
**Statistical methods in process
management — Capability and
performance —**

Part 7:
Capability of measurement processes

*Méthodes statistiques dans la gestion de processus — Aptitude et
performance —*

Partie 7: Aptitude des processus de mesure

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in product and process management*.

This second edition cancels and replaces the first edition (ISO 22514-7:2012), which has been technically revised.

The main changes compared to the previous edition are as follows:

- use of the MPE values in the calculations;
- revision of the calculation of the linearity, with amendments in the example in [Clause A.1](#);
- addition of a method to calculate the capability when the specifications of the characteristic of interest is defined as a one-sided specification (new [9.3](#)).

A list of all parts in the ISO 22514 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The purpose of a measurement process is to produce measurement results obtained from defined characteristics on parts or processes. The capability of a measurement process is derived from the statistical properties of measurements from a measurement process that is operating in a predictable manner.

Calculations of capability and performance indices are based on measurement results. The uncertainty of the measurement process used to generate capability and performance indices are estimated before the indices can be meaningful. The actual measurement uncertainty should be adequately small.

If the measurement process is used to judge whether a characteristic of a product conforms to a specification or not, the uncertainty of the measurement process is compared to the specification itself. If the measurement process is used for process control of a characteristic, the uncertainty should be compared with the process variation. Limits of acceptability are stated for both cases.

The quality of measurement results is given by the uncertainty of the measurement process. This is defined by the statistical properties of multiple measurements, or estimates of properties, based on the knowledge of the measurement process.

The methods specified in this document address the implementation uncertainty (for more information on implementation uncertainty, see ISO 17450-2). Therefore, they are only useful if it is known that the method uncertainty and the specification uncertainty are small compared to the implementation uncertainty. This document specifies methods to define and calculate capability indices for measurement processes based on estimated uncertainties. The approach given in ISO/IEC Guide 98-3 (GUM) is the basis of this approach.

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Statistical methods in process management — Capability and performance —

Part 7: Capability of measurement processes

1 Scope

This document defines a procedure to validate measuring systems and a measurement process in order to state whether a given measurement process can satisfy the requirements for a specific measurement task with a recommendation of acceptance criteria. The acceptance criteria are defined as a capability figure (C_{MS} , C_{MP}) or a capability ratio (Q_{MS} , Q_{MP}).

NOTE This document follows the approach taken in ISO/IEC Guide 98-3 (GUM), and establishes a basic, simplified procedure for stating and combining uncertainty components used to estimate a capability index for an actual measurement process.

This document is primarily developed to be used for simple one-dimensional measurement processes, where it is known that the method uncertainty and the specification uncertainty are small compared to the implementation uncertainty. It can also be used in similar cases, where measurements are used to estimate process capability or process performance. It is not suitable for complex geometrical measurement processes, such as surface texture and position measurements that rely on several measurement points or simultaneous measurements in several directions.

2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: General statistical terms and terms used in probability*

ISO 3534-2:2006, *Statistics — Vocabulary and symbols — Part 2: Applied statistics*

ISO 5725-1, *Accuracy (trueness and precision) of measurement methods and results — Part 1: General principles and definitions*

ISO 5725-2, *Accuracy (trueness and precision) of measurement methods and results — Part 2: Basic method for the determination of repeatability and reproducibility of a standard measurement method*

ISO 5725-3, *Accuracy (trueness and precision) of measurement methods and results — Part 3: Intermediate measures of the precision of a standard measurement method*

ISO 5725-4, *Accuracy (trueness and precision) of measurement methods and results — Part 4: Basic methods for the determination of the trueness of a standard measurement method*

ISO 5725-5, *Accuracy (trueness and precision) of measurement methods and results — Part 5: Alternative methods for the determination of the precision of a standard measurement method*

ISO 5725-6, *Accuracy (trueness and precision) of measurement methods and results — Part 6: Use in practice of accuracy values*

ISO/IEC Guide 98-3:2008, *Uncertainty of measurement — Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 3534-1, ISO 3534-2 and ISO 5725 (all parts), and the following apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

3.1 maximum permissible measurement error

maximum permissible error
limit of error

MPE

extreme value of measurement error, with respect to a known *reference quantity value* (3.15), permitted by specifications or regulations for a given measurement, measuring instrument, or measuring system

Note 1 to entry: Usually, the term “maximum permissible errors” or “limits of error” is used where there are two extreme values.

Note 2 to entry: The term “tolerance” cannot be used to designate “maximum permissible error”.

[SOURCE: ISO/IEC Guide 99:2007, 4.26, modified — The abbreviated term “MPE” has been added.]

3.2 measurand

quantity intended to be measured

Note 1 to entry: The specification of a measurand requires knowledge of the kind of quantity, description of the state of the phenomenon, body, or substance carrying the quantity, including any relevant component, and the chemical entities involved.

Note 2 to entry: In the second edition of the VIM and in IEC 60050-300:2001, the measurand is defined as the “quantity subject to measurement”.

Note 3 to entry: The measurement, including the measuring system and the conditions under which the measurement is carried out, might change the phenomenon, body, or substance such that the quantity being measured may differ from the measurand as defined. In this case, adequate correction is necessary.

EXAMPLE 1 The potential difference between the terminals of a battery may decrease when using a voltmeter with a significant internal conductance to perform the measurement. The open-circuit potential difference can be calculated from the internal resistances of the battery and the voltmeter.

EXAMPLE 2 The length of a steel rod in equilibrium with the ambient Celsius temperature of 23 °C is different from the length at the specified temperature of 20 °C, which is the measurand. In this case, a correction is necessary.

Note 4 to entry: In chemistry, “analyte”, or the name of a substance or compound, are terms sometimes used for “measurand”. This usage is erroneous because these terms do not refer to quantities.

[SOURCE: ISO/IEC Guide 99:2007, 2.3]

3.3**measurement uncertainty**

uncertainty of measurement

uncertainty

non-negative parameter characterizing the dispersion of the quantity values being attributed to a *measurand* (3.2), based on the information used

Note 1 to entry: Measurement uncertainty includes components arising from systematic effects, such as components associated with corrections and the assigned quantity values of measurement standards, as well as the definitional uncertainty. Sometimes estimated systematic effects are not corrected for but, instead, associated measurement uncertainty components are incorporated.

Note 2 to entry: The parameter may be, for example, a standard deviation called *standard measurement uncertainty* (3.6) (or a specified multiple of it), or the half-width of an interval, having a stated coverage probability.

Note 3 to entry: Measurement uncertainty comprises, in general, many components. Some of these may be evaluated by *Type A evaluation of measurement uncertainty* (3.4) from the statistical distribution of the quantity values from series of measurements and can be characterized by standard deviations. The other components, which may be evaluated by *Type B evaluation of measurement uncertainty* (3.5), can also be characterized by standard deviations, evaluated from probability density functions based on experience or other information.

Note 4 to entry: In general, for a given set of information, it is understood that the measurement uncertainty is associated with a stated quantity value attributed to the measurand. A modification of this value results in a modification of the associated uncertainty.

[SOURCE: ISO/IEC Guide 99:2007, 2.26]

3.4**Type A evaluation of measurement uncertainty**

Type A evaluation

evaluation of a component of *measurement uncertainty* (3.3) by statistical analysis of measurement quantity values obtained under defined measurement conditions

Note 1 to entry: For various types of measurement conditions, see repeatability condition of measurement, intermediate precision condition of measurement, and reproducibility condition of measurement.

Note 2 to entry: For information about statistical analysis, see e.g. ISO/IEC Guide 98-3.

Note 3 to entry: See also ISO/IEC Guide 98-3:2008, 2.3.2, ISO 5725 (all Parts), ISO 13528, ISO 21748, ISO/TS 21749.

[SOURCE: ISO/IEC Guide 99:2007, 2.28]

3.5**Type B evaluation of measurement uncertainty**

Type B evaluation

evaluation of a component of *measurement uncertainty* (3.3) determined by means other than a *Type A evaluation of measurement uncertainty* (3.4)

EXAMPLE Evaluation based on information

- associated with authoritative published quantity values,
- associated with the quantity value of a certified reference material,
- obtained from a calibration certificate,
- about drift,
- obtained from the accuracy class of a verified measuring instrument,
- obtained from limits deduced through personal experience.

Note 1 to entry: See also ISO/IEC Guide 98-3:2008, 2.3.3.

[SOURCE: ISO/IEC Guide 99:2007, 2.29]

3.6

standard measurement uncertainty

standard uncertainty of measurement

standard uncertainty

measurement uncertainty (3.3) expressed as a standard deviation

[SOURCE: ISO/IEC Guide 99:2007, 2.30]

3.7

combined standard measurement uncertainty

combined standard uncertainty

standard measurement uncertainty (3.6) that is obtained using the individual standard measurement uncertainties associated with the input quantities in a *measurement model* (3.11)

Note 1 to entry: In case of correlations of input quantities in a measurement model, covariances must also be taken into account when calculating the combined standard measurement uncertainty; see also ISO/IEC Guide 98-3:2008, 2.3.4.

[SOURCE: ISO/IEC Guide 99:2007, 2.31]

3.8

expanded measurement uncertainty

expanded uncertainty

product of a *combined standard measurement uncertainty* (3.7) and a factor larger than the number one

Note 1 to entry: The factor depends upon the type of probability distribution of the output quantity in a *measurement model* (3.11) and on the selected coverage probability.

Note 2 to entry: The term “factor” in this definition refers to a coverage factor.

Note 3 to entry: Expanded measurement uncertainty is termed “overall uncertainty” in paragraph 5 of Recommendation INC-1 (1980) (see the GUM) and simply “uncertainty” in IEC documents.

[SOURCE: ISO/IEC Guide 99:2007, 2.35]

3.9

measurement bias

bias

estimate of a systematic measurement error

[SOURCE: ISO/IEC Guide 99:2007, 2.18]

3.10

measurement result

set of quantity values being attributed to a *measurand* (3.2), together with any other available relevant information

Note 1 to entry: A measurement result generally contains “relevant information” about the set of quantity values, such that some can be more representative of the measurand than others. This can be expressed in the form of a probability density function (PDF).

Note 2 to entry: A measurement result is generally expressed as a single measured quantity value and a *measurement uncertainty* (3.3). If the measurement uncertainty is considered to be negligible for some purpose, the measurement result may be expressed as a single measured quantity value. In many fields, this is the common way of expressing a measurement result.

Note 3 to entry: In the traditional literature and in the previous edition of the ISO/IEC Guide 99 (VIM), measurement result was defined as a value attributed to a measurand and explained to mean an indication, or an uncorrected result, or a corrected result, according to the context.

3.11**measurement model**

model of measurement

model

mathematical relation among all quantities known to be involved in a measurement

Note 1 to entry: A general form of a measurement model is the equation $h(Y, X_1, \dots, X_n) = 0$, where Y , the output quantity in the measurement model, is the *measurand* (3.2), the quantity value of which is to be inferred from information about input quantities in the measurement model X_1, \dots, X_n .

Note 2 to entry: In more complex cases, where there are two or more output quantities in a measurement model, the measurement model consists of more than one equation.

[SOURCE: ISO/IEC Guide 99:2007, 2.48]

3.12**measurement task**

quantification of a *measurand* (3.2) according to its definition

Note 1 to entry: The measurement task is synonymous with the purpose of applying the measurement procedure.

Note 2 to entry: The measurement task can be used, e.g.:

- to compare the *measurement results* (3.10) with one or two specification limits in order to state whether the value of the measurand is an admissible value;
- to state whether the measurand characterizing a manufacturing process is within the specifications given;
- to obtain a confidence interval of given average length for the difference between two values of the same measurand.

3.13**measurement process**

set of operations to determine the value of a quantity

[SOURCE: ISO 9000:2015, 3.11.5]

3.14**resolution**

smallest change in a quantity being measured that causes a perceptible change in the corresponding indication provided by a measuring equipment

Note 1 to entry: Resolution can depend on, for example, noise (internal or external) or friction. It may also depend on the value of a quantity being measured.

Note 2 to entry: For a digital displaying device, the resolution is equal to the digital step.

Note 3 to entry: Resolution not necessarily linear.

[SOURCE: ISO/IEC Guide 99:2007, 4.14 modified - "provided by a measuring equipment" has been added in the definition, Notes 2 and 3 to entry have been added.]

3.15**reference quantity value**

reference value

quantity value used as a basis for comparison with values of quantities of the same kind

Note 1 to entry: A reference quantity value can be a true quantity value of a *measurand* (3.2), in which case it is unknown, or a conventional quantity value, in which case it is known.

Note 2 to entry: A reference quantity value with associated *measurement uncertainty* (3.3) is usually provided with reference to:

- a) a material, e.g. a certified reference material,

- b) a device, e.g. a stabilized laser,
- c) a reference measurement procedure,
- d) a comparison of measurement standards.

[SOURCE: ISO/IEC Guide 99:2007, 5.18]

3.16

measurement repeatability

repeatability

measurement precision under repeatability conditions of measurement

[SOURCE: ISO/IEC Guide 99:2007, 2.21]

3.17

measurement reproducibility

reproducibility

measurement precision under reproducibility conditions of measurement

[SOURCE: ISO/IEC Guide 99:2007, 2.25, modified — The Note has been deleted.]

3.18

stability of a measurement process

property of a *measurement process* (3.13), whereby its properties remain constant in time

3.19

item

entity

object

anything that can be described and considered separately

4 Symbols and abbreviated terms

4.1 Symbols

a	half width of a distribution of possible values of input quantity
a_{OBJ}	maximal form deviation
α	significance level
B_i	bias
$B_{i\max}$	maximal found bias
\bar{B}_i	arithmetic mean of biases
C_{MP}	measurement process capability index
$C_{MP\min}$	minimum measurement process capability index
C_{MS}	measuring system capability index
$C_{MS\min}$	minimum measuring system capability index
C_p	process capability index
C_{pk}	minimum process capability index

$C_{p\text{ obs}}$	observed process capability index
$C_{p\text{ real}}$	real process capability index
Δ	process dispersion/variation
Δ_U	50 % process dispersion/variation to upper limit
Δ_L	50 % process dispersion/variation to lower limit
d_{LR}	interval from the last reference value, for which all operators have assessed the result as unsatisfied to the first reference value, for which all operators have the result as approved
d_{UR}	interval from the last reference value, for which all operators have assessed the result as approved to the first reference value, for which all operators have the result as unsatisfied
k	coverage factor
K	total number of replicate measurements on one reference. The reference can be a reference standard or a reference workpiece
k_{CAL}	coverage factor from the calibration certificate
l	measured length
L	lower specification limit
M_{PE1}	maximum permissible error (of the first characteristic)
M_{PE2}	maximum permissible error (of the second characteristic)
M_{PE}	maximum permissible error (of the measuring system) (MPE-value)
N	number of standards
n	number of measurements
n_{ij}	ij th frequencies of measurement results
n_{ji}	ji th frequencies of measurement results
P	probability
P_p	process performance index
$P_{p\text{ obs}}$	observed process performance index
$P_{p\text{ real}}$	real process performance index
Q_{attr}	attributive measurement process capability ratio
$Q_{MS\text{ min}}$	minimum measuring system capability ratio
Q_{MS}	measuring system capability ratio
$Q_{MP\text{ min}}$	minimum measurement process capability ratio
Q_{MP}	measurement process capability ratio
R_E	resolution of measuring system

s	sample standard deviation (on the mean bias)
s_A	sample standard deviation (on the measuring system repeatability, ANOVA)
s_{EV}	sample standard deviation (on the measuring system repeatability)
s_{LIN}	sample standard deviation (from the linearity system repeatability)
s_{obs}	sample standard deviation (the observed standard deviation)
s_{RES}	sample standard deviation (on the mean bias)
T	temperature
ΔT	temperature difference
$t_{1-(\alpha/2)}$	the two-sided critical value of Student's t distribution
U	upper specification limit
u	standard uncertainty
u_α	standard uncertainty on the coefficient of expansion
u_{AV}	standard uncertainty from the operator's reproducibility
u_{BI}	standard uncertainty from the measurement bias
u_{CAL}	calibration standard uncertainty on a standard
u_{EV}	standard uncertainty from maximum value of repeatability or resolution
u_{EVR}	standard uncertainty from repeatability on standards
u_{EVO}	standard uncertainty from repeatability on test parts
u_{GV}	standard uncertainty from reproducibility of the measuring system
u_{IAi}	standard uncertainty from interactions
u_{LIN}	standard uncertainty from linearity of the measuring system
u_{MP}	combined standard uncertainty on measurement process
u_{MPE}	standard uncertainty calculated based on maximum permissible error
u_{MS}	combined standard uncertainty on measuring system
$u_{MS-REST}$	standard uncertainty from other influence components not included in the analysis of the measuring system
u_{OBJ}	standard uncertainty from test part inhomogeneity
u_{RE}	standard uncertainty from resolution of measuring system
u_{REST}	standard uncertainty from other influence components not included in the analysis of the measurement process
u_{STAB}	standard uncertainty from the stability of measuring system
u_T	standard uncertainty from temperature

u_{TA}	standard uncertainty from expansion coefficients
u_{TD}	standard uncertainty from temperature difference between workpiece and measuring system
U_{attr}	expanded uncertainty on an attributive measurement
U_{CAL}	expanded uncertainty on the calibration of a standard
U_{MS}	expanded uncertainty of the measuring system
U_{MP}	expanded uncertainty of the measurement process
U_{LIN}	expanded uncertainty from linearity of the measuring system
$X_{0,135\%}$	0,135 % quantile of process dispersion
$X_{50\%}$	50 % quantile of process dispersion
$X_{99,865\%}$	99,865 % quantile of process dispersion
x_i	i th measurement input quantity
x_m	reference quantity value
X_{mid}	process midpoint
X_{nom}	nominal value/operating point
\bar{x}_g	arithmetic mean of all the sample values
y_n	n th measurement value
y_{nj}	nj th measurement value
\bar{y}	average of all measurements

4.2 Abbreviated terms

ANOVA	analysis of variance
GPS	geometrical product specifications
GUM	guide to the expression of the uncertainty of measurement (ISO/IEC Guide 98-3)
MPE	maximum permissible error
MPL	maximum permissible error limit
SPC	statistical process control
VIM	international vocabulary of metrology (ISO/IEC Guide 99)

5 Basic principles

5.1 General

The method described in this document covers a large part of the estimation of measurement uncertainty that occurs in practice. In some cases, where the preconditions set out for this method (no correlation between influence components, no sensitivity factors, simple linear model present) are not

present, the user shall utilize the general current method for determining the measurement uncertainty that is described in ISO/IEC Guide 98-3 (GUM).

The following method addresses the implementation uncertainty (see also ISO 17450-2). Therefore, it shall be determined before the method is applied that the method uncertainty and the specification uncertainty is small compared to the implementation uncertainty. Further, the method is not suitable and shall not be used for complex geometrical measurement processes, such as surface texture and location measurements that rely on several measurement points or simultaneous measurements in several directions, or both.

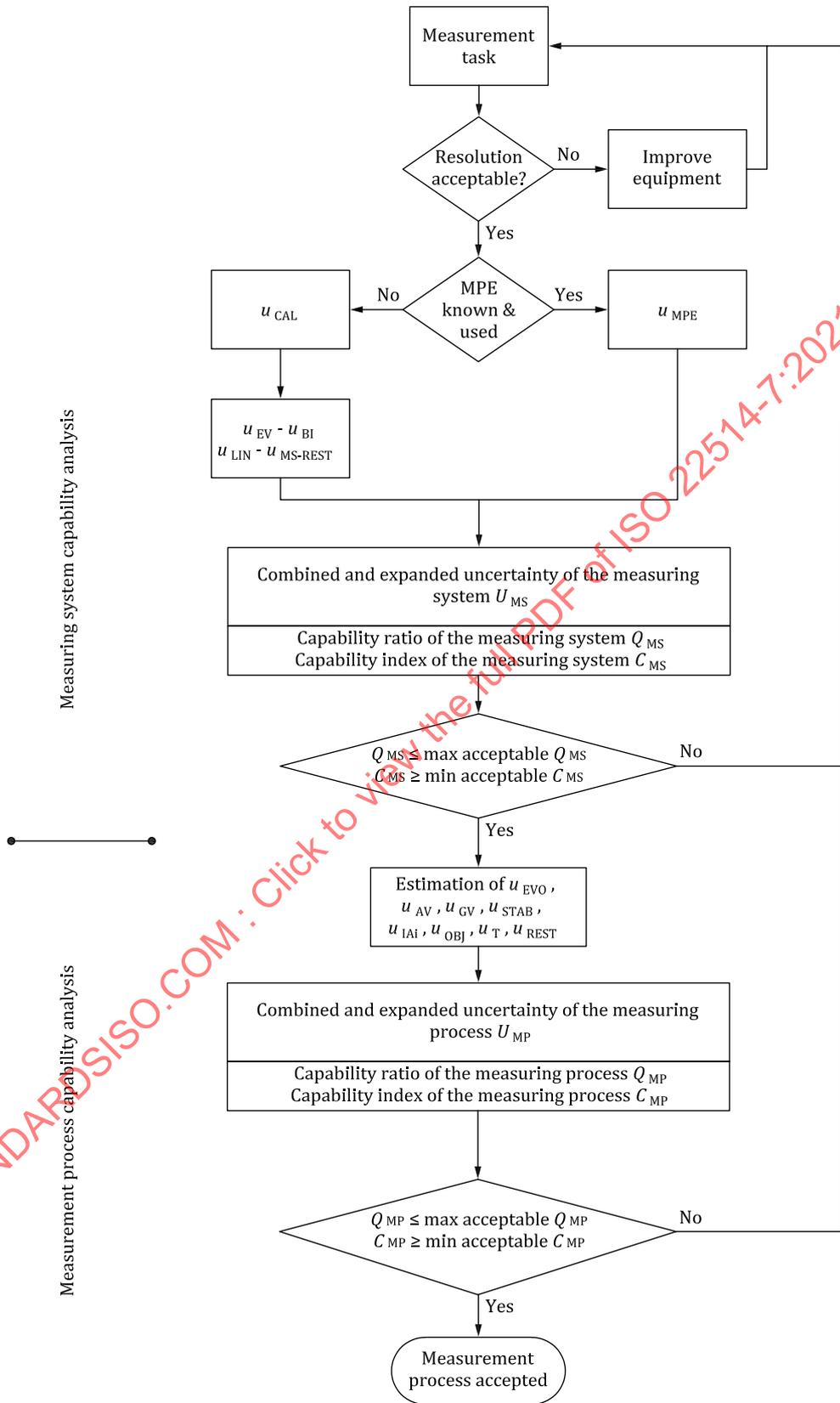
The ISO/IEC Guide 98-3 (GUM) permits the evaluation of standard uncertainties by any appropriate means. It distinguishes the evaluation by the statistical treatment of repeated observations as a Type A evaluation of uncertainty, and the evaluation by any other means as a Type B evaluation of uncertainty. In evaluating the combined standard uncertainty, both types of evaluation are to be characterized by squared standard uncertainties and treated in the same way. The standard uncertainties can be aggregated to obtain the (combined) standard measurement uncertainty. This evaluation of uncertainty is carried out, according to ISO/IEC Guide 98-3 (GUM), using the law of propagation of uncertainty. Full details of this procedure and the additional assumptions on which it is based are given in ISO/IEC Guide 98-3 (GUM).

To assess a measuring system or a measurement process, the capability ratio Q_{MS} or Q_{MP} or the capability index C_{MS} or C_{MP} can be calculated based on the combined standard measurement uncertainty and the specification.

The combined expanded uncertainty should be substantially smaller than the specification of the characteristic being measured.

If the uncertainty components estimated from an experiment (Type A evaluation) do not correspond to the expected spread of these components in the actual measurement process, then these components cannot be estimated experimentally. Instead, they should be derived through the use of a mathematical model (Type B evaluation e.g. constant temperature in a measuring laboratory when conducting a study and the normal temperature variations of the place of the future application). The practitioner should fully understand the model to be used.

[Figure 1](#) describes the step by step approach of the method. Linearity, repeatability and bias can be found using a reference standard as shown in the flowchart. Alternatively, bias can be found based on the MPE-value (maximum permissible error).



Measuring system capability analysis

Measurement process capability analysis

Figure 1 — Measurement process capability analysis

5.2 Resolution

The resolution is one of the contributors to the measurement uncertainty. It shall never be lower than the resolution effect. If the expanded uncertainty calculated based on only the actual resolution is bigger than the requirement to the measurement process, then the measuring system should be improved.

If a measurement system is used to evaluate processes in a bilateral specification without a specific rule being agreed between supplier and buyer, the resolution shall be less than 1/20 of the specification interval by default. In a one-sided specification, the resolution shall be less than 1/10 of the specification interval defined in 9.3.

When using a measurement system to control a manufacturing process using the SPC tools according to a bilateral specification, without a specific rule agreed between supplier and buyer, the resolution shall be lower than 1/10 of the process variation expressed by the reference interval (often 6 standard deviations, see ISO 22514-1) by default. Since the variation of the process can vary over time, it is recommended that the requirement be kept at 1/20 of the bilateral specification interval or 1/10 of the half-sided specification interval to avoid re-evaluation due to variable variation. This also applies to cases where the capability of the manufacturing process is very large and therefore the variation of the manufacturing process is very small in relation to the manufacturing tolerances.

5.3 Maximum permissible error known and used

5.3.1 General

If a standard measuring system is used, then a maximum permissible error (MPE), or more often a number hereof, should be defined for the actual system. The calibration system is used to document the compliance with the requirement to the defined metrological characteristic(s) given as one or more maximum permissible errors.

In this case, the MPE value or, if more than one metrological characteristic influences the measuring task, the combined result of the actual MPE values can be used to calculate the capability of the measuring system instead of the experimental method. If a population of different equipment should be used as measuring system, then the method using MPE is recommended. If only one defined measuring system can be used in connection with the measurement process, then the experimental method is preferable because the combined uncertainty is normally smaller.

5.3.2 MPE, maximum permissible deviation of the measuring system — u_{MPE}

The maximum permissible deviation (MPE) or the error limit (often called MPL) is the permitted extreme value of a measurement deviation in relation to a known reference value. The MPE always describes a half width of the allowed deviation.

If the MPE is proven by calibration and includes calibration uncertainty and is trustworthy, the determination of the individual uncertainty components of the measuring system can be omitted. To do this, it shall be ensured that the evidence provided by the calibration service provider contains at least the following additional information in addition to the defined MPE:

- reference to the applied accredited calibrated national/international standard;
- appropriate calibration methods that have been published and internationally accepted or similar methods shall be used;
- documentation of the standards (nominal values and calibration uncertainty), the used reference points, and how many repeated measurements were carried out;
- under what conditions (laboratory, the actual temperature deviations, range of air humidity, etc.) has the calibration been done;
- whether the usage decision is made with or without calibration uncertainty;

- the resolution should be significantly lower than the specified error limit value (reference value: $RE \leq 25 \% MPE$).

It is important that the used MPE be directly related to the actual measurement task. For example, the MPE of a micrometer specified in ISO 3611 explicitly refers to the maximum permissible length deviation under different use cases in practice. In other cases e.g. the MPE is determined according to ISO 10360-2 and ISO 10360-5 only refers to the conditions defined in these standards (stylus, environment, calibration ball, probing points, ...) and does not relate to the measurement processes and shall not be used in practice (e.g. measurement of parallelism) for the calculation of the Q_{MS} or C_{MS} .

5.4 Capability and performance limits for a measuring system and measurement process

If the measuring system shall be classified to a specific measurement process, it is important to set a limit on measurement uncertainty. In this way, the selection of a measuring system is simplified for upcoming measurement tasks.

If there is no requirement for a maximum Q_{MP} or a minimum C_{MP} , then proceed and calculate Q_{MS} or C_{MS} .

The following method is based on the precondition that some uncertainty components associated with the measurement process, such as non-homogeneity of the measured object, resolution and temperature should be modelled mathematically.

6 Implementation

6.1 General

As for other processes, the measurement process is under the influence of both random and systematic sources of variation. In order to estimate and control the variation of the measurement process, it is necessary to identify all important sources of the variation, and, if possible, to monitor them. In general, uncertainty components that are less than 10 % of the largest uncertainty component are considered to be unimportant and therefore are not taken into account.

6.2 Factors that influence the measurement process

6.2.1 General

In industrial practice, the reported uncertainty of the measurement process is usually limited to the uncertainty derived from repeatability of the measurement process on a reference standard, or an item typical of that to be produced, known as a workpiece. The uncertainties arising from any linearity deviation is either intentionally set to zero or acquired from the manufacturer's specification, e.g. in terms of adopted error limit (MPE values).

The use of the commonly known repeatability experiment on a reference standard to estimate the repeatability and bias of the measurement process is recommended. Based on this experiment, one can then estimate a measurement capability index. This method can be extended by the use of more than one reference standard, located near or inside the specification limits. In both cases, the measuring system can be corrected by use of the identified systematic error(s).

If the linearity of the measuring system shall be determined, it can be done by means of a linearity study based on at least three reference standards. The result of this investigation (the regression function) can then be used for correction of the measurement result. Hereby, the uncertainty caused by the linearity deviation is reduced.

6.2.2 Uncertainty components that belong to the measuring system

6.2.2.1 Types

The uncertainty components related to the measuring system (see [Table 4](#)) are either

- uncertainty estimated from maximum permissible error,
- or
- uncertainty estimated from the combination of:
 - calibration uncertainty,
 - repeatability or resolution,
 - bias,
 - linearity, and
 - other uncertainty components.

6.2.2.2 Estimation of uncertainty using the maximum permissible error value

When a measuring equipment or measuring standard is known to conform to stated MPE values for each of the metrological characteristics, these MPE values should be used to estimate the uncertainty component as shown in [Table 1](#).

Table 1 — Uncertainty from maximum permissible error

Uncertainty components	Symbol	Test/model
MPE value	u_{MPE}	Standard uncertainty due to maximum permissible error. $u_{MPE} = \frac{M_{PE}}{\sqrt{3}}$ where a rectangular distribution is assumed. In cases where more than one MPE value influences the measurement process, the combined standard uncertainty can be calculated from: $u_{MPE} = \sqrt{\frac{M_{PE1}^2}{3} + \frac{M_{PE2}^2}{3} \dots}$

6.2.2.3 Measuring system resolution

The actual proposed measuring system should have a high enough resolution so that the expanded uncertainty calculated from the standard uncertainty of the resolution is much lower (common practice is 5 %) than the specification interval for the characteristic to be measured (measurand).

The resolution of the measuring system, or the step in the last digit of a digital display, or rounded measured value, always causes an uncertainty component. When the repeatability uncertainty component is derived from experimental data, the effect from resolution, etc., is included if the repeatability uncertainty component (u_{EVR}) is greater than the component based on resolution.

If the uncertainty of the repeatability component is greater than that of the resolution component, then the resolution component is included in the repeatability component. If not, then the component u_{RE} should be added to the model as shown in [Table 2](#).

Table 2 — Uncertainty from resolution

Uncertainty components	Symbol	Test/model
Resolution of the measuring system	u_{RE}	$u_{RE} = \frac{1}{\sqrt{3}} \cdot \frac{R_E}{2} = \frac{R_E}{\sqrt{12}}$ where R_E is the resolution and is assumed to follow a rectangular distribution. If analogue scales are used, the actual distribution can be another, e.g. a normal distribution.

6.2.2.4 Calculation of repeatability, bias and linearity using reference standards or calibrated workpieces

The used reference standards or workpieces should be traceable to stated references, usually national or international standards, or so-called consensus standards (standards agreed by both customer and supplier). The present uncertainty during this calibration should be determined (see Table 3).

Table 3 — Uncertainty of standard calibration

Uncertainty components	Symbol	Test/model
Calibration	u_{CAL}	Standard deviation of uncertainty due to calibration (from certificate). In cases where the uncertainty in protocol is given as expanded uncertainty, it should be divided by the corresponding coverage factor: $u_{CAL} = U_{CAL} / k_{CAL}$

Linearity analyses should be made sufficiently often so that the estimated value for M_{PE} is not exceeded between two linearity analyses.

6.2.2.5 Experimental method

The experimental method considers how a relationship $Y = A + BX$ (describing how the dependent variable Y varies as a function of the independent variable X) can be determined from measurement data. The measurement data arise when a measuring system specified by (unknown) values A and B of the calibration function parameters is “stimulated” by standards with calibrated values of X_i , given in standard units, and the corresponding “responses”, or indications Y_i , of the instrument are recorded.

Table 4 — Uncertainty from measuring system

Uncertainty components	Symbol	Test/model
Uncertainty arising from linearity	u_{LIN}	Instance 1: $u_{LIN} = 0$ Instance 2: $u_{LIN} = \frac{a}{\sqrt{3}}$ where a is half width of the range of a rectangular distribution or the known MPE-value. Instance 3: u_{LIN} is determined experimentally together with u_{EVR} (see instance 2 in row u_{EVR} below). Instance 4: u_{LIN} is determined based on the results from the calibration certificate.

Table 4 (continued)

Uncertainty components	Symbol	Test/model
Uncertainty arising from bias	u_{BI}	<p>Instance 1: From the measurements on a reference standard, u_{BI} can be calculated based on the distance between the standard and the average of the measured values.</p> <p>Instance 2: K repeated measurements on each of the N (≥ 2) different reference standards with $N \times K \geq 30$, u_{BI} can be calculated from the maximum bias of all standards.</p> <p>Instance 3: u_{BI} is determined experimentally together with u_{EVR} (see instance 2 in row u_{EVR} below).</p> $u_{BI} = \frac{ \bar{x}_g - x_m }{\sqrt{3}}$
Repeatability using reference standards	u_{EVR}	<p>Instance 1: minimum 30 repeated measurements on a reference standard, whereby u_{EVR} can be estimated.</p> <p>Instance 2: K repeated measurements on each of the N (≥ 3) different reference standards with $N \times K \geq 30$.</p> <p>Estimate both u_{EVR} and u_{LIN} by the ANOVA method.</p>
Other uncertainty components not included in the above	$u_{MS-REST}$	E.g. scale shift (use of different measuring faces in a calliper).

6.2.3 Additional uncertainty components belonging to the measurement process

6.2.3.1 General

In an analysis of a defined measurement process under real conditions, an identification and determination of additional uncertainty components of the process (see [Table 5](#)) should be carried out together with the above described uncertainty components coming from the measuring system.

6.2.3.2 Determination of uncertainty components from experiments (Type A)

Table 5 — Uncertainty from repeatability and reproducibility of the measurement process

Uncertainty component	Symbol	Test/model
Repeatability using workpieces	u_{EVO}	<p>A minimum of 5 workpieces shall always be used in the analysis:</p> <ul style="list-style-type: none"> — measured by a minimum of 2 operators, or — measured by a minimum of 2 different measuring systems (if relevant). <p>Minimum total sample size: 30.</p> <p>Estimation of uncertainty components by the ANOVA method.</p> <p>[ISO/IEC Guide 99 (VIM), ISO/IEC Guide 98-3 (GUM), ISO 5725 (all Parts), ISO 13528, ISO 21748, ISO/TS 21749].</p> <p>If no operator influence is present, the number of workpieces should be increased.</p>
Effect of operators changing in reproducibility conditions of measurement	u_{AV}	
Reproducibility of the measuring system (Place of measurement)	u_{GV}	
Effect of changing over the times in reproducibility conditions of measurement	u_{STAB}	
Interactions	u_{IAi}	

NOTE 1 In special circumstances (e.g. high cost of test), two repetitions can be acceptable.

NOTE 2 If the number of samples is smaller than 30, the Student's t -test can be used to expand the extended uncertainty. See [Clause 8](#).

6.2.3.3 Determination of uncertainty components not included in the experiments (Type B)

In addition to the estimated uncertainty components of the measuring system (6.2.2), and the estimated uncertainty components of the measurement process (6.2.3.2), the following additional uncertainty components should be determined using mathematical models (see Table 6).

Table 6 — Other uncertainty on the measurement process

Uncertainty components	Symbol	Test/model
Non-homogeneity of the part	u_{OBJ}	$u_{OBJ} = \frac{a_{OBJ}}{\sqrt{3}}$ where a_{OBJ} is the maximum permitted or expected error due to the object (e.g. form deviation).
Temperature	u_T	The influence from temperature can be calculated using the formula:
		$u_T = \sqrt{u_{TD}^2 + u_{TA}^2}$ The uncertainty from temperature differences u_{TD} can e.g. be estimated as defined in ISO 14253-2.
		$u_{TD} = \frac{\Delta T \cdot \alpha \cdot l}{\sqrt{3}}$ where α is the expansion coefficient; ΔT is the difference in temperatures; and a rectangular distribution is assumed. The uncertainty on expansion coefficients can be estimated as defined in ISO 15530-3.
		$u_{TA} = \frac{ T - 20 \text{ }^\circ\text{C} \cdot u_\alpha \cdot l}{\sqrt{3}}$ where: T is the average temperature during the measurement; u_α is the uncertainty on the coefficient of expansion; l is the observed value for length measurement.
<p>T is temperature in the formulae above. It should not be confused with specification interval or target value, used elsewhere in this document.</p> <p>In the case that a compensation for temperature difference is not made, a contribution for this difference should be included in the estimation in the formula above.</p> <p>The part is the object to be measured, including object measured by embedded devices in production.</p>		

6.2.3.4 Impact of the deviation of workpiece on the measurement result

In many measurement processes, the surface of the workpiece is in contact with the measuring system during the measurement. Depending on the surface texture, form deviation and geometrical deviations from the nominal geometry, the contact between the measuring system and the workpiece results in an uncertainty component. Depending on the measurand and the repartition of the measuring on the workpiece, the impact of the form deviation does not have the same level. (If the measurand corresponds to the maximum value, and only one measure is taken, then the form deviation impacts directly. But if the workpiece is turned and the maximum observed value is taken, the form deviation is integrated in the evaluation, and does not impact in measurement uncertainty.)

The component a_{OBJ} can be found from requirements on the drawing or by experiments suitable to find the maximum form deviation or similar non-homogeneities.

Add the component u_{OBJ} to the model, as shown in Table 10.

6.2.3.5 Resolution

If the repeatability component using workpieces (u_{EVO}) is greater than that of the resolution component, then the resolution component is included in the repeatability component. If not, then the component u_{RE} should be added to the model as shown in [Table 1](#).

6.2.3.6 Temperature influence

6.2.3.6.1 Uncertainty calculation

The uncertainty from temperature influence u_T should be calculated based on the uncertainty component caused by temperature difference and uncertainty from unknown expansion coefficients.

$$u_T = \sqrt{u_{TD}^2 + u_{TA}^2}$$

6.2.3.6.2 Uncertainty component caused by temperature differences and expansion

The standard reference temperature for geometrical product specifications (GPS) and GPS measurements is 20 °C (see ISO 1). There can be reference temperatures for applications other than geometrical (e.g. electrical influences from temperature) that can be caused by absolute temperature as well as time and spatial temperature gradients result in linear expansion, bending, etc., of the measuring system. The measurement setup and the object being measured cause an uncertainty component u_{TD} .

The transformation from temperature to length is given by the linear expansion equation:

$$\Delta L = \Delta T \cdot \alpha \cdot l$$

where

- ΔT is the relevant temperature difference;
- α is the temperature expansion coefficient of the material;
- l is the effective length under consideration.

A known deviation in temperature from the reference temperature can be corrected as a systematic error component if appropriate.

The uncertainty u_{TD} can, for example, be estimated as defined in ISO 14253-2.

6.2.3.6.3 Uncertainty on the coefficient of expansion

An uncertainty contribution from the variation of the expansion coefficient of the measured workpieces is often present. In this case, the uncertainty u_{TA} is calculated by:

$$u_{TA} = \frac{|T - 20| \cdot u_\alpha \cdot l}{\sqrt{3}}$$

where u_α is the standard uncertainty of the expansion coefficient of the workpieces.

Alternatively, the uncertainty u_{TA} can be estimated as defined in ISO 15530-3.

7 Studies for calculating the uncertainty components

7.1 Measuring system

7.1.1 General

In order for a study to provide meaningful information, it is a prerequisite that the resolution of the measuring system be determined and adequate for the actual measurement process.

It should be confirmed that the standard uncertainty from repeatability is not smaller than the standard uncertainty from the resolution. Otherwise, the uncertainty from the resolution should be used instead of the repeatability ($\max\{u_{\text{EVR}}, u_{\text{EVO}}, u_{\text{RE}}\}$).

The method applied is based on knowledge to the linearity of actual measuring system. If the linearity shall be regarded as known, the repeatability and bias can be found using one (or more) standard(s).

7.1.2 Repeatability and bias based on one reference standard

7.1.2.1 General

If the uncertainty component u_{LIN} is equal to zero or estimated from the maximum permissible error (M_{PE}), the component u_{EVR} should be determined experimentally. The determination of the uncertainty u_{EVR} comes from the repeatability estimated from measurements on a reference standard or workpiece. It should be based on the spread of a minimum of 30 repeated measurements, to estimate the combined effect of bias and repeatability. In this case, the bias and the variation are used together as two different uncertainty components u_{BI} and u_{EVR} .

7.1.2.2 Preconditions

The reference quantity value of the reference standard or workpiece should have a quantity value close to the target value. The maximum deviation of the reference standard from target value depends on the characteristics of the measuring system.

The reference quantity value x_m of the reference standard or workpiece should be determined (normally by calibration).

The reference standard or workpiece shall be removed and replaced between each measurement.

In the case of physically one-sided specification ("natural limit"), the reference quantity value of the reference standard or workpiece should have a quantity value close to the specification value.

7.1.2.3 Procedure

Take at least 30 measurements on the reference standard or calibrated workpiece.

Based on the actual values, the measurement bias (B_i), the standard uncertainty of repeatability from reference standard and the standard uncertainty of the bias are estimated from:

$$u_{\text{EVR}} = s = \sqrt{\frac{1}{K-1} \cdot \sum_{i=1}^K (x_i - \bar{x}_g)^2} \quad \text{and} \quad B_i = |\bar{x}_g - x_m|$$

where

K is the number of repeated measurements;

x_i is the single value of the i -th measurement;

\bar{x}_g is the arithmetic mean of all the sample values.

$$u_{BI} = \frac{B_i}{\sqrt{3}} = \frac{|\bar{x}_g - x_m|}{\sqrt{3}}$$

This formula shall only be used in cases where the systematic and random errors cannot be distinguished.

If a zero setting of the measuring equipment can cause extra variation, it is recommended to set zero on the measuring system using the defined standard or workpiece between each attempt.

If more than one standard is used in the experiment to determine the repeatability, the largest mean deviation from the respective standard should be used as the bias value. If the variance is assumed to be constant, the average variance should be used.

7.1.3 Standard uncertainty from the linearity deviation — u_{LIN}

7.1.3.1 Applicable methods

The standard uncertainty from the linearity deviation can be determined using either method A or method B.

7.1.3.2 Method B

Data sheets, calibration certificates or other documents relating to the measuring system clearly show the maximum linearity deviation in the application area. The same requirements as for MPE apply to the clarity and reliability of the information (see 5.3). The following situations shall be distinguished.

- The linearity deviation is specified as maximum value a , i.e. it can lie in a range from $-a \dots +a$:

$$u_{LIN} = \frac{a}{\sqrt{3}} .$$

- The linearity deviation is given as standard deviation s_{LIN} :

$$u_{LIN} = s_{LIN} .$$

- The linearity deviation is given as expanded uncertainty U_{LIN} with coverage factor k :

$$u_{LIN} = \frac{U_{LIN}}{k} .$$

7.1.3.3 Method A

The linearity deviation is determined in the "Measuring system experiment" (see 6.2.2.5). In this case, the linearity deviation describes the portion of the bias that is variable over the application range, while the bias described in 7.1.2.3 is assumed to be constant. In practice, these constant and variable parts of the bias are difficult to separate with acceptable experimental effort.

In the "simple linearity assessment" (see Figure 2) described below, this separation is omitted, which in some cases can lead to an erroneous estimate of uncertainty. With the simple linearity evaluation, however, a reduced linearity study is possible to secure the tolerance limits with two standards (one in the range of each tolerance limit). More standards can be used at any time to increase the quality of the study. In total, at least 30 measurements should be available, i.e. with 2 standards, at least 15 measurements per standard; with 3 standards, 10 measurements per standard. In this case, linearity is not explicitly shown and is contained in u_{BI} .

The "linearity evaluation with ANOVA" (see Figure 3) estimates the constant bias u_{BI} and thus replaces the determination of the bias according to 7.1.2.3. Furthermore, the variable part u_{LIN} of the bias and

the repeatability at the reference part u_{EVR} is estimated by an analysis of variance. In this case, at least three reference parts/standards shall be measured several times under repeatability conditions, so that in total at least 30 measured values are available.

The actual values of the standards should be distributed approximately equidistantly over the application range of the measuring system, whereby the application range exceeds the tolerance range as far as parts outside the tolerance can be plausibly expected.

For both variants, the calibration uncertainty of the reference parts/standards should be significantly less than 5 % of the characteristic tolerance. The largest calibration uncertainty of the used reference parts/standards is used as u_{CAL} (see 6.2.2.4) for the determination of the combined uncertainty of the measuring system. The measurements shall be performed under typical conditions of use of the measuring system.

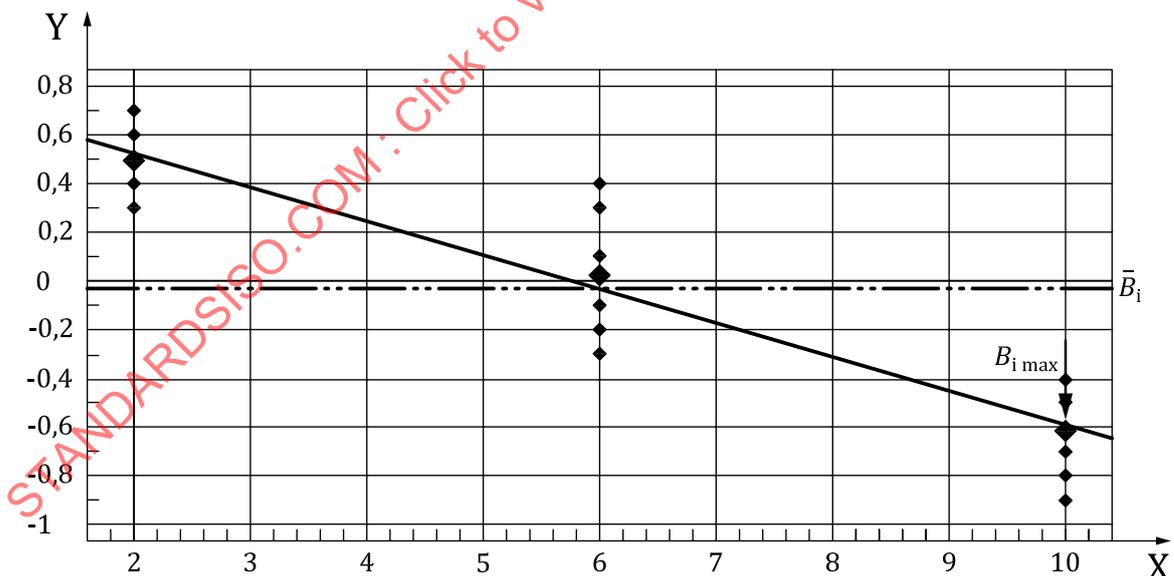
Simple linearity assessment

The bias is calculated for each reference part/standard. If the test is designed in such a way that the point of maximum linearity deviation is found in the test with the selection of reference part/standard, this is included in the maximum bias (see Figure 2). Therefore only u_{BI} is calculated and considered, an additional consideration of u_{LIN} is not necessary. For n reference parts/standards the following applies:

$$B_{i \max} = \max(B_{i1} \dots B_{in})$$

$$u_{\text{BI}} = \frac{1}{\sqrt{3}} B_{i \max}$$

$$u_{\text{LIN}} = 0$$



Key

- X value of reference parts
- Y bias

Figure 2 — Determination of linearity with maximum bias

Furthermore, the repeatability s_{EV} is calculated for each reference part/standard. The maximum repeatability is entered as u_{EVR} (see 6.2.2.5) in the calculation of the combined uncertainty of the measurement system.

This calculation of the linearity deviation corresponds to a "worst case" assumption for the case that the linearity deviation follows a kind of characteristic curve and the maximum deviation was determined in the test. The calculation is not applicable if the linearity deviation corresponds to a predominantly random variation.

7.1.3.4 Linearity evaluation with ANOVA

A mean bias is calculated over all determined deviations from the actual values of the reference parts/standards, which is converted to a standard uncertainty analogous to 6.2.2.5 under the assumption of an a-priori rectangular distribution.

$$u_{BI} = \frac{\overline{(B_i)}}{\sqrt{3}}$$

It shall be assumed that this mean bias cannot be corrected and can be changed in a random manner after a readjustment of the measuring system. Correctable portions shall be eliminated before the examination [see 5.1 and ISO/IEC Guide 98-3 (GUM)].

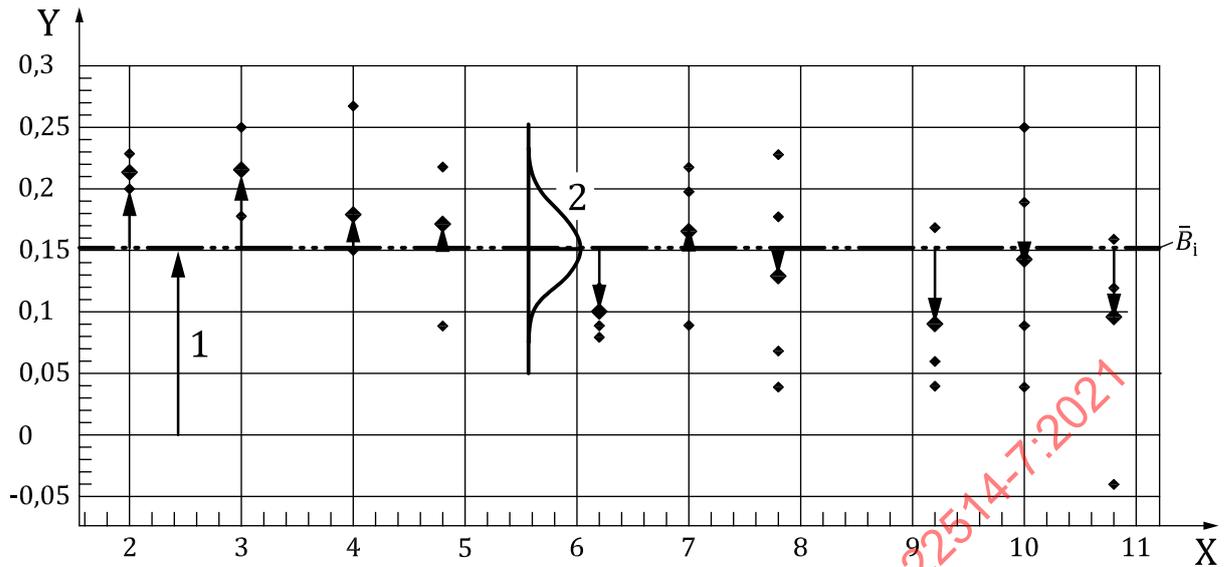
With the help of a "one-way analysis of variance" (see Reference [23]) the remaining scattered components are now determined. The variation of the mean bias (rest bias per reference part/normal) s_A results in the variable portion of the bias u_{LIN} :

$$u_{LIN} = s_A$$

The mean variation s_{RES} about the mean values of the individual standards (rest residuals) results in the repeatability on the reference part u_{EVR} :

$$u_{EVR} = s_{RES}$$

The raw data in Figure 3 are taken from ISO 11095:1996, there are 10 standards with 4 measurements each.



Key

- X value of reference part
- Y bias
- 1 mean bias over all reference parts
- 2 uncertainty from linearity
- ◆ individual error
- ◆ mean bias of the reference part

Figure 3 — Determination of linearity with ANOVA

8 Calculation of combined uncertainty

8.1 General

The combined uncertainty of the measuring system and the measurement process shall be calculated as given in Table 9. The calculation can only be carried out in the given way if there is no correlation between the components. Further information about the calculation can be found in ISO/IEC Guide 98-3:2008, Clause 5.

Table 9 — Calculation of the uncertainty

Uncertainty components	Symbol	Combined measurement uncertainty
Calibration of the standard or work-piece	u_{CAL}	$u_{MS} = \sqrt{u_{CAL}^2 + u_{LIN}^2 + u_{BI}^2 + u_{EVR}^2 + u_{MS-REST}^2}$ where $u_{EV} = \max\{u_{EVR}, u_{RE}\}$
Deviations from linearity	u_{LIN}	
Bias	u_{BI}	
Repeatability on standards	u_{EVR}	
Resolution	u_{RE}	
Other uncertainty components (measuring system)	$u_{MS-REST}$	

Table 9 (continued)

Uncertainty components	Symbol	Combined measurement uncertainty
Repeatability on workpiece	u_{EVO}	$u_{MP} = \sqrt{u_{CAL}^2 + u_{LIN}^2 + u_{BI}^2 + u_{EV}^2 + u_{MS-REST}^2 + u_{AV}^2 + u_{GV}^2 + u_{STAB}^2 + u_{OBJ}^2 + u_T^2 + u_{REST}^2 + \sum_i u_{IAi}^2}$ <p>where</p> $u_{EV} = \max\{u_{EVR}, u_{EVO}, u_{RE}\}$
Reproducibility of operator	u_{AV}	
Reproducibility of the measuring system (Different locations of the measurement process)	u_{GV}	
Reproducibility over time	u_{STAB}	
Interactions	u_{IAi}	
Inhomogeneity of measurand	u_{OBJ}	
Temperature	u_T	
Other uncertainty components (measurement process)	u_{REST}	

The combined standard uncertainty of the measuring system can be estimated using the formula from [Table 9](#):

$$u_{MS} = \sqrt{u_{CAL}^2 + u_{LIN}^2 + u_{BI}^2 + u_{EV}^2 + u_{MS-REST}^2}$$

where

$$u_{EV} = \max\{u_{EVR}, u_{RE}\}.$$

In a similar way, the combined standard uncertainty of the measurement process can be estimated using the formula from [Table 9](#):

$$u_{MP} = \sqrt{u_{CAL}^2 + u_{LIN}^2 + u_{BI}^2 + u_{EV}^2 + u_{MS-REST}^2 + u_{AV}^2 + u_{GV}^2 + u_{STAB}^2 + u_{OBJ}^2 + u_T^2 + u_{REST}^2 + \sum_i u_{IAi}^2}$$

where

$$u_{EV} = \max\{u_{EVR}, u_{EVO}, u_{RE}\}.$$

If the calculated combined measurement uncertainty of the measurement process u_{MP} contains the reproducibility of multiple measurement systems, it has to be taken care that all components which are used for the calculation of the combined measurement uncertainty of the measurement system u_{MS} are derived from the measurement system where u_{MS} is the largest. If the separate calculation of u_{MS} for the compared measuring systems is not possible, the maximum value of each component of the measuring system ($u_{RE}, u_{CAL}, u_{EVR}, u_{LIN}, \dots$) has to be taken. The same applies to the remaining components of the measurement system and measurement process if the influences show different effects on the measurement systems. The estimation over all maximum values can cause the estimator for u_{MP} to be too large.

If the MPE-values are used instead the experimental method, then calculation of combined uncertainties for measuring system a measurement process follows [Table 10](#).

Table 10 — Calculation of the uncertainty with maximum permissible error MPE

Uncertainty components	Symbol	Combined measurement uncertainty
Maximum permissible error of measuring system components	u_{MPE}	If the MPE-values are used instead the experimental method then $u_{MS} = u_{MPE} = \sqrt{\frac{M_{PE1}^2}{3} + \frac{M_{PE2}^2}{3} + \dots}$
Repeatability on workpiece	u_{EVO}	If the MPE-values are used instead the experimental method, then $u_{MPE} = \sqrt{\frac{M_{PE1}^2}{3} + \frac{M_{PE2}^2}{3} + \dots}$ and the uncertainty $u_{MP} = \sqrt{u_{MPE}^2 + u_{AV}^2 + u_{GV}^2 + u_{STAB}^2 + u_{OBJ}^2 + u_T^2 + u_{REST}^2 + \sum_i u_{IAi}^2}$
Reproducibility of operator	u_{AV}	
Reproducibility of the measuring system (Different locations of the measurement process)	u_{GV}	
Reproducibility over time	u_{STAB}	
Interactions	u_{IAi}	
Inhomogeneity of measurand	u_{OBJ}	
Temperature	u_T	
Other uncertainty components (measurement process)	u_{REST}	

In case of using maximum permissible error MPE as the only input for the measurement system, u_{MS} is calculated as:

$$u_{MS} = u_{MPE} = \sqrt{\frac{M_{PE1}^2}{3} + \frac{M_{PE2}^2}{3} + \dots}$$

Corresponding to that, the combined uncertainty for the measurement process is calculated as:

$$u_{MS} = \sqrt{u_{MPE}^2 + u_{AV}^2 + u_{GV}^2 + u_{STAB}^2 + u_{OBJ}^2 + u_T^2 + u_{REST}^2 + \sum_i u_{IAi}^2}$$

8.2 Calculation of expanded uncertainty

The expanded U_{MS} can be found from the standard uncertainty u_{MS} by multiplying the uncertainty by the coverage factor k :

$$U_{MS} = k \cdot u_{MS}$$

The same method is used to find the expanded U_{MP} from the standard uncertainty u_{MP} :

$$U_{MP} = k \cdot u_{MP}$$

Calculation of the expanded uncertainty is based on an approximate 95 % confidence interval; therefore, the coverage factor $k = 2$ is used.

NOTE If there is an experiment where the sample size n is smaller than the preferred 30, it is necessary to use Student's t distribution instead of the standard normal distribution to estimate the coverage factor:
 $k = t_{1-(\alpha/2), \nu}$

The number of degrees of freedom ν is obtained from the product of the number of workpieces (N), the number of operators (p), the number of gauges (g) and the number of repeatability measurements (K) minus 1 ($N \cdot p \cdot g \cdot (K-1)$).

EXAMPLE 1 3 workpieces, 2 operators, 2 gauges and 3 repeated measurements:

For $v = 3 \cdot 2 \cdot 2 \cdot (3-1) = 24$, one finds $t_{1-(\alpha/2), v} = 2,06$.

EXAMPLE 2 3 workpieces, 2 operators, 2 gauges and 2 repeated measurements:

For $v = 3 \cdot 2 \cdot 2 \cdot (2-1) = 12$, one finds $t_{1-(\alpha/2), v} = 2,18$.

9 Capability

9.1 Performance ratios

9.1.1 General

The capability of a measurement process can be calculated either as a performance ratio or a capability index. Calculating of indices is preferred.

To assess the measuring system or the measurement process, the performance ratio (Q_{MS} or Q_{MP}) shall be calculated based on the measurement uncertainties given in [Clause 8](#). According to [Clause 8](#), a distinction is made between the performance ratios for the measuring system (Q_{MS}) and the measurement process (Q_{MP}).

Q_{MS} should not exceed 15 %, and Q_{MP} should not exceed 30 % (by common practice).

The 95 % confidence interval should be calculated for the uncertainty of the calculated ratios.

The process spread (99,73 % interval of the production process) can be used as an alternative reference figure, when the measurement process is used as a part of SPG (statistical process control) system.

9.1.2 Performance ratio of the measuring system

$$Q_{MS} = \frac{2 \cdot U_{MS}}{U - L} \cdot 100 \text{ (\%)}$$

The formula is based on the specification as reference.

9.1.3 Performance ratio of the measurement process

$$Q_{MP} = \frac{2 \cdot U_{MP}}{U - L} \cdot 100 \text{ (\%)}$$

The formula is based on the specification as reference.

9.2 Capability indices

The capability indices for measuring system C_{MS} and process C_{MP} can be calculated based on the general definition of a capability index, which can be found in ISO 3534-2:2006, 2.7.

It is recommended that C_{MS} and C_{MP} exceed 1,33. As mentioned in [8.2](#) and [9.1](#), the 95 % confidence interval (therefore $k = 2$) should be used to calculate the ratios.

The formulas for calculating the capability indices are designed in such a way that a capability index of $C_{MS \min} = C_{MP \min} = 1,33$ is achieved if the recommended limits of the performance indices ($Q_{MS \min} \leq 15 \%$, $Q_{MP \min} \leq 30 \%$) are observed.

The capability of a measurement process can be expressed as a capability index C_{MS} .

$$\widehat{C}_{MS} = \widehat{C}_{MS \min} \cdot \widehat{Q}_{MS \min} \cdot \frac{(U-L)}{2 \cdot k \cdot \hat{u}_{MS}} = \frac{4}{3} \cdot 0,15 \cdot \frac{(U-L)}{2 \cdot k \cdot \hat{u}_{MS}} = 0,2 \cdot \frac{(U-L)}{2 \cdot k \cdot \hat{u}_{MS}}$$

The capability of a measurement process can be expressed as a capability index C_{MP} .

$$\widehat{C}_{MP} = \widehat{C}_{MP \min} \cdot \widehat{Q}_{MP \min} \cdot \frac{(U-L)}{2 \cdot k \cdot \hat{u}_{MP}} = \frac{4}{3} \cdot 0,30 \cdot \frac{(U-L)}{2 \cdot k \cdot \hat{u}_{MP}} = 0,4 \cdot \frac{(U-L)}{2 \cdot k \cdot \hat{u}_{MP}}$$

9.3 Capability of a measurement process with one-sided specifications

In case of one-sided specifications, there is either no upper specification limit or, alternatively, no lower specification limit (Figure 4). A tolerance zone cannot be defined, wherefore the standard formulas for C_{MS} and C_{MP} cannot be used. To estimate the capability in such cases, a modified formula is needed to handle one-sided (unilateral) specifications. Therefore, it only can be focused on just one side of the manufacturing and measurement process distribution.

NOTE In cases where a characteristic is limited by a so-called natural limit (e.g. geometrical tolerances that cannot be less than zero due to the definition of the measured quantity), this natural limit is treated as a specification limit and all indices can be calculated according to 9.1 or 9.2.

The process variation Δ (also called process or part variation PV), the process location X_{mid} and a required capability index C_p (or P_p) are used to plausibly define the missing limits.

The result from the production process in case of geometrical tolerances is based on an evaluation of a point cloud and therefore is an extreme value distribution model.

There are several ways to estimate the process dispersion Δ , with the following priority (see details in the following paragraphs):

1. process dispersion determined according to ISO 22514-1 and ISO 22514-2;
2. process dispersion calculated from the experiment described in 6.2.3.2;
3. process spread and process location estimated from historical data of similar processes.

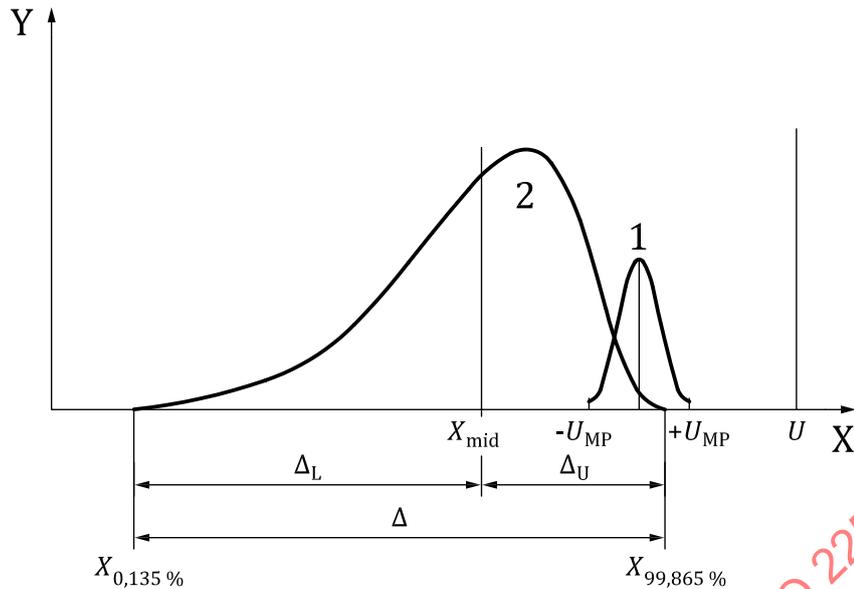
The process dispersion is preferably determined according to ISO 22514-1 and ISO 22514-2 using a minimum sample size of 100.

$$\Delta = X_{99,865\%} - X_{0,135\%}$$

From that, the one-sided process spread is calculated as:

$$\Delta_U = X_{99,865\%} - X_{mid}$$

$$\Delta_L = X_{mid} - X_{0,135\%}$$



- Key**
- X characteristics value
 - Y probability distribution value
 - 1 measurement uncertainty
 - 2 process distribution

Figure 4 — Measurement uncertainty in case of one-sided specification

If only fewer parts or smaller sample sizes are available, the experiment described in 6.2.3.2 can be used instead to calculate the process variation (PV) based on at least 10 parts.

The selection of parts leads to contradictory requirements. In terms of capability, the entire range of application should be examined, i.e. parts should be selected which cover the entire area of application. However, these specifically selected parts are not suitable for calculating the dispersion of the manufacturing process. For this purpose, a random sample would have to be taken from the manufacturing process.

If there are too few parts to determine a reliable distribution model, a normal distribution is assumed. The empirical standard deviation s_p based on these few parts will underestimate the process variation. An optimized estimate of the standard deviation based on the measured parts can be found using this formula:

$$s_{\text{eff}} = \sqrt{\frac{n-1}{n-3}} s_p$$

where

- s_{eff} is the estimated “true” standard deviation;
- n is the number of measurements used to calculate s_p ;
- s_p is the calculated standard deviation.

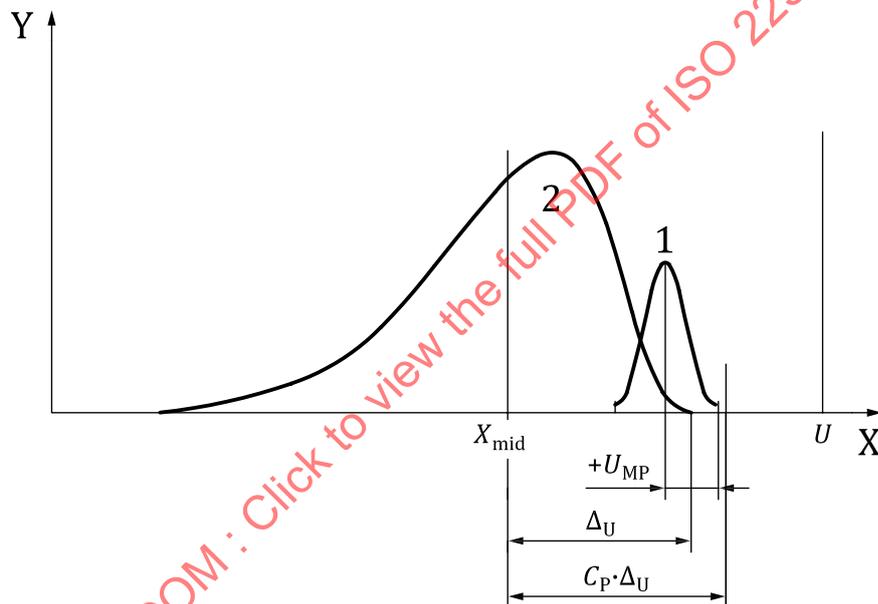
From that, the half-sided process variation Δ_U resp. Δ_L is calculated as:

$$\Delta_U = \Delta_L = 3 \cdot s_{\text{eff}} = 3 \cdot \sqrt{\frac{n-1}{n-3}} \cdot s_p$$

Alternatively, if there are no reliable data or enough parts available, process spread Δ and process location X_{mid} can be estimated from historical data of similar processes. This estimate shall be documented in a comprehensible way. The substituted limits depend on the required capability index for the manufacturing process and are estimating adequate tolerance limits where capability requirements are fulfilled.

The estimator for process location X_{mid} is the 50 %-quantile $X_{50\%}$ in case of an arbitrary distribution, or \bar{x} in case of non-skewed symmetric distribution (e.g. normal distribution).

The calculation of the substituted limits is depending on upper or lower one-sided limit (see [Figure 5](#) for upper specification limit).



Key

- X characteristics value
- Y probability distribution value
- 1 measurement uncertainty
- 2 process distribution

Figure 5 — Calculation of measuring system / measurement process capability in case of one-sided specification

If an upper one-sided specification limit is given, the capability indices and performance ratios are calculated as:

$$\widehat{C}_{\text{MS}} = 0,2 \cdot \frac{C_p \cdot \Delta_U}{k \cdot \hat{u}_{\text{MS}}} \quad \widehat{C}_{\text{MP}} = 0,4 \cdot \frac{C_p \cdot \Delta_U}{k \cdot \hat{u}_{\text{MP}}}$$

$$\widehat{Q}_{MS} = \frac{k \cdot \hat{u}_{MS}}{C_p \cdot \Delta_U} \quad \widehat{Q}_{MP} = \frac{k \cdot \hat{u}_{MP}}{C_p \cdot \Delta_U}$$

If a lower one-sided specification limit is given, the capability indices and performance ratios are calculated as:

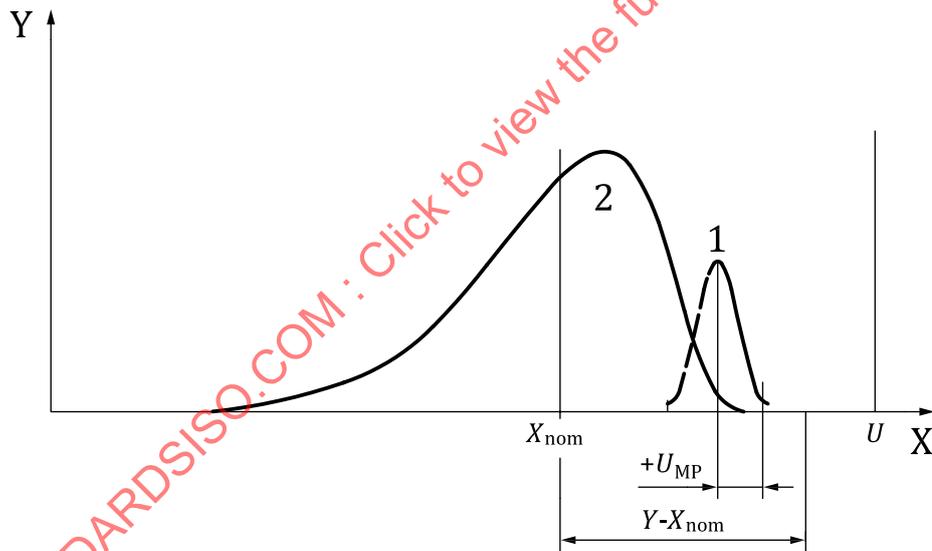
$$\widehat{C}_{MS} = 0,2 \cdot \frac{C_p \cdot \Delta_L}{k \cdot \hat{u}_{MS}} \quad \widehat{C}_{MP} = 0,4 \cdot \frac{C_p \cdot \Delta_L}{k \cdot \hat{u}_{MP}}$$

$$\widehat{Q}_{MS} = \frac{k \cdot \hat{u}_{MS}}{C_p \cdot \Delta_L} \quad \widehat{Q}_{MP} = \frac{k \cdot \hat{u}_{MP}}{C_p \cdot \Delta_L}$$

The resolution RE should be less than 1/10 of the half-sided specification interval ($C_p \times \Delta_U$) respectively ($C_p \times \Delta_L$) (see 5.2).

NOTE In special cases, where there is a specified operating point/nominal value X_{nom} for the manufacturing process, the term $C_p \times \Delta_L$, resp. $C_p \times \Delta_U$ can be substituted in the following way:

- in case of an upper one-sided specification limit $C_p \times \Delta_U = U - X_{nom}$;
- in case of a lower one-sided specification limit $C_p \times \Delta_L = X_{nom} - L$;
- (see Figure 6 for operating point and upper specification limit).



- Key**
- X characteristics value
 - Y distribution probability value
 - 1 measurement uncertainty
 - 2 process distribution

Figure 6 — Calculation of MS/MP capability and performance in case of one-sided specification with operating point/nominal value

10 Capability of the measurement process compared to the capability of the production process

10.1 Relation between observed process capability and measurement capability ratios

The following connection exists between an observed process capability or process performance ($C_{p \text{ obs}}, P_{p \text{ obs}}$), the actual real process capability or performance ($C_{p \text{ real}}, P_{p \text{ real}}$) and the capability ratio (Q_{MP}) of the measurement process:

$$C_{p \text{ real}} = \sqrt{\frac{1}{C_{p \text{ obs}}^2} - 2,25 \cdot Q_{MP}^2}$$

Details of the derivation of this formula are given in [Clause B.3](#).

The formula is based on the following assumptions:

- measurements of the manufactured characteristic are normally distributed;
- the production process is normally distributed and in a state of statistical control;
- the calculation of the C_p index is based on 99,73 % reference value estimated by 6 standard deviations;
- the distribution of the observed, empirical standard deviation is:

$$s_{\text{obs}}^2 \sim (\sigma_p^2 + \sigma_{MP}^2) \cdot \chi^2(\nu)$$

where

σ_p is the standard deviation of the production process;

σ_{MP} is the standard deviation of the measurement process;

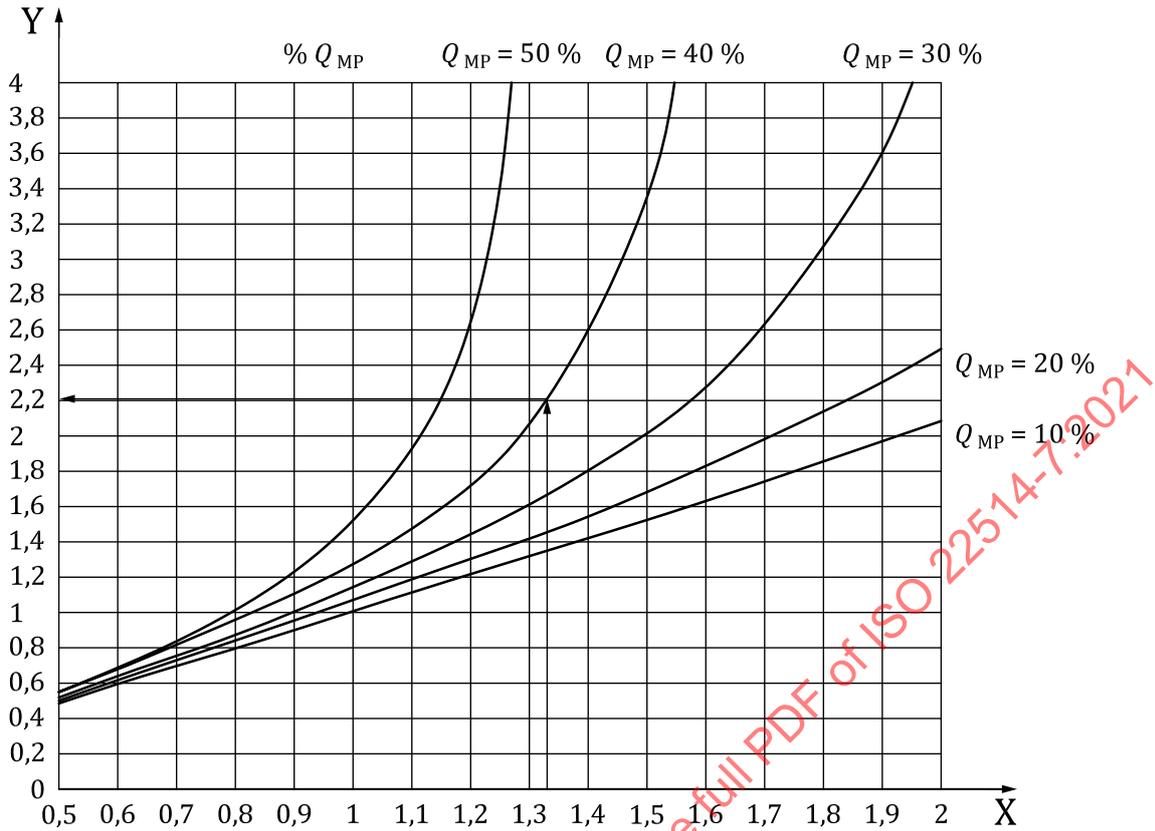
$\chi^2(\nu)$ is the chi-square distribution with ν degrees of freedom.

The area of uncertainty regarding the specification limits is symmetrical.

The coverage factor used to calculate the combined uncertainty is 2.

EXAMPLE The formula above, [Figure 7](#) and [Tables 11](#) and [12](#) below show that a real capability index of 2,21 from an actual production process when the measurement capability figure $Q_{MP} = 40$ % results in an observed capability index of 1,33.

NOTE This example is about theoretical capability indices. The estimated capabilities are random variables subject to error and an estimated observed capability index in this situation varies around 1,33 with a variation that depends on the sample size.



Key
 X observed C_p -value
 Y real C_p -value

Figure 7 — Capability index of the process alone as a function of the observed capability index for a range of capability fractions of the measurement process

Table 11 — Observed and real indices

Observed C -value	Real C -value for the process with...				
	$Q_{MP} = 10\%$	$Q_{MP} = 20\%$	$Q_{MP} = 30\%$	$Q_{MP} = 40\%$	$Q_{MP} = 50\%$
0,67	0,67	0,68	0,70	0,73	0,77
1,00	1,01	1,05	1,12	1,25	1,51
1,33	1,36	1,45	1,66	2,21	18,82
1,67	1,72	1,93	2,53	Na	Na
2,00	2,10	2,50	4,59	Na	Na

EXAMPLE A capability value $C_p = 1,00$ is calculated based on measurements from a production process and the measurement process has a $Q_{MP} = 30\%$.

$$C_{p\text{real}} = \sqrt{\frac{1}{C_{p\text{obs}}^2} - 2,25 \cdot Q_{MP}^2} = \sqrt{\frac{1}{1^2} - 2,25 \cdot 0,3^2} = 1,1198$$

10.2 Relation between observed process capability and measurement capability

The relation between the process and measurement capability can also be calculated (see [Table 12](#)). The following connection exists between an observed process capability or process performance

$(C_{p\text{ obs}}, P_{p\text{ obs}})$, the actual real process capability or performance $(C_{p\text{ real}}, P_{p\text{ real}})$ and the capability index (C_{MP}) of the measurement process:

$$C_{p\text{ obs}} = C_{p\text{ real}} \frac{1}{\sqrt{1 + (\sigma_{MP} / \sigma_p)^2}}$$

Table 12 — Observed and real indices

Observed C-value	Real C-value for the process when...				
	$C_{MP} = 2$	$C_{MP} = 1,66$	$C_{MP} = 1,33$	$C_{MP} = 1$	$C_{MP} = 0,5$
0,67	0,67	0,67	0,68	0,68	0,73
1,00	1,01	1,02	1,03	1,04	1,25
1,33	1,36	1,37	1,39	1,43	2,21
1,67	1,72	1,74	1,78	1,87	59
2,00	2,09	2,13	2,19	2,33	n/a

11 Ongoing review of the measurement process stability

11.1 Ongoing review of the stability

The short-term as well as the long-term stability shall be taken into account when the capability of the measurement process is calculated. However, a change in bias caused by drift, unintentional damage or new additional uncertainty components, which were not known by the time of calculation of the capability, can change the bias in the measurement process over time. A control chart should be used to determine those possible significant changes in the measurement process.

The following sequence is recommended.

Step 1:

Select an appropriate reference standard or calibrated workpiece with a known value for the test characteristic.

Step 2:

Carry out regular measurement of the reference standard (workpiece).

Step 3:

Plot the measured values on a control chart. Calculate the action limits in accordance with known methods of quality control charting techniques (in accordance with ISO 7870-1).

Step 4:

Check for out-of-control. If no out-of-control signal is detected, it is assumed that the measurement process has not changed significantly. If an out-of-control signal is detected, the measurement process is assumed to have changed and shall be reviewed. With this approach, the measurement process is continuously monitored, and significant changes can be detected.

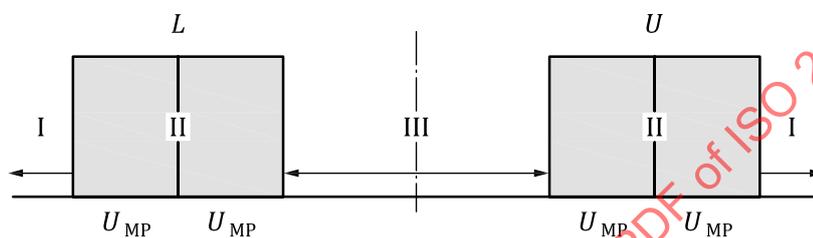
It is important to determine the qualification interval to be taken into account by the calibration of the measuring system.

12 Capability of attribute measurement processes

12.1 General

Because of the nature of attribute measurements, it is only possible to obtain an outcome that is either conforming to the specification or not. To establish the capability of the measurement process, a large number of measurements are required.

A suitable approach for calculating the capability of an attribute measurement process shall take into account that the probability of a particular test result is dependent on the characteristic type. For example, the probability of a correct test result is nearly 100 % of actual measured values that lie outside the uncertainty limits. For information about the specification limits, see [Figure 8](#). On the other hand, the probability is approximately 50 % if the measurement results lie in the middle of the uncertainty range (“Decision by pure chance”). The uncertainty zone should, as a rule of thumb, not exceed 20 % of the specification zone.



Key

- I values outside the specific limits
- II values around the specification limits
- III values inside the specification limits

Figure 8 — Uncertainty zone (marked II)

In principle, the proposed approach makes a distinction between calculation of measurement capability (cross-tab method), without or with reference values (signal detection approach). If reference values are available, a two-step approach is proposed.

In practise, there is no generally accepted test method, but a number of different methods for assessing operators' ability to obtain consistent results. This document specifies one of several methods for the study of test methods without reference parts and one with reference parts.

12.2 Capability calculations without using reference values

When a calculation of measurement capability shall be done without using reference values, only a test of whether or not there are significant differences between operators can be made. However, an assessment of whether or not the test has led to the correct result cannot be taken. This fact shall always be considered if no reference values are present.

The choice of test parts can have a decisive influence on the outcome of this test method, but it cannot be taken into account in this case. At least a proportion (e.g. 40 %) of the test parts should be in the uncertainty zone (zone II in [Figure 8](#)).

The following standard experiment is proposed.

At least 40 different test parts should be tested 3 times by 2 different operators, called A and B. Each of the 120 different measurement results on the 40 parts, which the operator A or operator B has achieved, is assigned to one of the following three classes.

Class 1: all three test results on the same part gave the result “good”.

Class 2: the three test results on the same part gave different results.

Class 3: all three test results on the same part gave the result “bad”.

An example of a test result is summarized in [Table 13](#).

Table 13 — Test result from an attribute measurement process

Frequency n_{ij}		Operator B		
		Class 1 Result "+++"	Class 2 Different results	Class 3 Result "---"
Operator A	Class 1 Result "+++"	7	3	1
	Class 2 Different results	10	4	7
	Class 3 Result "---"	2	1	5

The two operators in [Table 13](#) can now be tested using a Bowker Test of symmetry. If there are no significant differences between operators, the resulting frequencies in [Table 13](#) are sufficiently symmetrical with respect to main diagonal.

The hypothesis $H_0: n_{ij} = n_{ji}$ ($i, j = 1, \dots, 3$ with $i \neq j$) says that the frequencies n_{ij} and n_{ji} which lie symmetrical with respect to the main diagonal are identical.

$$\text{Test statistic } X^2 = \sum_{i>j} \frac{(n_{ij} - n_{ji})^2}{n_{ij} + n_{ji}} = \frac{(10-3)^2}{10+3} + \frac{(2-1)^2}{2+1} + \frac{(1-7)^2}{1+7} = 8,603$$

is compared to the $1-\alpha$ fractile in the χ^2 -distribution with 3 degrees of freedom.

The null hypothesis test states that changes from one category to another are random in nature. The hypothesis on symmetry is rejected on the level if the test value is greater than the $1-\alpha$ fractile in the χ^2 -distribution with 3 degrees of freedom. In this case, the hypothesis is rejected because the calculated value 8,603 is greater than the value 7,815 which is the 95 % fractile of the χ^2 (3) distribution.

In principle, this method is also to be used with more than two operators. In such cases, each operator makes three tests on the measured object and subsequently, all combinations of two combinations of operators should be tested individually.

NOTE In this case, the significance level is changed for the overall statements by these multiple tests.

12.3 Capability calculations using reference values

12.3.1 Calculation of the uncertainty range

This method is based on signal detections and therefore requires workpieces with known reference values. To address the area of risk around the specification limits, about 25 % of the workpieces should be at or close to the lower specification limit and 25 % of the workpieces at the upper specification limit.

The purpose of this method is to determine the uncertainty range, in which an operator is unable to make an unambiguous decision. [Figure 9](#) illustrates the test results of an attribute measurement process obtained from a set of reference values.

Part No. Char. No.	Ref. 1	1			Part Descr. Char. Descr.			MSA Third Edition Attribute Study			
		XA1	XA2	XA3	XB1	XB2	XB3	XC1	XC2	XC3	
25	0,599581	-	-	-	-	-	-	-	-	-	☺
48	0,587893	-	-	-	-	-	-	-	-	-	☺
3	0,576459	-	-	-	-	-	-	-	-	-	☺
5	0,570360	-	-	-	-	-	-	-	-	-	☺
42	0,566575	-	-	-	-	-	-	-	-	-	☺
4	0,566152	-	-	-	-	-	-	-	-	-	☺
30	0,561457	-	-	-	-	-	+	-	-	-	☹
12	0,559918	-	-	-	-	-	-	-	+	-	☹
26	0,547204	-	+	-	-	-	-	-	-	+	☹
22	0,545604	-	-	+	-	+	-	+	+	-	☹
6	0,544951	+	+	-	+	+	-	+	-	-	☹
36	0,543077	+	+	-	+	+	+	+	-	+	☹
13	0,542704	+	+	+	+	+	+	+	+	+	☺
18	0,531939	+	+	+	+	+	+	+	+	+	☺
23	0,529065	+	+	+	+	+	+	+	+	+	☺
29	0,523754	+	+	+	+	+	+	+	+	+	☺
28	0,521642	+	+	+	+	+	+	+	+	+	☺
19	0,520469	+	+	+	+	+	+	+	+	+	☺
17	0,519694	+	+	+	+	+	+	+	+	+	☺
15	0,517377	+	+	+	+	+	+	+	+	+	☺
10	0,515573	+	+	+	+	+	+	+	+	+	☺
24	0,514192	+	+	+	+	+	+	+	+	+	☺
41	0,513779	+	+	+	+	+	+	+	+	+	☺
2	0,509015	+	+	+	+	+	+	+	+	+	☺
32	0,505850	+	+	+	+	+	+	+	+	+	☺
31	0,503091	+	+	+	+	+	+	+	+	+	☺
27	0,502436	+	+	+	+	+	+	+	+	+	☺
8	0,502295	+	+	+	+	+	+	+	+	+	☺
40	0,501132	+	+	+	+	+	+	+	+	+	☺
35	0,496696	+	+	+	+	+	+	+	+	+	☺
46	0,493441	+	+	+	+	+	+	+	+	+	☺
11	0,483803	+	+	+	+	+	+	+	+	+	☺
38	0,488184	+	+	+	+	+	+	+	+	+	☺
33	0,487613	+	+	+	+	+	+	+	+	+	☺
47	0,486379	+	+	+	+	+	+	+	+	+	☺
18	0,484167	+	+	+	+	+	+	+	+	+	☺
49	0,483803	+	+	+	+	+	+	+	+	+	☺
20	0,477236	+	+	+	+	+	+	+	+	+	☺
1	0,476901	+	+	+	+	+	+	+	+	+	☺
44	0,470832	+	+	+	+	+	+	+	+	+	☺
7	0,465454	+	+	+	+	+	+	+	+	+	☹
43	0,462410	+	-	+	+	+	+	+	+	-	☹
14	0,454518	+	+	-	+	+	+	+	-	-	☹
21	0,452310	+	+	-	+	+	+	+	+	-	☹
34	0,449696	-	-	+	-	-	+	-	+	+	☹
50	0,446697	-	-	-	-	-	-	-	-	-	☺
9	0,437817	-	-	-	-	-	-	-	-	-	☺
39	0,427687	-	-	-	-	-	-	-	-	-	☺
45	0,412453	-	-	-	-	-	-	-	-	-	☺
37	0,409238	-	-	-	-	-	-	-	-	-	☺

Figure 9 — Test result of an attribute measurement process (example from AIAG MSA)

12.3.2 Symbols

In Figure 9, the reference measurement values are introduced in the form of a code. A plus sign means that the operator has indicated the result from the test piece as approved. A minus sign means that the operator has indicated the result from the test piece as not approved.

A good smiley means that all three operators have indicated the result from the test piece as approved or rejected in all three tests, and that this assessment is consistent with the reference value.

A bad smiley indicates a case where at least one of the operators has come to a test result which is not consistent with the reference value.

12.3.3 Working steps for determining the uncertainty range

Step 1:

Sort the table according to the measured reference size. In [Figure 9](#), a sorting in descending order is made from the highest reference value descending to the lowest reference value.

Step 2:

Select the last reference value for which all operators have assessed all the results as being unsatisfactory (not approved). This is the transition from symbol “-” to symbol “+”.

0,566 152	-
0,561 457	+

Step 3:

Select the first reference value for which all operators the first time assessed all results being approved. This is the transition from symbol “-” to the symbol “+”.

0,543 077	-
0,542 704	+

Step 4:

Select the last reference value for which all operators last time assessed all the results as being approved. This is the transition from the “+” symbol to the symbol “-”.

0,470 832	+
0,465 454	-

Step 5:

Select the first reference value for which every operator has again first assessed all the results as unsatisfactory (not approved). This is the transition from symbol “X” to the symbol “-”.

0,449 696	+
0,446 697	-

Step 6:

Calculate the d_{UR} interval from the last reference value, for which all operators have assessed the result as unsatisfied (not approved) to the first reference value, for which all operators have the result as approved.

$$d_{UR} = 0,566\ 152 - 0,542\ 704 = 0,023\ 448$$

Step 7:

Calculate the d_{LR} interval from the last reference value, for which all operators have assessed the result as approved to the first reference value, and for which all operators have the result as unsatisfied (not approved).

$$d_{LR} = 0,470\ 832 - 0,446\ 697 = 0,024\ 135$$

Step 8:

Calculate the average “d” of the two intervals:

$$d = (d_{UR} + d_{LR})/2 = (0,023\ 448 + 0,024\ 135)/2 = 0,023\ 791\ 5$$

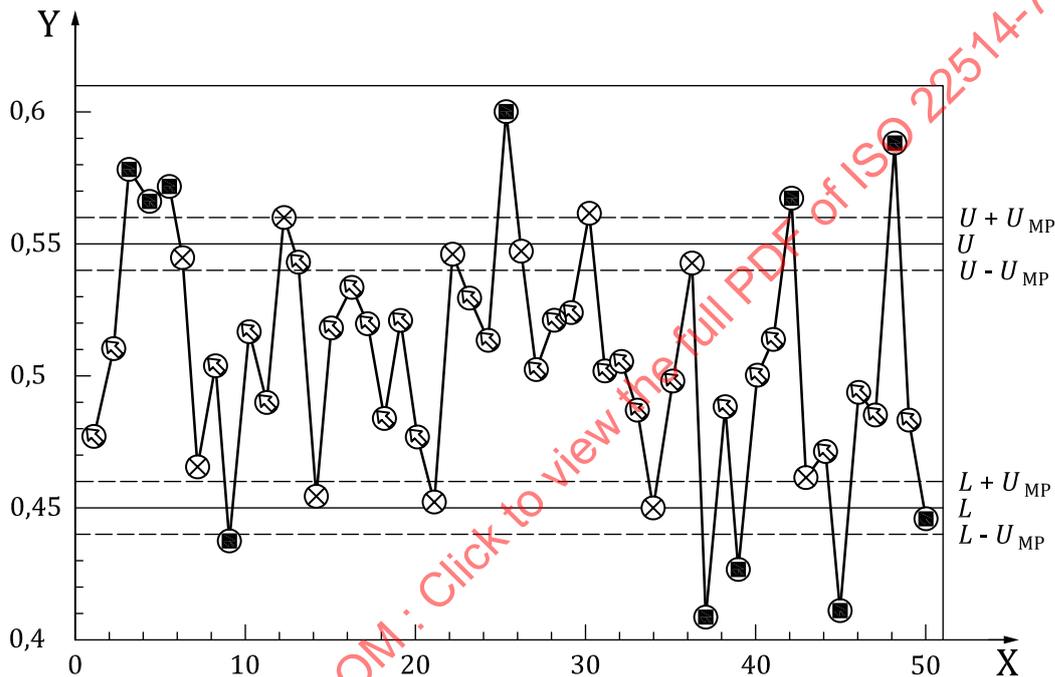
Step 9:

Calculate the uncertainty range:

$$U_{attr} = d/2 = (0,023\ 791\ 5)/2, \text{ and}$$

$$Q_{attr} = 2 \cdot U_{attr}/(U - L) = 2 \cdot [(0,023\ 791\ 5)/2]/0,1 = 0,24 \text{ where } U - L = 0,1 \text{ mm}$$

Thus, $Q_{attr} = 24\ %$.



Key

- X reference number
- Y reference values from the attribute study (mm)

Figure 10 — Value chart

Figure 10 shows another way of representing the measurement capability of all test results, all the reference values and the uncertainty range. Some practitioners prefer this display.

NOTE The effort for this method is considerable as, in this example, in addition to the 50 reference measurements, at least 450 other test measurements can be made and documented.

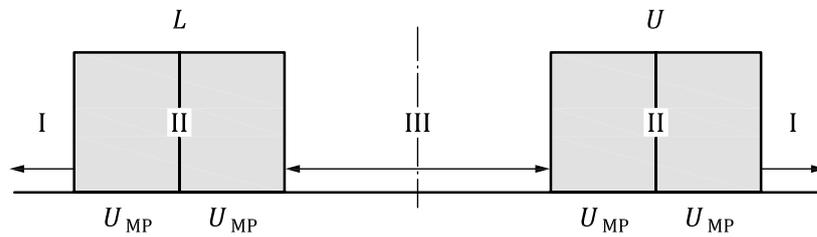
For the selection of workpieces, it shall be presumed that the uncertainty region is covered (see Figure 10).

12.4 Ongoing review

Because of the fact that the measuring system can change, e.g. caused by wear, it is necessary to periodically conduct a review of the system.

For ongoing monitoring of the measurement process, at least one operator should measure at least three workpieces all with defined reference values. The workpieces should be selected in a way that

the reference values are located outside the uncertainty ranges so that a clear result can be expected (all tests are consistent with the reference value; see [Figure 11](#), e.g. a workpiece in zone I (lower), a workpiece in zone III, and a workpiece in zone I (upper)).



Key

- I values outside the specific limits
- II values around the specification limits
- III values inside the specification limits

Figure 11 — Uncertainty range

The test is accepted if all three test results are consistent with the reference value. If this is not the case, the measuring system should not be used until it has been corrected or changed.

The size of the uncertainty range can either be determined experimentally (see [Clause 11](#)), or derived from the actual defined requirements for an appropriate measurement process (Q).

$$U_{MP\ max} = Q_{MP\ max} \cdot (U - L) / 2$$

Take into account that the extended uncertainty is usually given to be the 95 % level. In this test, it is not calculated.

Use binomial distribution to calculate the confidence interval.

Annex A (informative)

Examples

A.1 Example of linearity study with at least three standards

A.1.1 General

The raw data of this example has been taken from ISO 11095. The example describes an experiment carried out on an imaging system (an optical microscope with a measuring device). The data are measured values and true values of intervals in the range of 0,5 μm to 12 μm . It is assumed according to the calibration certificate that the calibration uncertainty u_{CAL} is 0,005 μm .

Table A.1 — Values from repeated measurements on reference materials

Conventional true values x_n of the 10 reference materials	Values y_{nj} from $K = 4$ repeatability measurements on $N = 10$ reference materials			
	y_{n1}	y_{n2}	y_{n3}	y_{n4}
6,19	6,31	6,27	6,31	6,28
9,17	9,27	9,21	9,34	9,23
1,99	2,21	2,19	2,22	2,20
7,77	8,00	7,81	7,95	7,84
4,00	4,27	4,15	4,15	4,15
10,77	10,93	10,73	10,92	10,89
4,78	4,95	4,87	5,00	5,00
2,99	3,24	3,17	3,21	3,21
6,98	7,14	7,07	7,18	7,20
9,98	10,23	10,02	10,07	10,17

Data in [Table A.1](#) are plotted in [Figure A.1](#).

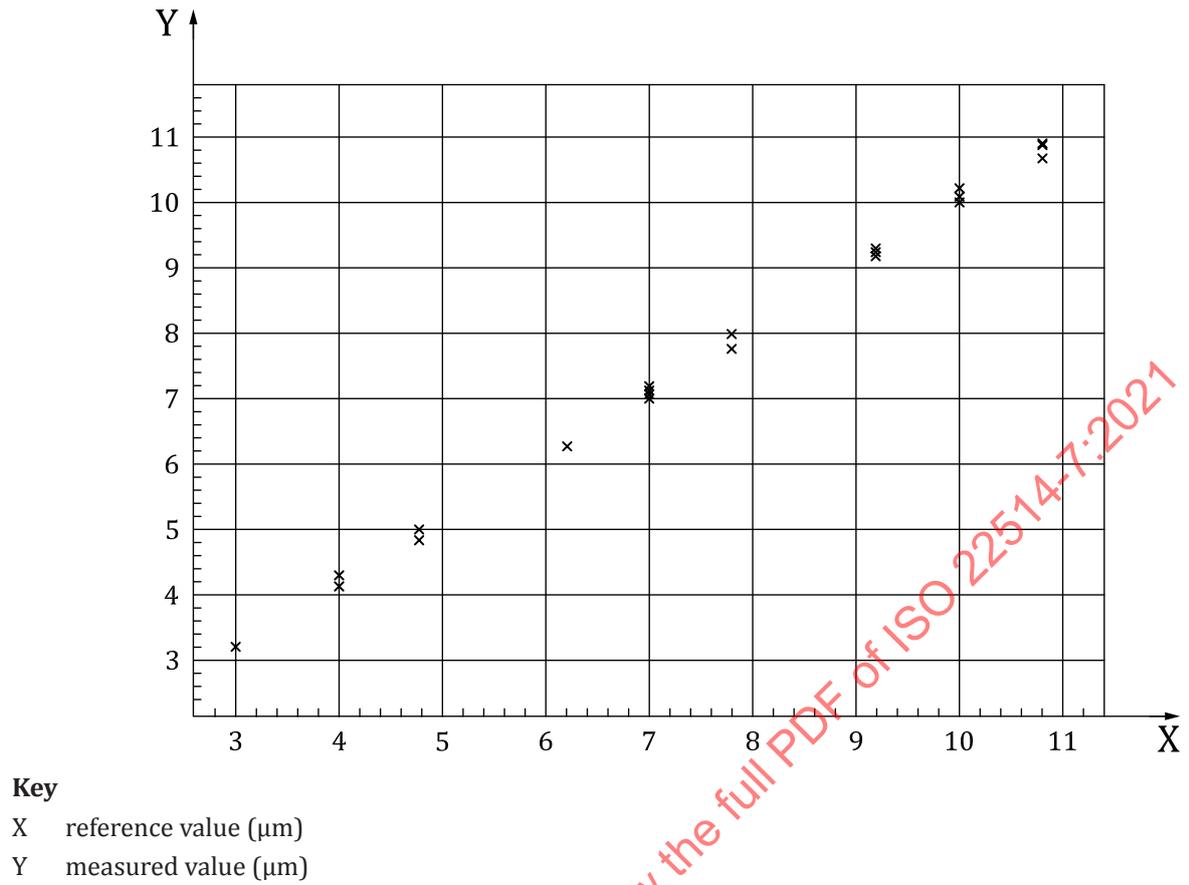


Figure A.1 — Plot of measured and true values

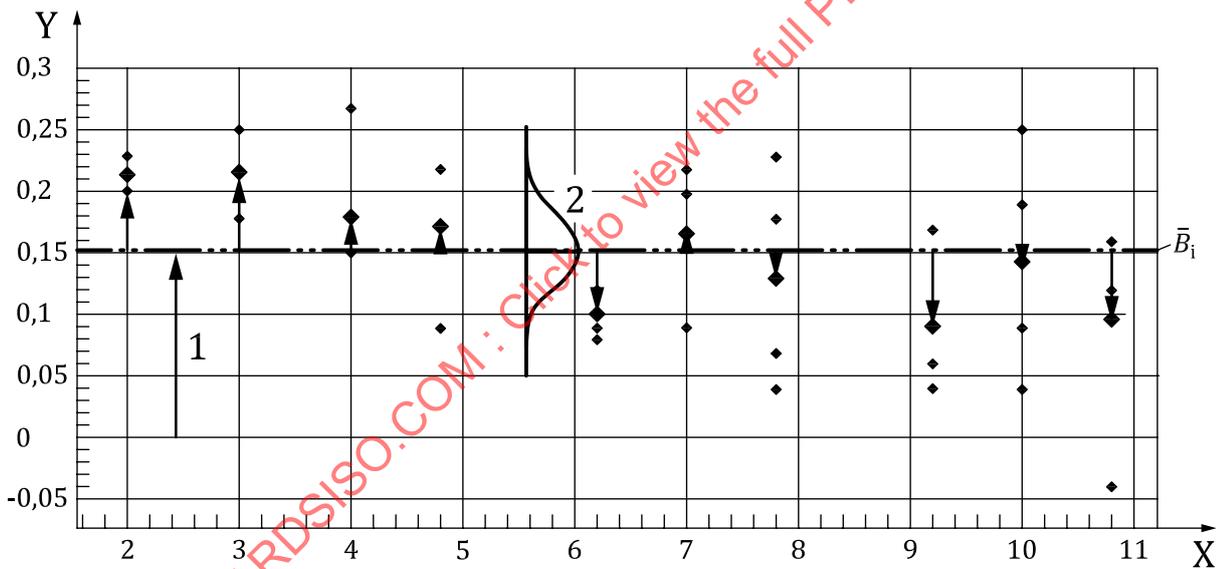
A.1.2 Calculation of means and residuals

See [Table A.2](#) for the calculations.

Table A.2 — Calculation of residuals

True values x_n of the 10 reference materials	Mean values \bar{y}_n	B_{in}	Residuals			
			e_{n1}	e_{n2}	e_{n3}	e_{n4}
6,19	6,292 5	0,102 5	0,017 5	-0,022 5	0,017 5	-0,012 5
9,17	9,262 5	0,092 5	0,007 5	-0,052 5	0,077 5	-0,032 5
1,99	2,205 0	0,215 0	0,005 0	-0,015 0	0,015 0	-0,005 0
7,77	7,900 0	0,130 0	0,100 0	-0,090 0	0,050 0	-0,060 0
4,00	4,180 0	0,180 0	0,090 0	-0,030 0	-0,030 0	-0,030 0
10,77	10,867 5	0,097 5	0,062 5	-0,137 5	0,052 5	0,022 5
4,78	4,955 0	0,175 0	-0,005 0	-0,085 0	0,045 0	0,045 0
2,99	3,207 5	0,217 5	0,032 5	-0,037 5	0,002 5	0,002 5
6,98	7,147 5	0,167 5	-0,007 5	-0,077 5	0,032 5	0,052 5
9,98	10,122 5	0,142 5	0,107 5	-0,102 5	-0,052 5	0,047 5

Data in [Table A.2](#) are plotted in [Figure A.2](#).



- Key**
- X value of reference part
 - Y bias
 - 1 mean bias over all reference parts
 - 2 uncertainty from linearity
 - ◆ individual error
 - ◆ mean bias of the reference part

Figure A.2 — Plot of deviations and conventional true values

A.1.3 ANOVA table:

Given values:

$N = 10$ Number of standards (Factor A)