code letter is found to be C; from Table 2, it is seen that the sample size, n, is 5 and the form k acceptance constant is 0,550. The diameters of five items from the first lot are measured, yielding diameters of 82,41 mm, 82,20 mm, 83,10 mm, 82,09 mm and 82,70 mm. Conformity with the acceptance criterion is to be determined.

**Information needed**

**Value obtained**

Sample size: *n*

5

Sample mean:  $\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j$

82,50 mm

Sample standard deviation:  $s = \sqrt{\sum_{j=1}^n (x_j - \bar{x})^2 / (n - 1)}$

0,408 2 mm

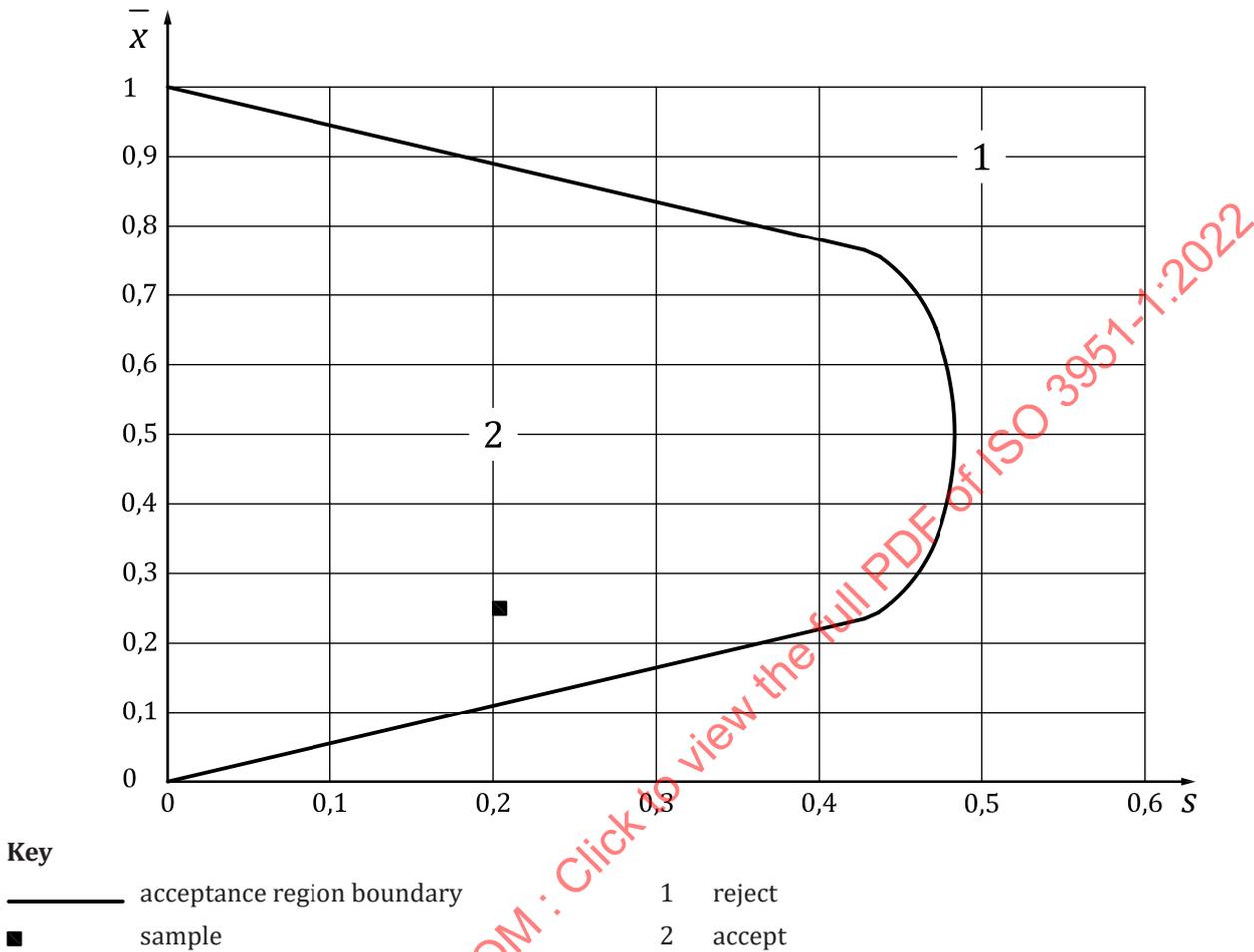
Upper specification limit: *U*

84,0 mm

Lower specification limit: *L*

82,0 mm

Determination of acceptance. Neither  $\bar{x} < L$  or  $\bar{x} > U$  so  $\left( s_S = \frac{s}{U-L}, \bar{x}_S = \frac{\bar{x}-L}{U-L} \right)$  can be plotted on the standardized acceptance diagram, as shown in [Figure 5](#). The point is not outside the acceptance region; therefore the lot is accepted.



**Figure 5 — Example of the use of a standardized acceptance diagram, s-method**

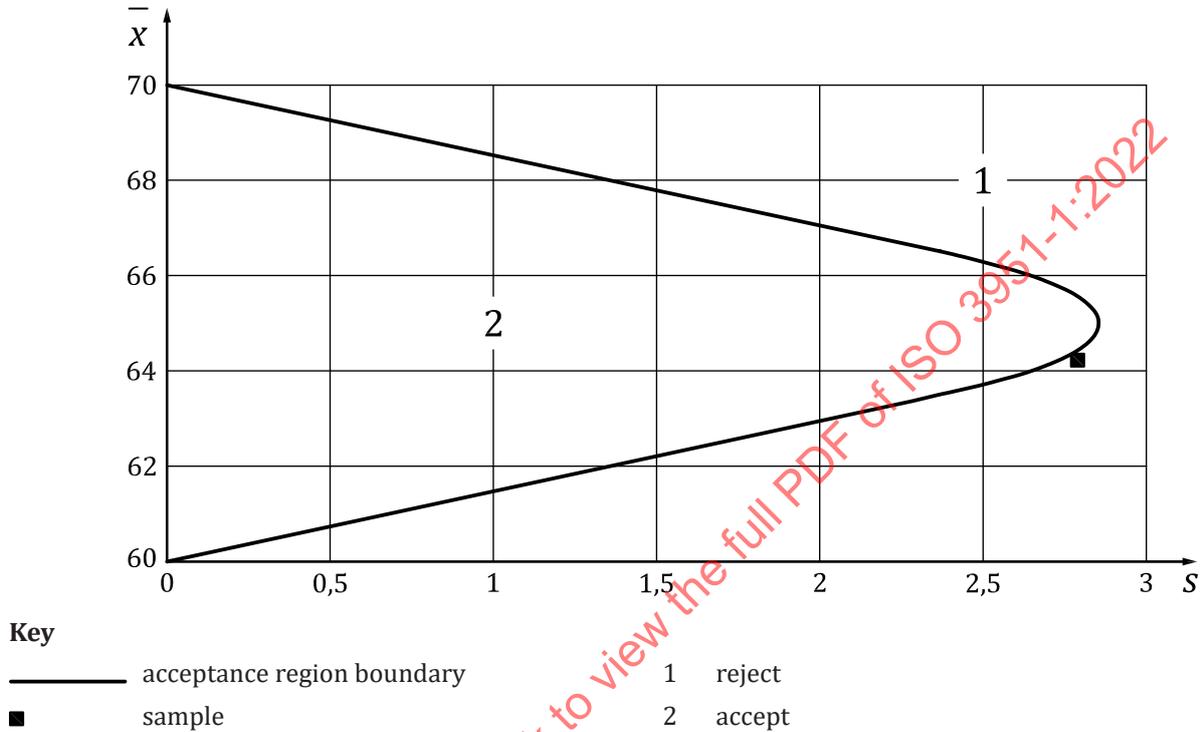
**EXAMPLE 5** Combined control of double specification limits using the s-method.

The minimum temperature of operation for a certain device is specified as 60 °C and the maximum temperature as 70 °C. Production is in inspection lots of 80 items. Inspection level II, normal inspection, with AQL of 2,5 %, is to be used. From Table 1, the sample size code letter is found to be E; from Table 2, it is seen that the sample size, n, is 13 and the form k acceptance constant is 1,475. Suppose the measurements obtained are as follows: 63,5 °C; 61,9 °C; 65,2 °C; 61,7 °C; 68,4 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C; 63,4 °C; 60,7 °C; 65,8 °C. Conformity with the acceptance criterion is to be determined.

Information needed	Value obtained
Sample size: $n$	13
Sample mean: $\bar{x} = \sum x / n$	64,223 °C
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$	2,789 9 °C

Upper specification limit:  $U$  70,0 °C  
 Lower specification limit:  $L$  60,0 °C

Determination of acceptance. Neither  $\bar{x} < L$  or  $\bar{x} > U$  so  $(s, \bar{x})$  can be plotted on the acceptance diagram, as shown in Figure 6. The point is outside the acceptance region; therefore, the lot is rejected.



**Figure 6 — Example of the use of an acceptance diagram, s-method**

EXAMPLE 6 Combined control of double specification limits using the s-method.

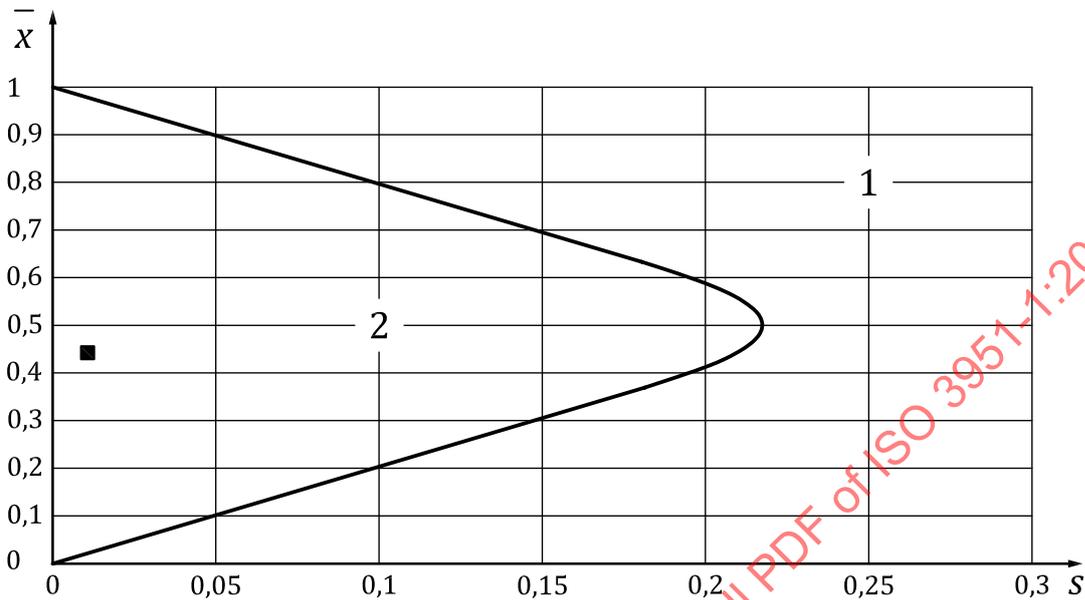
For a medical inhalation device, the cap removal force has a specification interval of [5 N, 10 N]. No reliable information on process standard deviation is available, so that the s-method has to be used. The usual lot size is ~100 000 units and with inspection level II, AQL = 1 %, from Table 1, the sample size code letter is found to be N; from Table 2, it is seen that the sample size,  $n$ , is 186 and the form  $k$  acceptance constant is 2,031. The observed measurement values were within a range of 7,03 N to 742 N and data normality can be assumed. Conformity with the acceptance criterion is to be determined.

**Information needed**

**Value obtained**

Sample size: $n$	186
Sample mean: $\bar{x} = \sum x/n$	7,21 °C
Sample standard deviation: $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$	0,053 8 °C
Lower specification limit: $L$	5 °N
Upper specification limit: $U$	10 °N

Determination of acceptance. Neither  $\bar{x} < L$  or  $\bar{x} > U$  so  $\left( s_S = \frac{s}{U-L}, \bar{x}_S = \frac{\bar{x}-L}{U-L} \right)$  can be plotted on the standardized acceptance diagram, as shown in [Figure 7](#). The point is not outside the acceptance region; therefore, the lot is accepted.



**Key**

—	acceptance region boundary	1	reject
■	sample	2	accept

**Figure 7 — Example of the use of a standardized acceptance diagram, s-method**

### 15.3 Examples for the $\sigma$ -method

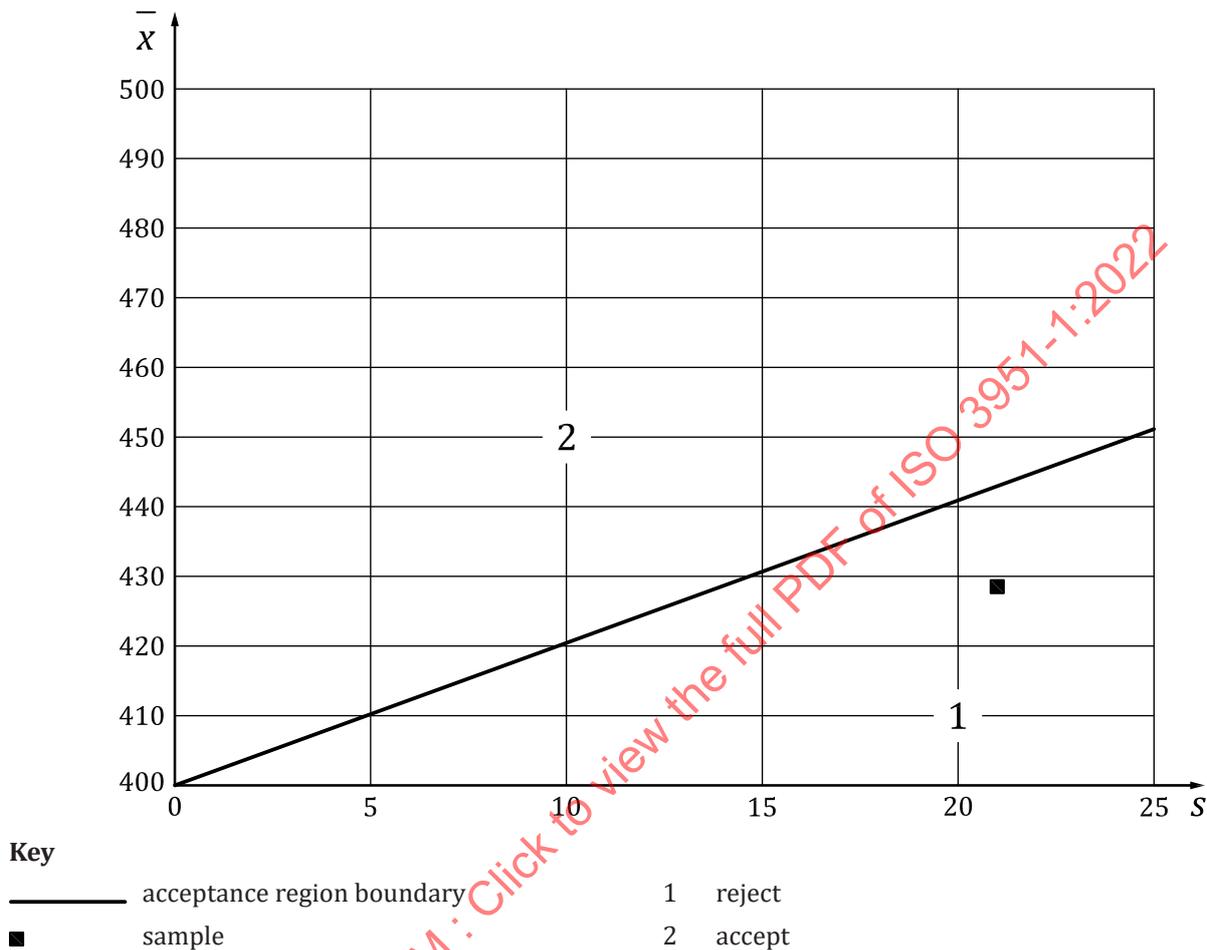
**EXAMPLE 1** Single, lower specification limit using the  $\sigma$ -method.

The specified minimum yield point for certain steel castings is 400 N/mm<sup>2</sup>. The next lot of 500 items is submitted for inspection. Inspection level II, normal inspection, with AQL of 0,65 %, is to be used. The value of  $\sigma$  is considered to be 21 N/mm<sup>2</sup>. From [Table 1](#), it is seen that the sample size code letter is H. From [Table 5](#) for an AQL of 0,65 %, it is seen that the sample size,  $n$ , is 11 and the form  $k$  acceptance constant is 2,046. Suppose the yield points of the sample specimens in N/mm<sup>2</sup> are: 431; 417; 469; 407; 450; 452; 427; 411; 429; 420; 400. Conformity with the acceptance criterion is to be determined.

<b>Information needed</b>	<b>Value obtained</b>
Sample size: $n$	11
Form $k$ acceptance constant: $k$	2,046
Sample mean: $\bar{x} = \sum x / n$	428,5 N/mm <sup>2</sup>
Known $\sigma$	21 N/mm <sup>2</sup>
Lower specification limit: $L$	400 N/mm <sup>2</sup>
Lower quality statistic: $Q_L = (\bar{x} - L) / \sigma$	1,355
Acceptance criterion: Is $Q_L \bar{x} \geq k$ ?	No (1,355 < 2,046)

The sample does not meet the acceptance criterion; therefore, the lot is rejected.

Note that the acceptance test,  $Q_L \geq k$ , is equivalent to checking that  $(s, \bar{x})$  is above the acceptance region boundary. This is shown in the following [Figure 8](#).



**Figure 8 — Example of the use of an acceptance diagram,  $\sigma$ -method**

EXAMPLE 2 Single, upper specification limit using the  $\sigma$ -method.

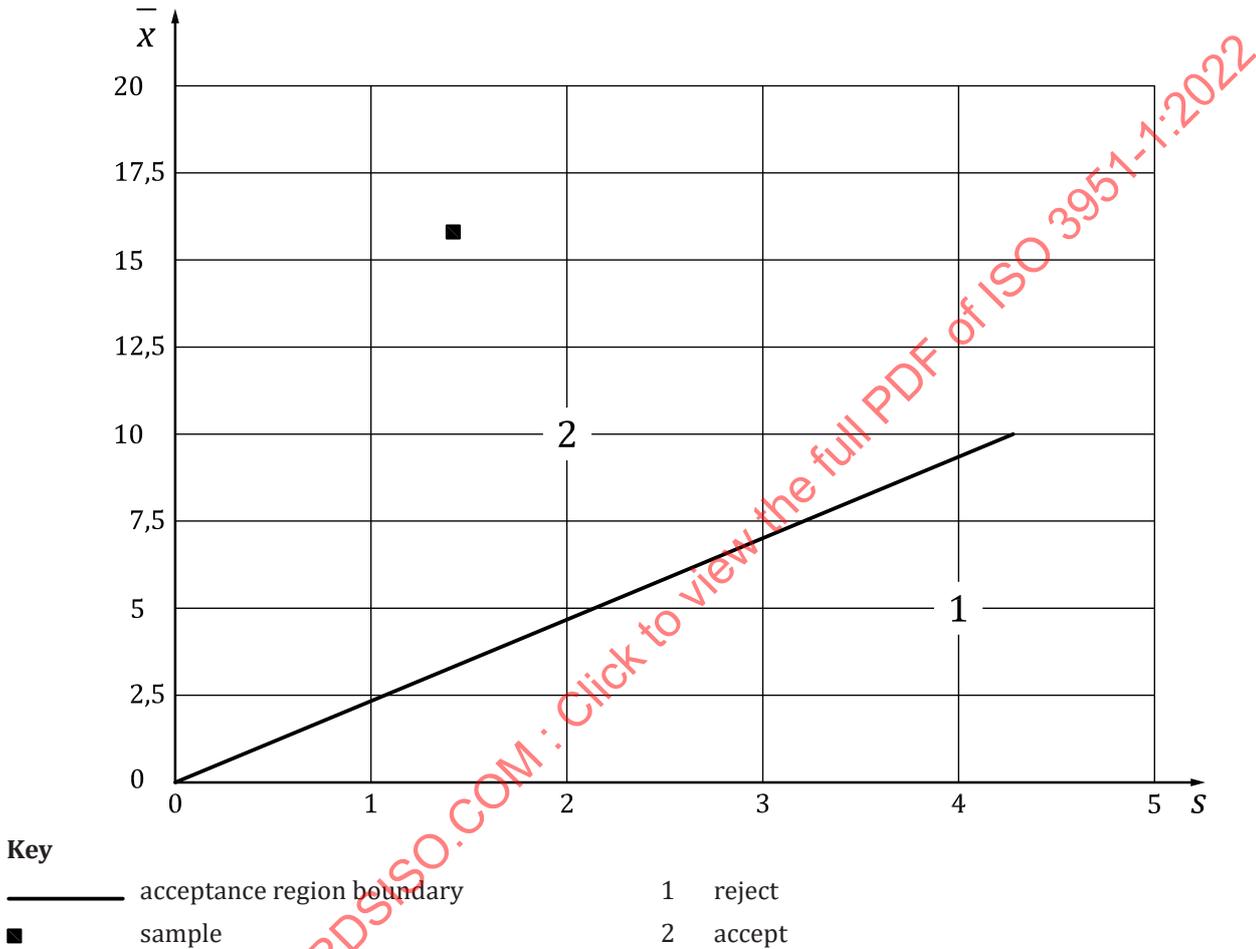
The actuation force of a medical inhalation device has to be below the upper specification limit of 20 N in order to guarantee usability. Based on an extensive process validation study, the process standard deviation can be assumed to be known, so that in this case the  $\sigma$  method can be applied. With an average lot size of ~200 000 units, with normal inspection level II with an AQL = 0,4 %, from [Table 1](#), the sample size code letter is found to be P; from [Table 5](#), it is seen that the sample size,  $n$ , is 51 and the form  $k$  acceptance constant is 2,337. The observed measurement values were within a range of 12,41 N to 18,98 N and data normality can be assumed. Conformity with the acceptance criterion is to be determined.

Information needed	Value obtained
Sample size: $n$	51
Form $k$ acceptance constant: $k$	2,337
Sample mean: $\bar{x}$	15,8 N
Known $\sigma$	1,42 N

Specification limit:  $U$  20 N  
 Upper quality statistic:  $Q_U = (U - \bar{x})/\sigma$  2,958 N  
 Acceptance criterion: Is  $Q_U \geq k$ ? Yes (2,958  $\geq$  2,337)

The sample meets the acceptance criterion; therefore, the lot is accepted.

Note that the acceptance test,  $Q_U \geq k$ , is equivalent to checking that  $(s, \bar{x})$  is below the acceptance region boundary. This is shown in the following [Figure 9](#).



**Figure 9 — Example of the use of an acceptance diagram,  $\sigma$ -method**

EXAMPLE 3 Combined control of double specification limits using the  $\sigma$ -method.

The specification for electrical resistance of a certain electrical component is  $(520 \pm 50) \Omega$ . Production is at a rate of 1 000 items per inspection lot. Inspection level II, normal inspection, with a single AQL of 1,5 %, is to be used for the two specification limits (470  $\Omega$  and 570  $\Omega$ ).  $\sigma$  is known to be 18,5  $\Omega$ .

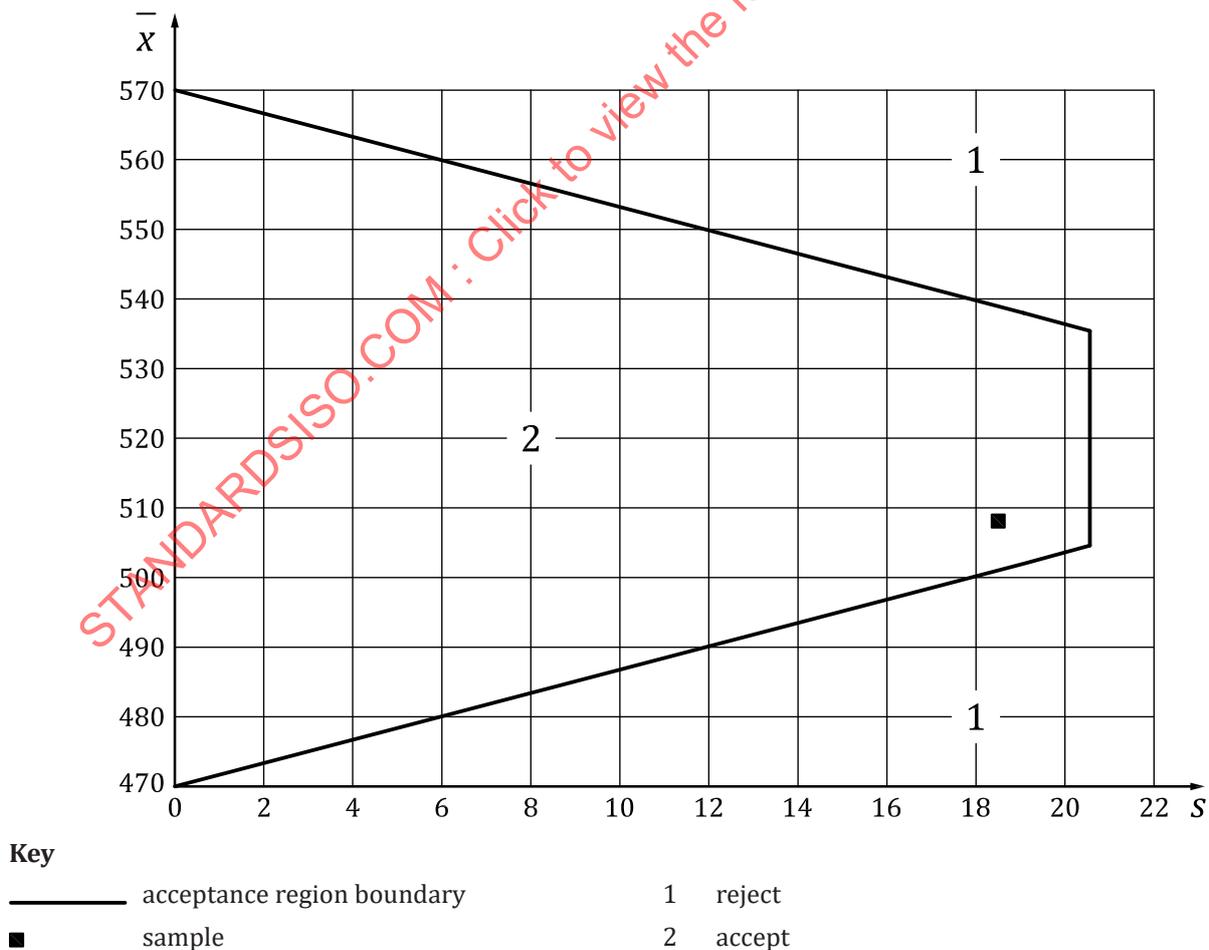
Information needed	Value obtained
Factor from <a href="#">Table 11</a> : $f_\sigma$	0,205 6
Upper specification limit: $U$	570 $\Omega$
Lower specification limit: $L$	470 $\Omega$
Known $\sigma$	18,5 $\Omega$

It is noted that the known  $\sigma$  of  $18,5 \Omega$  is less than  $(U - L) \times \text{MPSD} = 20,56 \Omega$  so there is evidence that there is a possibility that lots will be accepted so acceptance sampling can be undertaken. Entering [Table 1](#) with the lot size and inspection level, it is found that the sample size code letter is J; from [Table 2](#), it is seen that the sample size,  $n$ , is 19 and the form  $k$  acceptance constant is 1,677. Suppose that the 19 sample values of the resistance, in  $\Omega$ , are as follows: 515; 491; 479; 513; 521; 536; 483; 509; 514; 507; 484; 526; 532; 499; 530; 512; 492; 522; 488. Conformity with the acceptance criterion is to be determined.

Further information needed	Value obtained
Sample size: $n$	19
Acceptance constant (from <a href="#">Table 5</a> )	1,677
Sample mean: $\bar{x}$	508,1 $\Omega$
Lower quality statistic $Q_L = (\bar{x} - L)/\sigma$	2,059
Upper quality statistic $Q_U = (U - \bar{x})/\sigma$	3,346

$L \leq \bar{x} \leq U$ ;  $k < Q_L$  and  $k < Q_U$  with neither  $Q_L$  or  $Q_U$  close to  $k$ , even though  $\sigma > 0,75 \sigma_{\max} = 15,417$ ; therefore the lot is accepted.

Note that this acceptance test is equivalent to checking that  $(s, \bar{x})$  is inside the acceptance region boundary. This is shown in the following [Figure 10](#).



**Figure 10 — Example of the use of an acceptance diagram,  $\sigma$ -method**

15.4 Examples for the  $p^*$ -method

EXAMPLE 1 Single upper specification limit using the  $s$ -method. Example 1 from 15.1.

The maximum temperature of operation for a certain device is specified as 60 °C and the operating temperature is known from previous experience to be normally distributed. Production is inspected in lots of 100 items and the process standard deviation is unknown. Inspection level II, normal inspection with AQL of 2,5 % is to be used. From Table 1, the sample size code letter is found to be F. From Table 8, it is seen that the sample size,  $n$ , is 13 and that the form  $p^*$  acceptance constant, is 0,072 0. Suppose that the measurements are as follows: 53 °C; 57 °C; 49 °C; 58 °C; 59 °C; 54 °C; 58 °C; 56 °C; 50 °C; 50 °C; 55 °C; 54 °C; 57 °C. Conformity with the acceptance criterion is to be determined.

Information needed

Values obtained

Sample size:  $n$

13

Form  $p^*$  acceptance constant:  $p^*$

0,072 0

Sample mean:  $\bar{x} = \sum x / n$

54,62 °C

Sample standard deviation:  $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$

3,330 °C

Upper specification limit:  $U$

60 °C

Estimated fraction nonconforming

$$\hat{p}_U = F_{BETA}\left(\frac{n}{2}-1, \frac{n}{2}-1\right) \left( \max \left\{ 0, \frac{1}{2} - \frac{1}{2} \frac{U - \bar{x}}{s} \frac{\sqrt{n}}{n-1} \right\} \right)$$

0,046 3

Acceptance criterion: Is  $\hat{p}_U \leq p^*$

Yes (0,046 3  $\leq$  0,072 0)

The sample meets the acceptance criterion; therefore, the lot is accepted.

EXAMPLE 2 Combined control of double specification limits using the  $s$ -method. Example 5 from 15.1.

The minimum temperature of operation for a certain device is specified as 60 °C and the maximum temperature as 70 °C. Production is in inspection lots of 80 items. Inspection level II, normal inspection, with AQL of 2,5 %, is to be used. From Table 1, the sample size code letter is found to be E. From Table 8, it is seen that the sample size,  $n$ , is 13 and that the form  $p^*$  acceptance constant is 0,064 7. Suppose the measurements obtained are as follows: 63,5 °C; 61,9 °C; 65,2 °C; 61,7 °C; 68,4 °C; 67,1 °C; 60,0 °C; 66,4 °C; 62,8 °C; 68,0 °C; 63,4 °C; 60,7 °C; 65,8 °C. Conformity with the acceptance criterion is to be determined.

Information needed

Value obtained

Sample size:  $n$

13

Form  $p^*$  acceptance constant:  $p^*$

0,064 7

Sample mean:  $\bar{x} = \sum x / n$

64,223 °C

Sample standard deviation:  $s = \sqrt{\sum_j (x_j - \bar{x})^2 / (n-1)}$

2,789 9 °C

Upper specification limit:  $U$

70,0 °C

Lower specification limit:  $L$

60,0 °C

Estimated fraction nonconforming 0,059 2

$$\hat{p}_L = F_{\text{BETA}}\left(\frac{n}{2}-1, \frac{n}{2}-1\right) \left( \max \left\{ 0, \frac{1}{2} - \frac{1}{2} \frac{\bar{x} - L}{s} \frac{\sqrt{n}}{n-1} \right\} \right)$$

$$\hat{p}_U = F_{\text{BETA}}\left(\frac{n}{2}-1, \frac{n}{2}-1\right) \left( \max \left\{ 0, \frac{1}{2} - \frac{1}{2} \frac{U - \bar{x}}{s} \frac{\sqrt{n}}{n-1} \right\} \right) \quad 0,011 6$$

$$\hat{p} = \hat{p}_L + \hat{p}_U \quad 0,070 8$$

Acceptance criterion: Is  $\hat{p} \leq p^*$  No (0,070 8 > 0,064 7)

The sample does not meet the acceptance criterion; therefore the lot is rejected.

EXAMPLE 3 Single, lower specification limit using the  $\sigma$ -method. Example 1 from 15.2.

The specified minimum yield point for certain steel castings is 400 N/mm<sup>2</sup>. The next lot of 500 items is submitted for inspection. Inspection level II, normal inspection, with AQL of 0,65 %, is to be used. The value of  $\sigma$  is considered to be 21 N/mm<sup>2</sup>. From Table 1, it is seen that the sample size code letter is H. From Table 8, it is seen that for an AQL of 0,65 %, the sample size,  $n$ , is 11 and that the form  $p^*$  acceptance constant is 0,015 9. Suppose the yield points of the sample specimens in N/mm<sup>2</sup> are: 431; 417; 469; 407; 450; 452; 427; 411; 429; 420; 400. Conformity with the acceptance criterion is to be determined.

**Information needed**

**Value obtained**

Sample size:  $n$

11

Form  $p^*$  acceptance constant:  $p^*$

0,042 4

Sample mean:  $\bar{x}$

428,5 N/mm<sup>2</sup>

Known  $\sigma$

21 N/mm<sup>2</sup>

Lower specification limit:  $L$

400 N/mm<sup>2</sup>

Estimated fraction nonconforming

0,082 4

$$\hat{p}_L = \Phi \left( \frac{L - \bar{x}}{\sigma} \sqrt{\frac{n}{n-1}} \right)$$

Acceptance criterion: Is  $\hat{p}_L \leq p^*$  ?

No (0,082 4 > 0,072 0)

The sample does not meet the acceptance criterion; therefore, the lot is rejected.

EXAMPLE 4 Combined control of double specification limits using the  $\sigma$ -method. Example 3 from 15.2.

The specification for electrical resistance of a certain electrical component is (520 ± 50) Ω. Production is at a rate of 1 000 items per inspection lot. Inspection level II, normal inspection, with a single AQL of 1,5 %, is to be used for the two specification limits (470 Ω and 570 Ω).  $\sigma$  is known to be 18,5 Ω.

**Information needed**

**Value obtained**

Factor from Table 11:  $f_\sigma$

0,205 6

Upper specification limit:  $U$

570 Ω

Lower specification limit:  $L$

470 Ω

Known  $\sigma$

18,5 Ω

It is noted that the known  $\sigma$  of 18,5  $\Omega$  is less than  $(U - L) \times \text{MPSD} = 20,56 \Omega$  so there is evidence that there is a possibility that lots will be accepted so acceptance sampling can be undertaken. Entering [Table 1](#) with the lot size and inspection level, it is found that the sample size code letter is J; from [Table 8](#), it is seen that the sample size,  $n$ , is 19 and that the form  $p^*$  acceptance constant is 0,042 4. Suppose that the 19 sample values of the resistance, in  $\Omega$ , are as follows: 515; 491; 479; 513; 521; 536; 483; 509; 514; 507; 484; 526; 532; 499; 530; 512; 492; 522; 488. Sample acceptability is to be determined.

Further information needed	Value obtained
Sample size: $n$	19
Form $p^*$ acceptance constant: $p^*$	0,042 4
Sample mean: $\bar{x}$	508,0 $\Omega$
Lower specification limit: $L$	470 $\Omega$
Upper specification limit; $U$	570 $\Omega$
Estimated fraction nonconforming	0,017 4
$\hat{p}_L = \Phi \left( \frac{L - \bar{x}}{\sigma} \sqrt{\frac{n}{n-1}} \right)$	0,000 3
$\hat{p}_U = \Phi \left( \frac{\bar{x} - U}{\sigma} \sqrt{\frac{n}{n-1}} \right)$	0,017 7
$\hat{p} = \hat{p}_L + \hat{p}_U$	Yes (0,017 7 $\leq$ 0,042 4)
Acceptance criterion: Is $\hat{p} \leq p^*$	
The sample meets the acceptance criterion; therefore, the lot is accepted.	

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## Annex A (informative)

### Procedures for obtaining $s$ and $\sigma$

#### A.1 Procedure for obtaining $s$

##### A.1.1 Definition

The observed value of a sample standard deviation is generally denoted by the symbol  $s$ . It is defined in [Formula \(A.1\)](#).

$$s = \sqrt{\frac{\sum_{j=1}^n (x_j - \bar{x})^2}{n-1}} \quad (\text{A.1})$$

where

$x_j$  is the value of the quality characteristic of the  $j^{\text{th}}$  item in a sample of  $n$  articles, expressed as a decimal fraction;

$\bar{x}$  is the mean value of the  $x_j$ , i.e.

$$\bar{x} = \frac{1}{n} \sum_{j=1}^n x_j \quad (\text{A.2})$$

##### A.1.2 One-pass formula

[Formula \(A.1\)](#) for  $s$  is accurate but requires two passes through the sample data. A number of alternative methods exist and these are analysed in Reference [17] and are described here. An algebraically equivalent formula, which only requires a single pass through the sample data, is:

$$s = \sqrt{\frac{n \sum_{j=1}^n x_j^2 - \left( \sum_{j=1}^n x_j \right)^2}{n(n-1)}} \quad (\text{A.3})$$

##### A.1.3 Potential numerical inaccuracy

Although [Formula \(A.3\)](#) is well suited for manual calculations it can produce poor results for difficult problems where the variability is very small relative to the mean, i.e.  $s$  is very small in comparison with

$\bar{x}$ . Although [Formula \(A.3\)](#) can be improved by subtracting a suitable arbitrary constant,  $a$ , from all the values before computing  $s$ , i.e.

$$s = \sqrt{\frac{n \sum_{j=1}^n (x_j - a)^2 - \left[ \sum_{j=1}^n x_j - a \right]^2}{n(n-1)}} \tag{A.4}$$

it should be noted that there is no generally accepted guidance on how to choose this arbitrary constant and if either of these formulae is used the result may be suspect should  $s/\bar{x}$  turn out to be much less than 1.

**A.1.4 An updating algorithm for sequential data**

Both [Formula \(A.1\)](#) and [\(A.4\)](#) require complete recalculation in situations where additional sample data is to be added to the initial sample, for example in double sampling or sequential sampling. In these situations an updating algorithm due to West, which makes the sample mean and sample standard deviation at any stage in the updating process and which is almost as accurate as that for [Formula \(A.1\)](#) is preferable:

$$\begin{aligned} M_1 &= x_j, T_1 = 0 \\ M_j &= M_{j-1} + \left( \frac{x_j - M_{j-1}}{j} \right) \quad j = 2, \dots, n \\ T_j &= T_{j-1} + (j-1)(x_j - M_{j-1}) \left( \frac{x_j - M_{j-1}}{j} \right) \quad j = 2, \dots, n \\ \bar{x}_j &= M_j \\ s_j &= \sqrt{T_j / (j-1)} \end{aligned} \tag{A.5}$$

**A.1.5 Spreadsheets and pocket calculators**

Many spreadsheets and pocket calculators have a standard deviation function. Unfortunately, sometimes the sample size,  $n$ , is used by the machine in the denominator of [Formula \(A.1\)](#) instead of  $n - 1$ . If it is planned to use a spreadsheet or calculator, it is important to check that the formula used is equivalent to [Formula \(A.1\)](#). A simple check is to find the standard deviation of the three numbers 0, 1, and 2. The sample size  $n$  is 3, the sample mean is 1, the deviations from the mean are -1, 0, and 1, the squares of the deviations are 1, 0, and 1, the sum of squares of the deviations is 2. So from [Formula \(A.1\)](#), one obtains:

$$s = \sqrt{\frac{2}{2}} = \sqrt{1} = 1 \tag{A.6}$$

If the spreadsheet or calculator is erroneously using  $n$  instead of  $n - 1$  in the denominator, then the result of the calculation is:

$$s = \sqrt{\frac{2}{3}} = 0,816\ 5$$

Use of  $n$  in the denominator shall be avoided, for otherwise the acceptance criterion is weakened and any protection to the consumer is lost. The result obtained from the spreadsheet or calculator shall be scaled by  $(n - 1)/n$  to obtain the correct result.

NOTE It is instructive to work through the use of [Formula \(A.3\)](#) for this example. It is found that

$$s = \sqrt{\frac{3 \times (0^2 + 1^2 + 2^2) - (0 + 1 + 2)^2}{3 \times (3 - 1)}} = \sqrt{\frac{3 \times (0 + 1 + 4) - 3^2}{3 \times 2}} = \sqrt{\frac{3 \times 5 - 9}{6}} = \sqrt{\frac{6}{6}} = 1$$

as before.

## A.2 Procedure for obtaining $\sigma_{\text{root}}$

### A.2.1 Definition

If it appears from the control chart that the value of  $s$  is in control,  $\sigma_{\text{root}}$  can be presumed to be the weighted root mean square of  $s$  given by the following [Formula \(A.7\)](#):

$$\sigma_{\text{root}} = \sqrt{\frac{\sum_{i=1}^m (n_i - 1) s_i^2}{\sum_{i=1}^m (n_i - 1)}} \quad (\text{A.7})$$

where

$m$  is the number of lots;

$n_i$  is the sample size from the  $i^{\text{th}}$  lot;

$s_i$  is the sample standard deviation from the  $i^{\text{th}}$  lot.

### A.2.2 Simplification for equal sample sizes

If the sample sizes from each of the lots are equal, then [Formula \(A.7\)](#) simplifies to

$$\sigma_{\text{root}} = \sqrt{\frac{\sum_{i=1}^m s_i^2}{m}} \quad (\text{A.8})$$

## Annex B (informative)

### Accommodating measurement variability

#### B.1 General

The master tables, i.e. the tables in [Clause 14](#), are based on the assumption that the true values of the quality characteristic,  $x$ , of the items in the lots are normally distributed with unknown process mean,  $\mu$ , and either known or unknown process standard deviation,  $\sigma$ ; the assumption is also made that  $x$  can be measured without measurement error, i.e. that the measurement of an item with the true value,  $x_i$ , results in the value  $x_i$ . This annex explains how the tables in [Clause 14](#) can be used in the result of measurement error. For further technical background, see Reference [19].

In the case of measurement error, the measured value of an item with true value,  $x_i$ , differs from  $x_i$ . It is assumed that

- the measurement method is unbiased, i.e. the expected value of the measurement error is zero;
- measurement error inflates the perceived process variation and is independent of the actual process standard deviation;
- measurement error is normally distributed with known or unknown measurement standard deviation,  $\sigma_m$ .

It follows that the distribution of the measured values is a normal distribution with mean  $\mu$ , and standard deviation, given by [Formula \(B.1\)](#):

$$\sigma_{\text{total}} = \sqrt{\sigma^2 + \sigma_m^2} \quad (\text{B.1})$$

NOTE  $\sigma_{\text{total}}$  is always larger than  $\sigma$  if measurement error exists.

If it is known that  $\sigma_m < \sigma/10$ , i.e. the ratio  $\gamma = \sigma_m / \sigma$  of measurement standard deviation to process standard deviation is less than 10 %, the total standard deviation is given by [Formula \(B.2\)](#):

$$\sigma_{\text{total}} < \sqrt{\sigma^2 + (0,1\sigma)^2} = \sigma\sqrt{1+0,01} = 1,005\sigma \quad (\text{B.2})$$

i.e. the standard deviation is increased by less than 0,5 %, which is negligible and hence, the sampling plans do not need to be adjusted for measurement error.

In cases where  $\sigma_m \geq 0,1\sigma$ , the sampling plans of this document shall be used with the following adjustments.

- a) Increase the sample size,  $n$ , in order to compensate for the perceived inflated variability, but do not alter the acceptance constant,  $k$  or  $p^*$ .
- b) When the process standard deviation,  $\sigma$ , is known, use  $\sigma$  in calculating the test statistic,  $\bar{x} \pm k\sigma$  or  $\hat{p}$ ; otherwise, use an estimate  $s$  of  $\sigma$  in calculating the test statistic  $\bar{x} \pm ks$  or  $\hat{p}$ .

Further details are given in [B.2](#) to [B.4](#) for three distinct cases.

## B.2 Process standard deviation $\sigma$ and measurement standard deviation $\sigma_m$ are both known

- a) Increase the sample size,  $n$ , of the sampling plan to

$$n^* = n(1 + \gamma^2) \quad (\text{B.3})$$

- b) Use the process standard deviation,  $\sigma$ , in calculating the test statistic  $\bar{x} \pm k\sigma$  or  $\hat{p}$ .

## B.3 Process standard deviation $\sigma$ is unknown but measurement standard deviation $\sigma_m$ is known

- a) Increase the sample size,  $n$ , of the sampling plan as given by [Formula \(B.4\)](#):

$$n^* = n(1 + \tilde{\gamma}^2) \quad (\text{B.4})$$

where  $\tilde{\gamma}$  is an estimated upper bound of  $\gamma = \sigma_m / \sigma$ .

NOTE As  $\tilde{\gamma}$  increases, the operating characteristic curve of the sampling plan turns clockwise around the indifference quality point ( $p_{50} \%$ , 0,5), i.e. the point where the probability of acceptance of the lot is 50 %. If  $\tilde{\gamma}$  is overestimated ( $\tilde{\gamma}$  larger than  $\gamma$ ), the sampling plan is better than required, i.e. its probabilities of acceptance are larger than required for  $P > p_{50} \%$  and smaller than required for  $P < p_{50} \%$ . Hence, overestimation of  $\gamma$  ensures a sampling plan that is better than required.

- b) Use the estimate given by [Formula \(B.5\)](#):

$$s^* = \sqrt{s^2 - \sigma_m^2} \quad (\text{B.5})$$

of the process standard deviation instead of  $s$  in calculating the test statistic  $\bar{x} \pm ks$  or  $\hat{p}$ .

If  $s^2 - \sigma_m^2 < 0$ , use  $s^* = 0$ .

## B.4 Process standard deviation $\sigma$ and measurement standard deviation $\sigma_m$ are both unknown

Increase the sample size,  $n$ , in accordance with [Formula \(B.4\)](#), perform duplicate (or multiple) measurements on each sampled item, and use the measurement results to estimate the process standard deviation separately from the measurement standard deviation, as shown below. Use this estimate instead of  $s$  in calculating the test statistic  $\bar{x} \pm ks$  or  $\hat{p}$ .

- a) Estimation of the process and measurement standard deviations.

To denote the  $j$ th measurement on the  $i$ th item by  $x_{ij}$ , the mean for the  $i$ th item by  $\bar{x}_i$ , and the overall mean by  $\bar{\bar{x}}$ . The number of measurements for the  $i$ th item shall be denoted by  $n_i$ . The total sum of

squares of the measurements about their overall mean can be partitioned as given in the following [Formula \(B.6\)](#):

$$\begin{aligned}
 \sum_{i=1}^n \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_{..})^2 &= \sum_{i=1}^n \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i. + \bar{x}_i. - \bar{x}_{..})^2 \\
 &= \sum_{i=1}^n \sum_{j=1}^{n_i} [(x_{ij} - \bar{x}_i.)^2 + (\bar{x}_i. - \bar{x}_{..})^2 + 2(x_{ij} - \bar{x}_i.)(\bar{x}_i. - \bar{x}_{..})] \\
 &= \sum_{i=1}^n \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i.)^2 + \sum_{i=1}^n n_i (\bar{x}_i. - \bar{x}_{..})^2 + 2 \sum_{i=1}^n (\bar{x}_i. - \bar{x}_{..}) \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i.) \\
 &= \sum_{i=1}^n \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i.)^2 + \sum_{i=1}^n n_i (\bar{x}_i. - \bar{x}_{..})^2 + 0 \\
 &= \sum_{i=1}^n \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i.)^2 + \sum_{i=1}^n n_i (\bar{x}_i. - \bar{x}_{..})^2
 \end{aligned} \tag{B.6}$$

= W + B

where

W is the within-items sum of squares;

B is the between-items sum of squares.

The expectations of these sums of squares are given in [Formula \(B.7\)](#):

$$E(W) = \sigma_m^2 \sum_{i=1}^n (n_i - 1) = \sigma_m^2 (N - n) \tag{B.7}$$

where  $N = \sum_{i=1}^n n_i$  is the total number of observations, and following [Formula \(B.8\)](#):

$$E(B) = \sigma_m^2 (n - 1) + (N - n) \sigma^2 \tag{B.8}$$

Hence,  $\sigma_m^2$  can be estimated by [Formula \(B.9\)](#)

$$\sigma_m^2 = W / (N - n) \tag{B.9}$$

and  $\sigma^2$  can be estimated by [Formula \(B.10\)](#)

$$s^2 = \hat{\sigma}^2 = [B - (n - 1) \hat{\sigma}_m^2] / (N - n) \tag{B.10}$$

**EXAMPLE**

A manufactured component has a dimension with an upper specification limit of 13,05 cm. The process standard deviation,  $\sigma$ , and measurement standard deviation,  $\sigma_m$ , are unknown, but from previous experience, it is known that the ratio  $\sigma_m/\sigma$  is greater than 0,1 but less than 0,2. Lots of size 1 000 of these components are to be inspected. Normal inspection is to be instituted with an AQL of 0,15 %.

From [Table 1](#), it is found that the sample size code letter is J. As only one specification limit is being controlled, form k can be used; from [Table 2](#), the sampling plan for an AQL of 0,15 % in the absence of sampling error is  $n = 23, k = 2,425$ .

As  $\sigma_m / \sigma$  exceeds 0,1, it is necessary to adjust the sample size to allow for measurement uncertainty.

In the presence of the worst conceivable measurement error, the appropriate sample size (from [Formula B.3](#)) is given by

$$n^* = n(1 + \tilde{\gamma}^2) = 23(1 + (0,2)^2) = 23 \times 1,04 = 23,92.$$

The sample size should be an integer so, in order to provide at least the required AQL protection,  $n^*$  is rounded up to  $n^* = 24$ . A random sample of 24 of the components is taken from the next lot, and, in order to be able to assess the measurement uncertainty, each component is measured twice. The results for the sample from the first lot are as shown in [Table B.1](#).

**Table B.1 — Sample results for the example of B.4**

Item, $i$	$x_{i1}$	$x_{i2}$												
1	12,9972	12,9997	6	13,0231	13,0219	11	12,9562	12,9621	16	12,9578	12,9527	21	13,0009	12,9993
2	12,9848	12,9731	7	12,9930	12,9937	12	12,9886	12,9867	17	12,9765	12,9674	22	13,0034	12,9945
3	12,9646	12,9630	8	12,9589	12,9439	13	13,0071	13,0083	18	12,9991	13,0010	23	12,9651	12,9625
4	12,9543	12,9539	9	12,9589	12,9524	14	12,9787	12,9738	19	13,0029	13,0067	24	12,9865	12,9852
5	12,9763	12,9802	10	13,0150	13,0164	15	12,9274	12,9277	20	12,9688	12,9762			

The accuracy of subsequent calculations can be improved by subtracting an arbitrary constant that reduces the number of significant figures. Denote the constant by  $c$  and set  $c = 12,9$ . The resulting values of  $y_{ij} = x_{ij} - 12,9$  are as given in [Table B.2](#).

**Table B.2 — Sample results with subtraction of 12,9 for the example of B.4**

Item, $i$	$y_{i1}$	$y_{i2}$												
1	0,0972	0,0997	6	0,1231	0,1219	11	0,0562	0,0621	16	0,0578	0,0527	21	0,1009	0,0993
2	0,0848	0,0711	7	0,0930	0,0937	12	0,0886	0,0867	17	0,0765	0,0674	22	0,1034	0,0945
3	0,0646	0,0630	8	0,0589	0,0439	13	0,1071	0,1083	18	0,0991	0,1010	23	0,0651	0,0625
4	0,0543	0,0539	9	0,0589	0,0524	14	0,0787	0,0738	19	0,1029	0,0992	24	0,0865	0,0852
5	0,0763	0,0802	10	0,1150	0,1164	15	0,0274	0,0277	20	0,0688	0,0762			

The sum of the  $y_{ij}$  is  $\sum_{i=1}^{24} \sum_{j=1}^2 y_{ij} = 3,8399$ .

The sample mean value of  $y$  is  $\bar{y} = 3,8399 / 48 = 0,079998$ .

Hence, the sample mean value of  $x$  is  $\bar{x} = c + \bar{y} = 12,9 + 0,079998 = 12,979998$ .

The total sum of squares of  $y$  is  $T = \sum_{i=1}^{24} \sum_{j=1}^2 y_{ij}^2 = 0,33279115$ .

The total sum of squares,  $T$ , about the overall sample mean is given by [Formula \(B.11\)](#):

$$= \sum_{i=1}^{24} \sum_{j=1}^2 y_{ij}^2 - \sum_{i=1}^{24} \left[ \left( \sum_{j=1}^2 y_{ij} \right)^2 / 2 \right] \tag{B.11}$$

$$= 0,33279115 - 0,30718400$$

$$= 0,02560715$$

The within-items sum of squares,  $W$ , is given by [Formula \(B.12\)](#):

$$\begin{aligned}
 W &= \sum_{i=1}^{24} \sum_{j=1}^2 (y_{ij} - \bar{y}_i.)^2 \\
 &= \sum_{i=1}^{24} \sum_{j=1}^2 y_{ij}^2 - \sum_{i=1}^{24} \left( \sum_{j=1}^2 y_{ij} \right)^2 / 2 \\
 &= 0,332\ 791\ 15 - 0,332\ 407\ 52 \\
 &= 0,000\ 383\ 63
 \end{aligned}
 \tag{B.12}$$

By subtraction, the between-item sum of squares,  $B$ , is given by [Formula \(B.13\)](#):

$$\begin{aligned}
 B &= T - W \\
 &= 0,025\ 607\ 15 - 0,000\ 383\ 63 \\
 &= 0,025\ 223\ 52
 \end{aligned}
 \tag{B.13}$$

The measurement error variance is estimated as

$$\hat{\sigma}_m^2 = W / (N - n) = 0,000\ 383\ 63 / (48 - 24) = 0,000\ 015\ 984\ 6$$

The process variance is estimated as

$$\begin{aligned}
 s^2 &= \hat{\sigma}^2 = \left[ B - (n - 1)\hat{\sigma}_m^2 \right] / (N - n) \\
 &= \left[ 0,025\ 223\ 52 - 23 \times 0,000\ 015\ 984\ 6 \right] / (48 - 24) \\
 &= 0,024\ 855\ 87 / 24 \\
 &= 0,001\ 035\ 66
 \end{aligned}$$

so the process standard deviation is estimated as

$$\begin{aligned}
 s &= \sigma = \sqrt{0,001\ 035\ 66} = 0,032\ 182 \\
 U - 2,425s &= 13,05 - 2,425 \times 0,032\ 182 = 12,972
 \end{aligned}$$

As  $\bar{x} = 12,980 > 12,972$ , the lot is rejected.

## Annex C (informative)

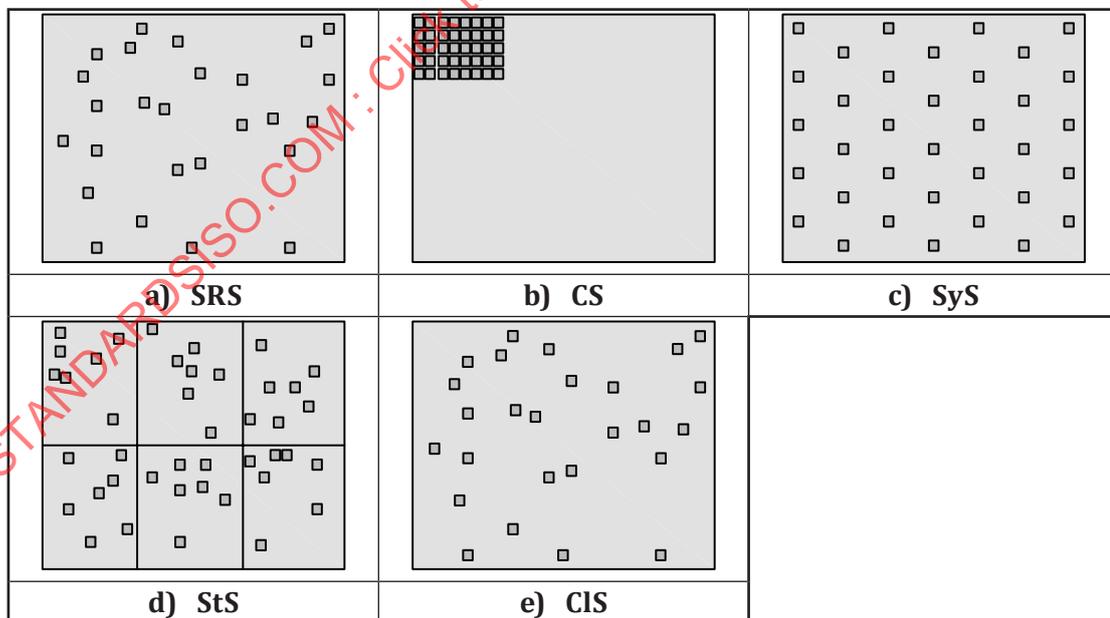
### Sampling strategies

The sampling strategy is an important part of statistical sampling. [Figure C.1](#) illustrates how the  $n$  sample units are obtained from the lot.

For industrial applications, the following sampling strategies can be differentiated.

- a) (Simple) Random sampling (SRS) – Each item in the lot has the same probability to be selected for the sample.
- b) Convenience sampling (CS) – Items that are most convenient and easy to select are taken.
- c) Systematic sampling (SyS) – The items in the sample are systematically (by time, number, etc.) spread over the lot.
- d) Stratified sampling (StS) – The lot is divided into sub lots (strata) from which (random) samples are taken. Usually sub lots and sub lot samples have equal size, but also different sizes of sub lots and/or samples are feasible.
- e) Cluster sampling (ClS) – The lot is divided into sub lots (clusters) from which sub lots are (randomly) selected. For these selected sub lots all items are sampled (100 % inspection).

The drawings in [Figure C.1](#) illustrate the different sampling strategies for a two-dimensional population, e.g. tablets spread open for a drying process.



**Figure C.1 — Examples of different sampling strategies with a two-dimensional population**

Generally, there is no statistical guidance on which sampling strategy is superior to the others. Some statistical comparisons of different sampling strategies has been carried out (see Reference [3]). However, this standard and associated probabilities does assume that random sampling is being used no matter which sampling strategy is selected. There is risk for the validity of the results if convenience (biased) sampling is used.

## ISO 3951-1:2022(E)

ISO 2859-4:2020 specifies: “The items selected for the sample shall be drawn from the lot by simple random sampling. However, when the lot consists of sub-lots or strata, identified by some rational criterion, stratified sampling shall be used in such way that the size of the subsample from each subplot or stratum is proportional to the size of that subplot or stratum”.

Under general conditions, i.e. if there are no technically or practically founded assumptions on the existence of specific nonconformity patterns, a sampling strategy, including a systematic as well as a random component, yields successful sampling in most practical applications.

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## Annex D (informative)

### Operating characteristics for the $\sigma$ -method

#### D.1 Probability of acceptance for the $\sigma$ -method

The exact probability of lot acceptance for a single specification limit at process fraction nonconforming,  $p$ , when the process standard deviation is known is given by [Formula \(D.1\)](#):

$$P_a = 1 - \Phi \left[ (k - K_p) \sqrt{n} \right] \quad (\text{D.1})$$

where

$\Phi(\cdot)$  is the distribution function of the standardized normal distribution;

$n$  is the sample size;

$K_p$  is the upper  $p$ -quantile of the standardized normal distribution;

$k$  is the  $\sigma$ -method acceptance constant.

#### D.2 Example

Consider the calculation of the probability of acceptance at a process quality of 2,5 % nonconforming for a  $\sigma$ -method plan with an AQL of 1,0 % and sample size code letter M under normal inspection. Entering [Table 5](#) with sample size code letter M and AQL of 1,0 %, it is found that the sample size,  $n$ , is 39 and the acceptance constant,  $k$ , is 1,963. The process fraction nonconforming under consideration is  $p = 0,025$  0, and  $K_p = 1,960$ . Hence,

$$P_a = 1 - \Phi \left[ (1,963 - 1,960) \sqrt{39} \right] = 1 - \Phi(-0,018\ 7) = 1 - 0,5075 = 0,492\ 5.$$

#### D.3 Comparison with tabulated value for the $s$ -method

It is instructive to observe that this probability of acceptance for the  $\sigma$ -method is very roughly in agreement with the corresponding probability of acceptance for the  $s$ -method. From the column of the table with code letter M in [Annex E](#) for AQL of 1,0 %, it is seen that a process quality level of 2,43 %, i.e.  $P = 0,024\ 3$ , corresponds to a probability of acceptance of 50 %, i.e. to  $P_a = 0,500$ .

## Annex E (informative)

### Operating characteristic for the s-method – tabulated values for single sampling plans, normal inspection

#### E.1 Probability of acceptance for the s-method

The exact probability of acceptance for a single specification limit at process fraction nonconforming  $p$ , when the process standard deviation is unknown is given by [Formula \(E.1\)](#). In addition, [Figure E.1](#) shows the operating characteristic curves for single sampling plans for normal inspection, Code Letter B.

$$P_a = 1 - F_{t(n-1, K_p \sqrt{n})}(k\sqrt{n}) \tag{E.1}$$

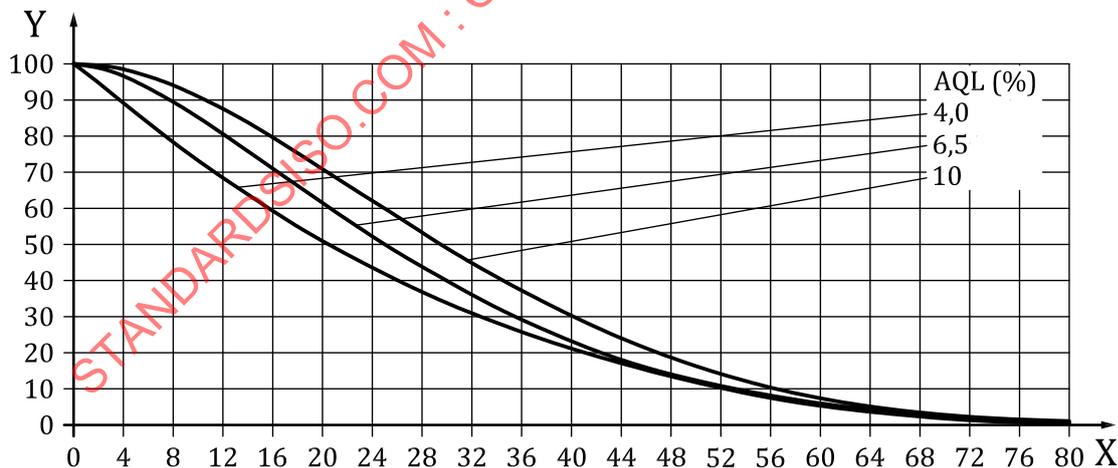
where

$F_{t(v, \delta)}(\cdot)$  is the distribution function of the noncentral  $t$ -distribution with  $v$  degrees of freedom and noncentrality  $\delta$ ;

$n$  is the sample size;

$K_p$  is the upper  $p$ -quantile of the standardized normal distribution;

$k$  is the  $\sigma$ -method acceptance constant.



**Key**

X process quality (in percent nonconforming)

Y percent of lots expected to be accepted ( $P_a$ )

**Figure E.1 — Operating characteristic curves for single sampling plans, normal inspection,  
Code Letter B**

**Table E.1 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter B: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter B					$P_a$ %
	4,0	—	6,5	10,0	—	
99,0	0,458	1,34	2,06	3,35	13,65	99,0
95,0	1,94	3,73	5,11	7,37	20,19	95,0
90,0	3,73	5,98	7,80	10,64	24,33	90,0
75,0	9,32	11,85	14,40	18,18	32,15	75,0
50,0	20,49	21,92	25,10	29,57	41,87	50,0
25,0	36,55	35,40	38,75	43,34	52,11	25,0
10,0	53,01	49,17	52,27	56,44	61,22	10,0
5,0	62,60	57,47	60,26	64,00	66,43	5,0
1,0	78,03	71,75	73,82	76,56	75,33	1,0
	6,5	—	10,0	—	—	
	Acceptance quality limit (tightened inspection) in percent – sample size code letter B					
	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) in percent – sample size code letter D					

**Table E.2 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter C: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter C					$P_a$ %	
	2,5	—	4,0	6,5	10,0		—
99,0	0,224	0,827	1,20	1,87	4,91	15,80	99,0
95,0	1,02	2,25	2,99	4,22	9,38	22,38	95,0
90,0	2,03	3,60	4,61	6,20	12,72	26,44	90,0
75,0	5,45	7,23	8,72	10,98	19,96	33,97	75,0
50,0	13,04	13,79	15,82	18,73	30,33	43,14	50,0
25,0	25,31	23,30	25,69	29,01	42,51	52,70	25,0
10,0	39,48	34,03	36,51	39,88	54,11	61,19	10,0
5,0	48,59	41,10	43,52	46,77	60,93	66,05	5,0
1,0	65,06	54,73	56,84	59,65	72,67	74,43	1,0
	4,0	—	6,5	10,0	—	—	
	Acceptance quality limit (tightened inspection) in percent – sample size code letter C						
	1,0	1,5	2,5	4,0	6,5	10,0	
	Acceptance quality limit (reduced inspection) in percent – sample size code letter E						

**Table E.3 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter D: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter D							$P_a$ %
	1,5	—	2,5	4,0	6,5	10,0	—	
99,0	0,194	0,488	0,828	1,23	2,23	6,34	14,51	99,0
95,0	0,746	1,35	1,97	2,69	4,84	11,05	20,81	95,0
90,0	1,40	2,18	2,98	3,93	6,99	14,39	24,74	90,0
75,0	3,53	4,49	5,56	6,95	12,05	21,33	32,10	75,0
50,0	8,24	8,89	10,11	12,01	20,07	30,92	41,18	50,0
25,0	16,26	15,70	16,75	19,08	30,50	41,97	50,76	25,0
10,0	26,37	23,94	24,51	27,08	41,37	52,48	59,36	10,0
5,0	33,48	29,70	29,85	32,49	48,20	58,72	64,32	5,0
1,0	47,88	41,62	40,84	43,42	60,87	69,71	72,95	1,0
	2,5	—	4,0	6,5	10,0	—	—	
Acceptance quality limit (tightened inspection) in percent – sample size code letter D								
	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
Acceptance quality limit (reduced inspection) in percent – sample size code letter F								

**Table E.4 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter E: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter E								$P_a$ %
	1,0	—	1,5	2,5	4,0	6,5	10,0	—	
99,0	0,168	0,313	0,549	0,785	1,36	3,79	7,26	18,19	99,0
95,0	0,552	0,839	1,25	1,68	2,92	6,70	11,44	23,93	95,0
90,0	0,975	1,35	1,87	2,44	4,22	8,83	14,25	27,36	90,0
75,0	2,28	2,77	3,46	4,31	7,36	13,40	19,89	33,57	75,0
50,0	5,13	5,56	6,30	7,51	12,55	20,06	27,50	41,03	50,0
25,0	10,08	10,05	10,58	12,16	19,74	28,31	36,27	48,83	25,0
10,0	16,68	15,80	15,84	17,70	27,80	36,81	44,82	55,90	10,0
5,0	21,59	20,02	19,62	21,62	33,22	42,23	50,06	60,05	5,0
1,0	32,44	29,31	27,87	30,01	44,13	52,64	59,79	67,51	1,0
	1,5	—	2,5	4,0	6,5	10,0	—	—	
Acceptance quality limit (tightened inspection) in percent – sample size code letter E									
	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
Acceptance quality limit (reduced inspection) in percent – sample size code letter G									

**Table E.5 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter F: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter F									$P_a$ %
	0,65	—	1,0	1,5	2,5	4,0	6,5	10,0	—	
99,0	0,102	0,231	0,365	0,552	0,940	2,57	4,67	10,19	13,02	99,0
95,0	0,339	0,584	0,821	1,14	1,95	4,43	7,33	14,24	17,73	95,0
90,0	0,605	0,918	1,22	1,63	2,79	5,79	9,13	16,76	20,63	90,0
75,0	1,45	1,84	2,25	2,84	4,81	8,72	12,84	21,60	26,06	75,0
50,0	3,35	3,63	4,13	4,92	8,21	13,09	18,00	27,80	32,84	50,0
25,0	6,82	6,57	7,04	8,01	13,05	18,71	24,25	34,73	40,23	25,0
10,0	11,70	10,45	10,74	11,79	18,74	24,78	30,70	41,44	47,19	10,0
5,0	15,50	13,39	13,48	14,54	22,71	28,83	34,86	45,58	51,41	5,0
1,0	24,31	20,15	19,69	20,67	31,17	37,07	43,08	53,43	59,23	1,0
	1,0	—	1,5	2,5	4,0	6,5	10,0	—	—	
Acceptance quality limit (tightened inspection) in percent – sample size code letter F										
	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
Acceptance quality limit (reduced inspection) in percent – sample size code letter H										

**Table E.6 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter G: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter G										$P_a$ %	
	0,40	—	0,65	1,0	1,5	2,5	4,0	6,5	—	10,0		—
99,0	0,077 2	0,144	0,231	0,335	0,601	1,58	2,88	6,02	7,85	9,85	11,96	99,0
95,0	0,236	0,362	0,514	0,697	1,23	2,73	4,51	8,54	10,76	13,12	15,56	95,0
90,0	0,406	0,568	0,762	1,00	1,75	3,57	5,63	10,16	12,59	15,13	17,74	90,0
75,0	0,932	1,14	1,41	1,76	3,00	5,41	7,97	13,34	16,09	18,93	21,79	75,0
50,0	2,10	2,28	2,60	3,10	5,16	8,24	11,32	17,57	20,65	23,75	26,84	50,0
25,0	4,25	4,21	4,49	5,14	8,32	11,99	15,53	22,53	25,86	29,16	32,40	25,0
10,0	7,37	6,85	6,97	7,73	12,17	16,21	20,04	27,56	31,05	34,46	37,76	10,0
5,0	9,86	8,90	8,86	9,67	14,96	19,12	23,06	30,81	34,34	37,78	41,09	5,0
1,0	15,94	13,82	13,30	14,13	21,13	25,29	29,30	37,26	40,81	44,22	47,47	1,0
	0,65	—	1,0	1,5	2,5	4,0	6,5	—	10,0	—	—	
Acceptance quality limit (tightened inspection) in percent – sample size code letter G												
	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	—	10,0	

**Table E.7 — Tabulated values for operating characteristic curves for single sampling plans – sample size code letter H: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter H												$P_a$ %
	0,25	—	0,40	0,65	1,0	1,5	2,5	4,0	—	6,5	—	10,0	
99,0	0,047 8	0,096 6	0,153	0,225	0,398	1,01	1,85	3,85	5,03	6,22	7,49	10,21	99,0
95,0	0,146	0,237	0,334	0,457	0,797	1,73	2,88	5,43	6,85	8,29	9,77	12,90	95,0
90,0	0,253	0,368	0,492	0,650	1,13	2,26	3,60	6,46	8,01	9,57	11,18	14,51	90,0
75,0	0,586	0,734	0,901	1,13	1,92	3,45	5,09	8,49	10,25	12,03	13,82	17,49	75,0
50,0	1,35	1,47	1,67	1,98	3,31	5,29	7,27	11,24	13,22	15,22	17,21	21,21	50,0
25,0	2,79	2,72	2,90	3,30	5,38	7,80	10,06	14,55	16,71	18,91	21,07	25,34	25,0
10,0	4,96	4,47	4,54	5,01	7,96	10,71	13,14	18,01	20,28	22,64	24,92	29,37	10,0
5,0	6,75	5,87	5,82	6,30	9,87	12,76	15,24	20,30	22,61	25,04	27,38	31,91	5,0
1,0	11,27	9,32	8,90	9,36	14,25	17,25	19,72	24,99	27,34	29,87	32,28	36,89	1,0
	0,40	—	0,65	1,0	1,5	2,5	4,0	6,5	—	—	10,0	—	
Acceptance quality limit (tightened inspection) in percent – sample size code letter H													
	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	—	6,5	10,0	
Acceptance quality limit (reduced inspection) in percent – sample size code letter K													

**Table E.8 — Tabulated values for operating characteristic curves for single sampling plans – sample size code letter J: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter J														$P_a$ %
	0,15	—	0,25	0,40	0,65	1,0	1,5	2,5	—	4,0	—	6,5	—	10,0	
99,0	0,033 4	0,0625	0,102	0,144	0,251	0,645	1,15	2,39	3,11	3,84	4,62	6,30	8,06	9,86	99,0
95,0	0,097 1	0,150	0,216	0,288	0,498	1,09	1,79	3,37	4,24	5,12	6,04	7,96	9,95	11,97	95,0
90,0	0,164	0,232	0,315	0,408	0,701	1,42	2,23	4,01	4,97	5,93	6,92	8,97	11,08	13,21	90,0
75,0	0,372	0,459	0,569	0,707	1,20	2,16	3,17	5,29	6,39	7,48	8,60	10,86	13,16	15,46	75,0
50,0	0,841	0,915	1,04	1,24	2,07	3,31	4,56	7,05	8,30	9,54	10,79	13,27	15,75	18,24	50,0
25,0	1,74	1,71	1,81	2,08	3,39	4,90	6,37	9,20	10,59	11,97	13,34	16,00	18,65	21,30	25,0
10,0	3,11	2,84	2,86	3,18	5,09	6,78	8,41	11,51	12,99	14,48	15,94	18,74	21,52	24,29	10,0
5,0	4,27	3,76	3,68	4,03	6,37	8,13	9,84	13,06	14,59	16,14	17,64	20,50	23,34	26,17	5,0
1,0	7,27	6,08	5,70	6,09	9,38	11,16	12,93	16,32	17,91	19,53	21,10	24,04	26,96	29,89	1,0
	0,25	—	0,40	0,65	1,0	1,5	2,5	4,0	—	—	6,5	—	10,0	—	
Acceptance quality limit (tightened inspection) in percent – sample size code letter J															
	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	—	4,0	6,5	—	—	
Acceptance quality limit (reduced inspection) in percent – sample size code letter L															

**Table E.9 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter K: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter K																$P_a$ %
	0,10	—	0,15	0,25	0,40	0,65	1,0	1,5	—	2,5	—	4,0	—	6,5	—	10,0	
99,0	0,022 7	0,042 2	0,066 4	0,094 4	0,169	0,420	0,741	1,53	1,98	2,45	2,94	3,99	5,09	6,22	8,64	10,49	99,0
95,0	0,064 0	0,098 8	0,139	0,187	0,328	0,703	1,14	2,15	2,70	3,27	3,84	5,06	6,30	7,57	10,23	12,25	95,0
90,0	0,107	0,151	0,202	0,263	0,457	0,912	1,43	2,56	3,16	3,78	4,40	5,71	7,03	8,37	11,16	13,27	90,0
75,0	0,239	0,295	0,364	0,453	0,772	1,38	2,02	3,38	4,08	4,78	5,48	6,93	8,38	9,84	12,83	15,09	75,0
50,0	0,539	0,585	0,667	0,794	1,32	2,12	2,92	4,52	5,32	6,11	6,91	8,51	10,09	11,69	14,88	17,29	50,0
25,0	1,12	1,09	1,16	1,33	2,17	3,15	4,10	5,93	6,83	7,71	8,59	10,33	12,04	13,76	17,14	19,68	25,0
10,0	2,01	1,82	1,85	2,05	3,27	4,39	5,45	7,46	8,45	9,39	10,33	12,18	13,98	15,81	19,33	21,98	10,0
5,0	2,78	2,43	2,39	2,61	4,10	5,29	6,41	8,50	9,53	10,51	11,49	13,39	15,24	17,12	20,72	23,43	5,0
1,0	4,83	3,98	3,75	3,99	6,11	7,35	8,53	10,73	11,82	12,84	13,88	15,87	17,79	19,76	23,47	26,27	1,0
	0,15	—	0,25	0,40	0,65	1,0	1,5	2,5	—	—	4,0	—	6,5	—	10,0	—	
Acceptance quality limit (tightened inspection) in percent – sample size code letter K																	
	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	—	2,5	4,0	—	—	—	—	
Acceptance quality limit (reduced inspection) in percent – sample size code letter M																	

**Table E.10 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter L: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter L																$P_a$ %
	0,065	—	0,10	0,15	0,25	0,40	0,65	1,0	—	1,5	—	2,5	—	4,0	—	6,5	
99,0	0,014 9	0,026 6	0,043 2	0,061 0	0,108	0,264	0,470	0,959	1,24	1,53	1,84	2,48	3,15	3,86	5,34	6,47	99,0
95,0	0,041 0	0,061 8	0,089 0	0,119	0,207	0,440	0,720	1,34	1,68	2,03	2,40	3,14	3,91	4,70	6,33	7,58	95,0
90,0	0,068 0	0,094 4	0,128	0,166	0,287	0,570	0,894	1,60	1,97	2,35	2,75	3,55	4,37	5,21	6,92	8,23	90,0
75,0	0,150	0,184	0,229	0,285	0,483	0,862	1,27	2,11	2,54	2,98	3,42	4,32	5,22	6,14	7,99	9,39	75,0
50,0	0,336	0,367	0,418	0,497	0,827	1,33	1,82	2,83	3,32	3,83	4,32	5,32	6,31	7,32	9,31	10,82	50,0
25,0	0,697	0,690	0,729	0,835	1,36	1,98	2,57	3,73	4,28	4,85	5,39	6,50	7,57	8,66	10,78	12,39	25,0
10,0	1,27	1,16	1,16	1,29	2,06	2,78	3,43	4,72	5,31	5,94	6,52	7,71	8,85	10,00	12,24	13,93	10,0
5,0	1,76	1,56	1,51	1,65	2,60	3,37	4,04	5,40	6,02	6,67	7,27	8,50	9,68	10,87	13,16	14,91	5,0
1,0	3,11	2,60	2,39	2,54	3,92	4,72	5,42	6,87	7,52	8,22	8,84	10,16	11,39	12,64	15,03	16,85	1,0
	0,10	—	0,15	0,25	0,40	0,65	1,0	1,5	—	—	2,5	—	4,0	—	6,5	—	
Acceptance quality limit (tightened inspection) in percent – sample size code letter L																	
	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	—	1,5	2,5	—	—	—	—	
Acceptance quality limit (reduced inspection) in percent – sample size code letter N																	

**Table E.11 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter M: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent - sample size code letter M																$P_a$ %
	0,04	—	0,065	0,10	0,15	0,25	0,40	0,65	—	1,0	—	1,5	—	2,5	—	4,0	
99,0	0,009 7	0,017 2	0,027 8	0,039 1	0,068 5	0,170	0,299	0,610	0,787	0,973	1,17	1,58	2,00	2,44	3,36	4,09	99,0
95,0	0,026 3	0,039 5	0,056 8	0,075 5	0,131	0,282	0,457	0,85	1,07	1,29	1,52	1,99	2,48	2,98	4,00	4,79	95,0
90,0	0,043 4	0,060 1	0,081 6	0,106	0,181	0,364	0,567	1,01	1,25	1,49	1,74	2,25	2,77	3,30	4,38	5,20	90,0
75,0	0,095 2	0,117	0,145	0,180	0,305	0,548	0,803	1,34	1,61	1,89	2,17	2,74	3,31	3,89	5,06	5,95	75,0
50,0	0,213	0,233	0,265	0,315	0,524	0,842	1,16	1,80	2,11	2,43	2,75	3,38	4,02	4,65	5,92	6,87	50,0
25,0	0,444	0,439	0,465	0,532	0,868	1,26	1,64	2,37	2,73	3,09	3,44	4,13	4,84	5,52	6,89	7,89	25,0
10,0	0,813	0,746	0,743	0,826	1,33	1,77	2,19	3,02	3,40	3,79	4,17	4,91	5,68	6,39	7,85	8,91	10,0
5,0	1,14	1,00	0,97	1,06	1,68	2,16	2,60	3,46	3,86	4,27	4,66	5,43	6,23	6,97	8,47	9,55	5,0
1,0	2,04	1,69	1,55	1,65	2,57	3,05	3,51	4,44	4,85	5,29	5,70	6,51	7,37	8,14	9,73	10,86	1,0
	0,065	—	0,10	0,15	0,25	0,40	0,65	1,0	—	—	1,5	—	2,5	—	4,0	—	
Acceptance quality limit (tightened inspection) in percent - sample size code letter M																	
	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	—	1,0	1,5	—	—	—	—	
Acceptance quality limit (reduced inspection) in percent - sample size code letter P																	

**Table E.12 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter N: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent - sample size code letter N																$P_a$ %
	0,025	—	0,04	0,065	0,10	0,15	0,25	0,40	—	0,65	—	1,0	—	1,5	—	2,5	
99,0	0,006 4	0,011 1	0,017 6	0,025 0	0,044 5	0,109	0,190	0,387	0,497	0,614	0,735	0,991	1,26	1,54	2,12	2,57	99,0
95,0	0,016 9	0,025 1	0,035 8	0,048 0	0,083 7	0,178	0,289	0,538	0,673	0,813	0,956	1,25	1,56	1,87	2,51	3,01	95,0
90,0	0,027 6	0,038 0	0,051 3	0,066 8	0,115	0,230	0,358	0,639	0,787	0,940	1,10	1,41	1,74	2,07	2,75	3,27	90,0
75,0	0,060 2	0,073 7	0,091 2	0,114	0,193	0,345	0,506	0,842	1,01	1,19	1,37	1,72	2,08	2,45	3,19	3,74	75,0
50,0	0,134	0,146	0,167	0,198	0,330	0,531	0,730	1,13	1,33	1,53	1,73	2,13	2,53	2,93	3,73	4,33	50,0
25,0	0,280	0,277	0,293	0,335	0,547	0,796	1,03	1,50	1,72	1,95	2,17	2,61	3,05	3,48	4,35	4,98	25,0
10,0	0,515	0,473	0,471	0,521	0,836	1,12	1,39	1,91	2,15	2,40	2,64	3,12	3,58	4,05	4,97	5,64	10,0
5,0	0,725	0,640	0,618	0,671	1,06	1,37	1,65	2,20	2,45	2,71	2,96	3,45	3,94	4,42	5,37	6,06	5,0
1,0	1,32	1,09	1,00	1,05	1,63	1,95	2,25	2,83	3,10	3,38	3,64	4,16	4,67	5,18	6,19	6,91	1,0
	0,04	—	0,065	0,10	0,15	0,25	0,40	0,65	—	—	1,0	—	1,5	—	2,5	—	
Acceptance quality limit (tightened inspection) in percent - sample size code letter N																	
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	—	0,65	1,0	—	—	—	—	
Acceptance quality limit (reduced inspection) in percent - sample size code letter Q																	

**Table E.13 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter P: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent - sample size code letter P															$P_a$ %	
	0,015	—	0,025	0,04	0,065	0,10	0,15	0,25	—	0,40	—	0,65	—	1,0	—		1,5
99,0	0,0041	0,0070	,0113	,0158	,0286	,0682	0,119	0,243	0,312	0,385	0,461	0,620	0,787	0,960	1,32	1,60	99,0
95,0	0,0108	0,0158	,0227	,0301	,0531	0,112	0,180	0,337	0,421	0,509	0,598	0,783	0,973	1,17	1,57	1,88	95,0
90,0	0,0175	0,0239	0,0323	,0419	,0728	0,143	0,223	0,399	0,492	0,588	0,685	0,883	1,09	1,29	1,72	2,04	90,0
75,0	0,0378	0,0461	0,0573	,0710	0,121	0,215	0,316	0,526	0,634	0,743	0,854	1,08	1,30	1,53	1,99	2,34	75,0
50,0	0,0838	0,0914	0,104	,124	0,206	0,331	0,456	0,705	0,831	0,956	1,08	1,33	1,58	1,83	2,33	2,70	50,0
25,0	0,175	0,173	0,183	,210	0,341	0,498	0,647	0,936	1,08	1,22	1,36	1,63	1,91	2,18	2,72	3,12	25,0
10,0	0,323	0,297	0,296	,328	0,522	0,705	0,873	1,19	1,35	1,50	1,66	1,95	2,25	2,54	3,11	3,53	10,0
5,0	0,456	0,404	0,389	,423	0,665	0,862	1,04	1,38	1,54	1,70	1,86	2,17	2,47	2,78	3,36	3,80	5,0
1,0	0,836	0,694	0,632	,668	1,03	1,24	1,42	1,78	1,95	2,12	2,30	2,62	2,94	3,27	3,88	4,35	1,0
	0,025	—	0,04	0,065	0,10	0,15	0,25	0,40	—	—	0,65	—	1,0	—	1,5	—	
Acceptance quality limit (tightened inspection) in percent - sample size code letter P																	
	—	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	—	0,40	0,65	—	—	—	—	
Acceptance quality limit (reduced inspection) in percent - sample size code letter R																	

**Table E.14 — Tabulated values for operating characteristic curves for single sampling plans - sample size code letter Q: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent - sample size code letter Q														$P_a$ %	
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	—	0,40	—	0,65	—	1,0		
99,0	0,0027	0,0073	0,0102	0,0184	0,0440	0,0767	0,156	0,246	0,295	0,396	0,504	0,614	0,845	1,02	99,0	
95,0	0,0070	0,0146	0,0193	0,0341	0,0715	0,116	0,216	0,325	0,383	0,500	0,622	0,747	1,00	1,20	95,0	
90,0	0,0113	0,0207	0,0269	0,0467	0,0919	0,143	0,256	0,376	0,438	0,565	0,695	0,827	1,10	1,30	90,0	
75,0	0,0242	0,0366	0,0455	0,0775	0,138	0,202	0,336	0,476	0,546	0,689	0,833	0,978	1,27	1,49	75,0	
50,0	0,0536	0,0667	0,0795	0,132	0,212	0,292	0,451	0,613	0,693	0,853	1,01	1,17	1,49	1,73	50,0	
25,0	0,112	0,117	0,135	0,219	0,319	0,415	0,599	0,782	0,872	1,05	1,22	1,40	1,74	2,00	25,0	
10,0	0,208	0,190	0,212	0,336	0,453	0,562	0,766	0,968	1,06	1,26	1,44	1,63	2,00	2,27	10,0	
5,0	0,294	0,250	0,274	0,430	0,555	0,670	0,883	1,10	1,20	1,40	1,59	1,79	2,16	2,44	5,0	
1,0	0,544	0,410	0,435	0,669	0,799	0,920	1,14	1,37	1,48	1,70	1,90	2,11	2,50	2,80	1,0	
	0,015	0,025	0,04	0,065	0,10	0,15	0,25	—	0,40	—	0,65	—	1,0	—		
Acceptance quality limit (tightened inspection) in percent - sample size code letter Q																

**Table E.15 — Tabulated values for operating characteristic curves for single sampling plans – sample size code letter R: s-method**

$P_a$ %	Acceptance quality limit (normal inspection) in percent – sample size code letter R														$P_a$ %
	—	0,01	0,015	0,025	0,04	0,065	0,10	0,15	—	0,25	—	0,40	—	0,65	
99,0	0,007 0	0,004 6	0,006 5	0,011 5	0,027 7	0,048 2	0,097 6	0,312	0,155	0,461	0,249	0,315	0,384	0,528	99,0
95,0	0,015 8	0,009 2	0,012 2	0,021 2	0,044 9	0,072 6	0,135	0,421	0,204	0,598	0,313	0,389	0,467	0,627	95,0
90,0	0,023 9	0,013 0	0,016 8	0,029 0	0,057 6	0,089 7	0,160	0,492	0,235	0,685	0,353	0,434	0,517	0,686	90,0
75,0	0,046 1	0,022 9	0,028 5	0,048 2	0,086 2	0,126	0,210	0,634	0,297	0,854	0,430	0,521	0,611	0,795	75,0
50,0	0,091 4	0,041 6	0,049 6	0,082 4	0,132	0,182	0,282	0,831	0,382	1,08	0,533	0,633	0,732	0,932	50,0
25,0	0,173	0,073	0,084	0,137	0,200	0,259	0,376	1,08	0,489	1,36	0,656	0,766	0,873	1,09	25,0
10,0	0,297	0,119	0,132	0,212	0,284	0,352	0,481	1,35	0,605	1,66	0,786	0,906	1,02	1,25	10,0
5,0	0,404	0,157	0,172	0,272	0,349	0,420	0,556	1,54	0,685	1,86	0,874	0,999	1,12	1,35	5,0
1,0	0,694	0,259	0,274	0,426	0,505	0,579	0,723	1,95	0,861	2,30	1,06	1,20	1,32	1,57	1,0
	0,010	0,015	0,025	0,04	0,065	0,10	0,15	—	0,25	—	0,40	—	0,65	—	
Acceptance quality limit (tightened inspection) in percent – sample size code letter R															

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## Annex F (informative)

### Consumer's risk qualities

#### F.1 The consumer's risk quality for the $s$ -method plans

For a given sampling plan, the consumer's risk quality is the process quality for which the probability of accepting a given lot is 10 %.

For the  $s$ -method, the consumer's risk quality is the solution in  $p$  such that the probability of acceptance is 0,10 i.e.

$$1 - F_{t(n-1, K_p \sqrt{n})}(k \sqrt{n}) = 0,10$$

where

$n$  is the sample size,

$k$  is the  $s$ -method acceptance constant,

$K_p$  is the upper  $p$ -quantile of the standardized normal distribution

$F_{t(n-1, K_p \sqrt{n})}(\cdot)$  is the distribution function of the non-central  $t$ -distribution with  $n-1$  degrees of freedom and non-centrality parameter  $K_p \sqrt{n}$ .

#### F.2 Tables for the consumer's risk qualities for the $s$ -method plans

Consumer's risk qualities for the  $s$ -method plans of this document are given in [Table F.1](#), [Table F.3](#), and [Table F.5](#) for normal, tightened, and reduced inspection, respectively.

#### F.3 The consumer's risk quality for the $\sigma$ -method plans

For the  $\sigma$ -method the consumer's risk quality is the solution in  $p$  such that the probability of acceptance is 0,10 and is given by the formula:

$$1 - \Phi \left[ \frac{k - K_{0,10}}{\sqrt{n}} \right] \equiv 1 - \Phi \left[ \frac{k - 1,2816}{\sqrt{n}} \right]$$

where

$n$  is the sample size,

$k$  is the  $\sigma$ -method acceptance constant

$\Phi(\cdot)$  is the distribution function of the standardized normal distribution.

#### F.4 Tables for the consumer's risk qualities for the $\sigma$ -method plans

Consumer's risk qualities for the  $\sigma$ -method plans of this document are given in [Table F.2](#), [Table F.4](#), and [Table F.6](#) for normal, tightened, and reduced inspection, respectively.

Table F.1 — Consumer's risk quality (in percent) for normal inspection: s-method

Code letter	Acceptance quality limit (in percent nonconforming)																
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
B																	
C																	
D												26,4	24,5	27,1	41,4	51,2	
E											16,7	15,8	17,7	27,8	36,8	44,8	
F										11,7	10,7	11,8	18,7	24,8	30,7	41,4	
G									7,37	6,97	7,73	12,2	16,2	20,0	27,6	34,5	
H								4,96	4,54	5,01	7,96	10,7	13,1	18,0	22,6	29,4	
J								3,11	3,18	5,09	6,78	8,41	11,5	14,5	18,7	24,3	
K								1,85	3,27	4,39	5,45	7,46	9,39	12,2	15,8	22,0	
L								1,29	2,78	3,43	4,72	5,94	7,71	10,0	13,9		
M				0,812	0,743	0,826	1,77	2,19	2,19	3,02	3,79	4,91	6,39	8,91			
N			0,515	0,471	0,521	0,830	1,39	1,91	1,91	2,40	3,12	4,05	5,64				
P		0,323	0,296	0,328	0,521	0,705	1,19	1,50	1,50	1,95	2,54	3,53					
Q	0,207	0,190	0,211	0,336	0,453	0,562	0,968	1,26	1,26	1,63	2,27						
R	0,119	0,132	0,209	0,284	0,352	0,481	0,768	1,02	1,02	1,42							
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection. When the sample size exceeds the lot size the consumer's risk will be greater than that given in the table.																
NOTE	There is no suitable plan in this area; use the last sampling plan in the appropriate column. The consumer's risk quality is the process fraction nonconforming at which 10 % of lots are expected to be accepted.																

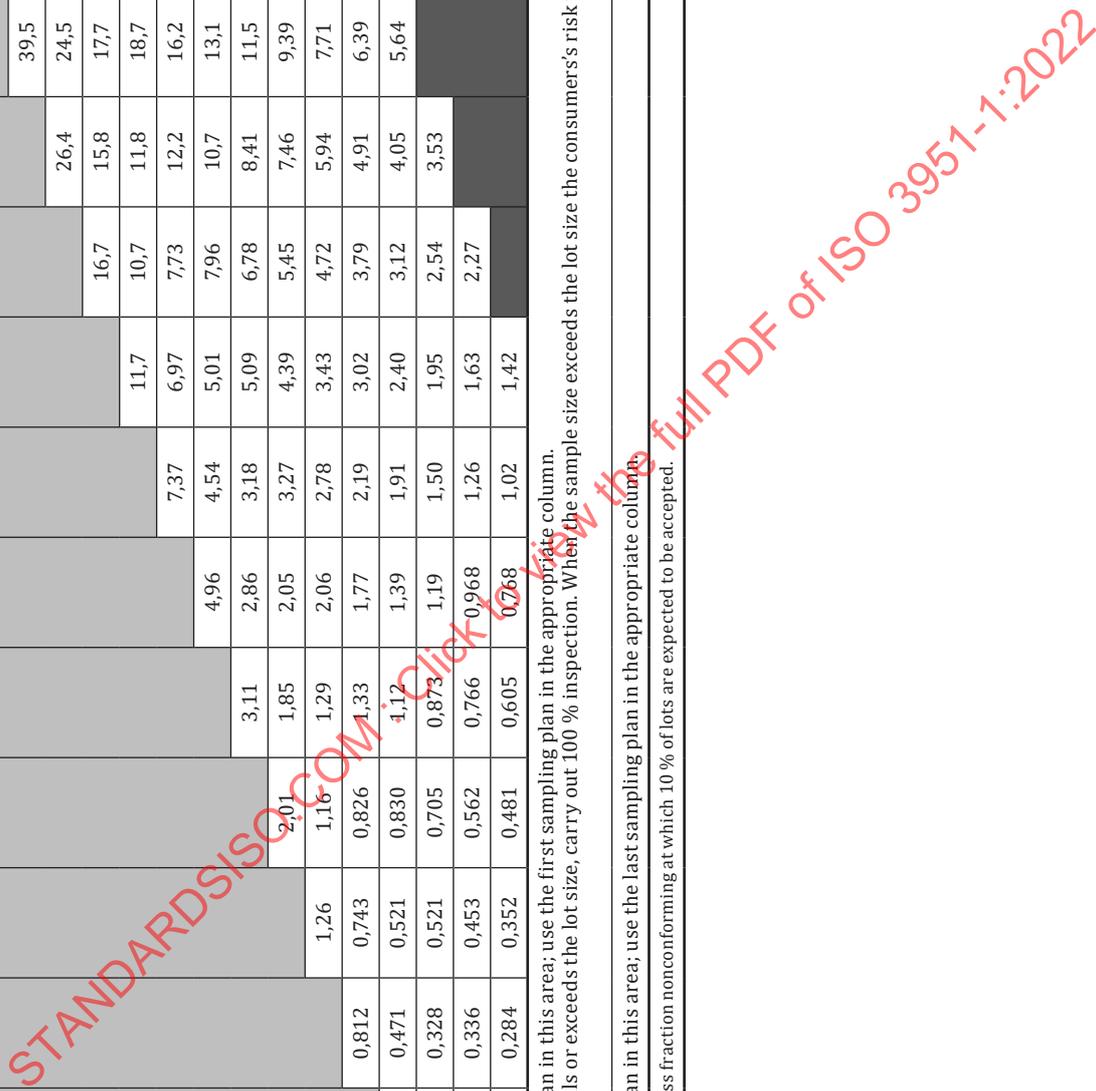


Table F.2 — Consumer's risk quality (in percent) for normal inspection:  $\sigma$ -method

Code letter	Acceptance quality limit (in percent nonconforming)																
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
B														51,2	52,8	62,7	
C													35,4	35,5	40,2	58,1	
D											22,2	23,7	27,3	27,3	42,2	55,3	
E											17,0	15,4	16,7	26,4	37,9	47,2	
F										10,2	10,3	11,4	18,1	24,6	32,6	43,0	
G									7,59	6,59	7,34	11,8	15,9	20,3	29,1	35,8	
H								4,50	4,30	4,85	7,85	10,7	13,2	19,2	23,7	30,2	
J							2,76	2,77	3,12	5,07	6,79	8,33	12,0	15,1	19,4	24,9	
K						1,98	1,80	2,04	3,18	4,39	5,45	7,54	9,76	12,6	16,2	22,3	
L					1,20	1,15	1,25	2,05	2,78	3,43	4,72	6,16	7,95	20,2	14,2		
M				0,840	0,738	0,807	1,32	1,78	2,18	3,02	3,94	5,05	6,55	9,06			
N				0,469	0,518	0,821	1,11	1,38	1,91	2,48	3,20	4,13	5,73				
P		0,311	0,284	0,317	0,523	0,696	0,865	1,19	1,55	2,00	2,59	3,59					
Q	0,193	0,184	0,206	0,330	0,448	0,562	0,768	0,990	1,29	1,66	2,30						
R	0,116	0,131	0,204	0,282	0,352	0,480	0,616	0,803	1,04	1,45							
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection. When the sample size exceeds the lot size the consumers's risk will be greater than that given in the table.																
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.																
NOTE	The consumer's risk quality is the process fraction nonconforming at which 10 % of lots are expected to be accepted.																

Table F.3 — Consumer's risk quality (in percent) for tightened inspection: s-method

Code letter	Acceptance quality limit (in percent nonconforming)																
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
B																	
C																	
D																	
E																	
F																	
G																	
H																	
J																	
K																	
L																	
M																	
N																	
P																	
Q																	
R																	
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection. When the sample size exceeds the lot size the consumer's risk will be greater than that given in the table.																
NOTE	The consumer's risk quality is the process fraction nonconforming at which 10 % of lots are expected to be accepted.																

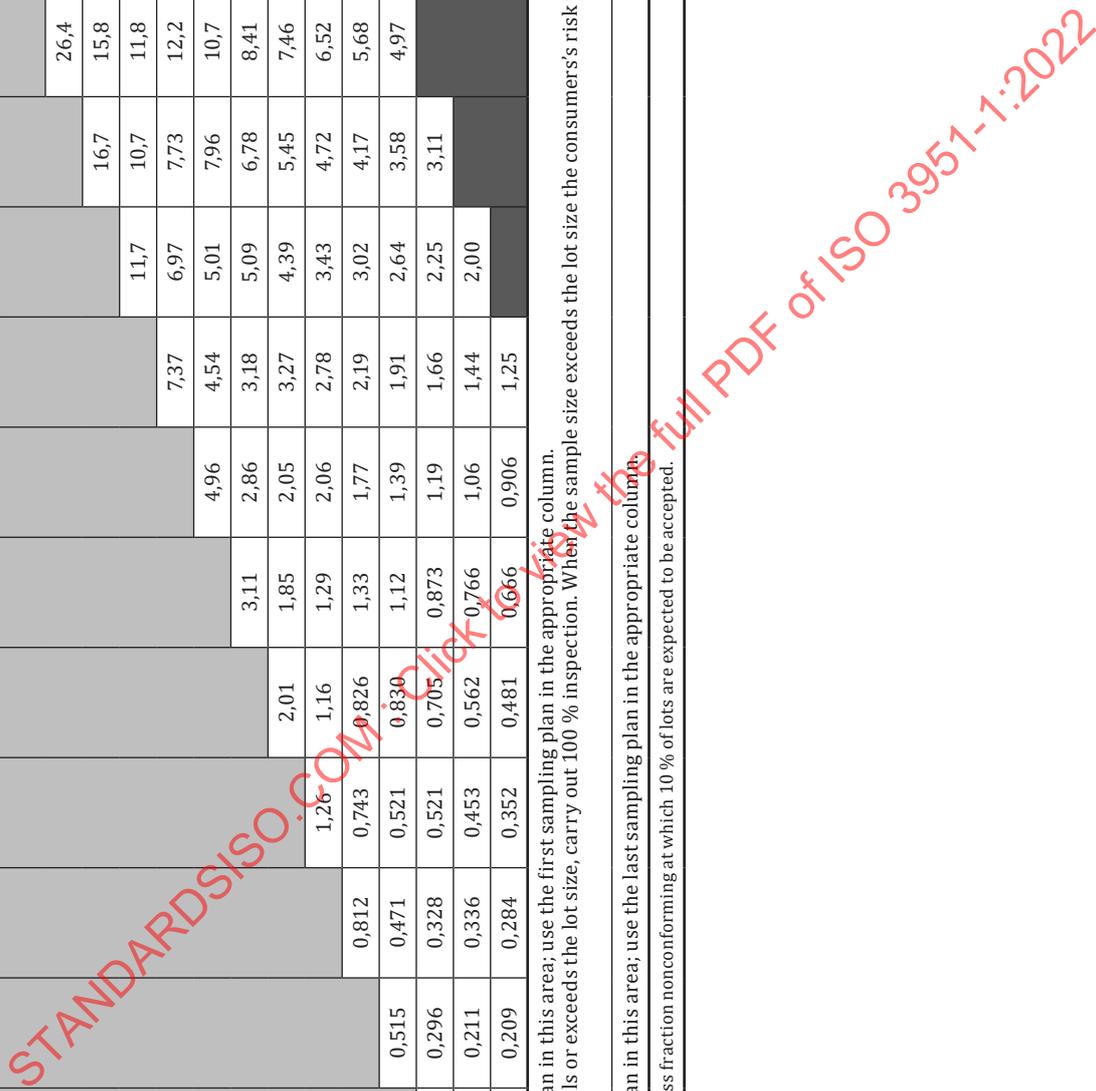


Table F.4 — Consumer's risk quality (in percent) for tightened inspection:  $\sigma$ -method

Code letter	Acceptance quality limit (in percent nonconforming)																
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
B																51,2	52,8
C															35,4	35,5	40,2
D													22,2	23,7	27,3	42,2	
E											17,0	17,0	15,4	16,7	26,4	37,9	
F											10,2	10,3	11,4	18,1	24,6	32,6	
G										7,59	6,59	7,34	11,8	15,9	20,3	29,1	
H									4,50	4,30	4,85	7,85	10,7	13,2	19,2	26,0	
J								2,76	2,77	3,12	5,07	6,79	8,33	12,0	16,7	22,1	
K								1,80	2,04	3,18	4,39	5,45	7,54	10,7	14,4	19,7	
L						1,20	1,15	1,25	2,05	2,78	3,43	4,72	6,75	9,10	12,5		
M						0,738	0,807	1,32	1,78	2,18	3,02	4,31	5,83	7,99			
N						0,510	0,821	1,11	1,38	1,91	2,72	3,67	5,05				
P			0,311			0,507	0,696	0,865	1,19	1,71	2,30	3,16					
Q		0,193	0,184	0,206		0,448	0,562	0,768	1,10	1,47	2,03						
R	0,115	0,116	0,131	0,204	0,282	0,352	0,480	0,684	0,924	1,27							
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection. When the sample size exceeds the lot size the consumers's risk will be greater than that given in the table.																
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.																
NOTE	The consumer's risk quality is the process fraction nonconforming at which 10 % of lots are expected to be accepted.																

Table F.5 — Consumer's risk quality (in percent) for reduced inspection: s-method

Code letter	Acceptance quality limit (in percent nonconforming)																
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
B-D																	
E											39,5	53,0	49,2	52,3	56,4	61,1	
F										26,4	23,9	34,1	36,5	39,9	54,1	60,8	
G									16,7	15,8	15,8	24,5	27,1	41,4	51,2	59,4	
H								11,7	10,5	10,7	11,8	17,7	27,8	36,8	44,8	55,7	
J							7,37	6,85	6,97	7,73	12,2	16,2	20,0	27,6	31,0	37,8	
K						4,96	4,48	4,54	5,01	7,96	10,7	13,1	18,0	20,3	24,9	29,4	
L						2,84	2,86	3,18	5,09	6,78	8,41	11,5	13,0	15,9	18,7		
M				2,01	1,82	1,85	2,05	3,27	4,39	5,45	7,46	8,45	10,3	12,2			
N			1,26	1,16	1,16	1,29	2,06	2,78	3,43	4,72	5,31	6,52	7,71				
P		0,812	0,745	0,743	0,826	1,33	1,77	2,19	3,02	3,40	4,17	4,91					
Q	0,515	0,473	0,471	0,521	0,830	1,12	1,39	1,91	2,15	2,64	3,12						
R	0,297	0,296	0,328	0,521	0,705	0,873	1,19	1,35	1,66	1,95							
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection. When the sample size exceeds the lot size the consumer's risk will be greater than that given in the table.																
	There is no suitable plan in this area; use the last sampling plan in the appropriate column.																
NOTE	The consumer's risk quality is the process fraction nonconforming at which 10 % of lots are expected to be accepted.																

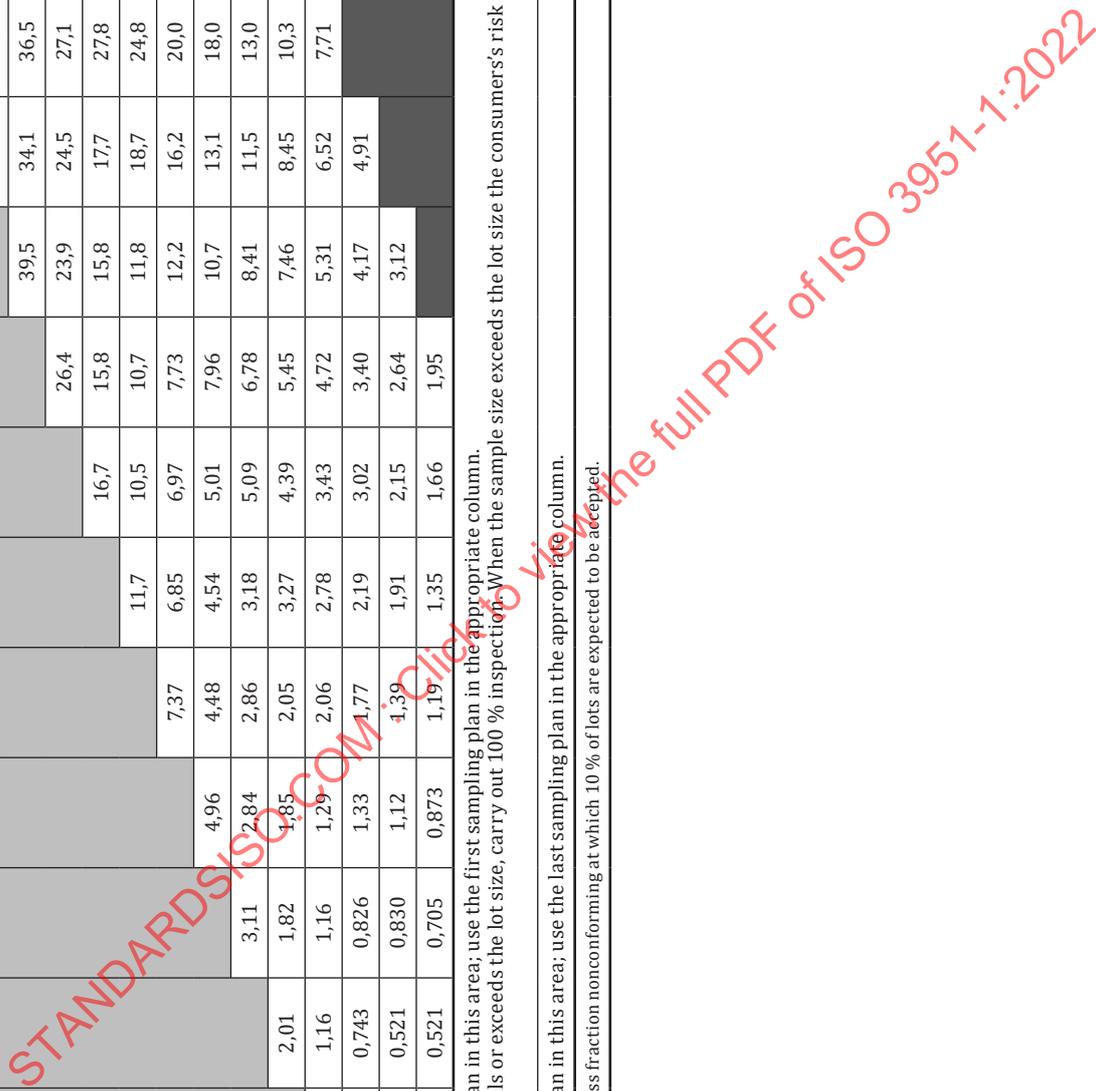


Table F.6 — Consumer's risk quality (in percent) for reduced inspection:  $\sigma$ -method

Code letter	Acceptance quality limit (in percent nonconforming)																
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0	
B-D												51,2	48,5	52,8	62,7	63,3	
E											35,4	31,8	35,5	40,2	58,1	62,8	
F										22,2	22,9	23,7	27,3	42,2	55,3	61,1	
G									17,0	14,5	15,4	16,7	26,4	37,9	47,2	56,8	
H								10,2	9,61	10,3	11,4	18,1	24,6	32,6	43,0	48,5	
J							7,59	6,71	6,59	7,34	11,8	15,9	20,3	29,1	32,5	39,0	
K						4,50	4,30	4,30	4,85	7,85	10,7	13,2	19,2	21,4	26,0	30,2	
L							2,69	3,12	5,07	6,79	8,33	12,0	13,7	16,7	19,4		
M							1,80	3,18	4,39	5,45	7,54	8,86	10,7	12,6			
N			1,20				1,25	2,78	3,43	4,72	5,54	6,75	7,95				
P		0,840	0,743				1,32	2,18	3,02	3,47	4,31	5,05					
Q	0,510	0,467	0,469				1,11	1,91	2,16	2,72	3,20						
R	0,292	0,284	0,317				0,865	1,36	1,71	2,00							
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection. When the sample size exceeds the lot size the consumer's risk will be greater than that given in the table.																
NOTE	There is no suitable plan in this area; use the last sampling plan in the appropriate column. The consumer's risk quality is the process fraction nonconforming at which 10 % of lots are expected to be accepted.																

## Annex G (informative)

### Producer's risks

#### G.1 The producer's risk for the s-method plans

The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL, i.e. 1 minus the probability of accepting a given lot when the process fraction nonconforming is equal to the AQL.

For the s-method, the producer's risk is given by the following [Formula \(G.1\)](#):

$$F_{t(n-1, K_p \sqrt{n})}(k\sqrt{n}) \tag{G.1}$$

where

$n$  is the sample size,

$p$  is the AQL expressed as a fraction nonconforming,

$k$  is the s-method acceptance constant,

$K_p$  is the upper  $p$ -quantile of the standard normal distribution

$F_{t(n-1, K_p \sqrt{n})}(\cdot)$  is the distribution function of the non-central  $t$ -distribution with degrees of freedom  $n - 1$  and non-centrality parameter  $\sqrt{n}K_p$ .

#### G.2 Tables for the producer's risks for the s-method plans

Producer's risks for the s-method plans of this document are given in [Table G.1](#), [Table G.3](#), and [Table G.5](#) for normal, tightened, and reduced inspection, respectively.

#### G.3 The producer's risk for the $\sigma$ -method plans

For the  $\sigma$ -method, the producer's risk is given by the following [Formula \(G.2\)](#):

$$\Phi[\sqrt{n}(k - K_p)] \tag{G.2}$$

where

$n$  is the sample size,

$p$  is the AQL expressed as a fraction nonconforming,

$k$  is the  $\sigma$ -method acceptance constant,

$K_p$  is the upper  $p$ -quantile of the standard normal distribution,

$\Phi(\cdot)$  is the distribution function of the standard normal distribution.

#### G.4 Tables for the producer's risk for the $\sigma$ -method plans

Producer's risks for the  $\sigma$ -method plans of this document are given in [Table G.2](#), [Table G.4](#), and [Table G.6](#) for normal, tightened, and reduced inspection, respectively.

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Table G.1 — Producer's risk (in percent) for normal inspection: s-method

Code letter	Acceptance quality limit (in percent nonconforming)															
	0,01	0,015	0,025	0,04	0,065	0,10	0,15	0,25	0,40	0,65	1,0	1,5	2,5	4,0	6,5	10,0
B														10,8	7,46	8,93
C													12,2	8,00	10,8	5,82
D											10,3	10,8	7,52	10,3	8,74	2,50
E											10,3	6,88	10,4	9,07	4,62	3,18
F										10,8	7,12	8,54	8,14	3,77	3,34	0,908
G									9,81	7,62	9,99	7,49	3,94	3,35	1,45	1,10
H								9,88	6,98	9,99	7,95	3,37	3,07	1,21	1,30	0,853
J									6,61	8,64	3,91	2,71	1,26	1,28	1,27	1,13
K									5,79	3,99	3,14	0,891	1,12	1,01	1,48	0,568
L									8,16	3,51	1,24	0,891	1,08	1,37	1,05	
M									6,77	1,39	1,19	0,685	1,23	0,787		
N			8,76	6,26	9,48	7,30	2,97	2,98	1,20	1,43	1,07	0,803	0,741			
P		8,09	6,12	9,15	7,88	3,60	2,55	1,18	1,27	1,42	1,44	0,462				
Q	8,47	5,32	8,68	7,20	3,74	2,93	0,806	1,10	1,07	1,66	0,759					
R	6,00	7,90	7,07	3,52	3,35	1,14	0,821	1,05	1,42	1,18						
Key	There is no suitable plan in this area; use the first sampling plan in the appropriate column. If the sample size equals or exceeds the lot size, carry out 100 % inspection. When the sample size exceeds the lot size the producer's risk will be greater than that given in the table.															
NOTE	There is no suitable plan in this area; use the last sampling plan in the appropriate column. The producer's risk is the probability of not accepting a given lot when the process fraction nonconforming is equal to the AQL.															