# INTERNATIONAL STANDARD

# ISO 19927

First edition
2018-08

# Fibre-reinforced plastic composites — Determination of interlaminar strength and modulus by double beam shear test

*Composites plastiques renforcés de fibres — Détermination de la résistance interlaminaire et du module par un double essai de cisaillement de faisceau*

# Contents

# Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights. Details of any patent rights identified during the development of the document will be in the Introduction and/or on the ISO list of patent declarations received (see www.iso.org/patents).

Any trade name used in this document is information given for the convenience of users and does not constitute an endorsement.

For an explanation on the voluntary nature of standards, the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the World Trade Organization (WTO) principles in the Technical Barriers to Trade (TBT) see the following URL: www.iso.org/iso/foreword.html.

This document was prepared by Technical Committee ISO/TC 61, *Plastics*, Subcommittee SC 13, *Composites and reinforcement fibres.*

Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

# Introduction

Interlaminar shear (ILS) properties of load-bearing composite laminates are important for structural design, stress analysis and numerical modelling, material development and selection, component manufacture as well as for repair of delaminated structural components. A number of test methods have been developed for the determination of shear properties of composite laminates and several have been standardized, including short beam strength (SBS) [ASTM D2344, ISO 14130, EN 2377[1]) and EN 2563[1])], V-notched beam (known also as Iosipescu shear), ASTM D5379, V-notched rail (ASTM D7078) and notched tension (ASTM D3846, BS 4994) or notched compression (BS 6464).

SBS methods are very popular due to their simplicity in specimen preparation, test operation and cost-effectiveness, though they are used to determine only an apparent ILS strength for QA purposes rather than for use in design. However, a loaded SBS specimen does not have a pure ILS region within its gauge section so it often fails in a number of modes, dependent on, amongst others, type of composite materials, lay-up and specimen thickness. Unacceptable SBS failure modes include flexural failure, local crushing (under central loading roller) and plastic deformation and roller-induced through-thickness shear band.

The DBS method is based on loading a composite beam on double support spans (i.e. between 5 loading points), which generates a stress state that minimises axial bending stresses while promoting interlaminar shear stresses. In DBS tests delamination occurred consistently within one of the pure ILS regions in the specimens, for a range of laminated composite materials or lay-ups.

Comparisons with the SBS ILS strength results indicated that the DBS ILS strengths were 20 % to >30 % higher. In addition, a value of the interlaminar shear modulus can be obtained, providing values of $E_{11}$, $E_{22}$, $G_{12}$ and $\nu_{12}$ are known or measured separately for the material under test.

---

1)  CEN Aerospace series.

v

# Fibre-reinforced plastic composites — Determination of interlaminar strength and modulus by double beam shear test

## 1 Scope

This document specifies a procedure for determining the interlaminar strength, and modulus, by double beam shear (DBS) tests of fibre-reinforced plastic composites.

The method is suitable for use with glass or carbon fibre-reinforced plastic composites with thermoset matrices, providing an acceptable interlaminar failure is obtained.

This document only applies to laminate with a symmetrical and balanced lay-up as it avoids bending/twisting or bending/extension coupling deformations (see 6.2). The preferred lay-up is unidirectional, with fibres aligned along the specimen (axial) length.

The suitability of the test for specific lay-ups, matrices and fibres (e.g. natural) are assessed by trial tests to ensure the correct delamination failures are obtained.

## 2 Normative references

The following documents are referred to in the text in such a way that some or all of their content constitutes requirements of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 291, *Plastics — Standard atmospheres for conditioning and testing*

ISO 472, *Plastics — Vocabulary*

ISO 1268 (all parts), *Fibre-reinforced plastics — Methods of producing test plates*

ISO 2602, *Statistical interpretation of test results — Estimation of the mean — Confidence interval*

ISO 5893, *Rubber and plastics test equipment — Tensile, flexural and compression types (constant rate of traverse) — Specification*

ISO 16012, *Plastics — Determination of linear dimensions of test specimens*

## 3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 472 and the following apply.

ISO and IEC maintain terminological databases for use in standardization at the following addresses:

— ISO Online browsing platform: available at https://www.iso.org/obp

— IEC Electropedia: available at http://www.electropedia.org/

**3.1**
**critical load**
$F_{crit}$
load at delamination, as defined in 9.5 for acceptable failure modes

Note 1 to entry: See Cases 1-3.

Note 2 to entry: It is expressed in newtons (N).

### 3.2
### interlaminar shear strength
$\tau_{13\ max}$
shear strength calculated from the critical load

Note 1 to entry: See 10.1.

Note 2 to entry: It is expressed in megapascals (MPa).

### 3.3
### interlaminar shear modulus
$G_{13}$
shear modulus calculated from the slope of the load-displacement curve

Note 1 to entry: See 10.2.

Note 2 to entry: It is expressed in gigapascals (GPa).

### 3.4
### span
$L$
distance between the outermost support rollers

Note 1 to entry: It is expressed in millimetres (mm).

### 3.5    Specimen coordinate axes

NOTE 1    The direction parallel with the plate longitudinal axis is the "1" direction and the direction perpendicular, is the "2" direction (see ISO 1268-4). The direction "3" is perpendicular to the plate (i.e. the through-thickness direction).

NOTE 2    See Figure 1.

### 3.5.1
### longitudinal tensile modulus
$E_{11}$
tensile modulus in the specimen longitudinal or 1 direction

Note 1 to entry: It is expressed in gigapascals (GPa).

### 3.5.2
### transverse tensile modulus
$E_{22}$
tensile modulus in the perpendicular or 2 direction

Note 1 to entry: It is expressed in gigapascals (GPa).

### 3.5.3
### in-plane shear modulus
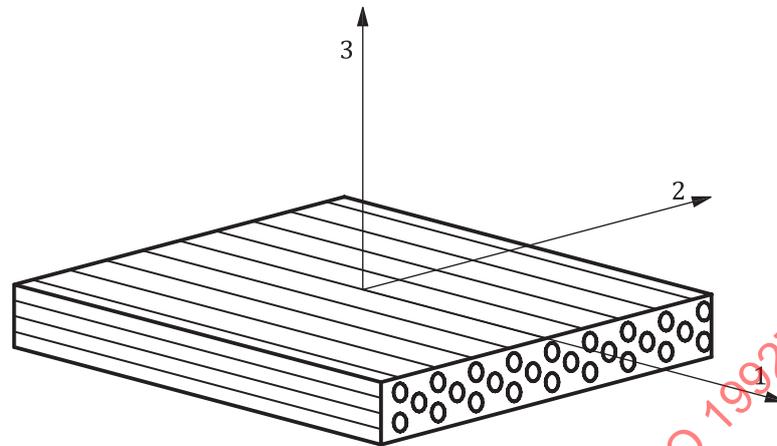$G_{12}$
shear modulus in the 1-2 plane

Note 1 to entry: It is expressed in gigapascals (GPa).

### 3.5.4
### in-plane poisson ratio
$\nu_{12}$
Poisson's ratio in the 1-2 plane

Note 1 to entry: It is dimensionless.

Note 2 to entry: Properties defined in 3.5.1 to 3.5.4 are required for calculation of the interlaminar shear modulus using either manufacturer's or measured data (see 10.2).



**Key**

1    direction parallel with the plate longitudinal axis
2    direction perpendicular with the plate longitudinal axis
3    direction perpendicular to the plate (i.e. the through-thickness direction, with plies layered perpendicular to this direction)

**Figure 1 — Unidirectional reinforced composites plate element showing symmetry axes**

## 4   Principle

A test specimen consisting of a beam of rectangular cross-section is loaded in 5-point flexure, as shown in Figure 2. The applied load generates regions of pure interlaminar shear stress, as indicated in Figure 2. The critical load sustained by the specimen when delamination failure occurs is used to determine the interlaminar shear strength. In addition, the deflection of the specimen under load is used to determine the interlaminar shear modulus. In order to calculate the interlaminar shear modulus, it is necessary to know or measure the following properties: $E_{11}$, $E_{22}$, $G_{12}$ and $\nu_{12}$.
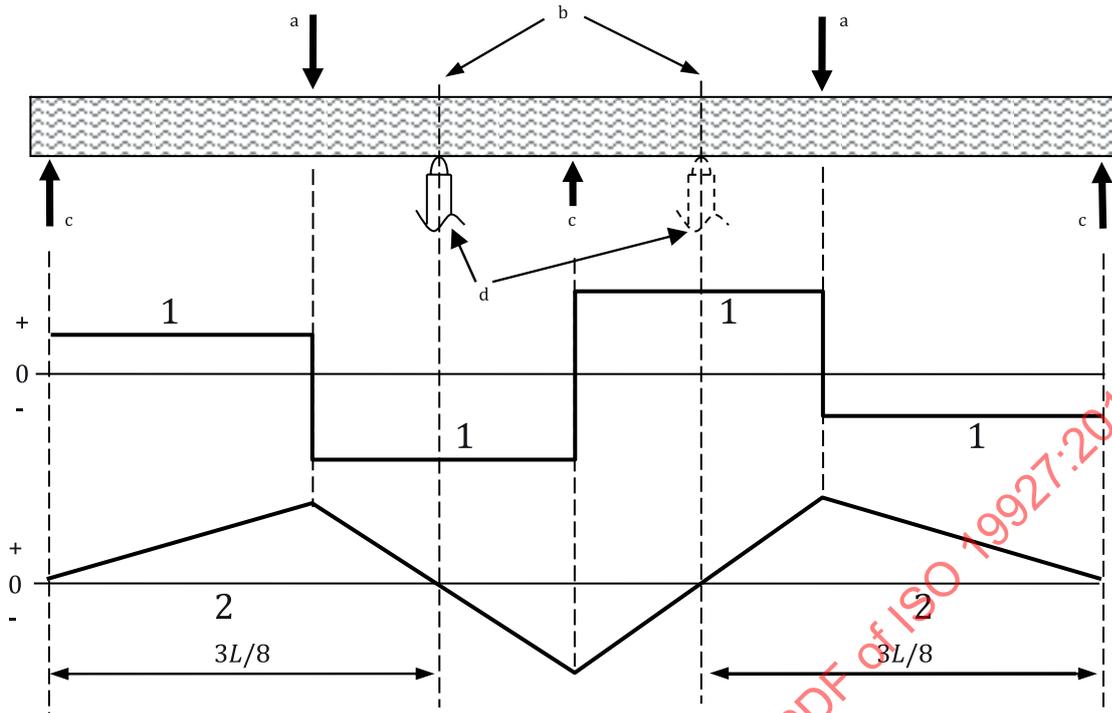
**Key**

1    ILS stress
2    bending stress
a    Load.
b    Sections of pure shear.
c    Support.
d    Displacement measurement points.

**Figure 2 — Stress distributions in DBS specimen**

## 5    Apparatus

### 5.1    Test machine

#### 5.1.1    General

The **test machine** conforms to ISO 5893 as appropriate to the requirements given in 5.1.2 to 5.1.3.

#### 5.1.2    Speed of testing, $v$

It shall be kept constant according to ISO 5893.

#### 5.1.3    Indicator for load

The error in the indicated force is less than ±1 % of the load to be measured (see Class 1 of ISO 7500-1).

**5.2    Micrometer or equivalent**, capable of reading to 0,01 mm, or less, and suitable for measuring the thickness, $h$ and width, $b$ of the test specimen. The micrometer shall have faces appropriate to the surface being measured (i.e. flat faces for flat, polished surfaces and hemispherical faces for irregular surfaces).

**5.3    Loading fixture**, a 5 point flexure jig with adjustable spans, as shown schematically in Figure 3. The loading/support rollers are uniformly positioned within the span (tolerance ± 0,2 mm), as shown in Figure 4. The loading/support rollers have a contact diameter of 6 mm (tolerance 0 mm to −0,1 mm). A typical set-up is shown in Figure 5.
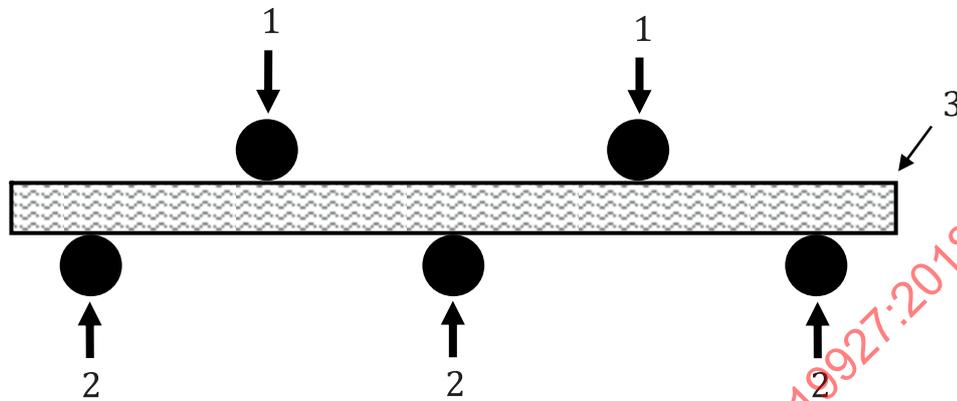


**Key**

1    loading rollers
2    support rollers
3    specimen

**Figure 3 — Schematic of double-beam shear fixture**



a) Side elevation          b) End elevation
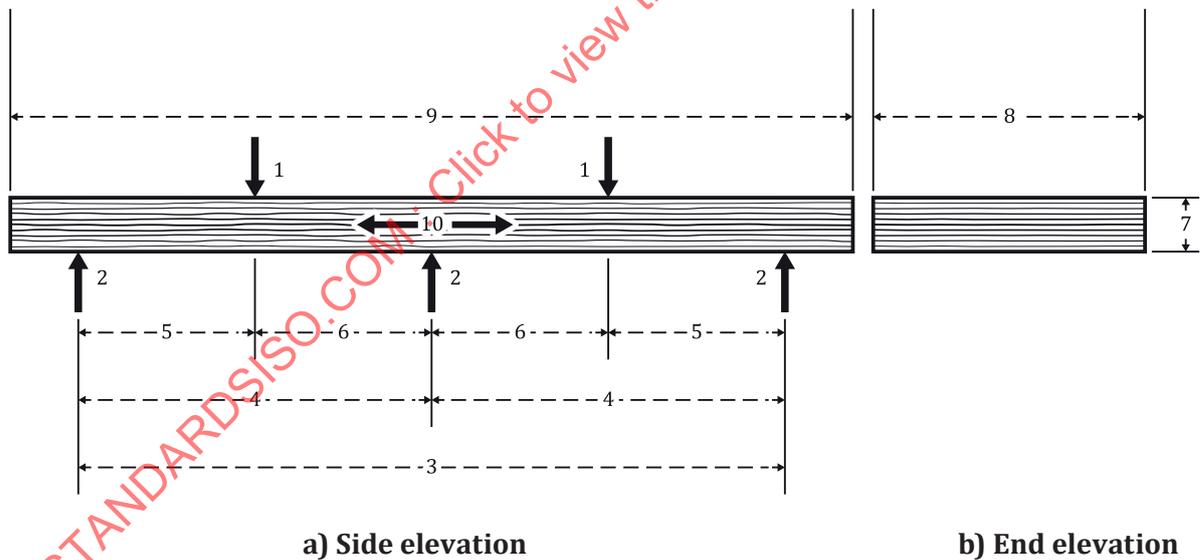
**Key**

1    loading points
2    support points
3    span
4    1/2 span
5    outer region

6    inner region
7    specimen thickness
8    specimen width
9    specimen length
10   Fibre direction (unidirectional specimen)

**Figure 4 — Test specimen and rollers lay-out for determination of interlaminar modulus and strength**
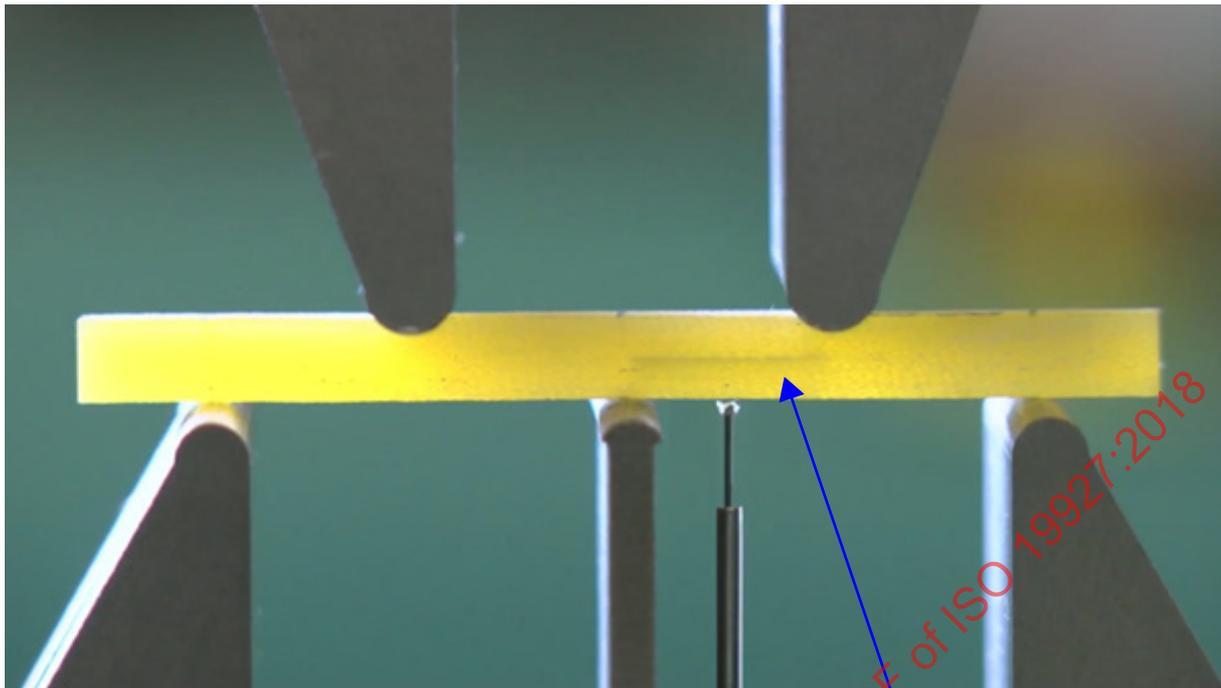
**Key**

1   interlaminar crack

**Figure 5 — Test specimen mounted in double-beam shear fixture used for strength and modulus determination**

## 5.4   Displacement measurement

A suitable displacement transducer (e.g. a DVRT-type of LVDT or single arm extensometer) is used to measure the deflection of the specimen at the mid-point of the inner span (i.e. 0,375 L from an outer support position). The equipment shall be calibrated for displacement. The unit should record continuously the displacement of the lower surface of the specimen to ±1 % of the displacement to be measured (i.e. Class 1 of ISO 9513). The displacement measurement equipment shall be calibrated and traceable.

## 6   Test specimens

### 6.1   Shape and dimensions

#### 6.1.1   Preferred specimen size

Specimen A shall have a width, $b$, of 10 mm ± 0,2 mm, a thickness, $h$, of 2 mm ± 0,1 mm and a length, $l$, of 30 mm. This is the same cross-section as used in ISO 14130.

Specimen B shall have a width, $b$, of 8 mm ± 0,2 mm, a thickness, $h$, of 4 mm ± 0,1 mm and a length, $l$, of 60 mm.

In cases of dispute, specimen A shall be used for strength measurements.

### 6.1.2 Alternative specimen size

Alternative specimens shall maintain the same specimen width/thickness ratio of 5 and the same specimen length/thickness ratio of 15 of the preferred specimen A.

## 6.2 Preparation of specimens

### 6.2.1 General

A test plate shall be prepared in accordance with ISO 1268 (all parts) or other specified/agreed procedure sufficient to manufacture a batch of test specimens (5 minimum), together with additional spare specimens (see 9.5). The laminate shall have a symmetrical and balanced lay-up to avoid bending/twisting or bending/extension coupling deformations. The plate configuration shall be normally fully unidirectional (UD) or 0/90 fabric, if not given by the materials specification or as agreed by the interested parties. Alternative configurations, such as multiaxial, are acceptable providing the required failure mode is obtained. The 0°, x or axial direction in the plate shall be marked.

### 6.2.2 Machining the test specimens

The test specimen shall be machined without causing damage. Guidance is given on cutting specimens in ISO 2818 and ISO 527-4:1997, Annex A.

## 6.3 Checking the test specimens

The specimens shall be flat and free of twist. The surfaces and edges shall be free from cracks, scratches, pits, sink marks and flashes. The specimens shall be checked if they comply with these requirements by visual observation against straight-edges, squares and flat plates, and by measuring using micrometer calipers. Specimens showing measurable or observable departure from one or more of these requirements shall be rejected or machined to the required size and shape before testing. The thickness along the length of the specimen shall be within ±5 % of the mean thickness.

# 7 Number of test specimens

At least five test specimens shall be tested and acceptable failures achieved (see 9.6 for checking valid failure modes).

The number of measurements may be more than five if greater precision of the mean value is required. It is possible to evaluate this by means of the confidence interval (95 % probability, see ISO 2602).

# 8 Conditioning

Where applicable, condition the test specimen as specified in the International Standard for the material under test. In the absence of such information, the most appropriate condition from ISO 291 shall be selected, unless otherwise agreed by the interested parties.

# 9 Procedure

## 9.1 Test conditions

Where applicable, the specimen shall be tested as specified in the International Standard for the material under test. In the absence of such information, the most appropriate condition from ISO 291 shall be selected, unless otherwise agreed by the interested parties (e.g. for testing at elevated or reduced temperatures).

## 9.2 Specimen dimensions

Measure the width and the thickness of the specimen to the nearest 0,02 mm at the mid-point of the test span, according to ISO 16012.

Set the test span to 10× thickness for Type A (i.e. 20 mm) and Type B specimens (i.e. 40 mm), as appropriate to the type of specimen to be tested.

## 9.3 Testing speed

Where applicable, set the speed of testing as given in the International Standard for the material being tested. In the absence of this information, the speed of testing, $v$, shall be 1 mm/min. See ISO 5893.

## 9.4 Data collection

Record the load and displacement continuously throughout the test, as shown in Figure 6.

## 9.5 Failure mode acceptance

Check and record the mode of failure as shown in Figure 7.

Test specimens shall fail by an interlaminar failure on or near the specimen mid-plane. Failure in Cases 1 to 3 is acceptable.

Failures in Cases 4 to 7 are unacceptable and these results should be excluded. Replacement specimens shall be tested in their place. The number of discarded specimen failures shall be recorded.

## 9.6 Critical load

Record the critical load, as the failure load, as indicated in Figures 6 and 7.

## 9.7 Slope of load-displacement curve

From the load-displacement curve calculate the slope of the secant to the initial linear response, as shown in Figure 6.
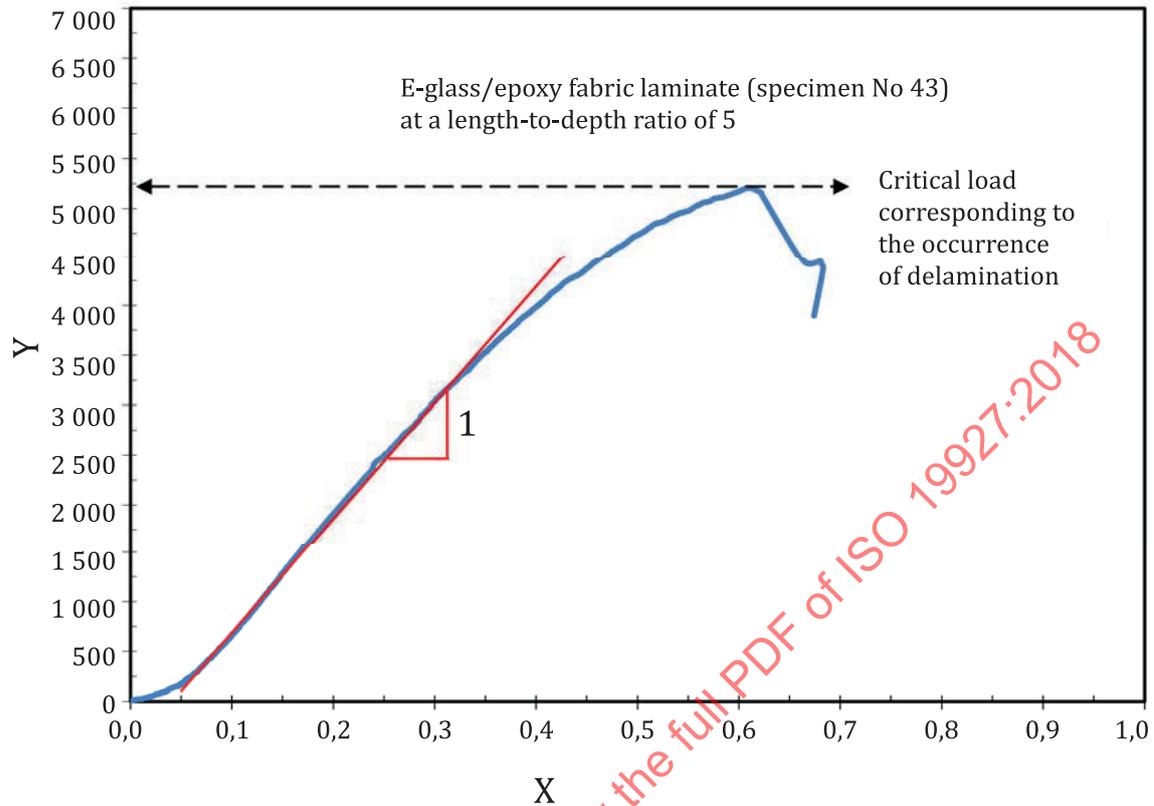
**Key**

Y    load (N)

X    deflection (mm)

1    slope

**Figure 6 — Typical load-displacement trace showing slope of initial tangent**

# 10 Calculation and expression of results

## 10.1 Interlaminar shear strength

Calculate the interlaminar shear strength, $\tau_{13max}$, using Formula (1).

$$\tau_{13\,max} = 33F_{crit}/64bh \qquad (1)$$

where

$F_{crit}$ is the critical load, in newtons;

$b$    is the specimen width, in millimetres;

$h$    is the specimen thickness, in millimetres.

Calculate the arithmetic mean of the individual determinations and, if required, the standard deviation and the 95 % confidence interval of the mean value according to the procedure given in ISO 2602.

Calculate the stress to three significant figures.

## 10.2 Interlaminar shear modulus

Calculate the interlaminar shear modulus, $G_{13}$ using Formula (2).

EXAMPLE

The values of the constants in Formulae (2) to (5) are given for a type B specimen of Material A for values of $E_{11}$ = 159 GPa; $E_{22}$ = 8,5 GPa; $G_{12}$ = 4,6 GPa, $v_{12}$ = 0,32, L = 42 mm, b = 9,42 mm and $h$ = 4,11 mm.[2]

| Term | Description | Units | Nominal values for Specimen B |
|---|---|---|---|
| $D_{11}^{-1}$ | first diagonal term of the inverted bending stiffness matrix of the beam specimen | $(\text{GPa}\cdot\text{mm}^3)^{-1}$ | 0,002 782 |
| $C_1$ | Constant | mm/kN | 1,708 6 |
| $C_2$ | Constant | 1/mm | 0,010 2 |

NOTE     Further details on the derivation of this analysis are given in Annex A.

$$G_{13} = \frac{C_2\left(1-\frac{43}{s}C_1\right)}{16C_1\left(1-\frac{25}{8s}C_1\right)}\left(-1\pm\sqrt{1+\frac{128C_1\left(1-\frac{25}{8s}C_1\right)}{s\left(1-\frac{43}{s}C_1\right)^2}}\right) \tag{2}$$

where $s$ denotes reciprocal of the slope of the load-deflection curve,

$$C_1 = \frac{L^3}{12\,288\sum_{i=1}^{n}(E_iI_i)} \tag{3}$$

$$C_2 = \frac{L}{128k_sA} \tag{4}$$

and

$$\sum_{i=1}^{n}(E_iI_i) = \frac{b}{D_{11}^{-1}} \tag{5}$$

where

  $A$     is the cross-sectional area ($bh$);

  $(E_iI_i)$   is the flexural rigidity of the beam specimen;

  $D_{11}^{-1}$   is the first diagonal term of the inverted bending stiffness matrix of the beam specimen.

Calculate the arithmetic mean of the individual determinations and, if required, the standard deviation and the 95 % confidence interval of the mean value according to the procedure given in ISO 2602.

Calculate the modulus to three significant figures.

---

2)   A convenient calculator for these values is available at http://www.lboro.ac.uk/enterprise/double-beam-shear/. This information is given for the convenience of users of this document and does not constitute an endorsement by ISO of the product named.

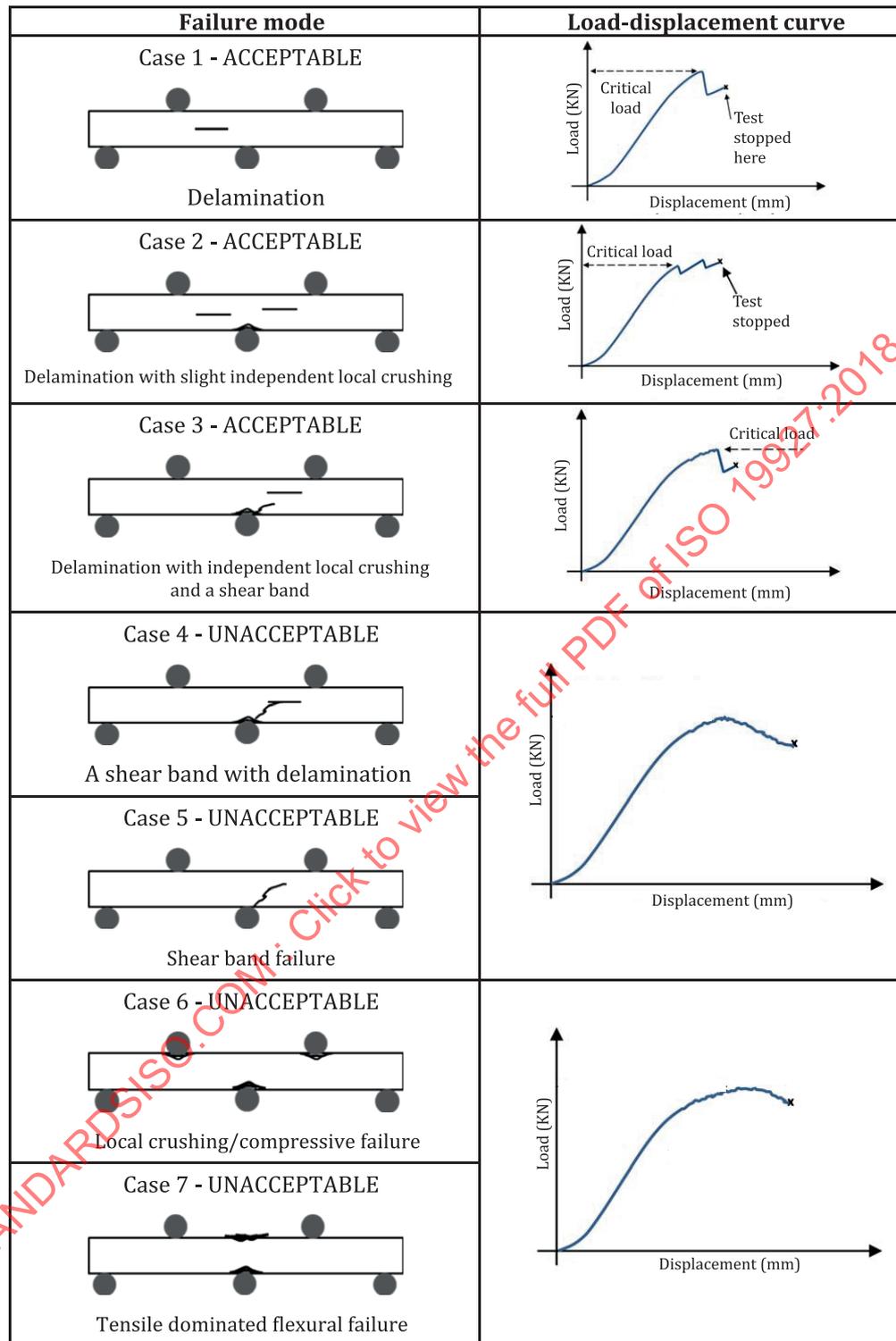| Failure mode | Load-displacement curve |
|---|---|
| Case 1 - ACCEPTABLE <br><br> Delamination | |
| Case 2 - ACCEPTABLE <br><br> Delamination with slight independent local crushing | |
| Case 3 - ACCEPTABLE <br><br> Delamination with independent local crushing and a shear band | |
| Case 4 - UNACCEPTABLE <br><br> A shear band with delamination | |
| Case 5 - UNACCEPTABLE <br><br> Shear band failure | |
| Case 6 - UNACCEPTABLE <br><br> Local crushing/compressive failure | |
| Case 7 - UNACCEPTABLE <br><br> Tensile dominated flexural failure | |

**Figure 7 — Acceptable and unacceptable failure modes and associated load-displacement graphs**

## 11 Precision

The precision of this test method obtained in an interlaboratory study is summarized below. The study involved 7 test sites and three different composite materials:

a)   UD carbon-fibre/epoxy ;

b) Carbon-fibre/epoxy in plain weave;

c) E-glass-fibre/epoxy in 8 harness satin weave.

Further details are given in Annex B. See also Tables 1 to 4.

**Table 1 — Repeatability, reproducibility and mean interlaminar shear strength for the three materials**

| Material | Thickness mm | Repeatability conditions | | Reproducibility conditions | | Mean, $\tau_{13max}$ |
|---|---|---|---|---|---|---|
| | | $S_r$ | $r$ | $S_R$ | $R$ | MPa |
| A. Carbon/epoxy 1 | 2 | 4,2 | 11,6 | 6,3 | 17,5 | 138,3 |
| | 4 | 2,6 | 7,2 | 3,0 | 8,5 | 131,7 |
| B. Carbon/epoxy 2 | 2 | 3,1 | 8,7 | 3,6 | 10,2 | 86,0 |
| | 4 | 2,8 | 7,9 | 4,1 | 11,5 | 87,2 |
| C. E-glass/epoxy | 2 | 1,3 | 3,6 | 2,4 | 6,9 | 61,7 |
| | 4 | 0,7 | 2,1 | 1,4 | 3,8 | 59,2 |

**Table 2 — Repeatability and reproducibility values as percentage of mean ILS strength values for the three materials**

| Material | Thickness mm | Repeatability conditions | | Reproducibility conditions | |
|---|---|---|---|---|---|
| | | $S_r$ (%) | $r$ (%) | $S_R$ (%) | $R$ (%) |
| A. Carbon/epoxy 1 | 2 | 3,0 | 8,4 | 4,6 | 12,7 |
| | 4 | 2,0 | 5,5 | 2,3 | 6,5 |
| B. Carbon/epoxy 2 | 2 | 3,6 | 10,1 | 4,2 | 11,9 |
| | 4 | 3,2 | 9,1 | 4,7 | 13,2 |
| C. E-glass/epoxy | 2 | 2,1 | 5,8 | 3,9 | 11,2 |
| | 4 | 1,2 | 3,5 | 2,4 | 6,4 |

**Table 3 — Repeatability, reproducibility and mean interlaminar shear modulus for the three materials**

| Material | Thickness mm | Repeatability conditions | | Reproducibility conditions | | Mean, $G_{13}$ |
|---|---|---|---|---|---|---|
| | | $S_r$ | $r$ | $S_R$ | $R$ | GPa |
| A. Carbon/epoxy 1 | 2 | 0,3 | 1,0 | 0,6 | 1,6 | 2,1 |
| | 4 | 0,2 | 0,4 | 0,2 | 0,5 | 2,1 |
| B. Carbon/epoxy 2 | 2 | 0,2 | 0,6 | 0,4 | 1,1 | 2,1 |
| | 4 | 0,2 | 0,5 | 0,2 | 0,7 | 2,1 |
| C. E-glass/epoxy | 2 | 0,2 | 0,6 | 0,3 | 0,7 | 1,9 |
| | 4 | 0,1 | 0,4 | 0,8 | 2,2 | 2,1 |

**Table 4 — Repeatability and reproducibility values as percentage of mean interlaminar shear modulus values for the three materials**

| Material | Thickness mm | Repeatability conditions | | Reproducibility conditions | |
|---|---|---|---|---|---|
| | | $S_r$ (%) | $r$ (%) | $S_R$ (%) | $R$ (%) |
| A. Carbon/epoxy 1 | 2 | 15,8 | 44,3 | 26,7 | 74,8 |
| | 4 | 7,6 | 21,3 | 8,8 | 24,7 |
| B. Carbon/epoxy 2 | 2 | 9,6 | 27,0 | 17,7 | 49,7 |
| | 4 | 8,7 | 24,3 | 11,9 | 33,3 |
| C. E-glass/epoxy | 2 | 11,1 | 31,0 | 14,1 | 39,4 |
| | 4 | 6,8 | 19,1 | 37,7 | 105,7 |

## 12 Test report

The test report shall include the following information:

a)  a reference to this document, i.e. ISO 19927:2018;

b)  complete identification of the material tested, including type, source, the manufacturer's code number, form and previous history, where these are known;

c)  the date of measurement;

d)  the dimensions of the test specimens;

e)  the method of preparing the specimens;

f)  the test conditions and conditioning procedures, if applicable;

g)  the number of specimens tested;

h)  the individual determinations, including load-displacement diagrams, if required;

i)  the mean values of the individual determinations;

j)  the type(s) of failure obtained;

k)  statement as to whether any test specimens have been rejected and, if so, the reasons;

l)  the standard deviations and the 95 % confidence intervals of the mean values, if required;

m)  any operation not specified in this document, as well as any incident likely to have affected the results.

# Annex A
## (informative)

# Additional details relating to calculation of shear modulus, $G_{13}$

## A.1 General

Interlaminar shear modulus is obtained using energy method. Figure 4 is duplicated here as Figure A.1 with additional notations, which are convenient for the analysis.
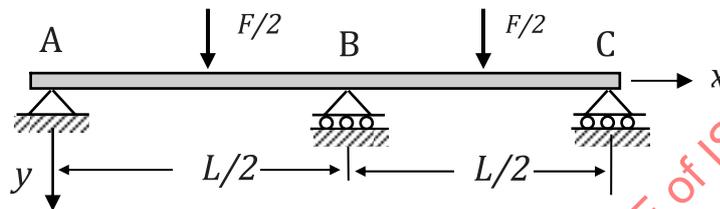


**Figure A.1 — Test specimen and rollers lay-out for determination of interlaminar modulus and strength**

## A.2 Application of the Castigliano's 2nd theorem

Because of symmetry in the loading and supporting configuration of the beam, the only one half of the four regions is needed. The total deflection at the central support (B) is obtained by Formulae (A.1) and (A.2):

$$\Delta_B = 2 \int_0^{L/4} \frac{M}{EI}\frac{dM}{dR_B}dx + 2\int_{L/4}^{L/2}\frac{M}{EI}\frac{dM}{dR_B}dx + 2\int_0^{L/4}\frac{V}{k_sGA}\frac{dV}{dR_B}dx + 2\int_{L/4}^{L/2}\frac{V}{k_sGA}\frac{dV}{dR_B}dx \qquad (A.1)$$

$$R_A = \frac{F}{2} - \frac{R_B}{2} \qquad (A.2)$$

Bending moment, shear force expressions and their derivatives with regard to $R_B$ for the first two regions of the beam are given respectively by

$$-R_A x + M = 0 \therefore M = R_A x = \frac{F}{2}x - \frac{R_B}{2}x \text{ for } 0 < x < \frac{L}{4} \qquad (A.3)$$

$$-R_A x + \frac{F}{2}\left(x - \frac{L}{4}\right) + M = 0 \therefore M = R_A x - \frac{F}{2}\left(x - \frac{L}{4}\right) = \frac{FL}{8} - \frac{R_B}{2}x \text{ for } \frac{L}{4} < x < \frac{L}{2} \qquad (A.4)$$

$$\frac{dM}{dR_B} = -\frac{x}{2} \text{ for } 0 < x < \frac{L}{4} \qquad (A.5)$$

$$\frac{dM}{dR_B} = -\frac{x}{2} \text{ for } \frac{L}{4} < x < \frac{L}{2} \qquad (A.6)$$

$$-R_A + V = 0 \therefore V = R_A = \frac{F}{2} - \frac{R_B}{2}x \text{ for } 0 < x < \frac{L}{4} \qquad (A.7)$$

$$-R_A + \frac{F}{2} + V = 0 \therefore V = R_A - \frac{F}{2} = -\frac{R_B}{2} \text{ for } \frac{L}{4} < x < \frac{L}{2}$$ (A.8)

$$\frac{dV}{dR_B} = -\frac{1}{2} \text{ for } 0 < x < \frac{L}{4}$$ (A.9)

$$\frac{dV}{dR_B} = -\frac{1}{2} \text{ for } \frac{L}{4} < x < \frac{L}{2}$$ (A.10)

Since the deflection at support B must be zero, the completion of all integration terms in Formula (A.1) along with Formulae (A.3) to (A.10) yields

$$R_B = \frac{11k_s GAL^2 + 96EI}{k_s GAL^2 + 12EI} \cdot \frac{F}{16} = \frac{1 + \dfrac{96EI}{11k_s GAL^2}}{1 + \dfrac{12EI}{k_s GAL^2}} \cdot \frac{11}{16} F$$ (A.11)

## A.3  Application of the unit load ($F_0$) method

To maintain the deformation characteristics of the beam, two unit loads $F_0$ are applied respectively at the two middle distances between the external loads and the central support, as shown in Figure A.2.
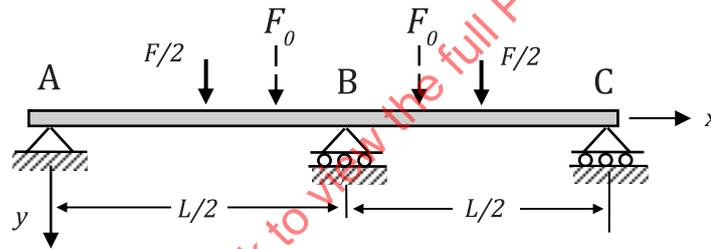


**Figure A.2 — Test specimen and rollers lay-out for determination of interlaminar modulus and strength (Unit load method)**

$$R_A = \frac{F}{2} - \frac{R_B}{2} + F_0$$ (A.12)

Shear force expressions for the first two regions and their derivatives with regard to $F_0$ are given respectively by Formulae (A.13) to (A.18):

$$V = R_A = \frac{F}{2} - \frac{R_B}{2} + F_0 \text{ for } 0 < x < \frac{L}{4}$$ (A.13)

$$V = R_A - \frac{F}{2} = -\frac{R_B}{2} + F_0 \text{ for } \frac{L}{4} < x < \frac{3L}{8}$$ (A.14)

$$V = R_A - \frac{F}{2} - F_0 = -\frac{R_B}{2} \text{ for } \frac{3L}{8} < x < \frac{L}{2}$$ (A.15)

$$\frac{dV}{dF_0} = 1 \text{ for } 0 < x < \frac{L}{4}$$ (A.16)

$$\frac{dV}{dF_0} = 1 \text{ for } \frac{L}{4} < x < \frac{3L}{8}$$ (A.17)

**15**

$$\frac{dV}{dF_0} = 0 \text{ for } \frac{3L}{8} < x < \frac{L}{2} \tag{A.18}$$

Bending moment expressions for the first two regions and their derivatives with regard to $F_0$ are given respectively by Formulae (A.19) to (A.24):

$$M = R_A x = \frac{Fx}{2} - \frac{R_B}{2} x + F_0 x \text{ for } 0 < x < \frac{L}{4} \tag{A.19}$$

$$M = R_A x - \frac{F}{2}\left(x - \frac{L}{4}\right) = \frac{FL}{8} - \frac{R_B}{2} x + F_0 x \text{ for } \frac{L}{4} < x < \frac{3L}{8} \tag{A.20}$$

$$M = R_A x - \frac{F}{2}\left(x - \frac{L}{4}\right) - F_0\left(x - \frac{3L}{8}\right) = \frac{FL}{8} - \frac{R_B}{2} x + \frac{3F_0 L}{8} \text{ for } \frac{3L}{8} < x < \frac{L}{2} \tag{A.21}$$

$$\frac{dM}{dF_0} = x \text{ for } 0 < x < \frac{L}{4} \tag{A.22}$$

$$\frac{dM}{dF_0} = x \text{ for } \frac{L}{4} < x < \frac{3L}{8} \tag{A.23}$$

$$\frac{dM}{dF_0} = \frac{3L}{8} \text{ for } \frac{3L}{8} < x < \frac{L}{2} \tag{A.24}$$

## A.4  Application of the Castigliano's 2nd theorem

Application of the Castigliano's 2nd Theorem yields

$$\begin{aligned}
\Delta = &\int_0^{L/4} \frac{M}{EI}\frac{dM}{dF_0}dx + \int_{L/4}^{3L/8} \frac{M}{EI}\frac{dM}{dF_0}dx + \int_{3L/8}^{L/2} \frac{M}{EI}\frac{dM}{dF_0}dx \\
&+ \int_0^{L/4} \frac{V}{kGA}\frac{dV}{dF_0}dx + \int_{L/4}^{3L/8} \frac{V}{kGA}\frac{dV}{dF_0}dx + \int_{3L/8}^{L/2} \frac{V}{kGA}\frac{dV}{dF_0}dx \\
&= \frac{41FL^3}{3\,072EI} - \frac{39R_B L^3}{2\,048EI} + \frac{FL}{8kGA} - \frac{3R_B L}{16kGA}
\end{aligned} \tag{A.25}$$

Re-arranging Formula (A.25) with $R_B$ being substituted by Formula (A.11), with $s$ (i.e. $\Delta/P$) being reciprocal of the slope of a load-deflection curve measured at the middle distance, yields

$$k_s^2 A^2 L^2 (98\,304EIs - 25L^3)G^2 + 32EIk_s A(36\,864EIs - 129L^3)G - 36\,864E^2 I^2 L = 0 \tag{A.26}$$

where $k_s$ is taken as 5/6, which is a standard, universal approach with regard to through-thickness variation of interlaminar shear stresses.

The roots of Formula (A.26) are given in Formula (2) and the positive root will be the value of ILS modulus, $G_{13}$.