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Acoustics — Experimental method for transposition of dynamic forces generated by an active component from a test bench to a receiving structure

Acoustique — Méthode expérimentale de transposition des forces dynamiques générées par un composant actif d'un banc d'essai vers une structure réceptrice

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Foreword

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For an explanation of the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT), see www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 43, *Acoustics*, Subcommittee SC 1, *Noise*.

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The vibroacoustic behaviour of products has become a major challenge not only in terms of user health protection through regulations, but also in terms of sound quality for safety, quality perception, and attractiveness.

At the same time, requirements on products development cycles are more and more stringent, reaching the point where component suppliers and integrators should work independently, without physical prototypes.

To master the transmission of dynamic forces (also called structure-borne noise), one needs to adapt the components to the receiving structure, and hence exchange information prior to manufacturing prototypes. This information will only be valuable for the integrator if it is clearly defined and intrinsic to the component.

This document, issued from a French experimental standard, addresses this issue. It is a user guidance to characterize an active source on a test bench and predict the effects of its integration on a passive structure. The component is characterized on its own, which makes the document complementary to the ISO 20270 that describes the measurement of “in-situ” characteristics (blocked forces), where the component is connected to its receiving structure.

The intrinsic characterization of an active source requires measuring two quantities (expressed as a function of the frequency): the first one characterizing the dynamic aspect, blocked forces, and the second one describing “static” behaviour, such as the impedance or the mobility.

The objective of this document is to help the user predict the component behaviour in a particular assembly. The theoretical background is laid in Annex A. The user is then guided (see 5.2) all along the experimental procedure enabling to reach this objective:

— Static characterization of the component, the test bench and the receiving structure.

— Force measurement: the standard proposes here direct and indirect methods. Indirect methods are generally easier to implement, but they need a particular focus on the measurement quality and matrix inversion.

— Interface integration (connecting device).

— Prediction of the behaviour of the component/receiving structure assembly.

This whole procedure is based on a general formula expressing the dynamic forces in the assembly as a function of blocked forces and static characteristics. Depending on these static characteristics, simplifications are proposed (see 5.6).

Annex B and C guide the user to measure both transfer functions and dynamic forces. It should be noted that, in general, these quantities are expressed in the 3 directions and 3 rotations, but the procedure can be applied on a number of degrees of freedom chosen by the user.

The Annex D informs about data processing. The Annex E contains a test example and the Annex F describes the method using a particular test bench (block sensor).

The data obtained and assessed in this document can be used:

— as part of a specification between suppliers and integrators;

— as input data of numerical vibroacoustic simulation models;

— to drive the modification of the physical structure or the interface in order to improve the vibroacoustic behaviour.

Acoustics — Experimental method for transposition of dynamic forces generated by an active component from a test bench to a receiving structure

1 Scope

This document specifies a method to predict the dynamic forces generated by an active component on a receiving structure from measurement on a test bench.

It sets out the requirements applicable to test benches and setup measurement conditions of dynamic forces: a criterion of validity of transfer functions measurements can be established for example.

The objective is to evaluate noise and vibrations generated by active components mounted on receiving structures, including the possibility to optimise vibration isolators.

It can be applied to different systems connected to a building, such as a compressor or a power generator, or to systems connected to a vehicle body, such as an engine powertrain or an electrical actuator, for example.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at https://www.electropedia.org/

3.1
active component
active substructure which generates dynamic forces

Note 1 to entry: See Figure 1.

3.2
connecting device
mechanical interface with a specific "spring like" matrix structure which allows connecting the *active component* (3.1) to the receiving structure

Note 1 to entry: See Figure 1, Key 2.

Note 2 to entry: Insulators at fixation points are typical "spring like" connecting devices.

Note 3 to entry: A "spring like" connecting device is a structure with no internal degrees of freedom and internal mass, see 3.10.

Note 4 to entry: In the case of a connecting point, active component and receiving structure share the same location.

Note 5 to entry: In the case of seals at contact surfaces, direct connections or any other connection type, the connecting device item 2 cannot be used, and a block diagram with items 1 and 3 and 4 shall be used (see Figure 1).

Note 6 to entry: In case of direct connection, the hypothetical surface between the active component and receiving structure is called interface.

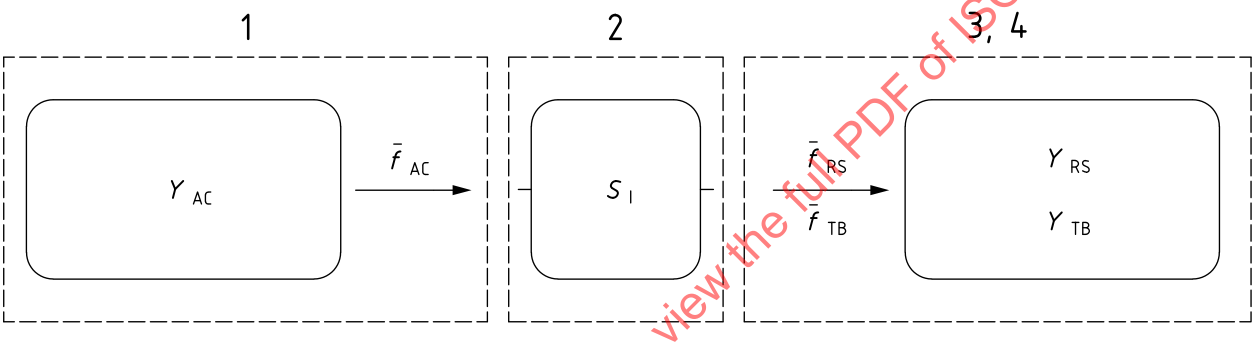
3.3
receiving structure
passive substructure to which the dynamic forces are transmitted

Note 1 to entry: See Figure 1, Key 3 and 4.

Note 2 to entry: The receiving structure can be a test bench or the structure for which the dynamic forces will be predicted.

Note 3 to entry: The "test bench" can be a specific structure designed to test the *active component* (3.1), or any other receiving structure.

Note 4 to entry: active device, connecting device and receiving structures are deformable structures.



Key

1 active component

2 connecting device

3 receiving structure

4 test bench

NOTE An active component (left) connected via a connecting device (centre) transmits dynamic forces to a receiving structure (right) which may vibrate and radiate sound.

Figure 1 — Schematic of the structure assembly

3.4
degree of freedom
n degrees of freedom through which structure-borne sound or vibration is transmitted from the *active component* (3.1) to the *receiving structure* (3.3)

EXAMPLE A connection point can have up to 6 degrees of freedom (dof).

3.5
dynamic force
$\bar{f}_d$
complex force associated to a structure or an interface with n degrees of freedom, arranged into a $n \times 1$ vector at each frequency, according to

$$\bar{f}_d(f) = \begin{bmatrix} \bar{f}_{d,1}(f) \\ \bar{f}_{d,2}(f) \\ \vdots \\ \vdots \\ \bar{f}_{d,n}(f) \end{bmatrix}$$

where $\bar{f}_{d,i}(f)$ is the complex Fourier transform of the i^{th} component of dynamic force at frequency f

Note 1 to entry: Forces $\bar{f}_d$ can be considered as generalised forces, that is, including moments.

Note 2 to entry: Generalised forces units are Newtons for dynamic forces and N·m for dynamic moments

Note 3 to entry: In case pseudo-random signals, statistical tools can help into describing the dynamic forces into set of amplitude and phase force vectors.

Note 4 to entry: In Table 1, the specific dynamic forces $\bar{f}_d$ applied at particular points used in this document are defined.

Table 1 — Dynamic forces symbols

Symbols and abbreviations	Definition
$\bar{f}_{AC}$	Force generated by the active component at the interface of the connecting device in operational conditions.
$\bar{f}_{RS}$ and $\bar{f}_{TB}$	Forces transmitted to the receiving structure: final receiving structure (RS) or test bench (TB) in operational conditions. An additional “pred” or “meas” give indication about how the force is obtained (predicted from formulae, or directly measured): $\bar{f}_{RS\_pred}$ and $\bar{f}_{TB\_meas}$. In the case of no presence of a connecting device, $\bar{f}_{AC} = \bar{f}_{RS}$.

3.6
blocked force
dynamic force (3.5) applied by an *active component* (3.1) transmitted to a rigid *receiving structure* (3.3)

Note 1 to entry: Blocked forces indirect measurement methods are detailed in Annex F and ISO 20270.

3.7
velocity
$\bar{v}_d$

complex vibration velocity associated to a structure or an interface with n degrees of freedom, arranged into a $n \times 1$ vector at each frequency, according to

$$\boldsymbol{v}_{\mathrm{d}}(f)=\begin{bmatrix} \boldsymbol{v}_{\mathrm{d},1}(f) \\ \boldsymbol{v}_{\mathrm{d},2}(f) \\ \vdots \\ \vdots \\ \boldsymbol{v}_{\mathrm{d},n}(f) \end{bmatrix}$$

where $\boldsymbol{v}_{\mathrm{d},i}(f)$ is the complex Fourier transform of the i^{th} velocity component at frequency f.

Note 1 to entry: Velocity units are meters per second (m/s).

Note 2 to entry: Associated complex acceleration $\boldsymbol{a}_{\mathrm{d}}(f)$ can be defined via derivation of the velocity.

Note 3 to entry: Associated complex displacement $\boldsymbol{x}_{\mathrm{d}}(f)$ can be defined via integration of the velocity.

Note 4 to entry: In Table 2, the displacement, velocity and acceleration $\boldsymbol{x}_{\mathrm{d}}, \boldsymbol{v}_{\mathrm{d}}, \boldsymbol{a}_{\mathrm{d}}$ generated at particular points used in this standard are defined.

Table 2 — Velocity, displacement and acceleration definitions

Symbols and abbreviations	Definition
$\boldsymbol{x}_{\mathrm{AC}}, \boldsymbol{v}_{\mathrm{AC}}$ and $\boldsymbol{a}_{\mathrm{AC}}$	dynamic displacement, velocity and acceleration generated by the active component.
$\boldsymbol{x}_{\mathrm{RS}}, \boldsymbol{v}_{\mathrm{RS}}$ and $\boldsymbol{a}_{\mathrm{RS}}$	dynamic displacement, velocity and acceleration on the receiving structure.

3.8
frequency response function
FRF
frequency dependent ratio of the Fourier transform of the response to the Fourier transform of the excitation of a linear system

Note 1 to entry: See ISO 2041.

Note 2 to entry: The FRF denomination and associated unit depends on the two vibration quantities of the ratio (See Table 3).

Note 3 to entry: In this document, any reference to mobility Y or impedance Z is related to:

— the free mobility, $\boldsymbol{Y}_{\mathrm{free}\,ij}$, which is defined as a ratio of a dynamic velocity response in degree of freedom i to an excitation force in degree of freedom j, with all degrees of freedom free, except the one of the excitation forces; or

— the blocked impedance $\boldsymbol{Z}_{\mathrm{blocked}\,ij}$, which is defined as a ratio of the response force in degree of freedom j to the dynamic velocity in degree of freedom i, with all degrees of freedom blocked, except the one of the excitation velocity $\bar{\boldsymbol{v}}_{\mathrm{d},j}$

Table 3 — Denomination of frequency response functions FRF for various vibration quantities (displacement, *x*, velocity, *v* and acceleration, *a*)

	Dynamic Compliance	Free Mobility	Accelerance	Dynamic stiffness	Blocked impedance	Effective Mass
Denomination	$\frac{\boldsymbol{x}_i}{\bar{\boldsymbol{f}}_J}$	$\boldsymbol{Y}_{\mathrm{free}\,ij}=\frac{\boldsymbol{v}_i}{\bar{\boldsymbol{f}}_J}$	$\frac{\boldsymbol{a}_i}{\bar{\boldsymbol{f}}_J}$	$\frac{\bar{\boldsymbol{f}}_J}{\boldsymbol{x}_i}$	$\boldsymbol{Z}_{\mathrm{blocked}\,ij}=\frac{\bar{\boldsymbol{f}}_J}{\boldsymbol{v}_i}$	$\frac{\bar{\boldsymbol{f}}_J}{\boldsymbol{a}_i}$

Table 3 *(continued)*

	Dynamic Compliance	Free Mobility	Accelerance	Dynamic stiffness	Blocked impedance	Effective Mass
Unit	$\frac{\text{m}}{\text{N}}$	$\frac{\text{m}}{\text{N}\cdot\text{s}}$	$\frac{\text{m}}{\text{N}\cdot\text{s}^2}$	$\frac{\text{N}}{\text{m}}$	$\frac{\text{N}\cdot\text{s}}{\text{m}}$	$\frac{\text{N}\cdot\text{s}^2}{\text{m}}$

Note 4 to entry: Thus, in terms of matrix writing, corresponding $\boldsymbol{Y}$ and $\boldsymbol{Z}$ matrices are related:

$$\boldsymbol{Z}_{\text{blocked}} = \boldsymbol{Y}_{\text{free}}^{-1}$$

3.9
transfer matrix
set of FRF between multiple degrees of freedom systems

Note 1 to entry: In this document, the terms $\boldsymbol{Y}_{\text{AC}}$, $\boldsymbol{Y}_{\text{RS}}$ and $\boldsymbol{Y}_{\text{TB}}$ (see Table 4) can be related to free mobility, dynamic compliance or accelerances, depending of the quantities that are commonly used by the reader, with the same boundary conditions as the free mobility.

Table 4 — Main transfer matrices used in the document

Symbols and abbreviations	Definition
$\boldsymbol{Y}_{\text{AC}}$	transfer matrix of the active component
$\boldsymbol{Y}_{\text{RS}}$ and $\boldsymbol{Y}_{\text{TB}}$	transfer matrix of the receiving structure or the test bench

3.10
connecting device transfer matrix
connecting device (case of insulators at fixation points) *transfer matrix* (3.9) (see Table 5) can be obtained via different methods

Note 1 to entry: Such methods are described in ISO 10846.

Table 5 — Different expressions of connecting device transfer matrix versus dynamic stiffness matrix

Formula	Unit	Homogeneous to
$\boldsymbol{S}_{\text{I}} = \omega^2 \boldsymbol{K}_{\text{I}}^{*-1}$	$\text{m}\cdot\text{N}^{-1}\text{s}^{-2}$	accelerance
$\boldsymbol{S}_{\text{I}} = j\omega \boldsymbol{K}_{\text{I}}^{*-1}$	$\text{m}\cdot\text{N}^{-1}\text{s}^{-1}$	mobility
$\boldsymbol{S}_{\text{I}} = \boldsymbol{K}_{\text{I}}^{*-1}$	$\text{m}\cdot\text{N}^{-1}$	compliance (or receptance)
with $\boldsymbol{K}_{\text{I}}^{*}$ homogeneous to a dynamic stiffness complex matrix of the connecting device.		

3.11
operational conditions
set of conditions under which the source operates for the operational test, including speed, load and any other settings or conditions particular to the source which might affect source operation

4 Principle of the method of transposition of the dynamic force

4.1 General matters

This subclause explains how to predict the forces generated by an active component (which comes with its own sources) on a receiving structure from a series of measurements on a test bench and on specific data about the receiving structure.

Predicted or measured dynamic forces are required when

— there is no opportunity to measure directly any dynamic force of any sort (e.g. heavy and high cost electrical machine to be duplicated in a new place),

— there is only the possibility to work on a test bench, because the final receiving structure is still not available,

— the active source is provided by a component supplier to an integrator, and the integrator defines a specification on a bench, with a target to comply, and

— internal forces matrix of a specific product is needed for noise comfort prediction, or for durability purpose.

4.2 General formulae

The first Formula (1) which detail is given in Annex A can be written as follows:

$$\bar{\boldsymbol{f}}_{\mathrm{RS}} = [\boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{AC}} + \boldsymbol{S}_{\mathrm{I}}]^{-1} \cdot [\boldsymbol{Y}_{\mathrm{TB}} + \boldsymbol{Y}_{\mathrm{AC}} + \boldsymbol{S}_{\mathrm{I}}] \cdot \bar{\boldsymbol{f}}_{\mathrm{TB}} \qquad (1)$$

where

$\bar{\boldsymbol{f}}_{\mathrm{RS}}$ is the transmitted force vector to the receiving structure;

$\boldsymbol{Y}_{\mathrm{RS}}$ is the transfer matrix of the receiving structure;

$\boldsymbol{Y}_{\mathrm{AC}}$ is the transfer matrix of the active component;

$\boldsymbol{Y}_{\mathrm{TB}}$ is the transfer matrix of the test bench;

$\boldsymbol{S}_{\mathrm{I}}$ is the spring-like matrix representation of the connecting device;

$\bar{\boldsymbol{f}}_{\mathrm{TB}}$ is the transmitted force vector to the test bench.

The purpose of Formula (2) is to enable the prediction of a force transmitted to a receiving structure from the measurement or the estimation of 4 different FRFs matrices:

$$\bar{\boldsymbol{f}}_{\mathrm{RS\_predict}} = [\boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{AC}} + \boldsymbol{S}_{\mathrm{I}}]^{-1} \cdot [\boldsymbol{Y}_{\mathrm{TB}} + \boldsymbol{Y}_{\mathrm{AC}} + \boldsymbol{S}_{\mathrm{I}}] \cdot \bar{\boldsymbol{f}}_{\mathrm{TB\_meas}} \qquad (2)$$

To build this formula, there are intermediate steps that are detailed in Annex A.

Instead of a link from bench to receiving structure, in certain cases, the need is to go from receiving structure to bench; Formula (2) is then given as Formula (3):

$$\bar{\boldsymbol{f}}_{\mathrm{TB\_predict}} = [\boldsymbol{Y}_{\mathrm{TB}} + \boldsymbol{Y}_{\mathrm{AC}} + \boldsymbol{S}_{\mathrm{I}}]^{-1} \cdot [\boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{AC}} + \boldsymbol{S}_{\mathrm{I}}] \cdot \bar{\boldsymbol{f}}_{\mathrm{RS\_meas}} \qquad (3)$$

4.3 Geometrical considerations

Sizes and quantities handled in this document are defined in a specific coordinate system, usually the geometric coordinate system related to the receiving structure.

During FRF functions measurements (see Annex B), it can be more practical to use a local coordinate system for certain attachment points. In this case, it will be necessary to re-project in a global reference system.

5 Operating mode

5.1 General

In this subclause, an operating mode to apply this document is proposed, as an example.

This procedure is based on the general Formula (2) allowing to transpose the dynamic forces generated by an active component from a test bench to a receiving structure. Depending on the assumptions on the different transfer functions, this operating mode allows the use of simplified versions of Formula (2).

The frequency range(s) for which the formulated hypotheses and steps presented below are considered as valid or invalid shall be mentioned.

5.2 Synopsis of procedure

The various special cases discussed below can be summarized in the form of a general diagram of the procedure (see Figure 2).

5.3 Tasks and preliminary operations

A number of tasks and processes shall be performed previous to the application of this procedure:

a) During the development of the product, the component will determine the active component transfer matrix at the connecting points. There are many cases for which the active component is not only connected to its fixation points, but interacts with its environment through cables, rotation axes, hoses, pipes, friction, which do not allow to measure the transfer matrix in free conditions for all degrees of freedom. In this case, different alternatives are proposed in the document.

b) During the development of the product, the component chooses the properties of the connecting device between the component and the receiving structure. It is remarked that this connecting device matrix properties are of first order influence on the final transmitted forces: the product shall ensure a perfect decoupling in order to minimize the vibration coupling between the component and the receiving structure. Some advices are given hereunder in this operating mode.

c) To apply the methodology to predict the forces transmitted to the receiving structure in order to check compliance with the specifications, a test bench is generally developed, transmitted forces to the bench are measured. Usually, in the field of noise and vibration, an infinitely rigid bench, such as a marble, is used, but this methodology is not mandatory. Therefore, the procedure covers the case of a not infinitely rigid test bench.

The operating mode starts with the analysis of the general equation [Formula (2)] and attempts to cover the different real cases that may be encountered in practice in the fields covered by the document. The choices in the flow chart not only depend on the possibilities offered by the product, but also on the relative order of scales of different transfer matrices in Formula (2). Three different examples are described in Annexes E and F, to scan a wide range of applications.

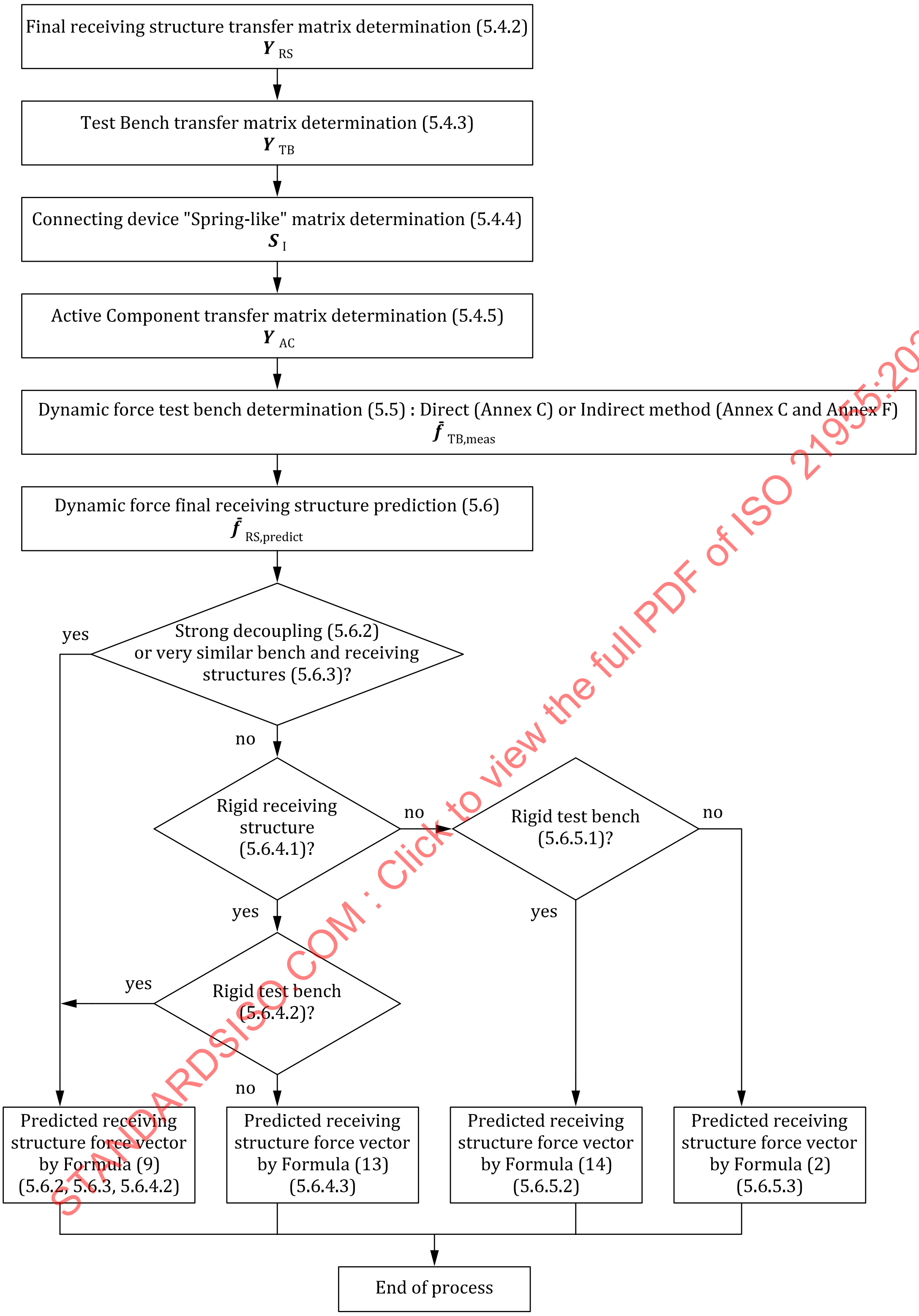


Figure 2 — Synoptic of the steps to determine predicted force

5.4 Transfer matrices determination

5.4.1 General

Annex B is dedicated to frequency transfer functions measurement. These measurements are generally performed with accelerometers and force sensors, leading to accelerance measurements.

5.4.2 Final receiving structure transfer matrix determination Y_{RS}

In Formula (2), the transfer matrix at the connecting points is required to predict the forces on the final structure; whenever there is a component integrator specification about the predicted forces, which means that the component integrator shall provide to the component supplier the transfer matrix values at the connecting points. These values are generally available at early stages of a project development via simulation tools.

In many cases, it is impossible to decouple the active component from the receiving structure to perform a measurement. ISO 20270 can be applied, with an indirect measurement of blocked forces.

5.4.3 Test bench transfer matrix determination, Y_{TB}

At design stage for the test bench, it shall be considered to let some space to position the sensors for the matrix determination.

5.4.4 Connecting device spring-like matrix properties determination, S_I

The connecting device matrix is generally determined on its own on a specific bench. In this case only diagonal terms of the matrix are measured; ISO 10846 can be used.

Taking into account a connecting device is not adapted when:

— the active component is rigidly coupled to the receiving structure or test bench;

— the connecting devices are not a set spring-like point-like devices.

Taking into account a connecting device is not mandatory:

— the active component can be directly coupled to the receiving structure or test bench;

— the connecting device can be integrated in the Active component or in the Test Bench/ Receiving structure part, whenever possible.

5.4.5 Active Component transfer matrix determination, Y_{AC}

Whenever possible to position the active component in free boundary conditions, the frequency response functions of the transfer matrix shall be measured.

In many cases, the active component main function is to deliver a mechanical function to a system via connections which are not fixation points or fixation surfaces. In this case, it is not possible to test the active component in free boundary conditions.

Two options are possible to determine the active component transfer matrix:

a) Usage of a specific test bench called block sensor designed to measure indirect active component transfer matrix and indirect blocked force (see Annex F);

b) Indirect determination of the active component transfer matrix associated with the connecting device.

In this second case, only the case with no connecting device is relevant to be studied (with no connecting device), and the Formula (2) can be written as:

$$\bar{f}_{RS\_predict} = [\boldsymbol{Y}_{RS} + \boldsymbol{Y}_{AC}]^{-1} \cdot [\boldsymbol{Y}_{TB} + \boldsymbol{Y}_{AC}] \cdot \bar{f}_{TB\_meas} \quad (4)$$

At this stage, to measure indirectly the $\boldsymbol{Y}_{AC}$ matrix, which should be tested in free conditions, but is not possible, it is needed to measure a transfer matrix of the coupled system: active component mounted on the test bench: $\boldsymbol{Y}_{AC:TB}$

It can be written that

$$\boldsymbol{Y}_{AC:TB} = \left[\boldsymbol{Y}_{AC}^{-1} + \boldsymbol{Y}_{TB}^{-1} \right]^{-1} \quad (5)$$

and then $\boldsymbol{Y}_{AC}$ can be extracted:

$$\boldsymbol{Y}_{AC} = \boldsymbol{Y}_{AC:TB} \cdot \left[\boldsymbol{I} - \boldsymbol{Y}_{TB}^{-1} \cdot \boldsymbol{Y}_{AC:TB} \right]^{-1} \quad (6)$$

Same formulae can be reached from

$$\boldsymbol{Z}_{AC:TB} = \boldsymbol{Z}_{AC} + \boldsymbol{Z}_{TB} \quad (7)$$

In the following subclauses, different cases of simplification are presented.

5.5 Measured dynamic forces transmitted to the test bench

Dynamic forces transmitted to the test bench can be measured by direct or indirect methods. Annex C and Annex F propose different ways to measure these forces.

5.6 Predicted dynamic forces transmitted to the final structure

5.6.1 General

After determination of the dynamic force vector $\bar{f}_{TB\_meas}$ and of the four transfer Matrices, the dynamic forces transmitted to the final receiving structure can be predicted. Depending on the characteristics of the test bench and/or the characteristics of the final receiving structure, Formula (2) can be simplified or not.

5.6.2 Strong decoupling

This case arises when the connecting device is very flexible (impedance mismatch between active component and connecting device transfer matrices, and between connecting device and receiving structures RS and TB), that is:

$$\boldsymbol{Y}_{AC} \ll \boldsymbol{S}_I \text{ and } \boldsymbol{Y}_{RS} \ll \boldsymbol{S}_I \text{ and } \boldsymbol{Y}_{BE} \ll \boldsymbol{S}_I \quad (8)$$

Generally, an impedance mismatch is achieved when there is an order of scale higher than 10 (20 dB) between each term of the transfer matrices for the different expressions of Formula (8).

This case is very useful because:

— the predicted force vector applied to the receiving structure is equal to the measured force vector applied on the test bench, as given by Formula (9):

$$\bar{f}_{\text{RS}_{\text{predict}}} = \bar{f}_{\text{TB}_{\text{meas}}} \quad (9)$$

With such decoupling, it is very easy to estimate the transmitted forces.

— it minimizes the receiving structure force vector, which allows to transmit as less energy as possible to the receiving structure.

5.6.3 Very similar bench and receiving structure

There is a particularly clear case for the prediction methodology that shall be addressed. As it is generally necessary to work on a test bench on the active source design part (example of a component supplier in charge of the development of the active component), choosing a test bench with a transfer matrix similar to the transfer matrix of the receiving structure is to be pointed out:

$$\boldsymbol{Y}_{\text{RS}} \sim \boldsymbol{Y}_{\text{TB}} \quad (10)$$

Then, from Formula (2), the predicted force vector applied to the receiving structure is equal to the measured force vector applied to the test bench – see Formula (9) above.

$$\bar{f}_{\text{RS}_{\text{predict}}} = \bar{f}_{\text{TB}_{\text{meas}}}$$

5.6.4 Case of a rigid receiving structure

5.6.4.1 General

It is common for the final receiving structure to be designed to have a transfer matrix much less influent than the transfer matrix constituted by the active component and the connecting device. In this case, it is given by Formula (11):

$$\boldsymbol{Y}_{\text{RS}} \ll \boldsymbol{Y}_{\text{AC}} + \boldsymbol{S}_{\text{I}} \quad (11)$$

and the receiving structure can be considered as rigid.

Then, the test bench transfer matrix shall be studied.

5.6.4.2 Rigid test bench (marble)

At this stage, it is worth to try and previously design the test bench in order to be in the case written in Formula (12). At the end, this formula is generally valid over certain frequency ranges, and not valid over other frequency ranges, but easy to apply.

Ability to get access to a rigid bench enables to write:

$$\boldsymbol{Y}_{\text{TB}} \ll \boldsymbol{Y}_{\text{AC}} + \boldsymbol{S}_{\text{I}} \quad (12)$$

Then, the prediction forces vector applied to the receiving structure is equal to the vector measurement of the forces applied to the test bench – see Formula (9) above.

$$\bar{f}_{\text{RS}_{\text{predict}}} = \bar{f}_{\text{TB}_{\text{meas}}}$$

5.6.4.3 Non- rigid test bench

If the test bench transfer matrix is not similar to the one of the receiving structure, then Formula (2) can only be slightly simplified in Formula (13):

$$\bar{\boldsymbol{f}}_{RS\_predict} = [\boldsymbol{Y}_{AC} + \boldsymbol{S}_{I}]^{-1} \cdot [\boldsymbol{Y}_{TB} + \boldsymbol{Y}_{AC} + \boldsymbol{S}_{I}] \cdot \bar{\boldsymbol{f}}_{TB\_meas} \quad (13)$$

5.6.5 Case of a non-rigid receiving structure

5.6.5.1 General

If the receiving structure cannot be considered as rigid (which means that Formula (11) is not satisfied), the test bench transfer matrix that shall be studied.

5.6.5.2 Rigid test bench (marble)

If the test bench can be considered as rigid (which means that Formula (12) is satisfied), the predicted force vector transmitted to the receiving structure is obtained by Formula (14):

$$\bar{\boldsymbol{f}}_{RS\_predict} = [\boldsymbol{Y}_{RS} + \boldsymbol{Y}_{AC} + \boldsymbol{S}_{I}]^{-1} . [\boldsymbol{Y}_{AC} + \boldsymbol{S}_{I}] \cdot \bar{\boldsymbol{f}}_{TB\_meas} \quad (14)$$

5.6.5.3 Non-rigid test bench

If the test bench cannot be considered as rigid (which means that Formula (12) is not satisfied), the predicted force vector transmitted to the receiving structure cannot be simplified and Formula (2) is to be applied:

$$\bar{\boldsymbol{f}}_{RS\_predict} = [\boldsymbol{Y}_{RS} + \boldsymbol{Y}_{AC} + \boldsymbol{S}_{I}]^{-1} . [\boldsymbol{Y}_{TB} + \boldsymbol{Y}_{AC} + \boldsymbol{S}_{I}] \cdot \bar{\boldsymbol{f}}_{TB\_meas}$$

5.6.5.4 Evaluation of the quality of the predicted forces

The quality of the final predicted forces highly depends on the measurements and hypothesis that are used:

- quality of the transfer functions that have been measured;
- quality of the operational measurements such as dynamic forces or accelerations;
- validity of the different hypothesis that enabled the formulae to be simplified;
- quality of matrix inversion.

These indicators of quality help to define frequency ranges for which the predicted force results are reliable, see Table 6.

Table 6 — Indicators to be checked for quality evaluation of the predicted forces

Final predicted forces depend on	Indicator	To be checked
transfer functions (Annex B)	FRFs coherence	Indicator of the frequency ranges for which non-linearities will not ensure mastered FRF measurement uncertainties and of the frequency ranges for which the signal to noise ratio is not high enough to ensure repeatable results.
	Reciprocity	The principle of reciprocity of swapping the excitation and response positions and comparing the transfer functions is a very good indicator of the reproducibility of the measurement.
	Positive damping factor	Based on the basic physical principle that the damping factor of any system is a positive quantity: if the real part of the complex mobility or the imaginary part of the accelerances is not positive at any frequency, the obtained measurement cannot be considered as acceptable.
operational measurements	reproducibility	The associated modulus of the operational quantity spectrum with the active component in operation shall be considered towards the same spectrum with active component not active. If the spectra show similar levels, the confidence in the resulting measured quantities could be low. Signal to noise ratio can be considered as reliable when it exceeds 20 dB.
quality of matrix inversion	Force measurement by indirect method	The conditioning number of the transfer matrix can be used as an indicator of the quality of the matrix inversion. To reach a good conditioning number, it is recommended to: — have a matrix size with a number of sensors higher than twice [ref] the number of matrix minimum size
	Predicted force on final receiving structure	Use a regularization method. The conditioning number of the transfer matrix should be used as an indicator of the quality of the matrix inversion. When very low values are obtained at certain frequencies for the terms of the $[\boldsymbol{Y}_{RS} + \boldsymbol{Y}_{AC} + \boldsymbol{S}_{I}]$ matrix, matrix inversion introduces very big uncertainties in these frequency ranges: results are probably not reliable in these frequency ranges. When a quantity used in the formula used to predict the receiving structure force is already obtained through an indirect measurement process involving matrix inversion, the opportunity of a second inversion shall be very carefully studied.

Table 6 *(continued)*

Final predicted forces depend on	Indicator	To be checked
quality of matrix inversion	Inverting a matrix obtained from measured values tends to amplify measurement uncertainties. It is generally recommended to discard the information detected as not reliable. The conditioning number of the transfer matrix can be used as an indicator of the quality of the matrix inversion: to reach a good conditioning number, it is recommended to: — have a matrix size with a number of sensors higher than twice the number of matrix minimum size — use a regularization method. As a good practice, it is recommended to validate that the final results are not too much dependent on the inversion matrix tools. For example, comparing or sharing the results of various inversions of a typical matrix is recommended.	

6 Requirements for data in test report

Besides the usual indications about the contents of a test report (see ISO 17025), some advices are listed hereunder.

Using this document, there are two types of test reports:

— Specifications of the integrator to the supplier;

— Data sent by the supplier to the integrator.

6.1 Specification of the integrator to the supplier

The test report shall include:

— a characterization of the final receiving structure (impedance, mobilities, or accelerances).

— if the integrator is responsible for the coupling elements at interface, the characteristics to be implemented in the formula shall be sent.

— a list of specifications of the predicted dynamic force levels.

6.2 Data sent by the supplier to the integrator

In this case, a very accurate description and characteristics of the test bench shall be written, which ensure a justification of the choice of bench design (impedance, mobilities, or accelerances).

The active component characteristics (impedance, mobilities, or accelerances….) shall be documented.

Forces measured on the test bench and the predicted forces on the final receiving structure shall be included in the test report.

To ensure the quality of the measurement it is recommended to include (or in an annex) to plot measured transfer functions and associated coherences; reciprocity and repeatability should be checked during the different phases of measurements.

In the case of block sensor method (see Annex F), an auto validation can be added to the final report.

It is highly recommended to express the reasons for:

— reducing the number of DOF;

— not taking into account measurements with very low signal to noise ratio;

— reducing the number of sensors.

with the help of some quality indicators listed in Clause 5.

Annex A
(informative)

Theoretical developments

A.1 Introduction

The purpose of this annex is to develop theoretical developments that enable to obtain Figure 1 and Formula (1). These theoretical developments will help the reader to better define how to design the vibroacoustics properties of each component of the formula such as test bench related to final receiving structure, or connecting device related to active component.

At first, direct mechanical coupling considerations are used for an active component coupled to a receiving structure via a connecting device at connecting points, in order to help the reader to get a better understanding of the different steps leading to this formula.

The electrical analogy is another pedagogical tool leading to unidimensional formulae.

Details about matrices organisation are given in Annex D.

An active component is coupled via a connecting device to a receiving structure, which does not include any active structure (see Figure 1). This receiving structure can be a test bench or the final real structure.

A.2 Components formulae

The following formulae are written with

— a dynamic movement u, can be indifferently an acceleration, a dynamic velocity or a displacement,

— frequency response functions Y which are respective to the dynamic movement accelerances, free mobilities or dynamic compliances.

The quantities are

— with exponent ^{i} for internal points, and with exponent ^{c} at the connecting points,

— with an index indicating the component.

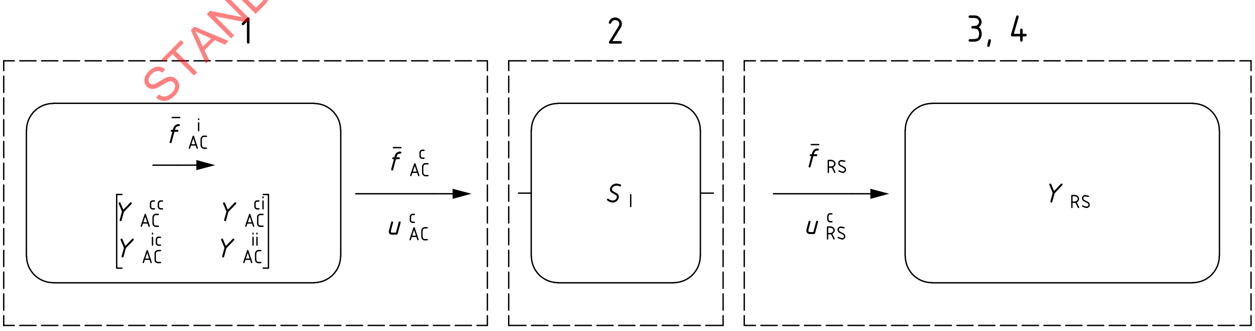


Figure A.1 — Detailed schematic of the structure assembly

The active component vibrational behaviour is given in the frequency domain by the following linear system given by Formula (A.1):

$$\begin{bmatrix} \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{cc}} & \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{ci}} \\ \boldsymbol{Y}_{\mathrm{AC}}^{ic} & \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{ii}} \end{bmatrix} \begin{bmatrix} \overline{\boldsymbol{f}}_{\mathrm{AC}}^{\mathrm{c}} \\ \overline{\boldsymbol{f}}_{\mathrm{AC}}^{i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}_{\mathrm{AC}}^{c} \\ \boldsymbol{u}_{\mathrm{AC}}^{i} \end{bmatrix} \quad \text{(A.1)}$$

As the receiving structure does not experience any internal excitations, its vibrational behaviour is given in the frequency domain by the following linear system given by Formula (A.2):

$$\boldsymbol{Y}_{\mathrm{RS}}^{c} \overline{\boldsymbol{f}}_{\mathrm{RS}}^{c} = \boldsymbol{u}_{\mathrm{RS}}^{c} \quad \text{(A.2)}$$

The following hypothesis are made at the connection between the two components:

— the connecting device is not a structure with internal degrees of freedom and internal mass;

— the connecting points of the active component and of the receiving structure share the same geometrical location.

Thus, with these hypothesis, the principle of action and reaction enables to obtain the following Formula (A.3):

$$\overline{\boldsymbol{f}}_{\mathrm{AC}}^{\mathrm{c}} = -\overline{\boldsymbol{f}}_{\mathrm{RS}}^{c} \quad \text{(A.3)}$$

The connecting device being “spring–like”, forces and movements at its interfaces are related by Formula (A.4):

$$\boldsymbol{u}_{\mathrm{AC}}^{c} - \boldsymbol{u}_{\mathrm{RS}}^{\mathrm{c}} = \boldsymbol{S}_{\mathrm{I}} \overline{\boldsymbol{f}}_{\mathrm{RS}}^{\mathrm{c}} \quad \text{(A.4)}$$

The different expressions of connecting device transfer matrix versus dynamic stiffness matrix are compiled in Table 5. In the case of a rigid connection, see Formula (A.5):

$$\boldsymbol{S}_{\mathrm{I}} = 0 \quad \text{(A.5)}$$

And of course a combination of rigid and elastic connections also guarantees the previous hypothesis.

A.3 Total system formulae

From Formula A.1:

$$\boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{ci}} \overline{\boldsymbol{f}}_{\mathrm{AC}}^{\mathrm{i}} = \boldsymbol{u}_{\mathrm{AC}}^{\mathrm{c}} - \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{cc}} \overline{\boldsymbol{f}}_{\mathrm{AC}}^{\mathrm{c}} \quad \text{(A.6)}$$

Using Formulae A.3 and A.4:

$$\boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{ci}} \overline{\boldsymbol{f}}_{\mathrm{AC}}^{\mathrm{i}} = \boldsymbol{u}_{\mathrm{RS}}^{\mathrm{c}} + \boldsymbol{S}_{\mathrm{I}} \overline{\boldsymbol{f}}_{\mathrm{RS}}^{\mathrm{c}} + \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{cc}} \overline{\boldsymbol{f}}_{\mathrm{RS}}^{\mathrm{c}} \quad \text{(A.7)}$$

Using Formula A.2:

$$\boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{ci}} \overline{\boldsymbol{f}}_{\mathrm{AC}}^{\mathrm{i}} = \left[\boldsymbol{Y}_{\mathrm{RS}}^{\mathrm{c}} + \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{cc}} + \boldsymbol{S}_{\mathrm{I}} \right] \overline{\boldsymbol{f}}_{\mathrm{RS}}^{\mathrm{c}} \quad \text{(A.8)}$$

It should be pointed out that the excitation term $\boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{ci}} \overline{\boldsymbol{f}}_{\mathrm{AC}}^{\mathrm{i}}$ is intrinsic to the active component.

In the case of a test bench, replacing the RS index by TB in Formula (A.8) leads to Formula (A.9):

$$\boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{ci}} \bar{\boldsymbol{f}}_{\mathrm{AC}}^{\mathrm{i}} = \left[\boldsymbol{Y}_{\mathrm{TB}}^{\mathrm{c}} + \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{cc}} + \boldsymbol{S}_{\mathrm{I}} \right] \bar{\boldsymbol{f}}_{\mathrm{TB}}^{\mathrm{c}} \tag{A.9}$$

A.4 Transposition to dynamic forces

From Formulae (A.8) and (A.9), the following Formula (A.10) is obtained:

$$\left[\boldsymbol{Y}_{\mathrm{RS}}^{\mathrm{c}} + \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{cc}} + \boldsymbol{S}_{\mathrm{I}} \right] \bar{\boldsymbol{f}}_{\mathrm{RS}}^{\mathrm{c}} = \left[\boldsymbol{Y}_{\mathrm{TB}}^{\mathrm{c}} + \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{cc}} + \boldsymbol{S}_{\mathrm{I}} \right] \bar{\boldsymbol{f}}_{\mathrm{TB}}^{\mathrm{c}} \tag{A.10}$$

In order to enable evaluating scale of orders of the various quantities, a formula issued from Formulae (A.8) and (A.10) is given, connecting the force vector transmitted to the final receiving structure $\bar{\boldsymbol{f}}_{\mathrm{RS}}$ to the internal force vector generated by the active component $\bar{f}_{\mathrm{AC}}$:

$$\bar{\boldsymbol{f}}_{\mathrm{RS}}^{\mathrm{c}} = \left[\boldsymbol{Y}_{\mathrm{RS}}^{\mathrm{c}} + \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{cc}} + \boldsymbol{S}_{\mathrm{I}} \right]^{-1} \cdot \boldsymbol{Y}_{\mathrm{AC}}^{\mathrm{ci}} \bar{\boldsymbol{f}}_{\mathrm{AC}}^{\mathrm{i}} \tag{A.11}$$

Formula (A.10) can also be written with simplified notations as Formula (1):

$$\bar{\boldsymbol{f}}_{\mathrm{RS}} = [\boldsymbol{Y}_{\mathrm{RS}} + \boldsymbol{Y}_{\mathrm{AC}} + \boldsymbol{S}_{\mathrm{I}}]^{-1} \cdot [\boldsymbol{Y}_{\mathrm{TB}} + \boldsymbol{Y}_{\mathrm{AC}} + \boldsymbol{S}_{\mathrm{I}}] \cdot \bar{\boldsymbol{f}}_{\mathrm{TB}}$$

and Figure A.1 can also be simplified into Figure 1.

Annex B
(informative)

Frequency response functions measurement

B.1 General

In the vibrations measurement field, measured FRFs are generally a ratio between dynamic accelerations versus forces, which are accelerances. As in the rest of the document, the term of accelerance used in this annex can be replaced by any other FRF type, respecting homogeneity of formulae, and the definition of free mobility boundary conditions.

The measurements of the different terms for the active component or for the two receiving structures are performed at component input force locations on each independent structure. If these input force locations are the only points of connection between the component and the host structure, the measurements are performed for the component, using the free/free boundary conditions. If extra connections are present, they shall be kept in place and the free/free boundary conditions shall be applied only at input force locations. For a parking brake example, there are n attachment points in addition to a cable connection, and the attachment points are the input force locations: boundary conditions for measurement shall be applied at input force locations, while the connection cable shall remain in service.

For the receiving structure, similar measurements are performed at input force locations without the presence of the active component.

Considering n fixation points between the active component and the receiving structure, each transfer matrix dimension to be used in Formula (2) is $6n \times 6n$ (each excitation point is defined by 6 dof). In many cases, only forces are able to be measured and the transfer matrix dimension is then $3n \times 3n$. Each point and direction of excitation corresponds to a column of the matrix and each point and direction of a response corresponds to a line. On the diagonal are located direct accelerances terms (same points and same directions for excitation and response). Assuming that the reciprocity hypothesis is valid (i.e. same response at a point in the direction j for an excitation at a second point in the direction i than response at the second point in the direction i for excitation at the first point in the direction j), it shall be checked that the accelerance matrix is symmetric.

NOTE 1 Definition of direct accelerance: any transfer function which force and acceleration are measured at the same point and in the same direction.

NOTE 2 Definition of local accelerance or local transfer accelerance: any transfer function whose force and acceleration are measured at the same point but in a different direction.

NOTE 3 Any transfer function for which force and acceleration are measured at different points will be qualified as far transfer accelerance.

B.2 Measurement of transfer functions $\frac{a}{f}$

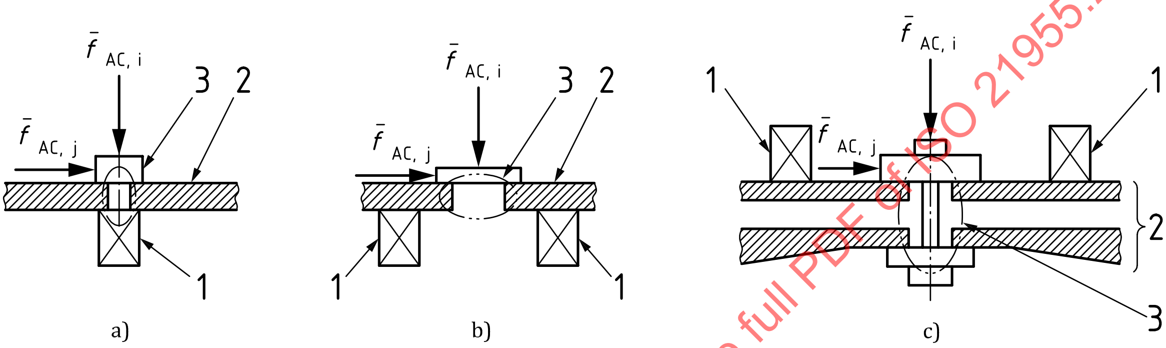
B.2.1 Implementation and fixtures

B.2.1.1 General

Several types of fixtures can be considered depending if the interface input force locations are of easy access or not. For the different fixtures that are presented hereunder, the structural response is measured by accelerometers and the excitation is performed using an impact hammer. In practice, it is generally the easiest and cheapest way to implement the measurement. A shaker could be considered as excitation force, and/or a response measurement via laser vibrometer.

B.2.1.2 Assembly 1 - One single 3D accelerometer by attachment point.

This accelerometer [see Figure B.1 a)] is glued at the same exact location of the attachment point. Depending on the excitation direction, the position of the impact of the hammer can be more or less distant from the attachment point depending in the accessibility. If the distance between excitation and response is considered as too far, a low-weight and high rigidity cube can be positioned at the point of attachment to apply forces on this cube. The size and mass of the cube should be minimized so as not to be too intrusive. The accelerometer shall always be glued to the structure. A particular care shall be focused on for the structure to properly taken into account in between excitation and response: for example, an accelerometer directly glued on the cube does not correctly measure the response of the structure.



Key

1 accelerometer

2 structure to be characterised

3 interface point

Figure B.1 — Examples of mounting an attachment point for measuring under accelerances matrices, Y_i

B.2.1.3 Assembly 2 - Using multiple 3D anchor accelerometers by attachment point

The accelerometers [Figure B.1 b) and c)] are glued symmetrically around the fixation point and the excitation is applied at the geometrical gravity centre of accelerometers. From the measurements of transfer functions between the various accelerometers and the applied force, it is possible to calculate the transfer function at the attachment point, averaging the 2 transfer functions again. For accessibility, it is sometimes necessary to add a small interface to have a better access for force application. As in the previous assembly, it is worth to have the structure to be characterised in between excitation and response.

B.2.2 Recommendations

The assembly 1 using a cube or not, is the fastest to implement because it requires the use of a single accelerometer attachment point. In addition, the added mass by the introduction of a cube and a sensor is low. Whenever possible this particular fixture is the one to be chosen.

The assembly 2 is to be used whenever the assembly 1 is not possible to implement. This fixture also enables eliminating the effect of rotations that can be of first order influence when the frequency increases. From the transfer functions measurements performed with the various accelerometers, the average transfer functions constitute the $3n \times 3n$ terms of an average matrix. The added mass, m, for this assembly is greater; the effect of this mass can be corrected only for direct FRFs terms.

EXAMPLE The correction term for a measured transfer function at point and direction k will be $-m$ to the inverse of the initial measured accelerance by the Formula (B.1)

$$\frac{1}{\boldsymbol{Y}_{i\_kk\_\text{corrected}}} = \frac{1}{\boldsymbol{Y}_{i\_kk}} - m \tag{B.1}$$

where $\boldsymbol{Y}_{i\_kk}$ corresponds to the measured direct accelerance and $\boldsymbol{Y}_{i\_kk\_\text{corrected}}$ corresponds to the accelerance after direct mass correction. The effect on the terms of local and distant transfer is not corrected.

In all cases, it is important to ensure consistency of measurements. For example, the direct accelerance measurements are correct if there are no phase rotations. Once a phase rotation appears, it means that it is not anymore a measurement of direct accelerance but a measurement of transfer. For local transfers or remote transfers, we can ensure consistency of measurements by checking good measurements reciprocity. Reciprocity is defined as the reciprocal transfer function of identity (same amplitudes and opposite phases). The reciprocal transfer function is obtained by inverting the point and excitation direction by the point and observation direction.

It should be remarked that a good reciprocity local transfer is difficult to achieve as soon as the frequency increases.

Annex C
(informative)

Dynamic forces measurement

C.1 General

Consider an active component connected to a test bench or a final receiving structure by n fixation points. In this annex, advice is given to measure the dynamic forces at these different locations in three directions to construct the forces components of the force vector, $\bar{\boldsymbol{f}}_{\mathrm{RS}}$ or $\bar{\boldsymbol{f}}_{\mathrm{TB}}$ in order to predict $\bar{\boldsymbol{f}}_{RS\_\mathrm{pred}}$. These vectors have 3$n$ components (n attachment points times 3 directions), if only forces are taken into account.

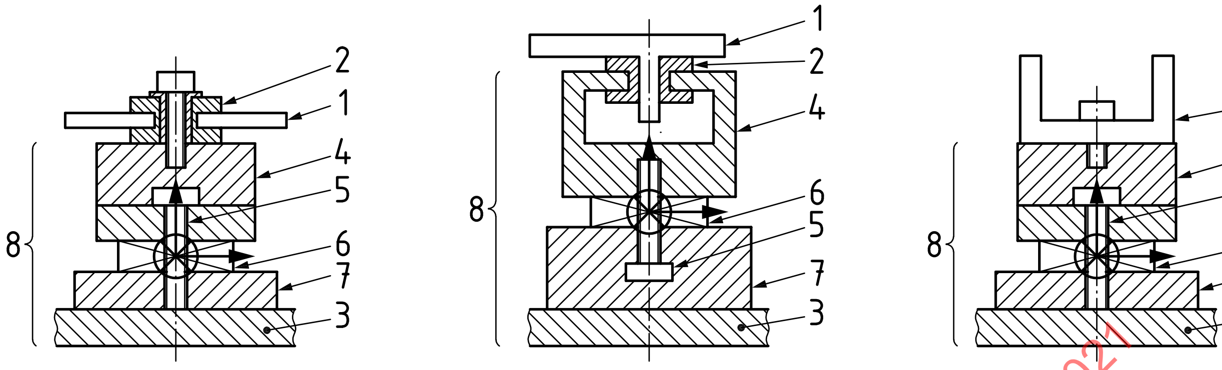
These measurements are performed with the component in operation (operational conditions) on the test bench or the receiving structure.

C.2 Direct method by force sensor

C.2.1 Test set-up and fixture

At each fixation point, three dimensions sensors are positioned, to measure simultaneously operational forces.

To get an accurate measurement of the shear forces, it is mandatory to preload the force sensor between two steel spacers, each one rectified on the surface in contact with the sensor. The objective is to minimize the height of the upper spacer so for the component to be as close as possible to the force sensor. The fixture between the component and the upper spacer shall be representative of the real final receiving structure characteristics. The lowest spacer is fixed on the receiving structure or on the test bench. Three examples for elastic or rigid connections are presented in Figure C.1. With such type of fixture, the attachment point between the component and the receiving structure (or test bench) is located at the top of the upper spacer: the two spacers and the force sensor are part of the considered receiving structure or test bench (host structure).



Key

1 active component
2 connecting device
3 receiving structure or test bench
4 upper spacer
5 screw to preload the sensor
6 force sensor
7 lower spacer
8 host structure

Figure C.1 — Examples of mounting of force sensors for elastic connections or rigid connections

C.2.2 Preliminary measurements of force/force transfers

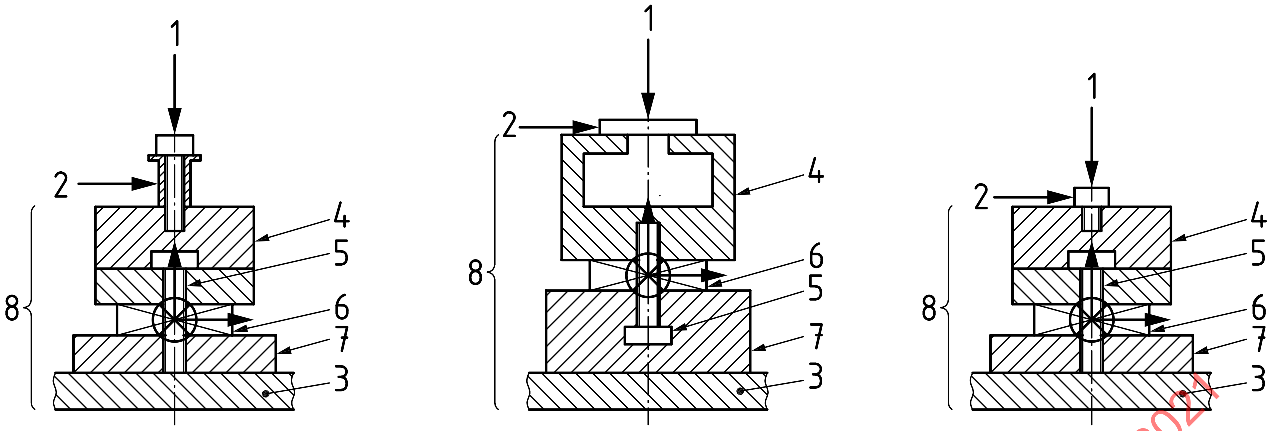
C.2.2.1 General

Due to the presence of the upper spacer, the force sensor is not directly positioned at the interface between the active component and the receiving structure. It is therefore necessary to characterize the transfer of the force measured by the force sensor on the force injected at the interface with the component to transfer thereafter the force measured by the force sensor to the force actually transmitted to the host structure. This is some kind of "calibration" of the force sensor.

Frequency transfer functions measurements shall be organised, between a force applied to the nearest interface between the active component and receiving structure, noted $\bar{\boldsymbol{f}}_{\text{exc}}$ and a force measured by the force sensor in each direction. The excitation is made with an impact hammer or a shaker in one direction when neither the component nor the possible connecting device are present (see Figure C.2). For m connecting points of interface, 3m excitations shall be applied and 3m × 3m FRF shall be recorded. These 3m × 3m FRF are the terms of a transfer matrix, noted $\boldsymbol{T}$, which links the force vector measured by the force sensors, noted $\bar{\boldsymbol{f}}_{\text{sens\_meas}}$ to the vector of forces actually injected by the component to the host structure by

$$\boldsymbol{T} \cdot \bar{\boldsymbol{f}}_{\text{exc}} = \bar{\boldsymbol{f}}_{\text{sens\_meas}} \tag{C.1}$$

A column of the matrix $\boldsymbol{T}$ corresponds to an excitation (point of application of force and direction) and each row corresponds to a response measured by the stress sensors (measuring point and direction) force. The scheduling of the various terms in the matrix is defined in Annex D.



Key

1 external force, $\bar{f}_{ext,i}$

2 external force, $\bar{f}_{ext,j}$

3 receiving structure or test bench

4 upper spacer

5 screw to preload the sensor

6 force sensor

7 lower spacer

8 host structure

Figure C.2 — Measurement principle of the calibration matrix, *T* (see Figure C.1)

C.2.2.2 Recommendations

For measurements on test bench (or marble), this measurement method is suitable because it provides directly and accurately the force. The intrusive side of the instrumentation is not annoying because the instrumentation is an integral part of the bench. However, it is necessary to check if the stiffness of the bench at the point of connection is large enough to neglect the transfer matrix $\boldsymbol{Y}_{TB}$ in the calculation process of the predicted force vector [Formula (12)]. For example, the addition of the instrumentation on a marble "softens" the marble at connection points.

For measurements on final receiving structure, this method of measurement of the forces is not recommended because the intrusive instrumentation strongly modifies the characteristics of the receiving structure and measured forces can then be very different than the forces actually injected into the receiving structure.

C.2.2.3 Equipment used

The force sensors are piezoelectric sensors for example, chosen with a dynamic range adapted to measure the phenomenon (the case studies and presented in Annex E required the implementation of sensors with ±5 kN range, sensitivity around 5 pC/N). When mounting, be careful in the direction in which the various forces will be measured. Unlike an accelerometer, turning over the sensor does not necessarily change the sign of the measured force. The best solution is to check when measuring the *T* matrix that forces are measured in the appropriate direction, with the correct sign.

C.3 Indirect method: calculation from accelerometer measurements

C.3.1 Principle

This method involves operational accelerations measurement at attachment points on the test bench $\boldsymbol{a}_{\mathrm{TB}_{\mathrm{op}}}$ or the final receiving structure $\boldsymbol{a}_{\mathrm{RS}_{\mathrm{op}}}$. Generally, this method is used to obtain indirectly operational force vector on the test bench, $\bar{\boldsymbol{f}}_{\mathrm{TB}_{\mathrm{ind}}}$, in order to be able to predict the force vector on the final receiving structure [Formula (2)]: in this sub-clause, all notations are with the test bench, but they can also be written for the final receiving structure. $\boldsymbol{Y}_{\mathrm{TB}}$ is the test bench transfer matrix, measured without the active component (see Annex B). It is then possible to calculate the forces transmitted from these measurements and measurements of transfer functions of the host structure. The matrix formula is used:

$$\bar{\boldsymbol{f}}_{\mathrm{TB}_{\mathrm{ind}}} = \boldsymbol{Y}_{\mathrm{TB}}^{-1} . \boldsymbol{a}_{\mathrm{TB}_{\mathrm{op}}} \tag{C.2}$$

It is mandatory that the accelerometers are at the same location for both operational and transfer matrix measurements.

C.3.2 Implementation and fixtures

Different arrangements are possible according to the accessibility of attachment points.

C.3.2.1 Assembly 1

A single 3D accelerometer is glued to the exact location of the attachment point [Figure C.3 a)]. when the active component is not present and when it is connected to the host structure.

For *n* attachment points, the vector $\boldsymbol{a}_{\mathrm{TB}_{\mathrm{op}}}$ therefore contains 3n components and the single matrix $\boldsymbol{Y}_{\mathrm{TB}}$ has $3n \times 3n$ terms.

With this arrangement, the $\boldsymbol{Y}_{\mathrm{TB}}$ matrix used in Formula (C.2) is the same as that used in the calculation of Formula (2).

C.3.2.2 Assembly 2

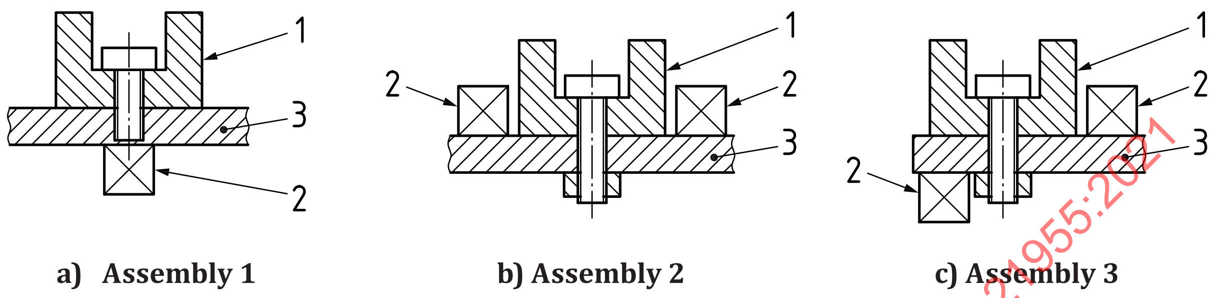
Multiple 3D accelerometers (or 1D) are placed on either side of the attachment point, for example symmetrically [Figure C.3 b)]. It is then possible to obtain the acceleration at the attachment point by averaging (for both operational measurements and FRF measurements). For calculation, the matrix $\boldsymbol{Y}_{\mathrm{TB}}$ is an average matrix with $3n \times 3n$ terms and vector $\boldsymbol{a}_{\mathrm{TB}_{\mathrm{op}}}$ is a mean vector with $3n$ components.

As in the previous assembly, the $\boldsymbol{Y}_{\mathrm{TB}}$ matrix used in Formula (C.2) is the same as the one used in the calculation of Formula (2).

C.3.2.3 Assembly 3

Several 3D accelerometers, called indicators, are positioned without any symmetry to each other [Figure C.3 (c)]. It is recommended for these sensors to ensure the highest possible distance, in order for them to give various information content in Formula (C.2), with a number of acceleration sensors at least twice higher than the number of forces to estimate. When these sensors are away from the attachment point, the injected force for FRF measurements shall be applied directly at the attachment point. With j 3D sensors per connecting point, the Formula (C.2) is then an overdetermined system that shall be solved in a least squares sense: the vector $\boldsymbol{a}_{\mathrm{TB}_{\mathrm{op}}}$ has $3 \times j \times n$ components while the matrix $\boldsymbol{Y}_{\mathrm{TB}}$ is a matrix of dimensions $3 \times j \times n \times 3 \times n$.

Unlike the two previous arrangements, the $Y_{TB}^{'}$ matrix used in Formula (C.2) is then not the same as the Y_{TB} one used for the predicted receiving structure force [Formula (2)]. Y_{TB} (or Y_{RS}) matrices shall be obtained exactly at attachment points between component and receiving structure. This could mean an assembly 1 or 2 for measuring Y_{TB} (or Y_{RS}), and assembly 3 could then be used to estimate the forces transmitted if the accelerometers placed in assembly 1 or 2 cannot remain during operational measurements.



Key

1 active component

2 accelerometer

3 receiving structure or test bench

Figure C.3 — Test set-ups to obtain force at the interface by an inverse method

C.3.3 Recommendations

In practice, it is rarely possible to consider installing assembly 1 for accessibility reasons. Generally, there are holes which allow to fix the active component to the host structure, with or without a connecting device. The accelerometer can generally be fixed for transfer matrices measurement, but cannot remain at the same place for the operational measurements. However, if this arrangement is possible, it should be applied in priority (low added mass, only one sensor, no additional calculation to consider, direct inversion of the matrix Y_{TB}, same matrix Y_i to estimate operational forces and calculate the predicted force vector).

Although assembly 2 requires more sensors, it cancels rotation effects and allows to get accelerations at the connection point. For accessibility reasons, it is not necessarily easy to install four accelerometers around each attachment point. The installation is longer and the number of sensors more important. But if the sensors can remain at the same place for the operational measures, the use of this arrangement allows to get the same matrices Y_{TB} for estimating operational force and for predicting the forces on the receiving structure.

Finally, in most cases, assembly 3 will be the one that will be easy to implement, the accelerometers do not have predefined positions. However, this arrangement requires the use of one or more additional accelerometers for measuring the matrix Y_{TB} at attachment points, which is used for prediction calculations.

C.4 Method of dynamic stiffness

C.4.1 Principle

As the connecting devices between the component and the host structure are elastic, a third method may be considered: it consists in calculating the operational forces at the interface points from accelerometer measurements upstream and downstream the connecting device, with previous knowledge of the connecting device transfer matrix.

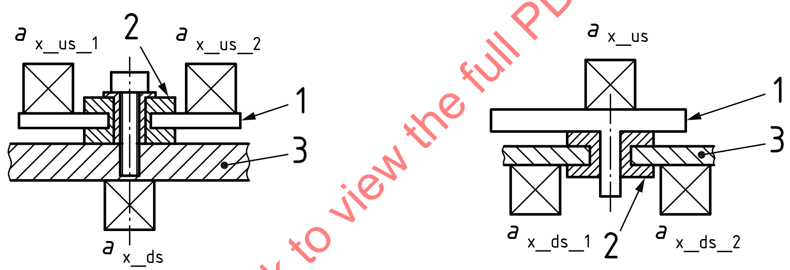
For this method, the different attachment points are considered as independent one from each other. For each elastic connecting device, it is also considered that only the direct FRF (excitation and response in the same direction) are considered. The connecting device transfer matrix is fully diagonal: no coupling between different points and directions.

With these assumptions, it is possible to calculate a dynamic force component in the x direction, $\bar{\boldsymbol{f}}_x$, from acceleration components measured upstream $\boldsymbol{a}_{x\_\mathrm{us}}$ and downstream $\boldsymbol{a}_{x\_\mathrm{us}}$, with already known dynamic stiffness of the elastic connecting device in the same direction, $\boldsymbol{K}_x^*$ by:

$$\bar{\boldsymbol{f}}_x = -\omega^2 \boldsymbol{K}_x^* (\boldsymbol{a}_{x\_\mathrm{us}} - \boldsymbol{a}_{x\_\mathrm{ds}}) \quad \text{(C.3)}$$

C.4.2 Implementation and fixtures

Depending on the active component fixation means, the connecting device set-up is variable. Two examples are detailed in Figure C.4 with the accelerometers positioning to use this method. In Figure C.4 a), the acceleration at the attachment point on the active component side is not directly accessible: two accelerometers bonded symmetrically to the attachment point are then used to obtain by averaging the acceleration $\boldsymbol{a}_{x\_us}$ necessary in Formula (C.3). In Figure C.4 b) the acceleration at the attachment point of the receiving structure is not accessible: the principle of averaging is used, but this time on the receiving structure side.



a) 2 accelerometers on the active component side

b) 2 accelerometers on the receiving structure side

Key

1	active component
2	connecting device
3	receiving structure or test bench
$\boldsymbol{a}_{x\_\mathrm{us}\_i}$	acceleration measured on the upstream side (active component side) at point x – sensor number i
$\boldsymbol{a}_{x\_\mathrm{ds}\_i}$	acceleration measured on the downstream side (receiving structure side) at point x – sensor number i

Figure C.4 — Mounting examples for the forces determination by the dynamic stiffness method

C.4.3 Recommendations

The main advantage of this method is that it is easy to implement it. However, it is necessary to previously know the transfer matrix of the elastic connecting devices, which implies strong assumptions. For example, it is well known that elastic connecting devices are dependent on the preload value, or on the active component amplitude: it is obviously not easy to pre-know these to two values, for each frequency of interest on the final real set-up. A second limitation is that this calculation does not take into account the couplings between different directions. Lastly, it is reminded that this method cannot be applied to rigid connections.

To summarise, this method may be useful whenever the direct terms of the connecting device transfer matrix are already well known, and whenever only direct transmitted forces are to be estimated.

Annex D
(informative)

Data processing

D.1 General

The purpose is to help pre-defining the matrices writing and dimensions for a system with:

— m connecting devices at m attachment points $\left[\boldsymbol{A}_1, .., \boldsymbol{A}_j, .., \boldsymbol{A}_m\right]$ to the test bench or receiving structure,

— measurements are performed in three directions,

— all terms are defined as in Annex C

D.2 Direct measurement (see C.1)

Dynamic force vector (3×m terms) measured on bench and transfer matrix force/force $\boldsymbol{T}_{\mathrm{TB}}$ (3m × 3m terms) can be expressed as:

$$\bar{f}_{\mathrm{TB}_{\mathrm{sens\_meas}}} = \begin{bmatrix} f_{\mathrm{TB\_}A_{1x}} \\ f_{\mathrm{TB\_}A_{1y}} \\ f_{\mathrm{TB\_}A_{1z}} \\ \cdots \\ \cdots \\ f_{\mathrm{TB\_}A_{mx}} \\ f_{\mathrm{TB\_}A_{my}} \\ f_{\mathrm{TB\_}A_{mz}} \end{bmatrix} \text{ and } \boldsymbol{T}_{\mathrm{TB}} = \begin{bmatrix} \boldsymbol{T}_{\mathrm{TB\_}A_{1x}/A_{1x}} & \cdots & \boldsymbol{T}_{\mathrm{TB\_}A_{1x}/A_{mz}} \\ \boldsymbol{T}_{\mathrm{TB\_}A_{1y}/A_{1x}} & & \boldsymbol{T}_{\mathrm{TB\_}A_{1y}/A_{mz}} \\ \boldsymbol{T}_{\mathrm{TB\_}A_{1z}/A_{1x}} & \cdot & \boldsymbol{T}_{\mathrm{TB\_}A_{1z}/A_{mz}} \\ \cdots & \cdot & \cdots \\ \cdots & \cdot & \cdots \\ \boldsymbol{T}_{\mathrm{TB\_}A_{mx}/A_{1x}} & \cdot & \boldsymbol{T}_{\mathrm{TB\_}A_{mx}/A_{mz}} \\ \boldsymbol{T}_{\mathrm{TB\_}A_{my}/A_{1x}} & & \boldsymbol{T}_{\mathrm{TB\_}A_{my}/A_{mz}} \\ \boldsymbol{T}_{\mathrm{TB\_}A_{mz}/A_{1x}} & \cdots & \boldsymbol{T}_{\mathrm{TB\_}A_{mz}/A_{mz}} \end{bmatrix}$$

Dynamic force vector (3m terms) measured on receiving structure and transfer matrix force/force $\boldsymbol{T}_{RS}$ (3m × 3m terms) can be expressed as:

$$\bar{f}_{\mathrm{RS}_{\mathrm{sens\_meas}}} = \begin{bmatrix} f_{\mathrm{RS\_}A_{1x}} \\ f_{\mathrm{RS\_}A_{1y}} \\ f_{\mathrm{RS\_}A_{1z}} \\ \cdots \\ \cdots \\ f_{\mathrm{RS\_}A_{mx}} \\ f_{\mathrm{RS\_}A_{my}} \\ f_{\mathrm{RS\_}A_{mz}} \end{bmatrix} \text{ and } \boldsymbol{T}_{\mathrm{RS}} = \begin{bmatrix} \boldsymbol{T}_{\mathrm{RS\_}A_{1x}/A_{1x}} & \cdots & \boldsymbol{T}_{\mathrm{RS\_}A_{1x}/A_{mz}} \\ \boldsymbol{T}_{\mathrm{RS\_}A_{1y}/A_{1x}} & & \boldsymbol{T}_{\mathrm{RS\_}A_{1y}/A_{mz}} \\ \boldsymbol{T}_{\mathrm{RS\_}A_{1z}/A_{1x}} & \cdot & \boldsymbol{T}_{\mathrm{RS\_}A_{1z}/A_{mz}} \\ \cdots & \cdot & \cdots \\ \cdots & \cdot & \cdots \\ \boldsymbol{T}_{\mathrm{RS\_}A_{mx}/A_{1x}} & \cdot & \boldsymbol{T}_{\mathrm{RS\_}A_{mx}/A_{mz}} \\ \boldsymbol{T}_{\mathrm{RS\_}A_{my}/A_{1x}} & & \boldsymbol{T}_{\mathrm{RS\_}A_{my}/A_{mz}} \\ \boldsymbol{T}_{\mathrm{RS\_}A_{mz}/A_{1x}} & \cdots & \boldsymbol{T}_{\mathrm{RS\_}A_{mz}/A_{mz}} \end{bmatrix}$$

D.3 Indirect method (see C.2)

Assembly 1 and 2: measured accelerations on receiving structure $\boldsymbol{a}_{\mathrm{RS}_{op}}$ and transfer matrix acceleration $\boldsymbol{Y}_{\mathrm{RS}}$

$$\bar{\boldsymbol{a}}_{\mathrm{RS_{op}}} = \begin{bmatrix} \boldsymbol{a}_{\mathrm{RS}\_A_{1x}} \\ \boldsymbol{a}_{\mathrm{RS}\_A_{1y}} \\ \boldsymbol{a}_{\mathrm{RS}\_A_{1z}} \\ \cdots \\ \cdots \\ \boldsymbol{a}_{\mathrm{RS}\_A_{mx}} \\ \boldsymbol{a}_{\mathrm{RS}\_A_{my}} \\ \boldsymbol{a}_{\mathrm{RS}\_A_{mz}} \end{bmatrix} \text{ and } \boldsymbol{Y}_{\mathrm{RS}} = \begin{bmatrix} \boldsymbol{Y}_{\mathrm{RS}\_A_{1x}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{RS}\_A_{1x}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{1y}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{RS}\_A_{1y}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{1z}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{RS}\_A_{1z}}/A_{\mathrm{mz}} \\ \cdots & \cdot & \cdots \\ \cdots & \cdot & \cdots \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{mx}}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{mx}}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{my}}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{my}}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{mz}}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{mz}}}/A_{\mathrm{mz}} \end{bmatrix}$$

Assembly 3: k acceleration vectors $\boldsymbol{a}_{\mathrm{RS}_{\mathrm{op}}\_k}$ and k transfer matrices $\boldsymbol{Y}_{\mathrm{RS}\_k}$ corresponding to k indicators.

$$\bar{\boldsymbol{a}}_{\mathrm{RS_{op}}} = \begin{bmatrix} \boldsymbol{a}_{\mathrm{RS}\_A_{1x}\_1} \\ \boldsymbol{a}_{\mathrm{RS}\_A_{1y}\_1} \\ \boldsymbol{a}_{\mathrm{RS}\_A_{1z}\_1} \\ \cdots \\ \boldsymbol{a}_{\mathrm{RS}\_A_{mx}\_1} \\ \boldsymbol{a}_{\mathrm{RS}\_A_{my}\_1} \\ \boldsymbol{a}_{\mathrm{RS}_{A_{mz}\_1}} \\ \cdots \\ \boldsymbol{a}_{\mathrm{RS}\_A_{1x}\_k} \\ \boldsymbol{a}_{\mathrm{RS}\_A_{1y}\_k} \\ \boldsymbol{a}_{\mathrm{RS}\_A_{1z}\_k} \\ \cdots \\ \boldsymbol{a}_{\mathrm{RS}\_A_{mz}\_k} \end{bmatrix} \text{ and } \boldsymbol{Y}_{\mathrm{RS}} = \begin{bmatrix} \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{1x}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{1x}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{1y}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{1y}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{1z}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{1z}}/A_{\mathrm{mz}} \\ \cdots & & \cdots \\ \cdots & \cdot & \cdots \\ \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{\mathrm{mx}}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{\mathrm{mx}}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{\mathrm{my}}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{\mathrm{my}}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{\mathrm{mz}}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{RS}\_1\_A_{\mathrm{mz}}}/A_{\mathrm{mz}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{Y}_{\mathrm{RS}\_k\_A_{1x}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{RS}\_k\_A_{1x}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_k\_A_{1y}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{RS}\_k\_A_{1y}}/A_{\mathrm{mz}} \\ \boldsymbol{Y}_{\mathrm{RS}\_k\_A_{1z}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{RS}\_k\_A_{1z}}/A_{\mathrm{mz}} \\ \cdots & \cdots & \cdots \\ \boldsymbol{Y}_{\mathrm{RS}\_k\_A_{\mathrm{mz}}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{RS}\_k\_A_{\mathrm{mz}}}/A_{\mathrm{mz}} \end{bmatrix}$$

D.4 Transfer matrices for dynamic force prediction

Accelerances three matrices: the bench $\boldsymbol{Y}_{\mathrm{TB}}$, the receiving structure $\boldsymbol{Y}_{\mathrm{RS}}$ and the component $\boldsymbol{Y}_{\mathrm{AC}}$.

$$\boldsymbol{Y}_{\mathrm{TB}}=\begin{bmatrix} \boldsymbol{Y}_{\mathrm{TB}\_A_{1x}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{TB}\_A_{1x}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{TB}\_A_{1y}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{TB}\_A_{1y}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{TB}\_A_{1z}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{TB}\_A_{1z}}/A_{\mathrm{m}z} \\ \cdots & \cdot & \cdots \\ \cdots & \cdot & \cdots \\ \boldsymbol{Y}_{\mathrm{TB}\_A_{\mathrm{m}x}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{TB}\_A_{\mathrm{m}x}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{TB}\_A_{\mathrm{m}y}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{TB}\_A_{\mathrm{m}y}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{TB}\_A_{\mathrm{m}z}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{TB}\_A_{\mathrm{m}z}}/A_{\mathrm{m}z} \end{bmatrix} \quad \boldsymbol{Y}_{\mathrm{RS}}=\begin{bmatrix} \boldsymbol{Y}_{\mathrm{RS}\_A_{1x}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{RS}\_A_{1x}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{1y}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{RS}\_A_{1y}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{1z}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{RS}\_A_{1z}}/A_{\mathrm{m}z} \\ \cdots & \cdot & \cdots \\ \cdots & \cdot & \cdots \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{m}x}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{m}x}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{m}y}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{m}y}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{m}z}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{RS}\_A_{\mathrm{m}z}}/A_{\mathrm{m}z} \end{bmatrix}$$

$$\boldsymbol{Y}_{\mathrm{AC}}=\begin{bmatrix} \boldsymbol{Y}_{\mathrm{AC}\_A_{1x}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{AC}\_A_{1x}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{AC}\_A_{1y}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{AC}\_A_{1y}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{AC}\_A_{1z}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{AC}\_A_{1z}}/A_{\mathrm{m}z} \\ \cdots & \cdot & \cdots \\ \cdots & \cdot & \cdots \\ \boldsymbol{Y}_{\mathrm{AC}\_A_{\mathrm{m}x}}/A_{1x} & \cdot & \boldsymbol{Y}_{\mathrm{AC}\_A_{\mathrm{m}x}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{AC}\_A_{\mathrm{m}y}}/A_{1x} & & \boldsymbol{Y}_{\mathrm{AC}\_A_{\mathrm{m}y}}/A_{\mathrm{m}z} \\ \boldsymbol{Y}_{\mathrm{AC}\_A_{\mathrm{m}z}}/A_{1x} & \cdots & \boldsymbol{Y}_{\mathrm{AC}\_A_{\mathrm{m}z}}/A_{\mathrm{m}z} \end{bmatrix}$$

Annex E
(informative)

Study of a wiper system

NOTE The cases in this annex are case studies conducted with industry on existing products.

E.1 General description of the case study

The practical and methodological interests to consider a car wiper system without arm and blades are:

— the wiper system is the location of broadband vibration. The movement of the linkage occurs at low frequency (about 1 second scan time-return). The vibrational excitation of the electric motor is around 600 Hz;

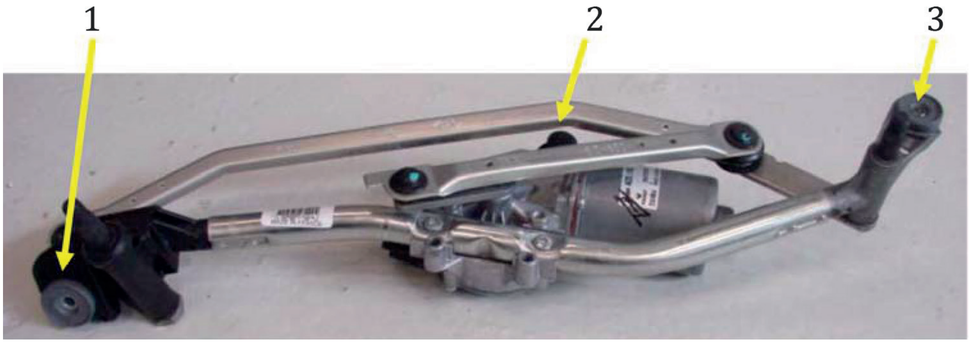
— there is need to adopt a non-stationary approach to the methodology due to the movement of the wiper linkage;

— in this case, it is not possible to mount force sensors between the wiper system and car structure. There is therefore no reference measurement when comparing the prediction result of the methodology;

— it is a three-dimensional case because there is no clear main direction of dynamic force transmission to the receiving structure;

— this case is a bit unusual with three attachment points and the points of contact of brushes with the windshield.

The active component is the wiper linkage with motor [Figure E.1 a)]. It is attached to the receiving structure, the chassis of the vehicle by resilient connecting devices at three attachment points [Figure E.1 b)], called fixing driver side (DFP) middle fixation point (MFP) and passenger fixing point (PFP). Surface transmission of forces associated to these three attachment points are not co-planar. We therefore chose to work in the local reference associated with each of its planes for force transmission.



a) Active component: wiper system

b) Receiving structure: vehicle frame

Key

1 passenger fixation point PFP

2 middle fixation point MFP

3 driver fixation point DFP

Figure E.1 — Active component and receiving structure

All recommendations transcribed in this document are followed. This is particularly the case with regard to the inverse method to estimate effort for which it is recommended that the accelerometers shall remain at the same position during transfer matrices measurements and operational measurements.

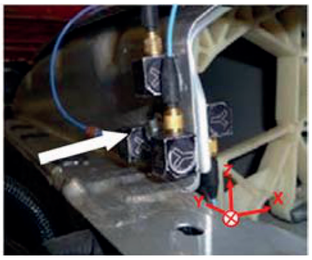
E.2 Transfer matrices measurement

In order to apply Formula (2), the chassis transfer matrix $\boldsymbol{Y}_{RS}$, the active component transfer matrix $\boldsymbol{Y}_{AC}$ and the test bench transfer matrix $\boldsymbol{Y}_{TB}$ are measured.

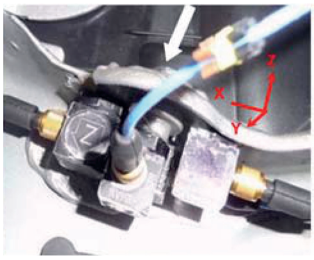
At first, the chassis transfer matrix is measured (Figure E.2). Four accelerometers are mounted at each attachment point and oriented in the associated local coordinate system. The system is excited with an impact hammer to the nearest point of attachment.

For PFP and DFP points, impact interfaces based on screw system are mounted in the receiving hole and tightened to the recommended torque (8 Nm). In this way it is believed to provide local stiffness of same order as the one of the existing system where the frame rigidity is secured on the frame. A cubic impact interface is stuck on the screw head.

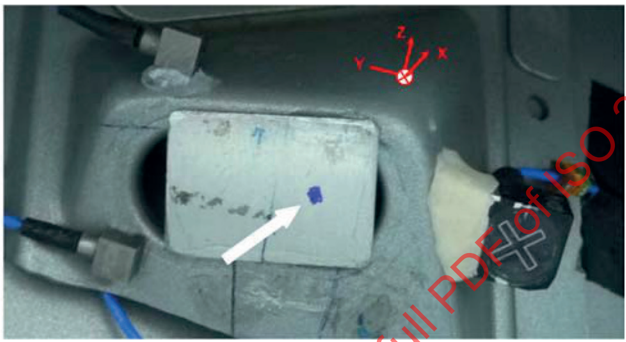
The MFP interface is bonded to a slot at the edges of a cube on which the impact interface is in turn bonded to a plate. For reasons of space linked to the movement of the linkage, the accelerometers are stuck far away from the entry point effort. We can therefore expect that reciprocity is difficult to obtain for local accelerances. The accelerances are estimated by averaging the components in each direction.



a) Driver side fixing point (DFP)



b) Passenger side fixing point (PFP)

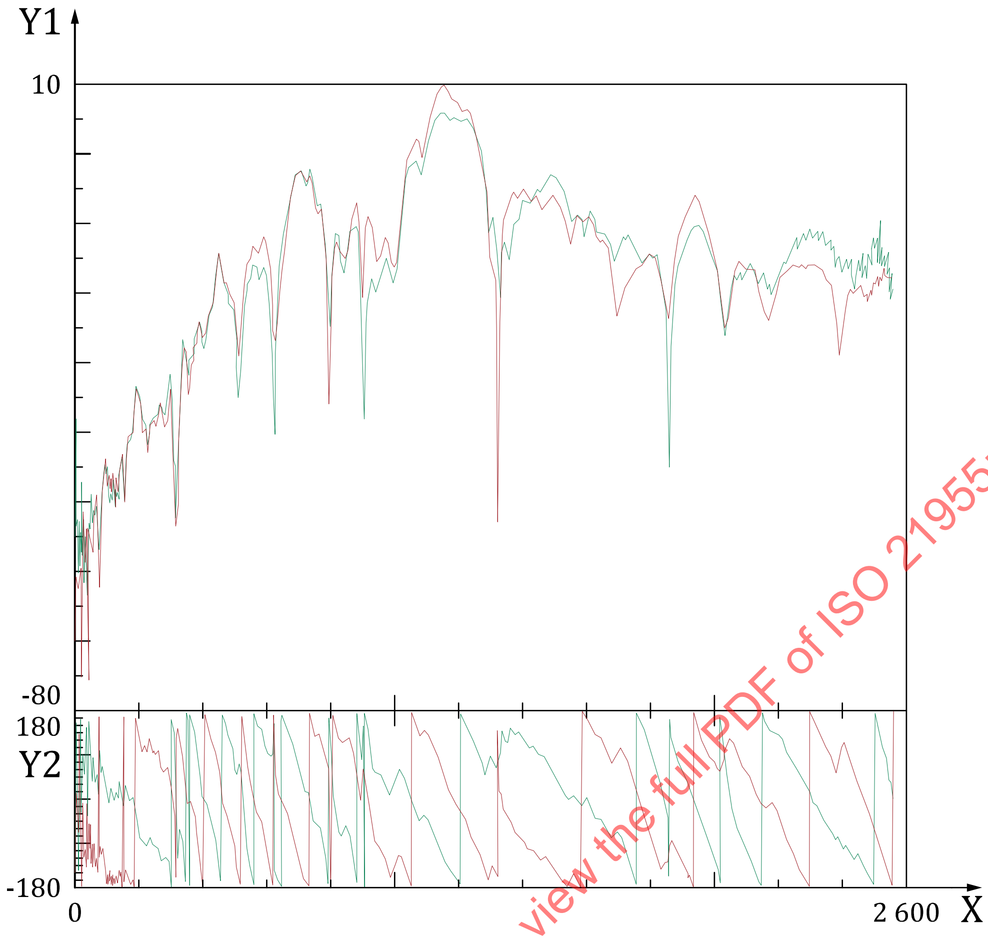


c) Medium fixing point (MFP)

NOTE The white arrows represent one out of the 3 impact force directions at each point.

Figure E.2 — Accelerance measurements of the passive component

The accelerances measurements are validated by FRF reciprocity check up to 2 kHz (direct accelerances) and up to 700 Hz for local accelerances (see Figure E.3).



Key

X frequency, expressed in Hz

Y1 reciprocity check, expressed in dB

Y2 Phase in degrees

—— DFP +Z / PFP +X

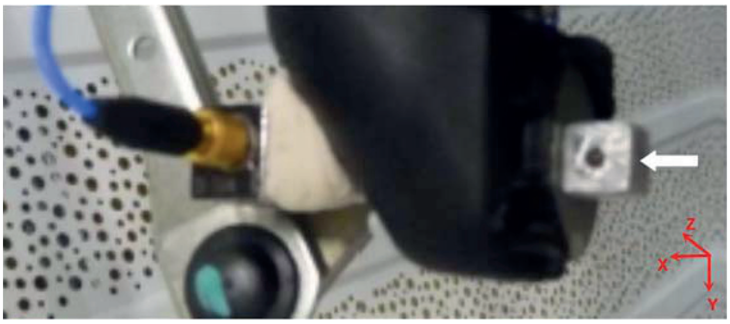
—— PFP +X / DFP -Z

Figure E.3 — Example of reciprocity check: case of remote transfer between the Z direction to the point DFP and the *X*-direction at the point PFP

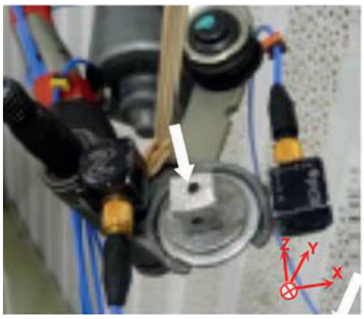
Concerning the active component, the linkage of wiper system (without wiper arms) is suspended in free-free condition, by various elastics. Three accelerometers are glued to each attachment point and oriented in the associated local coordinate system. The connecting devices, which are not considered part of the active component (no more than the passive component), are not mounted. Therefore, washers and cube strikes are glued to the point DFP and PFP (Figure E.4). At the point of MFP, the impact is applied directly to the pin pad support.

The characteristic of this active component is that it is driven by a pseudo-periodic motion. It is good to check how accelerances are affected by this movement. For this, the measurements are made at four positions in the linkage: fixed stop position, middle of opening phase, position opposite fixed stop and mid- closure phase. We note that the accelerances are affected from 200 Hz (Figure E.5). At first, we consider that transfer function is independent of wiping position and only fixed stop position measurements are considered.

The active component accelerances are validated by reciprocity check-up to 2 kHz (direct accelerances) and up to 700 Hz for local accelerances.



a) Driver side mounting point (DFP)

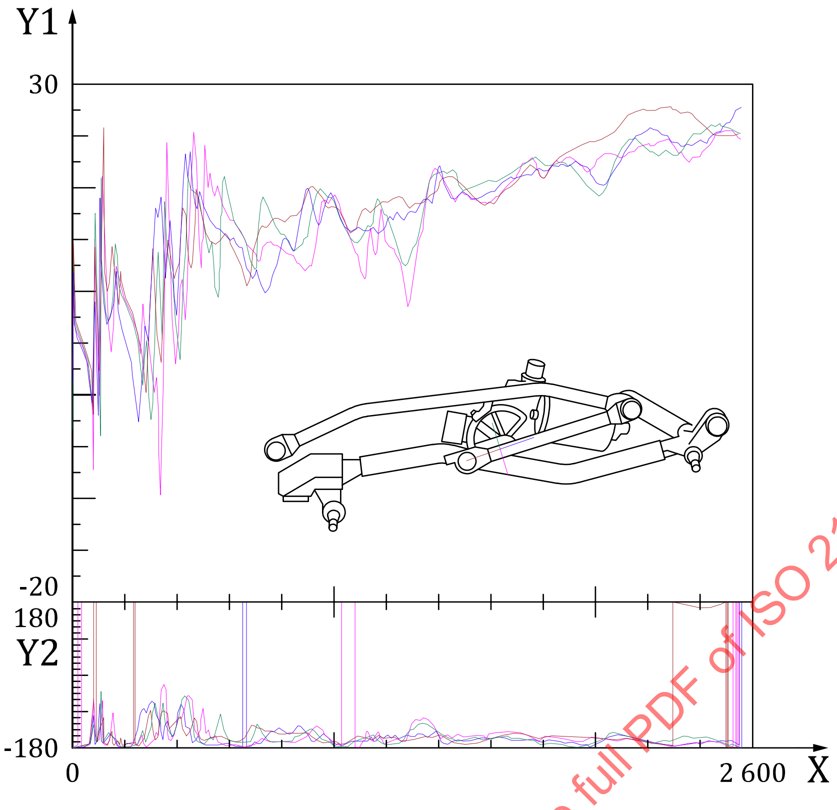


b) Passenger side mounting point (PFP)

c) Medium mounting point (MFP)

NOTE The white arrows represent one out of the 3 impact force directions at each point.

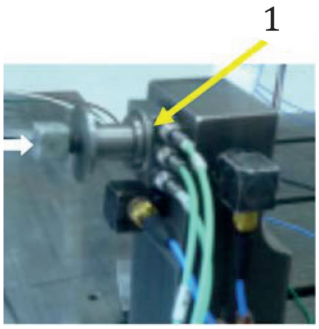
Figure E.4 — Accelerance measurements of the active component



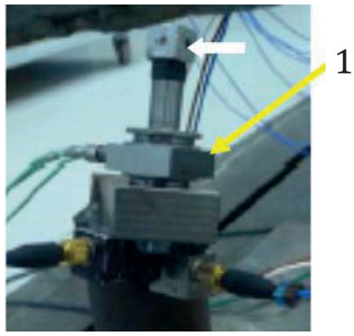
Key

X	frequency, expressed in Hz
Y1	accelerance, expressed in dB
Y2	phase in degrees
(red line)	opening phase
(green line)	InWipe zone phase in medium
(blue line)	OutWipe zone phase
(magenta line)	closing phase

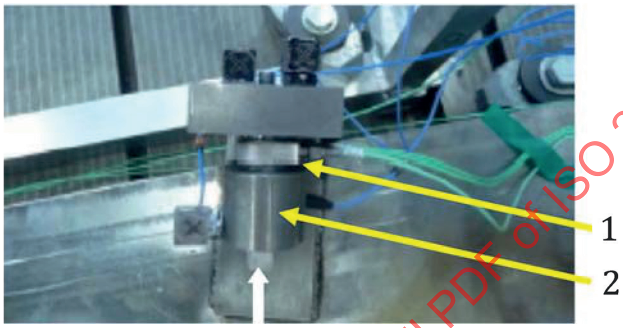
Figure E.5 — Active component accelerances for different positions of the linkage local accelerance point PFP in the X direction



a) **Passenger side fixation point (PFP)**



b) **Middle fixation point (MFP)**



c) **Driver side fixation point (DFP)**

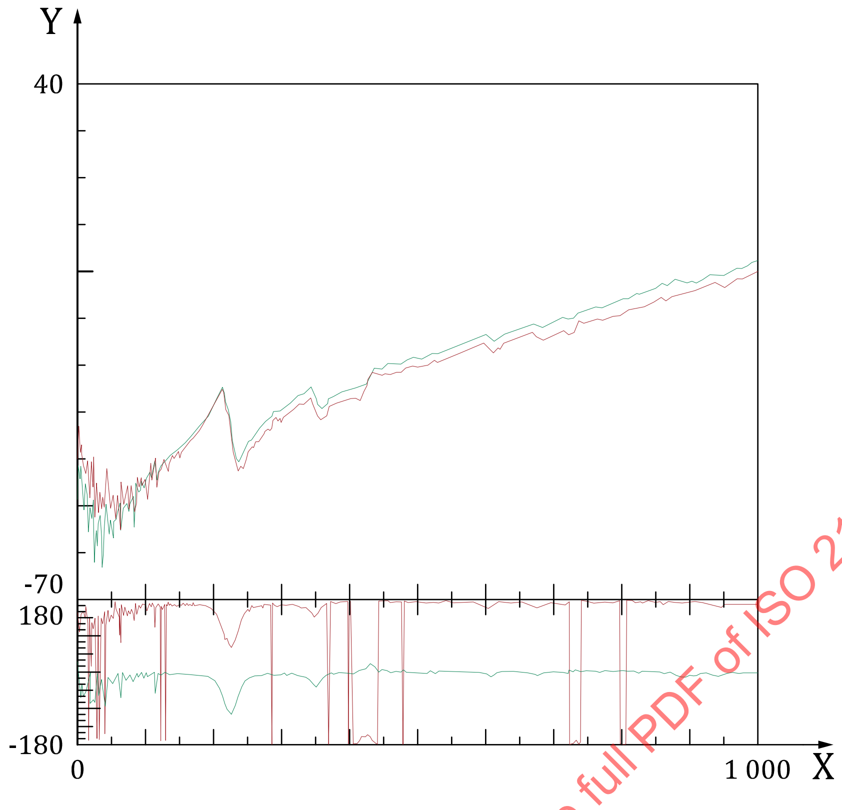
Key

1 force sensor

2 middle fixation point MFP grommet interface

NOTE The white arrows represent one out of the 3 impact force directions at each point.

Figure E.6 — Accelerance measurements on test bench



Key

X frequency, expressed in Hz

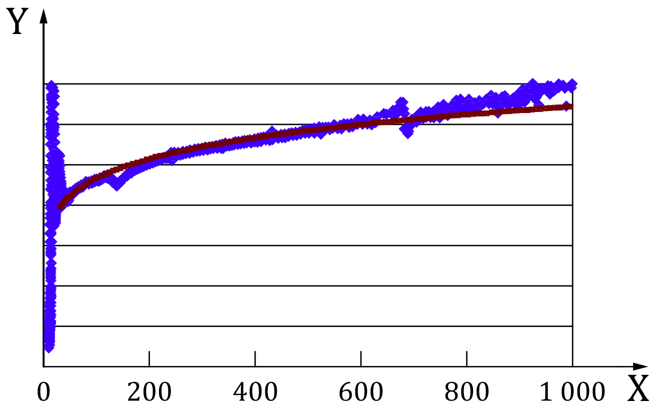
Y reciprocity check, expressed in dB

PFP +X/PFP +Z

PFP +Z/PFP -X

Figure E.7 — Example of reciprocity check: case of local transfer between the Z direction developed PFP and the X direction at the point PFP

For rigid bench accelerances, two accelerometers are glued to each attachment point and oriented in the local coordinate system associated with each point (Figure E.6). The bench is of course designed to receive the force sensors, which are therefore considered as part of the bench. The spacer studs DFP and PFP are used to be able to ensure the preload force sensors (25 kN). For MFP point, circular hollow spacer (reproducing simply the slot receiving the connecting device) has been implemented. Accelerances of the test bench are validated by reciprocity up to 2 kHz for direct accelerances and up to 1 kHz for local accelerances (Figure E.7).



Key

X frequency, expressed in Hz

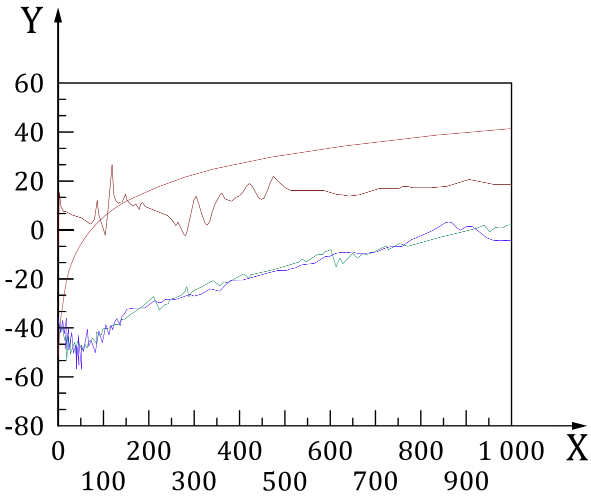
Y dynamic stiffness K_x , expressed in N/m

Figure E.8 — Example of dynamic stiffness (N/m) module in X direction measurement of a connecting device (PFP)

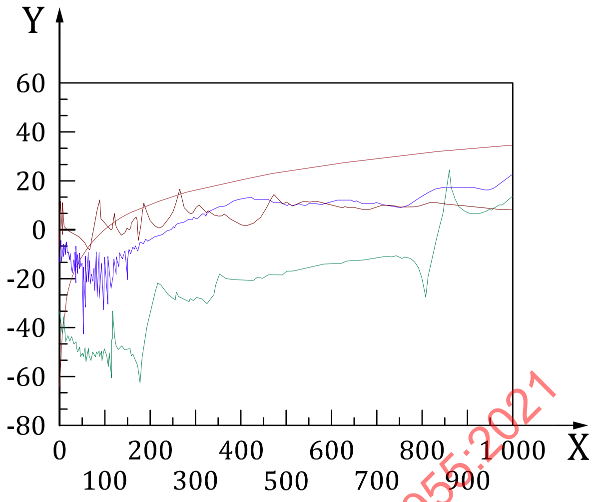
The last matrix of accelerances to take into account in Formula (2) is the equivalent term of the dynamic stiffness of the connecting devices. In this study, we choose, for practical reasons (adaptation to the frequency, ease of numerical implementation) to use estimated from measurements made by the manufacturer analytical law. The matrix should in theory be complex and is full real and diagonal here because of lack of information. There is no clear test methodology to totally and correctly estimate this matrix. An example of an equivalent term dynamic stiffness of the connecting device is given in Figure E.8.

It is interesting at this stage to assess the relative contribution of the different terms of transfer matrices (Figure E.9). Note that the equivalent term of the connecting device accelerance is prominent below 300 Hz, the prominent term being the active component $\boldsymbol{Y}_{\mathrm{AC}}$ over 300 Hz. Throughout the frequency range, we can consider the accelerances of the test bench and of the passive component as negligible.

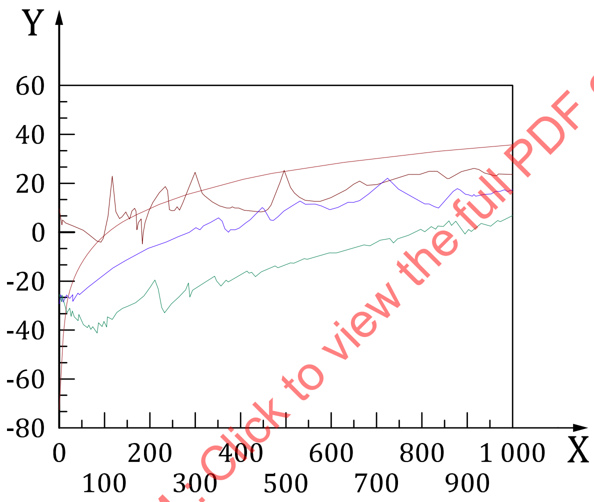
For the calculations, the raw matrices constructed from measurements are used. No properties (such as symmetry) are imposed.



a) **DFP in X direction**



b) **MFP point in Z direction**



c) **PFP point in X direction**

Key

X frequency, expressed in Hz

Y amplitude, expressed in dB

—— force sensor $\boldsymbol{Y}_{\mathrm{AC}}$

—— receiving structure accelerance $\boldsymbol{Y}_{\mathrm{RS}}$

—— test bench accelerance $\boldsymbol{Y}_{\mathrm{TB}}$

—— test bench accelerance $-\omega^2 \boldsymbol{K}_x$

Figure E.9 — Examples of accelerance

E.3 Direct measurement of the dynamic forces on test bench

$\bar{\boldsymbol{f}}_{TB}$ dynamic forces are needed for prediction on vehicle [Formula (2)]. $\bar{\boldsymbol{f}}_{TB}$ measurements are performed with direct method on the test bench. The test bench is built as to be able for the force sensor to be inserted between the active component and the test bench (Figure E.10).

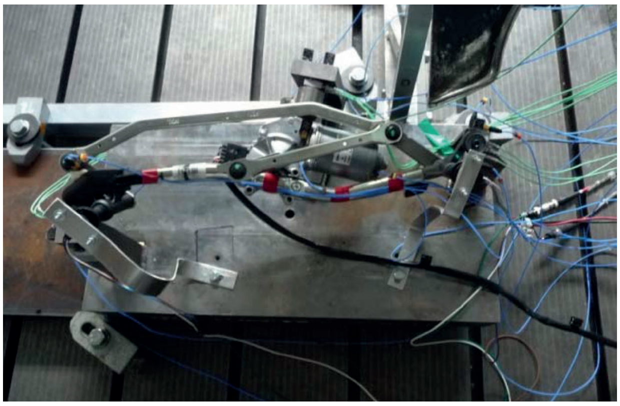
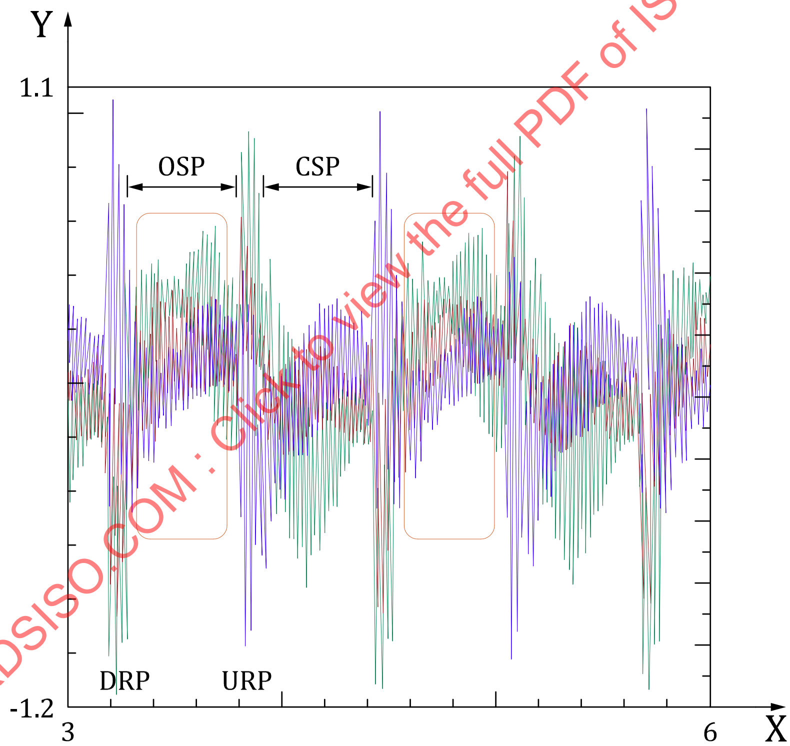


Figure E.10 — Active component mounted on the test bench clamped on the marble

While operating the wiper linkage and motor at low speed (1,25 s by wiping cycle), the motor is supplied by a stabilized voltage fixed at 13,5 V.



Key

X time, expressed in s

Y force signal, expressed in N

PFP X-Force in N

PFP Y-Force in N

PFP Z-Force

Figure E.11 — Example of time data force signals measured at test bench

The wiping cycle is divided (Figure E.11) into four phases: a fixed stop phase (DRP), an opening phase (OSP), a fixed stop opposite phase (URP) and a closing phase (CSP). In this work, we are interested only in the opening phase.

The time signals are acquired for 80 s which is about 64 cycles. Among the available signals, we chose one for which the four phases can be readily identified (Figure E.11). A semi -automatic procedure for cutting signals and selecting phases is used. A matrix of Fourier transform of the signal blocks is built.

E.4 Validation of estimated dynamic forces on bench

In this case, the conventional indirect methods for estimating efforts are used:

— the method of dynamic stiffness (see C.4) for which the dynamic stiffness matrix is used.

— the inverse method (C.3) implementing the inverse matrix of accelerance matrix of test bench. The inversion of the matrix is performed by a regularized inverse method, with a decomposition into truncated singular values associated with a technique called "L-curve."

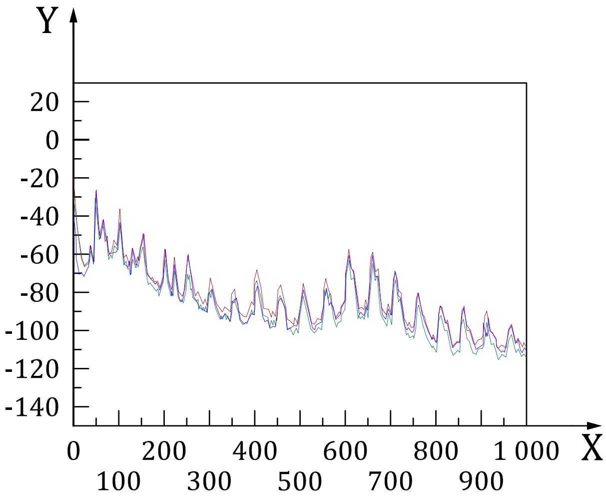
These two methods are first applied to operational data obtained on the test bench and compared with the direct measurement of forces (Figure E.11). This step shall be here considered as a validation step.

The results obtained on the test bench were very easy to post-process: the particular shape of the accelerance matrix, being almost diagonal, shall be attributed to the rigidity of the test bench which guarantees low values of cross terms accelerances.

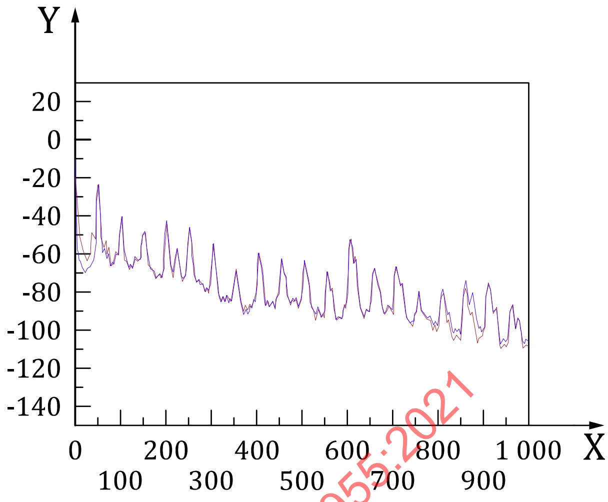
E.5 Estimated dynamic forces on vehicle

To validate the results of the methodology, the forces transmitted by the active component to the vehicle under operational conditions are estimated. In this, as in the previous sub-clause, it is necessary to measure the three attachment points accelerations simultaneously on active component (2 accelerometers glued to each attachment point) and on chassis (three accelerometers by attachment point). The operating conditions are identical to those previously chosen (motor powered with 13,5 V, low speed, no wiper arms). The same procedure of time signals post-processing is applied as in the case of operational measurements on the test bench (E.3).

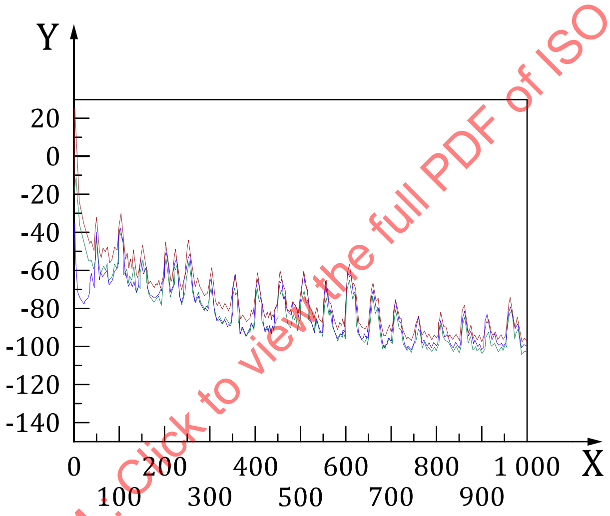
From these measurements, the dynamic forces transmitted by the active component to the chassis are estimated with the two indirect methods. These results represent the reference dynamic forces, which are compared to the predicted dynamic forces (Figure E.12).



a) DFP point at X direction



b) MFP point in Z direction



c) PFP point in X direction y curve

Key

X frequency, expressed in Hz

Y level, expressed in dB

—— Dynamic stiffness method-Force in dB ref. 1N

—— Inverse method method-Force in dB ref. 1N

—— Direct measurement method-Force in dB ref. 1N

Figure E.12 — Example of dynamic forces comparison on test bench

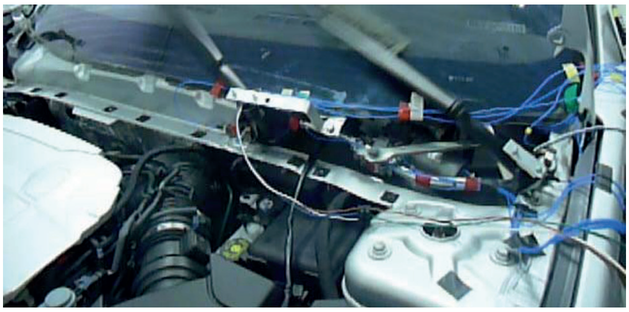


Figure E.13 — Active component mounted on the vehicle

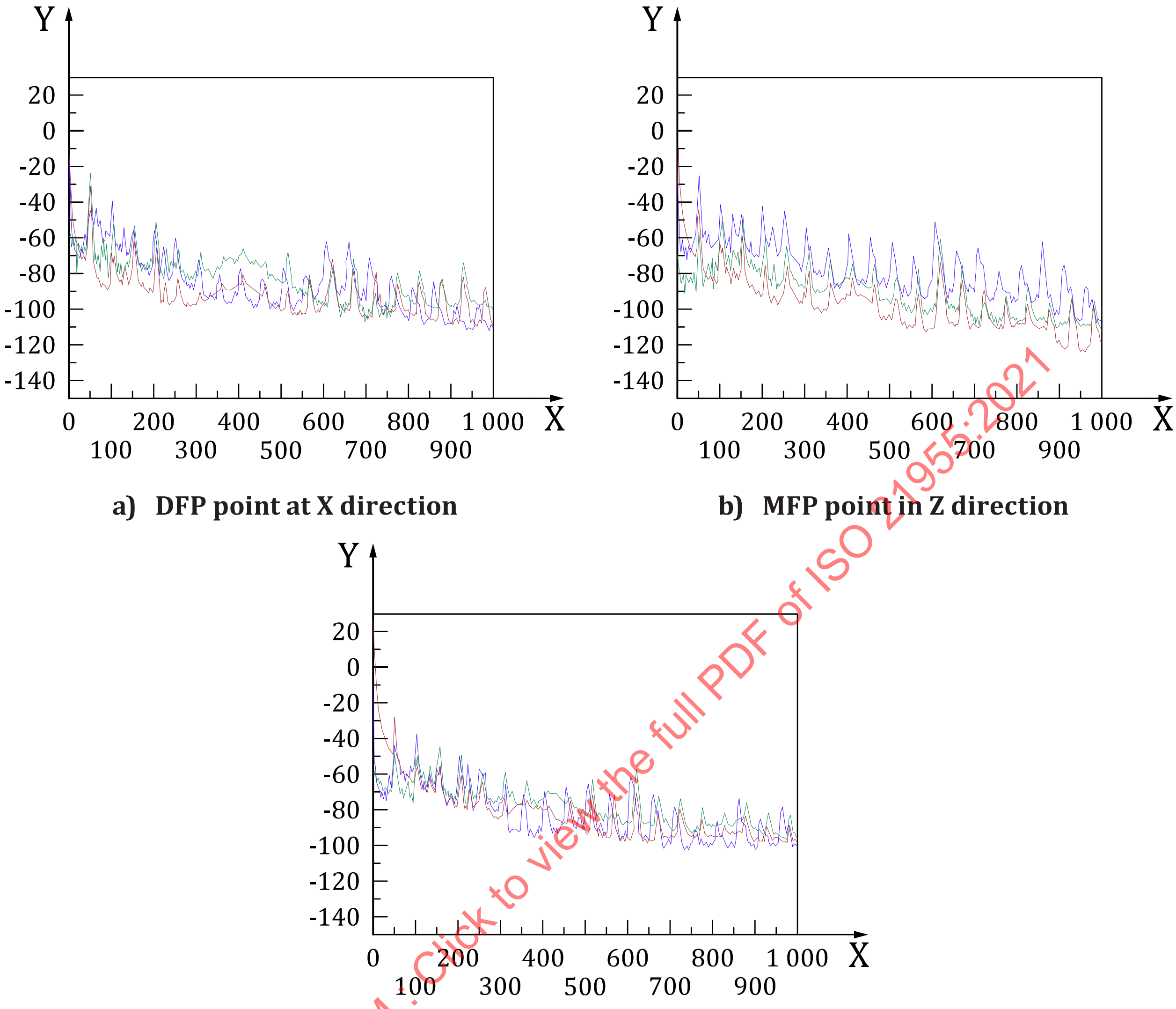
E.5.1 Prediction results

E.5.2 Validation of estimated dynamic force on bench

It should be remarked that the matrix inversion in Formula (2) is obtained using the pseudo-inverse method. The estimated results obtained are compared with reference data are not as good as hoped (Figure E.14). We note first that the estimated effort by two indirect methods is less consistent than those observed in the case of operational measurements on test bench: in this case, initial matrix was not at all diagonal and the inversion results in quite influent uncertainty.

It also should be remarked that the predicted method systematically overestimates the dynamic forces, and also that the gaps between the different methods are highly dependent on the attachment point and direction. In particular, the estimation of the forces at the midpoint (MFP) seems less accurate than elsewhere: when installing accelerometers for accelerances on chassis, there were many unknowns concerning the transmission mechanism through the connecting device.

Finally, a frequency offset between the different methods: predicted force on test bench were not performed at the same time, leading to a slight shift of motor rotating speed during the tests.



a) **DFP point at X direction**

b) **MFP point in Z direction**

c) **PFP point in X direction**

Key

X frequency, expressed in Hz

Y level, expressed in dB

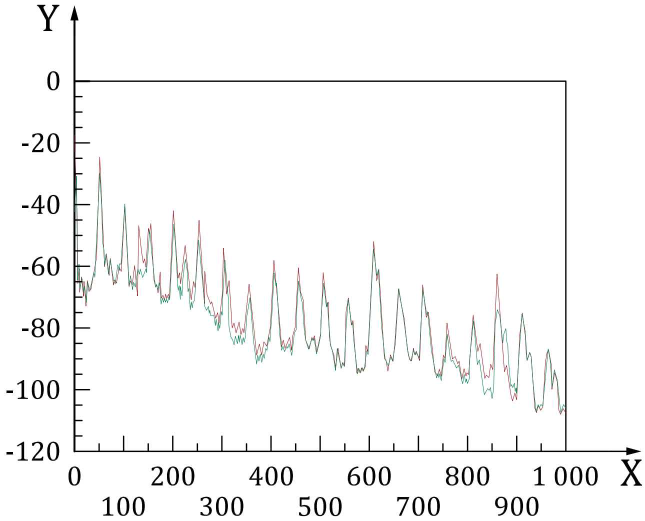
(red line) Dynamic stiffness method-Force in dB ref. 1N

(green line) Inverse method method-Force in dB ref. 1N

(purple line) Direct measurement method-Force in dB ref. 1N

Figure E.14 — Example of predicted results of dynamic forces

To conclude, for frequency ranges in which accelerances of test bench and host vehicle are negligible, then the predicted dynamic forces on vehicle are equal to the measured dynamic forces on bench (Figure E.15).



Key

X frequency, expressed in Hz

Y level, expressed in dB

—— F_{TB} in dB ref. 1 N

—— F_{RS} in dB ref. 1 N

Figure E.15 — Example of comparison between measured dynamic force on bench and predicted dynamic force on vehicle at MFP point in *Z* direction