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Mechanical vibration and shock — Signal processing —

Part 4:
Shock-response spectrum analysis

Vibrations et chocs mécaniques — Traitement du signal —

Partie 4: Analyse du spectre de réponse aux chocs

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 18431-4 was prepared by Technical Committee ISO/TC 108, *Mechanical vibration, shock and condition monitoring*.

ISO 18431 consists of the following parts, under the general title *Mechanical vibration and shock — Signal processing*:

— *Part 1: General introduction*

— *Part 2: Time domain windows for Fourier Transform analysis*

— *Part 4: Shock-response spectrum analysis*

The following parts are under preparation:

— a part 3, dealing with bilinear methods for joint time-frequency analysis

— a part 5, dealing with methods for time-scale analysis

Introduction

In the recent past, nearly all data analysis has been accomplished through mathematical operations on digitized data. This state of affairs has been accomplished through the widespread use of digital signal-acquisition systems and computerized data processing equipment. The analysis of data is, therefore, primarily a digital signal-processing task.

The analysis of experimental vibration and shock data should be thought of as a part of the process of experimental mechanics that includes all steps from experimental design through data evaluation and understanding.

ISO 18431 (all parts) assumes that the data have been sufficiently reduced so that the effects of instrument sensitivity have been included. The data covered in ISO 18431 (all parts) are considered to be a sequence of time samples of acceleration describing vibration or shock. Experimental methods for obtaining the data are outside the scope of ISO 18431 (all parts).

This part of ISO 18431 is concerned with methods for the digital calculation of a shock-response spectrum. The analysis is by no means restricted to signals that can be characterized as shocks. On the contrary, it is, in a strict sense, meaningless to analyze a shock according to the definition in ISO 2041, where a shock is defined as a sudden event, taking place in a time that is short compared with the fundamental periods of concern. Such a shock has no frequency characteristics in the frequency range of concern. It is only characterized by its time integral, the impulse, corresponding to constant frequency content. The notation “shock-response spectrum” has been kept, however, although a better term would be maximum-response spectrum.

Historically, the shock-response spectrum was initially used to describe transient phenomena, at the time called shocks.

Response analysis in general is a method to characterize a vibration or shock when other frequency analysis methods are inadequate. This can be the case, for instance, when different kinds of vibration are compared. Spectrum analysis based on the Fourier Transform produces spectra that are incompatible when the signals analyzed are of different kinds, such as periodic, random or transient.

The typical use of a shock-response spectrum is to characterize a dynamic mechanical environment. The vibration (or shock) characterized is recorded in digital form, commonly as acceleration. The data are analyzed into a shock-response spectrum. This spectrum can then be used to define a test for equipment that is required to endure the environment in question. There exist International Standards that describe how to design tests from given shock-response spectrum specifications, for example IEC 60068-2-81. (See the bibliography for additional information.)

When measurements to characterize a vibration and/or shock environment are performed, it is necessary to take certain measures, for instance to ascertain a proper dynamic load in the measurement points. These measures are beyond the scope of this part of ISO 18431. There are many good handbooks and recommended practices that are helpful in this area [1],[2].

Mechanical vibration and shock — Signal processing —

Part 4:
Shock-response spectrum analysis

1 Scope

This part of ISO 18431 specifies methods for the digital calculation of a shock-response spectrum (SRS) given an acceleration input, by means of digital filters. The filter coefficients for different types of shock-response spectra are given together with recommendations for adequate sampling frequency.

NOTE The definition of a shock-response spectrum given in ISO 2041, implies that a shock-response spectrum can be defined in terms of an acceleration, velocity or displacement transfer function. This part of ISO 18431 deals only with acceleration input.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 2041, *Vibration and shock — Vocabulary*

3 Terms and definitions

For the purposes of this document, the terms and definitions given in ISO 2041 and the following apply.

3.1
maximax shock-response spectrum
SRS where the maximum absolute value of the response is taken

3.2
negative shock-response spectrum
SRS where the maximum value is taken in the negative direction of the response

3.3
positive shock-response spectrum
SRS where the maximum value is taken in the positive direction of the response

3.4
primary shock-response spectrum
SRS where the maximum value is taken during the duration of the input

3.5
residual shock-response spectrum
SRS where the maximum value is taken after the duration of the input

4 Symbols and abbreviated terms

$a(s)$	Laplace transform of acceleration	(m/s^2)·s
c	damping constant in SDOF system	N/(m/s)
$d(s)$	Laplace transform of displacement	m·s
f_n	natural frequency for SDOF system	Hz
f_s	sampling frequency, sampling rate	Hz
$G(s)$	transfer function in s domain	
$H(z)$	transfer function in z domain	
k	spring constant in SDOF system	N/m
m	mass in SDOF system	kg, N/(m/s^2)
Q	Q-value, resonance gain	
s	Laplace variable, complex frequency	rad/s
SDOF	single-degree-of-freedom system	
SRS	shock-response spectrum	
T	sampling time interval	s
$v(s)$	Laplace transform of (vibration) velocity	(m/s)·s
z	z-transform variable	
α	digital filter denominator coefficient	
β	digital filter numerator coefficient	
ω_n	angular natural frequency	rad/s
ζ	damping ratio, fraction of critical damping	

5 Shock-response spectrum fundamentals

5.1 Introduction

In this part of ISO 18431, a shock-response spectrum is the response to a given acceleration of a set of single-degree-of-freedom, SDOF, mass-damper-spring oscillators. The given acceleration is applied to the base of all oscillators, and the maximum responses of each oscillator versus the natural frequency make up the spectrum; see Figure 1.

Each single-degree-of-freedom system in Figure 1 has a unique set of defining parameters; mass, m, damping constant, c, and spring constant, k. The parameters of the system are the conventional ones, given in Clause 4.

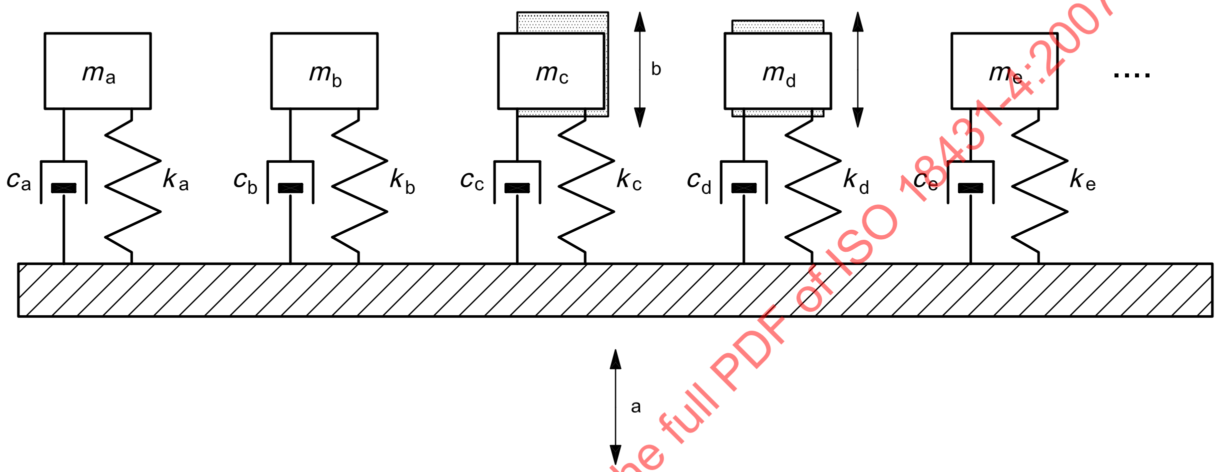
A given acceleration, a_1, is applied to the base. If the response is measured as acceleration, a_2, then the transfer function, $G(s)$, for a SDOF system is given by Equation (1):

$$G(s) = \frac{a_2(s)}{a_1(s)} = \frac{cs + k}{ms^2 + cs + k} \qquad (1)$$

where s is the Laplace variable (complex frequency) in radians per second. The single-degree-of-freedom system is normally characterized by its (undamped) natural frequency, f_n, in hertz, as given in Equation (2), and the resonance gain, Q (Q-factor), as given in Equation (3):

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad (2)$$

$$Q = \frac{\sqrt{km}}{c} \quad (3)$$



a input motion

b response motion

NOTE The responses of a set of single-degree-of-freedom (SDOF) mechanical systems define the shock-response spectrum. The combination of m, c and k differs among the systems.

Figure 1 — Responses of a set of single-degree-of-freedom (SDOF) mechanical systems

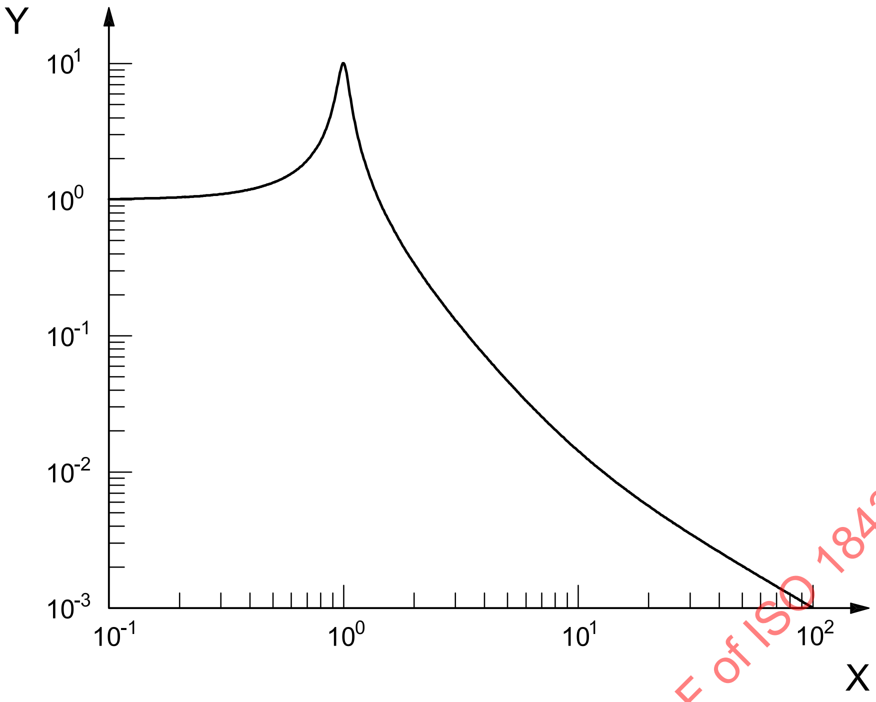
The transfer function may then be rewritten, as given in Equation (4):

$$G(s) = \frac{a_2(s)}{a_1(s)} = \frac{\frac{\omega_n s}{Q} + \omega_n^2}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2} \quad (4)$$

with $\omega_n = 2\pi f_n$ being the angular natural frequency in radians per second.

The transfer function is given versus frequency in Figure 2, where the natural frequency is set to 1 Hz and $Q = 10$ as an example. Note the gain of Q at resonance.

NOTE Equation (4) defines the transfer function used. The maximum is approximately Q and the maximum occurs approximately at f_n Hz. The larger the Q-value, the more accurate the approximation.



Key

X frequency, expressed in hertz

Y transfer function

Figure 2 — Transfer function of SDOF system as function of frequency

Instead of the resonant gain, Q, the damping ratio, fraction of critical damping, ζ, may be used. ζ is often expressed in “percent of critical damping,” as given in Equation (5):

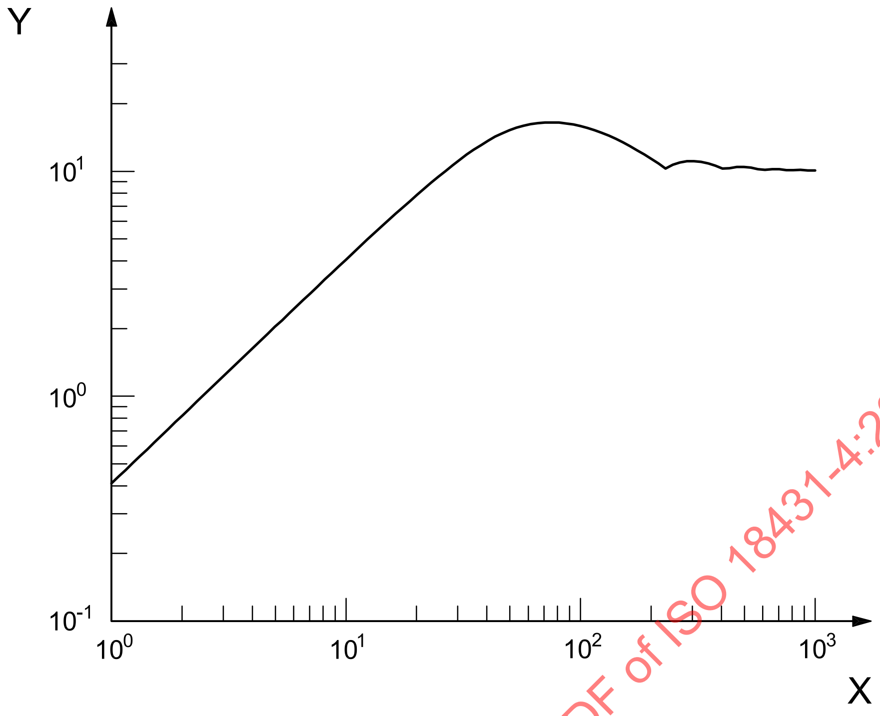
$$\zeta = \frac{1}{2Q} = \frac{c}{2\sqrt{km}} \tag{5}$$

NOTE Critical damping, c_{crit}, is defined as $c_{\text{crit}} = 2\sqrt{km}$.

To calculate the shock-response spectrum, the acceleration signal to be analyzed is applied to the base of a set of SDOF systems characterized by their natural frequencies and Q-values. The responses are calculated; the maximum responses as a function of the natural frequencies compose the shock-response spectrum. In a basic version of the shock-response spectrum, the maximum taken is the maximax, which is the maximum of the absolute value of the response.

In the calculation of a shock-response spectrum, the natural frequencies are selected in a logarithmic fashion. The same Q-value is used for all SDOF systems. The number of natural frequencies depends on the Q-value (or damping). For a common Q-value of 10, corresponding to a damping ratio of 5 %, a minimum of six frequencies per octave, corresponding to 20 frequencies per decade, is recommended. A finer resolution can be warranted if a smaller value of damping is assumed.

As an example, Figure 3 shows the (maximax) shock-response spectrum for a half-sine pulse with duration 11 ms and amplitude of 10 g_n.



Key

X frequency, expressed in hertz

Y maximax acceleration SRS, expressed in g-values

Figure 3 — Shock-response spectrum for a half-sine pulse with amplitude 10 g_n, **duration** 11 ms **and** Q = 10

5.2 Variations of the shock-response spectrum

5.2.1 Introduction

In the basic shock-response spectrum, the maximax value of the acceleration response of the SDOF system is calculated. Variations of the basic shock-response spectrum are created when other responses are considered, such as relative velocity or relative displacement. Apart from that, different maximum values may be considered, such as the maximum value (largest positive value) or the minimum value (largest negative value). If the maximum is taken in the positive direction, the spectrum is called a positive shock-response spectrum. If the maximum is taken in the negative direction, the spectrum is called a negative shock-response spectrum. If the maximum absolute value is taken, the spectrum is called a maximax shock-response spectrum.

In some cases, a distinction is made between the maximum value occurring during the duration of the input (especially if it has a pulse character) and the maximum value occurring after the pulse. The former is called primary shock-response spectrum while the latter is called residual shock-response spectrum.

To avoid confusion, the type of spectrum calculated should be stated, for instance “relative-displacement maximax response spectrum.”

5.2.2 Relative-velocity response spectrum

When the response of the SDOF system is calculated as relative velocity between the SDOF system mass and the base, the transfer function becomes as given in Equation (6) or (7):

$$G(s) = \frac{v_2(s) - v_1(s)}{a_1(s)} = \frac{-ms}{ms^2 + cs + k} \tag{6}$$

or

$$G(s) = \frac{v_2(s) - v_1(s)}{a_1(s)} = \frac{-s}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2} \tag{7}$$

5.2.3 Relative-displacement response spectrum

When the response of the SDOF system is calculated as relative displacement between the SDOF system mass and the base, the transfer function becomes as given in Equation (8) or (9):

$$G(s) = \frac{d_2(s) - d_1(s)}{a_1(s)} = \frac{-m}{ms^2 + cs + k} \tag{8}$$

or

$$G(s) = \frac{d_2(s) - d_1(s)}{a_1(s)} = \frac{-1}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2} \tag{9}$$

5.2.4 Pseudo-velocity response spectrum

The relative-displacement response may be multiplied by the angular natural frequency, ω_n, to create a pseudo-velocity response. In this case, the transfer function becomes as given in Equation (10) or (11):

$$G(s) = \frac{d_2(s) - d_1(s)}{a_1(s)} \cdot \omega_n = \frac{-m\omega_n}{ms^2 + cs + k} \tag{10}$$

or

$$G(s) = \frac{d_2(s) - d_1(s)}{a_1(s)} \cdot \omega_n = \frac{-\omega_n}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2} \tag{11}$$

5.2.5 Relative-displacement response spectrum expressed as equivalent static acceleration

The relative-displacement response may also be multiplied by the angular natural frequency squared, ω_n^2, to create an equivalent static acceleration response. In this case the transfer function becomes as given in Equation (12) or (13):

$$G(s) = \frac{d_2(s) - d_1(s)}{a_1(s)} \cdot \omega_n^2 = \frac{-m\omega_n^2}{ms^2 + cs + k} \tag{12}$$

or

$$G(s) = \frac{d_2(s) - d_1(s)}{a_1(s)} \cdot \omega_n^2 = \frac{-\omega_n^2}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2} \tag{13}$$

6 Shock-response spectrum calculation

6.1 Introduction

To calculate the shock-response spectrum, an input acceleration signal is fed to a set of digital filters approximating the transfer functions defined in Clause 5. The acceleration time signal shall be properly recorded. This means that apart from all the mechanical considerations (accelerometer mounting, etc.), adequate anti-aliasing protection is used. This part of ISO 18431 deals with the processing of the signal when it exists as a digital record, sampled with a sampling frequency of f_s Hz, corresponding to a time interval between samples of T s:

$$T = \frac{1}{f_s} \tag{14}$$

There are several methods to design a digital filter from a given analogue transfer function. The method stated in this part of ISO 18431 is the Ramp Invariant Method [6], [7]. Algorithms to calculate the filter coefficients for the different response types defined in 5.2.2 to 5.2.5 are given in 6.2 to 6.6.

The digital filters corresponding to different SDOF system responses are second-order filters, with the general z-transform expression as given in Equation (15):

$$H(z) = \frac{\beta_0 + \beta_1 \cdot z^{-1} + \beta_2 \cdot z^{-2}}{1 + \alpha_1 \cdot z^{-1} + \alpha_2 \cdot z^{-2}} \tag{15}$$

The filter expression corresponds to a different equation describing how to calculate the response time series, y_n, when the input acceleration time series, x_n, is as given in Equation (16):

$$y_n = \beta_0 \cdot x_n + \beta_1 \cdot x_{n-1} + \beta_2 \cdot x_{n-2} - \alpha_1 \cdot y_{n-1} - \alpha_2 \cdot y_{n-2} \tag{16}$$

It should be noted that, in the expressions for the digital filters corresponding to different kinds of responses, the denominator (alpha coefficient) is always the same. The difference comes in the beta coefficients.

6.2 Filter coefficients for absolute acceleration response

Sampling frequency: f_s, in hertz

Sampling time interval: $T = \frac{1}{f_s}$, in seconds

Natural frequency: f_n, in hertz

Natural angular frequency: $\omega_n = 2\pi f_n$, in radians per second

Resonance gain: Q

Transfer function: $G(s)$

$$G(s) = \frac{a_2(s)}{a_1(s)} = \frac{\frac{\omega_n s}{Q} + \omega_n^2}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2}$$

Digital filter:

$$H(z) = \frac{\beta_0 + \beta_1 \cdot z^{-1} + \beta_2 \cdot z^{-2}}{1 + \alpha_1 \cdot z^{-1} + \alpha_2 \cdot z^{-2}}$$

where the digital filter coefficients are defined as follows:

$$\beta_0 = 1 - \exp(-A) \cdot \sin(B) / B$$
$$\beta_1 = 2\exp(-A) \cdot \{\sin(B) / B - \cos(B)\}$$
$$\beta_2 = \exp(-2A) - \exp(-A) \cdot \sin(B) / B$$

$$\alpha_1 = -2\exp(-A) \cdot \cos(B)$$
$$\alpha_2 = \exp(-2A)$$

where

$$A = \frac{\omega_n \cdot T}{2Q}$$

$$B = \omega_n \cdot T \cdot \sqrt{1 - \frac{1}{4Q^2}}$$

6.3 Filter coefficients for relative-velocity response

Sampling frequency: f_s, in hertz

Sampling time interval: $T = \frac{1}{f_s}$, in seconds

Natural frequency: f_n, in hertz

Natural angular frequency: $\omega_n = 2\pi f_n$, in radians per second

Resonance gain: Q

Transfer function: $G(s)$

$$G(s) = \frac{v_2(s) - v_1(s)}{a_1(s)} = \frac{-s}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2}$$

Digital filter:

$$H(z) = \frac{\beta_0 + \beta_1 \cdot z^{-1} + \beta_2 \cdot z^{-2}}{1 + \alpha_1 \cdot z^{-1} + \alpha_2 \cdot z^{-2}}$$

where the digital filter coefficients are defined as follows:

$$\beta_0 = \frac{1}{\omega_n^2 \cdot T} \cdot \left\{ -1 + \exp(-A) \cdot \cos(B) + \frac{\exp(-A) \cdot \sin(B)}{\sqrt{4Q^2 - 1}} \right\}$$

$$\beta_1 = \frac{1}{\omega_n^2 \cdot T} \cdot \left\{ 1 - \exp(-2A) - \frac{2\exp(-A) \cdot \sin(B)}{\sqrt{4Q^2 - 1}} \right\}$$

$$\beta_2 = \frac{1}{\omega_n^2 \cdot T} \cdot \left\{ \exp(-2A) - \exp(-A) \cdot \cos(B) + \frac{\exp(-A) \cdot \sin(B)}{\sqrt{4Q^2 - 1}} \right\}$$

$$\alpha_1 = -2\exp(-A) \cdot \cos(B)$$

$$\alpha_2 = \exp(-2A)$$

where

$$A = \frac{\omega_n \cdot T}{2Q}$$

$$B = \omega_n \cdot T \cdot \sqrt{1 - \frac{1}{4Q^2}}$$

6.4 Filter coefficients for relative-displacement response

Sampling frequency: f_s, in hertz

Sampling time interval: $T = \frac{1}{f_s}$, in seconds

Natural frequency: f_n, in hertz

Natural angular frequency: $\omega_n = 2\pi f_n$, in radians per second

Resonance gain: Q

Transfer function: $G(s)$

$$G(s) = \frac{d_2(s) - d_1(s)}{a_1(s)} = \frac{-1}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2}$$

Digital filter:

$$H(z) = \frac{\beta_0 + \beta_1 \cdot z^{-1} + \beta_2 \cdot z^{-2}}{1 + \alpha_1 \cdot z^{-1} + \alpha_2 \cdot z^{-2}}$$

where the digital filter coefficients are defined as follows:

$$\beta_0 = \frac{1}{\omega_n^3 \cdot T} \cdot \left\{ \frac{1-\exp(-A)\cdot\cos(B)}{Q} - q\cdot\exp(-A)\cdot\sin(B) - \omega_n \cdot T \right\}$$

$$\beta_1 = \frac{1}{\omega_n^3 \cdot T} \cdot \left\{ 2\exp(-A)\cdot\cos(B)\cdot\omega_n\cdot T - \frac{1-\exp(-2A)}{Q} + 2q\cdot\exp(-A)\cdot\sin(B) \right\}$$

$$\beta_2 = \frac{1}{\omega_n^3 \cdot T} \cdot \left\{ -\exp(-2A)\cdot\left(\omega_n\cdot T + \frac{1}{Q}\right) + \frac{\exp(-A)\cdot\cos(B)}{Q} - q\cdot\exp(-A)\cdot\sin(B) \right\}$$

$$\alpha_1 = -2\exp(-A)\cdot\cos(B)$$

$$\alpha_2 = \exp(-2A)$$

where

$$A = \frac{\omega_n \cdot T}{2Q}$$

$$B = \omega_n \cdot T \cdot \sqrt{1 - \frac{1}{4Q^2}}$$

$$q = \frac{\frac{1}{2Q^2} - 1}{\sqrt{1 - \frac{1}{4Q^2}}}$$

6.5 Filter coefficients for pseudo-velocity response

Sampling frequency: f_s, in hertz

Sampling time interval: $T = \frac{1}{f_s}$, in seconds

Natural frequency: f_n, in hertz

Natural angular frequency: $\omega_n = 2\pi f_n$, in radians per second

Resonance gain: Q

Transfer function: $G(s)$

$$G(s) = \frac{d_2(s) - d_1(s)}{a_1(s)} \cdot \omega_n = \frac{-\omega_n}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2}$$

Digital filter:

$$H(z) = \frac{\beta_0 + \beta_1 \cdot z^{-1} + \beta_2 \cdot z^{-2}}{1 + \alpha_1 \cdot z^{-1} + \alpha_2 \cdot z^{-2}}$$

where the digital filter coefficients are defined as follows:

$$\beta_0 = \frac{1}{\omega_n^2 \cdot T} \cdot \left\{ \frac{1 - \exp(-A) \cdot \cos(B)}{Q} - q \cdot \exp(-A) \cdot \sin(B) - \omega_n \cdot T \right\}$$

$$\beta_1 = \frac{1}{\omega_n^2 \cdot T} \cdot \left\{ 2\exp(-A) \cdot \cos(B) \cdot \omega_n \cdot T - \frac{1 - \exp(-2A)}{Q} + 2q \cdot \exp(-A) \cdot \sin(B) \right\}$$

$$\beta_2 = \frac{1}{\omega_n^2 \cdot T} \cdot \left\{ -\exp(-2A) \cdot \left(\omega_n \cdot T + \frac{1}{Q} \right) + \frac{\exp(-A) \cdot \cos(B)}{Q} - q \cdot \exp(-A) \cdot \sin(B) \right\}$$

$$\alpha_1 = -2\exp(-A) \cdot \cos(B)$$
$$\alpha_2 = \exp(-2A)$$

where

$$A = \frac{\omega_n \cdot T}{2Q}$$

$$B = \omega_n \cdot T \cdot \sqrt{1 - \frac{1}{4Q^2}}$$

$$q = \frac{\frac{1}{2Q^2} - 1}{\sqrt{1 - \frac{1}{4Q^2}}}$$

6.6 Filter coefficients for relative-displacement response expressed as equivalent static acceleration

Sampling frequency: f_s, in hertz

Sampling time interval: $T = \frac{1}{f_s}$, in seconds

Natural frequency: f_n, in hertz

Natural angular frequency: $\omega_n = 2\pi f_n$, in radians per second

Resonance gain: Q

Transfer function: $G(s)$

$$G(s) = \frac{d_2(s) - d_1(s)}{a_1(s)} \cdot \omega_n^2 = \frac{-\omega_n^2}{s^2 + \frac{\omega_n s}{Q} + \omega_n^2}$$

Digital filter:

$$H(z) = \frac{\beta_0 + \beta_1 \cdot z^{-1} + \beta_2 \cdot z^{-2}}{1 + \alpha_1 \cdot z^{-1} + \alpha_2 \cdot z^{-2}}$$

where the digital filter coefficients are defined as follows:

$$\beta_0 = \frac{1}{\omega_n \cdot T} \cdot \left\{ \frac{1 - \exp(-A) \cdot \cos(B)}{Q} - q \cdot \exp(-A) \cdot \sin(B) - \omega_n \cdot T \right\}$$

$$\beta_1 = \frac{1}{\omega_n \cdot T} \cdot \left\{ 2\exp(-A) \cdot \cos(B) \cdot \omega_n \cdot T - \frac{1 - \exp(-2A)}{Q} + 2q \cdot \exp(-A) \cdot \sin(B) \right\}$$

$$\beta_2 = \frac{1}{\omega_n \cdot T} \cdot \left\{ -\exp(-2A) \cdot \left(\omega_n \cdot T + \frac{1}{Q} \right) + \frac{\exp(-A) \cdot \cos(B)}{Q} - q \cdot \exp(-A) \cdot \sin(B) \right\}$$

$$\alpha_1 = -2\exp(-A) \cdot \cos(B)$$

$$\alpha_2 = \exp(-2A)$$

where

$$A = \frac{\omega_n \cdot T}{2Q}$$

$$B = \omega_n \cdot T \cdot \sqrt{1 - \frac{1}{4Q^2}}$$

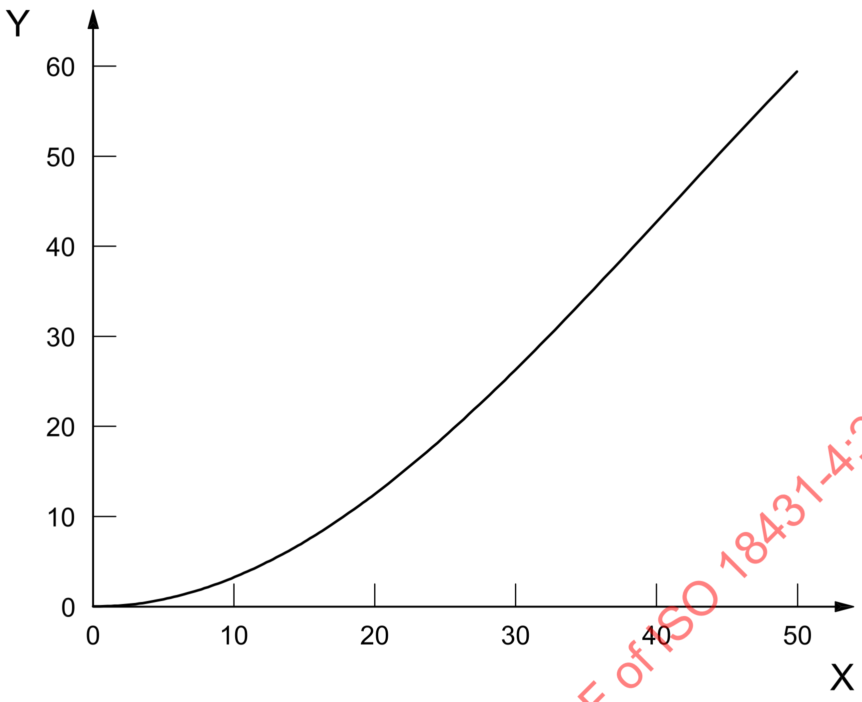
$$q = \frac{\frac{1}{2Q^2} - 1}{\sqrt{1 - \frac{1}{4Q^2}}}$$

7 Sampling frequency considerations

The ramp invariant algorithm contains a bias error which is dependent on the sampling frequency. The error, ε, as a function of frequency, f, is given by the following equation:

$$\varepsilon(f) = 1 - \left[\sin\left(\frac{\pi f}{f_s} \right) \bigg/ \frac{\pi f}{f_s} \right]^2 \qquad (17)$$

A plot of the bias error is given in Figure 4.



Key

X sampling frequency, expressed in percent

Y bias error, expressed in percent

Figure 4 — Bias error in ramp invariant algorithm

Figure 4 shows that the algorithm can be used only for frequencies well below the sampling frequency. This might seem to be a serious problem, but to make the maximum estimation simple, a high sampling frequency shall be used. If the maximum sample is selected as an estimate of the “true” maximum (which can appear between samples), the maximum error is as shown by the plot in Figure 5. In the worst case, the two errors may be combined as shown in Figure 6.

