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General principles on reliability for structures

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Foreword

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Users should note that all International Standards undergo revision from time to time and that any reference made herein to any other International Standard implies its latest edition, unless otherwise stated.

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General principles on reliability for structures

0 Introduction

This International Standard constitutes a common basis for defining rules for design relevant to the construction and use of all building and civil engineering works whatever the nature or combination of the materials used, for example, concrete, steel, wood, brick, etc. However, their application to each type of material will require specific adaptation to ensure a level of reliability which, as far as possible, is consistent with the objectives of the code drafting committees for each material.

NOTE — This International Standard is intended to serve as a basis for those committees responsible for the task of preparing national standards or codes of practice in accordance with the conditions, both technical and economic, in each particular country and which take into account the nature and type of structure, the properties of the materials during the intended life, with defined conditions of use. It will also provide a common basis for other International Standards dealing with loadbearing structures.

However, to allow continuing development of practice in different countries, the national standards or codes of practice may be simplified or more detailed by comparison with this International Standard.

It is important to recognise that structural safety is an overall concept comprising models for describing actions, design rules, safety elements, workmanship, quality control procedures and national requirements all of which are mutually dependent.

The modification of one factor in isolation could therefore disturb the balance of safety inherent in the overall concept.

In making progress towards harmonization, it is important that the modification of any one factor should be accompanied by a study of the implications involved in relation to the overall concept of safety.

1 Scope and field of application

This International Standard specifies general principles for the verification of reliability of structures subjected to known or foreseeable types of actions.¹⁾ Reliability is considered in relation to the performance of the structure throughout its intended life.

The general principles are applicable to the design of complete structures (buildings, bridges, industrial structures, etc.), the structural elements making up the structure and the foundations.

This International Standard is applicable also to the successive stages in construction, namely the fabrication of structural elements, the transport and handling of the structural elements, their erection and all work on site, and the use of the structure during its intended life.

Generally, the principles are also applicable to the redesign of existing structures when they are in the course of being repaired or reconstructed.

2 General requirements and conditions

2.1 Fundamental requirements

Structures and structural elements should be designed and constructed so that they are suited to their intended use. The structures should normally be designed so that they are not subsequently damaged disproportionately to the original cause, which implies a certain degree of robustness. In particular they should, with appropriate degrees of reliability, fulfil the following performance requirements:

- a) They should withstand all normal actions imposed upon them during their construction and anticipated use.
- b) They should, in general, retain sufficient integrity to withstand local failures and specified accidental events such as explosions, fire and vehicular impact.
- c) Structures and structural elements should perform adequately under normal use.

These requirements should be fulfilled during the intended life of the structures.

NOTE — This means that the durability of the structures in their working environment should be such that the deterioration of material properties will not lead to an unacceptable probability of failure. Thus length of intended life should be attained with an acceptable level of maintenance.

1) The term "load" may be used instead of "action". See note to 4.1.

A further requirement is that design solutions should be economical in their utilization of materials, energy, financial resources, manpower, space and time.

NOTES

- 1 The term "failure" as used in this document refers to either inadequate safety or serviceability of the structure.
- 2 The first two requirements refer to the safety of structures whilst the third requirement refers to their serviceability.
- 3 In the narrow sense implied by this document, the term "reliability" is used to describe by probabilities that neither inadequate safety nor inadequate serviceability will be encountered during the intended service life. In a general sense, the reliability of a structure is its ability to fulfil its design purpose for some specified time under the environmental conditions encountered.

Safety, serviceability throughout the intended service life, and durability are not simply functions of the design calculations but are also dependent on the quality control exercised in manufacture, the supervision on site and the manner in which the structure is used and maintained.

The choice of the various levels of reliability should take into account the possible consequences of failure in terms of risk to human life or injury, the potential economic losses and the degree of social inconvenience. It should also take into account the amount of expense and effort required to reduce the risk of failure and should depend on the cause of failure.

NOTE — Thus, as an example, structures or structural parts may be classified according to the consequences of failure as follows:

- a) risk to life negligible and economic and/or social consequences small or negligible;
- b) risk to life exists, economic and/or social consequences considerable;
- c) risk to life great, economic and/or social consequences very great.

As a result of such a classification structures or structural parts could be assigned to different safety classes for which the extent of the measures taken to ensure safety, and possibly serviceability, differs.

The consequences of a failure generally depend on the mode of failure, especially in those cases when the risk to human life or injury exists. Thus a structure which fails suddenly without warning should be designed for a higher level of reliability than one in which the failure is ductile (see 3.2.3).

2.2 Use and environmental conditions

The influences arising from the intended use of the structure and the environmental conditions can be described as the design situations associated with normal use of the structure.

NOTE — These influences and conditions can, for example, include actions due to specified normal use combined with all foreseeable environmental conditions which occur at the same time.

These influences and conditions form a basis for the design of the structure in the serviceability limit state and possibly in the ultimate limit state. They should be used *inter alia* as parameters for defining design situations (see 3.2.2).

2.3 Hazards

It is necessary to take into account hazardous circumstances which alone or in combination with normal conditions could cause the serviceability or ultimate limit state to be attained.

The hazards may occur due to

- consequence of an error such as lack of information, omission, misunderstanding, etc.;
- effects of extreme loading arising from the environment (climatic influences, geotechnical conditions, etc.).

The measures taken to counter such hazards would basically consist of

- avoiding the structural effects of the hazards by either eliminating the source or by bypassing and overcoming them;
- designing for hazards [see 3.2.3b)];
- accepting the risk of failure due to the hazards and trying to minimize the consequences [see 3.2.3a)].

If a specific hazard has to be considered it should be used to define a design situation (see 3.2.2). This design situation will normally be dominated by one hazardous occurrence which could be combined with other normal conditions.

2.4 Design considerations

In order that the properties of the completed structure should be consistent with the assumptions made during design, a number of conditions should be fulfilled.

2.4.1 Initial choice of structural system, material, detailing and method of construction

With regard to safety, the choice of the structural system should be made so that important parts of it are able to maintain sufficient structural integrity during and after accidents. Material, detailing and method of construction may also influence the structural integrity.

2.4.2 Responsibilities

For every foreseeable activity during the building process, as well as for the interfaces between them, the responsibility should clearly be defined. It is important that the meaning and scope of their responsibility and task is known by all concerned. This also refers to the transfer of information and documents.

NOTE — Many causes of structural failures can be traced back to those situations where such responsibilities are not clearly defined.

2.4.3 Measures against human mistake

The effects of human mistake should be eliminated to a reasonable extent by taking such measures as:

- a) improving conditions to the point that the occurrence of human mistakes may be reduced or avoided. This means, for example, selection of qualified staff, improved communications, precautionary measures against human mistake, such as an adequate checking system:
- b) improving working procedures. Particularly the places of work and their approach should be appropriately organized. These conditions are also necessary for separation and conservation.

NOTE — Experience indicates that structural failures are often due to human mistake arising from the incorrect selection of a structural system, choice of a material, design and construction.

2.4.4 Quality control

All steps in planning, design, construction and use of a structure should be controlled to an extent which depends on the possible consequences of error and unfavourable deviations and cost-effectiveness of control.

NOTE — Many cases of structural failures are caused by errors which slip through undetected by those engaged in the building process.

Control should also include the introduction, execution and supervision of measures required in cases where the results obtained do not agree with the original requirements.

NOTE — Control is treated in more detail in clause 7.

2.4.5 Maintenance and repair

Structures should be maintained in such a way that they can be used during their intended life for the purpose for which they were designed. When a structure has been damaged or has deteriorated to such an extent that its use would entail an unacceptable risk, then repairs should be undertaken.

NOTE — Under certain circumstances, demolition could be a better solution than repair.

Damage or deterioration may be discovered through inspection, which should be undertaken at regular intervals. The effort to be undertaken in the maintenance should be specified with regard to the importance and type of the actual structural part, the conditions of use, knowledge of the durability of the material, environmental conditions, the protection against external actions and the cost of investigations.

Structural elements which are essential to the stability of a structure should, as far as possible, be made accessible for inspection without the need for difficult or extensive dismantling of the structure.

3 Principles of limit state design

3.1 Limit states

3.1.1 The structural performance of a whole structure or part of it should be described with reference to a specified set of

limit states beyond which the structure no longer satisfies the design requirements.

NOTE — Limit states can be regarded as a discrete representation of a more general and often continuous loss function.

The limit states are divided into the following two categories which, in turn, may be subdivided:

- a) the ultimate limit states which generally correspond to the maximum load carrying capacity (safety related);
- b) the serviceability limit states which correspond to the criteria governing function related normal use.

3.1.2 Ultimate limit states correspond to, for example

- loss of static equilibrium of the structure, or of a part of the structure, considered as a rigid body (e.g. overturning),
- rupture of critical sections of the structure caused by exceeding the ultimate strength (in some cases reduced by repeated loading) or the ultimate deformation of the material,
- transformation of the structure into a mechanism (collapse),
- loss of stability (buckling, etc.).

3.1.3 Serviceability limit states correspond to, for example

- deformations which affect the efficient use or appearance of structural or non-structural elements,
- excessive vibrations producing discomfort or affecting non-structural elements or equipment (especially if resonance occurs),
- local damage (including cracking) which reduces the durability of a structure or affects the efficiency or appearance of structural or non-structural elements.

3.1.4 To control serviceability limit states by design, it is often necessary to use one or more constraints (C) which describe acceptable deformations, accelerations, crack widths, etc.

NOTE — These constraints may be determined by statistical methods but are normally introduced into codes with deterministic values.

3.2 Design

3.2.1 General design requirements

All relevant limit states should be considered in design. A calculation model should be established for each specific limit state; this model should incorporate all appropriate variables and also allow for the uncertainties with respect to actions; the response of the structure as a whole, the behaviour of individual elements and materials of the structure and the conditions of the environment. Appropriate measures should be taken to avoid any mistakes in the design interpretation. However, the design procedure should not be refined beyond a point that is compatible with the standard of workmanship likely to be achieved.

The purpose of design calculations or prototype testing is to keep the probability of a limit state being reached below a certain value prescribed for the type of structure in question.

To verify the reliability of the structural design, the format of partial coefficients, described in clause 6, is generally recommended. However, for certain design problems, e.g. in the case of dynamic analysis, direct reference to the respective design values may be more convenient.

NOTE — Not all limit states can be verified exclusively by calculation. This applies, for example, to limit states specified with regard to chemical or biological attack or to the prevention of brittle failure.

3.2.2 Design situations

For any structure it is generally necessary to consider several distinct design situations (see 2.2). Corresponding to each of these design situations, there may be different structural systems, different reliability requirements, different design values, different environmental conditions, etc. Separate reliability checking is required for each design situation with due regard to different consequences of failure.

The design situations may be classified as:

- a) persistent situations, having a duration of the same order as the life of the structures,

NOTE — A structure of a residential building which, for example, is subjected to loads from the weight of furniture and a normal number of persons.

- b) transient situations, having a shorter duration and a high probability of occurrence,

NOTE — Where a structure is, for example, subjected to loads caused by storage of material during construction or repair.

- c) accidental situations (during or after an accident) normally of short duration and low probability of occurrence.

NOTE — Where a structure is, for example, subjected to actions due to fire, explosions, impact or considerable local damage.

3.2.3 Structural integrity

Where appropriate, all parts of a structure and the structure as a whole should be designed for relevant ultimate limit states and relevant serviceability limit states. However, for accidental situations, generally, only the principal load-bearing system need be designed for relevant ultimate limit states.

NOTE — This does not preclude that verification of additional limit states may be required for certain accidental situations, e.g. thermal insulations and integrity to fire penetration, which may not only refer to members of the principal load bearing system.

The principal load bearing system should be designed for accidental situations when certain hazards occur in such a way that the probability of damage disproportionate to the original incident is sufficiently small (see 2.3).

NOTE — This requirement may, for example, be achieved by

- a) designing the structure in such a way as to ensure that, should a single member fail, then neither the whole nor a significant part would collapse immediately, thus allowing the necessary emergency measures — for example, evacuation of the building — to take place;
- b) ensuring (by design or by protective measures) that no essential load-bearing member can be made ineffective as a result of an accident.

In addition, the structure should have adequate resistance to lateral forces. Thus, if the true horizontal forces are unknown or are of insufficient magnitude to provide a robust structure, then the structure should sustain a horizontal force of magnitude equivalent to a specified proportion of the total vertical action.

4 Basic variables

4.1 General

The calculation model expressing each limit state considered should contain a specified set of basic variables. In general, the basic variables should correspond to measurable physical quantities. Normally, basic variables characterize:

- actions,¹⁾
- properties of materials and soils,
- geometrical parameters.

Basic variables are often considered as being random variables.

The basic variables are often affected by the environmental conditions. This influence should be taken into account by the specific codes for each special material and each special type of structure.

4.2 Actions

4.2.1 Definitions

An action is

- an assembly of concentrated or distributed forces acting on the structure (direct actions), or
- the cause of imposed or constrained deformations in the structure (indirect actions).

1) The term "load" (which is prevalent in some countries) may be used with essentially the same meaning as "action". In the past, it has often been used to describe direct actions only.

The term "action" was introduced to cover also the effects due to imposed deformation.

An action is considered to be one single action if it can be assumed to be stochastically independent, in time and space, of any other action acting on the structure.

NOTE — In reality, actions which are introduced simultaneously are often stochastically dependent to a certain extent. To simplify calculations, those which, where they are present, depend closely on each other and attain their upper values at the same time, are considered together as a single action. Actions where the dependence is small (low correlation) can be considered as independent.

In case of ambiguity, actions should be considered such that they result in the most unfavourable action effect resulting from the combination rule of concern.

To facilitate the calculation of the action effects, it may be convenient to regroup several analogous elementary actions into one composite action or to resolve certain actions into a sum or difference of several components.

4.2.2 Classification of actions according to the variation of their magnitude with time

Actions are divided — according to their variation in time — into:

- a) permanent actions (G) which are likely to act throughout a given design situation and for which variations in magnitude with time are negligible in relation to the mean value; or those for which the variation is in one sense and the actions attain some limiting values;
- b) variable actions (Q) which are unlikely to act throughout a given design situation or for which variations in magnitude with time are not monotonic and not negligible in relation to the mean value;
- c) accidental actions (A or F_a), the occurrence of which, with a significant value, is unlikely on a given structure over the period of time under consideration and also in most cases is of short duration. The occurrence of an accidental action could in many cases be expected to cause severe consequences unless special measures are taken.

NOTE — Examples of permanent, variable and accidental actions are given in annex A.

4.2.3 Classification of actions according to their variation in space

Actions are divided — according to their variation in space — into two groups:

- a) fixed actions are those which have a spatial distribution over the entire structure, such that the magnitude and orientation of the action is unambiguously determined for the entire structure if they are given at a single point of the structure;
- b) free actions which may have arbitrary spatial distribution over the structure within given limits.

Actions which cannot be defined as belonging to either of these two groups may be considered to consist of a fixed part and a free part.

The treatment of free actions requires the consideration of different load arrangements. A load case is determined by fixing the configuration of each of the free actions.

NOTE — In some cases it is necessary to distinguish between fixed actions and actions which are movable or act in a probabilistic way at certain parts of structures. In such cases and in the absence of a more detailed study, it is generally agreed to separate such actions into different elementary actions such as those applied to points or parts which are recognized as the most unfavourable and those applied to other parts.

4.2.4 Classification of actions according to the structural response

Actions are divided — according to the way in which the structure responds to an action — into:

- a) actions which may produce static action-effects without causing significant acceleration of the structure or structural member, so-called static action;
- b) dynamic actions which may produce dynamic action-effects in the structure.

NOTE — Whether or not the action is regarded as dynamic is dependent on the structure.

For simplicity, dynamic actions may often be treated as static actions in which the dynamic effects which depend on the behaviour of the structure are taken into account by an appropriate increase in the magnitude of the action.

4.3 Properties of materials and soils

The values describing the properties of materials and their random variations should be based on either specific tests, results of previous tests, or *in situ* observations in conjunction with other sources of information. Properties relating to special test specimens should be converted to the relevant properties of the actual material in the structure by the use of conversion factors or functions, which should take account of any scale effects and any dependence on time and temperature. The uncertainty in the properties of the material in the structure or of the soil should be derived from the uncertainties of the standard test results and of the conversion factor or function. Allowance should also be made for different standards of workmanship and control.

4.4 Geometrical parameters

Geometrical parameters describe the shape, size and overall arrangement of structures, elements and cross-sections. When the deviation of any of the geometrical parameters from their prescribed values may have a significant effect on the structural behaviour and the resistance of the structure, these parameters should be considered as random variables. The magnitudes and their variability should be determined by taking into account prescribed tolerance limits (see 6.4).

NOTE — In many cases, however, the random variability of the geometrical parameters may be considered to be small in comparison with the variability of the actions and of material properties. In such cases the geometrical parameters may be assumed to be non-random and as specified in the design. Some examples of random geometrical parameters are unintentional eccentricities, inclinations and curvature affecting columns and walls.

5 Analyses, calculations and testing

5.1 General

In many cases the design procedure consists of

- structural analysis which gives the action-effects (forces and moments) in the cross-sections, and
- analysis of cross-sections, joints etc. which gives their resistance and more generally their behaviour.

However, in some cases it is not possible to make this distinction. An example of such a case would be when the stability of an entire structure is studied.

The analysis of a structure can be made with the aid of calculation, model testing or prototype testing. In some cases a combination of these methods is useful.

It is important to recognize clearly the principal load-bearing system by which forces are safely transmitted to the foundations, and to identify those features (including the layout) of the structure which have a critical influence on its overall stability and integrity. Those features should then be consistently examined and maintained throughout all stages of the design and construction.

5.2 Calculation

Calculation models and basic assumptions for the calculation should express the structural response according to the limit state under consideration.

For the purpose of analysis, a structure can generally be described by a model consisting of one-dimensional elements (beams, columns, cables and arches), two-dimensional elements (slabs and shells) and three-dimensional elements.

For the serviceability limit states, linear elastic methods of analysis will usually be appropriate. However, sometimes non-linear methods have to be used.

For the ultimate limit states, linear elastic and geometrically and/or materially non-linear and plastic theories may be applied depending on the response of the material and the structure to the actions.

In treating free actions, it is necessary to define simplified spatial models for each action and to use them in order to define different load arrangements. For a given structure, it is necessary to select the load arrangement which is the most unfavourable. However, it will sometimes be justified, from statistical considerations, not to consider certain load arrangements with low probability of occurrence.

If the influence of the environmental conditions on the behaviour of materials, elements and structures is of a systematic nature it can be expressed directly in the analysis.

Examples of influence of environmental conditions which can be expressed directly in the analysis are:

- the influence of environmental humidity conditions on the strength of wood or on the shrinkage and creep deformations of concrete,

- the influence of high temperature during a fire on the strain distribution and the yield strength of steel.

The uncertainties in a calculation model can be included in the model itself, for example by use of one or more parameters, which may be treated as an additional basic variable with its own mean and variance. These may be determined by comparing predicted and observed data in relevant tests.

5.3 Model testing

A structure or part of it may be designed on the basis of results from appropriate model testing coupled with the use of model analysis to predict the behaviour of the actual structure.

NOTE — In most cases, model testing is used for structural analysis to verify calculations or as a substitute for them. Conversion of model test results to be compatible for use in the actual structures should be ensured.

5.4 Prototype testing

A structure or part of it may also be designed on the basis of results from testing prototype units relevant to the particular design under consideration.

NOTE — This type of testing is sometimes used as a substitute for calculation to verify the resistance of small units or details. It is important, therefore, that the prototype testing is conducted as nearly as possible under the same conditions and assumptions as for the actual structure regarding strength, dimensions, loading and environmental conditions. If only a few prototype test results are available, statistical uncertainties should be indicated.

6 Design format of partial coefficients

6.1 Principles

The partial coefficient format separates the influence of uncertainties and variabilities originating from different causes by means of partial coefficients assigned to basic variables.

The principles of the method are given in this clause. However in practical application slight modifications are sometimes necessary or convenient. (See 6.2.2 and 6.3.2.)

In the verification procedure the values assigned to the basic variables are called design values.

The design values for actions, F_d (see 6.2.2) are given by the equation

$$F_d = \gamma_f F_r$$

Strengths of materials are expressed by their design values f_d (see 6.3) by the equation

$$f_d = f_k / \gamma_m$$

Other relevant properties may be treated in a similar way or by introducing additive elements.

Geometrical parameters are expressed by their design values a_d (see 6.4.2) by the equation

$$a_d = a_k \pm \Delta a$$

where

F_r are the representative values of actions;

f_k are characteristic values of material properties, for example strength;

a_k are characteristic values of geometrical parameters;

γ_f are partial action coefficients. Their values reflect the uncertainties of the actions (see 6.2.2);

γ_m are partial material coefficients. Their values reflect the uncertainties of the material properties (see 6.3.2);

Δa are additive partial geometrical quantities. Their values reflect the uncertainties of the geometrical parameters (see 6.4.2).

The condition for a limit state not to be exceeded can be written as follows:

$$\theta(F_d, f_d, a_d, C, \gamma_n, \gamma_d) \geq 0 \quad \dots (1)$$

where

C are constraints, for example according to 3.1;

γ_n is a coefficient by which the importance of the structure and the consequences of failure, including the significance of the type of failure, are taken into account. The value of γ_n could be made dependent on the safety class (see 2.1) of the actual structure or structural part;

γ_d are coefficients related to model uncertainties or other circumstances which are not taken into account by the other γ -values;

F_d, f_d and a_d are the same as above.

The coefficients γ_d will take on different values depending on the degree of confidence in the design model as an accurate representation of the real structure or structural element. They may also cover the effect of the sensitivity of the structural system (underproportional or overproportional behaviour).

NOTE — Many sources of variability have been identified in design and construction, such as poor mathematical modelling, standards of construction, the difference between test and *in situ* material properties, human error and workmanship. The common feature of these uncertainties is that, while it is possible to identify them qualitatively, it is not always possible to quantify them.

In general, the limit state function is time-dependent. In many practical applications the effects are accounted for by assigning appropriate numerical values to the basic variables.

For practical applications the design condition can often be separated into one action effect function S and one resistance function R so that for ultimate states it can be expressed by

$$\gamma_n S(F_d, a_d, \gamma_{Sd}) \leq R(f_d, a_d, C, \gamma_{Rd}) \quad \dots (2a)$$

In many cases this equation can be changed to

$$\gamma_n \gamma_{Sd} S(F_d, a_d) \leq \frac{1}{\gamma_{Rd}} R(f_d, a_d, C) \quad \dots (2b)$$

γ_d has been split into one load effect part γ_{Sd} and one resistance part γ_{Rd} . Splitting of γ_d is not however a prerequisite for the form of equations (2a) or (2b).

In practical application the coefficient γ_n could be introduced on either side of the equation and also directly to the design values.

For the serviceability limit states the design condition can often be represented by an expression of the type

$$S(F_d, f_d, a_d, \gamma_n, \gamma_{Sd}) \leq \frac{C}{\gamma_{Rd}} \quad \dots (3a)$$

or

$$\gamma_n \gamma_{Sd} S(F_d, f_d, a_d) \leq \frac{C}{\gamma_{Rd}} \quad \dots (3b)$$

where C is a serviceability constraint (see 3.1).

Equations (1), (2) and (3) should be regarded only as a schematic description of the principles. Each symbol F, f, a and C may represent several variables. Thus in the equations used for a calculation, F may represent a set of actions entering into a combination. Furthermore, for example regarding a reinforced concrete construction, f may represent both the strength of concrete and the strength of steel.

Each of the equations (1), (2) and (3) could represent several equations which are valid simultaneously as a design condition. The equations could also be vectorial.

The problem of static equilibrium can be described by equation (2) where R is equal to zero.

NOTE — In some cases the character of the problem requires the design condition to be written according to equation (1). This could be the case if the problem involves a thorough study of the stability of a frame system.

If the actual problem is non-linear, special care is necessary. The non-linearity could, for example, imply that some action-effect varies with material properties.

For fatigue problems it may be necessary to make some adjustments to equations (1) or (2).

Equation (3) for the serviceability limit states could be used for most of the ordinary problems of deformations, cracking, etc. However for some types of problems, such as vibrations and fatigue, the equations may not be applicable.

6.2 Actions and their combinations

6.2.1 Representative values

For different purposes different values may be assigned to each action. These values are called representative values. They are given in codes or in other documents.

The main representative value is the characteristic value which is a value with an accepted probability of not being exceeded by unfavourable values during some reference period which has been determined with regard to the intended life of the structure or the duration of the design situation. In some cases an action may have two characteristic values, that is upper and lower. In cases where the effect of a reduction in load is more dangerous for the structure, the lower value should be taken as the more unfavourable.

The other representative values are chosen with regard to some features of the situation, for example duration, fatigue, rheology phenomena, and may be expressed as a particular part of the characteristic value by using a factor $\psi_i < 1$.

Whenever possible, actions and their random variations should be established on the basis of reliable observations, tests or from data supplied by producers of material, components, equipment, etc.

Other sources of information such as judgement on the basis of experience with the type of use, or physical constraints may also be taken into account. In existing documents values obtained within this group of information are described as "nominal values".

A permanent action G has, in general, a unique representative value. When the action consists of the self-weight of the structure, the value G_k should be obtained from the intended values of the geometrical parameters (in general, taken from drawings) and the mean unit weight of the material. In cases where the uncertainties in the permanent actions are important, the characteristic values may be determined so that the probability of their being exceeded is sufficiently small. In such cases it may sometimes be necessary to define both upper and lower characteristic values.

Variable actions Q may have the following representative values:

- a) the characteristic value Q_k
- b) the frequent value $\psi_1 Q_k$
- c) the quasi-permanent value (sustained value) $\psi_2 Q_k$.

For special purposes other representative values may be specified.

If characteristic values for variable actions cannot be determined from statistical data or where appropriate data are not available, the corresponding values may then be estimated on the basis of available information. The characteristic value is then a nominal value.

Accidental actions A (or F_a) generally have unique representative values A_r (or F_{ar}), specified by regulatory authorities or by the engineer considering various criteria of a general nature (economic and social policies, experience of design and use of structures, etc.).

6.2.2 Design values

An action is introduced into calculation by its design values which are obtained from the representative values by multiplication with partial coefficients γ_f (see 6.1).

Different design values are used and correspond to the different representative values, for example

- γ_f multiplied by the characteristic value,
- γ_f multiplied by the frequent value,
- γ_f multiplied by the quasi-permanent (sustained) value.

The partial coefficients γ_f take account of

- the possibility of unfavourable deviations of the actions from their representative values,
- uncertainty in the loading model.

NOTE — In codes the partial action coefficients sometimes include uncertainties of the load effect model, i.e. γ_{SD} . If this is the case, a distinction by use of another symbol, e.g. γ_F , shall be made and the code (or a note) shall specify the numerical value allocated to γ_f alone.

The partial coefficient γ_f depends on the limit state considered. In particular, coefficients chosen for ultimate limit states and for serviceability limit states are different. Also the partial factors γ_f may be different for different action values.

Generally, it can be assumed that the partial coefficient for accidental actions is equal to unity.

6.2.3 Combinations of actions

A combination of actions is an assembly of the design values of a set of different actions considered simultaneously in the verification of the structural reliability for a limit state.

Actions which are mutually exclusive should not enter together into a combination.

The actions should be combined so that they produce the most unfavourable effect on the structure for the limit state considered.

To take account of a reduced probability of simultaneous occurrence of unfavourable values of several independent actions, the values of the variable actions may be reduced by a coefficient ψ_0 . This should in principle be done so that the probability of exceeding the design action effect from a combination of actions is of the same order of magnitude as the probability of exceeding the design action effect caused by one action.

NOTE — A special combination value $\psi_0 F_k$ may also be specified.

In the ultimate limit states the following two types of combination may be applied:

- a) fundamental combinations: combinations of permanent actions and variable actions,
- b) accidental combinations: combinations of permanent actions, variable actions and one accidental action. An accidental combination should also cover effects associated with an accidental situation such as the decrease of resistance due to fire, instead of an accidental action.

Accidental actions should be included in accidental combinations only.

In the serviceability limit states different combinations corresponding to the fundamental combinations in the ultimate limit states may be applied for different types of design situations, such as:

- a) combinations of permanent actions and variable actions with frequent values, giving the short-term effect (frequent combinations),
- b) combinations of permanent actions and variable actions with quasi-permanent (sustained) values, giving the long-term effect (quasi-permanent combinations).

For special purposes other combinations may also be applied. For each of these types of combination, special sets of combination coefficients may be specified.

6.3 Properties of materials and soils

6.3.1 Characteristic values

In general, the characteristic value of material properties can be presented as that value which has a prescribed probability of not being attained in a hypothetical unlimited test series (corresponding to a fractile in the distribution of the material property taken from a homogeneous quantity of material). Often characteristic values are chosen as specified values in codes.

The method of quality control, including any acceptance rules, should be chosen to ensure a given probability of the required characteristic value.

In cases where the material properties may vary in time or where environmental conditions or action-effects may cause alterations to the material, the characteristic values used for the design should be chosen to take such alterations into account.

NOTE — For soils, as for existing structures, the materials are not produced but are found on the site from the beginning. Therefore, the characteristic values of the properties in principle have to be determined by tests for each project. This means that soils and existing structures have to be treated in a somewhat different way by comparison with, for example, future concrete or steel structures where generally specified values of the material properties are used and checked afterwards for quality.

6.3.2 Design values

The design value f_d of the strength of materials (or other material properties) is obtained from the characteristic value f_k by

$$f_d = f_k / \gamma_m$$

In this equation γ_m takes account of

- the possibility of unfavourable deviations of the strength of material, interpreted as a random variable, from the characteristic value,
- possible inaccurate assessment of the resistance of sections or load-carrying capacity of parts of the structure (if not included in γ_d),

- uncertainties in geometrical parameters, if they are not taken into account according to 6.4.2 or included in γ_d ,

- uncertainties in the relation between the material properties in the structure and those measured by tests on control specimens, i.e. uncertainties in the conversion factor or function according to 4.3.

NOTE — If partial material coefficients account for uncertainties usually represented by γ_d , then this should be identified by use of another symbol, e.g. γ_M .

The value of γ_m depends on the material property, the actual limit state, etc.

NOTES

1 The total partial coefficient γ_m may be resolved into several different factors, each one of which takes into account one or more of the uncertainties mentioned above.

2 It is not in accordance with the principles given in 6.1 to include the influence of uncertainties in geometrical parameters in the value of γ_m . However in practical applications this may be convenient, as it is then possible to use nominal values for the measurement of the cross-sections in the calculations.

In codes γ_m sometimes includes not only the uncertainties in the conversion factor but the conversion factor itself. However this is not satisfactory from a harmonization point of view.

For certain material properties, additive elements may be used to transform characteristic values into design values, where appropriate, as in the case of geometrical parameters (see 6.4.2).

Generally, a multiplication factor is appropriate when the coefficient of variation is constant while an additive element is appropriate when the standard deviation is constant.

6.4 Geometrical parameters

6.4.1 Characteristic values

For geometrical parameters the characteristic values a_k usually correspond to the nominal values specified in the design.

6.4.2 Design values

In cases where deviations of the geometrical parameters have insignificant effects or where the effects are accounted for by the γ values, a_k may be used as design value.

In other cases the design values a_d of geometrical parameters should be obtained from the characteristic (nominal) values a_k and the partial geometrical quantity Δa

$$a_d = a_k \pm \Delta a$$

In this equation, Δa takes account of

- the possibility of unfavourable deviations of the geometrical parameters from the characteristic (nominal) values,
- the importance of variations in a_d
- the given tolerance limits for a_d
- the cumulative effects of a simultaneous occurrence of several geometrical deviations.

In some cases where geometrical parameters are used, a multiplication factor may be substituted instead of Δa , but additive elements Δa are generally more suitable than multiplication factors γ (see 6.3.2).

6.5 Uncertainties of calculation models

The uncertainties in a calculation model may be evaluated by tests or by comparing approximate calculation models with more sophisticated models. The uncertainties may be expressed by one or several of the partial coefficients (preferably γ_d). The effect of the uncertainties may also be included in the model itself, i.e. the model is chosen so that it is "on the safe side".

6.6 Determination of values for partial coefficients

The format of partial coefficients, described above, is intended to be the normal method for the design procedure. In this format the appropriate level of structural reliability is provided by the partial coefficients or other safety elements related to some predefined values of the basic variables. These values of the basic variables (for example, characteristic values) should, if possible, be determined on a statistical basis.

There are, from a probabilistic point of view, more sophisticated methods for checking design reliability. These would include probabilistic design methods which use first order approximations of the functions expressing the limit state. The basic variables are represented by their known or postulated probability distributions (defined in terms of their type and relevant parameters such as mean and standard deviation). The reliability level is expressed by a prescribed failure probability or by a corresponding reliability index.¹⁾

NOTE — One example of a first order probabilistic method is given in annex B.

In the format of partial coefficients the values of the partial coefficients can be verified with the aid of such a probabilistic method if there is sufficient reliable basic data. The target notional failure probability or the reliability index may be established from a study of the values implicit in existing acceptable designs regarding the initial cost of structure and possible consequences of failure. Deviations of probabilities or indices associated with a proposed design criterion from the targets should be examined over the domain of application of the criterion.

The partial coefficients may also be determined on the basis of a semiprobabilistic approach in which each design value, considered separately, has a prescribed probability of being exceeded in the unfavourable sense. Such probabilities may be established from a study of the values implicit in existing acceptable codes.

Where human safety is not a determining factor and relevant data are available, the partial coefficient γ_n could be evaluated by optimization on the basis of economic criteria.

Finally, the partial coefficients may be determined by decisions taking into account current design practice. This is especially the case for the partial coefficients γ_d .

7 Quality control

7.1 General

An effective quality control programme during planning, design and construction of a structure is a necessary condition to achieve the required levels of reliability.

The purpose of quality control is to ensure an acceptable quality of design, materials, products and work on the construction site, and to detect gross errors and other hazards.

Quality control consists of

- collection of information,
- judgement based on this information,
- decision based on the judgement.

7.2 Control procedure

Regarding the control procedure within manufacturing and construction, a distinction can be made between

- a) production control which is a control of a production process. The purpose of this control is the steering of a production process and the guarantee of an acceptable result,
- b) compliance control which is a control of the result of a production process. The purpose of this control is to ensure that the result of a production process complies with given specifications.

NOTE — Properties of materials, components or structures that are the object of the control may not necessarily be the same in these two control procedures.

The content of these two kinds of control could be different for mass production and individual production.

7.3 Control criteria and acceptance rules

The control can be total or statistical.

If the control is total, every produced unit is inspected. The acceptance rules imply that a unit is judged as good (accepted) or bad (not accepted). Normally the criteria, if they are quantitative, refer to given tolerances.

A statistical control procedure generally consists of the following parts:

- a) batching the products;
- b) sampling within each batch;
- c) testing the samples;
- d) statistical judgement of the results;
- e) decision regarding acceptance.

1) Sometimes these methods have been called level-2-methods in literature while the method of partial coefficients and similar methods have been called level-1-methods.

A batch should be such that it may be regarded as homogeneous with regard to the properties which are the subject of the control, which means that all units within a batch are produced under essentially the same conditions. Within each batch a number of units are then taken out as a sample and subjected to testing. The judgement of a batch — if it acceptable or not — is made by comparing the test results with the given criteria. The judgement of the results should normally be made with regard to a given level of confidence and/or a given interval of confidence or by applying bayesian techniques.

7.4 Control process

Distinction can be made between the following different control steps depending on the person or organization supervising the control:

- a) individual self-checking,
- b) internal control,
- c) acceptance control handled by the project management.

NOTE — Internal control may be supplemented by external audits which check the reliable execution of internal control procedures.

There often exists an additional control, for instance initiated and executed by the public authority or the client.

In this International Standard it is assumed that participants in the building process intend to carry out their respective tasks with care and skill. This can only be exercised through individual self-checking measures which means that a certain amount of individual self-checking is assumed.

Self-control (internal control) may concern design work in an office, material production at a factory, production of components in a workshop and erection of a building at the construction site. It is executed in the same office, factory or workshop where the work which is the object of the control, goes on. However the work and the control are executed by separate bodies.

Those who have the responsibility for the project management should also be responsible for all control throughout the whole building process. Some of their control functions may be delegated to qualified specialists within their managerial domain of responsibility. Careful consideration shall be given to the interfaces between responsibilities within the entire building process. This may involve e.g. acceptance tests of subcontracted works and deliveries of materials to site.

An independent consultant may also be employed, especially in cases where

- a) very complicated problems have to be solved;
- b) a structural failure would endanger a very large number of persons.

Additional control executed by public authorities is based on building laws and/or codes. The aim is generally connected with the society having requirements for safety and a quality which is acceptable to the whole.

A rigorous control process includes all control steps described above and should be executed for the most important parts of a building. For parts of minor importance, from a structural point of view, it should be sufficient to limit the control to individual self-checking and self-control. The additional control could then consist of merely a check to ensure that the internal control has been executed acceptably.

If a control process consists of several steps, it is important for the final result that the activities of these steps, as far as possible, are mutually independent in a statistical sense, otherwise the efficiency of the control will decrease.

7.5 Planning of control

The whole building process from planning to use of the structure should be sub-divided by control stops. Such control stops should be introduced mainly at points where responsibility is transferred from one party to another or where one phase of the building process gives way to another. Each control stop contains control measures which should be exercised before the next stage of the building process can commence.

In many cases it is necessary to set up a control plan, especially in cases where control tasks are vested in different persons.

For the design control the plan should include

- a) checking that the requirements and conditions assumed during design are in accordance with those specified;
- b) checking that the relevant calculation models are used and that the numerical calculations are correct;
- c) checking that drawings and other design documents are in accordance with the design calculations and with given specifications.

For the control of material and products and for the control of the erection on the building site the plan should include:

- a) responsibilities regarding control;
- b) subject of control;
- c) time schedule including control intervals;
- d) control procedures;
- e) control criteria and acceptance rules;
- f) requirements of reports and documentation;
- g) procedure in the case of deviations from control criteria.

Annex A

Examples of permanent, variable and accidental actions

The following examples include the most common types of actions. In certain cases there may be other types; such actions shall be classified according to the basic definitions.

The permanent actions include

- a) the weight of structures themselves (except possibly certain parts of this weight during certain phases of construction);
- b) the weight of superstructures, including any permanent formwork or fixtures;
- c) the forces applied by earth pressure, resulting from the weight of the soil at their final values;
- d) the deformations imposed by the mode of construction of the structure at their final values;
- e) the actions resulting from shrinkage of concrete and distortions due to welding;
- f) the forces resulting from water pressure when appropriate;
- g) the actions resulting from support settlements and mining subsidence;
- h) prestressing forces.

The variable actions include

- a) loads due to use and occupancy, imposed loads;
- b) certain parts of the weight of structures themselves during certain phases of construction;

- c) erection loads;
- d) all moving loads and their effects;
- e) wind loads;
- f) snow loads;
- g) ice formation;
- h) earthquakes;¹⁾
- i) the effects of variable level of water surface, when appropriate;
- j) temperature changes;
- k) wave loads.

The accidental actions include

- a) collisions;
- b) explosions;
- c) subsidence of subsoil;
- d) tornadoes in regions not normally exposed to them;
- e) earthquakes;¹⁾
- f) fire;
- g) extreme erosion.

¹⁾ Earthquakes may be considered as either a variable action under specified conditions or an accidental action.

Annex B

Example of a first order probabilistic method

B.1 General

There are several methods for verification of the reliability of structures which could be designated as probabilistic. One of them is described in this annex.

B.2 Failure probability and safety index

Within probabilistic methods, the notional probability for attaining a limit state — referred to as acceptable failure probability p_f — is introduced. Acceptable failure probabilities, reflecting the accepted risk associated with the attainment of a certain limit state, may differ according to the design situation considered.

Within the method described herein, the failure probability p_f is substituted by a safety index β

$$p_f = \Phi(-\beta) \quad \dots (4)$$

where $\Phi(\dots)$ is the standardized normal distribution function.

The safety index β is a descriptive measure for the acceptable probability and allows more convenient handling. Because of the strict correspondence between β and p_f , specifications of the levels of reliability for different situations may also be given in terms of safety indices β .

Basically, failure probabilities could be related to the anticipated life-time of the structure. Other reference periods may sometimes be inevitable; they shall, however, be clearly defined.

B.3 Calculation model

It is assumed that the limit state considered can be specified by a calculation model in terms of one (or several) function(s) $\theta(\dots)$ of a set of variables X_1, X_2, \dots, X_n comprising actions, material properties, etc., so that a condition for the structure not to fail of the form

$$\theta(X_1, X_2, \dots, X_n) > 0 \quad \dots (5)$$

can be associated with the limit state. The design condition may then be written

$$\theta(x_1^*, x_2^*, \dots, x_n^*) > 0 \quad \dots (6)$$

where $x_1^*, x_2^*, \dots, x_n^*$ are design values [see equation (1)].

First order reliability theory provides a consistent method for deriving design values in the majority of practical design tasks.

Specific principles for application of first order reliability to limit state design are outlined below.

B.4 Variables governing the limit state

The set of variables X_1, X_2, \dots, X_n within the calculation model for the limit state may comprise random or deterministic basic variables.

B.4.1 Basic variables

Within this method basic variables X_i are presented as random variables (and not, for example, as stochastic processes) with a certain distribution function and distribution parameters. Generally, agreed assumptions on distribution functions are necessary.

Special distribution functions are the normal distribution with the parameter mean value μ_i and standard deviation σ_i , and the lognormal distribution with corresponding parameters. If distribution functions other than normal are used, transformation (modification) of the variable X_i to a new normally distributed variable with a modified mean value and standard deviation is required. If basic variables are correlated, special techniques have to be employed.

In this method the basic variables can be represented in the following way:

1. An action F is represented by its mean value μ_F and standard deviation σ_F . μ_F and σ_F are referred to the statistical distribution for the maximum value occurring during the reference period.
2. Strength of material, f , is represented by its mean value μ_f and standard deviation σ_f .
3. A geometrical parameter, a , is represented by its mean value μ_a and standard deviation σ_a . However the geometrical parameters may often be regarded as deterministic.

B.4.2 Uncertainties in the calculation model

The uncertainties in a calculation model can often be described by means of some coefficient or function which is introduced in the model. This coefficient or function may be treated as a basic variable X_i and is represented by its mean value μ_i and standard deviation σ_i .

B.5 Combination of actions

According to the definitions for values of actions given in clause 1, the statistical data for actions are related to the distribution for the maximum value occurring during the

reference period. However if two or more actions are combined, the effects related to instantaneous values of the actions should also be combined (added). Thus the values μ_F and σ_F which are used to represent actions cannot generally be used directly in combination including more than one variable action but have to be reduced (either or both). The magnitudes of the reductions depend on the parameters, describing the variations with time, of the variable actions in the combination. In most cases the relative duration of the actions and their number of repetitions may be regarded as the most important parameters.

B.6 Design values

The design value x_i^* of the variable X_i depends on

- the mean value μ_i and standard deviation σ_i of the variable X_i ,
- the assumed type of distribution,
- the safety index β for the limit state and design situation of concern,
- a factor α_i describing the sensitivity with regard to attaining the limit state for variations in X_i .

For an arbitrary distribution $F(x_i)$ the design value is given by

$$F(x_i^*) = \Phi(-\alpha_i \beta) \quad \dots (7)$$

If X_i is assumed to be normally distributed, then

$$x_i^* = \mu_i - \alpha_i \beta \sigma_i \quad \dots (8)$$

and a lognormal distribution gives

$$x_i^* = \mu_i \exp(-\alpha_i \beta V_i) \quad \dots (9)$$

provided the coefficient of variation $V_i < 0,25$,

otherwise μ_i should be substituted by

$$\frac{\mu_i}{\sqrt{1 + V_i^2}}$$

and V_i by

$$\sqrt{\ln(1 + V_i^2)}$$

Depending on uncertainties considered x^* may (ideally) correspond to the design value x_d used in practical design (see 6.3).

B.7 Sensitivity factors

Factors α_i should be chosen so that

$$\sum \alpha_i^2 > 1 \quad \dots (10)$$

where $\sum \alpha_i^2 = 1$ should correspond only to the situation where the proportion of risk associated with the individual variable is clearly identified. For dependent variables other relations are obtained.

Analytical definition of α_i is based on the limit state function, (see equation 5) and is given by

$$\alpha_i = \frac{x_i}{\sqrt{\sum x_i^2}} \quad \dots (11)$$

where

$$x_i = \sigma_i \left. \frac{\partial \theta}{\partial X_i} \right|_{X = x^*}$$

for normal distributions, and

$$x_i = x_i^* V_i \left. \frac{\partial \theta}{\partial X_i} \right|_{X = x^*}$$

for lognormal distributions, provided the respective derivatives exist at the design point x^* .

If $V_i > 0,25$, V_i should be substituted by $\sqrt{\ln(1 + V_i^2)}$ for lognormal distribution.

For the majority of limit state functions, this mathematical approach requires several steps of iteration for solution. Therefore a simplified approach by introducing fixed sensitivity factors within a certain domain of application, for example according to the guidelines given in B.9, may be considered.

B.8 Example

In many cases in practice, the load combination consists of one permanent load G and one variable load Q . Often the resistance of the structure R is dominated by one factor or could approximately be divided into several different resistances (bending, shear, etc.) each with one dominating factor. Thus, as the relations are often linear, a very common format of the condition for the structure not to fail is the simple linear equation

$$R - G - Q > 0 \quad \dots (12)$$

where the resistance has been expressed in the same units as the actions. This relation will be used as a basis for the following example and corresponds to equation (5).

The mean values μ_G of G and μ_Q of Q are assumed to be known (μ_G may be equal to the representative value for the self-weight).

The problem treated in this example is the determination of the required mean value μ_R .

R is assumed to be lognormally distributed whereas G and Q are assumed to be normally distributed.

The design values are obtained from equations (8) and (9):

$$G^* = \mu_G (1 - \alpha_G \beta V_G) \quad \dots (13)$$

$$Q^* = \mu_Q (1 - \alpha_Q \beta V_Q) \quad \dots (14)$$

$$R^* = \mu_R \exp(-\alpha_R \beta V_R) \quad \dots (15)$$

The values of x and α are obtained from equation (11)

$$x_G = -\sigma_G = -\mu_G V_G \quad \alpha_G = -\mu_G V_G / \sqrt{\sum x^2} \dots (16)$$

$$x_Q = -\sigma_Q = -\mu_Q V_Q \quad \alpha_Q = -\mu_Q V_Q / \sqrt{\sum x^2} \dots (17)$$

$$x_R = R^* V_R \quad \alpha_R = R^* V_R / \sqrt{\sum x^2} \dots (18)$$

Finally the design condition from equation (6) gives

$$R^* - G^* - Q^* > 0 \quad \dots (19)$$

These equations are sufficient to solve the problem, for example, in the following way assuming $\mu_G, \mu_Q, V_R, V_G, V_Q$ and β to be given:

- a) Calculate $v = \mu_Q / \mu_G$
- b) Assume a value of α_Q
- c) Calculate $\alpha_G = \frac{1}{v} \frac{V_G}{V_Q} \alpha_Q$

d) Calculate G^* / μ_G and Q^* / μ_Q from equations (13) and (14)

e) Calculate $R^* / \mu_G = G^* / \mu_G + v Q^* / \mu_Q$

f) Calculate

$$x_G / \mu_G = -V_G$$

$$x_Q / \mu_G = -V_Q v$$

$$x_R / \mu_G = V_R R^* / \mu_G$$

g) Calculate $\sqrt{\sum x_i^2 / \mu_G^2}$

h) Calculate $\alpha_Q = \frac{x_Q / \mu_G}{\sqrt{\sum x_i^2 / \mu_G^2}}$

If the calculated value does not agree with the assumed value from step b), repeat from step c) until agreement is reached.

i) Calculate $\alpha_R = \sqrt{1 - \alpha_Q^2 - \alpha_G^2}$

j) Calculate $\mu_R / \mu_G = \frac{R^* / \mu_G}{\exp(-\alpha_R \beta V_R)}$

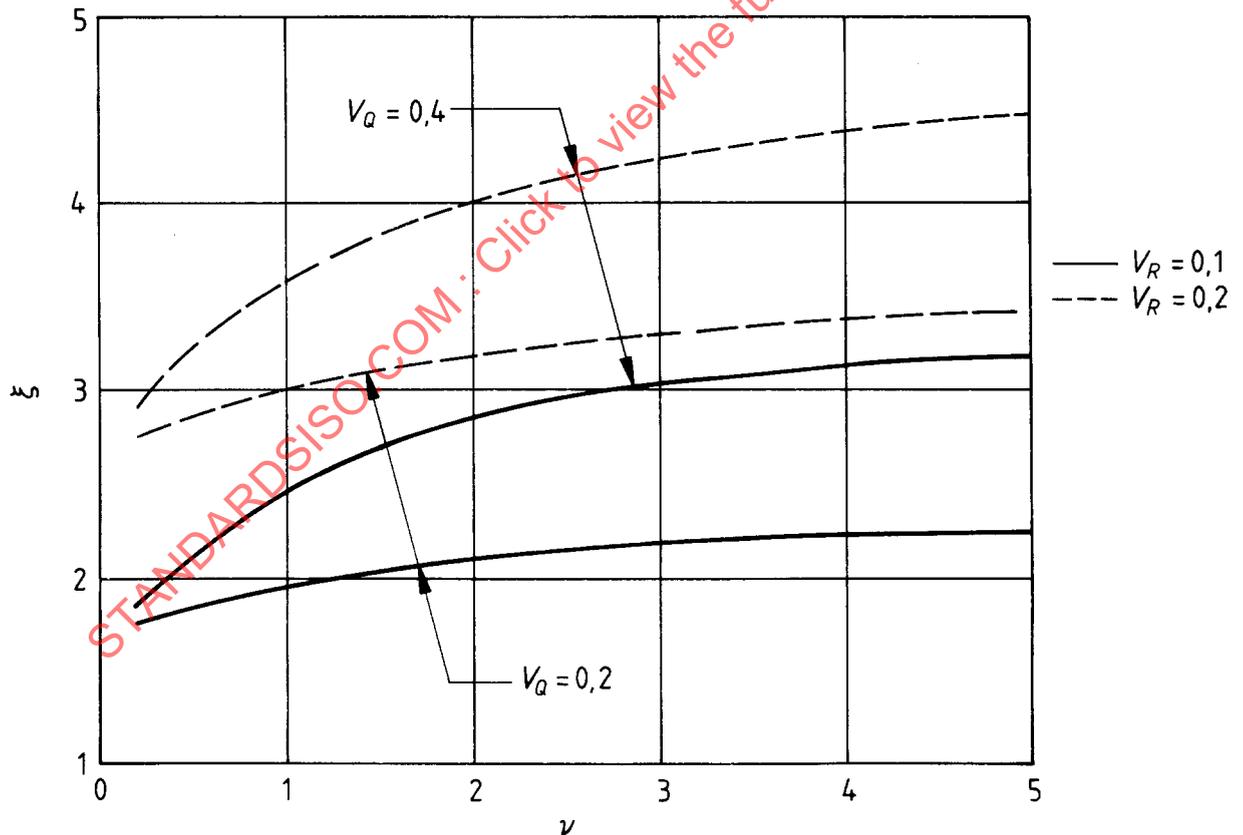


Figure 1 — Results of the calculation of $\xi = \frac{\mu_R}{\mu_G + \mu_Q}$ as a function of $v = \frac{\mu_Q}{\mu_G}$