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Statistical interpretation of test results – Estimation of the mean – Confidence interval

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FOREWORD

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Draft International Standards adopted by the Technical Committees are circulated to the Member Bodies for approval before their acceptance as International Standards by the ISO Council.

International Standard ISO 2602 was drawn up by Technical Committee ISO/TC 69, *Applications of statistical methods.*

It was approved in June 1972 by the Member Bodies of the following countries :

Australia	India	Portugal
Austria	Ireland	Romania
Belgium	Israel	South Africa, Rep. of
Czechoslovakia	Italy	Sweden
Egypt, Arab Rep. of	Japan	Switzerland
France	Netherlands	Thailand
Germany	New Zealand	United Kingdom
Hungary	Poland	U.S.S.R.

No Member Body expressed disapproval of the document.

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Statistical interpretation of test results – Estimation of the mean – Confidence interval

0 INTRODUCTION

The scope of this International Standard is limited to a special question. It concerns only the estimation of the mean of a normal population on the basis of a series of tests applied to a random sample of individuals drawn from this population, and deals only with the case where the variance of the population is unknown. It is not concerned with the calculation of an interval containing, with a fixed probability, at least a given fraction of the population (statistical tolerance limits).

It is recalled that ISO 2854 relates to the following collection of problems :

– estimation of a mean and of the difference between two means (the variances being either known or unknown);

– comparison of a mean with a given value and of two means with one another (the variances being either known or unknown);

– estimation of a variance and of the ratio of two variances;

– comparison of a variance with a given value and of two variances with one another.

The test methods with which this International Standard is concerned provide generally for several determinations which are carried out

– on the same item (where the test is not destructive);

– on distinct portions of a very homogeneous product (a liquid, for example);

– on distinct items sampled from an aggregate with a certain amount of variability.

In the first two cases, the deviations between the results obtained depend only upon the repeatability of the method. In the third case, they depend also on the variability of the product itself.

The statistical treatment of the results allows the calculation of an interval which contains, with a given probability, the mean of the population of results that would be obtained from an indefinitely large number of determinations, carried out under the same conditions. In the case of items with a variability, this International Standard assumes that the individuals on which the determinations are carried out constitute a random sample from the original population and may be considered as independent.

The interval so calculated is called the confidence interval for the mean. Associated with it is a confidence level (sometimes termed a confidence coefficient), which is the probability, usually expressed as a percentage, that the interval does contain the mean of the population. Only the 95 % and 99 % levels are provided for in this International Standard.

1 SCOPE

This International Standard specifies the statistical treatment of test results needed to calculate a confidence interval for the mean of a population.

2 FIELD OF APPLICATION

The test results are expressed by measurements of a continuous character. This International Standard does not cover tests of a qualitative character (for example presence or absence of a property, number of defectives, etc.).

The probability distribution taken as a mathematical model for the total population is the normal distribution with mean m and standard deviation σ.

This assumption is very widely satisfied : the distribution of the results obtained under test conditions is frequently a normal or nearly normal distribution.

It may, however, be useful to check the validity of the assumption of normality by means of appropriate methods[1)].

The calculations may be simplified by a change of the origin or the unit of the test results but it is dangerous to round off these results.

It is not permissible to discard any observations or to apply any corrections to apparently doubtful observations without a justification based on experimental, technical or other evident grounds which should be clearly stated.

The test method may be subject to systematic errors, the determination of which is not taken into consideration here. It should be noted, however, that the existence of

1) An International Standard on this subject is in preparation.

such errors may invalidate the methods which follow. In particular, if there is an unsuspected bias the increase of the sample size n may lead to spurious results. The methods that are treated in ISO 2854 may be useful in certain cases for disclosing systematic errors.

3 REFERENCES

ISO/R 645, *Statistical vocabulary and symbols – First series of terms and symbols – Part I : Statistical vocabulary.*

ISO/R 1786, *Statistical vocabulary and symbols – Second series of terms and symbols.*

ISO 2854, *Statistical treatment of data – Problems of estimation and tests of means and variances.*[1]

4 DEFINITIONS AND SYMBOLS

The vocabulary and symbols used in this International Standard are in conformity with ISO/R 645 and ISO/R 1786.

5 ESTIMATION OF THE MEAN

5.1 Case of ungrouped results

After the discarding of any doubtful results, the series comprises n measurements x_i (where $i = 1, 2, 3, ..., n$), some of which may have the same value.

The mean m of the underlying normal distribution is estimated by the arithmetic mean $\bar{x}$ of the n results :

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

5.2 Case of results grouped in classes

When the number of results is sufficiently high (above 50 for example), it may be advantageous to group them into classes of the same width. In certain cases, the results may also have been directly obtained grouped into classes.

The frequency of the ith class, i.e. the number of results in class i, is denoted by n_i.

The number of classes being denoted by k, we have :

$$n = \sum_{i=1}^{k} n_i$$

The midpoint of class i is designated by y_i. The mean m is then estimated by the weighted mean of all midpoints of classes :

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{k} n_i y_i$$

6 CONFIDENCE INTERVAL FOR THE MEAN

The confidence interval for the population mean is calculated from the estimates of the mean and of the standard deviation.

The alternative method of calculating the confidence interval by use of the range is given in the Annex.

6.1 Estimation of the standard deviation from the squares of the deviations from the arithmetic mean

6.1.1 *Case of ungrouped results*

The estimate of the standard deviation σ, calculated from the squares of the deviations from the arithmetic mean, is given by the formula :

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_1 - \bar{x})^2}$$

where

x_i is the value of the ith measurement ($i = 1, 2, 3, ..., n$);

n is the total number of measurements;

$\bar{x}$ is the arithmetic mean of the n measurements, calculated as in clause 5.1.

For ease of calculation, the use of the following formula is recommended :

$$s = \sqrt{\frac{1}{n-1} \left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n} \left(\sum_{i=1}^{n} x_i \right)^2 \right]}$$

6.1.2 *Case of grouped results*

In the case of grouping by classes, the formula for the estimate of the standard deviation is written :

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{k} n_i (y_i - \bar{y})^2}$$

1) At present at the stage of draft.

For ease of calculation, the use of the following formula is recommended :

$$s = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{k} n_i y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{k} n_i y_i\right)^2\right]}$$

where

y_i is the mid-point of the ith class ($i = 1, 2, 3, ..., k$);

n is the total number of measurements;

$\bar{y}$ is the weighted mean of all midpoints of classes calculated as in clause 5.2.

6.2 Confidence interval for the mean

For a chosen confidence level (95 % or 99 %), according to the specific case, a two-sided or a one-sided confidence interval has to be determined.

6.2.1 *Two-sided confidence interval*

The two-sided confidence interval for the population mean is defined by the following double inequality :

a) at the confidence level 95 % :

$$\bar{x} - \frac{t_{0,975}}{\sqrt{n}} s < m < \bar{x} + \frac{t_{0,975}}{\sqrt{n}} s$$

b) at the confidence level 99 %

$$\bar{x} - \frac{t_{0,995}}{\sqrt{n}} s < m < \bar{x} + \frac{t_{0,995}}{\sqrt{n}} s$$

6.2.2 *One-sided confidence interval*

The one-sided confidence interval for the population mean is defined by one or other of the following inequalities :

a) at the confidence level 95 % :

$$m < \bar{x} + \frac{t_{0,95}}{\sqrt{n}} s$$

or

$$m > \bar{x} - \frac{t_{0,95}}{\sqrt{n}} s$$

b) at the confidence level 99 % :

$$m < \bar{x} + \frac{t_{0,99}}{\sqrt{n}} s$$

or

$$m > \bar{x} - \frac{t_{0,99}}{\sqrt{n}} s$$

with $\bar{x}$, if necessary, replaced by $\bar{y}$, in the case of results grouped in classes.

The coefficients $\frac{t_{0,975}}{\sqrt{n}}$, $\frac{t_{0,995}}{\sqrt{n}}$, $\frac{t_{0,95}}{\sqrt{n}}$, $\frac{t_{0,99}}{\sqrt{n}}$ are derived from Student's t distribution and are given in Table 1.

TABLE 1

	Confidence level Two-sided case		Confidence level One-sided case	
	95 %	99 %	95 %	99 %
n	$\frac{t_{0,975}}{\sqrt{n}}$	$\frac{t_{0,995}}{\sqrt{n}}$	$\frac{t_{0,95}}{\sqrt{n}}$	$\frac{t_{0,99}}{\sqrt{n}}$
2	8,985	45,013	4,465	22,501
3	2,484	5,730	1,686	4,021
4	1,591	2,920	1,177	2,270
5	1,242	2,059	0,953	1,676
6	1,049	1,646	0,823	1,374
7	0,925	1,401	0,734	1,188
8	0,836	1,237	0,670	1,060
9	0,769	1,118	0,620	0,966
10	0,715	1,028	0,580	0,892
11	0,672	0,956	0,546	0,833
12	0,635	0,897	0,518	0,785
13	0,604	0,847	0,494	0,744
14	0,577	0,805	0,473	0,708
15	0,554	0,769	0,455	0,678
16	0,533	0,737	0,438	0,651
17	0,514	0,708	0,423	0,627
18	0,497	0,683	0,410	0,605
19	0,482	0,660	0,398	0,586
20	0,468	0,640	0,387	0,568
21	0,455	0,621	0,376	0,552
22	0,443	0,604	0,367	0,537
23	0,432	0,588	0,358	0,523
24	0,422	0,573	0,350	0,510
25	0,413	0,559	0,342	0,498
26	0,404	0,547	0,335	0,487
27	0,396	0,535	0,328	0,477
28	0,388	0,524	0,322	0,467
29	0,380	0,513	0,316	0,458
30	0,373	0,503	0,310	0,449
40	0,320	0,428	0,266	0,384
50	0,284	0,379	0,237	0,340
60	0,258	0,344	0,216	0,309
70	0,238	0,317	0,199	0,285
80	0,223	0,295	0,186	0,265
90	0,209	0,277	0,175	0,250
100	0,198	0,263	0,166	0,236
200	0,139	0,184	0,117	0,166
500	0,088	0,116	0,074	0,104

When n becomes large, interpolation is easier for t than for $\frac{t}{\sqrt{n}}$. Table 2 gives the corresponding values for t.

TABLE 2

n	$\frac{120}{n}$	$t_{0,975}$	$t_{0,995}$	$t_{0,95}$	$t_{0,99}$
60	2	2,000	2,660	1,671	2,390
120	1	1,980	2,617	1,658	2,358
∞	0	1,960	2,576	1,645	2,326

If $\frac{120}{n}$ is used as argument, linear interpolation for t at any value of $n > 60$ is possible.

7 PRESENTATION OF THE RESULTS

Express the results in the form of the double inequality of 6.2.1 or one of the inequalities of 6.2.2, stating the confidence level (95 % or 99 %) and giving the number n of results effectively used as well as the number of results discarded as being doubtful and the reasons for discarding.