

---

---

**Statistical methods in process  
management — Capability and  
performance —**

**Part 3:  
Machine performance studies for  
measured data on discrete parts**

*Méthodes statistiques dans la gestion de processus — Aptitude et  
performance —*

*Partie 3: Études de performance de machines pour des données  
mesurées sur des parties discrètes*

STANDARDSISO.COM : Click to view the full PDF of ISO 22514-3:2020



STANDARDSISO.COM : Click to view the full PDF of ISO 22514-3:2020



**COPYRIGHT PROTECTED DOCUMENT**

© ISO 2020

All rights reserved. Unless otherwise specified, or required in the context of its implementation, no part of this publication may be reproduced or utilized otherwise in any form or by any means, electronic or mechanical, including photocopying, or posting on the internet or an intranet, without prior written permission. Permission can be requested from either ISO at the address below or ISO's member body in the country of the requester.

ISO copyright office  
CP 401 • Ch. de Blandonnet 8  
CH-1214 Vernier, Geneva  
Phone: +41 22 749 01 11  
Email: [copyright@iso.org](mailto:copyright@iso.org)  
Website: [www.iso.org](http://www.iso.org)

Published in Switzerland

# Contents

Page

|   |           |
|---|-----------|
| <b>Foreword</b> .....   | <b>iv</b> |
| <b>Introduction</b> .....   | <b>v</b>  |
| <b>1 Scope</b> .....  | <b>1</b>  |
| <b>2 Normative references</b> .....   | <b>1</b>  |
| <b>3 Terms and definitions</b> .....  | <b>1</b>  |
| <b>4 Symbols</b> .....  | <b>1</b>  |
| <b>5 Pre-conditions for application</b> .....                               | <b>2</b>  |
| 5.1 General.....  | 2         |
| 5.2 Number of parts to be used in the study.....                            | 2         |
| 5.3 Materials to be used.....   | 3         |
| 5.4 Measurement system.....   | 3         |
| 5.5 Running the study.....  | 3         |
| 5.6 Special circumstances.....  | 3         |
| <b>6 Data collection</b> .....  | <b>3</b>  |
| 6.1 Traceability of data.....   | 3         |
| 6.2 Retention of specimens.....   | 4         |
| 6.3 Data recording.....   | 4         |
| <b>7 Analysis</b> .....   | <b>4</b>  |
| 7.1 General.....  | 4         |
| 7.2 Run chart.....  | 4         |
| 7.2.1 Purpose.....  | 4         |
| 7.2.2 Review the plot.....  | 4         |
| 7.3 Analyse the pattern of the data.....                                    | 5         |
| 7.3.1 Software approach.....  | 5         |
| 7.3.2 Check the pattern of the data.....                                    | 6         |
| 7.3.3 Summarize the data.....   | 6         |
| 7.3.4 Manual approach.....  | 6         |
| 7.4 Produce a probability plot.....   | 9         |
| 7.4.1 General.....  | 9         |
| 7.4.2 Analyse the data.....   | 9         |
| 7.5 Special cases.....  | 10        |
| 7.5.1 Data indicate a skewed distribution.....                              | 10        |
| 7.5.2 Bimodal data.....   | 11        |
| 7.5.3 Truncated data.....   | 12        |
| 7.5.4 Censored data.....  | 13        |
| 7.6 Calculation of machine performance indices.....                         | 13        |
| 7.6.1 General procedure.....  | 13        |
| 7.6.2 Data following a normal distribution.....                             | 14        |
| <b>8 Reporting</b> .....  | <b>14</b> |
| 8.1 Test report.....  | 14        |
| 8.2 Confidence intervals.....   | 15        |
| 8.2.1 General.....  | 15        |
| 8.2.2 Indices calculated with the data following a normal distribution..... | 15        |
| 8.2.3 Indices calculated with data following a non-normal distribution..... | 16        |
| <b>9 Actions following a machine performance study</b> .....                | <b>16</b> |
| <b>Annex A (informative) Tables and worksheets</b> .....                    | <b>17</b> |
| <b>Bibliography</b> .....   | <b>19</b> |

## Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see [www.iso.org/directives](http://www.iso.org/directives)).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see [www.iso.org/patents](http://www.iso.org/patents)).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see [www.iso.org/iso/foreword.html](http://www.iso.org/iso/foreword.html).

This document was prepared by Technical Committee ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in product and process management*.

This second edition cancels and replaces the first edition (ISO 22514-3:2008), which has been technically revised.

The main changes compared to the previous edition are as follows:

- updated and improved figures and computer outputs.

A list of all parts in the ISO 22514 series can be found on the ISO website.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at [www.iso.org/members.html](http://www.iso.org/members.html).

## Introduction

This document has been prepared to provide guidance in circumstances where a study is necessary to determine if the output from a machine, for example, is acceptable according to some criteria. Such circumstances are common in engineering when the purpose for the study is part of an acceptance trial. These studies can also be used when diagnosis is required concerning a machine's current level of performance or as part of a problem-solving effort. The method is very versatile and has been applied to many situations.

Machine performance studies of this type provide information about the behaviour of a machine under very restricted conditions such as limiting, as far as possible, external sources of variation that are commonplace within a process, e.g. multi-factor and multi-level situations. The data gathered in a study might come from items made consecutively, although this may be altered according to the study requirements. The data are assumed to have been, generally, gathered manually.

The study procedure and reporting are of interest to engineers, supervisors and management wishing to establish whether a machine should be purchased or put in for maintenance, to assist in problem-solving or to understand the level of variation due to the machine itself.

STANDARDSISO.COM : Click to view the full PDF of ISO 22514-3:2020

[STANDARDSISO.COM](https://standardsiso.com) : Click to view the full PDF of ISO 22514-3:2020

# Statistical methods in process management — Capability and performance —

## Part 3: Machine performance studies for measured data on discrete parts

### 1 Scope

This document describes the steps for conducting short-term performance studies that are typically performed on machines (including devices, appliances, apparatuses) where parts produced consecutively under repeatability conditions are considered. The number of observations to be analysed vary according to the patterns the data produce, or if the runs (the rate at which items are produced) on the machine are low in quantity. The methods are not considered suitable where the sample size produced is less than 30 observations. Methods for handling the data and carrying out the calculations are described. In addition, machine performance indices and the actions required at the conclusion of a machine performance study are described.

This document is not applicable when tool wear patterns are expected to be present during the duration of the study, nor if autocorrelation between observations is present. The situation where a machine has captured the data, sometimes thousands of data points collected in a minute, is not considered suitable for the application of this document.

### 2 Normative references

There are no normative references in this document.

### 3 Terms and definitions

No terms and definitions are listed in this document.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <http://www.electropedia.org/>

### 4 Symbols

|            |                                   |
|------------|-----------------------------------|
| $P$        | probability                       |
| $P_m$      | machine performance index         |
| $P_{mk_L}$ | lower machine performance index   |
| $P_{mk_U}$ | upper machine performance index   |
| $P_{mk}$   | minimum machine performance index |
| $f$        | absolute frequency                |

|                   |   |
|-------------------|---|
| $\Sigma f$        | cumulative absolute frequency   |
| $\Sigma f\%$      | cumulative relative frequency in percent                                    |
| $i$               | control variable, subscript used to identify the values of a variable       |
| $L$               | lower specification limit   |
| $n$               | sample size   |
| $X_{\alpha\%}$    | $\alpha\%$ distribution fractile, percentile                                |
| $X_i$             | $i^{\text{th}}$ value in a sample   |
| $\sigma$          | standard deviation, population  |
| $S$               | standard deviation, sample statistic  |
| $U$               | upper specification limit   |
| $z_{\alpha}$      | fractile of the standardized normal distribution from $-\infty$ to $\alpha$ |
| $\mu$             | population mean value in relation to the machine location                   |
| $\bar{X}$         | arithmetic mean value, sample   |
| $\chi_{\alpha}^2$ | fractile of the chi-squared distribution                                    |

## 5 Pre-conditions for application

### 5.1 General

The pre-conditions given in 5.2 and 5.6 are the minimum and may be exceeded if needed. In this type of study, it is important to maintain constant all factors, other than the machine, which can influence the results, if the study is to properly represent the machine itself, e.g. the same operator, same batch of material, etc.

### 5.2 Number of parts to be used in the study

The number specified is usually 100. However, if the pattern of variation is expected to form a non-normal distribution, the number of parts should be at least 100. The methods given within this document may also be used when conducting audits of a process, in which case the number of measurements taken might be less than the above number, e.g. 50.

NOTE 1 This is to ensure that a reasonably narrow confidence interval can be calculated for the machine performance indices when a normal distribution has been used. The interval is approximately  $\pm 12\%$  of the estimated index with a confidence of 90 % for samples of 100.

Some machines have very slow cycle times and a 'run' cannot produce 100 parts. In such circumstances, it is necessary to proceed with available data. The minimum number that this document recommends with the methods described herein is 30.

NOTE 2 Special techniques beyond the scope of this document exist for smaller sample sizes.

By contrast, for a machine that produces parts at a very high rate, e.g. a rivet-making machine, the sampling strategy can require alteration since 100 parts can be produced in a few seconds. In circumstances such as these, several studies can be required each allowing a different sampling approach to examine the machine's behaviour.

### 5.3 Materials to be used

Ensure all input materials to be used in the study have been checked, conform to specifications and belong to the same batches. It is not advised that a study be conducted with materials that are outside specification since this could lead to unrepresentative results.

Care should be exercised not to introduce any other sources of variation other than those to be studied. A typical example is where a machine run has to change to another batch of a particular material within a single process batch, and batch material variation is not included in the study. In this instance, only data taken while the first batch of that particular material was in use should be used in the analysis.

### 5.4 Measurement system

Ensure the measurement system used during the study has adequate properties and is calibrated, and the measurement system variation has been quantified and minimized. Special studies on the measurement system should be undertaken to establish the amount of variation present due to measuring. The measurement system should ideally have a combined standard uncertainty  $u_{MS}$  of less than 10 % of the standard deviation of the characteristic that the machine study is to investigate, as determined through a properly conducted measurement systems analysis. This analysis should address the issues of bias, calibration, linearity and discrimination. The resolution shall be lower than 1/20 of the specification interval.

It is appropriate to calculate the expanded uncertainty  $U_{MP}$  of the measurement process and to express the result as a percentage of a given tolerance. If the expanded uncertainty  $U_{MP}$  does not exceed 15 % of the tolerance, it may be regarded as acceptable, dependent upon application. If it exceeds 15 %, the measurement process should be regarded as inappropriate. Should a study be performed using a measurement process with an uncertainty worse than these recommendations, some wrong conclusions can be drawn from the study. Refer to ISO 22514-7 for more information about the calculation of the measurement system and measurement process capability. Users who prefer doing measurement systems analysis and gauge repeatability and reproducibility can refer to ISO/TR 12888 for more information.

### 5.5 Running the study

Ensure an uninterrupted run takes place, under normal operating conditions. This includes any warm-up time for the machine necessary to bring it up to its usual operating condition and with the machine set at nominal for the characteristic to be studied. If the machine is stopped during the study for whatever reason, either re-run the study or analyse the data collected, as long as sufficient data have been collected and as long as the repeatability conditions have not been violated. Under no circumstance shall less than 30 consecutive results be used, to conclude the acceptance of the machine performance.

### 5.6 Special circumstances

In a multiple fixture set-up, multiple-cavity or multi-stream situation, each station, fixture, cavity or stream should be treated as a separate machine for machine performance purposes since those streams can violate the repeatability conditions.

In the case of a multiple-cavity tool, some extra studies may be performed to examine the between-cavity and within-cavity variation, see ISO 22514-8.

## 6 Data collection

### 6.1 Traceability of data

It is important for all data to be traceable so that unexpected values can be investigated. The collection sequence should be preserved so that a time series can be plotted of the data that might indicate unexpected variations. Such occurrences should be explained, and a decision taken about the

admissibility of such data. A 'log-book' would be suitable for recording all machine settings, including any prior work on the machine, e.g. maintenance, and for recording all events during the study, such as adjustments.

## 6.2 Retention of specimens

Unless the tests performed are destructive in their nature, all specimens should be retained so that all necessary examinations can be made. They should only be disposed of once the study is complete and all conclusions determined.

## 6.3 Data recording

Data should be clearly recorded either electronically or on the appropriate analysis sheet in numerical form to the appropriate number of significant digits, often one significant digit more than that of the tolerance. This should be determined prior to the measuring process and is dependent on the resolution of the measuring instrument.

# 7 Analysis

## 7.1 General

The analysis of the data generated in the study is often performed using computer programs, or by manual means, examples of which are given within this clause.

## 7.2 Run chart

### 7.2.1 Purpose

When conducting a machine study, it is important to understand whether the data collected form a single and stable pattern or not. There are occasions when the conditions within the machine under study lead to a drift in its settings that influence the pattern of data produced. There might be occasions when an unauthorized adjustment has been made to the machine, or data have been mixed in some way. Such an event should stop the study and a new study should be begun. A run chart is helpful to identify such circumstances. The pattern on the run chart in [Figure 1](#) (see also [Table 1](#)) might have been caused by a slight trend within the first 25 items or something might have gone wrong with the machine itself or it is being used wrongly.

If such a systematic influence had been proven, it would have been necessary to take special measures according to the circumstances. These might range between repeating the whole study to analysing the data in its separate parts or eliminating certain results.

ISO 7870-1 contains guidance about the application of control charts and their associated statistical tests that should be applied to plots such as that shown in [Figure 1](#) to assist with the interpretation of the plots.

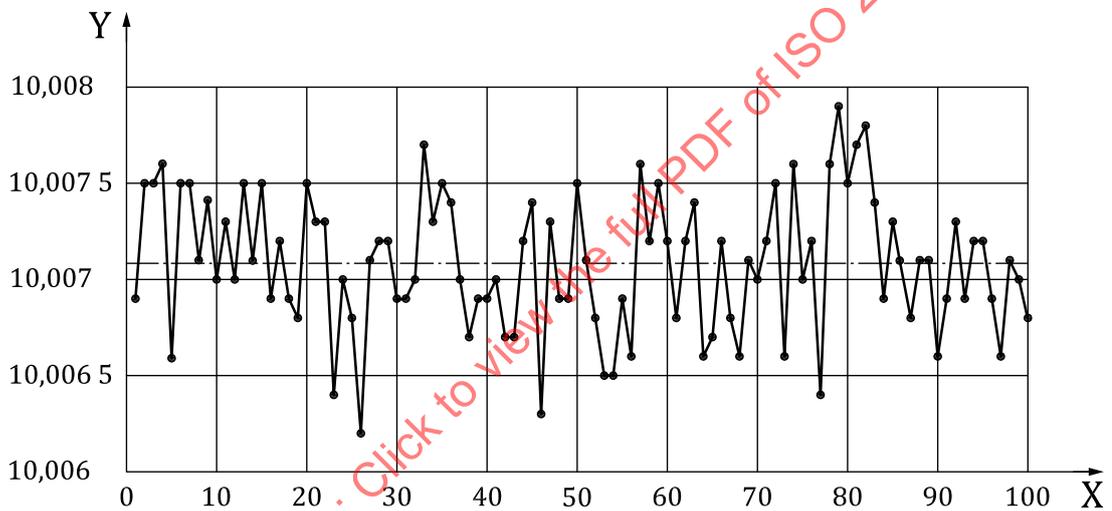
### 7.2.2 Review the plot

Inspect the plot for evidence of instability. This can appear as a step change in the data. Other patterns might appear such as a drift. It is possible to use control limits and control chart rules to assess, easily, for any other assignable causes in the data. The data might be put into an individual and moving range chart to check for potential outliers in the data. (See ISO 7870-2 for further information about such limits and rules.)

There exists a number of software products that can replace the manual methods. These have become popular because they produce the graphs mentioned above quickly and easily.

**Table 1 — Example 1 — Example of observed values**

| Sample No.     | 1 to 10  | 11 to 20 | 21 to 30 | 31 to 40 | 41 to 50 | 51 to 60 | 61 to 70 | 71 to 80 | 81 to 90 | 91 to 100 |
|----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----------|
| Diameter in mm | 10,006 9 | 10,007 3 | 10,007 3 | 10,006 9 | 10,007 0 | 10,007 1 | 10,006 8 | 10,007 2 | 10,007 7 | 10,006 9  |
|                | 10,007 5 | 10,007 0 | 10,007 3 | 10,007 0 | 10,006 7 | 10,006 8 | 10,007 2 | 10,007 5 | 10,007 8 | 10,007 3  |
|                | 10,007 5 | 10,007 5 | 10,006 4 | 10,007 7 | 10,006 7 | 10,006 5 | 10,007 4 | 10,006 6 | 10,007 4 | 10,006 9  |
|                | 10,007 6 | 10,007 1 | 10,007 0 | 10,007 3 | 10,007 2 | 10,006 5 | 10,006 6 | 10,007 6 | 10,006 9 | 10,007 2  |
|                | 10,006 6 | 10,007 5 | 10,006 8 | 10,007 5 | 10,007 4 | 10,006 9 | 10,006 7 | 10,007 0 | 10,007 3 | 10,007 2  |
|                | 10,007 5 | 10,006 9 | 10,006 2 | 10,007 4 | 10,006 3 | 10,006 6 | 10,007 2 | 10,007 2 | 10,007 1 | 10,006 9  |
|                | 10,007 5 | 10,007 2 | 10,007 1 | 10,007 0 | 10,007 3 | 10,007 6 | 10,006 8 | 10,006 4 | 10,006 8 | 10,006 6  |
|                | 10,007 1 | 10,006 9 | 10,007 2 | 10,006 7 | 10,006 9 | 10,007 2 | 10,006 6 | 10,007 6 | 10,007 1 | 10,007 1  |
|                | 10,007 4 | 10,006 8 | 10,007 2 | 10,006 9 | 10,006 9 | 10,007 5 | 10,007 1 | 10,007 9 | 10,007 1 | 10,007 0  |
|                | 10,007 0 | 10,007 5 | 10,006 9 | 10,006 9 | 10,007 5 | 10,007 2 | 10,007 0 | 10,007 5 | 10,006 6 | 10,006 8  |



**Key**

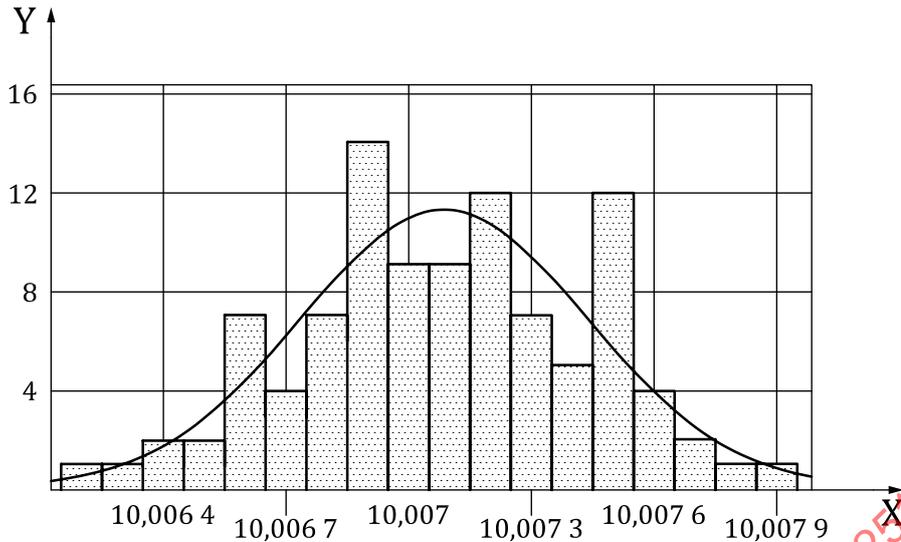
- X observation number (*i*)
- Y diameter in mm

**Figure 1 — Example 1 — Run chart**

**7.3 Analyse the pattern of the data**

**7.3.1 Software approach**

The data should be entered into a software tool and a histogram produced of the data. There exist a number of suitable software products that carry out such analysis. [Figure 2](#) shows the histogram of the data from [Figure 1](#).



**Key**  
 X diameter in mm  
 Y absolute frequency (*f*)

**Figure 2 — Example 1 — Histogram for normally distributed data**

**7.3.2 Check the pattern of the data**

Study the pattern of the data to see if it conforms to a known distribution. Investigate the cause if the data appear to form a quite different pattern. If the data do not form a normal distribution, it can become necessary to employ a different distribution model. An analysis carried out on non-normal data using the normal distribution can produce inaccurate results. Non-normality can occur from circumstances where the data are limited in some way, such as the results of measurements of stress or of concentricity. There might be some anticipation of non-normal data if geometrical tolerances have been specified for a dimension or characteristic, for example. Consult the Bibliography for assistance in determining if the data follow a known distribution model (e.g. ISO 5479) as well as in using other statistical procedures beyond the scope of this document.

Special cases, such as skewed distributions and bimodal data, are discussed in 7.5.

If similar studies have been conducted prior to the current one, there can be a certain expectation of what the distribution might be. Engineering knowledge might also suggest what the pattern ought to be and this can be an important reference should the pattern appear unusual. It can be that something has happened to induce a non-random pattern and an investigation should be conducted.

Misleading results can occur if the computer program used does not check for normality.

**7.3.3 Summarize the data**

Report the sample mean ( $\bar{X}$ ) and the sample standard deviation (*S*). For the mean value, this is usually one decimal place more than the resolution of the raw data, and three more digits for the standard deviation. If the distribution is non-normal, report the sample statistics corresponding to the relevant parameters for the assumed distribution.

**7.3.4 Manual approach**

A simple manner to begin analysing the shape of the frequency distribution is to construct a tally chart.

The data are arranged into 'classes'. If the number of classes is not pre-determined by resolution of the measurement device, the number of classes should be between 5 and 20.

To find the 'class width', the range of the data shall be determined and divided by the number of classes. A common recommendation is to use a number of classes of about  $\sqrt{n}$ . The result shall be rounded to the next convenient integer multiple of the resolution.

As the lower limit of the first class, the minimum data entry minus 0,5 resolutions may be used — or a convenient smaller natural rounding boundary. To get the upper limit of the class and the lower limit of the next class, the class width is added. Continue until the last class is reached. To make sure classes don't overlap, the lower class limits are exclusive ( $>$ ) and the upper ones inclusive ( $\leq$ ).

For scaling purposes, the 'midpoint' or 'class mark' of each class can be calculated as (lower class limit + upper class limit)/2.

The convention of counting the data into groups of five is often used, and an example of this can be seen in [Figure 3](#). In this example, the data have been tallied using a class width of 5  $\mu\text{m}$ . As the tally chart has a bell shape, the values can be considered normally distributed.

Additional information on the form of distribution can be obtained by transferring the data to a probability plot.

The 'cumulative absolute frequencies'  $\Sigma f$  are the running totals of the 'absolute frequencies'  $f$  through all classes of the frequency distribution. The 'cumulative relative frequency'  $\Sigma f\%$  of each class is the proportion of the data that falls below the upper class limit in percent. It can be calculated for a data set of size  $n$  by  $\Sigma f\% = \Sigma f/n \cdot 100$ . The cumulative relative frequencies  $\Sigma f\%$  are plotted in the probability plot above the upper class limits.

If a straight line of best fit can be laid through the set of points in the probability plot without constraint, the values may be considered normally distributed. In this case, the sample mean ( $\bar{X}$ ) and the sample standard deviation ( $S$ ) may be used to determine the performance indices according to [7.6.2](#).

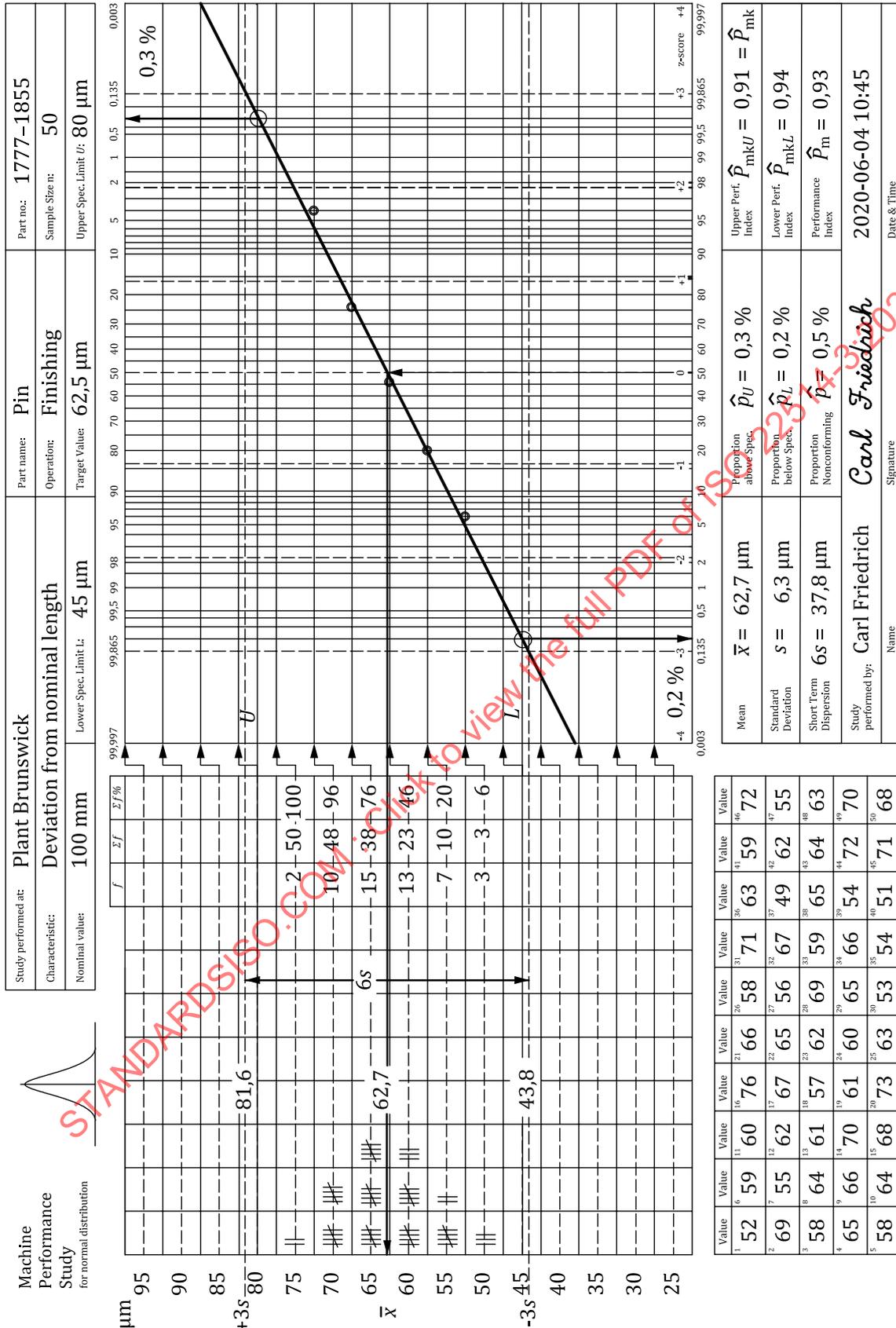
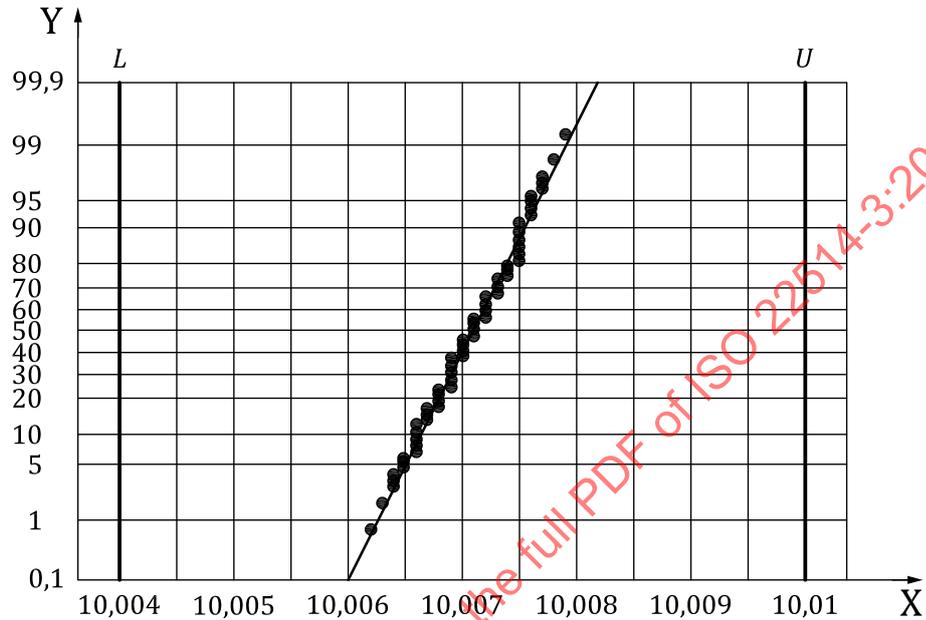


Figure 3 — Example 2 — Worksheet for normally distributed data

## 7.4 Produce a probability plot

### 7.4.1 General

A probability plot should be produced from the data (see [Table 1](#)). This may be achieved by using either a software tool or by using the manual method described in [7.3.4](#). An example of the output of one software package can be seen in [Figure 4](#).



#### Key

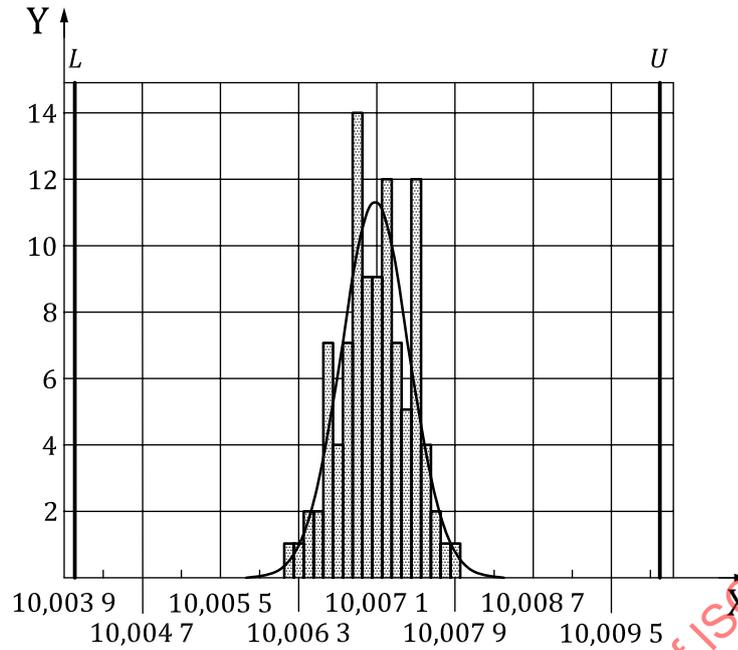
X diameter in mm

Y probability ( $P$ ) in percent

**Figure 4 — Example 1 — Probability plot for normally distributed data**

### 7.4.2 Analyse the data

[Figure 5](#) shows a typical output from a software package that shows the performance of the machine. The data are those given earlier in [Table 1](#).



**Key**  
 X diameter in mm  
 Y absolute frequency (*f*)

**Figure 5 — Example 1 — Machine performance study for normally distributed data**

**7.5 Special cases**

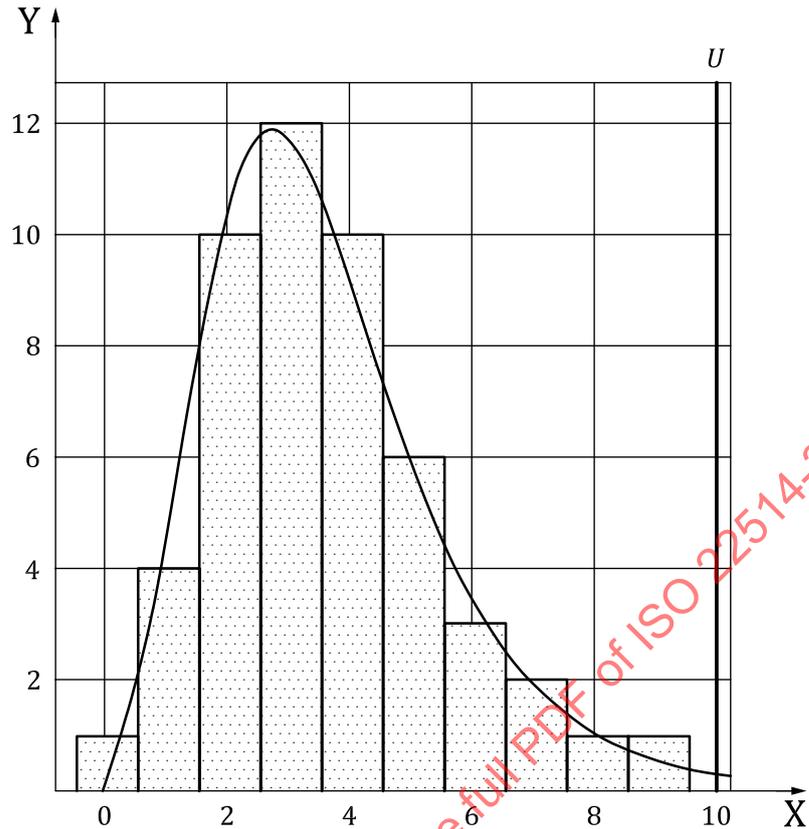
**7.5.1 Data indicate a skewed distribution**

There are occasions when machine studies produce data that indicate a skewed distribution. These often arise when a natural limit exists beyond which data cannot occur. An example of this is the measurement of concentricity where it is impossible to obtain readings less than zero.

To analyse the skewed distribution, it is necessary to select a different probability distribution that is based on the same skewed distribution as that from which the data have come. An example of the method is given in [Table 2](#) and [Figure 6](#) for an extreme value distribution.

**Table 2 — Example 3 — Example of observed values**

| Sample No.                     | 1 to 5 | 6 to 10 | 11 to 15 | 16 to 20 | 21 to 25 | 26 to 30 | 31 to 35 | 36 to 40 | 41 to 45 | 45 to 50 |
|--------------------------------|--------|---------|----------|----------|----------|----------|----------|----------|----------|----------|
| Concentricity in $\mu\text{m}$ | 2      | 2       | 7        | 0        | 1        | 1        | 3        | 5        | 5        | 2        |
|                                | 6      | 2       | 4        | 5        | 6        | 3        | 2        | 3        | 4        | 3        |
|                                | 4      | 3       | 4        | 3        | 3        | 2        | 3        | 2        | 5        | 3        |
|                                | 6      | 4       | 4        | 7        | 5        | 1        | 5        | 2        | 4        | 2        |
|                                | 1      | 3       | 3        | 4        | 2        | 3        | 4        | 4        | 9        | 8        |



#### Key

- X concentricity in  $\mu\text{m}$   
 Y absolute frequency ( $f$ )

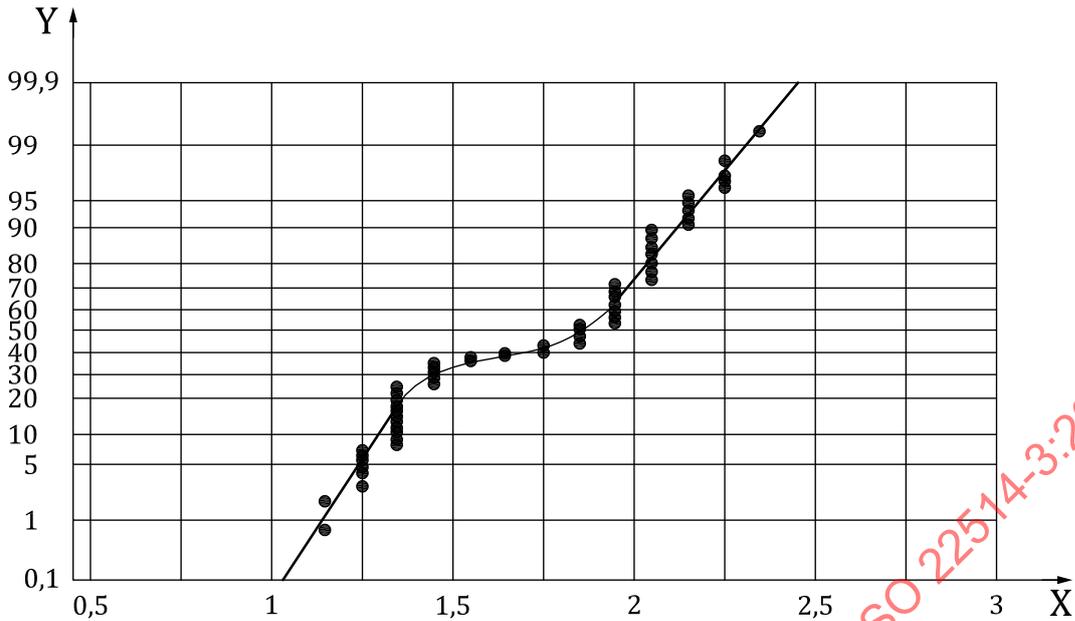
**Figure 6 — Example 3 — Machine performance study using an extreme value distribution**

- For skewed distributions, the percentiles  $\hat{X}_{99,865\%}$ ,  $\hat{X}_{0,135\%}$  and  $\hat{X}_{50\%}$  may be used to determine the performance indices according to 7.6.1.
- When there is only one specification limit given, it is only possible to calculate the  $\hat{P}_{mk}$  index. The index is calculated using the appropriate limit, either an L or a U.

#### 7.5.2 Bimodal data

If the machine is adjusted during the study, the adjustment is likely to affect the results and often produces a distribution with more than one mode. For example, the distribution possesses two modes (sometimes more), each mode representing the different settings of the machine. It can be caused if the machine has multiple tools or cavities and the samples from each are mixed together. It can also be symptomatic of something wrong with the machine.

Bimodal (or multi-modal) data, when plotted onto a normal probability plot, do not produce a straight-line plot. Instead the plot shows a different shape, as shown in Figure 7.



**Key**  
 X distance in mm  
 Y probability (P) in percent

**Figure 7 — Example 4 — Probability plot with bimodal data**

**NOTE** The ‘tied’ values are plotted one above each other as can be seen in [Figure 7](#). This is so that all of the data can be seen and not ‘hidden’ as can sometimes be the case with some methods of representing tied values.

When the data have come from different tools or cavities, if at all possible, the data should be segregated into their respective separate sets and the analysis carried out as described in [7.4.2](#).

[Figure 7](#) gives an example of such a plot of data forming a bimodal distribution. If such a pattern emerges following a study, the assignable cause should be established and, if possible, eliminated. There might be several reasons for a bimodal pattern, one of which could be that an adjustment to the machine was made during the study that resulted in two modes in the data. A simple run chart of the data can illustrate such a shift. [Figure 1](#) is an example of a run chart. Once the assignable cause has been identified, it might be necessary to repeat the study. A run chart is an excellent tool to show problems during data collection. As an alternative to a run chart, a control chart can be used. A control chart has a number of advantages over a run chart as it has the ability to provide statistical signals about changes within the data. However, just as with a run chart, the sequence with which the data were created is required.

- For bi- or multimodal distributions, the percentiles  $\hat{X}_{99,865\%}$ ,  $\hat{X}_{0,135\%}$  and  $\hat{X}_{50\%}$  may be used to determine the performance indices according to [7.6.1](#).

**7.5.3 Truncated data**

Some studies may be made with data that have been truncated in some way. This can be due to some display of the data. Such an example would be if a test or a control instrument measures, for example, resistance (ohms) of an insulator, but only displays values above 5 Ω. Specialist advice should be sought, if this situation exists, to analyse the data.

- To avoid potentially misleading results, the formulae from [7.6](#) for calculating machine performance indicators should not be applied to truncated data.

#### 7.5.4 Censored data

Censored data occur if certain values are ignored. This can be introduced by some sampling strategy, such as taking results only from the first two cavities of a four-cavity tool. Values from cavities three and four are excluded from the analysis. This could be brought about by some machine control device and not necessarily known to the experimenter or analyst. Another example occurs if the parts have been sorted prior to analysis and any oversize or undersize parts have been eliminated from the data set.

The consequence of using censored data can be an erroneous conclusion about the performance of the machine. Such data produce an unexpected plot on the worksheet and the analyst should seek specialist advice.

- To avoid potentially misleading results, the formulae from 7.6 for calculating machine performance indicators should not be applied to censored data.

### 7.6 Calculation of machine performance indices

#### 7.6.1 General procedure

##### 7.6.1.1 General

The actual distribution function shall be identified and the percentile values  $X_{0,135\%}$  and  $X_{99,865\%}$  estimated from it. They can be substituted into formulae to give numerical values for the indices. Alternatively, they can be estimated from the probability paper plot that, although simple and quick, is subject to error.

NOTE Historically, these indices have been recorded as  $C_m$ ,  $C_{mk_L}$  and  $C_{mk_U}$ . Because no attempt is made to establish statistical control, the symbols  $\hat{P}_m$ ,  $\hat{P}_{mk_L}$  and  $\hat{P}_{mk_U}$  are used instead to be compatible with process performance indices.

##### 7.6.1.2 Estimation of $P_m$ index

The following expression is used to estimate the index.

$$\hat{P}_m = \frac{U - L}{\hat{X}_{99,865\%} - \hat{X}_{0,135\%}}$$

##### 7.6.1.3 Estimation of $P_{mk}$ index

The following expressions are used to estimate the indices.

$$\hat{P}_{mk_U} = \frac{U - \hat{X}_{50\%}}{\hat{X}_{99,865\%} - \hat{X}_{50\%}}$$

or

$$\hat{P}_{mk_L} = \frac{\hat{X}_{50\%} - L}{\hat{X}_{50\%} - \hat{X}_{0,135\%}}$$

The  $P_{mk}$  index is given as the minimum of the following:

$$\hat{P}_{mk} = \min \left\{ \hat{P}_{mk_U}, \hat{P}_{mk_L} \right\}$$

NOTE The 'hats' above the indices indicate these are estimated values.

**7.6.2 Data following a normal distribution**

**7.6.2.1 Estimation of  $P_m$  index**

The following expression is used to estimate the index.

$$\hat{P}_m = \frac{U-L}{6S}$$

**7.6.2.2 Estimation of  $P_{mk}$  index**

The following expressions are used to estimate the indices.

$$\hat{P}_{mk_U} = \frac{U-\bar{X}}{3S}$$

or

$$\hat{P}_{mk_L} = \frac{\bar{X}-L}{3S}$$

The  $P_{mk}$  index is given as the minimum of the following:

$$\hat{P}_{mk} = \min \left\{ \hat{P}_{mk_U}, \hat{P}_{mk_L} \right\}$$

NOTE The 'hats' above the indices indicate these are estimated values.

**7.6.2.3 Estimation of proportion out of specification**

Having calculated the  $\hat{P}_{mk}$  values, the proportion out of specification can be read from [Table A.1](#). For example, if  $\hat{P}_{mk_U}$  is 0,85, the estimated proportion is 0,005 4.

**8 Reporting**

**8.1 Test report**

The study report shall contain the following information:

- a) the place where the study was performed, and the type of process the machine is part of;
- b) the persons who performed the study, and who took the measurements;
- c) when the study was performed, including the date, times of start and finish, log of any interruptions;
- d) the machine reference number;
- e) the part's name and reference number;
- f) the characteristic(s) measured;
- g) the specification for the characteristic(s) and which factors were held constant;
- h) the ambient conditions;
- i) the raw data;
- j) non-standard conditions.